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Working Paper

Seigniorage

Economics Discussion Papers, No. 2007-8

Provided in Cooperation with:
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Suggested Citation: Buiter, Willem H. (2007) : Seigniorage, Economics Discussion Papers, No. 2007-8, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:
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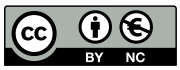

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Economics Discussion Papers
The Open-Access, Open-Assessment E-Journal

Discussion Paper 2007-8
March 1, 2007

Seigniorage**

Willem H. Buiter

Professor of European Political Economy European Institute,
London School of Economics and Political Science

Abstract:

Governments through the ages have appropriated real resources through the monopoly of the 'coinage'. In modern fiat money economies, the monopoly of the issue of legal tender is generally assigned to an agency of the state, the Central Bank, which may have varying degrees of operational and target independence from the government of the day.

In this paper I analyse four different but related concepts, each of which highlights some aspect of the way in which the state acquires command over real resources through its ability to issue fiat money. They are (1) seigniorage (the change in the monetary base), (2) Central Bank revenue (the interest bill saved by the authorities on the outstanding stock of base money liabilities), (3) the inflation tax (the reduction in the real value of the stock of base money due to inflation and (4) the operating profits of the central bank, or the taxes paid by the Central Bank to the Treasury.

To understand the relationship between these four concepts, an explicitly intertemporal approach is required, which focuses on the present discounted value of the current and future resource transfers between the private sector and the state. Furthermore, when the Central Bank is operationally independent, it is essential to decompose the familiar consolidated 'government budget constraint' and consolidated 'government intertemporal budget constraint' into the separate accounts and budget constraints of the Central Bank and the Treasury. Only by doing this can we appreciate the financial constraints on the Central Bank's ability to pursue and achieve an inflation target, and the importance of cooperation and coordination between the Treasury and the Central Bank when faced with financial sector crises involving the need for long-term recapitalisation or when confronted with the need to mimick Milton Friedman's helicopter drop of money in an economy faced with a liquidity trap.

JEL: E4, E5, E6, H6

Keywords: inflation tax; central bank budget constraint; coordination of monetary and fiscal policy

Correspondence:
Tel.: + 44 (0)20 7955 6959 Fax: + 44 (0)20 7955 7546 E-mail: w.buiter@lse.ac.uk

**I would like to thank Charles Goodhart, Michael Bordo, Marc Flandreau and Anne Sibert for helpful comments.

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I. Introduction

Seigniorage refers historically, in a world with commodity money, to the difference between the face value of a coin and its costs of production and mintage. In fiat money economies, the difference between the face value of a currency note and its marginal printing cost are almost equal to the face value of the note – marginal printing costs are effectively zero. Printing fiat money is therefore a highly profitable activity – one that has been jealously regulated and often monopolized by the state.

Although the profitability of printing money is widely recognized, the literature on the subject contains a number of different measures of the revenue appropriated by the state through the use of the printing presses. In this paper, I discuss four of them and consider the relationship between them in an intertemporal setting. There also is the empirical institutional regularity, that the state tends to assign the issuance of fiat money to a specialized agency, the Central Bank, which has some degree of independence from the other organs of the state and from the government administration of the day. This institutional arrangement has implications for the conduct of monetary policy that cannot be analysed in the textbook macroeconomic models, which consolidate the Central Bank with the rest of the government.

In the next four Sections, the paper addresses the following four questions. (1) What resources does the state appropriate through the issuance of base money (currency and commercial bank balances with the Central Bank)? (2) What inflation rate would result if the monetary authority were to try to maximise these resources? (3) Who ultimate appropriates and benefits from these resources, the Central Bank or the Treasury/Ministry of Finance? (4) Does the Central Bank have adequate financial resources to pursue its monetary policy mandate (taken to be price stability) and its financial stability mandate. Specifically, for inflation-targeting Central

Banks, is the inflation target financeable? The first two questions receive preliminary answers in Section II of the paper, confirming results that can be found e.g. in Walsh (2003) and Romer (2006). The second half of Section II contains an analysis of the relationship between three of base money issuance revenue measures (seigniorage, central bank revenue and the inflation tax) in real time, that is, outside the steady state and without the assumptions that the Fisher hypothesis holds and that the velocity of circulation of base money is constant over time. It derives the 'intertemporal seigniorage identity' relating the present discounted value of seigniorage and the present discounted value of Central Bank revenue.

The government's period budget constraint and its intertemporal budget constraint have been familiar components of dynamic macroeconomic models at least since the late 1960s (see e.g. Christ (1968), Blinder and Solow (1973) and Tobin and Buiter (1976)). The 'government' in question is invariably the consolidated general government (central, state and local, henceforth the 'Treasury') and Central Bank. When the Central Bank has operational independence, it is useful, and at times even essential, to disaggregate the general government accounts into separate Treasury and Central Bank accounts. Section III of the paper presents an example of such a decomposition, extending the analysis of Walsh (2003). In Section IV, a simple dynamic general equilibrium model with money is presented, which incorporates the Treasury and Central Bank whose accounts were constructed in Section III. It permits all four questions to be addressed. Section V raises two further issues prompted by the decomposition of the government's accounts into separate Central Bank and Treasury accounts: the need for fiscal resources to recapitalise an financially stretched or even insolvent Central Bank and the institutional modalities of 'helicopter drops of money'.

The analysis of the sources of Central Bank revenue or seigniorage is part of a tradition that is both venerable and incomplete. It starts (at least) with Thornton (1802) and includes such classics as Bresciani-Turroni (1937) and Cagan (1956). Milton Friedman (1971), Phelps (1973), Sargent (1982, 1987) and Sargent and Wallace (1981) have made important contributions. Empirical investigations include King and Plosser (1985), Dornbusch and Fischer (1986), Anand and van Wijnbergen (1989), Buiter (1990), Kiguel and Neumeyer (1995) and Easterly, Mauro and Schmidt-Hebbel (1995). Recent theoretical investigations include Sims (2004, 2005) and Buiter (2004, 2005). Modern advanced textbooks/treatises such as Walsh (2003 and Romer (2006) devote considerable space to the issue. The explicitly multi-period or intertemporal dimension linking the various notions of seigniorage has not, however, been brought out and exploited before.

II. Three faces of seigniorage

There are two common measures of 'seigniorage', the resources appropriated by the monetary authority through its capacity to issue zero interest fiat money. The first, S_1, is the change in the monetary base, $S_{1,t} = \Delta M_t = M_t - M_{t-1}$, where M_t is the stock of nominal base money outstanding at the end of period t and the beginning of period $t-1$. The term *seigniorage* is sometimes reserved for this measure (see e.g. Flandreau (2006), and Bordo (2006)) and I shall follow this convention, although usage is not standardised. The second measure, S_2, is the interest earned by investing the resources obtained though the past issuance of base money in interest-bearing assets: $S_{2,t} = i_{t,t-1} M_{t-1}$, where $i_{t,t-1}$ is the risk-free nominal interest rate on financial instruments other than base money between periods *t-1* and *t*. Flandreau refers to this as *Central Bank revenue* and again I shall follow this usage.

It is often helpful to measure seigniorage and Central Bank revenue in real terms or as a share of GDP. Period *t* seigniorage as a share of GDP, $s_{1,t}$, is defined as $s_{1,t} = \frac{\Delta M_t}{P_t Y_t}$ and period *t* Central Bank revenue as a share of GDP, $s_{2,t}$, as $s_{2,t} = i_{t,t-1}\frac{M_{t-1}}{P_t Y_t}$, where P_t is the period *t* price level and Y_t period *t* real output.

A distinct but related concept to seigniorage and Central Bank revenue is the inflation tax, S_3. The inflation tax is the reduction in the real value of the stock of base money caused by inflation.[1] Let $\pi_{t,t-1} = \frac{P_t}{P_{t-1}} - 1$ be the rate of inflation between periods *t-1* and *t*, then the period *t* inflation tax is $S_{3,t} = \pi_{t,t-1} M_{t-1}$. The inflation tax as a share of GDP will be denoted $s_{3,t} = \pi_{t,t-1}\frac{M_{t-1}}{P_t Y_t}$.

Let $\gamma_{t,t-1} = \frac{Y_t}{Y_{t-1}} - 1$ be the growth rate of real GDP between periods *t-1* and *t*. The real interest rate between periods *t-1* and *t* is denoted $r_{t,t-1}$ where

$$(1 + r_{t,t-1})(1 + \pi_{t,t-1}) = 1 + i_{t,t-1} \tag{1}$$

The growth rate of the nominal stock of base money between periods *t-1* and *t* is denoted $\mu_{t,t-1} = \frac{M_t}{M_{t-1}} - 1$. Finally, let the ratio of the beginning-of-period base money stock to nominal GDP in period *t* be denoted $m_t = \frac{M_{t-1}}{P_t Y_t}$.

[1] This is sometimes called the 'anticipated inflation tax', to distinguish it from the 'unanticipated inflation tax', the reduction in the real value of outstanding fixed interest rate nominally denominated debt instruments caused by an unexpected increase in the rate of inflation which causes their price and real value to decline.

Steady-state seigniorage

Assume that in a deterministic steady state, the ratio of base money to nominal GDP is constant, that is,

$$1+\bar{\mu}=(1+\bar{\pi})(1+\bar{\gamma}) \tag{2}$$

where variables with overbars denote deterministic steady-state values. In steady state,

$$\begin{aligned}\bar{s}_1&=\bar{\mu}\bar{m}\\ \bar{s}_2&=\bar{i}\bar{m}\\ \bar{s}_3&=\bar{\pi}\bar{m}\end{aligned}$$

or, using (1) and (2)

$$\begin{aligned}\bar{s}_1&=\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\bar{m}\\ \bar{s}_2&=\left((1+\bar{\pi})(1+\bar{r})-1\right)\bar{m}\\ \bar{s}_3&=\bar{\pi}\bar{m}\end{aligned} \tag{3}$$

Let $\eta(\bar{\pi})\equiv-\dfrac{\ell'(\bar{\pi})}{\ell(\bar{\pi})}$ be the semi-elasticity of long-run money demand with respect to the inflation rate. In what follows I will only consider steady-state money demand functions $\bar{m}=\ell(\bar{\pi})$, $\ell'<0$ that have the property that $\bar{s}_i$, $i=1,2,3$ is continuously differentiable, increasing in $\bar{\pi}$ when $\bar{\pi}=\bar{r}=\bar{\gamma}=0$ and has a unique maximum.[2] Such unimodal long-run seigniorage Laffer curves are consistent with the available empirical evidence (see Cagan (1956), Anand and van Wijnbergen (1989), Easterly, Mauro and Schmidt-Hebbel (1995) and Kiguel and Neumeyer (1995)).

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[2] For $\bar{s}_1$ this means that for $\bar{\pi}<\hat{\bar{\pi}}_1$, $\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\eta(\bar{\pi})<1+\bar{\gamma}$ and for $\bar{\pi}>\hat{\bar{\pi}}_1$, $\left((1+\bar{\pi})(1+\bar{\gamma})-1\right)\eta(\bar{\pi})>1+\bar{\gamma}$. For $\bar{s}_2$ this means that for $\bar{\pi}<\hat{\bar{\pi}}_2$, $\left((1+\bar{\pi})(1+\bar{r})-1\right)\eta(\bar{\pi})<1+\bar{r}$ and for $\bar{\pi}>\hat{\bar{\pi}}_2$, $\left((1+\bar{\pi})(1+\bar{r})-1\right)\eta(\bar{\pi})>1+\bar{r}$. For $\bar{s}_3$ this means that for $\bar{\pi}<\hat{\bar{\pi}}_3$, $\bar{\pi}\eta(\bar{\pi})<1$ and that for $\bar{\pi}>\hat{\bar{\pi}}_3$, $\bar{\pi}\eta(\bar{\pi})>1$, the familiar condition that when price falls total revenue increases (decreases) if and only if the price elasticity of demand is less than (greater than) one.

