Mash, Richard

Working Paper

Endogenous Indexing and Monetary Policy Models

Kiel Working Paper, No. 1358

Provided in Cooperation with:
Kiel Institute for the World Economy – Leibniz Center for Research on Global Economic Challenges

Suggested Citation: Mash, Richard (2007) : Endogenous Indexing and Monetary Policy Models, Kiel Working Paper, No. 1358, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:
https://hdl.handle.net/10419/17870

Kieler Arbeitspapiere • Kiel Working Papers

1358

Endogenous Indexing and Monetary Policy Models

Richard Mash

June 2007

This paper is part of the Kiel Working Paper Collection No. 2

"The Phillips Curve and the Natural Rate of Unemployment"
June 2007

http://www.ifw-kiel.de/pub/kap/kapcoll/kapcoll_02.htm

Institut für Weltwirtschaft an der Universität Kiel
Kiel Institute for the World Economy

Kiel Institute for World Economics
Duesternbrooker Weg 120
24105 Kiel (Germany)

Kiel Working Paper No. 1358

Endogenous Indexing and Monetary Policy Models

by

Richard Mash

June 2007

Endogenous Indexing and Monetary Policy Models

Richard Mash[1]

Department of Economics and New College
University of Oxford

January 2007

Abstract

Models in which firms use rules of thumb or partial indexing in their price setting have become prominent in the recent monetary policy literature. The extent to which these firms adjust their prices to lagged inflation has been taken as fixed. We consider the implications of firms choosing the optimal degree of indexation so these simple pricing rules deliver prices as close as possible to those which would be chosen optimally. We find that the degree of indexation depends on the extent of persistence in the economy such that models with constant indexation are vulnerable to the Lucas critique. We also study the interactions between firms price setting and the macroeconomic environment finding that, for the models which appear most plausible on microeconomic grounds, the Nash equilibrium between firms and the policy maker is characterised by zero indexation and zero macroeconomic persistence.

JEL E52, E58, E22

Key Words: Indexing, Monetary Policy, Phillips curve, Inflation persistence, Microfoundations

[1]Correspondence: New College, Oxford, OX1 3BN, UK; Tel. +44-1865-289195; Fax +44-1865-279590; email: richard.mash@economics.ox.ac.uk.

Introduction

Models in which decision or optimisation costs introduce frictions to price changing and give rise to (partial) indexing or rule of thumb behavior by firms have been a central feature of the recent monetary policy literature. This contrasts with earlier models which emphasised price stickiness while assuming that price setting was optimal subject to constraints on when prices could be changed. The two most prominent models with indexing behaviour are Gali and Gertler (1999) and Christiano, Eichenbaum and Evans (2005), the pricing mechanisms in which have also been used in Eichenbaum and Fisher (2003), Steinsson (2003) and Smets and Wouters (2003) amongst many others since these have become workhorse models (Woodford, 2006). They differ as to whether indexing behavior is combined with price stickiness or not but both give rise to the standard hybrid New Keynesian Phillips curve with forward looking expected inflation and lagged inflation.

A feature of the existing indexing and rule of thumb models is that the degree to which firms index to past inflation, which we refer to as their indexing parameter, has been treated as an exogenous constant. The contribution of this paper is to explore the implications of varying that assumption. It is natural to suppose that firms might consider the optimal or at least near optimal value of their indexing parameter so as to achieve higher profits by more closely matching their prices to the prices which would be set in the absence of decision or optimisation costs and thus come closer to constrained optimal behavior. It may not be appropriate to assume that firms would necessarily optimise their indexing parameter continuously, since infrequent re-optimisation is a maintained assumption of these models, but it seems plausible that they may review their indexing parameter periodically.

