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## Asymmetric Expectation Effects of Regime Shifts and the Great Moderation

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# Asymmetric Expectation Effects of Regime Shifts and the Great Moderation

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**Kiel Working Paper No. 1357**

**Asymmetric Expectation Effects of Regime  
Shifts and the Great Moderation**

**by**

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# ASYMMETRIC EXPECTATION EFFECTS OF REGIME SHIFTS AND THE GREAT MODERATION

ZHENG LIU, DANIEL F. WAGGONER, AND TAO ZHA

ABSTRACT. We assess the quantitative importance of expectation effects of regime shifts in monetary policy in a DSGE model that allows the monetary policy rule to switch between a “bad” regime and a “good” regime. When agents take into account such regime shifts in forming expectations, the expectation effect is asymmetric. In the good regime, the expectation effect is small despite agents’ disbelief that the regime will last forever. In the bad regime, however, the expectation effect on equilibrium dynamics of inflation and output is quantitatively important, even if agents put a small probability that monetary policy will switch to the good regime. Although the expectation effect dampens aggregate fluctuations in the bad regime, a switch from the bad regime to the good regime can still substantially reduce the volatility of both inflation and output, provided that we allow some “reduced-form” parameters in the private sector to change with monetary policy regime.

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[Lucas Jr. (1976)] has expressed the view that it makes no sense to think of the government as conducting one of several possible policies while at the same time assuming that agents remain certain about the policy rule in effect.

*Cooley, LeRoy, and Raymon (1984, p.468)*

Explicit modelling of the connection of expectation-formation mechanisms to policy [regime] in an accurately identified model would allow better use of the data.

*Sims (1982, p.120)*

## I. INTRODUCTION

There is a broad consensus that U.S. monetary policy regime has shifted over time, notably since the early 1980s. In an important strand of literature that studies the macroeconomic effects of changes in monetary policy regime, the prevailing assumption is that private agents form rational expectations with respect to all shocks and underlying uncertainties. At the same time, perhaps paradoxically, it is also assumed that whenever monetary policy enters a particular regime, agents will naively believe that the regime will last forever. For example, the influential work by Clarida, Galí, and Gertler (2000), along with Lubik and Schorfheide (2004) and Boivin and Giannoni (2006), studies macroeconomic effects of two different monetary policy rules, corresponding to the pre-Volcker regime and the post-Volcker regime. By studying the two sub-sample periods separately, they reach a conclusion that changes in monetary policy help explain the substantial decline in macroeconomic volatility observed in the post-war U.S. economy. The practice of splitting the sample into sub-samples reflects the simplifying assumption that after observing a regime shift, agents believe that the current regime will prevail permanently.

Such a simplification does not square well with possible changes in future monetary policy regime. This point has been forcibly elaborated by Sims (1982), Cooley, LeRoy, and Raymon (1984), and Sims (1987), among others. These authors argue that in an economy where past changes in monetary policy rules are observable and future changes are likely, rational agents will form a probability distribution over possible policy shifts in the future when forming expectations. The difference in equilibrium outcomes between a model that ignores probabilistic switches in future policy regime and a model that takes into account such expected regime switches reflects the key expectation-formation aspect of

the Lucas critique, as implied by the first epigraph at the start. We call this difference the “expectation effect of regime shifts” in monetary policy.

There are two important questions. The first question is how significant the expectation effect of regime shifts can be. If such an effect is small, the equilibrium outcome obtained under the assumption that rules out future regime changes can be a good approximation to the rational-expectations equilibrium. If the expectation effect turns out to be large, however, it will be crucial to assess the equilibrium consequences of expected regime changes in monetary policy. The second question is whether large expectation effects will diminish the model’s ability to predict the Great Moderation observed in the data (Stock and Watson, 2003).

The goal of this paper is to (1) assess the quantitative importance of the expectation effect of regime shifts in monetary policy and (2) study whether a standard dynamic stochastic general equilibrium (DSGE) model is able to predict the Great Moderation when potentially large expectation effects are accounted for. For this purpose, we build a DSGE model that explicitly connects the expectation-formation mechanism to regime shifts in the systematic component of monetary policy. Our model features nominal rigidities in the form of staggered price setting and dynamic inflation indexation, and real rigidities in the form of habit formation (e.g., Christiano, Eichenbaum, and Evans 2005, henceforth CEE). Monetary policy follows a Taylor rule, under which the nominal interest rate is adjusted to respond to its own lag and deviations of inflation from its target value and of output from its trend. We generalize the standard DSGE model by allowing coefficients in the monetary policy rule as well as the duration of price contracts and the degree of inflation indexation to change over time. These regime changes follow a Markov-switching process, as in Hamilton (1994). We view this kind of regime-switching structural model as a starting point to study the quantitative importance of expectation effects of regime switching in monetary policy, as emphasized by Sims and Zha (2006) and Cecchetti, Hooper, Kasman, Schoenholtz, and Watson (2007).<sup>1</sup>

