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Mathematical modelling of uncertainty in a process of provisioning at a commercial enterprise

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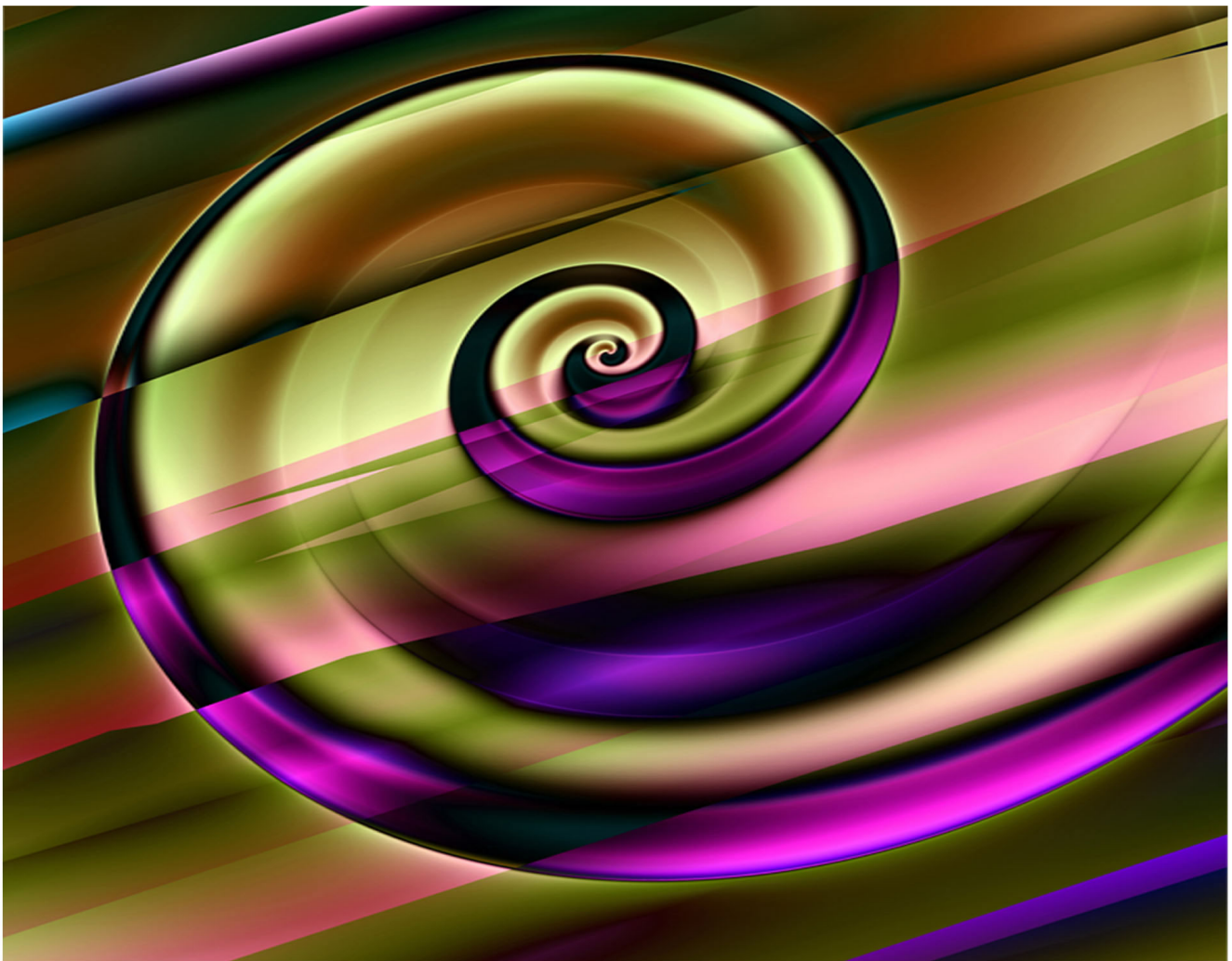
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Mathematical modelling of uncertainty in a process of provisioning at a commercial enterprise

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Abstract

The survival of a business is nowadays more than ever a matter of utmost importance. The process of globalization and current economic crisis are the two main reasons that caused the situation of uncertainty and instability on the world market. In this paper we made a theoretical approach and practical development of the study case related with the food inventory process at a small or medium-sized enterprises commercial enterprise. We used a mathematical tool based on one of the well-known classical methods that consists of calculating an inventory/supply rate with the transition into uncertain environment using some elements of fuzzy arithmetic. This model is sensible to many economical factors and gives a level of inventory stock that is optimal for the company at each moment.

Keywords: Globalization, Economic Crisis, Logistics, Supply Chain, Fuzzy Mathematics

1. Introduction

The survival of the business is nowadays more than ever a matter of utmost importance. The process of globalization and economic crisis are the two main reasons that caused the situation of uncertainty and instability in the world society (O'Brien, 2006; Soros, 2008; Yang, 2011).

This complex situation requires that the small and large companies change their performance and business strategies to guarantee the adequacy of the decisions made in a new economic environment.

These approaches are adapted to all areas of a business that pretends to achieve its objectives with a minimal chance for success. These measures affect the areas starting from the housekeeping department, which importance seems to be trivial, to the marketing or human resources departments, which, on the contrary, put all the emphasis on managing the production process in a modern enterprise. By this manner during the last decades the logistics became a highlight point in the company management assuming all the importance and responsibility for the company's daily performance. See (Hobday et al., 2005; OECD, 2008; Wang & Belardo, 2009; Liu & Wu, 2009; Baud & Durand, 2011).