We study the implications of "endogenous indexing" behavior of this kind within the two established models while adding a third, a more effective variant of the Christiano, Eichenbaum and

Evans (2005) model. We follow the literature in assuming a Calvo (1983) constant hazard structure, the Calvo signal being interpreted either as an opportunity to change price or as an opportunity to reoptimise prices depending on the model. The Gali and Gertler (1999) model assumes optimisation costs for a proportion of firms, who apply a rule of thumb in the setting of a new price when they may do so, while retaining the Calvo model's assumption that the firm's price remains fixed until the next Calvo signal. The rule of thumb determined price depends on a measure of lagged aggregate prices plus the product of an indexing parameter and lagged inflation. We find that this structure is constrained optimal and hence focus on the size of the indexing parameter within the rule of thumb. The scope of the Christiano, Eichenbaum and Evans (2005) model is different in that firms may change price every period, the friction being that they do so fully optimally only when a Calvo signal arrives. Christiano et. al. assume an indexing structure in which the firm takes its own lagged price as a base and adds the product of an indexing parameter and lagged inflation. We refer to this as lagged own price indexing (LOPI) and find that this structure is not constrained optimal and a variant in which the firm uses the lagged aggregate price as a base is generally superior. We refer to this as lagged aggregate price indexing (LAPI) and find that this apparently minor change in specification matters a great deal for the results obtained..

A result common to all the models is that the optimal value of the indexing parameter depends on firms' beliefs about the degree of persistence in the economy. Intuitively if inflation is strongly persistent it is constrained optimal for an indexed or rule of thumb price to be strongly influenced by lagged inflation whereas weak persistence means that lagged inflation should not feature so prominently. This has two immediate implications for the standard versions of these models in which the indexing parameters are assumed fixed. Firstly they are vulnerable to the Lucas critique since the value of the indexing parameters influences the coefficients in the Phillips curve and hence a change in monetary policy regime which changes the degree of persistence will lead to changes in those coefficients under endogenous indexing. Secondly, the derivation of microfounded loss

functions for the indexing/rule of thumb models as a guide to optimal monetary policy on the assumption of fixed indexing parameters is questionable for the same reason that changes to the monetary policy regime may alter them. A further point is simply that the models with endogenous indexing allow one to check the plausibility of the particular values of the indexing parameters typically assumed in the literature.

The second set of results concerns the stability of the models and their ability to predict inflation persistence once we combine firm's choices with a model of policy. Here we focus on the Gali and Gertler (1999) and LAPI models which appear to be the most plausible from the micro analysis. Under endogenous indexing one may think of a firm's choice of indexing parameter in relation to perceived persistence as a reaction function. Collectively they determine the Phillips curve which in turn acts as a constraint on the policy maker. Policy in part determines persistence given the Phillips curve (even if the degree of persistence is not a primary objective) so it may also be characterised as a reaction function specifying realised persistence as a function of the Phillips curve and thus indirectly firms' beliefs about persistence. Hence a natural question is the nature of the Nash equilibrium at which firms behavior is optimal given the degree of persistence at the macro level and policy is optimal given the Phillips curve which results from the behavior of firms. In particular it is interesting to ask at what level of persistence the models are stable in the sense that reaction functions intersect and a fixed point between firm and policy behavior is achieved. We find strong results with very mild restrictions on policy behavior that amount to ensuring stability. For both the Gali and Gertler (1999) and LAPI models the fixed point is zero persistence.

Section 1 analyses the choice of indexing/rule of thumb structures and indexing parameters at the level of a single firm. Section 2 aggregates these to give the Phillips curves and Section 3 studies the interactions between firm and policy behavior. Section 4 concludes.

1. Constrained Optimal Indexing Behavior

We consider the optimal choices of indexing/rule of thumb structures and indexing parameters for individual firms who take the macroeconomic environment as given. The framework is standard with monopolistically competitive firms and a Calvo constant hazard structure. The optimisation problem for the firm is given in general form by (1) where V is the firm's expected net present value of profits over the relevant decision horizon, β is the discount factor, α is one minus the (constant) probability of the Calvo signal (and hence the probability of the price not changing if prices are sticky or the probability of not reoptimising if they are flexible), X is the firm's price in levels, Q its output and C(Q) its cost function. The optimisation problem (1) is subject to the usual constraint (2) which is the firm's demand curve given standard Dixit-Stiglitz preferences in which P is the aggregate price index and D an index of aggregate demand.

