Long, Ngo Van; Raff, Horst; Stähler, Frank

Working Paper
Innovation and Trade with Heterogeneous Firms

Kiel Working Paper, No. 1430

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Long, Ngo Van; Raff, Horst; Stähler, Frank (2008) : Innovation and Trade with Heterogeneous Firms, Kiel Working Paper, No. 1430

This Version is available at:
http://hdl.handle.net/10419/17893

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Innovation and Trade with Heterogeneous Firms
by Ngo Van Long, Horst Raff and Frank Stähler

No. 1430 | June 2008
Innovation and Trade with Heterogeneous Firms*

Ngo Van Long, Horst Raff and Frank Stähler

Abstract: This paper examines how trade liberalization affects the innovation incentives of firms, and what this implies for industry productivity and social welfare. For this purpose we develop a reciprocal dumping model of international trade with heterogeneous firms and endogenous R&D. We identify two effects of trade liberalization on productivity: a direct effect through changes in R&D investment, and a selection effect due to inefficient firms leaving the market. We show how these effects operate in the short run when market structure is fixed, and in the long run when market structure is endogenous. Among the robust results that hold for any market structure are that trade liberalization (i) increases (decreases) aggregate R&D for low (high) trade costs; (ii) increases expected industry productivity; and (iii) raises expected social welfare if trade costs are low.

Keywords: international trade, firm heterogeneity, R&D, productivity, market structure

JEL classification: F12, F15

Ngo Van Long
Department of Economics,
McGill University,
Montreal H3A 2T7, Canada
E-mail: ngo.long@mcgill.ca

Horst Raff
University of Kiel and
Kiel Institute for the World Economy
Department of Economics
University of Kiel
Wilhelm-Seelig-Platz 1
D-24098 Kiel, Germany
E-mail: raff@econ-theory.uni-kiel.de

Frank Stähler
Department of Economics
University of Otago
PO Box 56
Dunedin 9001, New Zealand
E-mail: fstaehler@business.otago.ac.nz

*We would like to thank Jonathan Eaton, Julien Prat, Nicolas Schmitt and participants at ERWIT, CESifo Venice Summer Institute, the Midwest International Economics Meetings, the German Economic Association and seminar audiences at the Universities of Bamberg, Otago and Tübingen for helpful comments and suggestions.

The responsibility for the contents of the working papers rests with the author, not the Institute. Since working papers are of a preliminary nature, it may be useful to contact the author of a particular working paper about results or caveats before referring to, or quoting, a paper. Any comments on working papers should be sent directly to the author.

Coverphoto: uni_com on photocase.com
Innovation and Trade with Heterogeneous Firms\textsuperscript{1}

Ngo Van Long\textsuperscript{2}  
Horst Raff\textsuperscript{3}  
Frank Stähler\textsuperscript{4}  

25th June 2008

\textsuperscript{1}We would like to thank Jonathan Eaton, Julien Prat, Nicolas Schmitt and participants at ERWIT, CESifo Venice Summer Institute, the Midwest International Economics Meetings, the German Economic Association and seminar audiences at the Universities of Bamberg, Otago and Tübingen for helpful comments and suggestions.

\textsuperscript{2}Department of Economics, McGill University, Montreal H3A 2T7, Canada, email: ngo.long@mcgill.ca

\textsuperscript{3}University of Kiel and Kiel Institute for the World Economy. Address: Department of Economics, University of Kiel, Wilhelm-Seelig-Platz 1, D–24098 Kiel, Germany, email: raff@econ-theory.uni-kiel.de

\textsuperscript{4}Department of Economics, University of Otago, PO Box 56, Dunedin 9001, New Zealand, email: fstaehler@business.otago.ac.nz
Abstract

This paper examines how trade liberalization affects the innovation incentives of firms, and what this implies for industry productivity and social welfare. For this purpose we develop a reciprocal dumping model of international trade with heterogeneous firms and endogenous R&D. We identify two effects of trade liberalization on productivity: a direct effect through changes in R&D investment, and a selection effect due to inefficient firms leaving the market. We show how these effects operate in the short run when market structure is fixed, and in the long run when market structure is endogenous. Among the robust results that hold for any market structure are that trade liberalization (i) increases (decreases) aggregate R&D for low (high) trade costs; (ii) increases expected industry productivity; and (iii) raises expected social welfare if trade costs are low.

JEL classification: F12, F15
Keywords: international trade, firm heterogeneity, R&D, productivity, market structure
1 Introduction

This paper examines how trade liberalization affects the incentive of firms to innovate. Specifically we study how a reduction in trade barriers affects firms’ investment in process R&D, and what this implies for industry productivity and social welfare. Process R&D refers to investment designed to reduce production costs, thereby making the firm more productive. A key feature of process R&D is that its outcome is stochastic. Higher R&D spending only raises the likelihood that the firm will realize a higher level of productivity. However, it is the realized level of productivity that determines the firm’s performance, including its domestic sales, export sales and profitability. Only productive firms will be able to survive in the market-place, and only the most productive will be able to bear the cost of exporting. Hence the type of R&D decision we focus on is one where firms choose their investment level with a view to boosting their chance of success in both domestic and export markets.

Innovation incentives depend on such factors as market size, toughness of existing competition, and the potential for entry and exit of competitors. Trade liberalization affects all of these factors simultaneously. Firms face tougher import competition at home and may lose market share to imports, which may reduce the benefit of undertaking R&D. On the other hand, they gain easier access to export markets and hence may gain market share abroad. This may lead firms to raise their R&D spending. Trade liberalization may also affect market structure, thus changing the number not only of foreign but also of domestic competitors. Obviously, then, trade policy has non-trivial effects on R&D incentives. Disentangling these effects is the first task of the paper.

Changes in R&D investment represent one channel through which trade policy affects industry productivity and social welfare. Another is the se-

---

1 See, for example, the seminal paper by Dasgupta and Stiglitz (1980), as well as the more recent work by Aghion et al. (2004, 2005).
lection of firms into domestic and export markets.\textsuperscript{2} By this we mean that trade liberalization may force the least efficient firms to exit the market, but provides export opportunities to firms that before found exporting too costly. However, since R&D investment as well as domestic and export market participation are endogenous, and since all of these decisions are directly affected by trade liberalization, the R&D effect and the selection effect will interact to determine industry productivity and social welfare. Examining this interaction is the second task.