The logistics demonstrates its efficiency in achieving the optimal levels of objects and materials movements inside and outside of the company as well as their more adequate levels of handling costs in order to get a maximum agility of the whole process and the shortest possible time in achieving a final customer (Langevin & Riopel, 2005; Wang & Shu, 2005; O'Donnel et al., 2006; Alex, 2007; Mangan et al., 2008).

In this paper we considered only one aspect of the logistics such as an inventory process of the food products that occupies, due to our opinion, a central place in every commercial enterprise. In order to develop this theme we chose one of the most utilized tools of the classical mathematics such as a calculus of the inventory rate with the transition into uncertain environment using some elements of fuzzy arithmetic. The method is based on the probabilistic studies.

Over the past few decades a large number of mathematical models have been elaborated. They provide a solution to the problems that come from the inventory process. This extensive range of the useful tools for the company has been offered by A. Kaufmann in the book entitled *Méthodes et Modèles de la Recherche Opérationnelle* (Kaufmann, 1970). See other authors (Petrovic, D. & Petrovic, R., 2001; Langevin & Riopel, 2005; Graves & Willems, 2008).

The recent developments made by A. M. Gil Lafuente that have been included in the book such as *The analysis of fixed assets in uncertainty* (A.M. Gil Lafuente, 2004) represent a special importance for our study. See other authors (Giannoccaro et al., 2003; Lin et al., 2010; Kristianto, 2012)

It is essential for this paper to mention J. Gil Aluja and other researchers who transferred the models of determinism into mathematics of uncertainty that was a field little explored until two decades but widely expanded currently. These tools can treat and solve more complex problems with a purpose to achieve the expectations of an unpredictable future (Kaufmann & Gil Aluja, 1992; Gil Aluja, 1999, 2004; Gil Aluja, et al., 2008; Gil Lafuente & Barcellos Paula, 2010; Gil Lafuente, et al., 2012; Klimova & Gil Lafuente, 2012; Merigó, et al., 2011).

2. Commerce globalization and world economic crisis

Globalization is an historical process which initially referred to the commercial interchange between different international economies. There is a large number of products which circulate on this global market that have extremely varied origin and destination. This fact increases the foreign trade of those countries which favor an opening their borders towards the outside taking in account the particular impact of the technological revolution in the process of the liberalization of the markets.

The process of globalization started in the decade of the 60s when companies in search of new markets extended their scope of actions beyond national borders, passing from being the producers in their countries of origin to the establishment of its manufacturing plants in the foreign countries (Vernon, 1966; Hirsch, 1976; Johanson & Vahlne, 1977; Fletcher, 2001).

From the decade of the 80s the links between different international economies started to tighten having as a result the participation of foreign shareholders in the companies. This phenomenon increased due to the transnational corporate mergers and corporate investment that took place in the 90s of last century (Hakansson, 1982; Johanson & Vahlne, 1990; 1993).

The most noticeable effects of globalization are the following: the growing spending on goods and services by the national citizens due to the large volume of the import and also the increasing volume of the export of domestic products under the continuous and increasingly varied exchange between different countries. On the other hand, globalization establishes a new framework for the competitiveness of the small and medium-sized companies (SMEs). These enterprises have a number of competitive advantages and disadvantages in order to cope with the national and international competition, which arise from a relative resource endowments and capabilities, and that is somewhat conditioned by the intrinsic strengths and weaknesses due to the size (Informe España, 1994, 2012; Izryadnova, O. et al. 2009).

Structural changes of the modern economy have created some favorable conditions for SMEs, due to their dynamic nature, their business models are more flexible and represent shorter maturity periods.

By the contrary, some of the advantages inherent to a small enterprise size have been removed due to uncertainty and turbulence of an increasingly globalized world. The evolution of the forces such as a globalization of the markets and increasing technological change as well as their increased vulnerability to the environmental turbulence, place the SME in a position of a strong risk competitive exposure with the serious obstacles in exploiting their strengths.

This is the scenario where a systematic logistics activity obtains the maximum importance becoming a very useful tool in the economic relations (Gil Lafuete, 1997; Bramel, J., 2005; Mira, 2006; Mangan et al., 2008).

Another no less important subject that it is necessary to mention is a current financial crisis. The crisis causes havoc in all sectors of the world economy. The devastating effect of this phenomenon has attacked mainly the activity of small and medium enterprises. A large number of these companies has disappeared from the economic horizon and another part is about to submerge.

Nowadays the economic turbulence and the situation of uncertainty occur constantly in the market. This is characterized not only by a constantly changing environment which affects the way the companies perform their activity but also by the fact of permanent technological revolution which make everything become obsolete in a

short time. Despite of this worrying perspective some businesses manage not only to survive but emerge from the crisis stronger and with the increasing volumes of sales (Informe España, 1994, 2012; Zhavoronkov, S. 2009).

According to the numerous studies made by the economists in the recent years the following conclusion was obtained: the key variable for the growth of a business consist in a global change of its strategy and management. It is necessary to focus more on the innovation, on the satisfaction and customer loyalty, on new marketing strategies, internationalization, acquisitions and fusion (Zaheer, 2000; UNCTAD, 2005; Hobday et al., 2005).

We dedicated our paper to developing a specific case, that is, a proposal that could serve as an effective and regulatory tool for the company's profitability both for the enterprise of general characteristics and also for the commercial companies. This is a mathematical tool that is used in situations of uncertainty and economic instability that extend significantly the horizon of future prospects for the current business.

3. Logistic and environmental changes

Nowadays a term known as a logistics has begun to be used in business since 1960s when the National Council of Physical Distribution Management began to use it to define this new branch of industry. This concept can be determined as follows:

"A set of activities that deals with the total-flow of materials that starts with the supply of raw materials and ends with the delivery of finished products to the customers. The logistics management is responsible for planning, performance and monitoring all activities related to the total-flow through the company of raw materials, components, intermediate products and finished products, and all the information associated to it" (J. Gil Lafuete, 1997; Bramel, J., 2005; Mira, 2006).