The economy we consider has two monetary policy regimes. The first regime represents a policy that responds to inflation weakly (a bad regime) and the second represents a policy that responds to inflation aggressively (a good regime). To address the

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<sup>1</sup>There has been a growing literature on Markov-switching rational expectations models. See, for example, Andolfatto and Gomme (2003), Leeper and Zha (2003), Schorfheide (2005), Svensson and Williams (2005), Davig and Leeper (2006), and Farmer, Waggoner, and Zha (2007). Those studies do not tackle the issues that we address in this paper.

quantitative importance of regime shifts in monetary policy, we simulate the two-regime DSGE model and obtain the following key findings.

- The expectation effect of regime change is *asymmetric* across regimes. Under the bad policy regime, the volatilities of inflation and output are significantly lower when agents take account of the probability of a switch to the good policy regime than when they naively believe that the bad regime will persist indefinitely. Under the good policy regime, however, the expectation effect is small. The asymmetric expectation effects arise because equilibrium dynamics are *nonlinear* functions of the model parameters.
- The importance of the expectation effect depends more on how strong the propagation mechanisms are and less on how persistent the prevailing regime is. The stronger the propagation mechanisms are, the more impact the expectation of future regime change will have on the equilibrium evolution of inflation and output. While in theory the expectation effect disappears if the prevailing regime last indefinitely, we find that in practice the expectation effect under the bad policy regime is quantitatively important even if the regime is very persistent.
- Although expectations of regime switches dampen the fluctuations in inflation and output under the bad regime, we find that a switch from the bad regime to the good regime can nonetheless lead to a sizable reduction in the volatility of both inflation and output if firms' pricing behaviors (characterized by the price-stickiness and inflation-indexation parameters) vary with policy regime.

Understanding the expectation effects of regime shifts helps bridge the gap between two polar approaches in the DSGE literature: one that does not allow for any switch in the systematic component of monetary policy and one that allows for switches in monetary policy regimes but does not allow private agents to form expectations about possible changes in future policy. Since the expectation effect under the bad regime can considerably alter the dynamics of key macroeconomic variables, caution needs to be taken in interpreting empirical models that are used to fit a long sample that covers the period with the bad regime. In the good policy regime, on the other hand, the expectation effect is small even if agents expect that the regime will shift to the bad regime with a non-trivial probability. Thus, even if a newly instituted good regime is not perfectly credible, such as the Volcker regime studied by Goodfriend and King (2005), inflation fluctuations can still be effectively stabilized. These theoretical findings have important empirical implications. Fitting a regime-switching DSGE model

to the data takes into account the potentially important expectation effects of regime shifts. Because it does not require splitting a long sample into short sub-samples, one can obtain more precise estimates of the “deep” parameters that do not vary with policy regimes.

## II. A SIMPLE MONETARY MODEL

In this section we study a simple monetary model with monetary policy switching regimes. The model is simple enough for us to obtain closed-form analytical results. These results help us to gain intuition of how asymmetric expectation effects of regime switches can occur.

**II.1. The simple model.** Consider an endowment economy in which a one-period risk-free nominal bond is traded. The representative agent maximizes the utility

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$P_t c_t + B_t = P_t y_t + R_{t-1} B_{t-1},$$

where  $c_t$  denotes consumption,  $y_t$  denotes the endowment,  $P_t$  denotes the price level,  $B_t$  denotes the agent’s holdings of the bond, and  $R_{t-1}$  denotes the nominal interest rate between period  $t-1$  and  $t$ . The parameter  $\beta \in (0, 1)$  is a subjective discount factor and the parameter  $\gamma > 0$  measures the relative risk aversion. The endowment follows the exogenous stochastic process

$$y_t = y_{t-1} \lambda \exp(z_t), \quad z_t = \rho z_{t-1} + \varepsilon_t, \quad (1)$$

where  $\lambda \geq 1$  measure the average growth rate of the endowment,  $\rho \in (0, 1)$  measures the persistence of the endowment shock, and  $\varepsilon_t$  is an i.i.d. normal process with mean zero and variance  $\sigma_z^2$ .

The first order condition with respect to the bond holdings is given by

$$\frac{c_t^{-\gamma}}{P_t} = \beta \mathbb{E}_t \frac{c_{t+1}^{-\gamma}}{P_{t+1}} R_t, \quad (2)$$

which describes the tradeoff between spending a dollar today for current consumption and saving a dollar for future consumption.

