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Working Paper
Flattening of the Short-run Trade-off between Inflation and Domestic Activity: The Analytics of the Effects of Globalization

Kiel Working Paper, No. 1363

Provided in Cooperation with:
Kiel Institute for the World Economy (IfW)

Suggested Citation: Razín, Assaf; Binyamini, Alon (2007) : Flattening of the Short-run Trade-off between Inflation and Domestic Activity: The Analytics of the Effects of Globalization, Kiel Working Paper, No. 1363, Kiel Institute for the World Economy (IfW), Kiel

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Flattening of the Short-run Trade-off between Inflation and Domestic Activity: The Analytics of the Effects of Globalization

Assaf Razin and Alon Binyamini

June 2007

This paper is part of the Kiel Working Paper Collection No. 2

“The Phillips Curve and the Natural Rate of Unemployment”
June 2007

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Preliminary Draft
May 27, 2007

Abstract

This paper reviews the analytics of the effects of globalization on the Phillips curve and the utility-based objective function of the central bank. It demonstrates that in an endogenous-policy set up, when trade in goods is liberalized, financial openness increases, and in- and out-labor migration are allowed, policymakers become more aggressive on inflation and less responsive to the output gap. In other words, globalization induces the monetary authority, when guided in its policy by the welfare criterion of a representative household, to put more emphasis on the reduction of inflation variability, at the expense of an increase in the output gap variability.
1. Introduction

Fredric Mishkin (2007) writes about the slope of the US inflation-output tradeoff: “The finding that inflation is less responsive to the unemployment gap, suggests that fluctuations in resource utilization will have smaller implications for inflation than used to be the case. From the point of view of policymakers, this development is a two-edged sword: On the plus side, it implies that an overheating economy will tend to generate a smaller increase in inflation. On the negative side, however, a flatter Phillips curve also implies that a given increase in inflation will be more costly to wring out of the system.”

A massive globalization process also swept emerging markets in Latin America, European transition economies, and East Asia. The 1992 single market reform in Europe and the formation of the Euro zone are important episodes of globalization in this period, as well.

The average annual inflation rate among developing countries was 41 percent in the early 1980s and came down to 13 percent towards the end of the 1990s. Global inflation in the 1990s has dropped from 30 percent a year to about 4 percent a year. Thus, disinflation and globalization go hand in hand.

Ken Rogoff (2003, 2004) elaborates on some favorable factors that have been helping to drive down global inflation in the last two decades. The hypothesis, which he put forth, is

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1 Charles Bean (2006) writes about the UK tradeoff: “One of the most notable developments of the past decade or so has been the apparent flattening of the short-run trade-off between inflation and activity. That is particularly obvious in the case of the United Kingdom, but can also be observed in many other countries. The seventies were characterized by an almost vertical relationship in the United Kingdom, in which attempt to hold unemployment below its natural rate resulted in rising inflation. In the eighties, the downward sloping relationship reappears, as inflation was squeezed out of the system by the slack of the economy. However, since the early Nineties, the relationship looks to have been rather flat. Three factors - increased specialization; the intensification of product market competition; and the impact of that intensified competition and migration on the behavior of wages--should all work to flatten the short-run tradeoff between inflation and domestic activity.”
that the “globalization—interacting with deregulation and privatization—has played a strong supporting role in the past decade’s disinflation.”

In the EU a potential significant effect on the Phillips curve is attributable to migration. There is, for example, a fundamental change affecting the Spanish labor market over the last decade. In 1995 the percentages of foreigners in the Spanish population and in the Spanish labor force were, respectively, below 1% and below 0.5%. At the end of 2006, these rates were around 9% and 14%, respectively. The immigration boom impact on the Phillips curve has been recently addressed by Bentolila, Dolado and Jimeno (2007).

Studies that present evidence of a marked decline in the sensitivity of U.S. inflation to unemployment and other measures of resource utilization include Roberts (2006) and Williams (2006). Unpublished work by staff at the Federal Reserve Board in the USA indicates that this result generally holds across a variety of regression specifications, estimation methods, and data definitions. Other studies find similar declines in many foreign industrial economies; see, among others, Borio and Filardo (2006) and Ihrig and others (forthcoming). Studies that present evidence on the effect of globalization on the slope of the Phillips curve include Loungani, Razin and Yuen (2001) and Razin and Loungani (2007). Previously, Romer (1993, 1998), and Lane (1997) show that inflation and trade liberalization are negatively, and significantly, correlated in large (flexible exchange rate) OECD economies. Gali and Monacelli, (2003) analyze the effect of exchange rate movements on inflation. More recently, Chen, Imbs and Scott (2004) investigate the competitive effects of increased international trade in goods and services on prices, productivity and markups. Using disaggregated data for EU manufacturing over the period 1988-2000 they find that increased openness exerts a negative and significant impact on sectors prices. Increased openness lowers prices by reducing markups and by raising productivity. Their results suggest that the increase in the trade volume could account for as much as a quarter of European disinflation over the sample period. On the effect of productivity on the tradeoff see Ball and Moffitt (2001).

The purpose of this paper is to review the analytics of this remarkable phenomenon, the flattening effects of globalization on the aggregate supply in the New Keynesian framework. Globalization incorporates international capital mobility, international trade in goods, and international migration. An important implication of the flattening of the inflation-activity relationship is that to the extent that monetary authorities are guided by
the objective function of the representative household, these globalization forces provide a good reason for central banks to become aggressive with regard to inflation. The organization of the paper is as follows. Section 2 describes the analytical framework. Section 3 derives the aggregate supply relationship. Section 4 discusses welfare and monetary policy implications. Section 5 concludes.

2. Analytical Framework

The analytical framework draws on the New Keynesian macroeconomics literature. Main features of the model are as follows.

(1) The domestic economy produces a continuum of varieties. The decisions of the representative household are governed by Dixit-Stiglitz preferences over varieties (generating fixed elasticities). Purchasing power parity condition prevails and foreign firms' prices are exogenous.

(2) There is a representative household whose utility is defined over consumption and leisure, as in the standard micro-based welfare analysis.

(3) Price updates are staggered (see Calvo (1983)). Producers update prices upon receiving a price-update signal drawn from a stochastic time interval distribution.

(4) Labor supply of the domestic household is divided between domestic and foreign destinations. Exported labor receive wage premium over unskilled foreign labor wage. There exists greater disutility in supplying labor to the rest of world, compared to supplying labor domestically.

(5) Domestic output is produced with domestic (skilled) and foreign (unskilled) labor.

(6) There is international trade in goods and bonds.

2.1. Household

The representative household preferences are represented by the utility function:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(C_t, \xi_t) - \int_0^1 \left[ h^\text{home} (j) + \phi \cdot h^\text{exp} (j) \right]^{1+\varphi} \cdot dj + \Gamma \left( \frac{M_t}{P_t}, \xi_t \right) \right]$$

(1)
Where $\beta$ is a discount factor. The instantaneous utility function consists of consumption composite $C_t$, domestic labor supply, employed by domestic producers, $h_t^{\text{hom}}(j)$, domestic labor supply employed by the foreign country, $h_t^{\text{exp}}(j)$, and of real balances $\frac{M_t}{P_t}$, the ratio of money holdings and the price level. The parameter $\phi > 1$ indicates the home bias in the supply of labor (as originally suggested by Engler (2007)). $\xi_t$ is a preferences shock. The consumption composite, $C_t$, is a Dixit-Stiglitz aggregate of goods produced at home and imported goods:

$$C_t = \left[ \int_0^n (c_{H,j})^{\phi-1} \cdot dj + \int_1^n (c_{W,j})^{\phi-1} \cdot dj \right]^{\frac{1}{\phi-1}}$$  (2)

Where $n$ is the number of domestically produced varieties, $1-n$ is the number of foreign produced varieties. Subscripts $H$ & $W$ indicate the home country and the representative foreign country, respectively. Thus, the variable $c_{i,j}(j)$ is the consumption level of variety $j$, which is produced in country $i \in \{H,W\}$. All goods are tradable. The infinite continuum of goods is uniformly distributed over the unit interval: $j \in [0,1]$. The parameter $\theta > 1$ is the Constant Elasticity of Substitution in consumption.