$$Max V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [X_{t+j} Q_{t+j} - C(Q_{t+j})] \tag{1}$$

$$Q_{t+j} = \left(\frac{X_{t+j}}{P_{t+j}}\right)^{-\eta} D_{t+j} \tag{2}$$

The models differ according to the restrictions placed on the evolution of the firm's price in (1). In a standard Calvo model it is simply fixed and determined fully optimally at time t. With Gali and Gertler (1999) firms it is also fixed but set according to a rule of thumb applied at time t. In the Christiano, Eichenbaum and Evans (2005) model and the LAPI variant derived below the initial price, X_t is set optimally and then varies according to an indexing function. We refer to these mechanisms collectively as simple pricing rules and follow the literature in assuming that a simple rule uses lagged information only and hence cannot respond to the current value of any shock variables. In the framework that follows, lagged inflation is the only past dated state variable and hence conditioning a pricing rule on it remains "simple" in the sense of using only a small

information set. In more general models with many past dated state variables an issue would arise of whether a simple rule can be conditioned on all of them or only a subset. If all past dated state variables may be included as here (and certainty equivalence applies) the optimal simple pricing rule will determine a price equal to the time t-1 rational expectation of the time t optimal price since the only constraint is the inability of the simple rule to react to contemporaneous (and assumed zero mean) shocks.

Before turning to the individual models we place further structure on the firms decision problem by assuming that inflation, π_t (=ln P_t-ln P_{t-1}), and the output gap in log deviation form, y_t, follow the processes (3) and (4) in which ϵ_t is a shock to marginal cost introduced below. For the time being we assume that the firms know the ρ parameters in (3)-(4) with certainty.

$$\pi_t = \rho_\pi \pi_{t-1} + k\varepsilon_t \tag{3}$$

$$y_t = \rho_y \pi_{t-1} + c\varepsilon_t \tag{4}$$

At this stage the assumption of these processes is arbitrary but later we show that they correspond to the reduced form of these variables in these models under plausible restrictions on policy. A remaining common element to the models is the assumption that the log deviation of real marginal cost from its steady state, m, is linear in the output gap and the cost push shock to give (5).

$$m_t = \phi y_t + \varepsilon_t \tag{5}$$

1.1 Decision Costs with Sticky Prices

We consider the solution to (1) when price stickiness means that the firm's price remains fixed over the decision horizon at X_t but decision or optimisation costs give rise to that price being set by a rule of thumb. This corresponds to the scope of the Gali and Gertler (1999) model. As a baseline we report the solution to (1) for a fully optimising Calvo firm and then derive the optimal rule of thumb which delivers a price equal in expectation to the optimised price given the constraint of not using

contemporaneous information.

For a standard Calvo firm, substituting (2) into (1) and imposing the restriction that the price remains fixed gives the problem (6) with first order condition (7).

$$Max_{X_t} V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [X_t^{1-\eta} D_{t+j} P_{t+j}^{\eta} - C([\frac{X_{t+j}}{P_{t+j}}]^{-\eta} D_{t+j})] \tag{6}$$

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j D_{t+j} P_{t+j}^{\eta} [(1-\eta) X_t + \eta \frac{\partial C(Q_{t+j})}{\partial Q_{t+j}}] \tag{7}$$

Following Goodfriend and King (1997), for example, (7) log linearises to (8) where lower case implies the log of the variable.

$$x_t = (1-\beta\alpha) E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [p_{t+j} + m_{t+j}] \tag{8}$$

Following the argument above, the optimal rule of thumb will efficiently use past dated information to achieve a price as close as possible to that in (8) and hence will amount to the time t-1 rational expectation of (8). An interim step is given by (9) which makes use of (3)-(5) and substituting (9) into the t-1 dated expectation of (8) and summing gives (10) where x^r indicates the optimal rule of thumb price and γ^r the indexing parameter with value shown by (11).

$$E_{t-1}[p_{t+j} + m_{t+j}] = p_{t-1} + \pi_{t-1}[\rho_\pi (\frac{1-\rho_\pi^{j+1}}{1-\rho_\pi}) + \phi\rho_y \rho_\pi^j] \tag{9}$$

$$x_t^r = p_{t-1} + \gamma^r \pi_{t-1} \tag{10}$$

$$\gamma^r = \frac{\rho_\pi + (1-\beta\alpha)\phi\rho_y}{1-\beta\alpha\rho_\pi} \tag{11}$$

We next relate (10)-(11) to Gali and Gertler (1999) framework. In their model the rule of thumb formula is expressed slightly differently but is equivalent to (10). A generalised version of the Gali-Gertler formula is (12), in which n_{t-1} are prices that were newly set the previous period, which reproduces their (23) except that we introduce the parameter γ^G which is implicitly set to unity in the original form.