I will also assume that the long-run money demand function has the property that the semi-elasticity of long-run money demand with respect to the inflation rate is non-decreasing: $\eta'(\bar{\pi}) \geq 0$; this is, again, a property shared by the empirically successful base money demand functions. Probably the most familiar example is the semi-logarithmic long-run base money demand function, made popular by Cagan's studies (Cagan (1956)) of hyperinflations, with its constant semi-elasticity of money demand ($\eta(\bar{\pi}) = \eta$):

$$\begin{aligned} &\ln \bar{m} = \alpha - \eta\bar{\pi} \\ &\eta > 0 \end{aligned} \tag{4}$$

Taking steady state output growth as exogenous, the constant inflation rate that maximises steady-state seigniorage as a share of GDP is given by:

$$\hat{\bar{\pi}}_1 = \arg\max \left((1+\bar{\pi})(1+\bar{\gamma})-1\right) f(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_1)} - \frac{\bar{\gamma}}{1+\bar{\gamma}} \tag{5}$$

Taking the steady-state real rate of interest as given, the constant inflation rate that maximises steady-state Central Bank revenue as a share of GDP is given by:

$$\hat{\bar{\pi}}_2 = \arg\max \left((1+\bar{\pi})(1+\bar{r})-1\right) f(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_2)} - \frac{\bar{r}}{1+\bar{r}} \tag{6}$$

The constant inflation rate that maximises steady-state inflation tax revenue as a share of GDP is given by

$$\hat{\bar{\pi}}_3 = \arg\max \bar{\pi} f(\bar{\pi}) = \frac{1}{\eta(\hat{\bar{\pi}}_3)} - 1 \tag{7}$$

The following proposition follows immediately:

Proposition 1:

Assume that the long-run seigniorage Laffer curve is increasing at $\bar{\pi} = 0$ and unimodal and that the semi-elasticity of money demand with respect to the inflation rate is non-decreasing in the inflation rate. The inflation rate that maximises steady-state seigniorage as a share of GDP is lower than the inflation rate that maximises steady-state Central Bank revenue as a share of GDP if and only if the growth rate of

real GDP is greater than the real interest rate. The inflation rate that maximises the steady-state inflation tax as a share of GDP is higher than the inflation rate that maximises steady-state seigniorage as a share of GDP (Central Bank revenue as a share of GDP) if and only if the growth rate of real GDP (the real interest rate) is positive.[3]

Corollary 1:

The ranking of the maximised values of $\overline{s}_1$, $\overline{s}_2$ *and* $\overline{s}_3$ *is the same as the ranking of the magnitudes of* $\hat{\overline{\pi}}_1$, $\hat{\overline{\pi}}_2$ *and* $\hat{\overline{\pi}}_3$.

Seigniorage in real time

I shall generalise these three measures of Central Bank resource appropriation to allow for a non-zero risk-free nominal interest rate on base money, $i^M_{t,t-1}$ for the rate on base money between periods *t-1* and *t*. This generalised seigniorage measure is defined by $S_{1,t} = M_t - (1+i^M_{t,t-1})M_{t-1}$ and the generalised measure of Central Bank revenue is defined by $S_{2,t} \equiv \left(i_{t,t-1} - i^M_{t,t-1}\right) M_{t-1}$

Expressed as shares of GDP, these two seigniorage measures become:

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[3] It suffices to show that $\hat{\overline{\pi}}_1$ is decreasing in $\overline{\gamma}$. Since $\frac{d\hat{\overline{\pi}}_1}{d\overline{\gamma}} = -\left(\frac{1}{1+\overline{\gamma}}\right)^2 \left(\frac{\left(\eta(\hat{\overline{\pi}}_1)\right)^2}{\eta'(\hat{\overline{\pi}}_1)+\left(\eta(\hat{\overline{\pi}}_1)\right)^2}\right)$, $\eta' \geq 0$ is sufficient but not necessary for the result. This result applies to a large number of empirically plausible base money demand functions. For the linear demand function found e.g. in Sargent and Wallace's Unpleasant Monetarist Arithmetic model (Sargent and Wallace (1981)) $m = \alpha - \beta(1+\pi),\ \beta > 0,\ m > 0$, for instance, we have

$\hat{\overline{\pi}}_1 = \arg\max\left((1+\overline{\pi})(1+\overline{\gamma})-1\right)\left(\alpha-\beta(1+\overline{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta}+\frac{1}{1+\overline{\gamma}}\right)-1,$

$\hat{\overline{\pi}}_2 = \arg\max\left((1+\overline{\pi})(1+\overline{r})-1\right)\left(\alpha-\beta(1+\overline{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta}+\frac{1}{1+\overline{r}}\right)-1$ and

$\hat{\overline{\pi}}_3 = \arg\max\overline{\pi}\left(\alpha-\beta(1+\overline{\pi})\right) = \frac{1}{2}\left(\frac{\alpha}{\beta}+1\right)-1$. Proposition 1 applies here also, with

$\eta(\overline{\pi}) = \frac{\beta}{\alpha-\beta(1+\overline{\pi})}$.(see Buiter (1990)).

$$s_{1,t} \equiv \frac{M_t - (1+i^M_{t,t-1})M_{t-1}}{P_t Y_t}$$

and

$$s_{2,t} \equiv (i_{t,t-1} - i^M_{t,t-1})\frac{M_{t-1}}{P_t Y_t}$$

The following notation will be needed to define the appropriate intertemporal relative prices or stochastic discount factors: I_{t_1,t_0} is the nominal stochastic discount factor between periods t_1 and t_0, defined by

$$\begin{aligned} I_{t_1,t_0} &= \prod_{k=t_0+1}^{t_1} I_{k,k-1} \quad \text{for } t_1 > t_0 \\ &= 1 \qquad\qquad\quad \text{for } t_1 = t_0 \end{aligned}$$

The interpretation of I_{t_1,t_0} is the price in terms of period t_0 money of one unit of money in period $t_1 \geq t_0$. There will in general be many possible states in period t_1, and period t_1 money has a period t_0 (forward) price for each state. Let E_t be the mathematical expectation operator conditional on information available at the beginning of period t. Provided earlier dated information sets do not contain more information than later dated information sets, these stochastic discount factors satisfy the recursion property

$$E_{t_0}\left(I_{t_1,t_0} E_{t_1} I_{t_2,t_1}\right) = E_{t_0} I_{t_2,t_0} \text{ for } t_2 \geq t_1 \geq t_0$$

Finally, the risk-free nominal interest rate in period t, $i_{t+1,t}$, that is, the money price in period t of one unit of money in every state of the world in period $t+1$ is defined by

$$\frac{1}{1+i_{t+1,t}} = E_t I_{t+1,t} \tag{8}$$

For future reference I also define the real stochastic discount factor between periods t_0 and t_1, R_{t_1,t_0}. Let the inflation factor between period t_0 and t_1, Π_{t_1,t_0}, be defined by

$$\begin{aligned}\Pi_{t_1,t_0} &= \frac{P_{t_1}}{P_{t_0}} = \sum_{k=t_0+1}^{t_1} (1+\pi_{k,k-1}) \quad \text{for } t_1 > t_0 \\ &= 1 \qquad\qquad\qquad\qquad \text{for } t_1 = t_0\end{aligned}$$

The real stochastic discount factor is defined by

$$R_{t_1,t_0} = I_{t_1,t_0}\Pi_{t_1,t_0}$$

It is easily checked that it has the same recursive properties as the nominal discount factor:

$$\begin{aligned}R_{t_1,t_0} &= \prod_{k=t_0+1}^{t_1} R_{k,k-1} \quad \text{for } t_1 > t_0 \\ &= 1 \qquad\qquad\qquad\quad \text{for } \mathrm{t}_1 = t_0\end{aligned}$$

$$E_{t_0}\left(R_{t_1,t_0}E_{t_1}R_{t_2,t_1}\right) = E_{t_0}R_{t_2,t_0} \text{ for } t_2 \geq t_1 \geq t_0$$

The risk-free real rate of interest between periods t and t+1 , $r_{t+1,t}$, is defined as

$$\frac{1}{1+r_{t+1,t}} = E_t R_{t+1,t}\,.$$

Note that the real GDP growth-corrected discount factors satisfy:

$$E_{t_0}\left[R_{t_1,t_0}Y_{t_1,t_0}E_{t_1}\left(R_{t_2,t_1}Y_{t_2,t_1}\right)\right] = E_{t_0}\left(R_{t_2,t_0}Y_{t_2,t_0}\right) \text{ for } t_2 \geq t_1 \geq t_0$$

The Intertemporal Seigniorage Identity

Acting in real time, the monetary authority will be interested in the present discounted value of current and future seigniorage, rather than in just its current value or its steady-state value. A focus on the current value alone would be myopic and an exclusive concern with steady state seigniorage would not be a appropriate if the traverse to the steady state is non-

instantaneous and involves transitional seigniorage revenues that are different from their steady state values. The present discounted value of the nominal value of seigniorage is given by:

$$PDV_t(S_1) \equiv E_t \sum_{j=t}^{\infty} I_{j,t} \left(M_j - (1+i^M_{j,j-1}) M_{j-1} \right) \tag{9}$$

The present discounted value of nominal Central Bank revenue is given by:

$$\begin{aligned} PDV_t(S_2) &\equiv E_t \sum_{j=t}^{\infty} I_{j,t} (i_{j+1,j} - i^M_{j+1,j}) M_j \\ &= E_t \sum_{j=t}^{\infty} \left(I_{j,t} - I_{j+1,t-1}(1+i^M_{j+1,j}) \right) M_j \end{aligned} \tag{10}[4]$$

Through the application of brute force (or in continuous time, through the use of the formula for integration by parts), and using the second equality in (10), it is easily established that the following relationship holds identically (see Buiter (1990)):

$$\begin{aligned} E_t \sum_{j=t}^{\infty} \mathrm{I}_{j,t} \left(M_j - (1+i^M_{j,j-1}) M_{j-1} \right) \equiv E_t \sum_{j=t}^{\infty} I_{j,t} (i_{j+1,j} - i^M_{j+1,j}) M_j - \left(1+i^M_{t,t-1}\right) M_{t-1} \\ + \lim_{N \to \infty} E_t I_{N,t} M_N \end{aligned} \tag{11}$$

I will refer to (11) as the 'intertemporal seigniorage identity' or ISI.