The budget constraint of the domestic representative household is:

$$P_t C_t - P_t T_t + M_t + B_{H,t} + \epsilon_t B_{W,t} - (1 + i_{H,t-1})B_{H,t-1} - (1 + i_{W,t-1})\epsilon_t B_{W,t-1} =$$

$$M_{t-1} + \mu_t^{H} w_t^{H} \int_0^n h_t^{\text{hom}}(j) \cdot dj + \epsilon_t \cdot \mu_t^{w} w_t^{w} \int_1^n h_t^{\text{exp}}(j) \cdot dj + \int_0^1 \Pi_t(j) dj$$  (3)

Where:

- $B_{H,t} = $ Bond holdings at the beginning of date $t$ (denominated in the domestic currency).
- $B_{W,t} = $ Bond holdings at the beginning of date $t$ (denominated in the foreign currency).
- $M_t = $ Money holdings in the end of period $t$.
- $P_t = $ The Consumer price level.
- $w_t^{H} = $ Wage rate of unskilled labor in the domestic market.
$w_i^w = \text{ Wage rate of unskilled labor in the foreign market.}$

$\mu_i^H = \text{ Skill premium of the native born labor.}$

$\mu_i^w = \text{ Skill premium, for the domestic native born labor, in the foreign market.}$

$i_{H,t} = \text{ The interest rate in the domestic economy.}$

$i_{w,t} = \text{ The world interest rate.}$

$\Pi_i(j) = \text{ Profit of the domestic j firm.}$

$\epsilon_t = \text{ Exchange rate in period t.}$

$T_t = \text{ Government lump-sum transfers.}$

Free migration of unskilled labor implies that $w_i^H = \epsilon_t \cdot w_i^w$.

### 2.2. Producers

Domestic firms produce with the aid of a non-increasing return to scale CES production function, using labor input of native born and immigrants:

$$y_i(j) = A_i \cdot \left[ (1 - \nu) \frac{v}{v+1} \cdot h_i^{\text{hom}}(j)^{\frac{1}{v}} + \left( \psi \right)^{\frac{v-1}{v}} \cdot h_i^{\text{imp}}(j)^{\frac{1}{v}} \right]^{\frac{1}{v-1}}$$

(4)

Where $y_i(j)$ is the output level of the $j^{th}$ firm, $A_i$ is an exogenous aggregate shock, common to all firms. The parameter $\nu > 1$ is inversely proportional to the elasticity of substitution between imported and local labor inputs. That is, the elasticity is given by $\frac{V}{V-1}$; the degree of diminishing return to scale is given by $\chi \leq 1$. The variable $h_i^{\text{imp}}(j)$ is endogenously determined amount of immigrant employed by firm $j$. We assume that natives are more skilled then the migrants. Hence, $\psi \in \left[ 0, \frac{1}{2} \right]$. It follows that the marginal productivity of native labor is higher function then that of immigrant labor, hence the wage of the native workers includes skill premium over the wage of immigrants, $\mu_i^H > 1$. 
2.3. Labor Market

The representative domestic household allocates time between leisure, labor in the domestic market, and labor in the foreign market. Labor supply satisfies the following first order conditions:

\[
u_t(C_t; \xi_t) \cdot \frac{\mu^H \epsilon_t W^H_t}{P_t} = \left[ h_t^{\text{home}}(j) + \phi h_t^{\text{exp}}(j) \right] \tag{5}\]

\[
u_t(C_t; \xi_t) \cdot \frac{\mu^W \epsilon_t W^W_t}{P_t} = \phi \cdot \left[ h_t^{\text{home}}(j) + \phi h_t^{\text{exp}}(j) \right] \tag{6}\]

Here, \( y_x(X_t) \) denotes the first derivative of \( y \) with respect to \( x \), evaluated at \( x = X_t \).

Dividing (5) by (6) relates the foreign to domestic skill premium ratio, to the home bias. \(^2\)

\[\frac{\mu^W_t}{\mu^H_t} = \phi \tag{7}\]

Whenever the foreign income has higher (lower) purchasing power in the domestic economy, the domestic labor supply shifts leftward (rightward), so that the domestic skill premium increases (decreases). This equalizing process will hold until equation (7) is satisfied.

We assume technology in the rest of the world uses both skilled and unskilled labor inputs, the production function in the representative foreign country is:

\[
\gamma^w(j) = A_t \left[ (1 - \psi^w) \cdot h_t^{\text{Skilled}}(j)^{\frac{1}{\nu}} + \psi^w \cdot h_t^{\text{UnSkilled}}(j)^{\frac{1}{\nu}} \right] \tag{8}\]

Where \( h_t^{\text{Skilled}} \) (\( h_t^{\text{UnSkilled}} \)) denotes the employment of skilled (unskilled) workers by the foreign firms. Those can include both natives & immigrants. Farther assume that the skill dispersion in the world, is higher than the dispersion between locals (skilled) & immigrants (unskilled) in the domestic economy; that is \( \psi^w < \psi \).

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\(^2\) Such relation would prevail in the short run, only in frictionless labor market. Such assumption, although not realistic, is useful simplification which serves us while discussing the intuition behind the mechanism at work.
In appendix A we show that the skill premium paid to native in the domestic economy, is exogenously determined by:

\[
\mu_t^h = \frac{1}{\phi} \cdot \mu_t^w = \frac{1}{\phi} \left[ \frac{1}{\tau_t} \cdot \left(\frac{1 - \psi^w}{\psi^w}\right)^{\frac{1}{1 - v}} \right]^{\frac{1}{1 - v}}
\]  \hspace{1cm} (9)

Where \( \tau_t \equiv \frac{h_t^{\text{Skilled}}(j)}{h_t^{\text{Unskilled}}(j)} \) is the world ratio between skilled and unskilled workers.

As already seen from (7), the skill premium in the domestic economy, \( \mu_t^h \), is negatively related to the home bias, \( \phi \), and positively related to the skill premium in the foreign market, \( \mu_t^w \). (9) shows that the skill premium in the foreign market, is negatively related to the share of skilled workers in the foreign market (captured by \( \tau_t \)), and positively related to productivity dispersion between skilled and unskilled workers (captured by the term \( \frac{1 - \psi^w}{\psi^w} \)).

It further follows that the optimal ratio of migrants to natives, employed by local firms (denoted \( R_t^{i/N} \)), is exogenously determined by:

\[
R_t^{i/N} \equiv \frac{h_t^{\text{imp}}(j)}{h_t^{\text{hom}}(j)} = \frac{1}{\phi^{\frac{1}{1 - v}}} \cdot \frac{1}{\tau_t} \cdot \frac{\psi^w}{(1 - \psi^w)} \frac{1 - \psi^w}{(1 - \psi)}
\]  \hspace{1cm} (10)

The real marginal cost of local producers is therefore given by:

\[
MC_t(j) = Z_t \cdot q_t^j \cdot \frac{1}{A_t} \cdot \frac{1}{\rho} \cdot y_t(j)^{\frac{1 - \chi}{\chi}}
\]  \hspace{1cm} (11)

Where \( Z_t \) is an exogenous variable, defined as \( \frac{1}{\chi} \cdot \frac{1}{\phi} \left[ (1 - \psi)^{\frac{v - 1}{v}} + \psi^w \frac{v - 1}{v} (R_t^{i/N})^{\frac{1}{v}} \right]^{\frac{1 - \chi}{v}} \) and the \textbf{wage based} real exchange rate is \( q_t^j \equiv \frac{\epsilon_t \cdot \mu_t^w \cdot w_t^w}{P_t} \).
As a result of the openness, the labour supply is flattened. The term $Z_t$, and the wage-based real exchange rate, $q_{t}^{i}$, determine the domestic real wage. The only endogenous source, for increased real Marginal Cost, is diminishing Marginal Productivity of Labour. However, (11) shows that in the limiting case of Constant Return to Scale, that is $\chi \to 1$, this source is eliminated as well, and the firms end up having real Marginal Cost, which is invariant to the output level.