$$x^G = n_{t-1} + \gamma^G \pi_{t-1} \tag{12}$$

New prices may be related to aggregate prices by (13) given the Calvo pricing rule.

$$p_t = (1-\alpha)n_t + \alpha p_{t-1} \tag{13}$$

Lagging (13) one period and substituting into (12) gives (14).

$$x^G = p_{t-1} + (\gamma^G + \tfrac{\alpha}{1-\alpha})\pi_{t-1} \tag{14}$$

Comparison of (14) with (10) confirms that the Gali-Gertler rule of thumb structure (12) is constrained optimal in the sense of corresponding to the efficient forecasting structure of (10), while (10) and (11) permit an assessment of the appropriate size of γ^G . Setting this parameter to unity implies that the coefficient on lagged inflation in (14) is 1/(1-α). This will be optimal if the ρ parameters in (11) are such that the right hand side of that expression equals 1/(1-α) which implies the following condition.

$$\rho_\pi = \frac{1-(1-\alpha)(1-\beta\alpha)\phi\rho_y}{1-\alpha(1-\beta)}$$

With β<1 and ρ_y<0 (as we argue below) this implies ρ_π>1 so inflation would have to be unstable for γ^G =1 to be optimal. A value of ρ_π in the more plausible range below unity would imply a lower value of γ^G and, as will be seen below, a smaller coefficient on lagged inflation in the Phillips curve for this model.

1.2 Decision Costs with Flexible Prices

We turn to the situation where decision/optimisation costs continue to motivate simple pricing rule behavior but prices are flexible. The Calvo signal is now interpreted as an opportunity to optimize the firm's price but prices may still change (according to the simple pricing rule) each period in the absence of that signal. This corresponds to the scope of the Christiano, Eichenbaum and Evans (2005) model but we derive the constrained optimal solution first before contrasting it with that framework.

If prices are flexible the optimisation (1) can reduce to a static problem since with flexible prices there is no inherent reason for the choice of price in one period to affect future periods. Hence a baseline result is the fully optimal flexible price using full information which, in log linear form, is given by x^f in (15). This is a standard result and may be derived following the steps above or simply taken from (8) with $\alpha=0$.

$$x_t^f = p_t + m_t \tag{15}$$

Given (15) the constrained optimal indexed price, x^a, will be the t-1 expectation of (15) given by (16) and (17) which make use of (3)-(5).

$$x_t^a = p_{t-1} + \gamma^a \pi_{t-1} \tag{16}$$

$$\gamma^a = \rho_\pi + \phi\rho_y \tag{17}$$

Hence firms will set the fully optimal flexible price in (15) when the Calvo signal permits them to optimise with contemporaneous information and the indexed price in (16) between signals.

We note that the constrained optimal indexing formula (16) involves the use of the lagged aggregate price as a base to which the indexing term is added and hence refer to this model as lagged aggregate price indexing (LAPI). This contrasts with the indexing formula assumed in Christiano, Eichenbaum and Evans (2005) in which the firm sets a price, X^c, when the Calvo signal arrives and subsequently indexes to its own lagged price rather than the aggregate lagged price until the next signal, hence the term lagged own price indexing (LOPI). The indexing formula takes the form (18) in levels or (19) in logs as in Woodford's (2003, chapter 3, eqn. 3.4) version of the model, also used in Smets and Wouters (2003), which generalises the original Christiano et. al. formulation (2005, eqn 8) in which γ^c was implicitly set to unity.

$$X^c_{t+j} = X^c_{t+j-1}\left(\frac{P_{t+j-1}}{P_{t+j-2}}\right)^{\gamma^c} \tag{18}$$

$$x^c_{t+j} = x^c_{t+j-1} + \gamma^c \pi_{t-1} \tag{19}$$

The LOPI firm's problem is hence (1) subject to (2), plus the need to use a simple pricing rule based on t-1 information, and (18) where (18) is an additional constraint not present in deriving the LAPI results immediately above. Here (18), or (19) in logs, is imposed as a constraint whereas in the LAPI model the indexing formula (16) emerged as part of the solution. This constraint binds and hence we find results for the LOPI model which differ from the LAPI framework. Given that the latter maximised (1) subject to (2) and the need to use a simple pricing rule, the LOPI outcome for firms will be inferior due to the presence of the extra constraint. This questions the use of the LOPI model if we seek constrained optimal behavior unless it is argued that the firm's own lagged price is more readily observable than the lagged aggregate price and hence simpler to use. That simplicity, however, comes at the cost both of the inferior performance already noted and a considerably more complex optimisation problem. Using (18) with (2), the LOPI firm's optimisation problem (1) is given by (20).