If we impose the boundary condition that the present value of the terminal base money stock is zero in the limit as the terminal date goes to infinity, that is,

$$\lim_{N \to \infty} E_t I_{N,t} M_N = 0, \tag{12}$$

the ISI becomes

[4] The equality of the last two expressions in (10) is established as follows. For $j \geq t+1$, $E_t \left(I_{j-1,t} - I_{j,t}(1+i^M_{j,j-1}) \right) M_{j-1} = E_t I_{j-1,t} \left(1 - I_{j,j-1}(1+i^M_{j,j-1}) \right) M_{j-1}$. Therefore, $E_t I_{j-1,t} \left(1 - I_{j,j-1}(1+i^M_{j,j-1}) \right) M_{j-1} = E_t I_{j-1,t} \left(1 - E_{j-1} I_{j,j-1}(1+i^M_{j,j-1}) \right) M_{j-1}$. From (8) $E_t I_{j-1,t} \left(1 - E_{j-1} I_{j,j-1}(1+i^M_{j,j-1}) \right) M_{j-1} = E_t I_{j-1,t} \left(1 - (1+i_{j,j-1})^{-1}(1+i^M_{j,j-1}) \right) M_{j-1}$.

$$E_t \sum_{j=t}^{\infty} \mathrm{I}_{j,t} \left(M_j - (1+i^M_{j,j-1}) M_{j-1} \right) = E_t \sum_{j=t}^{\infty} I_{j,t} (i_{j+1,j} - i^M_{j+1,j}) M_j - \left(1+i^M_{t,t-1}\right) M_{t-1}$$

or (13)

$$PDV_t(S_1) = PDV_t(S_2) - \left(1+i^M_{t,t-1}\right) M_{t-1}$$

There are no additional interesting relationships that can be established between the inflation tax and the other two monetary resource appropriation measures – seigniorage and Central Bank revenue, beyond the familiar identity that seigniorage revenue as a share of GDP equals the inflation tax plus the 'real growth bonus' (that is, the increase in the demand for real money balances associated, *cet. par.* with real GDP growth) plus the change in the ratio of base money to GDP:

$$\frac{\Delta M_t}{P_t Y_t} = \pi_{t+1,t} m_{t+1} + \left(1+\gamma_{t+1,t}(1+\pi_{t+1,t})\right) m_{t+1} + \Delta m_{t+1} \qquad (14)^5$$

When the nominal interest rate on base money is zero, (13) becomes:

$$E_t \sum_{j=t}^{\infty} \mathrm{I}_{j,t} \Delta M_j = E_t \sum_{j=t}^{\infty} I_{j,t} i_{j+1,j} M_j - M_{t-1}$$

or (15)

$$PDV_t(S_1) = PDV_t(S_2) - M_{t-1}$$

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[5] Consider, for sake of brevity, the continuous time analogue of (14): $\mu m = (\pi + \gamma) m + \dot{m}$. Taking present discounted values on both sides of this relationship yields: $\int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \mu(s) m(s) ds = \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} [\pi(s)+\gamma(s)] m(s) + \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \dot{m}(s) ds$. Applying integration by parts to the second term on the r.h.s. of the last equation yields $\int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} \mu(s) m(s) ds = \int_t^{\infty} e^{-\int_t^s [r(u)-\gamma(u)]du} [\pi(s)+r(s)] m(s) - m(t)$. With $r(s)+\pi(s) = i(s)$, this is simply the continuous time version of the ISI.

When there is no uncertainty, (15) simplifies to:

$$\sum_{j=t}^{\infty}\prod_{k=t+1}^{j}\frac{1}{1+i_{k,k-1}}\Delta M_j \equiv \sum_{j=t+1}^{\infty}\prod_{k=t+1}^{j}\frac{1}{1+i_{k,k-1}} i_{j,j-1}M_{j-1} - M_{t-1} \tag{16}$$

When the risk-free nominal interest rate is constant from period t on, this further simplifies to:

$$\sum_{j=t}^{\infty}\left(\frac{1}{1+i}\right)^{j-t}\Delta M_j = i\sum_{j=t+1}^{\infty}\left(\frac{1}{1+i}\right)^{j-t} M_{j-1} - M_{t-1}$$

Using real GDP units as the numéraire rather than money, the ISI in equation (13) becomes

$$E_t\sum_{j=t}^{\infty}R_{j,t}\Gamma_{j,t}\left(\frac{M_j-(1+i^M_{j,j-1})M_{j-1}}{P_jY_j}\right) = E_t\sum_{j=t}^{\infty}R_{j,t}\Gamma_{j,t}(i_{j+1,j}-i^M_{j+1,j})m_j - (1+i^M_{t,t-1})m_t$$

or (17)

$$\sigma_{1,t} = \sigma_{2,t} - (1+i^M_{t,t-1})m_t$$

Where

$$\begin{aligned}\sigma_{1,t} &= PDV_t(s_1) = E_t\sum_{j=t}^{\infty}R_{j,t}\Gamma_{j,t}\left(\frac{M_j-(1+i^M_{j,j-1})M_{j-1}}{P_jY_j}\right)\\ \sigma_{2,t} &= PDV_t(s_2) = E_t\sum_{j=t}^{\infty}R_{j,t}\Gamma_{j,t}(i_{j+1,j}-i^M_{j+1,j})m_j\end{aligned} \tag{18}$$

With a zero nominal interest rate on base money, (17) becomes (19):

$$E_t\sum_{j=t}^{\infty}R_{j,t}\Gamma_{j,t}\frac{\Delta M_j}{P_jY_j} \equiv E_t\sum_{j=t}^{\infty}R_{j+1,t}\Gamma_{j+1,t}i_{j+1,j}m_j - m_t \tag{19}$$

When there is no uncertainty, this simplifies to:

$$\sum_{j=t}^{\infty}\prod_{k=t+1}^{j}\frac{1+\gamma_{k,k-1}}{1+r_{k,k-1}}\frac{\Delta M_j}{P_jY_j} \equiv \sum_{j=t+1}^{\infty} i_{j,j-1}m_j\prod_{k=t+1}^{j}\frac{1+\gamma_{k,k-1}}{1+r_{k,k-1}} - m_t \tag{20}$$

When the risk-free real interest rate and the growth rate of real GDP are constant, this further simplifies to:

$$\sum_{j=t}^{\infty}\left(\frac{1+\gamma}{1+r}\right)^{j-t}\frac{\Delta M_j}{P_jY_j} = i\sum_{j=t+1}^{\infty}\left(\frac{1+\gamma}{1+r}\right)^{j-t} m_j - m_t \tag{21}$$

In general, the infinite sums in (21) will exist only if the real interest rate exceeds the growth rate of real GDP ($r > \gamma$). From equation (13) it is clear that maximizing the present discounted value of current and future nominal seigniorage ($M_j - M_{j-1}(1+i^M_{j,j-1})$) according to the S_1 definition, is equivalent to maximizing the present discounted value of current and future nominal Central Bank revenues according to the S_2 definition ($\left(i_{j,j-1} - i^M_{j,j-1}\right)M_{j-1}$). The two differ only by the inherited value of the nominal stock of base money gross of interest on base money, $(1+i^M_{t,t-1})M_{t-1}$, which is not a choice variable in period *t.* I summarise this as **Proposition 2.**

Proposition 2:

Acting in real time, and therefore treating the initial nominal stock of base money as predetermined, maximising the present discounted value of current and future nominal seigniorage is equivalent to maximising the present discounted value of current and future nominal Central Bank revenue.

The same result cannot be inferred quite as readily for either the present discounted values of current and future real seigniorage $\frac{M_j - (1+i^M_{j,j-1})M_{j-1}}{P_j}$ and future real Central Bank revenues $\left(i_{j,j-1} - i^M_{j,j-1}\right)\frac{M_{j-1}}{P_j}$, or for the present discounted values of current and future seigniorage as a share of GDP $\frac{M_j - (1+i^M_{j,j-1})M_{j-1}}{P_j Y_j}$ and future Central Bank revenues as a share of GDP $\left(i_{j,j-1} - i^M_{j,j-1}\right)\frac{M_{j-1}}{P_j Y_j}$. The reason is that both the initial value of the general price level, P_t, and the initial level of real GDP, Y_t, are, in principle, endogenous and could be choice variables of or influenced by the monetary authority. This suggests Corollaries 2 and 3:

Corollary 2:

Acting in real time, maximising the present discounted value of current and future real seigniorage is equivalent to maximising the present discounted value of current and future real Central Bank revenue if and only if the current price level is given.

Classes of models for which the current general price level is predetermined, exogenous or constant for other reasons include the following: (1) Old-Keynesian and New-Keynesian models, for which price level is predetermined); (2) any model of a small open economy with only traded goods, all of which obey the law of one price.

Corollary 3:

Acting in real time, maximising the present discounted value of current and future seigniorage as a share of GDP is equivalent to maximising the present discounted value of current and future Central Bank revenue as a share of GDP if and only if the current value of nominal GDP is given.

I will assume that a level of nominal GDP that is predetermined, exogenous or constant for other reasons requires both an initial general price level and an initial level of real GDP that are predetermined, exogenous or constant for other reasons. The New Keynesian model presented in Section IV has a predetermined price level. In this model, the current value of real GDP is invariant to the policy actions under consideration provided only the price level but not the rate of inflation is predetermined. Most Old-Keynesian models have both a predetermined price level and a predetermined rate of inflation, so maximising σ_1 in real time will not be equivalent to maximising σ_2 in real time. The equivalence result applies also for any model of a small open economy with only traded goods, all of which obey the law of one price, and an exogenous level of real GDP.

It is important to note that maximising, in real time, the present discounted value of current and future seigniorage when the inflation rate determined in the current period and in all other future periods is constant, and when current and future real interest rates and real growth

rates are constant, is not the same as maximising the present discounted value of steady state seigniorage. To clarify the difference, consider for simplicity an economy that, starting in period *t*, is in steady state, although the initial ratio of base money to GDP, m_t, need not be the same as the subsequent steady-state values. When the system is in a deterministic steady state starting from period *t,* the following hold for $j \geq t+1$:

$$(1+\pi_{j,j-1})(1+\gamma_{j,j-1}) = (1+\overline{\pi})(1+\overline{\gamma}) = (1+\overline{\mu}) = 1+\mu_{j,j-1} \tag{22}$$

$$(1+r_{j,j-1})(1+\pi_{j,j-1}) = (1+\overline{r})(1+\overline{\pi}) = 1+\overline{i} = 1+i_{j,j-1} \tag{23}$$

For simplicity, assume that the nominal interest rate on base money is zero. For simplicity I also assume that $\mu_{t,t-1} = \mu$. It does not follow, however, that $(1+\pi_{t,t-1})(1+\gamma_{t,t-1}) = 1+\mu_{t,t-1} = (1+\overline{\mu}) = (1+\overline{\pi})(1+\overline{\gamma})$.The ISI now simplifies to (24):

$$\overline{\mu}\left(m_t + \overline{m}\left(\frac{1+\overline{\gamma}}{\overline{r}-\overline{\gamma}}\right)\right) = \overline{i}\overline{m}\left(\frac{1+\overline{\gamma}}{\overline{r}-\overline{\gamma}}\right) - m_t$$

or (24)

$$\sigma_{1,t} = \sigma_{2,t} - m_t$$

If the the monetary authority cannot choose or influence the initial ratio of money to GDP, maximizing the present discounted value of current and future s_1 is equivalent to maximising $\left[(1+\overline{r})(1+\overline{\pi})-1\right]\left(\frac{1+\overline{\gamma}}{r-\overline{\gamma}}\right)\overline{m}$, which is the present discounted value of present and future s_2. If the initial value of the money-GDP ratio could be chosen, subject to the constraint that it is equal to the steady-state value of the ratio of the stock of base money to GDP from period t onward, and if (23) also holds for $j = t$, then the two maximization problems are not equivalent. When the initial value of base money velocity is a choice variable, in the sense that it can be set to equal to steady-state value of velocity for period *t* and beyond, the following holds:

$$\bar{\sigma}_1 = \left[(1+\bar{\gamma})(1+\bar{\pi})-1\right]\left(\frac{1+\bar{r}}{\bar{r}-\bar{\gamma}}\right)\bar{m}$$
$$\bar{\sigma}_2 = \left[(1+\bar{r})(1+\bar{\pi})-1\right]\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right)\bar{m} \qquad (25)$$

Consider again the semi-logarithmic base money demand function in (4), or any long-run money demand function that results in a well-behaved unimodal long-run seigniorage Laffer curve. It is clear that, if the steady-state growth rate of GDP and the steady-state real rate of interest are independent of monetary policy, maximising $\bar{\sigma}_1$ subject to (4) yields the same result as maximising $\bar{s}_1$, and maximising $\bar{\sigma}_2$ subject to (4) yields the same result as maximising $\bar{s}_2$. It is also obvious that maximising the present discounted value of the inflation tax $\bar{\sigma}_3 = \bar{\pi}\left(\frac{1+\bar{\gamma}}{\bar{r}-\bar{\gamma}}\right)\bar{m}$ subject to (4) yields the same result as maximising $\bar{s}_3$. However, because the steady-state present discounted values in (25) only exist if $\bar{r} > \bar{\gamma}$, the case where the inflation rate that maximises the steady-state value of seigniorage as a share of GDP is below the inflation rate that maximises the steady-state value of Central Bank revenue as a share of GDP ($\hat{\bar{\pi}}_1 < \hat{\bar{\pi}}_2$) has no counterpart in the maximisation of the present discounted values of steady-state seigniorage as a share of GDP and of steady-state Central Bank revenue as a share of GDP.