**Staggered Prices**

We employ the Calvo (1983) staggered pricing set up, according to which producers update prices, only upon receiving the price update signal; The price update signal evolves through stochastic time intervals. Every period there is a constant probability of $(1-\gamma) \in (0,1)$ for receiving such a signal. The probability is assumed to be independent of the time that has elapsed since the last price update, and of the current level of the price. We show in appendix A, that upon receiving the price update signal, the firm selects a new price level, $P_{H,t}^{opt}$, so as to satisfy the following non linear first order condition:

$$E_{t} \sum_{s=0}^{\infty} (\gamma \beta)^s \cdot u_{c}(C_{t+s}) \cdot y_{t+s} \cdot P_{H,t}^{opt} \cdot \frac{P_{t}}{P_{t+s}} = \frac{\theta}{\theta - 1} E_{t} \sum_{s=0}^{\infty} (\gamma \beta)^s \cdot y^{t+s} \cdot u_{c}(C_{t+s}) \cdot y_{t+s} \cdot y^{t+s} \cdot MC_{t+s}$$

(12)

This condition means that the weighted path of real prices should be marked up, over the weighted path of real marginal costs. The discount weights are consist of the corresponding probability $\gamma^s$, of the time discount factor $\beta^s$, of the corresponding marginal utility from consumption $u_{c}(C_{t+s})$ and of the corresponding output $y_{t+s}$. The last two introduce contrasting considerations: Higher consumption reduces the weight, due to diminishing marginal utility, while higher output increases the weight, by increasing the share of the relevant period in the overall profit.
3. Aggregate Supply

3.1 Open Capital Account, Open Trade Account, and in-and out-migration

Substituting the marginal cost (11) in the first order condition (12), accounting for the law of motion for the consumer price index and log linearizing around purely deterministic steady state, we show in appendix A that the New-Keynesian aggregate supply curve, for small open economy, with open labor market, is:

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1 + \sigma \theta} \left[ \sigma_c \cdot n \left( y_t^H - y_t^N \right) + \sigma_c \cdot (1 - n) \left( y_t^F - y_t^N \right) + q_c^t \left( C_t - \hat C_t^n \right) \right] \\
+ \frac{(1 - n)}{n} \left[ \left( \frac{1}{\gamma} q_c^t - q_c^{t-1} \right) + \gamma \beta \left( q_c^t - E_t \hat q_c^{t+1} \right) \right]
\]

Where, upper hat denotes proportional deviation from the purely deterministic steady state, and the superscript \( N \) denotes the "natural" value of real variables, that is, the value of a variable that would have prevailed under completely flexible prices. Hence, \( \left( y_t^H - y_t^N \right) \) is the domestic output gap; \( \left( y_t^F - y_t^N \right) \) is the foreign output gap; \( q_c^t \) and \( q_c^{t-1} \) are the wage-based and consumption-based real exchange rates respectively; and the parameter \( \omega_p \equiv \left( \frac{1 - \chi}{\chi} \right) y_{s.s.} \geq 0 \) is the elasticity of the marginal cost with respect to output.\(^3\)

We farther show (in appendix A) that if the labor market is closed, then the aggregate-supply curve becomes:

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1 + \sigma \theta} \left[ \sigma_c \cdot n \left( y_t^H - y_t^N \right) + \sigma_c \cdot (1 - n) \left( y_t^F - y_t^N \right) + \sigma \cdot \left( C_t - \hat C_t^n \right) \right] \\
+ \frac{(1 - n)}{n} \left[ \left( \frac{1}{\gamma} q_c^t - q_c^{t-1} \right) + \gamma \beta \left( q_c^t - E_t \hat q_c^{t+1} \right) \right]
\]

Where \( \left( C_t - \hat C_t^n \right) \) is the gap of private consumption from its natural level; and \( \sigma \equiv \sigma_p + \sigma_w \); where the parameter \( \omega_w \equiv \frac{\phi}{\chi} y_{s.s.} > 0 \) is the elasticity of the desired

\(^3\) This is non negative, due to non increasing return to scale.
real wage with respect to the output level (which is positive due to convex disutility from labor).

In (14), substitution and income effects, on labour supply, are captured by \( \sigma_w \) & \( \sigma \) respectively. As opposed to (13), in which the open labor market, causes a completely flexible labour supply.

3.2. **Open Capital Account, Open Trade Account; Closed Labor Market**

If capital is perfectly mobile, then the domestic agent has a costless access to the international financial market. As a consequence, household can smooth consumption similarly in the rigid price and flexible price cases. That is, \( \hat{C}_t = \hat{C}_t^N \). The Aggregate-Supply curve is still (13) for the case of open labor market, while for the case of closed labor market we get:  

\[
\hat{r}_t = \beta E_t \hat{r}_{t+1} + \frac{\kappa}{1 + \sigma \theta} \left[ \sigma \cdot n (\hat{y}_t^H - \hat{y}_t^N) + \sigma \cdot (1-n) (\hat{y}_t^F - \hat{y}_t^N) \right] \\
+ \frac{1-n}{n} \left[ \frac{(1-\gamma)}{\gamma} \hat{q}_t - \hat{q}_{t-1} \right] + \gamma \beta \left[ \hat{q}_t^c - \frac{(1-\gamma)}{\gamma} E_t \hat{q}_{t+1}^c \right]
\]

3.3. **Open Trade Account; Closed Capital Account and Closed Labor Market**

If the domestic economy is not integrated to the international financial market, then there is no possibility of consumption smoothing, and we have that:

\[
\hat{P}_{Ct} \hat{C}_t = \hat{P}_{yt} \hat{Y}_t \quad ; \quad \hat{P}_{Ct} \hat{C}_t^N = \hat{P}_{yt} \hat{Y}_t^N
\]

In this case, the aggregate-supply curve is:

---

4 Razin and Yuen (2002) extended this closed-economy framework to an open economy. Specifically, they derive the slope of the aggregate supply relationship for various openness regimes.
\[ \dddot{\pi}_t = \beta E_t \dddot{\pi}_{t+1} \]
\[ + \frac{\kappa}{1 + \sigma \theta} \left[ (n \cdot \sigma + \sigma) \cdot (\dddot{y}_t^H - \dddot{y}_t^N) + \sigma \cdot (1 - n) \cdot (\dddot{y}_t^F - \dddot{y}_t^N) \right] \]
\[ + \frac{(1 - n)}{n} \left[ \left( \frac{1 - \gamma}{\gamma} q^c_t \right) \cdot q^c_{t-1} + \gamma \beta \left( \frac{1 - \gamma}{\gamma} E_t q^c_{t+1} \right) \right] \]

(16)

### 3.4. Closed Economy

Closed trade account implies a complete diversification in production, \( n = 1 \). If also the capital account is closed and in- and out-migration is not possible, the aggregate-supply curve is reduced to:

\[ \dddot{\pi}_t = \beta E_t \dddot{\pi}_{t+1} \]
\[ + \frac{\kappa}{1 + \sigma \theta} \left[ (\sigma + \sigma) \cdot (\dddot{y}_t^H - \dddot{y}_t^N) \right] \]

(17)

### 3.5. Slope of the Aggregate Supply Curve

The slope of the aggregate supply curve (the inverse of the sacrifice ratio), in each one of the openness scenarios, is given by:

\[ \psi_1 = \kappa \cdot \frac{n \cdot \sigma_p}{(1 + \theta \sigma_p)} \]  
(Perfect mobility of Labor, Capital and Goods).

\[ \psi_2 = \kappa \cdot \frac{n \sigma}{(1 + \theta \sigma)} \]  
(Closed Labor Market; Perfect mobility of Capital and Goods).

\[ \psi_3 = \kappa \cdot \frac{(n \sigma + \sigma)}{(1 + \theta \sigma)} \]  
(Closed Labor Market; closed Capital Account; Open Trade account).

\[ \psi_4 = \kappa \cdot \frac{\sigma + \sigma}{(1 + \theta \sigma)} \]  
(Closed economy).
It can be easily seen, that $\psi_1 < \psi_2 < \psi_3 < \psi_4$.

This means that in every successive round of opening the economy, globalization contributes to flatten the aggregate supply curve. The intuition is as follows.