Monetary policy follows the interest rate rule

$$R_t = \kappa \left( \frac{\pi_t}{\pi^*} \right)^{\phi_{s_t}}, \quad (3)$$



where  $\pi_t = P_t/P_{t-1}$  is the inflation rate,  $\pi^*$  denotes the inflation target,  $s_t$  denotes the realization of monetary policy regime in period  $t$ ,  $\phi_{s_t}$  is a regime-dependent parameter that measures the aggressiveness of monetary policy against deviations of inflation from its target, and  $\kappa$  is a constant. Monetary policy regime follows a Markov-switching process between two states: a bad regime characterized by  $s_t = 1$  and  $0 \leq \phi_1 < 1$  and a good regime by  $s_t = 2$  and  $\phi_2 > 1$ . The transition probability matrix  $Q = [q_{ij}]$  is a  $2 \times 2$  matrix with  $q_{ij} = \text{Prob}(s_{t+1} = i | s_t = j)$ . Each column of  $Q$  sums to 1 so that  $q_{21} = 1 - q_{11}$  and  $q_{12} = 1 - q_{22}$ .

Market clearing implies that  $c_t = y_t$  and  $B_t = 0$  for all  $t$ . Using the goods market clearing condition, we can rewrite the intertemporal Euler equation as

$$\beta E_t \left( \frac{y_{t+1}}{y_t} \right)^{-\gamma} \frac{R_t}{\pi_{t+1}} = 1. \quad (4)$$

Thus, higher consumption (or income) growth requires a higher real interest rate.

**II.2. Steady state and equilibrium dynamics.** Given the stochastic process (1) for the endowment, an equilibrium in this economy is summarized by the Euler equation (4) and the monetary policy rule (3). The variables to interest include the inflation rate  $\pi_t$  and the nominal interest rate  $R_t$ .

A steady state is an equilibrium in which all shocks are shut off (i.e.,  $\varepsilon_t = 0$  for all  $t$ ). The Euler equation implies that, in the steady state, we have

$$\frac{R}{\pi} = \frac{\lambda^\gamma}{\beta}.$$

Let  $\kappa = \frac{\lambda^\gamma}{\beta} \pi^*$ . It follows from the Euler equation (4) and the interest rate rule (3) that the steady-state solution is

$$\pi = \pi^*, \quad R = \frac{\lambda^\gamma}{\beta} \pi^*.$$

Although monetary policy regime switches between the two regimes, the steady-state solution does not depend on policy regime and thus allows us to log-linearize the equilibrium conditions around the constant steady state.

Log-linearizing the Euler equation (4) around the steady state results in

$$\hat{R}_t = E_t \hat{\pi}_{t+1} + \gamma \rho z_t, \quad (5)$$

where  $\hat{R}_t$  and  $\hat{\pi}_t$  denote the log-deviations of the nominal interest rate and the inflation rate from steady state. Log-linearizing the interest rate rule (3) around the steady state leads to

$$\hat{R}_t = \phi_{s_t} \hat{\pi}_t. \quad (6)$$

The linearized Euler equation (5) implies that, following a positive shock to  $z_t$ , the real interest rate will rise. This result reflects that an increase in  $z_t$  leads to a rise in expected consumption growth and thus a rise in the real interest rate. Combining (5) and (6), we obtain the single equation that describes inflation dynamics:

$$\phi_{s_t} \hat{\pi}_t = E_t \hat{\pi}_{t+1} + \gamma \rho z_t, \quad s_t \in \{1, 2\}. \quad (7)$$

**II.3. The MSV solution.** We now discuss our approach to solving the model (7) for equilibrium dynamics of inflation. Throughout this paper we use the minimum-state-variable (MSV) solution advocated by McCallum (1983, 1998), a bubble-free solution in the spirit of King and Watson (1998).<sup>2</sup>

The state variable in the simple model (7) is the shock  $z_t$ . Thus the solution takes the form  $\pi_t = \alpha_{s_t} z_t$ , where  $\alpha_{s_t}$  is to be solved for  $s_t \in \{1, 2\}$ . The following proposition gives the analytical solution.

*Proposition 1.* The MSV solution to the regime-switching model (7) is given by

$$\hat{\pi}_t = \alpha_{s_t} z_t, \quad s_t \in \{1, 2\},$$

where

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \phi_1 - \rho q_{11} & -\rho q_{21} \\ -\rho q_{12} & \phi_2 - \rho q_{22} \end{bmatrix}^{-1} \begin{bmatrix} \gamma \rho \\ \gamma \rho \end{bmatrix}, \quad (8)$$

with the implicit assumption that the matrix above is invertible.

*Proof.* See Appendix A.1. □

The solution represented by (8) implies that the standard deviation of inflation is given by

$$\sigma_{\pi,1} = \frac{|\alpha_1|}{1 - \rho^2} \sigma_z, \quad \sigma_{\pi,2} = \frac{|\alpha_2|}{1 - \rho^2} \sigma_z.$$

The following proposition establishes that the volatility of inflation in the bad regime decreases with the probability of switching to the good regime and that the volatility of inflation in the good regime increases with the probability of switching to the dovish regime. Thus, the expectation of regime switch affects inflation dynamics.