The main message of this section is, however, that maximisation of seigniorage, Central Bank revenue and the inflation tax should be viewed from an explicitly intertemporal and real-time perspective.

III. The intertemporal budget constraints of the Central Bank and the Treasury

To obtain a full understanding of the constraints the Central Bank is subject to in the conduct of monetary policy in general and in its use of seigniorage in particular, it is essential to have a view of the Central Bank as an economic agent with a period budget constraint and an intertemporal budget constraint or solvency constraint. This requires us to decompose the consolidated Government's financial accounts and solvency constraint into separate accounts and solvency constraints for the Central Bank and the Treasury (see also Buiter (2004), Sims (2004), (2005) and Ize (2005)).[6] In this Section, I therefore introduce a stylized set of accounts for a small open economy. Separate period budget constraints for the Central Bank and Treasury are also considered in Walsh (2003) and in Buiter (2003, 2004 and 2005). The latter also considers the solvency constraints and intertemporal budget constraints of the two state sectors separately. Walsh leaves out the payments made by the Central Bank to the Treasury. While this does not, of course, affect the fiscal-financial-monetary options available to the consolidated Government, it does prevent the consideration of how the Treasury can, through its fiscal claims on the Central Bank, facilitate or prevent the Central Bank from implementing its monetary and supervisory mandates.

The Central Bank has only the monetary base $M \geq 0$ on the liability side of its financial balance sheet.[7] The marginal cost of base money issuance is equal to zero. On the asset side it has the stock of international foreign exchange reserves, R^f, earning a risk-free nominal interest rate in terms of foreign currency, i^f, and the stock of domestic credit, which consists of Central Bank holdings of nominal, interest-bearing Treasury bills, D, earning the risk-free domestic-

[6] The term 'government' as used in 'government budget constraint' refers to the consolidated general government and central bank. 'State' would be a better term, to avoid confusion with the particular administration in office at a point in time. The unfortunate usage is, however, too firmly ensconced to try to dislodge it here.

[7] In the real world this would be currency plus commercial bank reserves with the Central Bank. In many emerging markets and developing countries, the central bank also has non-monetary interest-bearing liabilities. These could be added easily to the accounting framework.

currency nominal interest rate, i, and Central Bank claims on the private sector, L, with risk-free domestic-currency nominal interest rate i^L;[8] e is the value of the spot nominal exchange rate (the domestic currency price of foreign exchange); T^b is the real value of taxes paid by the Central Bank to the Treasury; it is a choice variable of the Treasury and can be positive or negative; H is the real value of the transfer payments made by the Central Bank to the private sector ('helicopter drops'). I assume H to be a choice variable of the Central Bank. It is true that in most countries the Central Bank is not a fiscal agent. I can neither tax nor make transfer payments. While I shall deny the Central Bank the power to tax, $H \geq 0$, I will until further notice allow it to make transfer payments. This is necessary for 'helicopter drops of money' to be implementable by the Central Bank on its own, without Treasury support; $C^g \geq 0$ is real current consumption spending by the Central Bank; the stock of Treasury debt (assumed to be denominated in domestic currency) held outside the Central Bank is B; it pays the risk-free nominal interest rate i; T^p is the real value of the tax payments by the domestic private sector to the Treasury; it is a choice variable of the Treasury and can be positive or negative; total real taxes net of transfer payments received by the Government, that is, the consolidated Treasury and Central Bank are $T = T^p - H$; $C^g \geq 0$ is the real value of Treasury spending on goods and services and $C^b \geq 0$ the real value of Central Bank spending on goods and services. Public spending on goods and services is assumed to be public consumption only.

Equation (26) is the period budget identity of the Treasury and equation (27) that of the Central Bank.

$$\frac{B_t + D_t}{P_t} = C_t^g - T_t^p - T_t^b - (1 + i_{t,t-1})\frac{B_{t-1} - D_{t-1}}{P_t} \tag{26}$$

[8] For simplicity, I consider only short maturity bonds. Generalisations to longer maturities, index-linked debt or foreign-currency denominated debt are straightforward.

$$\frac{M_t - D_t - L_t - e_t R_t^f}{P_t} = C_t^b + T_t^b + H_t + \frac{(1+i_{t,t-1}^M)M_{t-1} - (1+i_{t,t-1})D_{t-1} - (1+i_{t,t-1}^L)L_{t-1} - (1+i_{t,t-1}^f)e_t R_{t-1}^f}{P_t} \tag{27}$$

The solvency constraints of, respectively, the Treasury and Central Bank are given in equations (28) and (29):

$$\lim_{j\to\infty} E_t I_{j,t-1}\left(B_j + D_j\right) \le 0 \tag{28}$$

$$\lim_{j\to\infty} E_t I_{j,t-1}\left(D_j + L_j + e_j R_j^f\right) \ge 0 \tag{29}$$

When there exist complete contingent claims markets, and the no-arbitrage condition is satisfied, these solvency constraints, which rule out Ponzi finance by both the Treasury and the Central Bank, imply the following intertemporal budget constraints for the Treasury (equation (30)) and for the Central Bank (equation (31)).

$$B_{t-1} + D_{t-1} \le E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j (T_j^p + T_j^b - C_j^g) \tag{30}$$[9]

$$D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f \le E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(P_j\left(C_j^b + T_j^b + H_j + Q_j\right) - \left(M_j - (1+i_{j,j-1}^M)M_{j-1}\right)\right) \tag{31}$$

where

$$P_j Q_j = (i_{j,j-1} - i_{j,j-1}^L)L_{j-1} + \left(1 + i_{j,j-1} - (1+i_{j,j-1}^f)\frac{e_j}{e_{j-1}}\right)e_{j-1}R_{j-1}^f \tag{32}$$

The expression Q in equation (32) stands for the real value of the quasi-fiscal implicit interest subsidies made by the Central Bank. If the rate of return on government debt exceeds that on loans to the private sector, there is an implicit subsidy to the private sector equal in period t to $\left(i_{t,t-1} - i_{t,t-1}^L\right)L_{t-1}$. If the rate of return on foreign exchange reserves is less than what would be

[9] Note that $E_t E_{t-1} I_{t,t-1} = E_{t-1} I_{t,t-1} = \frac{1}{1+i_{t,t-1}}$.

implied by Uncovered Interest Parity (UIP), there is an implicit subsidy to the issuers of these reserves, given in period t by $\left(1+i_{t,t-1}-(1+i^{f}_{t,t-1})\frac{e_t}{e_{t-1}}\right)e_{t-1}R^{f}_{t-1}$.

The solvency constraint of the Central Bank only requires that the present discounted value of its net *non-monetary* liabilities be non-positive in the long run. Its monetary liabilities are liabilities only in name, as they are *irredeemable*: the holder of base money cannot insist at any time on the redemption of a given amount of base money into anything else other than the same amount of itself (base money).

Summing (26) and (27) gives the period budget identity of the consolidated Government (the consolidated Treasury and Central Bank) in equation (33); summing (28) and (29) gives the solvency constraint of the consolidated Government in equation (34) and summing (30) and (31) gives the intertemporal budget constraint of the Government in equation (35).

$$\begin{aligned} M_t + B_t - L_t - e_t R^{f}_t \equiv\ & P_t(C^{g}_t + C^{b}_t - T_t) \\ & +(1+i^{M}_{t,t-1})M_{t-1} + (1+i_{t,t-1})B_{t-1} - (1+i^{L}_{t,t-1})L_{t-1} - e_t(1+i^{f}_{t,t-1})R^{f}_{t-1} \end{aligned} \tag{33}$$

$$\lim_{j\to\infty} E_t I_{j,t-1}\left(B_j - L_j - e_j R^{f}_j\right) \le 0 \tag{34}$$

$$B_{t-1} - L_{t-1} - e_{t-1}R^{f}_{t-1} \le E_t \sum_{j=t}^{\infty} I_{j,t-1}\left(P_j\left(T_j - Q_j - C^{g}_j - C^{b}_j\right) + M_j - (1+i^{M}_{j,j-1})M_{j-1}\right) \tag{35}$$

Consider the conventional financial balance sheet of the Central Bank in Table 1, that of the Treasury in Table 2, and that of the Government in Table 3. Loans to the private sector and international reserves are valued at their notional or face values.[10]

[10] If the outstanding stock of loans to the private sector were marked-to-market, its fair value would be $L_t\left(\frac{1+i^{L}_{t+1,t}}{1+i_{t,t+1}}\right)$, the fair value of the international reserves would be. $e_t R^{f}_t\left(\frac{(1+i^{f}_{t+1,t})e_{t+1}/e_t}{i_{t+1,t}}\right)$ and the fair value of

Table 1 Central Bank Conventional Financial Balance Sheet	
Assets	**Liabilities**
D	M
L	
eR^f	
	W^b

Table 2 Treasury Conventional Financial Balance Sheet	
Assets	**Liabilities**
	D
	B
	W^t

---

the stock of base money would be $M_t\left(\frac{1+i^M_{t+1,t}}{1+i_{t+1,t}}\right)$. As a store of value, base money is a perpetuity paying $1+i^M_{j,j-1}$ in each period $j>t$ for each unit of money acquired in period t. The marked-to-market or fair value of a unit of base money acquired in period *t* (ex-dividend, that is, after period *t* interest due has been paid) is therefore $E_t\sum_{t+1}^{\infty} I_{j,j-1}i^M_{j.j-1}$. In the deterministic case, this becomes $\sum_{j=t+1}^{\infty}\prod_{k=t+1}^{j}\frac{i^M_{j,j-1}}{1+i_{k,k-1}}$. If follows that, as a store of value, the fair value of currency, which has a zero interest rate, is zero, as it is effectively a consol with a zero coupon.

Table 3 Government Conventional Financial Balance Sheet	
Assets	**Liabilities**
L	B
eR^f	M
	W^g

The Central Bank's financial net worth, $W^b \equiv D + L + eR^f - M$, is the excess of the value of its financial assets, Treasury debt, D, loans to the private sector, L and foreign exchange reserves, eR^f, over its monetary liabilities, M. The Treasury's conventional financial net worth is denoted W^t, the Government's by W^g.

To make the relationship between the intertemporal budget constraints of the Treasury and the Central Bank and their conventional balance sheets more apparent, it is helpful to use the ISI, given in equation (13), to rewrite the intertemporal budget constraint of the Central Bank as in equation (36):

$$M_{t-1} - \left(D_{t-1} + L_{t-1} + e_{t-1}R^f_{t-1}\right) \leq E_t \sum_{j=t}^{\infty} I_{j,t-1}\left[P_j\left(-C^b_j - T^b_j - H_j - Q_j\right) + (i_{j+1,j} - i^M_{j+1,j})M_j\right] \quad (36)$$

III.1 Can Central Banks survive with 'negative equity'?