When an economy opens up to trade in goods, it tends to specialize in production and diversify in consumption, as is well known. This means the number of domestically produced goods (which is equal to n), is less then the number of domestically consumed goods (which is equal to 1). Consequently, the commodity composition of the consumption and output baskets, which are identical in a closed economy, are different when trade in goods is possible. As a result, the correlation between the fluctuations in output and consumption (which is equal to 1 in the case of a closed economy) is weakened if the economy is opened to international trade in goods. This means that the effects of supply shocks on inflation are also weakened under the open good market regime.

When the economy is financially open, then the correlation between the fluctuations in consumption and domestic output is farther weakened, since the representative household can smooth consumption through international borrowing and lending. The inflation effects of shocks to marginal costs are reduced, because the fluctuations in labor supply are smoothed, as a consequence of consumption smoothing.

When the economy opens up, for in- and out-labor migration, the labor supply and demand elasticities increase; which help to moderate the response of inflation to variability of output gap.\(^5\)

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\(^5\) If the economy imports intermediate goods there is also a real exchange rate effect. The real exchange rate affects the output inflation tradeoff, even in the absence of other cost push shocks. Clarida Gali and Gertler (2000) discuss this effect.
4. Welfare Analysis and Monetary Policy

A simple one-period optimization problem, of a discretionary central bank, can serve to illustrate our findings. Assume that the central bank minimizes the level of the utility-based quadratic loss function, subject to the aggregate supply constraint. In Appendix C we derive the following utility based loss function, along the lines of Woodford (2003):

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \tilde{\pi}_t^2 + \lambda \left( y_t^H - y_t^N \right)^2 \right]$$

(18)

Where $\lambda$, the relative weight of output gap in the loss function, is $\lambda = \frac{1}{\theta \cdot SR}$ ($SR$ denotes the sacrifice ratio, and $\frac{1}{\theta}$ is proportional to the flexible-price mark up).

It follows from our previous discussion that this weight gets smaller with openness. The relative weight of output stabilization in the loss function is equal to:

$$\lambda_1 = \frac{\kappa}{\theta} \cdot \frac{n \cdot \sigma_p}{(1 + \theta \sigma_p)}$$

(Perfect mobility of Labor, Capital and Goods).

$$\lambda_2 = \frac{\kappa}{\theta} \cdot \frac{n \sigma}{(1 + \theta \sigma)}$$

(Closed Labor Market; Perfect mobility of Capital and Goods).

$$\lambda_3 = \frac{\kappa}{\theta} \cdot \frac{(n \sigma + \sigma)}{(1 + \theta \sigma)}$$

(Closed Labor Market; closed Capital Account; Open Trade account).

$$\lambda_4 = \frac{\kappa}{\theta} \cdot \frac{(\sigma + \sigma)}{(1 + \theta \sigma)}$$

(Closed economy).

Where $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$.

The intuition is as follows.

In general, the distortions in the New-Keynesian equilibrium can be grouped into two types:
(1) Consumption fluctuations are welfare-reducing; therefore output gap fluctuations which are correlated with consumption fluctuations are also welfare-reducing.

(2) Efficient allocation of labor supply is the equal allocation of labor across product varieties, because varieties have the same technologies and the representative household preferences, concerning varieties, are assumed to be symmetric. Therefore, any cross-variety output (and labor input) dispersion tends to be distortionary. An inflation surprise, given that not all prices are updated instantaneously, which generate cross-variety dispersion is distortionary.

An implication is that in all three regimes of openness, albeit for different reasons, the correlation between the fluctuations in the output gap and the fluctuations in aggregate consumption is reduced. Because consumption welfare depends on consumption, not on output, the weight of the output gap in the loss function falls with trade and capital openness.

Under discretion, whenever inflation is above (below) target, the monetary authority contracts (expands) demand, so as to drive inflation back to its target. The optimality condition, linking the output gap and inflation, varies with the degree of openness (for derivations, see Appendix D):

\[
\left(\hat{y}_t^H - \hat{y}_t^N\right) = -\theta \left( 1 + \frac{\Omega \sigma}{\psi_1} \right) \cdot \pi_t \quad \text{(Perfect mobility of Labor, Capital and Goods).}
\]

\[
\left(\hat{y}_t^H - \hat{y}_t^N\right) = -\theta \left( 1 + \frac{\Omega \sigma}{\psi_2} \right) \cdot \pi_t \quad \text{(Closed Labor Market; Perfect mobility of Capital and Goods).}
\]

\[
\left(\hat{y}_t^H - \hat{y}_t^N\right) = -\theta \left( 1 + \frac{\Omega \sigma}{\psi_3} \right) \cdot \pi_t \quad \text{(Closed Labor Market; closed Capital Account; Open Trade account).}
\]

\[
\left(\hat{y}_t^H - \hat{y}_t^N\right) = -\theta \cdot \pi_t \quad \text{(Closed economy).}
\]
Where \( \Omega_\gamma \equiv \frac{(1-n)}{n} \left( \frac{1-\gamma}{\gamma} + \gamma \beta \right) \) is the elasticity of inflation, with respect to the consumption based real exchange rate. Since \( \psi_1 < \psi_2 < \psi_3 \) it follows the optimal monetary policy gets more aggressive with respect to inflation, with the opening up of the economy.

5. Conclusion

This paper reviews the analytics of the effects of globalization on the Phillips curve and the utility-based objective function of the central bank. It demonstrates that in an endogenous-policy set up, when trade in goods is liberalized, financial openness increases, and in- and out-labor migration are allowed, policymakers become more aggressive on inflation and less responsive to the output gap. In other words, globalization induces the monetary authority, when guided in its policy by the welfare criterion of a representative household, to put more emphasis on the reduction of inflation variability, at the expense of an increase in the output gap variability.

Note, however that if globalization would encourage businesses to update their prices more frequently, which will tend to steepen (rather than flatten) the slope of the aggregate supply curve, the above conclusion may reverse. More frequent price changes will also mitigate the inflation-based distortions in the New-Keynesian model, and lead, as by itself, to a reduction in the weight placed on inflation in the welfare-based loss function.
References


Appendixes

A. The Model

A.1. **Households**

The representative household maximizes utility from expected paths of consumption and leisure:

\[
\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(C_t; \xi_t) - \int_0^t \left[ h_{t,\text{hom}}^r(j) + \phi \cdot h_{t,\text{exp}}^r(j) \right]^{1+\phi} \cdot dj + \Gamma \left( \frac{M_t}{P_t} \right) \cdot \xi_t \right\} \quad (A. 1)
\]

Where \( \beta \) is a constant time discount factor, and the instantaneous utility function is separable in utility from a consumption composite \( C_t \), in disutility from domestic & exported labour effort \( h_{t,\text{hom}}^r(j) \) & \( h_{t,\text{exp}}^r(j) \) respectively and in utility from real balances \( \frac{M_t}{P_t} \). The parameter \( \phi > 1 \) indicates a home bias in preferences for working location, as proposed by Engler (2007). \( \xi_t \) is preferences shock. The space of goods is normalized, so the number of domestically produced goods is \( n \), and the number of foreign produced goods is \( 1-n \).

\[
C_t \equiv \left[ \int_0^n \left( c_{H,j} \right)^{\theta-1} \cdot dj + \int_n^1 \left( c_{W,j} \right)^{\theta-1} \cdot dj \right]^{\frac{\theta}{\theta-1}} \quad (A. 2)
\]

Subscripts \( H \) & \( W \) indicate the home country and the representative foreign country respectively. Thus, the variable \( c_{i,j}(j) \) is the consumption level of variety \( j \), which is produced in country \( i \in \{H,W\} \). All goods are tradable. The infinite continuum of goods is uniformly distributed over the unit interval: \( j \in [0,1] \). The parameter \( \theta > 1 \) is the Constant Elasticity of Substitution in consumption.