$$Max_{X_t^c,\gamma^c} V_t = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [(X_t^c (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma^c})^{1-\eta} D_{t+j} P_{t+j}^{\eta} - C(Q_{t+j})] \quad (20)$$

The first order condition for the initial price set when the Calvo signal is received , X^c at time t, is given by (21) whch linearises to (22).

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j P_{t+j}^{\eta} D_{t+j} (\frac{P_{t+j-1}}{P_{t-1}})^{-\gamma^c \eta} [(1-\eta) X_t^c (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma^c} + \eta P_{t+j} M_{t+j}] \quad (21)$$

$$0 = (1-\beta\alpha) E_t \sum_{j=0}^{\infty} \beta^j \alpha^j [-x_t^c + p_{t+j} + m_{t+j} - \gamma^c (p_{t+j-1} - p_{t-1})] \quad (22)$$

The first order condition for the indexing parameter, γ^c, is given by (23) which linearises to (24).

$$0 = E_t \sum_{j=0}^{\infty} \beta^j \alpha^j P_{t+j}^{\eta} D_{t+j} \ln(\frac{P_{t+j-1}}{P_{t-1}})(\frac{P_{t+j-1}}{P_{t-1}})^{-\gamma^c \eta} [(1-\eta) X_t^c (\frac{P_{t+j-1}}{P_{t-1}})^{\gamma^c} + \eta P_{t+j} M_{t+j}] \quad (23)$$

$$0 = (1-\beta\alpha) E_t \sum_{j=0}^{\infty} \beta^j \alpha^j (p_{t+j-1} - p_{t-1})[-x_t^c + p_{t+j} + m_{t+j} - \gamma^c (p_{t+j-1} - p_{t-1})] \quad (24)$$

Solving this model amounts to deriving the simultaneous solution to (22) and (24) which may readily be done using (3)-(5). The solution clearly involves the ρ parameters in (3)-(4), which is important for Proposition 1 below, but we do not present it since it is clear that the solution will generally differ from the LAPI model results of (15)-(17). Hence in addition to adding complexity to the firm's optimisation problem, the binding nature of the extra constraint (18) means that (19) will give rise to inferior outcomes compared with (16), making this model arguably less plausible without strong motivation for the need to index to the firm's own price.

Intuition for the worse performance under own price indexing may be seen by comparing (16) with (19). In (16) the indexed price will have automatically "caught up" with aggregate price developments up to t-1 by the first term, and the second indexing term plays a forecasting role of

developments between t-1 and t (necessitated by the simple rule making use of information up to t-1 only). If the ρ parameters in (3)-(4) are zero lagged inflation is not helpful for forecasting and the γ parameter is zero by (17). In (19) the use of the firm's own lagged price as a base in the first term implies that pricing errors from the past may still be present, there is no automatic error correction, and we find that the γ parameter is positive even if the ρ parameters are non-zero since it partly allows the firm's price to catch up with aggregate price developments to t-1. The second term also plays the forecasting role if inflation is persistent (the γ parameter increases) and hence this term has to balance the catch-up and forecasting roles whereas these are separated in (16).

Proposition 1. Under endogenous indexing, the constrained optimal price setting behavior of firms with significant decision or optimisation costs depends on their perception of the degree of persistence in inflation and the output gap/marginal cost.

In the models above, significant decision or optimisation costs leads firms to employ a rule of thumb or indexing formula when not resetting their prices optimally. From (11), (17) and (22)-(24) the indexing parameters depend on the coefficients in (3)-(4).