*Proposition 2.* Assume that the matrix

$$A = \begin{bmatrix} \phi_1 - \rho q_{11} & -\rho q_{21} \\ -\rho q_{12} & \phi_2 - \rho q_{22} \end{bmatrix}$$

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<sup>2</sup>In the case of indeterminacy, the MSV solution is selected for the reasons argued by McCallum (2003). Furthermore, Boivin and Giannoni (2006) show that the MSV solution to their DSGE model can explain the persistence and volatility of U.S. inflation observed in the pre-Volcker period.

is positive definite. Then the MSV solution given by (8) has the property that  $\alpha_j > 0$  for  $j \in \{1, 2\}$  and that

$$\frac{\partial \alpha_1}{\partial q_{21}} < 0, \quad \frac{\partial \alpha_2}{\partial q_{12}} > 0. \quad (9)$$

*Proof.* See Appendix A.2.  $\square$

**II.4. Expectation effects.** The solution (8) takes into account possible switches of future policy regime. This solution in general differs from that obtained under the simplifying assumption that agents believe that the current regime will continue permanently. The difference between these two solutions is what we call the expectation effect of regime switching.

To examine the underlying forces that drive the expectation effect, we consider the solution that rules out any change in future policy, which is equivalent to solving the following model

$$\phi_j \hat{\pi}_t = E_t \hat{\pi}_{t+1} + \gamma \rho z_t, \quad (10)$$

where  $\phi_j$  ( $j = 1, 2$ ) does not depend on the time  $t$ . The equilibrium condition (10) is a special case of the condition (7) with  $q_{11} = 1$  for  $j = 1$  and with  $q_{22} = 1$  for  $j = 2$ . The solution to (10) is given by the following proposition.

*Proposition 3.* The MSV solution to the model (10) is

$$\hat{\pi}_t = \bar{\alpha}_j z_t, \quad \bar{\alpha}_j = \frac{\gamma \rho}{\phi_j - \rho}, \quad j \in \{1, 2\}, \quad (11)$$

where it is assumed that  $\phi_j \neq \rho$ .

*Proof.* See Appendix A.3.  $\square$

The solution represented by (11) implies that the standard deviation of inflation under the assumption that rules out changes in future policy regime is given by

$$\bar{\sigma}_{\pi,1} = \frac{|\bar{\alpha}_1|}{1 - \rho^2} \sigma_z, \quad \bar{\sigma}_{\pi,2} = \frac{|\bar{\alpha}_2|}{1 - \rho^2} \sigma_z.$$

The expectation effect of regime switches can be measured by the magnitude  $|\alpha_j - \bar{\alpha}_j|$  for  $j = 1, 2$ . Because  $\bar{\alpha}_j$  does not depend on transition probabilities, Proposition 2 implies that the less persistent the regime  $j$  is, the more significant the expectation effect  $|\alpha_j - \bar{\alpha}_j|$  becomes. Similarly, it follows from the solutions (8) and (11) that if the endowment growth follows an i.i.d. process ( $\rho = 0$ ), we have  $\alpha_j = \bar{\alpha}_j = 0$  for  $j \in \{1, 2\}$ . In other words, if the shock has no persistence, inflation will be completely stabilized regardless of monetary policy regimes. There is no expectation effect of

regime shifts. With the persistent shock, the solutions (8) and (11) will be different, and the expectation effect will exist.

**II.5. Asymmetry.** As one can see from (8),  $\alpha_j$  is the nonlinear function of the model parameters. This nonlinearity implies that when the probabilities of switching are the same for both regimes (i.e., when  $q_{11} = q_{22}$ ), the expectation effect may *not* be symmetric across the two regimes. This result is formally stated in the following proposition.

*Proposition 4.* Assume that  $q_{11} = q_{22}$ . If  $\phi_1 > \rho$ , then

$$\frac{|\alpha_1 - \bar{\alpha}_1|}{|\alpha_2 - \bar{\alpha}_2|} = \frac{\phi_2 - \rho}{\phi_1 - \rho} > 1. \quad (12)$$

*Proof.* See Appendix A.4. □

Proposition 2 shows that the expectation of regime switching out of the bad regime stabilizes inflation fluctuations, whereas the expectation of regime switching out of the good regime destabilizes the inflation process. Proposition 4 shows that the stabilizing effect in the bad regime exceeds the destabilizing effect in the good regime. Moreover, the expectation effect becomes more asymmetric if the shock is more persistent, if monetary policy takes a stronger hawkish stance against inflation in the good regime, or if policy is less responsive to inflation in the bad regime.