The current paper makes progress on both tasks by providing a very simple international trade model in which these effects of trade liberalization can be studied. Our model is a variant of the reciprocal dumping model (Brander and Krugman, 1983), in which firms are ex post heterogeneous a la Melitz (2003). Firms decide on entry and their R&D spending before observing their marginal cost. R&D simply shifts the cost distribution. Firms then individually learn their marginal cost, and finally play a Bayesian Cournot game determining their domestic and foreign sales. The model allows us to derive the comparative static effects of a reduction in trade costs on R&D, domestic output, and exports at the firm level. It also lets us determine how trade liberalization affects the cut-off levels of firm productivity that separate firms that are not able to sell any output from the more productive ones that serve the domestic market, and the latter from the most productive ones that also export. From the changes in firm-level decisions and the selection effects induced by changes in the cut-off values we can then compute how trade liberalization affects aggregate industry productivity and social welfare.

The complementarity between innovation and exporting captured by our model—namely that firms are more likely to export if they innovate, and are more likely to innovate when they see good export opportunities—is well documented by recent empirical studies (Lileeva and Trefler (2007), Aw,\textsuperscript{2}

\textsuperscript{2}The selection effect is a feature of heterogeneous firm models, such as Melitz (2003). Both adjustment channels are empirically important. See Lileeva and Trefler (2007), Greenaway and Kneller (2005), and Wagner (2007) for recent surveys of the literature.
Roberts and Winston (2007), and Bustos (2007)). There is also clear empirical evidence that firms indeed try to boost their productivity to increase their market opportunities, as envisaged by our model (see Emami-Namini and Lopez (2006), Alvarez and Lopez (2005), and Hallward-Driermeier et al. (2002)).

We examine two scenarios, one with an exogenous and one with an endogenous market structure. The case of an exogenous market structure can be interpreted as a short-run scenario in which firms react to trade liberalization by adjusting domestic and foreign sales (possibly to zero) and R&D investment, but there is no entry of new firms. The case of an endogenous market structure may serve as a long-run scenario, in which expected profits are driven to zero by free entry and exit of firms. Firms may still respond to trade liberalization by adjusting output and R&D expenditure. However, part of the adjustment will be in the form of entry and exit. We are especially interested in identifying trade liberalization effects that are robust in that they hold across different market structures and can therefore be expected to occur across a wide range of industries independent of (often unobserved) sector-specific entry and exit costs.

We show, among other things, that trade liberalization (i) raises (reduces) expected aggregate R&D spending if trade costs are low (high); and (ii) forces firms at the bottom of the productivity distribution to produce zero expected output. The two effects determining how industry productivity reacts to trade liberalization may hence go in the same or in opposite directions. In particular, the R&D effect counteracts (reinforces) the selection effect when trade costs are low (high). However, we are able to prove that the selection effect dominates so that expected industry productivity rises unambiguously as trade costs fall. A sufficient condition for social welfare to increase with trade liberalization under any market structure is for trade costs to be low.

Our paper is most closely related to the recent work of Costantini and Melitz (2007), and Atkeson and Burstein (2006) who also examine innova-
tion and export decisions in a model with heterogeneous firms. Both papers have in common that they start from a situation in which firms already differ in their initial productivity before an innovation opportunity arises (a binary choice in the former, a continuous choice in the latter paper), and then study how productivity differences evolve over time when trade costs fall. The former paper examines the transition dynamics between two steady states, and finds that productivity effects depend on whether liberalization is anticipated and on how quickly it is implemented. The latter paper studies the long-run dynamics. It shows that a reduction in trade costs induces (less) productive firms to spend more (less) on innovation, thus becoming even more (less) productive over time. By contrast, firms in our model decide on innovation investment before they know their productivity. This assumption allows us to isolate the innovation- and selection-induced changes in productivity from effects generated by initial conditions. The simplicity of our model has the added advantage that we are able to perform classic comparative static analysis, which makes the economics behind these changes very transparent. Costantini and Melitz, and Atkeson and Burstein, on the other hand, have to rely on numerical simulation for (most) of their results.

The remainder of the paper is structured as follows. Section 2 introduces the model. Section 3 contains the results both in the case of a fixed market structure and in the case of an endogenous market structure. Section 4 concludes. The Appendix contains proofs.

3Navas and Sala (2006), and Gustafson and Segerstrom (2006) also introduce innovation into the Melitz (2003) model. However, in the open-economy version of the former paper the amount of innovation investment is held constant. In the latter, R&D is carried out in an innovation sector and depends crucially on the presence of intertemporal knowledge spillovers in the innovation sector.

4Other related papers include Ederington and McCalman (2007) and Yeaple (2005) who examine the effect of trade liberalization on technology adoption. The adoption process also leads to ex-post differences in firm productivity. Haaland and Kind (2008) employ a model in which R&D and exports are determined simultaneously, but their focus is on the effect of R&D subsidies.
2 The Model

We consider a reciprocal dumping model of trade with two segmented markets: the home and the foreign market. Firms in the two markets produce a homogeneous good and engage in Cournot competition. Consumers in each market have quadratic quasi-linear preferences that give rise to a linear inverse demand function,

\[ p_j = A - Q_j, \] (1)

where \( p_j \) and \( Q_j \) denote price and total sales in market \( j \). Labor is the only factor of production and comes in fixed supply. Assuming that the numeraire good is produced under constant returns to scale at unit cost and traded freely on a competitive world market, the equilibrium wage in each country is equal to one, and trade is always balanced.

The per-unit trade cost on shipments between countries is denoted by \( t \). We treat \( t \) as a resource cost, such as the cost of transporting goods or overcoming non-tariff barriers. Trade liberalization is modelled as a marginal fall in \( t \) in both countries.

