ECONSTOR
Make Your Publications Visible.

A Service of
ZBW
Leibniz-Informationszentrum Wirtschaft
Leibniz Information Centre for Economics

Vasilev, Aleksandar

Article — Accepted Manuscript (Postprint)

Aggregation with non-convex labor supply, unobservable effort, and efficiency wages of the no-shirking type

Journal of Mathematical Economics and Finance

Suggested Citation: Vasilev, Aleksandar (2018) : Aggregation with non-convex labor supply, unobservable effort, and efficiency wages of the no-shirking type, Journal of Mathematical Economics and Finance, ISSN 2458-0813, ASERS Publishing House, Craiova, Vol. IV, Iss. 1(6), pp. 19-33,
https://doi.org/10.14505/jmef.v3.1(6).03

This Version is available at:
https://hdl.handle.net/10419/178690

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

WWW.ECONSTOR.EU

Mitglied der Leibniz-Gemeinschaft

Aggregation with non-convex labor supply, unobservable effort, and efficiency wages of the no-shirking type

Aleksandar Vasilev*

May 24, 2018

Abstract

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micro-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply increases from unity to infinity.

Keywords: Aggregation, Indivisible labor, Non-convexities, Shirking, Efficiency Wages
JEL Classification Codes: E1, J22, J41

*Independent Researcher. E-mail: alvasilev@yahoo.com

1 Introduction and Motivation

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micto-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply increases from unity to infinity.

2 Model setup

The theoretical setup follows to a great extent Vasilev (2017). There is a unit mass of households, indexed by i and distributed uniformly on the $[0, 1]$ interval, as well as a representative firm. In the exposition below, we will use small case letters to denote individual variables and suppress the index i to save on notation. To simplify the analysis, the model economy is static, without physical capital, and agents will face a non-convex labor supply decision. The firm produces output using labor and capital, but cannot observe the effort exerted by workers. Given that effort is not directly contractible (due to its unobservability on the firm's side), the firm sets a reservation wage to induce an optimal level of effort.

2.1 Description of the model

Each household maximizes the following utility function:

$$U = \ln c + \eta \ln(1 - eh - \xi), \tag{1}$$

where $\eta > 0$ is the weight attached to leisure, and as in Burnside *at al.* (1993, 1996), $\xi > 0$ is some fixed cost of working.[1] Variable c denotes household i's consumption, h denotes hours worked, and e is the amount of effort exerted. The time available to each worker is

[1]Parameter ξ is to be interpreted as some kind of organizational or planning cost, e.g the time spent on planning how to spent the day productively. Note that if the household decides to supply zero hours of labor, then $\xi = 0$.

normalized to unity. In addition, we assume that that worker's effort will be imperfectly observable by firms.

All households have equal share in the firm's profit. Total profit is pooled together (within the "family" of households), and then distributed equally among all households. In this way, households can partially insure one another against unfavorable outcomes in the labor market, *e.g.* not being selected for work. The common consumption can be represented as

$$c^h = \pi, \tag{2}$$

$\pi = \Pi$, or the sum of individual profit income equals firm's total profit. The other type of income is the labor income, and households would differ in each period depending on their employment status.

From the perspective of firms, all individuals are identical, so employment outcome could be viewed as random, *i.e.* the firm will choose a certain share of households for work, and leave the rest unemployed. Since the level of effort is not directly observable by firms, some of the employed workers will work and exert the required effort level, e, stipulated in the contract, while others may decide to shirk. If caught, which happens with probability d due to the imperfect technology of detection, the individual is fired and receives a fraction $0 < s < 1$ of the wage.[2]

The labor contract that the firms then needs to offer provide is to be one that induces workers not to cheat in equilibrium. The contract would specify a wage rate, an effort level, and an implementable rule that a worker caught cheating on the job will be fired and paid only a fraction s of the wage, $0 < s < 1$. All workers know this in advance, and take the terms of the contract and the labor demand as given. In general, the supply of labor will exceed labor demand, so in equilibrium there is going to be involuntary unemployment.

In addition, each employed transfers/contributes T units of income to the unemployment pool, where the proceeds are used to payout to the unemployed. The level of transfers is

[2] As in Burnside *et al.* (2000), the household does not observe whether the others shirked, or were fired, only the initial employment status.

such that individuals who are not selected for work by the firm are at least as well off as employed workers who are caught shirking. The consumption of an employed worker who does not decide to shirk then equals:

$$c = c^h + wh - T, \tag{3}$$

where w is the hourly wage rate. Note that an employed worker who decided to shirk, but is not caught, obtains the same consumption as the conscientious worker, but a higher utility of leisure due to the zero effort exerted and thus no fixed cost of work is incurred.