On the left-hand side of (36) we have (minus) the equity of the Central Bank – the excess of its monetary liabilities over its non-monetary financial assets. On the right-hand side of (36) we have, in addition to the present discounted value of the Central Bank's primary surpluses, $E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j\left(-C^b_j - T^b_j - H_j - Q_j\right)$, the present discounted value Central Bank revenue,

$E_t \sum_{j=t}^{\infty} I_{j,t-1}(i_{j+1,j} - i^M_{j+1,j})M_j$, that is, the present discounted value of future interest payments saved by the Central Bank because of its ability to issue monetary liabilities bearing an interest rate $i^M_{j,j-1}$.

It should be noted that order to obtain the Central Bank's intertemporal budget constraint (31), I imposed the no-Ponzi game terminal condition $\lim_{j\to\infty} E_t I_{j,t-1}\left(D_j + L_j + e_j R^f_j\right) \geq 0$, that is, the present value of the terminal net *non-monetary* liabilities had to be non-negative. I did not impose the condition $\lim_{j\to\infty} E_t I_{j,t-1}\left(D_j + L_j + e_j R^f_j - M_j\right) \geq 0$, that is, that the present value of the terminal total net liabilities, *monetary and non-monetary,* had to be non-negative. To get from (31) to (36), I need the ISI of equation (13), which is the basic ISI given in equation (11) plus the terminal boundary condition (12), that is, $\lim_{N\to\infty} E_t I_{N,t} M_N = 0$.

Although I will maintain condition (12) until Section V.2, it should be noted that, from the point of view of the monetary authorities, this assumption is by no means obvious. The reason is that the monetary 'liabilities' of the Central Bank are not in any meaningful sense liabilities of the Central Bank. The owner (holder) of currency notes worth *X* units of currency have a claim on the Central Bank for currency notes worth *X* units of currency – nothing more. The monetary liabilities of the Central Bank are irredeemable or inconvertible into anything other than the same amount of itself. While in most well-behaved economies, $\lim_{j\to\infty} E_t I_{j,t-1} M_j = 0$, this will not be the case, for instance, in a permanent liquidity trap where $\lim_{j\to\infty} E_t I_{j,t-1} M_j = \lim_{j\to\infty} E_t M_j \neq 0$ unless the monetary authorities adopt a policy of (asymptotically) demonetising the economy in nominal terms. Such asymptotic demonetization (in nominal terms) characterises the efficient stationary liquidity trap equilibrium of the Bailey-Friedman Optimal Quantity of Money rule, when the

interest rate on base money is zero and the risk-free nominal interest rate on non-monetary assets is kept at zero throughout. The nominal stock of base money shrinks at a proportional rate equal to the real interest rate and the rate of time preference.

Even if the conventionally defined net worth or equity of the Central Bank is negative, that is, if $W^b_{t-1} \equiv D_{t-1} + L_{t-1} + e_{t-1}R^f_{t-1} - M_{t-1} < 0$, the Central Bank can be solvent provided $W^b_{t-1} \geq E_t \sum_{j=t}^{\infty} I_{j,t-1}\left[P_j\left(C^b_j + T^b_j + H_j + Q_j\right) - (i_{j+1,j} - i^M_{j+1,j})M_j\right]$. Conventionally defined financial net worth or equity excludes the present value of anticipated or planned future non-contractual outlays and revenues (the right-hand side of equation (36). It is therefore perfectly possible, for the central bank to survive and thrive with negative financial net worth. This might, however, require the central bank to raise so much seigniorage in real terms, $\frac{M_j - (1+i^M_{j,j-1})M_j}{P_j}$, through current and future nominal base money issuance, that, given the demand function for real base money, unacceptable rates of inflation would result.

The financial net worth of the Treasury, $W^t = -(B+D)$ is negative in most countries. The financial net worth of the Government, that is, the consolidated Treasury and Central Bank $W^g \equiv W^t + W^b = eR^f + L - M - B$, is also negative for most countries. None of this need be a source of concern, unless the gap between the outstanding contractual non-monetary debt of the state and the present discounted value of the future primary (non-interest) surpluses of the state, $T_j - C^g_j - C^b_j - Q_j$, $j \geq t$ is so large, that it either cannot be filled at all at all (the maximum value of the discounted future real seigniorage stream is too low) and the state defaults, or can only be closed at unacceptably high rates of inflation.

The only intertemporal budget constraint that ought to matter, that is, the only one that would matter in a well-managed economy, is that of the consolidated Treasury and Central Bank, given in

equation (35). Its breakdown into the Treasury's intertemporal budget constraint (equation (30)) and the Central Bank's intertemporal budget constraint (equation (31)) is without macroeconomic interest, unless there is a failure of cooperation and coordination between the monetary and fiscal authorities, that is, between the Central Bank and the Treasury. Operational independence for Central Banks may have raised the risk of such mishaps occurring.

The separation of the accounts of the Treasury and the Central Bank allows us to recognise a fourth measure of the revenues extracted by the state through its monopoly of the issuance of base money. This is the conventionally measured operating profits of the Central Bank (before payment of taxes to the Treasury), which will be denoted $S_{4,t}$. It consists of its net interest income minus its operating expenses:

$$S_{4,t} = i_{t,t-1}D_{t-1} + i^{L}_{t,t-1}L_{t-1} + e_t i^{f}_{t,t-1}R^{f}_{t-1} - i^{M}_{t,t-1}M_{t-1} - P_t H_t - P_t C^{b}_{t} \tag{37}$$

From equation (27) it follows that

$$\Delta M_t - \Delta D_t - \Delta L_t - e_t \Delta R^{f}_{t} \equiv P_t T^{b}_{t} - S_{4,t}$$

If we make the further assumption that the operating profits of the Central Bank are paid in taxes to the Treasury[11], that is,

$$P_t \mathrm{T}^{b}_{t} = S_{4,t}\,, \tag{38}$$

then, and only then, does the textbook identity hold that the change in the stock of base money, ΔM_t, equals domestic credit expansion, $\Delta D_t + \Delta L_t$, plus the value of the increase in the stock of foreign exchange reserves, $e_t \Delta R^{f}_{t}$:

$$\Delta M_t = \Delta D_t + \Delta L_t + e_t \Delta R^{f}_{t} \tag{39}$$

A little rearranging of the identities in (36) and (37) yields:

[11] The profits of the Bank of England (after Corporation Tax) are split fifty-fifty between the Treasury and additions to the Bank of England's reserves.

$$E_t \sum_{j=t}^{\infty} I_{j,t-1} S_{4,j} \leq M_{t-1} - (D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f)$$
$$+ E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j T_j^b \tag{40}$$
$$+ E_t \sum_{j=t}^{\infty} I_{j,t-1} i_{j,j-1} \left(D_{j-1} + L_{j-1} + e_{j-1} R_{j-1}^f - M_{j-1} \right)$$

So if the Treasury always taxes away all the operating profits of the Central Bank (equation (38) holds), then

$$M_{t-1} - (D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f) \leq E_t \sum_{j=t}^{\infty} I_{j,t-1} i_{j,j-1} \left(D_{j-1} + L_{j-1} + e_j R_{j-1}^f - M_{j-1} \right)$$
$$+ E_t \sum_{j=t}^{\infty} I_{j,t-1} i_{j,j-1} \left(e_{j-1} - e_j \right) R_{j-1}^f$$

From (39) it then follows that $D_{j-1} + L_{j-1} + e_{t-1} R_{j-1}^f - M_{j-1} = D_j + L_j + e_{j-1} R_j^f - M_j$ for $j \geq t$, so

$$E_t \sum_{j=t}^{\infty} I_{j,t-1} i_{j,j-1} \left(e_{j-1} - e_j \right) R_{j-1}^f \geq 0$$

So unless the Central Bank experiences, on average, capital gains (through currency depreciation) rather than capital losses (through currency appreciation) on its foreign exchange reserves, the Central Bank's solvency constraint will be violated when the Treasury taxes away its operating profits.

Regardless of the tax rule the Treasury imposes on the Central Bank, it is always the case that the present discounted value of the taxes paid by Central Bank to the Treasury can be written as

$$
\begin{aligned}
PDV_{t-1}(PT^b) &\equiv E_t \sum_{j=t}^{\infty} I_{j,t-1} P_j T_j^b \\
&= D_{t-1} + L_{t-1} + e_{t-1} R_{t-1}^f - M_{t-1} \\
&\quad + E_t \sum_{j=t}^{\infty} I_{j,t-1} \left[P_j \left(-C_j^b - H_j - Q_j \right) + (i_{j+1,j} - i_{j+1,j}^M) M_j \right]
\end{aligned}
$$

So unless the Central Bank can influence the present discounted value of its primary (non-interest) deficits (before taxes paid to the Treasury), maximising the present discounted value of the profits of the Central Bank, $PDV_{t-1}(S_4)$ is equivalent to maximising the present discounted value of central bank revenues, $PDV_{t-1}(S_2)$ and of the present discounted value of seigniorage, $PDV_{t-1}(S_1)$.

We now turn to the consideration of the question as to whether the Central Bank has the financial resources to successfully pursue its inflation target.

IV. Is the inflation target independently financeable by the Central Bank?

I consider here whether and under what conditions an inflation target, π^*, is consistent with the Central Bank's intertemporal budget constraint. Consider a closed economy model of an endowment economy, whose Treasury and Central Bank can be represented by a simplified version of the accounting framework developed in the previous Section. There are no international reserves, $R_t^f = 0$, no Central Bank loans to the private sector, $L_t = 0$, and therefore no quasi-fiscal subsidies by the Central Bank, $Q_t = 0$.

The intertemporal budget constraint of the Treasury remains as in equation (30), those of the Central Bank, respectively the Government (the consolidated Central Bank and Treasury) are given below:

$$-D_{t-1} \le \sum_{j=t}^{\infty} E_t I_{j,t-1} \left(P_j \left(-C_j^b - T_j^b - H_j \right) + M_j - (1 + i_{j,j-1}^M) M_{j-1} \right)$$

and

$$B_{t-1} \le \sum_{j=t}^{\infty} E_t I_{j,t-1} \left(P_j \left(T_j - C_t^g - C_t^b \right) + M_j - (1 + i_{j,j-1}^M) M_{j-1} \right)$$

Let the ratio of the stock of domestic credit to GDP be $d_t = \frac{D_{t-1}}{P_t Y_t}$, and let

$c_t^b = \frac{C_t^b}{Y_t}$, $c_t^g = \frac{C_t^g}{Y_t}$, $\tau_t^b = \frac{T_t^b}{Y_t}$, $\tau_t^p = \frac{T_t^p}{Y_t}$, $h_t = \frac{H_t}{Y_t}$; $\tau_t = \frac{T_t}{Y_t} = \tau_t^p - h_t$

We can re-write the intertemporal budget constraints of the Treasury, the Central Bank and the consolidated Government as, respectively:

$$b_t + d_t \le E_t \sum_{j=t}^{\infty} R_{j,t-1} \Gamma_{j,t-1} (\tau_j^p + \tau_j^b - c_j^g) \tag{41}$$

$$-d_t \le \sum_{j=t}^{\infty} E_t R_{j,t-1} \Gamma_{j,t-1} \left(-c_j^b - \tau_j^b - h_j + (\mu_{j,j-1} - i_{j,j-1}^M) m_j \right) \tag{42}$$

$$b_t \le \sum_{j=t}^{\infty} E_t R_{j,t-1} \Gamma_{j,t-1} \left(\tau_j - c_t^g - c_t^b + (\mu_{j,j-1} - i_{j,j-1}^M) m_j \right). \tag{43}$$

The period budget constraint of the representative household is given in (44) and its solvency constraint in (45); C_t is real household consumption in period t, A_t is the nominal value of its non-monetary assets (inclusive of period t interest or similar payments): The nominal value of total household financial wealth is denoted W_t where

$$W_t \equiv A_t + (1 + i_{t,t-1}^M) M_{t-1}$$

$$E_t I_{t+1,t} W_{t+1} \equiv W_t - \left(\frac{i_{t+1,t} - i^M_{t+1lt}}{1+i_{t+1,t}} \right) M_t + P_t(Y_t - \mathrm{T}^p_t - C_t) \qquad (44)^{12}$$

$$C_t \geq 0$$

$$\lim_{N\to\infty} E_t I_{j+1,t} W_{j+1,t} \geq 0 \qquad (45)$$

Note that while the Central Bank does not, in its solvency constraint (29), view irredeemable base money as an effective liability, households do view base money as an asset in their solvency constraint. This asymmetry is the formal expression of the view that fiat money is an asset of the holder but not a liability of the issuer.