Let \( n \) denote the number of active oligopolists in each market. Firms produce under constant (but ex-ante unknown) marginal cost, equal to the unit labor requirement. We assume that the marginal cost of firm \( i = 1, \ldots, n \), denoted by \( c_i \), is revealed to the firm only after it has incurred a sunk set-up cost \( f > 0 \) and invested an amount \( r_i \geq 0 \) in R&D. By conducting R&D a firm increases its chances to become a lower-cost firm. The probability that firm \( i \)'s marginal cost is less than or equal to \( c_i \) is given by \( G(c_i) \), where

\[ G(c_i) = g(r_i)F(c_i), \quad g(0) = 1, \quad g' > 0, \quad g'' \leq 0. \] (2)

The ex-ante cumulative distribution \( F(c_i) \) has support on the interval \([0, \bar{c}]\). Obviously, expression (2) is defined only as long as \( G(c_i) \leq 1 \).\(^5\) The cost of R&D is given by

\(^5\)Precisely, \( G(c_i) = \min(g(r_i)F(c_i), 1) \).
\[ \rho(r_i) : \rho(0) = 0, \rho' > 0, \rho'' \geq 0. \] (3)

We assume that both the level of R&D and the marginal-cost realization are private information of each firm. Hence output decisions are made under asymmetric information, and the R&D investment has no effect on the output choice of rival firms.\(^6\) Upon learning its marginal cost, firm \(i\) will produce a quantity \(y(c_i)\) for the domestic market and \(x(c_i)\) for the export market. This output decision will depend on the expected output of all rival firms in the domestic market, denoted by \(\hat{Q}_{-i}\). Firm \(i\)'s first-order condition for its domestic sales \(y_i(c_i)\) is

\[ p(y_i(c_i) + \hat{Q}_{-i}) + y_i(c_i)p'(y_i(c_i) + \hat{Q}_{-i}) - c_i \leq 0, (= 0 \text{ if } y_i(c_i) > 0). \] (4)

From (4), we may derive the critical marginal cost, \(\tilde{c}_{y_i} = A - \hat{Q}_{-i}\), for which firm \(i\)'s domestic sales become zero. Then the first-order conditions give rise to the decision rule\(^7\)

\[ y_i(c_i) = \begin{cases} 
0 & \text{if } c_i \geq \tilde{c}_{y_i}, \\
\frac{1}{2} (\tilde{c}_{y_i} - c_i) & \text{if } c_i < \tilde{c}_{y_i},
\end{cases} \] (5)

and the ex-post profit in the domestic market is equal to

\[ \pi_i(c_i) = \begin{cases} 
0 & \text{if } c_i \geq \tilde{c}_{y_i}, \\
\frac{1}{4} (\tilde{c}_{y_i} - c_i)^2 & \text{if } c_i < \tilde{c}_{y_i}.
\end{cases} \] (6)

Similarly, let \(\hat{Q}_{-i}^*\) denote the expected output of all rivals in the export market. Firm \(i\)'s first-order condition for its exports \(x_i(c_i)\) is

\[ p(x_i(c_i) + \hat{Q}_{-i}^*) + x_i(c_i)p'(x_i(c_i) + \hat{Q}_{-i}^*) - t - c_i \leq 0, (= 0 \text{ if } x_i(c_i) > 0), \] (7)

and the critical marginal cost for which its exports become zero is \(\tilde{c}_{x_i} = A - \hat{Q}_{-i}^* - t\). Hence the quantity of exports is

\[ x_i(c_i) = \begin{cases} 
0 & \text{if } c_i \geq \tilde{c}_{x_i}, \\
\frac{1}{2} (\tilde{c}_{x_i} - c_i) & \text{if } c_i < \tilde{c}_{x_i},
\end{cases} \] (8)

\(^6\)An increase in R&D therefore cannot serve as a commitment device to be more aggressive in both markets. This is similar to the model of Haaland and Kind (2008) which assumes that outputs and R&D are determined simultaneously by each firm.

\(^7\)See also Cramton and Palfrey (1990), Lemma 5 (p. 26 and pp. 41-2).
and the ex-post export profit is
\[ \pi^*_i(c_i) = \begin{cases} 
0 & \text{if } c_i \geq \bar{c}_{x_i}, \\
\frac{1}{4}(\bar{c}_{x_i} - c_i)^2 & \text{if } c_i < \bar{c}_{x_i}.
\end{cases} \] (9)

Using (6) and (9) we may write the total expected profit of firm \( i \) as
\[ \Pi_i(r_i) = \frac{g(r_i)}{4} \Omega_i - (f + \rho(r_i)), \] (10)
where
\[ \Omega_i = \frac{1}{4} \int_0^{\bar{c}_{y_i}} (\bar{c}_{y_i} - c_i)^2 dF(c) + \frac{1}{4} \int_0^{\bar{c}_{x_i}} (\bar{c}_{x_i} - c_i)^2 dF(c). \] (11)

Each entrant chooses its R&D level according to the following first-order condition:
\[ \frac{\partial \Pi_i}{\partial r_i} = g'(r_i) \frac{\Omega_i}{4} - \rho'(r_i) = 0. \] (12)

Since firms are identical prior to learning their cost realization, equilibrium R&D spending will be the same for all entering firms. For future convenience, let us denote the optimal common level of R&D by \( \hat{r} \), where
\[ g'(\hat{r}) \Omega - 4\rho'(\hat{r}) = 0. \] (13)

The following assumption guarantees that \( \hat{r} > 0 \):

**Assumption 1**
\[ \Omega > 4\rho'(0). \]

Since each firm does the same R&D, the expected outputs of firms will coincide in equilibrium. Furthermore, since the two countries are identical, the expected domestic and export sales of home firms will be identical to those of foreign firms. In its local market firm \( i \) will face \( n - 1 \) domestic rivals, each expected to produce and sell \( \hat{y} \) units, and \( n \) rivals from abroad, each expected to sell \( \hat{x} \) units. Hence, \( Q_{-i} = (n - 1)\hat{y} + n\hat{x} \). Similarly in its export market, the firm competes with \( n - 1 \) other exporters and \( n \) local firms so that \( \hat{Q}^*_{-i} = n\hat{y} + (n - 1)\hat{x} \). Using symmetry, the following Lemma shows that the expected local and export sales of a firm are determined by a system of only two equations:
Lemma 1  Expected sales are

\[
\hat{y} = \frac{g(\hat{r})}{2} \int_0^{\tilde{c}_y} F(c) dc, \\
\hat{x} = \frac{g(\hat{r})}{2} \int_0^{\tilde{c}_x} F(c) dc.
\]

Proof: See Appendix A.1. □

We may also use symmetry to rewrite the equilibrium profits as follows

\[
\hat{\Pi} = \frac{g(\hat{r})}{4} \Omega - (f + \rho(\hat{r})),
\]

where

\[
\Omega \equiv \int_0^{A-(n-1)\tilde{y}-n\tilde{x}} [A - (n - 1)\hat{y} - n\hat{x} - c]^2 dF(c) + \\
\int_0^{A-(n-1)\tilde{x}-n\tilde{y}-t} [A - (n - 1)\hat{x} - n\hat{y} - t - c]^2 dF(c).
\]

Furthermore, the critical cost levels will coincide across firms and are given by

\[
\tilde{c}_y \equiv A - (n - 1)\hat{y} - n\hat{x} \\
\tilde{c}_x \equiv A - (n - 1)\hat{x} - n\hat{y} - t.
\]

In our analysis below we will refer to the effect of trade liberalization on industry productivity, which we define as the inverse of the conditional expectation of the marginal cost of the firms that produce positive output. The conditional expectation is

\[
E(c \mid c \leq \tilde{c}_y) = \frac{1}{G(\tilde{c}_y)} \int_0^{\tilde{c}_y} cdG
\]

Of particular interest will be whether the effect coming from possible changes in \(\hat{r}\) will reinforce or offset changes induced through \(\tilde{c}_y\).