In contrast, a worker who is employed, decides to cheat, and is caught, receives

$$c^s = c^h + swh - T. \tag{4}$$

Alternatively, as proposed in Alexopoulos (2004), this is identical to a case where the firm pays swh upfront, and $(1-s)wh$ at the end of the period, which is retained in case the worker is caught cheating.

Note that not everyone will be employed, thus the employment rate $n < 1$, and $0 < 1-n < 1$ would denote the mass of unemployed. The consumption of unemployed individuals, c^u, is then

$$c^u = c^h + \frac{n}{1-n}T, \tag{5}$$

where the transfer received by each unemployed equals $\frac{n}{1-n}T$.[3] Note that if a household is selected for work and rejects the job offer, there will be no unemployment insurance, or it would receive just the common consumption c^h. Therefore, no household selected for work would have an incentive to reject, so the participation constraint will be trivially satisfied.

Depending on whether a household is selected for work or not, the corresponding instantaneous utility levels are:

$$u(c^u, e^u = 0, h^u = 0) = \ln c^u + \eta \ln 1 = \ln c^u, \tag{6}$$

[3]It is straightforward to reformulate the model so that a self-financing unemployment insurance program is provided by the government rather than the household. Therefore, this setup is very close to the one using unemployment lotteries in Rogerson (1988) and Hansen (1985)

if unemployed,

$$u(c, e, h) = \ln c + \eta \ln(1 - eh - \xi), \tag{7}$$

if employed and the worker does not shirk,

$$u(c, e, h) = \ln c + \eta \ln(1) = \ln c, \tag{8}$$

if the person shirks, but is not caught, and

$$u(c^s, e^s = 0, h^s = 0) = \ln c^s + \eta \ln(1) = \ln c^s, \tag{9}$$

if the person shirks, and is caught.

Let n^s be the proportion of shirkers and given a detection probability d of a shirker being caught, this implies dn^s would be the proportion of shirkers being caught, and $(1-d)n^s$ are the shirkers not being caught. In turn, $n - n^s$ are the employed individuals who decide not to shirk.

Finally, note that the leisure (in efficiency units) of shirkers that are caught, and leisure enjoyed by unemployed individuals is the same. Thus, the lump-sum transfer should be chosen so that the consumption levels of the two groups is equalized, or

$$c^s = c^u \tag{10}$$

$$c^h + swh - T = c^h + \frac{n}{1-n}T. \tag{11}$$

or

$$T = (1-n)swh. \tag{12}$$

In this setup the aggregate household takes as given the effort level and wage rate $\{e, w\}$, which are specified in the contracts that the firm offers. This means that the household takes firm's labor demand as given, which would produce involuntary unemployment. Thus, the household chooses $\{c^h\}$ to maximize (where we have already used the fact that $c^u = c^s$)

$$(n - n^s)[\ln c + \eta \ln(1 - \xi - eh)] + n^s[(1-d)\ln c + d \ln c^s] + (1-n)\ln c^s \Big\} \tag{13}$$

s.t

$$(n - dn^s)c + (dn^s + 1 - n)c^s = \\ (n - dn^s)wh + dn^s swh. \tag{14}$$

The first-order condition (FOCs) is as follows:

$$c^h \quad : \quad \frac{(n - dn^s)}{c} + \frac{(dn^s + 1 - n)}{c^s} = \lambda. \tag{15}$$

2.2 Firm

There is a perfectly competitive representative firm that produces output via the following Cobb-Douglas production function

$$y_t = (nhe)^{1-\alpha}. \tag{16}$$

The firm chooses the employment rate, capital input, wage rate (and thus effort level) to maximize

$$(nhe)^{1-\alpha} - wnh \tag{17}$$

s.t. "no shirking condition" (the ICC):

$$\ln c + \eta \ln(1 - \xi - h) \geq (1 - d) \ln c + d \ln c^s \tag{18}$$

or

$$d \ln c + \eta \ln(1 - \xi - h) \geq d \ln c^s \tag{19}$$

In equilibrium, the firm chooses the optimal employment. In addition the firm offers an efficiency wage rate w to induce a certain optimal effort level, i.e. $e = e(w)$.[4]

$$n : wh = (1 - \alpha)\frac{y}{n}. \tag{20}$$

$$w : nh = (1 - \alpha)\frac{y}{e}e'(w) \tag{21}$$

[4]As in Solow (1979), we assume that the wage rate is a function of effort.