This implies the following intertemporal budget constraint for the household:

$$W_t \geq E_t \sum_{j=t}^{\infty} I_{j+1,t} \left[P_j(C_j + T_j - Y_j) + \left(\frac{i_{j+1,j} - i^M_{j+1lj}}{1+i_{j+1,j}} \right) M_j \right] \qquad (46)$$

The household optimizes the following utility function:

$$U_t = E_t \sum_{j=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{j-t} u(C_j, \frac{M_j}{P_{j+1}}; .) \qquad (47)$$

$$\delta > 0$$

where $u(C_t, \frac{M_t}{P_{t+1}}; .) = v\left(\frac{C_t}{Y_t} \right) + w\left(\frac{M_t}{P_{t+1} Y_{t+1}} \right)$ is twice continuously differentiable, increasing in consumption, increasing in real money balances for low values of the stock of real money balances, strictly concave and satisfies the Inada conditions for consumption. Preferences are assumed separable in consumption and real money balances and homothetic in consumption, real money balances and the exogenous level of real output, so as to permit the existence of a steady

---

[12] $E_t \left(I_{t+1,t} A_{t+1} \right) + M_t = E_t \left[I_{t+1,t} \left(A_{t+1} + (1+i^M_{t+1,t}) M_t \right) \right] + \left[1 - E_t \left(I_{t+1,t} (1+i^M_{t+1,t}) \right) \right] M_t$

$$= E_t \left[I_{t+1,t} \left(A_{t+1} + (1+i^M_{t+1,t}) M_t \right) \right] + \left(\frac{i_{t+1,t} - i^M_{t+1,t}}{1+i_{t+1,t}} \right) M_t$$

state with non-zero real growth. Let $c_t = C_t / Y_t$. For expositional simplicity I will use the following parametric example: $v(c_t) = \ln(c_t)$ and $w(m_{t+1}) = (m_{t+1} - \theta)\ln(\theta - m_{t+1}) - m_{t+1}$; $\theta > m_{t+1} + 1$. These yield a money demand function close to the textbook semi-logarithmic one (I assume that the value of the parameter θ is sufficiently large to ensure an interior solution for the stock of real money balances, in the range where the marginal utility of real money balances is positive).[13] The household's intertemporal budget constraint (46) will hold with equality and the interior optimality conditions are:

$$u_m(c_t, m_{t+1}) = w'(m_{t+1}) = \left(\frac{i_{t+1,t} - i^M_{t+1,t}}{1 + i_{t+1,t}}\right) u_c(c_t, m_{t+1}) = \left(\frac{i_{t+1,t} - i^M_{t+1,t}}{1 + i_{t+1,t}}\right) v'(c_t) \tag{48}$$

$$\frac{1}{1+\delta} E_t\left(R_{t+1,t} \frac{u_c(c_{t+1}, m_{t+2})}{u_c(c_t, m_{t+1})}\right) = \frac{1}{1+\delta} E_t\left(R_{t+1,t} \frac{v'(c_{t+1})}{v'(c_t)}\right) = 1 \tag{49}$$

For the specific functional forms chosen for the sub-utility functions for consumption and real money balances, (48) and (49) become:

$$\ln(\theta - m_{t+1}) = \left(\frac{i_{t+1,t} - i^M_{t+1,t}}{1 + i_{t+1,t}}\right) c_t^{-1} \tag{50}$$

$$\frac{1}{1+\delta} E_t\left(R_{t+1,t} \frac{c_t}{c_{t+1}}\right) = 1 \tag{51}$$

Output is demand-determined, so

$$1 = c_t + c_t^g + c_t^b \tag{52}$$

Financial asset market equilibrium requires that

[13] In discrete time money-in-the-utility function models, a choice has to be made as to whether the end-of-period stock of nominal money balances is to be deflated by this period's price level (the backward looking opportunity cost approach, M_t / P_t) or next period's price level, when these money balances will actually available (the forward-looking purchasing power approach, M_t / P_{t+1}). Little of substance depends on this choice, but the algebra is a beat neater with the forward-looking approach, which is adopted in this paper.

$$A_t = B_t \tag{53}$$[14]

Pricing behaviour is given by the slightly modified New-Keynesian Phillips curve in (54)

$$\begin{aligned} &\pi_{t,t-1} - \omega_{t,t-1} = \varphi E_{t-1}(Y_t - Y_t^*) + \frac{1}{1+\delta} E_{t-1}(\pi_{t+1,t} - \omega_{t+1,t}) \\ &\varphi > 0 \end{aligned} \tag{54}$$

Here $Y_t^* > C_t^g + C_t^b$ is the exogenously given level of capacity output or potential output. Its proportional growth rate is denoted $\gamma_{t,t-1}^* = \frac{Y_t^*}{Y_{t-1}^*} - 1$.

The Phillips curve in (54) combines Calvo's model of staggered overlapping nominal contracts (see Calvo (1983) and Woodford (2003)) with the assumption that even those price setters who are free to set their prices have to do so one period in advance.[15] The current price level, P_t is therefore predetermined. The variable $\omega_{t,t-1}$ is the inflation rate chosen in period *t-1* for period *t* by those price setters that follow a simple behavioural rule or heuristic. In the original Calvo (1983) model, $\omega_{t,t-1} = 0$. I will assume that the period *t* inflation heuristic is the future expected deterministic steady state rate of inflation of the model expected at time *t-1*:

$$\omega_{t,t-1} = E_{t-1}\bar{\pi} \tag{55}$$

Thus, while the price level in period *t*, P_t, is predetermined, the rate of inflation in period *t*, $\pi_{t+1,t}$ and in later periods in flexible. It is therefore possible to achieve an immediate transition

[14] The household solvency constraint (45) and the consolidated Government solvency constraint government intertemporal budget constraint (34) (with $R_j^f = 0$ for the closed economy special case) together with $A_t = B_t$ and $W_t = A_t + (1 + i_{t,t-1}^M)M_{t-1}$ imply that $E_t I_{j+1,t} M_{j+1} \geq 0$, which, when holding with equality, was the assumption made to obtain the version of the ISI given in (13).

[15] Without the assumption that the optimising price setters have to set prices one period in advance, the Phillips curve would be $\pi_{t,t-1} - \omega_{t,t-1} = \varphi(Y_t - \bar{Y}_t) + \frac{1}{1+\delta} E_t(\pi_{t+1,t} - \omega_{t+1,t})$. Although prices would not be fully flexible, unless $\pi_{t,t-1} = \omega_{t,t-1}$ for all *t,* there can be some response of the period *t* price level to events and news in period *t*.

to a different rate of inflation without any effect on real output, provided the change in monetary policy is unexpected, immediate and permanent.

Economic decisions are made and equilibrium is established for periods $t \geq 1$. Initial financial asset stocks, M_0, D_0 and M_0, B_0, D_0 are given. Central Bank instruments are $i^M_{t,t-1}$, h_t, c^b_t and $\mu_{t,t-1}$.[16] Fiscal policy instruments are c^g_t, τ^b_t and τ^p_t.

It is clear that in the model developed here, as in any model with a predetermined price level, Corollary 2 holds: maximising the present discounted value of current and future real seigniorage is equivalent to maximising the present discounted value of future real Central Bank revenues.

In the Neo-Keynesian model, the actual level of current output is demand-determined and can therefore be influenced by past, present and anticipated future policy. In what follows I will consider the deterministic special case of the model developed here. All exogenous variables and policy instruments are constant. In period 0 the system starts off in a deterministic steady state. Then, in period, $t = 1$, the monetary authorities announce a constant growth rate for the nominal money stock, $\mu_{t+1,t} = \mu$, which they will adhere to forever afterwards. If this growth rate for the nominal money stock is different from the growth rate of the nominal money stock that supported the original deterministic steady state, the announcement is unexpected but fully credible. For this policy experiment to support an immediate transition to the new steady state, despite the predetermined price level, the nominal money stock held at the end of period 1 (the beginning of period *2*) has to be set at the level that supports monetary equilibrium in period 1 with the new steady-state stock of real money balances. This will, in general require a growth rate of the

[16] It would be more descriptively realistic to make $i_{t,t-1}$ a monetary policy instrument rather than $\mu_{t,t-1}$. None of the results of this paper depend on this choice of monetary policy instrument and for expositional simplicity an exogenous growth rate of the nominal money stock is best here.

nominal money stock in period 1, $\mu_{1,0}$ that is different from the subsequent steady state growth rate of the nominal money stock μ. This would certainly be the case if the demand for real money balances in period t were to be defined in terms of M_t / P_t. It may also be required when instead, as in the present paper, it is defined in terms of M_t / P_{t+1}.

The stationary equilibrium is characterised by the following conditions for $t \geq 1$:

$$c_t = 1 - c^g - c^b \tag{56}$$

equilibrium:

$$r_{t+1,t} = \rho \tag{57}$$

$$1 + \pi_{t+1,t} = \frac{1+\mu}{1+\gamma^*} \tag{58}$$

$$1 + i_{t+1,t} = (1+\rho)(1+\pi_{t+1,t}) \tag{59}$$

$$v'(m_{t+1}) = \left(\frac{i_{t+1,t} - i^M}{1 + i_{t+1,t}} \right) v'(c_t)$$

or (60)

$$m_{t+1} = \theta - e^{\left(\frac{i_{t+1,t} - i^M}{1+i_{t+1,t}} \right) c_t^{-1}}$$

$$Y_t = Y_t^* \tag{61}$$

$$\omega_{t+1,t} = \pi_{t+1,t} = \bar{\pi} \tag{62}$$

I am only considering equilibria where $i \geq i^M$ and $\rho > \gamma^*$.