3 The Effects of Trade Liberalization

In this section we examine how trade liberalization in the form of a marginal reduction in \(t\) affects the equilibrium of the model. We start with the case
of a fixed market structure. That is, we determine how trade liberalization affects expected local sales, expected exports and R&D, holding fixed the number of firms. One may interpret this as a short-run scenario, in which the number of firms has not yet had time to adjust.\footnote{Note that this does not preclude the fact that, ex post, firms with a bad draw will produce zero output.} We then turn to the case of an endogenous market structure, where entry and exit occur until expected profits are equal to zero. In this case we want to know how trade liberalization affects expected local sales, expected exports, R&D, as well as the equilibrium number of firms.

### 3.1 Fixed Market Structure

In the case of a fixed market structure the equilibrium $\hat{y}$, $\hat{x}$ and $\hat{r}$ are determined by equations (13) - (15). To derive the comparative static effects of a reduction in $t$ we totally differentiate these equilibrium conditions. This yields the following comparative static results:

**Proposition 1** If the number of firms is fixed, trade liberalization (i) increases expected exports; (ii) decreases expected local sales if trade costs are high, and has an ambiguous effect on local sales if trade costs are low; (iii) increases (decreases) R&D if trade costs are low (high); and (iv) increases the expected total output of each firm.

Proof: see Appendix A.2. □

To develop intuition for the results consider the effect of trade liberalization on the threshold values of the marginal cost, $\tilde{c}_y$ and $\tilde{c}_x$. For $t = 0$ we obviously have $\tilde{c}_y = \tilde{c}_x$: there is only one critical value such that firms with marginal cost draws below this value are active on the integrated home and foreign markets, whereas firms with higher marginal costs do not produce any output. For $t > 0$, we must have $\tilde{c}_y > \tilde{c}_x$. The most efficient firms—those with cost draws below $\tilde{c}_x$—produce for the both the domestic and export markets, firms with cost draws between $\tilde{c}_y$ and $\tilde{c}_x$ sell only on the domestic market; firms
with marginal costs above $\tilde{c}_y$ do not sell anything. We show in Appendix A.2 that $d\tilde{c}_y/dt > 0$ and $d\tilde{c}_x/dt < 0$. This implies that as trade costs decline, the threshold cost level $\tilde{c}_x$ rises, so that more firms will now be able to export. On the other hand, the threshold cost level $\tilde{c}_y$ falls, meaning that firms that before were efficient enough to sell on their local market are now forced to produce zero output.

Consider first how trade liberalization affects a firm’s expected sales holding fixed the level of R&D expenditure. Expected export sales rise, since trade liberalization raises the probability that any given firm will be efficient enough to be able to export, and allows those firms that do export to increase their shipments abroad. Expected domestic sales decrease, since firms respond to import competition by reducing local sales. In addition, the likelihood that a given firm will be able to sell on its local market falls. These arguments explain the increase in export sales (part (i) of the Proposition), but not why domestic sales might increase if trade costs are low (see part (ii)). The ambiguity in the case of domestic sales is due to changes in R&D spending. Specifically, expected domestic sales can only rise after trade liberalization, if increased R&D leads to a big enough downward shift in the marginal cost distribution.

How does R&D respond to a reduction in the trade cost? A firm selling only on the domestic market would want to reduce its R&D spending, since tougher competition from imports decreases its output and hence also the marginal benefit from R&D. An exporter would want to increase R&D, since the increase in its export sales more than compensates for the decrease in local market share, meaning that it has a greater incentive to invest in cost-reducing R&D. If $t$ is sufficiently close to the prohibitive level, the probability of being an exporter is very low ($\tilde{c}_x$ is small), whereas there is a large probability of selling only on the domestic market ($\tilde{c}_y$ and $(\tilde{c}_y - \tilde{c}_x)$ are big). Furthermore, the probability that some of the potential foreign rivals will be an exporter (and compete in the domestic market) is larger than the individual firm’s probability of being an exporter. This implies that for high
trade costs, R&D spending falls as trade is liberalized. If \( t \) is close to zero, almost all active firms will have access to the export market and therefore be able to expand output as trade is liberalized. Hence for low trade costs, R&D spending increases with trade liberalization. This explains the non-monotonic relationship between trade costs and R&D in part (iii). The effect of trade liberalization on total sales of a firm is unambiguously positive (part (iv)), as the expected increase in exports more than compensates even an expected decrease in domestic sales.

Proposition 1 indicates that trade liberalization has two effects on industry productivity: (i) a direct effect due to changes in R&D investment, and (ii) a selection effect due to the fact that the least efficient firms will produce zero output. When the trade cost is low, these two effects reinforce each other, so that trade liberalization unambiguously raises industry productivity. When the trade cost is high, however, the two effects go in the opposite direction, as trade liberalization weakens the incentives to invest in R&D. However, we are able to show:

**Proposition 2** If the number of firms is fixed, trade liberalization raises industry productivity. It raises expected social welfare provided that the trade cost is sufficiently low.

Proof: see Appendix A.2 and Appendix A.4. □

Consider how trade liberalization affects consumer surplus and social welfare. Since expected output increases with trade liberalization, it follows that consumer surplus must rise. To determine how social welfare is affected, we have to take into account the change in the domestic firms’ expected profits.\(^9\) A sufficient condition for trade liberalization to raise social welfare is for \( t \) to be sufficiently close to zero. In this case, the usual pro-competitive effect of

\(^9\)If we treated the trade cost not as a pure resource cost but as a tariff, then tariff revenue would also enter the social welfare function. To see where this matters consider a marginal increase in \( t \) starting from \( t = 0 \). This increase generates positive tariff revenue and hence raises welfare. This is the well know result that the optimal tariff in a reciprocal dumping model is positive.
trade liberalization dominates, meaning that the increase in consumer surplus caused by tougher competition more than compensates for the decline in industry profit.\footnote{If \( t \) is near the prohibitive level, this pro-competitive effect may be offset by the fact that exporters have to bear high trade costs so that profits may fall by more than consumer surplus rises.}

Note, however, that Proposition 2 does not claim that trade liberalization reduces welfare for high trade costs as it is the case in the standard reciprocal dumping model. Appendix A.2 shows that the marginal welfare effect at the prohibitive trade cost level is ambiguous in sign. Hence, it is possible that trade liberalization improves welfare also for high trade cost levels.