The budget constraint is:
\[
P_t C_t - P_t T_t + M_t + B_{H,t} + \varepsilon_t B_{W,t} - \left(1 + i_{H,t-1}\right) B_{H,t-1} - \left(1 + i_{W,t-1}\right) \varepsilon_t B_{W,t-1} = \\
M_{t-1} + \mu_t^H w_t \int_0^1 h_t^{\text{hom}}(j) \cdot dj + \varepsilon_t \cdot \mu_t^W w_t \int_0^1 h_t^{\exp}(j) \cdot dj + \frac{1}{0} \Pi_t(j) dj
\]

(3.4)

Where \( P_t \) is the Consumer Price Index in the home country; \( B_{H,t} \) \( (B_{W,t}) \) is the amount of a domestic risk-less bond, which pays 1 unit of domestic currency (foreign currency) in the next period; \( \varepsilon_t \) is the nominal exchange rates; \( w_t^\text{w} \) is the exogenous wage in the foreign market paid for one hour of unskilled labour effort. Assumed to be skilled workers, natives get skill premium captured by \( \mu_t^H \) at home and by \( \mu_t^W \) at the foreign labour market. Since unskilled labor can immigrate into the modeled small open economy, just as well as to anywhere else in the foreign market, it follows that the wage of unskilled labour is exogenously given by \( w_t^\text{w} \); The variable \( \Pi_t(j) \) is the level of profits of the \( j^{th} \) firm, paid as dividend to the representative household who owns all the firms in this economy.

The Demand Side of the Goods Market

Utility maximizing allocation between goods, yields demand functions:

\[
c_{H,t}(j) = \left[ \frac{p_{H,t}(j)}{P_t} \right]^{-\theta} C_t ; \quad c_{W,t}(j) = \left[ \frac{\varepsilon_t p_{W,t}(j)}{P_t} \right]^{-\theta} C_t
\]

(A.4)

Where \( p_{i,j}(j) \) is the price of variety \( j \) produced by country \( i \in \{H,W\} \).

The price composite \( P_t \) is defined as the cost, of obtaining one unit of the sub-utility maximizing bundle; Namely it is the total minimizing spending over the consumption bundle divided by the total "quantity" (that is, the utility composite \( C_t \)):

\[
P_t \equiv \frac{\int_0^n p_{H,t}(j) \cdot c_{H,t}(j) + \int_0^n \varepsilon_t p_{W,t}(j) \cdot c_{W,t}(j)}{C_t}.
\]

Substituting (A.4) and rearranging we get:

\[
P_t = \left[ \int_0^n \left[ p_{H,t}(j) \right]^{-\theta} \cdot dj + \int_0^n \left[ \varepsilon_t p_{W,t}(j) \right]^{-\theta} \cdot \right]^{\frac{1}{1-\theta}}
\]

(A.5)
Assuming intra country symmetry, the index \( j \) can be dropped, and the price composite will be written as:

\[
P_t = \left[ n \cdot P_{H,t}^{1-\theta} + (1-n) \cdot (P_{W,t}^{1-\theta}) \right]^{\frac{1}{1-\theta}}
\]  

(A. 6)

\( P_{H,t} \) and \( P_{W,t} \) are price index of domestic and imported goods, nominated in the currency of the country of origin.

**The Supply Side of the Labour Market**

The representative household allocates time between leisure, labor in the domestic market and labour in the foreign market. Labor supply satisfies the following first order conditions:

\[
u_{t} \left( C_{t}, \xi_{t} \right) \cdot \frac{\mu^{H}e_{t}W_{i,t}^{H}}{P_{t}} = \left[ h_{t}^{\text{home}}(j) + \phi h_{t}^{\text{exp}}(j) \right]^{\theta}
\]  

(A. 7)

\[
u_{t} \left( C_{t}, \xi_{t} \right) \cdot \frac{\mu^{W}e_{t}W_{i,t}^{H}}{P_{t}} = \phi \cdot \left[ h_{t}^{\text{home}}(j) + \phi h_{t}^{\text{exp}}(j) \right]^{\theta}
\]  

(A. 8)

Here, \( y_{,x}(X_{t}) \) denotes the first derivative of \( y \) with respect to \( x \), evaluated at \( x = X_{j} \).

Dividing (A.7) by (A.8) relates the foreign to domestic skill premiums ratio, to the home bias.\(^6\)

\[
\frac{\mu^{W}}{\mu^{H}} = \phi
\]  

(A. 9)

Whenever the foreign income has higher (lower) purchasing power in the domestic economy, the domestic labor supply of locals shifts leftward (rightward), so the local skill premium increases (decreases). This process will hold till (A.9) is satisfied.

---

\(^6\) Such relation would prevail in the short run, only in frictionless labor market. Such assumption, although not realistic, is useful simplification which serves us while discussing the intuition behind the mechanism at work.
A.2. The Supply Side

The Marginal Cost

Domestic firms produce by a non-increasing return to scale, CES production function, using labor input of natives & immigrants:

\[
y_j(j) = A_j \cdot \left[ (1-\psi)^{\frac{\nu-1}{\nu}} \cdot h_{\text{hom\_e}}^j(j)^{\frac{1}{\nu}} + (\psi)^{\frac{\nu-1}{\nu}} \cdot h_{\text{imp}}^j(j)^{\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}}
\]  

(A. 10)

Where \( y_j(j) \) is the total output level of the \( j \)'th firm, \( A_j \) is an exogenous aggregate shock, common to all firms. The parameter \( \nu > 1 \) is inversely proportional to the elasticity of substitution between imported and local labor inputs. That is, the elasticity is given by \( \frac{\nu}{\nu-1} \); The degree of diminishing return to scale is given by \( \chi \leq 1 \). The variable \( h_{\text{imp}}^j(j) \) is endogenously determined amount of immigrant employed by firm \( j \);

We assume that natives are more skilled than the migrants. Hence \( \psi \in \left[ 0, \frac{1}{2} \right] \). It follows that the marginal productivity of native labour is higher function than that of immigrant labour, hence the wage of the native workers includes skill premium over the wage of immigrants. The ratio of immigrant's to native's labor employed by the domestic producers, is positively related to their skill ratio & to the global skill premium, and negatively related to the home bias:

\[
\frac{h_{\text{imp}}^j(j)}{h_{\text{hom\_e}}^j(j)} = \frac{\psi}{1-\psi} \left[ \frac{\mu^w_{\text{skilled}}}{\phi} \right]^{\frac{\nu}{\nu-1}}
\]

(A. 11)

Assume technology in the rest of the world uses both skilled and unskilled labor inputs, the production function in the representative foreign country is:

\[
y^w_j(j) = A^w_j \cdot \left[ (1-\psi^w)^{\frac{\nu-1}{\nu}} \cdot h_{\text{skilled}}^j(j)^{\frac{1}{\nu}} + (\psi^w)^{\frac{\nu-1}{\nu}} \cdot h_{\text{unskilled}}^j(j)^{\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}}
\]

(A.10')

Where \( h_{\text{skilled}}^j(j) \) (\( h_{\text{unskilled}}^j(j) \)) denotes the employment of skilled (unskilled) workers by the foreign firms. Those can include both natives & immigrants. Further assume that the skill
dispersion in the world, is higher than the dispersion between locals (skilled) & immigrants (unskilled) in the domestic economy; that is $\psi^w < \psi$.

It follows that the ratio between unskilled to skilled working efforts in the foreign market, denoted $\frac{1}{\tau_i}$, is:

$$
\frac{1}{\tau_i} \equiv \frac{h^{\text{UnSkilled}}(j)}{h^{\text{Skilled}}(j)} = \frac{\psi^w}{1-\psi^w} \left[ \frac{\mu^w}{\tau_i} \right]^{\frac{1}{v-1}}
$$

(A.11')

From which it follows, that the skill premium in the domestic economy is exogenously determined by:

$$
\mu^H_i = \frac{1}{\phi} \cdot \mu^w_i = \frac{1}{\phi} \left[ \frac{1}{\tau_i} \frac{(1-\psi^w)^{\frac{v-1}{v}}}{\psi^w} \right]
$$

(A.11'')

Recall that the wage for immigrants in the domestic economy is exogenously given by the wage they face in the foreign market. Since the wage of the natives is anchored by (A.11''), it follows that the immigrants to natives ratio in the domestic economy, denoted by $R_i^{1/N}$, is exogenously given by:

$$
R_i^{1/N} \equiv \frac{h_{\text{imp}}^i(j)}{h_{\text{hom}}^i(j)} = \frac{1}{\phi^{\frac{1}{1-v}}} \cdot \frac{\psi^w}{(1-\psi^w)}
$$