I want to consider which constant rate(s) of inflation, $\pi = \bar{\pi}$, this economy can support, with a Central Bank whose intertemporal budget constraint is given by equation (42). With the economy in steady state from period *1,* it follows that the Central Bank's intertemporal budget constraint can be rewritten as follows:

$$-d_t + \left(\frac{1+\gamma^*}{\rho - \gamma^*} \right) \left(c^b + \tau^b + h \right) \leq \left(\frac{1+\gamma^*}{\rho - \gamma^*} \right) \left(\mu - i^M \right) m = \bar{\sigma}_1(\pi) \tag{63}$$

where

$$m = \ell(\pi; c, \rho, i^M);\ \ell_\pi = \frac{v'(c)}{w''(m)}\left(\frac{1+i^M}{(1+\rho)(1+\pi)^2}\right) < 0;\ \ell_c = \left(\frac{i-i^M}{1+i}\right)\frac{v''(c)}{w''(m)} > 0 \tag{64}$$

For the specific functional form $m = \theta - e^{\left(\frac{(1+\rho)(1+\pi)-(1+i^M)}{(1+\rho)(1+\pi)c}\right)}$, we have

$$\frac{d\bar{\sigma}_1}{d\pi} = \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left[(1+\gamma^*)m - \left((1+\pi)(1+\gamma^*) - (1+i^M)\right)(\theta - m)\left(\frac{1+i^M}{(1+\rho)(1+\pi)^2 c}\right)\right]$$

Consider the case where the nominal interest rate on base money is zero, so

$$\frac{d\bar{\sigma}_1}{d\pi} = \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left[(1+\gamma^*)\left(\theta - e^{\left(\frac{(1+\rho)(1+\pi)-1}{(1+\rho)(1+\pi)c}\right)}\right) - e^{\left(\frac{(1+\rho)(1+\pi)-1}{(1+\rho)(1+\pi)c}\right)}\left(\frac{(1+\pi)(1+\gamma^*)-1}{(1+\rho)(1+\pi)^2 c}\right)\right] \tag{65}$$

Assume both the long-run nominal interest rate and the long-run growth rate of nominal GDP are non-negative. Then $\frac{d\bar{\sigma}_1}{d\pi} > 0$ when $\pi = 0$ provided the demand for real money balances is sufficiently large at a zero rate of inflation. A sufficiently large value of $c = 1 - c^b + c^g$, steady state consumption as a share of GDP, will ensure that. I assume this condition is satisfied. The long-run seigniorage Laffer curve has a single peak at

$$m = \hat{m}_1 = \frac{\left((1+\hat{\pi}_1)(1+\gamma^*) - (1+i^M)\right)(1+i^M)\theta}{\left((1+\rho)(1+\hat{\pi}_1)^2 c\right)(1+\gamma^*) + \left((1+\hat{\pi}_1)(1+\gamma^*) - (1+i^M)\right)(1+i^M)}$$

where

$$\hat{\pi}_1 = \arg\max \bar{\sigma}_1$$

Let $\pi^b_{\min}$ be the lowest constant inflation rate that is consistent with the Central Bank's intertemporal budget constraint, given in (63), for given values of $d_t, c^b \geq 0, \tau^b$ and $h \geq 0$.[17] If there is a long-run Seigniorage Laffer curve, $\pi^b_{\min}$ may not exist: there may be no constant inflation rate that

[17] That is, $\pi^b_{\min}$ is the lowest value of π that solves $-d_t + \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(c^b + \tau^b + h\right) = \bar{\sigma}_1(\pi)$.

would generate enough real seigniorage to satisfy (63). If the value of the inflation target, π^*, is less than $\pi^b_{\min}$, then the Central Bank cannot achieve the inflation target, because doing so would bankrupt it. The most it could do would be to set both c^b and h equal to zero: there would be no Central Bank-initiated helicopter drops of money and Central Bank staff would not get paid. If that is not enough to cause the weak inequality in (63) to be satisfied with $\pi = \bar{\pi} = \pi^*$, I will call this a situation where the inflation target is not *independently financeable* by the Central Bank. The value of the Central Bank's holdings of Treasury debt, d_t, is determined by history; the net tax paid by the Central Bank to the Treasury, τ^b is determined unilaterally by the Treasury. I summarise this as follows:

Proposition 3:

If either $\pi^b_{\min}$ does not exist or $\pi^* < \pi^b_{\min}$, the inflation target is not independently financeable by the Central Bank.

If the Treasury decides to support the Central Bank in the pursuit of the inflation objective, the inflation target is *jointly financeable* by the Central Bank and the Treasury, as long as the consolidated intertemporal budget constraint of the Treasury and the Central Bank can be satisfied with the seigniorage revenue generated by the implementation of the inflation target. The intertemporal budget constraint of the Treasury and of the consolidated Government for this simple economy are given by, respectively:

$$b_t + d_t \le \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(\tau^p + \tau^b - c^g\right) \tag{66}$$

$$b_t + \left(\frac{1+\gamma^*}{\rho-\gamma^*}\right)\left(c^g + c^b - \tau\right) \le \sigma_1(\pi) \tag{67}$$

Let $\pi^g_{\min}$ be the lowest constant inflation rate that is consistent with the intertemporal budget constraint of the consolidated Government, given in (67), for given values of

$b_t, c^g \geq 0, c^b \geq 0$ and τ. Again, $\pi^g_{\min}$ could either not exist or could exceed the inflation target π^*. This suggests the following:

Proposition 4:

If either $\pi^g_{\min}$ does not exist or if $\pi^* < \pi^g_{\min}$, the inflation target is not financeable, even with cooperation between Treasury and Central Bank. The inflation target in that case is not feasible.

If (67) is satisfied with $\pi = \pi^*$, the inflation target is financeable by the consolidated Treasury and Central Bank – that is, the inflation target is feasible with cooperation between Treasury and Central Bank. It may of course (if (63) is satisfied as well as (67)), also be independently financeable by the Central Bank. Note that the feasibility condition for the inflation target, equation (67), is independent of τ^b (which is a transfer payment within the consolidated Treasury and Central Bank) and of d_t which is an internal liability/asset within the consolidated Treasury and Central Bank. What matters is the net debt of the consolidated Treasury and Central Bank, b_t, and the taxes net of transfers of the consolidated Treasury and Central Bank, τ. If the feasibility condition (67) is satisfied, the Treasury can always provide the Central Bank with the resources it requires to implement the inflation target. All it has to do is reduce taxes on the Central Bank (or increase transfer payments to the Central Bank), in an amount sufficient to ensure that equation (63) is also satisfied.[18]

If (67) is satisfied with $\pi = \pi^*$, but (63) is not, then the inflation target is only financeable by the Treasury and Central Bank jointly, not independently by the Central Bank. Note that this can only happen if the Treasury has 'surplus' resources, that is, (66) holds as a strict inequality. In that case, a reduction in τ^b can permit the Central Bank's intertemporal budget constraint (63)

[18] This could be achieved through a one-off capital transfer rather than through a sequence of current transfers.

to be satisfied without violating the Treasury's intertemporal budget constraint (67). I summarise this as follows:

Proposition 5:

If $\pi^b_{\min} < \pi^* < \pi^g_{\min}$, the inflation target is only cooperatively financeable by the Central Bank and the Treasury jointly.

This discussion provides an argument in support of the view that the Central Bank should not have *operational target independence* (freedom to choose a quantitative inflation target) even when it has *operational independence* (the freedom to set the monetary instrument (typically a short nominal interest rate, but in this paper the growth rate of the nominal stock of base money, as it sees fit). The reason is that if the political authorities choose the operational target, there is less of a risk of 'mandating without funding'. On its own, the Central Bank cannot be guaranteed to have the right degree of financial independence. Without Treasury support, there can be no guarantee that the minimal amount of seigniorage required to ensure the solvency of the Central Bank can support the inflation target. If $\pi^b_{\min} < \pi^* < \pi^g_{\min}$, only the Treasury can make sure that the Central Bank has enough resources to make the inflation target financeable by the Central Bank. The Treasury, through its ability to tax the Central Bank, is effectively constrained only by the consolidated intertemporal budget constrained in (67), even though formally it faces the intertemporal budget constraint given in equation (66).

V. Other aspects of necessary co-operation and co-ordination between Central Bank and Treasury

Even if the Treasury supports the Central Bank's inflation target and provides it with the financial resources to implement it, there are at least two other economic contingencies for which active Central Bank and Treasury co-ordination and co-operation is desirable.

V.1 Recapitalizing the central bank

The first case occurs when the (threat of) a serious banking crisis or financial crisis with systemic implications forces the Central Bank to act as a lender of last resort, and the problem turns out to be (or becomes), for a significant portion of the banking/financial system, a solvency crisis as well as a liquidity crisis. It could happen that recapitalising the insolvent banks or financial institutions with only the financial resources of the Central Bank (including a given sequence of net payments to the treasury, T^b) would require the Central Bank to engage in excessive base money issuance, which would result in unacceptable rates of inflation. As long as the resources of the consolidated Treasury and Central Bank are sufficient, the Treasury could either recapitalise the Central Bank (if the Central Bank recapitalised the private banking/financial system in the first instance), or the Treasury could directly recapitalise the banking/financial system. In the accounts set out above, recapitalising the Central Bank would amount to one or more large negative realisations of T^b, with as counterparts an increase in Central Bank holdings of Treasury debt, D (see Ize (2005)).

Special problems occur when the insolvency of (part of) the financial system is due to an excess of foreign-currency liabilities over foreign-currency assets. In that case the Treasury, in order to recapitalise the Central Bank (or some other part of the financial sector directly), has to be able to engineer both an internal fiscal transfer and an external transfer of resources of the required magnitude. If the external credit of the state is undermined, this may only be possible gradually, if and as the state can lay claim to (part of) the current and future external primary surpluses of the nation.

In the usual nation state setting, a single Treasury or national fiscal authority stands behind a single central bank. Unique complications arise in the EMU, where each national fiscal

authority stands financially behind its own national central bank (NCB), but no fiscal authority stands directly behind the ECB. The lender of last resort function in the EMU is assigned to the NCB members of the ESCB (see Padoa-Schioppa (2004) and Goodhart (2002)). This will work fine when a troubled or failing bank or other financial institution deemed to be of systemic importance has a clear nationality, as most Eurozone-domiciled banks and other financial institutions do today. Likewise, banks that are subsidiaries of institutions domiciled outside the EMU will be the responsibility of their respective national Central Banks and of the national fiscal authority that stands behind each of these Central Banks.

Trouble arises as and when Eurozone-domiciled banks emerge that do not have a clear national identity, say banks incorporated solely under European Law. As there is no fiscal authority, national or supranational, standing behind the ECB, who would organise and fund the bail-out and recapitalisation of such a 'European bank'? Whether this potential vulnerability will in due course be remedied by the creation of a serious supra-national fiscal authority at the EMU level that would stand behind the ECB, or by implicit or explicit agreements between the ECB, the NCBs (the shareholders of the ECB) and the national fiscal authorities is as yet unclear.

V.2 Helicopter drops of money

The second set of circumstances when cooperation and coordination between the monetary and fiscal authorities is essential is when an economy is confronting the need to avoid unwanted deflation or, having succumbed to it, to escape from it. In principle, the potential benefits from cooperation between the monetary and fiscal authority apply to stabilisation policy in general, that is to counter-inflationary as well as to counter-deflationary policies. The issue is particularly

urgent, however, when deflation is the enemy and conventional monetary policy has run out of steam.

Faced with deflation, the Central Bank on its own can cut the short nominal interest rate - the primary monetary policy instrument in most economies with a floating exchange rate. It can engage in sterilised foreign exchange market operations. If there are reserve requirements imposed on commercial banks or other financial institutions, these can be relaxed, as can the collateral standards in Repos and the eligibility requirements that must be met by potential counterparties.

Once the short nominal interest rate is at the zero floor, conventional monetary policy is effectively exhausted. The Central Bank can then engage in generalised open market purchases, monetising the outstanding stock of non-monetary public debt, of all maturities, nominally denominated or index-linked, held outside the Central Bank. Once all outstanding public debt has been absorbed by the Central Bank, it could turn its attention to the purchase and monetisation of private securities, from foreign currency-denominated securities, to stocks and shares, land, property or contingent claims. Clearly, such socialisation of private wealth would be subject to all kinds of moral hazard, adverse selection and governance problems.