Dividing the FOC for employment and wages, we obtain the standard Solow (1979) condition

$$\frac{we'(w)}{e} = 1 \tag{22}$$

or

$$\frac{w}{e(w)} = (1-\alpha)\frac{y}{nh}. \tag{23}$$

In other words, this is an equation that characterizes firm's labor demand. Note that the firm minimizes cost per efficiency unit here.[5] Firms want to hire labor as cheaply as possible, and $w/e(w)$ is the cost per unit of effective labor.

Next, for a given wage rate, the "no-shirking" condition indicated a maximum effort level the firm can obtain from each worker. Rearranging further the constraint, we obtain

$$e \leq e(w) = \frac{1-\xi}{h} - \frac{1}{h}(c^s/c)^{d/\eta}. \tag{24}$$

The firm takes T as given, so the right-hand side is only a function of w, since

$$\frac{c}{c^s} = \frac{c^h + wh - T}{c^h + swh - T} \tag{25}$$

Also

$$e'(w) = -\frac{d}{\eta}\frac{1}{h}(\frac{c^s}{c})^{d/\eta-1}(\frac{c^s - sc}{(c^s)^2})h. \tag{26}$$

and

$$w = \frac{c - c^s}{(1-s)h} \tag{27}$$

since the ratio of consumptions is a function of the wage rate, a result that follows from the Solow condition, the effort equation and the wage expression above. Combining the Solow condition, the effort equation, and the wage expression above, it follows that there is only one value for $\frac{c^s}{c}$ that solves this equation and produces a positive level of effort in

[5] If the firm pays higher efficiency wages to induce more effort, that decreases labor demand (because of the wage premium incorporated in the efficiency wage) and produces involuntary unemployment. Also note that the firm adjusts the extensive margin (employment rates), while hours per person are not changing.

equilibrium. Thus the ratio of consumptions is a constant (denoted by χ), and a function of model parameters, *i.e.*

$$\frac{c}{c^s} = \frac{c^h + wh - T}{c^h + swh - T} = \chi > 1 \tag{28}$$

In general, the optimal level of employment, will not coincide with the proportion of workers wishing to accept the contract $(w, e(w))$. As long as firm's demand for labor is less than the labor supply, the "no-shirking" constraint will be binding (hold with equality), and there will be involuntary unemployment in equilibrium.

2.3 Stand-in firm

There is a representative firm in the model economy. It produces a homogeneous final product using a production function that requires labor H as the only input. For simplicity, output price will be normalized to unity. The production function $f(H)$ features decreasing returns to scale (for any effort level): $f'(eH) > 0$, $f''(eH) < 0$, $f'(0) = \infty$, $f'(e\bar{h}) = 0$. The representative firm acts competitively by setting the wage rate w and choosing H to maximize profit by stimulating optimal effort:

$$\pi = f(eH) - wH \quad \text{s.t.} \quad 0 \leq H \leq \bar{h}. \tag{29}$$

and

$$e = f'(e)g(w). \tag{30}$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

2.4 Decentralized competitive equilibrium (DCE): Definition

A DCE is defined by allocations $\{c^w, c^u, e, H\}$, wage rate $\{w\}$, and aggregate profit $(\Pi = \pi)$ s.t. (1) all households maximize utility; (2) the stand-in firm maximizes profit; (3) all markets clear.

3 Characterization the DCE and derivation of the aggregate utility function

It will be shown that in the DCE, if it exists, only some of the households will be employed and work full-time, while the rest will be unemployed. Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that polar cases in which each household either, or a case in whichnoboly works, cannot not be equilibrium outcomes. Therefore, it must be the case that a only proportion of the agents in the economy are working. Denote this mass of employed by n. Workers in the official sector will receive consumption c^w, while those working in the unofficial sector will consume c^u. Note that n can be interpreted also as the probability of being chosen to work: This probability is determined endogenously in the model, as workers would seek for the optimal balance between the net return from working in terms of increased utility of consumption, which, however,comes at the expense of lower utility out of leisure. Note that no matter of the employment outcome, ex post every household enjoys the same utility level. Thus, in equilibrium $H = n\bar{h}$. As derived earlier, the wage is set equal to:

$$\ln w = \frac{e^{2-\alpha}}{\alpha} + \varphi \ln \frac{(e\bar{h}n)^{\alpha}}{n}$$

Firm's profit is then

$$\pi = \Pi = f(n\bar{h}e) - wn\bar{h} > 0, \tag{31}$$

which follows from the decdreasing returns to scale featured by the production function. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point $n \in (0, 1)$ by analyzing a non-linear equation using the fact that in equilibrium utility is the same for all households. Note that from the result that grey-sector hours are a function of total market hours, it follows now that unofficial hours are a function of the proportion of households employed in the market sector (since $\bar{h}$ is now a parameter).