Nowadays logistics management is characterized by the global development of the activities; meanwhile previously they were carried out in a dispersed and fragmented manner, thus the area of business is rationalized completely in keeping with the times.

The global logistics chain contributes an added value and that is because it relates a company with its suppliers and customers, so that at the core of all process there are internal operations of the company which facilitate operations and movements of the goods both inside and outside of the company.

There are some external changes of the company that have motivated the selection of logistics as one of the essential competitive factor for the performance of the companies (Bramel, J., 2005; Mira, 2006; Mangan et al., 2008):

- A lifecycle of the products shortens continuously, so the management of the goods stock has become a key element for combining the production and the optimal distribution of the products in a perfect symbiosis. Thus the risk of obsolescence increases gradually and logistics functions are of great importance.
- Increase of the references in a company's catalogue that is indispensable to maintain a competitive offer. Thus the functions of the logistics are supposed to make records of a large number of the goods, to manage the client's orders, to administrate warehouses and manufacturing flows of the goods.
- A decisive word in a decision-making process of a logistics chain depends on the intermediates and customers. That is because of the changes produced in a distribution system due to the appearance of the wholesale distribution and big supermarkets that have obstructed the decisive ability of the production companies in terms of number of the product to be produced.
- Globalization of the markets has removed the barriers between countries and the governments stimulate international trade.
- Information technology has had the great advances in the recent years which have been implemented efficiently in a real-time control of goods movement and the relationships between different stages of distribution channels.

These factors together with the fact that the logistics costs form between 10% and 25% of the total cost of an international trade transaction, have made the logistics to be an object of a big concern for the companies that have the interests in international markets, reducing total costs and playing an indispensable role for the optimal customer satisfaction.

From this global perspective the modern companies struggle in combining the systems that seek greater efficiency providing, by this manner, the cost reduction without reducing the speed of exchange of goods and also without

worsening of a good attention to the customer adapting their distribution and manufacturing systems to the different demands of the international market.

4. Mathematical tools of uncertainty in the field of company's commerce management

For a few last decades in different areas of economic knowledge as well as in a commercial area there have been manifested some changes related to the content and objectives of the decision making process. These changes represent a fundamental pillar for the development of scientific and technical activity.

In the commercial sector a new conceptualization as well as a new methodology have been incorporated that use the results of the researches made in the field of numerical mathematics and non-numerical mathematics of uncertainty, the contribution of which represents an essential aspect for the development of new models and algorithms (Gil Aluja, 1999; Gil Aluja, 2004; A.M. Gil Lafuente, 2004).

Indeed, the constant interest of the entrepreneur on trade issues that are supposed to give the possibilities in evaluation of the company's future represent a stimulus for the scientific and technical development of marketing, in general.

The omnipresent evolutionary economic process, which has been expanding over the years, has added new realities in our environment. Classical schemes that were useful in the creation of the economic science are now less appropriate in describing and dealing with a new management and economic phenomena. Among the first mathematicians and philosophers who have tried to find a solution for these anomalies were Roussel, Lukaciewicz, Lee, Gödel, Moissil, among others.

Nevertheless a real success in solving so complex objectives has been achieved in 1965 when Lotfi Zadeh published the article with a general title such as "Fuzzy sets" (Zadeh, 1965). Among other scientists who put their talent and experience in the study and development of management problems, in general, it is important to mention A. Kaufmann and J. Gil Aluja (Kaufmann and Gil Aluja, 1992; Gil Aluja, 2004).

There is a large number of studies developed by these scientists which focus on the new techniques and mathematical tools able to manage the questions of uncertainty in economics and business management. In 1986 it was published the first book devoted to the introduction of the theory of fuzzy subsets theory into management (Kaufmann and Gil Aluja, 1987).

Another remarkable work was developed by A. M. Gil Lafuente in 2004 titled as *The analysis of fixed assets under uncertainty*. The author says that over the last decades the noticeable changes were produced in the management of fixed assets among them: extension of the production horizon, minimization of the wastage and incorporation of new technologies (A.M. Gil Lafuente, 2004).

The author proposes the new tools for the treatment of the current problems in the assets' management. The necessary processes have been considered to transform the schemes that are valid in the area of determinism into the formal structures that were suitable for the adoption of the decisions under uncertainty.

There have been developed the models for searching the optimal time of renewal of fixed assets, where the time represents a supporting role and, on the contrary, a price of property and maintenance costs have gained a high importance.

There is another group of models where the phenomenon of time is introduced through the interest rate as an essential and active factor. The restrictive nature of these models has led to the development of the formal structures oriented to the uncertain numbers.

A special attention is paid to the famous "Adverse Minimum Method" that revolutionized the studies on fixed assets. It was introduced into the field of uncertainty through one of the models of FAMM in which the parameters are replaced by triplets of confidence (Terborgh, 1967).

Another solution for describing and treating the problems of management, in general, as well as the themes of renewal, in particular, that the author proposes in her study is the using of possibilities introduced by neural networks.

One part of this work is devoted to recall the probabilistic studies that allow to determine the concepts ("law of life", "type of fault", etc.) on which the most of the models is based. They help to solve the issues related to the substitution and usage of the fixed assets under uncertainty.

The author demonstrates that the new mathematics of uncertainty together with its logical and technical tools is very useful in solving the growing problems created by the tangible assets, more complex and sophisticated each time.

5. Inventory process of the finished products under uncertainty

It is a very common situation when the company has a necessity to maintain certain levels of production. For this purpose it is necessary to make a stock of different items in sufficient quantities to guarantee those levels that satisfy a “use function” (A.M. Gil Lafuente, 2004).