Should this too fail to boost aggregate demand and end deflation, the monetary authority on its own has one remaining exotic instrument and the combined monetary and fiscal authorities have one conventional but truly effective instrument. The unconventional instrument is to lower the zero floor on nominal interest rates (which is a result of the zero nominal interest rate paid on currency and often on all base money), by paying a negative nominal interest rate on base money. For commercial banks' reserves with the central bank, paying a negative nominal interest rate is technically and administratively trivial. Imposing a 'carry tax' on currency (setting a negative

value for i^M) is administratively cumbersome and intrusive, but not impossible. Silvio Gesell (1916) recommended it many years ago, and as great an economist as Irving Fisher (1933) thought the proposal had merit (see also Goodfriend (2000) and Buiter and Panigirtzoglou (2001, 2003)).

There is, however, a very conventional policy alternative. Milton Friedman referred to it as (base) money dropped from a helicopter (Friedman (1968)). If the recipients of this largesse do not expect it to reversed (in present discounted value terms) in the future, this would, at a given price level, represent an increase in the real net wealth of the private sector (see Buiter (2003)). Because base money does not have to be redeemed *ever*, it does not constitute an effective liability of the state. The increase in net private wealth is also in the most liquid form possible.[19]

An example of a helicopter drop, in the UK context, would be for the Governor of the Bank of England issue a £1,000 cheque, drawn upon the Bank of England, to every man, woman and child in the country. On the balance sheet of the Bank this would show up as an increase in the stock of base money and a corresponding reduction in the financial net worth of the Bank. In its budget constraint it would be a one-off transfer payment to the private sector (*h* in our notation).

Would it work? If the money rain is not expected to be reversed in present value, it surely would. It does not rely on the strength of the intertemporal substitution effect in private consumption or on the interest sensitivity of private investment demand. All that it requires is that aggregate consumption today is a normal good. If the wealth effect is weak and the

[19] In this case, the boundary condition $E_t I_{j+1,t} M_{j+1} \geq 0$ would hold as a strict inequality.

£1,000.00 cheque does not do the job, the Governor can add zeros in front of the decimal point on the cheque until the private consumer surrenders and goes out and spends.

Even if the economic mechanism of the helicopter drop of money is straightforward, its practical implementation cannot be done by the Central Bank alone. The reason is that in reality central banks do not have an instrument like *H* in their arsenals. Making transfer payments to the private sector is not something Central Banks are legally permitted to do, because they are not fiscal agents of the state. So the economically equivalent action has to be coordinated between the Treasury and the Central Bank. The treasury will implement a tax cut or increase in transfer payments (a cut in T^p) and will finance this by selling debt to the Central Bank (increasing *D*). The acquisition of Treasury debt by the central bank is financed through the issuance of base money, an increase in *M*.

VI. Conclusion

Governments through the ages have appropriated real resources through the monopoly of the 'coinage'. In modern fiat money economies, the monopoly of the issue of legal tender is generally assigned to an agency of the state, the Central Bank, which may have varying degrees of operational and target independence from the government of the day.

In this paper I analyse four different but related concepts, each of which highlights some aspect of the way in which the state acquires command over real resources through its ability to issue fiat money. They are (1) seigniorage (the change in the monetary base), (2) Central Bank revenue (the interest bill saved by the authorities on the outstanding stock of base money liabilities), (3) the inflation tax (the reduction in the real value of the stock of base money due to

inflation and (4) the operating profits of the central bank, or the taxes paid by the Central Bank to the Treasury.

To understand the relationship between these four concepts, an explicitly intertemporal approach is required, which focuses on the present discounted value of the current and future resource transfers between the private sector and the state. Furthermore, when the Central Bank is operationally independent, it is essential to decompose the familiar consolidated 'government budget constraint' and consolidated 'government intertemporal budget constraint' into the separate accounts and budget constraints of the Central Bank and the Treasury. Only by doing this can we appreciate the financial constraints on the Central Bank's ability to pursue and achieve an inflation target, and the importance of cooperation and coordination between the Treasury and the Central Bank when faced with financial sector crises involving the need for long-term recapitalisation or when confronted with the need to mimick Milton Friedman's helicopter drop of money in an economy faced with a liquidity trap.

References

Anand, Ritu and Sweder van Wijnbergen (1989), "Inflation and the Financing of Government Expenditure: an Introductory Analysis with an Application to Turkey", *World Bank Economic Review*, Vol. 3, No. 1, pp. 17-38. *Further information in IDEAS/RePEc*

Blinder, Alan S. and Robert M. Solow (1973), "Does fiscal policy matter?", *Journal of Public Economics*, 2, pp. 319-337. *Further information in IDEAS/RePEc*

Bordo, Michael (2006), "Comment on Marc Flandreau, 'Pillars of Globalization: A History of Monetary Policy Targets, 1797-1997'", Prepared for the Fourth ECB, Monetary Policy conference, Frankfurt November 9-10 2006.
http://www.ecb.int/events/pdf/conferences/cbc4/Discussion_Bordo.pdf

Bresciani-Turroni, Constantino (1937), *The Economics of Inflation: A Study of Currency Depreciation in Post-War Germany*. London, Allen and Unwin. *Further information*

Buiter, Willem H. (1990), Principles of Budgetary and Financial Theory, MIT Press, Cambridge, Massachusetts. *Further Information*

Buiter, Willem H. (2003), "Helicopter Money: Irredeemable Fiat Money and the Liquidity Trap", NBER Working Paper No. W10163, December. *Further information in IDEAS/RePEc*

Buiter, Willem H. (2004), "Two naked emperors? Concerns about the Stability and Growth Pact and second thoughts about Central Bank independence", *Fiscal Studies*, Vol. 25(3), pp. 249-77.
Further information in IDEAS/RePEc

Buiter, Willem H. (2005), "New Developments in Monetary Economics: two ghosts, two eccentricities, a fallacy, a mirage and a mythos", Royal Economic Society 2004 Hahn Lecture, *The Economic Journal*, Conference Papers, Vol. 115, No. 502, March 2005, pp. C1-C31.
Further information in IDEAS/RePEc

Buiter, Willem H. and Nikolaos Panigirtzoglou (2001), "Liquidity Traps: How to Avoid Them and How to Escape Them", with Nikolaos Panigirtzoglou, in *Reflections on Economics and Econometrics, Essays in Honour of Martin Fase*, edited by Wim F.V. Vanthoor and Joke Mooij, 2001, pp. 13-58, De Nederlandsche Bank NV, Amsterdam.
Further information in IDEAS/RePEc

Buiter, Willem H. and Nikolaos Panigirtzoglou (2003), "Overcoming the Zero Bound on Nominal Interest Rates with Negative Interest on Currency: Gesell's Solution", *Economic Journal*, Volume 113, Issue 490, October 2003, pp. 723-746.
Further information in IDEAS/RePEc

Cagan, Philip (1956), "Monetary Dynamics of Hyperinflation", in Milton Friedman, Editor, *Studies in the Quantity Theory of Money*, University of Chicago Press, Chicago, Illinois.

Calvo, Guillermo (1983), "Staggered Contracts in a Utility-Maximizing Framework", *Journal of Monetary Economics*, September.

Christ, Carl (1968), "A Simple Macroeconomic Model with a Government Budget Restraint," *Journal of Political Economy* 76, No. 1, January/February, pp. 53-67.

Dornbusch, Rudiger, and Staley Fischer (1986), "Stopping Hyperinflations Past and Present", *Weltwirtschaftliches Archiv* 122, pp. 1-47. *Further information in IDEAS/RePEc*

Easterly, William R., Paolo Mauro, Klaus Schmidt-Hebbel (1995), **"**Money Demand and Seigniorage-Maximizing Inflation", *Journal of Money, Credit and Banking***,** Vol. 27, No. 2 (May), pp. 583-603. *Further information in IDEAS/RePEc*

Fisher, Irving (1933), *Stamp Scrip*, Adelphi Company, New York.
Further information

Flandreau, Marc (2006), "Pillars of Globalization: A history of monetary policy targets, 1797-1997", Revised Draft : November 2, 2006, *Paper prepared for the Fourth ECB Central Banking Conference,* Frankfurt-Am-Main, November 9-10 2006. *Further information*

Friedman, Milton (1968), "The Role of Monetary Policy." *American Economic Review* 58, no. 1, pp. 1-17.

Friedman, M.(1971), "Government Revenue from inflation", *Journal of Political Economy*, vol. 79, N° 4, pp. 846-56. *Further information in IDEAS/RePEc*

Gesell, Silvio (1916, 1949), *Die Natuerliche Wirtschaftsordnung*, Rudolf Zitzman Verlag, available in English as *The Natural Economic Order*, Peter Owen Ltd, London, 1958.
Further information

Goodfriend, Marvin (2000), "Overcoming the Zero Bound on Interest Rate Policy, *Journal of Money, Credit and Banking*, Vol. 32, No. 4, Pt. 2, November, pp. 1007-1035.
Further information in IDEAS/RePEc

Goodhart, Charles (2002), "Recent Developments in Central Banking." In *Monetary Policy, Capital Flows and Exchange Rates: Essays in Honour of Maxwell Fry*. Edited by Fry, M. J.; Dickinson, D. G.; Allen, B. Routledge,
Further information

Ize, Alain (2005), "Capitalising Central Banks: A Net Worth Approach", IMF Working Paper, WP/05/15 January 2005 *Further information in IDEAS/RePEc*

Kiguel, Miguel A. and Pablo A. Neumeyer (1995), "Seigniorage and Inflation: The Case of Argentina", *Journal of Money, Credit and Banking*, Vol. 27, No. 3 (Aug.), pp. 672-682.
Further information in IDEAS/RePEc

King, Robert G. and C. I. Plossser (1985), "Money, Deficits and Inflation", *Carnegie-Rochester Conference Series on Public Policy*, 22, Spring, pp. 147-196.

Padoa-Schioppa, Tommaso (2004). The Euro and Its Central Bank, The MIT Press, Cambridge, Massachusetts. *Further information*

Phelps, Edmund (1973), Inflation in the Theory of Public Finance", *Swedish Journal of Economics*, 75, March, pp. 67-82.

Romer, David (2006), *Advanced Macroeconomics*, Third Edition, Chapter 10. McGraw-Hill, New York. *Further information*

Sargent, Thomas J. (1982), "The End of Four Big Inflations", in Robert E. Hall, ed., *Inflation*, pp. 41-98, University of Chicago Press, Chicago Illinois. *Further information in IDEAS/RePEc*

Sargent, Thomas J. (1987), *Dynamic Macroeconomic Theory*, Harvard University Press, Cambridge, Massachusetts. *Further information*

Sargent, Thomas J. and Neil Wallace (1981), "Some unpleasant monetarist arithmetic", Federal Reserve Bank of Minneapolis Quarterly Review, 5(3), pp. 1-17.
Further information in IDEAS/RePEc

Sims, C. A. (2004): "Fiscal Aspects of Central Bank Independence," Chapter 4, p.103-116, in *European Monetary Integration*, Hans-Werner Sinn, Mika Widgrén, and Marko Köthenbürger, editors, MIT Press. *Further information in IDEAS/RePEc*

Sims, C. A. (2005), "Limits to Inflation Targeting", Chapter 7 in *The Inflation-Targeting Debate*, Ben S. Bernanke and Michael Woodford, editors, NBER Studies in Business Cycles Volume32, p. 283-310. *Further information*

Thornton, H. (1802), *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain.*

Tobin, James and Willem H. Buiter (1976), " Long-run effects of fiscal and monetary policy on aggregate demand", in J. Stein ed. *Monetarism*, North Holland, Amsterdam, pp. 273-309.
Further information in IDEAS/RePEc

Walsh, Carl E. (2003), Monetary Theory and Policy, 2nd Edition, Chapter 4, pp. 135-197, The MIT Press, Cambridge Massachusetts. *Further information*

Woodford, Michael (2003), Interest & Prices; Foundations of a Theory of Monetary Policy, Princeton University Press, Princeton and Oxford. *Further information*