It is trivial to show that everyone working in the market sector ($n = 1$) is not an equilibrium, since then $w = 0$. From the ex ante symmetry assumption for households, market consumption would be the same for workers and those not selected for work, while the latter

would enjoy higher utility out of leisure, hence there is no benefit of working. Similarly, nobody working ($n = 0$) is not an equilibrium outcome either, since the firm would then offer a very high wage for the first unit of labor, and by taking a full-time job a marginal worker could increase his/her utility a lot.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If $n \in (0, 1)$ is an equilibrium, then total utility for households that work should equal to the utility of households that do not supply any hours. This equation is monotone in n, as the utility function is a sum of monotone functions. Thus we can explore the behavior of that function (the difference between the utility of working and not working) as we let n vary in the $(0, 1)$ interval. As $n \to 0$, the left-hand-side dominates (utility of working is higher), while when $n \to 1$ the right-hand-side dominates (utlity of not working is higher), where the results follow from the concavity of the utility functions and the production technology. In addition, from the continuity of those functions, $\exists n \in (0, 1)$, which is consistent with equilibrium. The unique value of n follows from the monotonicuty of the utility and production functions. Let c^w and c^u denote equilibrium consumption allocations of individuals selected for work, and those who will work in the informal sector.

Given the indivisibility of the labor supply, the equilibrium allocation obtained above is not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a social planner (SP) could make everyone better off by using an employment lottery in the first stage and choosing the fraction n of individual households to work in the market sector and give everyone consumption $nc^w + (1 - n)c^u$. In order to show this, we need to check that such an allocation is feasible, and that it provides a higher level of total utility. Showing feasibility is trivial as total market labor input and total consumption are identical to the corresponding individual equilibrium values.

Next, we will show that the new allocation, which is independent of households employment status in the market sector, makes households better off since it generates higher utility on average. This is indeed the case, where the strict inequality follows from the convexity of the CES aggregation and the concavity of the logarithmic function. Thus, the SP is indeed

giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation. If households can pool income together and doing so, they will be able to equalize consumption across states, when a worker is caught shirking, and when unemployed, i.e., $c = c^s = c^u$, so aggregate utiltiy becomes:

$$(n - n^s)[\ln c + \eta \ln(1 - \xi - eh)] + n^s[(1 - d) \ln c + d \ln c^s] + (1 - n) \ln c^s, \tag{32}$$

which is the representation used in Vasilev (2017). On the aggregate, when each household faces an indivisible labor choices, the representative agent obtained from the aggregation features different preferences of work: as in Hansen (1985), the disutility of work in the market sector is now linear.

4 Conclusions

The purpose of this note is to explore the problem of non-convex labor supply decision in an economy with imperfect observability of work effort, and the need to use efficiency wages to prevent shirking as in Shapiro and Stiglitz (1984). In addition, the paper and explicitly performs the aggregation presented in Vasilev (2017) without a formal proof, and thus provide - starting from micto-foundations - the derivation of the expected utility functions used for the aggregate household. We show how lotteries as in Rogerson (1988) can be used to convexify consumption sets, and aggregate over individual preferences. With a discrete labor supply decisions, the elasticity of aggregate labor supply increases from unity to infinity.

References

Alexopoulos, M. (2004) "Unemployment and the business cycle," *Journal of Monetary Economics* 51: 277-298.

Burnside, C., Eichenbaum, M. and Rebelo, S. (1993) "Labor Hoarding and the Business Cycle," *Journal of Political Economy*, vol. 101(2), pp. 245-73.

Burnside, C. and M. Eichenbaum (1996) "Factor Hoarding and the Propagation of Business Cycle Shocks," *American Economic Review* 86, 1154-74.

Burnside, C., Eichenbaum, M., and J. Fisher (2000) "Fiscal Shocks in an Efficiency Wage Model," *NBER Working Paper* No. 7515.

Hansen, G. (1985) "Indivisible labor and the business cycle," *Journal of Monetary Economics*, 16(3), 309-328.

Rogerson, R. (1988) "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, vol. 21(1), pp. 3-16.

Shapiro, C. and Stiglitz, J.E. (1984) "Equilibrium unemployment as a worker discipline device," *American Economic Review* 36, pp. 1291-1303.

Vasilev, A.Z. (2017) "A Real-Business-Cycle model with efficiency wages and fiscal policy: the case of Bulgaria," *Bulgarian Economic papers* 01-2017, *Center for Economic Theories and Policies*, Sofia University St. Kliment Ohridski, Sofia, Bulgaria.