(A.11''')

Substituting (A.11''') in (A.10), and rearranging, we can express $h_{\text{hom}}^i(j)$, the native labor effort employed by the domestic firm by:

$$
h_{\text{hom}}^i(j) = \left[ (1-\psi^w)^{\frac{v-1}{v}} + \psi^w \left( R_i^{1/N} \right)^{\frac{1}{v}} \right] \left[ \frac{y_i(j)}{A_i} \right]^{\frac{1}{v}}
$$

(A. 12)

Recall that $MC = \frac{W}{MPL}$; using the results obtained thus far, as well as (A.9) to substitute for the real wage, we get the general equilibrium real marginal cost:

$$
MC_i(j) = Z_i \cdot q_i^j \cdot \frac{1}{A_i} \cdot y_i(j)^{1-\frac{1}{v}}
$$

(A. 13)
Where the exogenous variable $Z_t$ is defined as
\[
\frac{1}{\phi} \left[ \left( 1 - \psi \right)^{\psi - \phi} + \psi^{\psi - \phi} \left( R^{I/N} \right)^{1 - \psi} \right]^{(\psi - 1)}
\]
the \textit{labour based} Real exchange rate is $q^l_t \equiv \frac{\epsilon^l_t \cdot \mu^w_t \cdot W^w_t}{P_t}$.

If, on the other hand, the economy is closed, it follows that $h^j_{t, exp} = h^j_{t, imp} = 0 \; \forall \; t \geq 0 \; j \in (0,1)$. In that case, firms employ natives only, and the level of employment is given by
\[
h^j_{t, hom} = \left[ \left( 1 - \psi \right)^{-\psi} - \left( 1 - \psi \right)^{\psi - \phi} \right]^{z}
\]
(A.12')

Note that this result corresponds to the private case of $R^{I/N} = 0$ in (A.12). The marginal cost here, is derived in similar way to the case of open labour market. Just this time, instead of substituting (A.9) for the real wage, we substitute (A.7). Hence, the general equilibrium real marginal cost under closed labour market, denoted by $MC^c_t$, is given by:
\[
MC^c_t(j) = Z^c \cdot \frac{1}{u^c_t(C_t)} \cdot \frac{1}{A_t} \cdot y_t(j)^{1-\phi - \chi}
\]
(A.13')

With $Z^c = \frac{(1 + \phi)}{\chi} \cdot (1 - \psi)^{(1-\psi)/(1+\phi)}$. Hence, under closed labour market, we get the familiar result of upward sloping labour supply. Hence:

- Elasticity of Marginal Cost with respect to own output level, is now increased by $\frac{\phi}{Z}$ (The effect of increasing disutility from work).

- The Marginal Cost is positively related to the aggregate consumption level (Income effect on labour supply).

\textit{Staggered Pricing}

The demand for the $j^{th}$ firm product consists of both the demand by domestic consumers and by foreign consumers. By open trade, consumers in different location face
the same goods price. We assume that elasticity of substitution between goods, is the same across different open economies.\(^7\) That is, preferences of the foreign representative consumer can be expressed identically to those of the local consumer.\(^8\) Define the composite of total world output by: \(Y^w_t \equiv \left[ \int_0^n (y_{H,t}\right)^{\theta - 1} dj + \int_n^1 (y_{F,t}\right)^{\theta - 1} dj \right]^{\theta / \theta - 1}.\) Global demand for the goods produced by the \(j^{th}\) domestic producer is then given by:

\[
y_t(j) = \left[ \frac{p_t(j)}{p_t} \right]^{\theta} Y^w_t
\]  

(A. 14)

We employ the Calvo (1983) staggered pricing set up, according to which producers update prices, only upon receiving the price update signal; The price update signal evolves through stochastic time intervals. Every period there is a constant probability of \((1 - \gamma) \in (0,1)\) for receiving such a signal. The probability is assumed to be independent of the time that has elapsed since the last price update, and of the current level of the price.

Receiving the price update signal, the firm selects a new price level so as to maximize the infinite conditional path of nominal profits, given by:\(^9\)  

\[
\text{Max}_{p^0_{t+1}} \sum_{s=0}^\infty \gamma^s Q_{t+s} \left[p_{H,t+s}(j) \cdot y_{t+s}(j) - \mu^H_{t+s} \varepsilon_{t+s} w^H_{t+s} h^\text{hom}_t(j) - \varepsilon_t w^w_{t+s} h^\text{imp}_t(j) \right] (A. 15)
\]

Where the nominal profits are weighted by the probability, that they will remain unchanged \(\gamma^s\), and by the corresponding nominal interest rate  

\[
E_t Q_{t+s} = E_t \prod_{i=0}^{s-1} \frac{1}{1 + i_{H,t+i}}
\]

Substitute (A.14), (A.11”), (A.12) & (A.11”’) to eliminate \(y_{t+s}(j)\), \(\mu^H_{t+s}, h^\text{hom}_t(j)\) & \(h^\text{imp}_t(j)\) respectively. Differentiate with respect to \(P^0_{H,t}\), and rearrange,

\footnotesize

\(^7\) This is to say that \(\theta\) is also the CES parameter of foreign consumers.

\(^8\) This assumption simplifies the analysis, by introducing unique solution for the problem of the local firm, both with regard to the local market & to export.

\(^9\) Since the firms’ distribution over the unit interval is such that the total number of firms is high enough, as in Dixit & Stiglitz (1977) the single firm is assumed to be small enough and hence ignores its influence on the aggregate CPI and on the total export of the economy. It follows that when optimizing, firms regard those variables as exogenous.
to get the price update first order condition. The condition means that upon getting a price update signal, the producer selects the maximizing value price, denoted $P_{H,t}^{opt}$, to satisfy:

$$E_t \sum_{s=0}^{\infty} \gamma^s \cdot Q_{t+s} \cdot q_{t+s}(j) \cdot P_{H,t}^{opt}(j) = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{\infty} \gamma^s \cdot Q_{t+s} \cdot y_{t+s} \cdot P_{t+s} \cdot MC_{t+s}(j)$$

Using recursive substitution of the usual Euler condition, we get

$$E_t Q_{t+s} = \beta^s \frac{P_t}{P_{t+s}} \cdot E_t \frac{u_c(C_{t+s})}{u_c(C_t)}$$

Substituting to eliminate $E_t Q_{t+s}$, and dropping the firm index ($j$) (assuming symmetry among firms), we get:

$$E_t \sum_{s=0}^{\infty} (\gamma \beta)^s \cdot u_c(C_{t+s}) \cdot y_{t+s} \frac{P_{H,t}^{opt}}{P_t} \cdot \frac{P_t}{P_{t+s}} = \frac{\theta}{\theta - 1} E_t \sum_{s=0}^{\infty} (\gamma \beta)^s \cdot u_c(C_{t+s}) \cdot y_{t+s} \cdot MC_{t+s}$$

(A. 16)

The consumer price index (which incorporates the exogenous process for the prices of imported goods) by the law of large numbers becomes:

$$P_t^{1-\theta} = (1 - n) \cdot \left( \bar{\epsilon}_t \cdot P_{W,j}^{1-\theta} \right) + n \cdot \left[ (1 - \gamma) \cdot P_{H,t}^{opt} + \gamma \cdot P_{H,t-1}^{opt} \right]^{1-\theta}$$

(A. 17)

### A.3. The Log Linearized System of Equations

In this subsection we log linearize the basic equations of the model: The first order condition for optimal price (A.16), the CPI law of motion (A.17), the MC under open labor market (A.13) and the MC under Closed labor market (A.13').