Hence we obtained a new concept such as inventory rate. For the specific cases such as shopping centres the main supply of the products will be represented by the number of finished products. The amount or level of supply will depend on the law of predicted demand. Let's study a basic theory of this approach taking into account the context of uncertainty in which the company operates.

Let's consider $N_0 = n(0) / n(0)$ identical elements of one product that are put for sale at time $t = 0$. Let's $v(t) = n(t) / n(0)$ be a law of survival of this product taking into account that there is no any replacement of the product (where $n(t)$ is a number of goods items that are in good condition for the sale at time t). In this case, at time t , the following number of items will be at disposal:

$$n(t) = n(0) \cdot v(t) = N_0 \cdot v(t) \quad (1)$$

Let's establish that the number of elements of the product offered for sale from time $t = 0$ is determined by a law called “use law” designated by $\varphi(t)$. Therefore it will be necessary to replace the items of this product that are expired or no longer fit for sales. Now let's look for the law $r(t)$ in order to provide the number of replacements to be made from the moment $t = 0$ to t .

We will treat this problem using the discrete values of the temporal measure such as: $t \in \{0, 1, 2, 3, \dots\}$.

If $r(u)$, $u \in N_0$ is the number of the elements of a particular type of the product that were replaced from $t = 0$ to $t = u$, then, in the interval $u-1$ to u , it will be replaced the following number of the expired products for new ones:

$$\tau(u) = r(u) - r(u-1) \quad (2)$$

A magnitude $\tau(u)$ is called a “supply rate”. From this situation at time u , some elements won't be useful any longer at time $t > u$ and the number of goods survived from $r(u)$ will be calculated as follows:

$$\tau(u) \cdot v(t-u) = (r(u) - r(u-1)) \cdot v(t-u) \quad (3)$$

Definitely, at the moment t , the number of survived items taking into account the initial and replaced number of goods, when the “use law” is $\varphi(t)$, will be:

$$N_0 \cdot v(t) + \sum_{u=1}^t \tau(u) \cdot v(t-u) = \varphi(t) \quad (4)$$

and this is, as a result of getting a sum for all instances u when the replacements have been done. This expression can be represented in a following recurring form:

For $t=1$:

$$\begin{aligned} \varphi(1) &= N_0 \cdot v(1) + \tau(1) \cdot v(0) = N_0 \cdot v(1) + \tau(1), \text{ where } v(0) = 1 \\ \tau(1) &= \varphi(1) - N_0 \cdot v(1) \end{aligned} \quad (5)$$

For $t=2$:

$$\begin{aligned}\varphi(2) &= N_0 \cdot v(2) + \tau(1) \cdot v(1) + \tau(2) \cdot v(0) \\ \tau(2) &= \varphi(2) - N_0 \cdot v(2) - \tau(1) \cdot v(1)\end{aligned}\quad (6)$$

where $v(t)$ and $\varphi(t)$ are known and $\tau(1)$ has been found previously.
For $t=3$:

$$\begin{aligned}\varphi(3) &= N_0 \cdot v(3) + \tau(1) \cdot v(2) + \tau(2) \cdot v(1) + \tau(3) \cdot v(0) \\ \tau(3) &= \varphi(3) - N_0 \cdot v(3) - \tau(1) \cdot v(2) - \tau(2) \cdot v(1)\end{aligned}\quad (7)$$

And so, we have for all necessary periods t :

$$\begin{aligned}\varphi(t) &= N_0 \cdot v(t) + \sum_{u=1}^{t-1} \tau(u) \cdot v(t-u) + \tau(t) \cdot v(0) = N_0 \cdot v(t) + \sum_{u=1}^t \tau(u) \cdot v(t-u) \\ \tau(t) &= \varphi(t) - N_0 \cdot v(t) - \sum_{u=1}^{t-1} \tau(u) \cdot v(t-u)\end{aligned}\quad (8)$$

Thus, $\tau(t)$ will be found if the terms such as N_0 , $v(t)$ and $\varphi(t)$ are known. As for $r(u)$, it will be sufficient to sum all replacements done:

$$r(t) = \sum_{u=1}^t \tau(u) \quad (9)$$

Regarding the function $\varphi(t)$, it can be increasing and decreasing. It can also be negative, but when it happens it is necessary to use the rule of randomness separating the goods which are in excess, and then we have:

$$\tau(t) < 0 \quad \text{will be substituted for} \quad \tau(t) = 0 \quad (10)$$

The redundant products won't be used for sale again after being removed at random previously.

6. Study case

For this study case we decided to consider three alimentary products that, on the one hand, are of the first necessity goods and, on the other hand, are perishable that require special storage conditions. Finally, these products can be easily found by the consumers in any shop or supermarket.

These products are fresh white bread, chocolate muffins, unpasteurized yogurt and bananas. The useful life of these products is from 2 to 20 days.

The stock management of these goods involves a high degree of a control from the part of a company in order to guarantee the sales before their expiry date avoiding, by this manner, the yield losses. Thus, the correct storage and a well-planned supply process are two issues of a vital interest for the companies' administration. There are some other factors that increase the importance of these issues, among them: constant changes in the preferences of final consumers, growing competition between the companies, internationalization of the market and others.

Here we consider a media-size surface commercial company that is represented by a "supermarket". The supermarket is an urban commercial store that sells different consumer goods under self-service system. These kind of stores offer products at low prices, that, in turn, implies that in order to generate profits the supermarkets attempt to counteract the low profit margin with high sales volume. The aim is thus a high turnover of the products.

Let's develop a scheme for the inventory process that represents a very important component for the logistics system of the supermarket. We analyse four different alimentary products in order to demonstrate a sensibility of the results represented by their big difference that our mathematical technique provides.