### 3.2 Endogenous Market Structure

Now consider the case of an endogenous market structure. Free entry and exit of firms ensures that expected profits (16) are zero, which implies that

\[
\frac{\Omega}{4} = \frac{\rho(r) + f}{g(r)}.
\]

Since \( \Omega \) is a function of \( r \), \( t \) and \( n \), this equation implicitly defines \( r \) as a function of \( t \) and \( n \). Using (21), we may therefore rewrite the first-order condition for R&D, (12), as:

\[
g'(r(t,n)) = \frac{\rho'(r(t,n))}{\rho(r(t,n)) + f}.
\]

Assuming that this equation has a unique positive solution, \( r(t,n) = \hat{r} > 0 \), we obtain:

**Proposition 3** Firm-level R&D is independent of the trade cost if market structure is endogenous.

Proposition 3 implies that in a free-entry-and-exit equilibrium any change in the trade cost leads to an adjustment in the number of firms such that
the incentive to undertake R&D remains unchanged.\footnote{Atkeson and Burstein (2006) and Eaton and Kortum (2001) also feature results that trade liberalization leaves firm-level R&D unchanged. In both papers a reduction in trade costs, per se, raises the incentive to innovate. In Atkeson and Burstein, however, the wage of managers required for innovation also rises. When all firms export, it rises so much that the innovation effort remains constant. In Eaton and Kortum the offsetting effect comes from the fact that trade liberalization raises the likelihood that a foreign competitor makes an innovation and captures the whole market.} This does not, however, mean that trade liberalization has no effect on aggregate R&D, since the equilibrium number of firms may change. To determine the effects of trade liberalization, we may treat R&D expenditures as a fixed cost and use equations (14), (15) and (21) to solve for the remaining endogenous variables \((n, \hat{x}, \hat{y})\). We may rewrite these equations as

\[
2\hat{y} - \int_{0}^{A-(n-1)\hat{y}-n\hat{x}} G(c)dc = 0, \quad (23)
\]

\[
2\hat{x} - \int_{0}^{A-(n-1)\hat{x}-n\hat{y}-t} G(c)dc = 0, \quad (24)
\]

\[
\int_{0}^{A-(n-1)\hat{y}-n\hat{x}} [A - (n - 1)\hat{y} - n\hat{x} - c]^2 dG(c) + \int_{0}^{A-(n-1)\hat{x}-n\hat{y}-t} [A - (n - 1)\hat{x} - n\hat{y} - t - c]^2 dG(c) - 4(f + \rho(r^*)) = 0. \quad (25)
\]

Total differentiation of (23), (24) and (25) yields the following comparative static results:

**Proposition 4** If market structure is endogenous, trade liberalization (i) increases expected exports and decreases expected local sales of each firm; (ii) increases the expected output of each firm if the trade cost is high; and (iii) increases (decreases) the number of firms and hence aggregate R&D if the trade cost is low (high).

Proof: see Appendix A.3. □

Trade liberalization has the same effects on the threshold levels of marginal cost as in the fixed market structure case (see Appendix A.3). The impact
of trade liberalization on expected domestic and export sales is therefore straightforward: the probability that a given firm exports rises as do sales of each exporting firm abroad. Increased competition from abroad reduces both the probability that a firm will sell positive output and local sales of those firms that do sell. The intuition for parts (ii) and (iii) is straightforward when trade costs are high. Trade liberalization increases expected import competition, making each firm’s residual demand more elastic. Firms hence reduce their mark-up and are forced to raise their output to keep the expected profit at zero. As firms become bigger, the number of firms has to fall. To see that we observe a different effect for low trade costs, consider an infinitesimal increase in the trade cost starting at free trade. Due to the additional cost, expected profit falls, and the number of firms has to decrease so as to keep expected profit equal to zero. Hence at free trade, and by continuity sufficiently close to it, trade liberalization will raise the number of firms and therefore also industry-level R&D.

Finally consider the effects of trade liberalization on industry productivity and social welfare. Since R&D per firms remains constant, trade liberalization affects industry productivity only through the selection effect. Hence industry productivity rises unambiguously when trade is liberalized. Since expected profits are zero due to free entry, the effect of trade liberalization on social welfare is equal to the effect on consumer surplus. We are able to show that consumer surplus unambiguously increases with trade liberalization. These results are summarized in the following Proposition:

**Proposition 5** If market structure is endogenous, trade liberalization raises industry productivity and expected social welfare.

Proof: see Appendix A.3 and Appendix A.4. □

4 Conclusions

In this paper we developed a simple model of international trade with heterogeneous firms to explore the effects of trade liberalization on firms’ innovation
incentives, as well as on industry productivity, and social welfare. We found a U-shaped relationship between the trade cost and expected industry-level R&D spending. That is, trade liberalization raises (reduces) industry R&D expenditure if trade costs are low (high). When the market structure is fixed, this is due to the underlying changes in firms’ R&D investments. In the case of an endogenous market structure, trade liberalization induces changes in the number of firms such that each individual firm has no incentive to alter its R&D spending. The industry-level R&D pattern then arises due to the U-shaped relationship between the trade cost and the number of firms.

The impact of trade liberalization on industry productivity comes from two sources, namely from changes in R&D spending, and from a selection effect (the least efficient firms are forced to produce zero output). Whereas the selection effect raises industry productivity, the R&D effect may go in the opposite direction depending on the level of trade costs. Specifically, when trade costs are low (high), the R&D effect counteracts the selection effect. Still, we were able to show that the total effect of trade liberalization on industry productivity is positive.

This result is important because the productivity enhancing effect of trade is often portrayed as one of the main reasons why trade liberalization may raise social welfare. When the market structure is endogenous, the higher industry-level productivity indeed translates into higher consumer surplus and social welfare. However, our paper also showed that this may not be true in the short run when market structure is fixed.

In addition to introducing R&D and thereby endogenizing firm productivity, our paper offers a novel approach to modelling firm heterogeneity. In particular, we allow firms to interact strategically in an oligopolistic market instead of relying on monopolistic competition. An important benefit of our approach, other than its simplicity, is that it explicitly reproduces output and mark-up adjustments by firms that are among the most robust empirical regularities of trade liberalization (see Tybout (2003) and Wag-
Specifically, we are able to derive sufficient conditions under which trade liberalization reduces price-cost margins, lowers domestic sales of import-competing firms, expands export markets for very efficient firms, and increases efficiency at the plant level.