Appendix B presents a step by step log linearization of equation (A.16) and the CPI equation (A.17) around a purely deterministic steady state. This yields:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \cdot m_c + \frac{(1 - n)}{n} \left[ \left( \frac{1 - \gamma}{\gamma} \right) \bar{q}_t^e - \bar{q}_{t-1}^e \right] + \alpha \beta \left( \bar{q}_t^c - \left( \frac{1 - \gamma}{\gamma} \right) E_t \bar{q}_{t+1}^e \right)$$

Where:

$$\kappa = \frac{(1 - \gamma \beta)(1 - \gamma)}{\gamma}$$

(A. 18)

Hatted variables indicate logarithmic deviations from the steady state's level. $\bar{q}_t^c \equiv \bar{\epsilon}_t + \bar{P}_{F,j} - \bar{P}_t$ is the logarithmic deviation of the consumption based real exchange rate from its steady state level.
Linearizing the marginal costs equations (A.13) and (A.13'), and expressing them in terms of gaps from the flexible price benchmark, we get:

\[
\hat{mc}_t(j) = \sigma_p \left[ \ddot{y}_t(j) - \ddot{y}^n_t(j) \right] + \ddot{q}_t^i
\]

(A.19)

\[
\hat{mc}_t^c(j) = (\sigma_w + \sigma_p) \left[ \ddot{y}_t(j) - \ddot{y}^n_t(j) \right] + \sigma \left( C_t - C^n_t \right)
\]

(A.19')

Where \( \sigma \equiv \frac{u_{cc}}{u_c} \); \( \omega_w \equiv \frac{\phi}{\chi} \); \( \omega_p \equiv (\frac{1-\chi}{\chi}) y_{s,s} \).

Substituting (the log linearizing) (A.14) and rearranging, we get the logarithmic deviations of the real Marginal Cost under open labour market:

\[
\hat{mc}_t(j) = \frac{1}{1 + \sigma_p \theta} \left\{ \sigma_p \cdot n \left( \ddot{y}_t^H - \ddot{y}_t^N \right) + \sigma_p \cdot (1-n) \left( \ddot{y}_t^F - \ddot{y}_t^N \right) + \ddot{q}_t^i \right\}
\]

(A.20)

Under closed labour market, the result derived is:

\[
\hat{mc}_t^c(j) = \frac{1}{1 + \sigma \theta} \left\{ \sigma \cdot n \left( \ddot{y}_t^H - \ddot{y}_t^N \right) + \sigma \cdot (1-n) \left( \ddot{y}_t^F - \ddot{y}_t^N \right) + \sigma \left( C_t - C^N_t \right) \right\}
\]

(A.20')

Where \( \sigma \equiv \sigma_p + \sigma_w \).

(A.20) and (A.20') express the marginal cost of the \( j^{th} \) firm, as function of the aggregate output gap. These are logarithmic deviations of the marginal cost, from its steady state value \( \left( \frac{\theta}{\theta - 1} \right)^{-1} \).

The log linearized marginal costs (A.20) or (A.20') can be substituted in the NKPC (A.18) so as to express the New Keynesian Phillip Curve in terms of the domestic & foreign real activities under open labour market:

\[
\hat{\pi}_t = \beta \bar{E}_t \hat{\pi}_{t+1} + \frac{\kappa}{1 + \sigma_p \theta} \left[ \sigma_p \cdot n \left( \ddot{y}_t^H - \ddot{y}_t^N \right) + \sigma_p \cdot (1-n) \left( \ddot{y}_t^F - \ddot{y}_t^N \right) + \ddot{q}_t^i \right]
\]

(A.21)

\[
+ \frac{(1-n)}{n} \left[ \left( \frac{1-\gamma}{\gamma} \ddot{q}_t^c - \ddot{q}_{t-1}^c \right) + \gamma \beta \left( \ddot{q}_t^c - \frac{(1-\gamma)}{\gamma} E_t \ddot{q}_{t+1}^c \right) \right]
\]

10 While rearranging we use \( \hat{mc}_t = \hat{p}_t(j) - \hat{p}_t \).
Under closed labour market the NKPC becomes:

\[
\hat{\pi}_t = \beta \hat{E}_t \hat{\pi}_{t+1} + \frac{\kappa}{1 + \alpha \theta} \left[ \sigma \cdot n \left( \hat{y}_t^m - \hat{y}_t^N \right) + \alpha \cdot (1 - n) \left( \hat{y}_t^w - \hat{y}_t^N \right) + \sigma \cdot (\hat{C}_t - \bar{C}_t) \right] \\
+ \left( \frac{1 - n}{n} \right) \left[ \left( 1 - \gamma \right) \hat{q}_t^c - \hat{q}^c_{t-1} \right] + \gamma \beta \left( \hat{q}_t^c - \left( \frac{1 - \gamma}{\gamma} \right) E_t \hat{q}^c_{t+1} \right)
\]

(A.21')

**B. Log linearization**

**B.1. Linearizing the Producer's First Order Condition**

Log linearization of the first order condition (A.16) around zero inflation steady state, and subtraction of terms that canceled out from both sides, yields (Hatted variables for logarithmic deviation from steady state):

\[
\frac{1}{1 - \gamma \beta} \hat{p}^*_{H,t} - E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \left( \hat{p}^*_{t+s} - \hat{P}_t \right) = E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \left[ \hat{m}_{c,t+s} \right] 
\]

(B.1)

Where \( \hat{p}^*_{H,t} \equiv \ln(P^*_t) - \ln(P_t) \). (B.1) can be rewritten as:

\[
\frac{1}{1 - \gamma \beta} \hat{p}^*_{H,t} - E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \left( \sum_{i=1}^{s} \hat{\pi}_{t+i} \right) = E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \left[ \hat{m}_{c,t+s} \right] 
\]

(B.2)

The second element in the left hand side of (B.2) can be rearranged by writing it explicitly for every \( s \in (0, \infty) \) and for every \( i \in (0, s) \). Then it can be collapsed down to:

\[
\frac{1}{1 - \gamma \beta} \hat{p}^*_{H,t} - \frac{1}{1 - \gamma \beta} E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \hat{\pi}_{t+s} = E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \left[ \hat{m}_{c,t+s} \right] 
\]

(B.3)

This, after rearrangement, becomes:

\[
\hat{p}^*_{H,t} = E_t \sum_{i=1}^{\infty} \left( \gamma \beta \right)^i \left( \hat{\pi}_{t+i} \right) + (1 - \gamma \beta) E_t \sum_{s=0}^{\infty} \left( \gamma \beta \right)^s \hat{m}_{c,t+s} 
\]

(B.4)

Taking the first period out of the \( \sum[i] \) we get (again – note the change in the first value of \( s \) in both \( \sum[i] \) );
\[ \dot{P}_{H,t}^* = E_t \left\{ \gamma \beta \hat{\pi}_{t+1} + (1 - \gamma \beta) mc_t + \sum_{s=2}^{\infty} \left[ (\gamma \beta)^s \hat{\pi}_{t+s} \right] \right\} \]  

(B.5)

This can simply be written as:

\[ \dot{P}_{H,t}^* = \gamma \beta E_t \hat{\pi}_{t+1} + (1 - \gamma \beta) mc_t + \gamma \beta \cdot E_t \dot{P}_{H,t+1}^* \]  

(B.6)

### B.2. Integrating the Linearized CPIs’ Law of Motion

Log linearizing the CPI (A.6) and rearranging, we get: \( \dot{P}_{H,t} = \frac{1}{n} \dot{P}_t - \frac{(1-n)}{n} (\hat{\varepsilon}_t + \hat{P}_{w,t} ). \)

Adding \( \left[ -\frac{(1-n)}{n} \dot{P}_t + \frac{(1-n)}{n} \hat{P}_t = 0 \right] \) to the right hand side, then rearranging and taking one period backward we get:

\[ \dot{P}_{H,t-1} = \hat{P}_{t-1} - \frac{(1-n)}{n} \dot{q}_{t-1}^c \]  

(B.7)

Where \( \dot{q}_{t}^c \) is the logarithmic deviation of the consumption based real exchange rate from its steady state level, formally defined as: \( \dot{q}_{t}^c \equiv \hat{\varepsilon}_t + \hat{P}_{w,t} - \hat{P}_t \). The (Purchasing Power Parity) steady state level is \( \dot{q}_{t,s}^c = 0 \).

Substituting (B.7) to the log linearization of (A.17) around steady state with zero inflation and Purchasing Power Parity, subtraction of \( \hat{P}_t \) from both sides and rearranging yields:

\[ \dot{P}_{H,t}^* = \gamma \frac{\dot{\pi}_t}{1-\gamma} - \frac{(1-n)}{n} \left[ \dot{q}_{t}^c - \frac{\gamma}{1-\gamma} \dot{q}_{t-1}^c \right] \]  

(B.8)

The last equation of annex B.4 of Woodford (2003) becomes the private case of (B.8) under closed economy; that is, when \( n = 1 \).