2. Phillips Curves

This section presents the Phillips curves for the three simple pricing models considered above. First, the Gali and Gertler (1999) Phillips curve is given by (25) which is simply their equation (24) with notation adjusted to allow for the more general γ^r in (10) rather than their value of unity for γ^g in (12) which would imply $\gamma^r=1/(1-\alpha)$ as above. This is a notational change and generalisation, the difference in the framework above being the potential endogeneity of the indexing parameter. In the Gali-Gertler model, rule of thumb firms are mixed with standard Calvo firms in the proportions ω and (1-ω) and hence if ω=0, (25) reduces to the standard New Keynesian Phillips curve.

$$\pi_t = \frac{\beta\alpha E_t[\pi_{t+1}] + \omega\gamma^r(1-\alpha)\pi_{t-1} + (1-\omega)(1-\alpha)(1-\beta\alpha)(\phi y_t + \varepsilon_t)}{\alpha + \omega(1-\alpha)(1+\beta\alpha\gamma^r)} \tag{25}$$

We briefly note two properties of (25). First it fails the weak form of the natural rate hypothesis, in the sense of the sum of coefficients on inflation on each side of the equation not being equal, even if $\beta=1$ unless γ^r equals $1/(1-\alpha)$ which is the value obtained from Gali and Gertler's assumption of $\gamma^G=1$ in (12). It was argued that this value was implausible under endogenous indexing unless inflation was believed to be extremely persistent. Second, if empirical estimates of the coefficient on lagged inflation are used to infer possible values of the structural parameters as in Gali and Gertler (1999), a lower value of γ^r (reflecting more plausible beliefs about inflation persistence) would require a larger implied proportion of rule of thumb firms, ω.

The LAPI model is new but its Phillips curve may readily be derived. All prices may change each period and are set optimally according to (15) if the Calvo signal occurs (with probability $1-\alpha$) and set according to the indexing function (16) if not (with probability α). If we assume that there are a large number of firms these probabilities translate into proportions and hence the price level is given by (26) which, using (16), gives rise to the Phillips curve (27).

$$p_t = (1-\alpha)x_t^f + \alpha x_t^a \tag{26}$$

$$\pi_t = \gamma^a \pi_{t-1} + \tfrac{1-\alpha}{\alpha}(\phi y_t + \varepsilon_t) \tag{27}$$

We note that the very simple form of (27) without any forward looking expectations reflects the absence of any price staggering when all prices are flexible and backward looking firms use the lagged aggregate price as an indexing base. It also violates the weak form of the natural rate hypothesis unless $\gamma^a=1$ which is implausible from (17) given $\rho_y<1$ unless inflation is again unstable with $\rho_\pi>1$.

For the LOPI model with an indexing parameter not restricted to unity, the Phillips curve is presented by Woodford (2003, chapter 3, eqn 3.6) and may be expressed as (28) using the notation above.

$$\pi_t = \frac{\beta E_t[\pi_{t+1}] + \gamma^c \pi_{t-1} + (\frac{1-\alpha}{\alpha})(1-\beta\alpha)(\phi y_t + \varepsilon_t)}{1+\beta\gamma^c} \tag{28}$$

Proposition 2. Under endogenous indexing the structure and coefficients of the Phillips curve depend on firms' perceptions of the degree of persistence in inflation and the output gap.

From Proposition 1, perceptions of the degree of persistence determine the indexing parameters in (11), (17) and (24). These appear in the Phillips curves respectively (25), (27) and (28) and hence perceptions of the degree of persistence partly determine the Phillips curve coefficients. In the special case where the γ parameters are zero, lagged inflation no longer appears in the Phillips curve.

Corollary 1. Under endogenous indexing, the indexing or rule of thumb models are vulnerable to the Lucas critique.

A change in monetary policy regime which affects the persistence parameters in (3)-(4) will change the γ parameters by Proposition 1 and hence the Phillips curve by Proposition 2 in which case the Phillips curves for these models are not invariant to monetary policy.

Remark 1. Under endogenous indexing, the derivation of appropriate microfounded loss functions for these models would be more complex than if the indexing parameters are fixed as has typically been assumed (Steinsson 2003, Woodford 2003). We leave this issue to future research.

3. Policy and Stability Analysis

We consider the interaction between firms price setting behavior from Section 1, which depends on macroeconomic persistence, and the degree of persistence which depends on the Phillips curve and

hence the underlying price setting behavior as shown in Section 2. In particular we derive the fixed point between the ρ parameters in (3)-(4) which guide price setting and the values which must obtain given the Phillips curve. This is done for the Gali and Gertler (1999) and LAPI models only since the LOPI framework appears to be less compelling at the microeconomic level.