Appendix

A.1 Proof of Lemma 1

Expected output for the home market is

$$E[y(c)] = \hat{y} = g(\hat{\tau}) \int_0^{\hat{c}_y} y(c) dF(c) = \frac{g(\hat{\tau})}{2} \int_0^{\hat{c}_y} [\hat{c}_y - c] dF(c) \quad (A.1)$$

and expected exports to the foreign market are

$$E[x(c)] = \hat{x} = g(\hat{\tau}) \int_0^{\hat{c}_x} x(c) dF(c) = \frac{g(\hat{\tau})}{2} \int_0^{\hat{c}_x} [\hat{c}_x - c] dF(c). \quad (A.2)$$

Evaluating the integral on the right-hand side of (A.1) by parts, and defining \( \phi(c) \equiv [\hat{c}_y - c] \), we have

$$\int_0^{\hat{c}_y} [\hat{c}_y - c] dF(c) = \int_0^{\hat{c}_y} \phi(c) F'(c) dc$$

$$= [\phi(\hat{c}_y) F(\hat{c}_y) - \phi(0) F(0)] - \int_0^{\hat{c}_y} \phi'(c) F(c) dc$$

$$= \int_0^{\hat{c}_y} F(c) dc,$$

because \( \phi(\hat{c}_y) = F(0) = 0 \) and \( \phi'(c) = -1 \). A similar derivation leads to the expected export level. □

A.2 Proofs of Propositions 1 and 2

Differentiating (14), (15) and (13) totally, we obtain

$$\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33}
\end{bmatrix}
\begin{bmatrix}
\frac{dr}{dt} \\
\frac{d\hat{x}}{d\hat{y}}
\end{bmatrix}
= \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix}
\frac{dt}{dt}$$

\(^{12}\)Output and mark-up effects are typically absent in monopolistic competition models. See Melitz and Ottaviano (2008) for an exception.
where

\[ \alpha_{11} \equiv -\frac{2g'\hat{y}}{g}, \quad \alpha_{12} \equiv gnF(\hat{c}_y), \quad \alpha_{13} \equiv 2 + g(n - 1)F(\hat{c}_y), \]
\[ \alpha_{21} \equiv -\frac{2g'\hat{x}}{g}, \quad \alpha_{22} \equiv 2 + g(n - 1)F(\hat{c}_x), \quad \alpha_{23} \equiv gnF(\hat{c}_x), \]
\[ \alpha_{31} \equiv \hat{\Pi}_{rr}, \quad \alpha_{32} = -\frac{4g'}{g}((n - 1)\hat{x} + n\hat{y}), \quad \alpha_{33} = -\frac{4g'}{g}((n - 1)\hat{y} + n\hat{x}). \]

\[ \beta_1 = 0, \quad \beta_2 = -gF(\hat{c}_x), \quad \beta_3 = \frac{4g'}{g}\hat{x}. \]

Expanding along the first column yields the determinant

\[ \Delta = \frac{8g'}{g^2} \left( \hat{x}^2[(2n - 1)(1 - gF(\hat{c}_y)) - 1] + \hat{y}^2[(2n - 1)(1 - gF(\hat{c}_y)) - 1] + 4n\hat{x}\hat{y} \right) \]
\[ + \hat{\Pi}_{rr} \left( g^2n^2F(\hat{c}_x)F(\hat{c}_y) - (2 + g(n - 1)F(\hat{c}_y))(2 + g(n - 1)F(\hat{c}_x)) \right) \]

We first establish that \( \Delta > 0 \). Since \( gnF(\hat{c}_x) < 2 + g(n - 1)F(\hat{c}_x) \) and \( gnF(\hat{c}_y) < 2 + g(n - 1)F(\hat{c}_y) \), \( \Delta_2 < 0 \) and hence \( \hat{\Pi}_{rr}\Delta_2 > 0 \). Thus, \( \Delta > 0 \) will hold true if we can show that \( \Delta_1 > 0 \). We will show that \( \Delta_1 > 0 \) by contradiction. We observe first that \( \Delta_1 > 0 \) if \( (2n - 1)(1 - gF(\hat{c}_y)) - 1 \geq 0 \) and \( (2n - 1)(1 - gF(\hat{c}_x)) - 1 \geq 0 \). Thus, \( \Delta_1 < 0 \) requires that \( (2n - 1)(1 - gF(\hat{c}_y)) - 1 < 0 \) and/or \( (2n - 1)(1 - gF(\hat{c}_x)) - 1 < 0 \). Since \( gF(\hat{c}_y) \geq gF(\hat{c}_x) \), \( (2n - 1)(1 - gF(\hat{c}_x)) - 1 \geq (2n - 1)(1 - gF(\hat{c}_y)) - 1 \), and we have to consider two possible cases:

**Case 1:** \( (2n - 1)(1 - gF(\hat{c}_x)) - 1 > 0, (2n - 1)(1 - gF(\hat{c}_y)) - 1 < 0 \)

In this case,

\[ \Delta_1 > \hat{x}^2[(2n - 1)(1 - gF(\hat{c}_y)) - 1] + 4n\hat{x}\hat{y} = \hat{x}[(2n - 1)(1 - gF(\hat{c}_y)) - 1] + 4n\hat{y} > 0 \]

because \( \hat{y} > \hat{x} \) and \( 4n > -(2n - 1)(1 - gF(\hat{c}_y)) + 1 \).