6.1. Product of interest: bread and pastries

Bread is a basic food that forms a part of a traditional diet of the most countries. This product can be bought at any store and at any department of a supermarket.

To confirm the efficiency and sensibility of the method for searching the product supply rate under the context of uncertainty, we decided to make the calculus for two different cases. Introducing different initial data for N_0 and $v(t)$ we observed significant changes produced in the results. Let's start from the following information presented in the Table 6.1.

Table 6.1 Fresh white bread. Cases I and II

t	$v(t) = n(t) / n(0)$	$\varphi(t)$	t	$v(t) = n(t) / n(0)$	$\varphi(t)$
0	(1; 1; 1)	1200	0	(1; 1; 1)	700
1	(0,80; 0,85; 0,90)	1200	1	(0,60; 0,65; 0,68)	1200
2	(0,50; 0,55; 0,60)	1250	2	(0,30; 0,35; 0,38)	1250
3	(0,15; 0,20; 0,20)	1300	3	(0,10; 0,15; 0,17)	1300
4	(0; 0; 0)	1400	4	(0; 0; 0)	1400
5	-	1500	5	-	1500
6	-	1650	6	-	1650
7	-	1200	7	-	1200
8	-	1250	8	-	1250
9	-	1400	9	-	1200
10	-	1450	10	-	1250

Where t is a period of time considered in days; $\varphi(t)$ is a demand function and $v(t)$ is the survival function.

Taking into account all necessary information and a scheme proposed previously, that is:

$$\varphi(t) = N_0 \cdot v(t) + \sum_{u=1}^{t-1} \tau(u) \cdot v(t-u) + \tau(t) = N_0 \cdot v(t) + \sum_{u=1}^t \tau(u) \cdot v(t-u)$$

and

$$\tau(t) = \varphi(t) - N_0 \cdot v(t) - \sum_{u=1}^{t-1} \tau(u) \cdot v(t-u)$$

we obtained the following results:

For case I:

$$\begin{aligned} t=1: \varphi(1) &= 1200; N_0 = 1200; v(0) = (1; 1; 1); v(1) = (0,80; 0,85; 0,90) \\ \varphi(1) &= N_0 \cdot v(1) + \tau(1) \cdot v(0) = N_0 \cdot v(1) + \tau(1) \\ \tau(1) &= \varphi(1) - N_0 \cdot v(1) = 1200 - 1200 \cdot (0,80; 0,85; 0,90) = (240; 180; 120) \end{aligned}$$

$$\begin{aligned} t=2: \varphi(2) &= N_0 \cdot v(2) + \tau(1) \cdot v(1) + \tau(2) \\ \tau(2) &= \varphi(2) - N_0 \cdot v(2) - \tau(1) \cdot v(1) = 1250 - 1200 \cdot (0,50; 0,55; 0,60) - \\ &\quad - (240; 180; 120) \cdot (0,80; 0,85; 0,90) = (422; 437; 458) \dots \end{aligned}$$

$$\begin{aligned} t=3: \varphi(3) &= N_0 \cdot v(3) + \tau(1) \cdot v(2) + \tau(2) \cdot v(1) + \tau(3) \\ \tau(3) &= \varphi(3) - N_0 \cdot v(3) - \tau(1) \cdot v(2) - \tau(2) \cdot v(1) = 1300 - 1200 \cdot \end{aligned}$$

$$\begin{aligned} & \cdot (0,15;0,20;0,20) - (240;180;120) \cdot (0,50;0,55;0,60) - (422;437;458) \cdot \\ & \cdot (0,80;0,85;0,90) = (575,80;589,55;662,40) \\ & \dots\dots \end{aligned}$$

For case II:

$$\begin{aligned} t=1: \varphi(1) &= 1200; N_0 = 700; v(0) = (1;1;1); v(1) = (0,60;0,65;0,68) \\ \varphi(1) &= N_0 \cdot v(1) + \pi(1) \cdot v(0) = N_0 \cdot v(1) + \pi(1) \\ \pi(1) &= \varphi(1) - N_0 \cdot v(1) = 1200 - 700 \cdot (0,60;0,65;0,68) = (780;745;724) \end{aligned}$$

$$\begin{aligned} t=2: \varphi(2) &= N_0 \cdot v(2) + \pi(1) \cdot v(1) + \pi(2) \\ \pi(2) &= \varphi(2) - N_0 \cdot v(2) - \pi(1) \cdot v(1) = 1250 - 700 \cdot (0,30;0,35;0,38) - \\ & - (780;745;724) \cdot (0,60;0,65;0,68) = (491,68;520,75;572,00) \end{aligned}$$

$$\begin{aligned} t=3: \varphi(3) &= N_0 \cdot v(3) + \pi(1) \cdot v(2) + \pi(2) \cdot v(1) + \pi(3) \\ \pi(3) &= \varphi(3) - N_0 \cdot v(3) - \pi(1) \cdot v(2) - \pi(2) \cdot v(1) = 1300 - 700 \cdot \\ & \cdot (0,10;0,15;0,17) - (491,68;520,75;572,00) \cdot (0,30;0,35;0,38) - \\ & - (491,68;520,75;572,00) \cdot (0,60;0,65;0,68) = (516,92;595,76;700,99) \\ & \dots\dots \end{aligned}$$