A strong propagation mechanism, be it exogenous or endogenous, is an important driving force behind the expectation effect as well as its asymmetry. To understand the role of endogenous propagation, consider the generalized version of the equilibrium condition (7)

$$\phi_{s_t} \hat{\pi}_t + \kappa_{s_t} \hat{\pi}_{t-1} = E_t \hat{\pi}_{t+1} + \gamma \rho z_t, \quad s_t \in \{1, 2\}, \quad (13)$$

where monetary policy responds to both current and lagged inflation rates. In this setup, even if the shock  $z_t$  is not persistent, the presence of the endogenous propagation mechanism through the coefficient  $\kappa_{s_t}$  can potentially make the expectation effect significant and asymmetric both in magnitude and in percentage change.

### III. THE DSGE MODEL

The theoretical results obtained in the previous section provide insight into why the expectation effect exists and how it can be asymmetric across regimes. The important questions still remain. How important can the asymmetric expectation effect of regime switches be quantitatively? How does it affect equilibrium dynamics when monetary policy shifts out of the bad regime to the good regime? To answer these

questions, one needs to have a serious dynamic model of the kind that has been a workhorse for empirical monetary analysis. We study a standard DSGE model, following Galí and Gertler (1999), Ireland (2001), Lubik and Schorfheide (2004), CEE, Boivin and Giannoni (2006), and Del Negro et al. 2007, among others.

The model economy is populated by a continuum of households, each endowed with a unit of differentiated labor skill indexed by  $i \in [0, 1]$ ; and a continuum of firms, each producing a differentiated good indexed by  $j \in [0, 1]$ . Households consume a composite of differentiated goods. Firms use a composite of differentiated labor skills as an production input. The composites of goods and labor skills are produced in a perfectly competitive aggregation sector. A monetary authority follows an interest rate rule, in which the policy parameters depend on the realization of a particular policy regime (denoted by  $s_t$ ). There are  $h$  distinct policy regimes that follow a stationary Markov process with a transition matrix  $Q$ . A typical element of  $Q$  is given by  $q_{ij} = \text{Prob}(s_{t+1} = i | s_t = j)$  for  $i, j \in \{1, \dots, h\}$ .

**III.1. The Aggregation Sector.** The aggregation sector produces a composite labor skill denoted by  $L_t$  to be used in the production of each type of intermediate goods and a composite final good denoted by  $Y_t$  to be consumed by each household. Production of the composite skill requires a continuum of differentiated labor skills  $\{L_t(i)\}_{i \in [0,1]}$  as inputs, and production of the composite final good requires a continuum of differentiated intermediate goods  $\{Y_t(j)\}_{j \in [0,1]}$  as inputs. The aggregate technologies are given by

$$L_t = \left[ \int_0^1 L_t(i)^{\frac{\theta_{wt}-1}{\theta_{wt}}} di \right]^{\frac{\theta_{wt}}{\theta_{wt}-1}}, \quad Y_t = \left[ \int_0^1 Y_t(j)^{\frac{\theta_p-1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p-1}}, \quad (14)$$

where  $\theta_{wt} \in (1, \infty)$  and  $\theta_p \in (1, \infty)$  are the elasticity of substitution between the skills and between the goods, respectively. We allow the elasticity of substitution between differentiated skills to be time-varying to capture inefficient labor market wedges, as we will explain further below.

Firms in the aggregation sector face perfectly competitive markets for the composite skill and the composite good. The demand functions for labor skill  $i$  and for good  $j$  resulting from the optimizing behavior in the aggregation sector are given by

$$L_t^d(i) = \left[ \frac{W_t(i)}{\bar{W}_t} \right]^{-\theta_{wt}} L_t, \quad Y_t^d(j) = \left[ \frac{P_t(j)}{\bar{P}_t} \right]^{-\theta_p} Y_t, \quad (15)$$

where the wage rate  $\bar{W}_t$  of the composite skill is related to the wage rates  $\{W_t(i)\}_{i \in [0,1]}$  of the differentiated skills by  $\bar{W}_t = \left[ \int_0^1 W_t(i)^{1-\theta_{wt}} di \right]^{1/(1-\theta_{wt})}$ , and the price  $\bar{P}_t$  of the

composite good is related to the prices  $\{P_t(j)\}_{j \in [0,1]}$  of the differentiated goods by  $\bar{P}_t = \left[ \int_0^1 P_t(j)^{1-\theta_p} dj \right]^{1/(1-\theta_p)}$ .