**Case 2:** \( (2n - 1)(1 - gF(\hat{c}_x)) - 1 < 0, (2n - 1)(1 - gF(\hat{c}_y)) - 1 < 0 \)
First observe that for zero trade costs, $\hat{x} = \hat{y}$, $F(\bar{c}_x) = F(\bar{c}_y)$ and

$$
\Delta_1 = 2\bar{g}^2(2n - 1)(2 - gF(\bar{c}_y)) > 0
$$

Hence, $\Delta_1 < 0$ warrants the existence of a critical $\bar{x} < \hat{y}$ such that

$$
\ddot{x}^2[(2n - 1)(1 - gF(\bar{c}_y)) - 1] + \bar{g}^2[(2n - 1)(1 - gF(\bar{c}_x)) - 1] + 4n\ddot{x}\hat{y} = 0.
$$

Solving for quadratic equation yields the two solutions

$$
\bar{x}_{1,2} = \frac{-4n\hat{y} \pm \sqrt{8n^2\hat{y}^2 - 4[(2n - 1)(1 - gF(\bar{c}_y)) - 1][2(2n - 1)(1 - gF(\bar{c}_x)) - 1]\bar{g}^2}}{2n - 1}(1 - gF(\bar{c}_y)) - 1
$$

Note carefully that $(2n - 1)(1 - gF(\bar{c}_y)) - 1 \in [-1, 0]$ so that $\bar{x}$ is larger than the numerator in absolute terms. The negative solution is irrelevant as it implied $\bar{x} > 4n\hat{y}$ which violates $\bar{x} < \hat{y}$. The positive solution fulfills $\bar{x} < \hat{y}$ only if

$$
\sqrt{8n^2\hat{y}^2 - 4[(2n - 1)(1 - gF(\bar{c}_y)) - 1][2(2n - 1)(1 - gF(\bar{c}_x)) - 1]\bar{g}^2} > (4n - 1)\hat{y}.
$$

However,

$$
\sqrt{8n^2\hat{y}^2 - 4[(2n - 1)(1 - gF(\bar{c}_y)) - 1][2(2n - 1)(1 - gF(\bar{c}_x)) - 1]\bar{g}^2} < \sqrt{8n^2\hat{y}^2 - 2\sqrt{2}n\hat{y} < (4n - 1)\hat{y},}
$$

so that no solution exists in the relevant range and $\Delta_1 > 0$ holds also for that case. This proves that $\Delta > 0$.

We can now derive the comparative-static effects:

$$
\frac{dr}{dt} = \frac{8g'}{g\Delta} (gn(\hat{y}F(\bar{c}_x) - \hat{x}F(\bar{c}_y)) - \hat{x}(2 - gF(\bar{c}_y))),
$$

$$
\frac{dr}{dt} < 0 \text{ at } t = 0 \Leftrightarrow \hat{x} = \hat{y} \Leftrightarrow F(\bar{c}_x) = F(\bar{c}_y), \frac{dr}{dt} > 0 \text{ at } x = 0,
$$

$$
\frac{d\hat{x}}{dt} = -\frac{8g'^2}{g^2\Delta}(\ddot{x}^2(2 + g(n - 1)F(\bar{c}_y)) + (n\hat{y} - \hat{x})g\hat{y}F(\bar{c}_x)) + \frac{\hat{n}_r}{\Delta}gF(\bar{c}_x)(2 + g(n - 1)F(\bar{c}_y)) < 0,
$$

$$
\frac{d\hat{y}}{dt} = \frac{8g'^2}{g^2\Delta}(F(\bar{c}_x)[(n - 1)g\hat{y}^2F(\bar{c}_x) + g\hat{x}(n\hat{x}F(\bar{c}_y) + \hat{y}F(\bar{c}_x))] - 2\hat{x}\hat{y}) - \frac{\hat{n}_r}{\Delta}g^2nF(\bar{c}_x)F(\bar{c}_y),
$$

18
\[
\frac{dy}{dt}(\hat{x} = 0) = \frac{8g^2}{g^2 \Delta} (n-1)g\hat{y}^2F(\hat{c}_x) - \frac{\hat{\Pi}_{rr}}{\Delta} g^2nF(\hat{c}_x)F(\hat{c}_y) > 0,
\]
\[
\frac{dy}{dt}(\hat{x} = \hat{y}) = -2g^2(2-n(g(\hat{F}(\hat{c}_x) + F(\hat{c}_y)))) \frac{8g^2}{g^2 \Delta} - \frac{\hat{\Pi}_{rr}}{\Delta} g^2nF(\hat{c}_x)F(\hat{c}_y) \text{ is ambiguous.}
\]
However, we can sign the change in total expected output per firm \( \hat{q} \equiv \hat{y} + \hat{x} \),
\[
\frac{d\hat{q}}{dt} = -\frac{8g^2}{g^2 \Delta} (2\hat{x}\hat{y}(1 - gF(\hat{c}_x)) - \hat{x}^2(2 - gF(\hat{c}_y)) - g\hat{y}^2F(\hat{c}_x) + \hat{\Pi}_{rr}(g(2 - gF(\hat{c}_y))) < 0,
\]
and therefore in expected industry output \( (Q = n\hat{q}) \):
\[
\frac{dQ}{dt} = n\frac{d\hat{q}}{dt} < 0.
\]
As for the critical values of marginal costs, \( \frac{dc_y}{dt} \) can be rewritten as
\[
\frac{dc_y}{dt} = -(n-1)\frac{d\hat{q}}{dt} - \frac{dc_x}{dt} > 0.
\]
Differentiating \( \frac{dc_x}{dt} \) yields
\[
\frac{dc_x}{dt} = \frac{2}{g^2} \left(2\hat{x}\hat{y}(1 - gF(\hat{c}_x)) - \hat{x}^2(2 - gF(\hat{c}_y)) - g\hat{y}^2F(\hat{c}_x) + \frac{\hat{\Pi}_{rr}}{\Delta} (g(2 - gF(\hat{c}_y))) \right) < 0.
\]
The welfare effect of integration consists of the effect on aggregate expected profits and consumer surplus. The change in expected profit (16) is
\[
\frac{d\widehat{\Pi}}{dt} = \frac{g(r)}{4} \left( \frac{\partial \Omega}{\partial \hat{y}} \frac{d\hat{y}}{dt} + \frac{\partial \Omega}{\partial \hat{x}} \frac{d\hat{x}}{dt} \right) = -(n-1)\frac{d\hat{q}}{dt}\hat{q} + \frac{d\hat{y}}{dt}\hat{x} - \frac{d\hat{x}}{dt}\hat{y} - \hat{x},
\]
taking into account that \( \frac{\partial \widehat{\Pi}}{\partial r} = 0 \). Let \( \widehat{CS} \equiv (n\hat{q})^2/2 \) denote expected consumer surplus. Its change with \( t \) is
\[
\frac{d\widehat{CS}}{dt} = n^2\hat{q}\frac{d\hat{q}}{dt} < 0,
\]
since \( \frac{d\hat{q}}{dt} < 0 \). The total expected welfare change is determined as
\[
\frac{d\widehat{W}}{dt} = \frac{d\widehat{CS}}{dt} + n\frac{d\widehat{\Pi}}{dt} = n \left( \frac{d\hat{q}}{dt}\hat{q} + \frac{d\hat{y}}{dt}\hat{x} - \frac{d\hat{x}}{dt}\hat{y} - \hat{x} \right).
\]
19
For $t = 0 \Leftrightarrow \hat{y} = \hat{x} \Leftrightarrow d\hat{y}/dt = d\hat{x}/dt$, we find
\[
\frac{d\hat{W}}{dt}(t = 0) = n \begin{pmatrix} \frac{d\hat{q}}{dt} - \hat{x} \\ -\frac{d\hat{x}}{dt} \end{pmatrix} < 0,
\]
whereas the marginal welfare effect at the prohibitive trade cost level, i.e., for $\hat{x} = 0$, is ambiguous:
\[
\frac{d\hat{W}}{dt}(\hat{x} = 0) = n \begin{pmatrix} \frac{d\hat{q}}{dt} - \frac{d\hat{x}}{dt} \\ -\frac{d\hat{x}}{dt} + \frac{d\hat{y}}{dt} \end{pmatrix}.
\]