Substituting (B.8) into (B.6) to eliminate both \( \dot{P}_{H,t}^* \) \& \( E_t \dot{P}_{H,t+1}^* \) and rearranging we get:

\[ \hat{\pi}_t = \beta \cdot E_t \hat{\pi}_{t+1} + \kappa \cdot mc_t + \frac{(1-n)}{n} \left[ \left( \frac{1-\gamma}{\gamma} \hat{q}_{t}^c - \hat{q}_{t-1}^c \right) + \gamma \beta \left( \frac{1-\gamma}{\gamma} E_t \hat{q}_{t+1}^c \right) \right] \]

Where:

\[ \kappa = \frac{(1-\gamma \beta)(1-\gamma)}{\gamma} \]

Which is exactly (A.18).
C. Utility Based Loss Function

In this appendix we derive a utility based quadratic approximation for the welfare criterion of the Central Bank, following Woodford (2003).\(^{11}\)

D. Discretionary Monetary Policy

In this appendix we derive the optimal monetary policy under discretion, in small open economy. We begin by deriving the aggregate demand block; then the interest parity; finally, we derive the policy rule that minimizes the loss function subject to the model.

**D.1. The Aggregate Demand**

Maximizing the utility (1) with respect to bond holding (nominated in the local currency), subject to the budget constraint (3) we get the familiar Euler equation. After log linearization, the result is:

\[
\bar{C}_t = E_t \bar{C}_{t+1} - \sigma^{-1} (\bar{i}_{H,t} - E_{t} \bar{\pi}_{t+1}) + g_t - E_t g_{t+1}
\]  

(D. 1)

Where \(\sigma \equiv -\frac{u_{cc}}{u_c} c_{s,t}\) as in appendix A; and \(g_t = -\frac{u_{cc}}{u_{cc}} \xi_t\) captures the effect of demand shock over the inter-temporal marginal rate of substitution.

\(^{11}\) While doing so, we abstract from the money in the utility of the households. That is, we focus at the limiting case of what Woodford (2003) names Bookkeeping Cashless Economy.
Market clearing condition implies that \( C_t = \bar{y}_t^H + \bar{y}_t^F - C_t^F \). Substituting in (D. 1), subtracting \( [\bar{y}_t^N + \bar{y}_{t+1}^N] \) from both sides and collecting exogenous terms we get the aggregate demand:

\[
\left( \bar{y}_t^H - \bar{y}_t^N \right) = E_t^U \left( \bar{y}_{t+1}^H - \bar{y}_{t+1}^N \right) - \sigma^{-1} \left( \bar{r}_{t,0} - E_t \bar{r}_{t+1} - \bar{r}_t^n \right)
\]  \hspace{1cm} (D. 2)

Where \( \bar{r}_t^n \equiv \sigma \left( g_t - E_t g_{t+1} \right) + \left( \bar{y}_t^n - E_t \bar{y}_{t+1}^n \right) + \left[ \bar{C}_t^F - \bar{y}_t^F - E_t \left( \bar{C}_{t+1}^F - \bar{y}_{t+1}^F \right) \right] \) is the exogenous deviation of the Wicksellian natural real interest rate from steady state, augmented for the small open economy. We assume that the domestic economy is small enough, so the foreign output and consumption are exogenous. The extension to the small open economy is that in addition to transitory local demand shocks, the natural real interest rate increases, responding to transitory exogenous increase in export demand.

**D.2. The Interest Parity**

Maximizing the utility (1) with respect to bond holding of both types, one gets two first order conditions. Dividing through and log linearizing, we get the following familiar result, for Uncovered Interest Parity:

\[
E_t \bar{c}_{t+1} + \bar{e}_{t+1} = \bar{e}_t - \bar{z}_{t+1} \]  \hspace{1cm} (D. 3)

Subtracting \( E_t \bar{z}_{t+1} + \bar{e}_{t+1} \) from both sides we get the parity in real terms:

\[
\bar{q}_t = E_t \bar{q}_{t+1} + \left( \bar{z}_{t+1} - E_t \bar{z}_{t+1} \right) - \left( \bar{z}_{t+1} - E_t \bar{z}_{t+1} \right) \]  \hspace{1cm} (D. 4)

**D.3. The Policy Rule**

In this appendix we derive the optimal rule under discretion. We do so by small open economy extension, to the treatment of Clarida et al (1999).
The monetary rule maximizes theLoss function (18) subject to:\(^\text{12}\)

- The aggregate supply under different degrees of openness (13)-(17).
- The aggregate demand (D.2).
- The Real Uncovered Interest Parity (D.4)

Optimizing in a discretionary way means that the Central Bank takes the expectations as given.

The Lagrangeian is:

\[
\min \quad \frac{1}{2} E_i \sum_{t=r}^{\infty} \beta^t \left( \tilde{\pi}_t^2 + \frac{\Omega_y}{\theta} \cdot (\tilde{y}_t^H - \tilde{y}_t^N)^2 \right)
\]

\[- \varphi_{1,i} \left[ \tilde{\pi}_t - \beta E_i \tilde{\pi}_{t+1} - \Omega_y \cdot (\tilde{y}_t^H - \tilde{y}_t^N) - \sum_i \Omega_i \cdot i_i \right] \]

\[- \varphi_{2,i} \left[ (\tilde{y}_t^H - \tilde{y}_t^N) - E_i (\tilde{y}_{t+1}^H - \tilde{y}_{t+1}^N) + \sigma^{-1} \left( \tilde{i}_{H,i} - E_i \tilde{i}_{t+1}^H + \tilde{i}_t^H \right) \right] \]

\[- \varphi_{3,i} \left[ q_{i,c}^c - E_i q_{c,t+1}^c - (\tilde{i}_{F,i} - E_i \tilde{i}_{t+1}^F + \tilde{i}_t^H) \right] \]

(D. 5)

Where \( \Omega_i \) is the elasticity\(^\text{13}\) of the inflation with respect to variable \( i \in \{ (\tilde{y}_t^F - \tilde{y}_t^N), q_{i,c}^c, (\tilde{C}_t - \tilde{C}_t^c), q_{c,t-1}^c, q_{c,t+1}^c, E_i q_{c,t+1}^c \} \); and \( \varphi_{1,i}, \varphi_{2,i} \) & \( \varphi_{3,i} \) are the Lagrange multipliers. The corresponding first order conditions with respect to \( \tilde{\pi}_t, (\tilde{y}_t^H - \tilde{y}_t^N), q_{i,c}^c \) & \( i \) are as follows:

\[
\begin{align*}
\tilde{\pi}_t - \varphi_{1,i} &= 0 \\
\frac{\Omega_y}{\theta} \cdot (\tilde{y}_t^H - \tilde{y}_t^N) + \varphi_{1,i} - \varphi_{2,i} &= 0 \\
0 &= \varphi_{1,i} - \varphi_{2,i} \\
- \varphi_{2,i} \sigma^{-1} &= \varphi_{3,i} \\
- \varphi_{2,i} &= 0
\end{align*}
\]

(D. 6)

\(^\text{12}\) At this version we endogenize only the consumption based real exchange rate. In the next version we will endogenize the labor based real exchange rate as well.

\(^\text{13}\) Of course, there is different elasticity under different degree of openness.
The unique solution to (D.6) is qualitatively isomorphic to the case of closed economy:

\[
\left( \bar{y}_t^H - \bar{y}_t^N \right) = -\theta \left( 1 + \frac{\Omega_q}{\Omega_y} \sigma \right) \pi_t
\]  \hspace{1cm} (D.7)

Note that the limiting case of closed economy, is the private case of \( \Omega_q = 0 \), from which the familiar\(^{14}\) result \( \left( \bar{y}_t^H - \bar{y}_t^N \right) = -\theta \cdot \pi_t \) follows. Farther note that the ratio \( \frac{\Omega_q}{\Omega_y} \) increases with openness, which makes the optimal response to inflation more aggressive, in response to openness!

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