**III.2. The Intermediate Good Sector.** Production of a type  $j$  good requires labor as the only input, with the production function

$$Y_t(j) = Z_t L_t(j)^\alpha, \quad 0 < \alpha \leq 1, \quad (16)$$

where  $L_t(j)$  is the input of the composite skill used by the producer of intermediate good  $j$  and  $Z_t$  is an exogenous productivity shock identical across intermediate-good producers, and follows the stochastic process

$$Z_t = Z_{t-1} \lambda \nu_t, \quad (17)$$

where  $\lambda$  measures the deterministic trend of  $Z_t$  and  $\nu_t$  is a stochastic component of  $Z_t$ . The stochastic component follows the stationary process

$$\log \nu_t = \rho_\nu \log \nu_{t-1} + \varepsilon_{\nu t}, \quad (18)$$

where  $\rho_\nu \in (0, 1)$  and  $\varepsilon_{\nu t}$  is an i.i.d. white noise with a zero mean and a finite variance  $\sigma_\nu^2$ .

Each firm in the intermediate-good sector is a price-taker in the input market and a monopolistic competitor in the product market, where it can set a price for its product, taking the demand schedule in (15) as given. We follow Calvo (1983) and assume that pricing decisions are staggered across firms. We generalize the standard Calvo (1983) framework in two dimensions. First, we allow the frequency of price adjustments to depend on monetary policy regime. In particular, we assume that the probability that a firm cannot adjust its price is given by  $\eta_{t-1} \equiv \eta(s_{t-1})$ , where  $s_t$  denotes the period- $t$  monetary policy regime. Under this specification,  $\eta_t$  is a random variable that follows the same stationary Markov process as does the monetary policy regime. A special case with  $\eta_t = \eta$  for all  $t$  corresponds to the standard model with Calvo (1983) price-setting. Second, following Woodford (2003) and CEE (2005), we allow a fraction of firms that cannot re-optimize their pricing decisions to index their prices to the overall price inflation realized in the past period. Unlike Woodford (2003) and others, however, we assume that the fraction of indexation varies with monetary policy regime. More specifically, if a firm  $j$  cannot set a new price, its price is automatically updated according to

$$P_t(j) = \pi_{t-1}^{\gamma_{t-1}} \pi^{1-\gamma_{t-1}} P_{t-1}(j), \quad (19)$$

where  $\pi_t = \bar{P}_t/\bar{P}_{t-1}$  is the price inflation between  $t-1$  and  $t$ ,  $\pi$  is the steady-state inflation rate, and  $\gamma_t \equiv \gamma(s_t)$  measures the regime-dependent degree of indexation. We view these extensions of the Calvo (1983) framework essential to study the effects of potential changes in monetary policy regime, especially in light of the Lucas Jr. (1976) critique.<sup>3</sup>

Under this generalized Calvo (1983) framework, a firm that can renew its price contract chooses  $P_t(j)$  to maximize its expected discounted dividend flows given by

$$E_t \sum_{i=0}^{\infty} \prod_{k=1}^i \eta_{t+k-1} D_{t,t+i} [P_t(j) \chi_{t,t+i} Y_{t+i}^d(j) - V_{t+i}(j)], \quad (20)$$

where  $D_{t,t+i}$  is the period- $t$  present value of a dollar in a future state in period  $t+i$ , and  $V_{t+i}(j)$  is the cost of production. The term  $\chi_{t,t+i}$  comes from the price-updating rule (19), and is given by

$$\chi_{t,t+i} = \begin{cases} \pi_{t+i-1}^{\gamma_{t+i-1}} \pi_{t+i-2}^{\gamma_{t+i-2}} \cdots \pi_t^{\gamma_t} \pi^{\prod_{k=0}^{i-1} (1-\gamma_{t+k})} & \text{if } i \geq 1 \\ 1 & \text{if } i = 0. \end{cases} \quad (21)$$

In maximizing its profit, the firm takes as given the demand schedule  $Y_{t+i}^d(j) = \left( \frac{P_t(j) \chi_{t,t+i}}{\bar{P}_{t+i}} \right)^{-\theta_p} Y_{t+i}$ .

Solving the profit-maximization problem yields the optimal pricing decision rule

$$P_t(j) = \frac{\theta_p}{\theta_p - 1} \frac{E_t \sum_{i=0}^{\infty} \prod_{k=1}^i \eta_{t+k-1} D_{t,t+i} Y_{t+i}^d(j) \Phi_{t+i}(j)}{E_t \sum_{i=0}^{\infty} \prod_{k=1}^i \eta_{t+k-1} D_{t,t+i} \chi_{t,t+i} Y_{t+i}^d(j)}, \quad (22)$$

where  $\Phi_{t+i}(j)$  denotes the nominal marginal cost of production, which can be obtained by solving the firm's cost-minimizing problem. Given the production function (16),