A.3 Proofs of Propositions 4 and 5

Differentiating (23), (24) and (25) totally, we get
\[
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \frac{dn}{dt} \\ \frac{d\hat{x}}{dt} \\ \frac{d\hat{y}}{dt} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} dt,
\]
where
\[
\begin{align*}
a_{11} &\equiv (\hat{x} + \hat{y})G(\bar{c}_y), & a_{12} &\equiv nG(\bar{c}_y), & a_{13} &\equiv 2 + (n - 1)G(\bar{c}_y), \\
a_{21} &\equiv (\hat{x} + \hat{y})G(\bar{c}_x), & a_{22} &\equiv 2 + (n - 1)G(\bar{c}_x), & a_{23} &\equiv nG(\bar{c}_x), \\
a_{31} &\equiv -4(\hat{x} + \hat{y})^2, & a_{32} &\equiv -4((n - 1)\hat{x} + n\hat{y}), & a_{33} &\equiv -4((n - 1)\hat{y} + n\hat{x}), \\
 b_1 &\equiv 0, & b_2 &\equiv -G(\bar{c}_x), & b_3 &\equiv 4\hat{x}.
\end{align*}
\]
The determinant is
\[
\Delta = 8(\hat{x} + \hat{y})[\hat{x}(2 - G(\bar{c}_y)) + \hat{y}(2 - G(\bar{c}_x))] > 0.
\]
The comparative-static effects are given by
\[
\frac{dn}{dt} = \frac{n(\hat{y}G(\bar{c}_x) - \hat{x}G(\bar{c}_y)) - (2 - G(\bar{c}_y))\hat{x}}{\Delta},
\]
where
\[
\frac{dn}{dt} < 0 \text{ at } t = 0 \Leftrightarrow \hat{x} = \hat{y} \Leftrightarrow G(\bar{c}_y) = G(\bar{c}_x), \frac{dn}{dt} > 0 \text{ at } \hat{x} = 0;
\]
\begin{align*}
\frac{d\hat{x}}{dt} &= -\frac{8\hat{y}(\hat{x} + \hat{y})G(\tilde{c}_x)}{\Delta} < 0, \\
\frac{d\hat{y}}{dt} &= \frac{8\hat{x}(\hat{x} + \hat{y})G(\tilde{c}_y)}{\Delta} > 0, \\
\frac{d\hat{\tilde{c}}}{dt} &= \frac{(\hat{x} + \hat{y})(\hat{x}G(\tilde{c}_y) - \hat{y}G(\tilde{c}_x))}{\Delta}, \\
\frac{d\hat{\tilde{c}}}{dt} &= 0 \text{ at } t = 0 \iff x = y \iff G(\tilde{c}_y) = G(\tilde{c}_x), \frac{d\hat{\tilde{c}}}{dt} < 0 \text{ at } \hat{x} = 0.
\end{align*}

The effect on consumption is
\[
\frac{dQ}{dt} = -\frac{8\hat{x}(\hat{x} + \hat{y})(2 - G(\tilde{c}_y))}{\Delta} = \frac{\hat{x}(2 - G(\tilde{c}_y))}{\hat{x}(2 - G(\tilde{c}_y)) + \hat{y}(2 - G(\tilde{c}_y))} < 0.
\]

Furthermore, noting that
\[
\begin{align*}
\frac{d\tilde{c}_y}{dt} &= -(n-1)\frac{d\hat{y}}{dt} - n\frac{d\hat{x}}{dt} - (\hat{y} + \hat{x})\frac{dn}{dt}, \\
\frac{d\tilde{c}_x}{dt} &= -(n-1)\frac{d\hat{x}}{dt} - n\frac{d\hat{y}}{dt} - (\hat{y} + \hat{x})\frac{dn}{dt} - 1,
\end{align*}
\]

we obtain
\[
\begin{align*}
\frac{d\tilde{c}_y}{dt} &= \frac{2\hat{y}}{2(\hat{x} + \hat{y}) - \hat{x}G(\tilde{c}_y) - \hat{y}G(\tilde{c}_x)} > 0, \\
\frac{d\tilde{c}_x}{dt} &= -\frac{2\hat{x}}{2(\hat{x} + \hat{y}) - \hat{x}G(\tilde{c}_y) - \hat{y}G(\tilde{c}_x)} < 0.
\end{align*}
\]

**A.4 Industry productivity**

The effect of trade liberalization on the conditional expectation of marginal cost of firms that survive is calculated as follows. Note that
\[
dG(c) = g(\hat{r})dF(c) \equiv g(\hat{r})f(c)
\]
so that
\[
\frac{d}{dt}E(c \mid \tilde{c}_y) = \frac{d}{dt} \frac{1}{G(\tilde{c}_y)} \int_{\tilde{c}_y}^{\infty} cdG
\]
\[
= \frac{1}{G(\tilde{c}_y)} \tilde{c}_y g(\hat{r})f(\tilde{c}_y) \frac{d\tilde{c}_y}{dt} - \left[ \int_{\tilde{c}_y}^{\infty} cdG \right] \frac{1}{G(\tilde{c}_y)^2} \left( g(\hat{r})f(\tilde{c}_y) \right) \frac{d\tilde{c}_y}{dt}
\]
\[
= \frac{1}{G(\tilde{c}_y)} g(\hat{r})f(\tilde{c}_y) [\tilde{c}_y - E(c \mid c \leq \tilde{c}_y)] \frac{d\tilde{c}_y}{dt} > 0
\]
because $d\tilde{c}_y/dt > 0$ for both a fixed and an endogenous market structure. Thus, a fall in $t$ will reduce the conditional expectation of marginal cost of firms that survive.

References