Table 6.2 Results of $\pi(t)$ for cases I and II

$\pi(1)$	(240;180;120)	$\pi(1)$	(780;745;724)
$\pi(2)$	(422;437;458)	$\pi(2)$	(491,68;520,75;572,00)
$\pi(3)$	(575,80;589,55;662,40)	$\pi(3)$	(516,92;595,76;700,99)
$\pi(4)$	(505,04;622,53;692,36)	$\pi(4)$	(582,89;718,74;864,34)
$\pi(5)$	(387,84;559,19;744,77)	$\pi(5)$	(548,63;746,19;946,02)
$\pi(6)$	(431,81;714,38;1000,84)	$\pi(6)$	(559,08;824,05;1094,26)
$\pi(7)$	(0;160,71;584,88)	$\pi(7)$	(0;295,39;641,67)
$\pi(8)$	(500,54;745,25;975,92)	$\pi(8)$	(673,36;849,65;1027,41)
$\pi(9)$	(0;487,05;934,79)	$\pi(9)$	(79,00;532,12;940,08)
$\pi(10)$	(0;593,97;1199,73)	$\pi(10)$	(311,25;762,44;1200,59)

It is possible to observe that there are significant differences in the obtained results. For the first case, if the supermarket can provide or has available the exact amount of bread, which is estimated to be sold the first day of the week, then the volume of inventory for the first day won't be high taking into consideration that the level of useful life (or survival function) of the product for this day is (0.80;0.85 0.90) but not 1.

For the second case, assuming that the company was not been able to estimate precisely the volume of the total sales of bread or, because of other reason it could not store 100% of the food volume to cover the sales for the first day, it is observed, as a consequence, the increase of the rate of inventory/ supply that the company should provide continuously starting from the first day. The fact of the decrease of survivability of the product will demonstrate that the inventory rate that the company should provide will increase noticeably.

It is also observed that if the demand function changes, in this case the company makes the estimates of sales (increasing or decreasing), the differences of the inventory rate are significant. If the sales are reduced significantly than the levels of food supply are also reduced. If the company plans to sell more then this fact will be reflected in the growth of the level of inventory.

Incorporating a probability of survival represented by fuzzy numbers (triplets of confidence) into the calculus we saw that the margins of the inventory rates were wider then if we would use the precise numbers. This detail helps us to look for the supply rate that is closer to the real situation. We calculated three different amounts of supply: the minimum supply rate, the maximum one and the level of maximum presumption. This approach of the problem allows to predict the necessary costs in order to perform the worst and best cases of the sales' scenarios. It is more reasonable to have a range of supply rate forecasts represented in the form of scenarios in order to react to unpredictable situations with less risk.

Basing on the following initial information we made the same approach to the problem with fresh chocolate muffins. We considered two different cases using the same scheme of calculus. See Table 6.3.

Table 6.3 Fresh chocolate muffins. Cases I and II

t	$v(t) = n(t) / n(0)$	$\varphi(t)$	t	$v(t) = n(t) / n(0)$	$\varphi(t)$
0	(1; 1; 1)	230	0	(1; 1; 1)	100
1	(1; 1; 1)	230	1	(1; 1; 1)	230
2	(0,92; 0,95; 0,97)	250	2	(0,86; 0,90; 0,93)	250
3	(0,85; 0,90; 0,93)	300	3	(0,75; 0,80; 0,83)	300
4	(0,77; 0,82; 0,85)	320	4	(0,66; 0,70; 0,73)	320
5	(0,55; 0,60; 0,62)	360	5	(0,55; 0,60; 0,63)	360
6	(0,42; 0,45; 0,48)	400	6	(0,36; 0,40; 0,43)	400
7	(0,30; 0,35; 0,37)	220	7	(0,16; 0,20; 0,22)	220
8	(0,05; 0,10; 0,10)	250	8	(0,03; 0,08; 0,10)	250
9	(0; 0; 0)	290	9	(0; 0; 0)	290
10	-	310	10	-	310
11	-	350	11	-	350
12	-	380	12	-	380

For case I:

$$t=1: \varphi(t)=230; N_0=230; v(0)=(1;1;1); v(1)=(1;1;1)$$

$$\varphi(1)=N_0 \cdot v(1) + \pi(1) \cdot v(0) = N_0 \cdot v(1) + \pi(1)$$

$$\pi(1)=\varphi(1)-N_0 \cdot v(1)=230-230 \cdot (1;1;1)=(0;0;0)$$

$$t=2: \varphi(2)=N_0 \cdot v(2) + \pi(1) \cdot v(1) + \pi(2)$$

$$\pi(2)=\varphi(2)-N_0 \cdot v(2) - \pi(1) \cdot v(1) = 250 - 230 \cdot (0,92;0,95;0,97) -$$

$$- (0;0;0) \cdot (1;1;1) = (26,90;31,50;38,40)$$

$$t=3: \varphi(3)=N_0 \cdot v(3) + \pi(1) \cdot v(2) + \pi(2) \cdot v(1) + \pi(3)$$

$$\pi(3)=\varphi(3)-N_0 \cdot v(3) - \pi(1) \cdot v(2) - \pi(2) \cdot v(1) = 300 - 230 \cdot$$

$$\cdot (0,85;0,90;0,93) - (0;0;0) \cdot (0,92;0,95;0,97) - (26,90;31,50;38,40) \cdot$$

$$\cdot (1;1;1) = (47,70;61,50;77,60)$$

.....

For case II:

$$t=1: \varphi(t)=230; N_0=100; v(0)=(1;1;1); v(1)=(1;1;1)$$

$$\varphi(1)=N_0 \cdot v(1) + \pi(1) \cdot v(0) = N_0 \cdot v(1) + \pi(1)$$

$$\pi(1)=\varphi(1)-N_0 \cdot v(1)=230-100 \cdot (1;1;1)=(130;130;130)$$

$$t=2: \varphi(2)=N_0 \cdot v(2) + \pi(1) \cdot v(1) + \pi(2)$$

$$\pi(2)=\varphi(2)-N_0 \cdot v(2) - \pi(1) \cdot v(1) = 250 - 100 \cdot (0,86;0,90;0,93) -$$

$$- (130;130;130) \cdot (1;1;1) = (25;30;34)$$

$$t=3: \varphi(3)=N_0 \cdot v(3) + \pi(1) \cdot v(2) + \pi(2) \cdot v(1) + \pi(3)$$

$$\pi(3)=\varphi(3)-N_0 \cdot v(3) - \pi(1) \cdot v(2) - \pi(2) \cdot v(1) = 300 - 100 \cdot$$

$$\cdot (0,75;0,80;0,83) - (130;130;130) \cdot (0,86;0,90;0,93) -$$

$$- (25;30;34) \cdot (1;1;1) = (57,50;73,00;88,20)$$

.....