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<sup>3</sup>The standard Calvo model with a constant fraction of re-optimizing firms is, in our view, not suitable for studying the effects of potentially large shifts in monetary policy regime. Our concern is not so much about the time-dependent nature of price setting in the Calvo model. Indeed, some studies show that in an environment with low and stable inflation the main implications of the Calvo model can be well approximated by a model with state-dependent price setting since most of the price adjustments occur at the intensive margin while the fraction of firms adjusting prices remains relatively stable (e.g., Gertler and Leahy (2006) and Klenow and Kryvtsov (2005)). Such approximations are likely to break down in an environment with highly variable inflation (such as that in the 1970s) or if changes in monetary policy regime are large (such as the change from the pre-Volcker regime to the Volcker-Greenspan-Bernanke regime). In these situations, the fraction of price-adjusting firms is likely to vary with the rate of inflation or the policy regime. Allowing the fraction of adjusting firms to depend on monetary policy regime, an approach that we adopt here, essentially captures this idea and meanwhile maintains the tractability of the standard Calvo model.

the marginal cost function facing firm  $j$  is given by

$$\Phi_{t+i}(j) = \frac{1}{\alpha} \frac{W_{t+i}}{Z_{t+i}} \left( \frac{Y_{t+i}(j)^d}{Z_{t+i}} \right)^{1/\alpha-1}. \quad (23)$$

According to the optimal price-setting equation (22), the optimal price is a markup over an average of the marginal costs for the periods in which the price will remain effective. Clearly, if  $\eta_t = 0$  for all  $t$ , that is, if prices are perfectly flexible in all periods, then the optimal price would be a constant markup over the contemporaneous marginal cost.

**III.3. The Households.** There is a continuum of households, each endowed with a differentiated labor skill indexed by  $i \in [0, 1]$ . A household  $i$  derives utility from consumption, real money balances, and leisure. The utility function is given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t a_t \left\{ U \left( C_t(i) - b C_{t-1}, \frac{M_t(i)}{\bar{P}_t} \right) - V(L_t(i)) \right\}, \quad (24)$$

where  $\beta \in (0, 1)$  is a subjective discount factor,  $C_t(i)$  denotes the household's consumption of the final composite good,  $C_{t-1}$  denotes aggregate consumption in the previous period,  $M_t(i)/\bar{P}_t$  is the real money balances, and  $L_t(i)$  represents hours worked. The parameter  $b$  measures the importance of habit formation in the utility function (e.g., Campbell and Cochrane (1999)). The variable  $a_t$  denotes a preference shock that follows the stationary process

$$\log a_t = \rho_a \log a_{t-1} + \varepsilon_{at}, \quad (25)$$

where  $0 \leq \rho_a < 1$  and  $\varepsilon_{at}$  is an i.i.d. normal process with mean zero and variance  $\sigma_a^2$ .

In each period  $t$ , the household faces the budget constraint

$$\bar{P}_t C_t(i) + \mathbb{E}_t D_{t,t+1} B_{t+1}(i) + M_t(i) \leq W_t(i) L_t^d(i) + B_t(i) + M_{t-1}(i) + \Pi_t(i) + T_t(i), \quad (26)$$

for all  $t \geq 0$ . In the budget constraint,  $B_{t+1}(i)$  is a nominal state-contingent bond that represents a claim to one dollar in a particular event in period  $t+1$ , and such a claim costs  $D_{t,t+1}$  dollars in period  $t$ ;  $W_t(i)$  is a nominal wage for  $i$ 's labor skill,  $\Pi_t(i)$  is its profit share, and  $T_t(i)$  is a lump-sum transfer from the government.

The household takes all prices and wages but its own as given and chooses  $C_t(i)$ ,  $B_{t+1}(i)$ ,  $M_t(i)$ , and  $W_t(i)$  to maximize (24) subject to (26), a borrowing constraint  $B_{t+1} \geq -\underline{B}$  for some large positive number  $\underline{B}$ , and the labor demand schedule  $L_t^d(i)$  described in (15).



The optimal wage-setting decision implies that

$$\frac{W_t(i)}{\bar{P}_t} = \mu_{wt} \frac{V_{lt}(i)}{U_{ct}(i)}, \quad (27)$$

where  $V_{lt}(i)$  and  $U_{ct}(i)$  denote the marginal utilities of leisure and of consumption, respectively, and  $\mu_{wt} = \frac{\theta_{wt}}{\theta_{wt}-1}$  measures the wage markup. Since the wage-setting decisions are synchronized across households, in a symmetric equilibrium, all households set an identical nominal wage and make identical consumption-saving decisions as well. Henceforth, we drop the household index  $i$ .

The wage markup  $\mu_{wt}$  follows the stochastic process

$$\log \mu_{wt} = (1 - \rho_w) \log \mu_w + \rho_w \log \mu_{w,t-1} + \varepsilon_{wt}, \quad (28)$$

with  $\rho_w \in (0, 1)$  and  $\varepsilon_{wt}$  being a white noise process with a zero mean and a finite variance  $\sigma_w^2$ . We further assume that  $\varepsilon_{zt}$  and  $\varepsilon_{wt}$  are independent. Note that the wage markup  $\mu_{wt}$  can also be interpreted as a time-varying wedge in the optimal labor-supply decision.