A prior step is to note that from the Phillips curves the relevant state variables are lagged inflation and the current value of the cost push shock. We assume that policy itself does not introduce additional state variables. This will be valid if a simple Taylor rule combined with an IS relationship without additional lagged variables is used, or a simple quadratic loss function in inflation and the output gap is minimised under discretion. This assumption supports the original assumption of the processes for inflation and the output gap in (3)-(4). It may be noted that it would be violated if policy is implemented under commitment since the presence of the forward looking inflation term in the Phillips curves will give rise to the lag of the output gap in the reduced form of the system. Efficient simple pricing rules would then incorporate the lagged output gap in firms' indexing formulae so that variable would appear in the Phillips curve which would in turn affect the nature of optimal policy and so on. This example suggests that there may be rich interactions between the set of variables included in simple pricing rules and the set of state variables in the reduced form of the system as policy interacts with those pricing rules via the Phillips curve. We focus on the simplest fixed point of that interaction in which firms perceive the reduced form processes (3)-(4) and policy does not add additional state variables. This case is the simplest with which to explore the dynamics of the models under endogenous indexing and corresponds to the standard indexing/rule of thumb models.

Beyond the assumption concerning the relevant state variables we find that results may be derived by placing very little additional structure on the policy process. In particular two restrictions are required. First that if the coefficient on lagged inflation in the Phillips curve is zero the ρ parameters

in (3)-(4) will also be zero. This may be justified on standard MSV grounds (McCallum 1983, 1999). Second we assume that if the coefficient on lagged inflation is positive, ρ_π will be positive and ρ_y negative. The former simply follows from inflation generally being brought back to target gradually when lagged inflation appears in the Phillips curve and the latter is necessary to ensure that. Given that these mild restrictions are sufficient for our results we do not present a formal representation of the policy process.

Hence we proceed by seeking the simultaneous solution to the relevant Phillips curve from Section 2 together with the relevant γ parameter from Section (1) and the processes (3)-(4). As a first step we substitute (3)-(4) into the relevant Phillips curve until only terms in the two state variables remain. Since that equation must be satisfied for any values of the state variables the coefficient on each of them must be zero which gives two equations of which that from the lagged inflation term is informative for the ρ parameters. This may then be compared with the determinants of the γ parameters from the micro analysis.

This procedure is most easily introduced using the LAPI model. Substituting (3)-(4) into (27) until only terms in the two state variables remain gives (29) in which the two square bracketed expressions must be zero for a solution to obtain.

$$0 = \pi_{t-1}[\rho_\pi - \gamma^a - (\tfrac{1-\alpha}{\alpha})\phi\rho_y] + \varepsilon_t[k - (\tfrac{1-\alpha}{\alpha})(1 + c\phi\rho_y)] \qquad (29)$$

The first square bracket in (29) is informative about the relationship between the ρ parameters in (3)-(4) and the ρ parameters in (17) which must be equal at a fixed point between the price setting behavior of firms and the degree of persistence at the macro level. Substituting (17) into the first term in (29) shows that the only values of the ρ parameters at which this is the case is if both are zero. In this case the γ parameter in (17) is zero so lagged inflation disappears from the Phillips

curve (27) and there is no macroeconomic persistence by (3)-(4). Furthermore if firms use incorrect values of the ρ parameters in (17) such that γ is too high versus the full information case the first term in (29) may be factorised to show that the true value of γ will always be lower than the incorrectly perceived value. Hence if firms observe the true value of γ over time their indexing parameter will tend to fall under learning so the zero persistence equilibrium is likely to be stable under imperfect information and learning.

For the Gali-Gertler framework, substituting (3)-(4) into (25) until only terms in the state variables remain yields (30).

$$0 = \begin{array}{c} \epsilon_t[\ k[\beta\alpha(\rho_\pi - \omega\gamma^r(1-\alpha) - [1-(1-\alpha)(1-\omega)]] + (1+\phi c)(1-\alpha)\alpha[1-\beta\alpha] \\ \pi_{t-1}[\rho_\pi[\beta\alpha(\rho_\pi - \omega\gamma^r(1-\alpha) - [1-(1-\alpha)(1-\omega)]] + \phi\rho_y(1-\alpha)(1-\omega)[1-\beta\alpha] + \gamma^r\omega(1-\alpha)] \end{array} \quad (30)$$