Having done all necessary calculus we obtained the following results.

Table 6.4 Results of $\pi(t)$ for cases I and II

$\pi(1)$	(0;0;0)	$\pi(1)$	(130;130;130)
$\pi(2)$	(26,90;31,50;38,40)	$\pi(2)$	(25;30;34)
$\pi(3)$	(47,70;61,50;77,60)	$\pi(3)$	(57,50;73,00;88,20)
$\pi(4)$	(9,65;39,98;70,45)	$\pi(4)$	(14,00;46,00;77,50)
$\pi(5)$	(35,96;95,25;157,10)	$\pi(5)$	(7,31;73,30;137,00)
$\pi(6)$	(0,82,09;197,30)	$\pi(6)$	(0,87,90;213,53)
$\pi(7)$	(0;0;58,18)	$\pi(7)$	(0;0;88,71)
$\pi(8)$	(0;0;162,97)	$\pi(8)$	(0;0;170,85)
$\pi(9)$	(0;75,33;228,90)	$\pi(9)$	(0;95,17;248,88)
$\pi(10)$	(0;67,54;270,51)	$\pi(10)$	(0;73,92;290,99)
$\pi(11)$	(0;98,64;329,61)	$\pi(11)$	(0;93,33;343,40)
$\pi(12)$	(0;75,13;368,73)	$\pi(12)$	(0;90,51;378,41)

The results of the calculus for this product slightly differ from those we've seen for the case of bread. There are two main reasons. The first one is that the lifecycle of the product is longer than in the case of bread and, the second one is that the estimations of demand function are smaller in comparison with the sales volumes of bread. The inventory rate decreases if both the lifecycle increases and the demand decreases. If the survival rate of the product decreases slightly and at the same time the company doesn't have enough stock of this product at the initial moment then the level of the supply rate increases causing by this manner the extremes of the supply intervals are more distant (with greater uncertainty).

As in the case of bread, the introduction of the triplets of confidence into the calculus that has made possible to find the supply rate most likely to occur. We could calculate three different supply amounts. This approach will give the company a broader view of the future prospects for its inventory process, so as to bring out more wisely their actions in the planning process and to react faster to the changes of the market.

6.2. Product of interest: unpasteurized yogurt

Yogurt is a dairy product obtained by the bacterial fermentation of milk. There are two types of yogurt: "live" (unpasteurized) yogurt and pasteurized yogurt. A recommended period of consumption of the "live" yogurt is 3 weeks while it is much longer for the pasteurized yogurt. We considered only the unpasteurized yogurt.

Table 6.5 Unpasteurized yogurt. Cases I and II

t	$v(t) = n(t) / n(0)$	$\varphi(t)$	t	$v(t) = n(t) / n(0)$	$\varphi(t)$
0	(1; 1; 1)	420	0	(1; 1; 1)	250
1	(1; 1; 1)	420	1	(1; 1; 1)	420
2	(1; 1; 1)	390	2	(1; 1; 1)	390
3	(0,97; 0,98; 0,99)	410	3	(0,92; 0,95; 0,96)	410
4	(0,95; 0,96; 0,97)	420	4	(0,90; 0,92; 0,94)	420
5	(0,93; 0,94; 0,95)	430	5	(0,85; 0,88; 0,89)	430
6	(0,91; 0,92; 0,93)	440	6	(0,81; 0,84; 0,85)	440
7	(0,88; 0,90; 0,91)	390	7	(0,75; 0,78; 0,80)	390
8	(0,85; 0,86; 0,87)	410	8	(0,66; 0,70; 0,73)	410
9	(0,82; 0,83; 0,84)	410	9	(0,60; 0,62; 0,65)	410
10	(0,78; 0,80; 0,81)	420	10	(0,52; 0,56; 0,60)	420
11	(0,74; 0,75; 0,77)	420	11	(0,45; 0,49; 0,52)	420
12	(0,70; 0,72; 0,73)	430	12	(0,40; 0,42; 0,44)	430
13	(0,67; 0,68; 0,69)	380	13	(0,33; 0,36; 0,39)	380
14	(0,63; 0,65; 0,66)	390	14	(0,26; 0,30; 0,32)	390

Having done all necessary calculus we obtained the following results.