Furthermore we may factorise the second line of (30) using (11) to give (31) in which γ^r is the firm's indexing parameter and γ^d is the same expression but with the empirical values in (3)-(4) rater than the values used by the firm in (11). In equilibrium these must be equal in which case (31) shows that ρ_y and ρ_π must be zero also. Hence this replicates the zero persistence result found for the LAPI model above and lagged inflation will once again disappear from the Phillips curve.

$$0 = \begin{array}{c} \phi\rho_y(1-\alpha)[1-\beta\alpha] - \rho_\pi\alpha(1-\beta\rho_\pi) \\ + (\gamma^r - \gamma^d)\omega(1-\alpha)[1-\beta\alpha\rho_\pi] \end{array} \quad (31)$$

The factorisation in (31) is also informative about the potential stability of this equilibrium under imperfect information and learning becase if the ρ parameters are non-zero, ρ_y. will be negative and hence $\gamma^d<\gamma^r$ so a "too high" value of the firms indexing parameter is likely to be pulled down under learning as firms observe its smaller empirical counterpart

Proposition 3. In the Gali and Gertler (1999) and LAPI models the unique Nash equilibrium between the pricing behavior of firms based on the degree of macroeconomic persistence and the latter resulting from that pricing behavior is one with zero persistence and lagged inflation is no longer present in the Phillips curve.

This follows from the discussion above with the stability restrictions imposed on the ρ parameters.

4. Conclusion

The paper has analysed the implications of endogenising the extent to which firms adjust their prices to lagged inflation when applying a rule of thumb or indexing formula. The motivation for this behavior by firms simply being to achieve a simple pricing rule which would deliver outcomes as close as possible to those which would obtain under full optimisation. The wider motivation of the paper was to explore the suitability of these models for monetary policy analysis when firms behavior is constrained optimal given the need to employ a simple pricing rule rather than them simply choosing arbitrary values for their indexing parameters.

The key results were i) that constrained optimal rules of thumb/indexing formulae depend on the degree of persistence in th macroeconomic environment, ii) following that these models are vulnerable to the Lucas critique (and microfounded loss functions which assume constant indexing parameters appear questionable), iii) the indexing structure in Christiano, Eichenbaum and Evans (2005) appears less plausible on microeconomic grounds than the simpler lagged aggregate price indexing framework, and iv) the Gali and Gertler and LAPI models are unable to reproduce inflation persistence under endogenous indexing when a fixed point is reached between pricing behavior and macroeconomic persistence. These results appear to strongly question the suitability of these models for monetary policy analysis, at least in the long run when learning dynamics may have worked themselves out.

References

Calvo, G. A. (1983), Staggered prices in a utility maximizing framework, Journal of Monetary Economics 12, 383-398.

Christiano, L. J., M. Eichenbaum and C. Evans (2005), Nominal rigidities and the dynamic effects of a shock to monetary policy, Journal of Political Economy, Vol. 113 No. 1., 1-45.

Eichenbaum, M. and J. Fisher (2003), Evaluating the Calvo model of sticky prices, Federal Reserve Bank of Chicago Working Paper 2003-23.

Gali, J. and M. Gertler (1999), Inflation dynamics: a structural econometric analysis, Journal of Monetary Economics 44, 195-222.

Goodfriend, M. and R. G. King (1997), The new neoclassical synthesis and the role of monetary policy, NBER Macroeconomics Annual 12, MIT Press, Cambridge.

McCallum, B. T. (1983), On non-uniqueness in rational expectations models: An attempt at perspective, Journal of Monetary Economics 11, 139-168.

McCallum, B. T. (1999), Role of the minimal state variable criterion in rational expectations models, International Tax and Public Finance 6, 621-639.

Smets, F. and R. Wouters (2003), An estimated stochastic dynamic general equilibrium model of the Euro area, Journal of the European Economic Association Vol 1 No. 5, September, 1123-1175.

Steinsson, J. (2003), Optimal monetary policy in an economy with inflation persistence, Journal of Monetary Economics 50(7), October, 1425-1456.

Woodford, M (2003), Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton, Princeton University Press.

Woodford, M. (2006), Interpreting inflation persistence: Comments on the conference on "Quantitative evidence on price determination", mimeo, available from www.columbia.edu.