Table 6.6 Results of $\pi(t)$ for cases I and II

$\pi(1)$	(0;0;0)	$\pi(1)$	(170;170;170)
$\pi(2)$	(0;0;0)	$\pi(2)$	(0;0;0)
$\pi(3)$	(0;0;2,60)	$\pi(3)$	(0;2,50;10,00)
$\pi(4)$	(10,00;16,80;21,00)	$\pi(4)$	(11,80;26,00;38,60)
$\pi(5)$	(7,40;18,40;29,40)	$\pi(5)$	(0;25,10;52,70)
$\pi(6)$	(0;18,40;40,40)	$\pi(6)$	(0;26,93;81,20)
$\pi(7)$	(0;0;3,30)	$\pi(7)$	(0;0;53,94)
$\pi(8)$	(0;0;36,32)	$\pi(8)$	(0;25,51;106,88)
$\pi(9)$	(0;9,91;49,27)	$\pi(9)$	(0;36,84;137,77)
$\pi(10)$	(0;23,67;76,42)	$\pi(10)$	(0;41,60;178,44)
$\pi(11)$	(0;22,07;93,67)	$\pi(11)$	(0;32,82;210,25)
$\pi(12)$	(0;24,21;120,99)	$\pi(12)$	(0;46,87;245,71)
$\pi(13)$	(0;0;84,11)	$\pi(13)$	(0;0;222,42)
$\pi(14)$	(0;0;111,53)	$\pi(14)$	(0;33,44;262,76)

Due to the fact that the lifetime of pasteurized yoghurt is longer than for bread and pastries, the survival function is distributed more gradually over the time, it is to say, that the function decreases as the three weeks pass. Taking into account this detail it is possible to see that the levels of supply rates do not differ significantly among themselves for two different cases. If, for the first case, the amount of yogurt stored at the moment 0 is equal to the estimated level of demand and, for the second case, this amount is smaller than the demand level at the moment 1, than the company, in the second case, would have a greater cost in the first period of sales and over the next 3 weeks would have to make a control and greater expenditure to replenish the inventory of yoghurt.

6.3. Product of interest: bananas

Fruits are a set of plant products that comes from the cultivation of the certain plants, whether it can be herbs such as melon or apricot trees. The fruits have their characteristic flavour and aroma; they possess special nutritional properties and have a chemical composition which distinguishes them from other foods. The fruit is a perishable product and its useful life is from 5 to 10 days. In this paper we considered only a ripe banana, and the obtained results we represent below.

Table 6.7 Ripe banana. Cases I and II

t	$v(t) = n(t) / n(0)$	$\phi(t)$	t	$v(t) = n(t) / n(0)$	$\phi(t)$
0	(1; 1; 1)	250	0	(1; 1; 1)	150
1	(1; 1; 1)	250	1	(0,93; 0,95; 0,97)	250
2	(0,94; 0,95; 0,97)	260	2	(0,87; 0,90; 0,93)	250
3	(0,86; 0,90; 0,93)	270	3	(0,76; 0,80; 0,84)	280
4	(0,77; 0,80; 0,82)	280	4	(0,65; 0,70; 0,74)	260
5	(0,66; 0,70; 0,72)	300	5	(0,53; 0,57; 0,60)	330
6	(0,55; 0,60; 0,63)	330	6	(0,41; 0,46; 0,48)	300
7	(0,45; 0,50; 0,52)	310	7	(0,30; 0,35; 0,40)	310
8	(0,35; 0,40; 0,42)	320	8	(0,20; 0,25; 0,29)	300
9	(0,26; 0,30; 0,32)	320	9	(0,10; 0,15; 0,17)	340
10	(0,05; 0,10; 0,13)	340	10	(0; 0,05; 0,05)	320

Having done all necessary calculus we obtained the following conclusions.

Table 6.8 Results of $\pi(t)$ for cases I and II

$\pi(1)$	(0;0;0)	$\pi(1)$	(110,50;107,50;104,50)
$\pi(2)$	(17,50;22,50;25,00)	$\pi(2)$	(9,14;12,88;16,74)
$\pi(3)$	(12,50;22,50;37,50)	$\pi(3)$	(40,58;51,02;61,37)
$\pi(4)$	(13,25;36,13;38,55)	$\pi(4)$	(0;8,94;32,83)
$\pi(5)$	(1,83;47,25;94,95)	$\pi(5)$	(59,69;104,54;136,43)
$\pi(6)$	(0;60,18;154,00)	$\pi(6)$	(0;12,54;87,64)
$\pi(7)$	(0;13,67;163,21)	$\pi(7)$	(0;51,85;136,54)
$\pi(8)$	(0;62,15;202,85)	$\pi(8)$	(0;88,70;166,23)
$\pi(9)$	(0;27,18;230,10)	$\pi(9)$	(0;40,83;244,72)
$\pi(10)$	(0;93,33;307,26)	$\pi(10)$	(0;42,73;263,31)

8. Conclusions

In this study we developed a theme such as inventory process that represents a very important component for the logistics system of a modern company. We made a theoretical approach and practical development of the case related with the food inventory in the commercial enterprise. We analysed four different alimentary products in order to demonstrate a sensibility of the results that were represented by their big difference. We used a mathematical tool, which is one of the well-known classical methods of the operational calculus that calculates an inventory rate under uncertainty using some elements of fuzzy arithmetic. This technique is very useful when the large number of products is offered. The mathematical model deals with the lifecycle of different products. For this type of problems it was proposed the following solution: a given “use function”, which shows quantitatively the needs of the food, we find a “inventory rate” for each period of time, that is to say, the number of the products which is necessary to incorporate together with the existing stocks to cover the quantity fixed by the demand function. The process of formalization of the model under uncertainty has been made using a classical equation, which obtains the rate of the products supply at a particular moment using the use function, the survival law and the supply rate of the previous moments. The sequential process allows finding a rate for each of the different moments taking into account previously established economic horizon. The uncertainty is incorporated into the process using the triplets of confidence.

Using this approach we tried to demonstrate the importance of this issue for the modern company. The use of the uncertain numbers in the models can provide not only the maximum and minimum supply levels at a particular time but also a large “description” of the company’s performance, that is to say, the values of the function between two extreme points in order to increase the range of possibilities at the moment of making decisions.

As a conclusion it is necessary to say that the mathematical model under uncertainty developed in this paper can give a big contribution into logistics management of any company. This can give an opportunity to continue a business with the highest profitability levels and maximum satisfaction of the final customers and also to fulfil their expectations and new plans in a dynamic environment and globalized economy.

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