The optimal choice of bond holdings leads to the equilibrium relation

$$D_{t,t+1} = \beta \frac{a_{t+1} U_{c,t+1}}{a_t U_{ct}} \frac{\bar{P}_t}{\bar{P}_{t+1}}, \quad (29)$$

and the optimal choice of real balances implies that

$$\frac{U_{mt}}{U_{ct}} = 1 - \frac{1}{R_t}, \quad (30)$$

where  $R_t = [E_t D_{t,t+1}]^{-1}$  is the nominal risk-free rate.

**III.4. Monetary Policy.** Monetary policy is described by an interest rate rule that allows the possibility of regime switching. Denote  $s_t$  the monetary policy regime in period  $t$ . The interest rate rule we consider is given by

$$R_t = \kappa(s_t) R_{t-1}^{\rho_r(s_t)} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\phi_\pi(s_t)} \tilde{Y}_t^{\phi_y(s_t)} \right]^{1-\rho_r(s_t)} e^{\varepsilon_{rt}}, \quad (31)$$

where  $\tilde{Y}_t = Y_t/Z_t$  is detrended output,  $\pi^*$  is a target rate of inflation, and the policy parameters  $\kappa(s_t)$ ,  $\rho_r(s_t)$ ,  $\phi_\pi(s_t)$ , and  $\phi_y(s_t)$  are regime dependent. The term  $\varepsilon_{rt}$  is a shock to monetary policy and follows an i.i.d. normal process with a zero mean and a finite variance  $\sigma_r^2$ . The state  $s_t$  represents monetary policy regime and its stochastic process is given in Section II.1.

Given monetary policy, an *equilibrium* in this economy consists of prices and allocations such that (i) taking all prices and nominal wages but its own as given, each household's allocation and nominal wage solve its utility maximization problem; (ii)

taking wages and all prices but its own as given, each firm's allocation and price solve its profit maximization problem; (iii) markets clear for bond, money balances, composite labor, and composite final goods.

#### IV. EQUILIBRIUM DYNAMICS

We now describe the equilibrium dynamics. In the model, the productivity shock contains a trend. We focus a stationary equilibrium (i.e., the balanced growth path). To be consistent with balanced growth, we assume that the utility functions take the form

$$\begin{aligned} U\left(C_t(i) - bC_{t-1}, \frac{M_t(i)}{\bar{P}_t}\right) &= \log(C_t(i) - bC_{t-1}) + \chi \log\left(\frac{M_t(i)}{\bar{P}_t}\right), \\ V(L_t(i)) &= \frac{\Psi}{1+\xi} L_t(i)^{1+\xi}. \end{aligned}$$

Further, we make appropriate transformations of the relevant variables to induce stationarity. The variables to be transformed include aggregate output, consumption, real money balances, and the real wage. In equilibrium, all these variables grow at the same rate as does the productivity shock, so we divide each of these variables by  $X_t$  and denote the resulting stationary counterpart of the variable  $X_t$  by  $\tilde{X}_t = X_t/Z_t$ .

**IV.1. The Steady State.** We now describe the steady state equilibrium, where all shocks are turned off. The steady-state equilibrium can be summarized by the solution to 4 equilibrium conditions, including (1) optimal pricing decision, (2) optimal wage-setting decision, (3) the intertemporal Euler equation, and (4) the Taylor rule. Once consumption and the nominal interest rate are solved from these equilibrium conditions, we can obtain the real money balances from (30).

The optimal pricing equation (22) implies that, in a steady state, the real marginal cost is equal to the inverse markup. That is,

$$\frac{1}{\mu_p} = \frac{1}{\alpha} \tilde{W} \tilde{Y}^{1/\alpha-1}, \quad (32)$$

where  $\tilde{W} = \frac{W}{\bar{P}_A}$  denotes the transformed real wage and  $\tilde{Y} = \frac{Y}{A}$  denotes the transformed output.

The wage-setting decision (27) implies that the real wage in the steady state is given by a constant markup over the MRS:

$$\tilde{W} = \mu_w \Psi L^\xi \left( \tilde{Y} - \frac{b}{\lambda} \tilde{C} \right), \quad (33)$$

where we have used the market clearing condition that aggregate consumption equals aggregate output in equilibrium.

The household's optimal intertemporal decision (29) implies that, in the steady-state equilibrium, we have

$$\frac{R}{\pi} = \frac{\lambda}{\beta}. \quad (34)$$

Finally, the Taylor rule in the steady-state equilibrium implies that

$$R = \kappa(s)^{1/(1-\rho_r(s))} \left( \frac{\pi}{\pi^*} \right)^{\phi_\pi(s)} \tilde{Y}^{\phi_y(s)}. \quad (35)$$

In a steady-state equilibrium, there is a classical dichotomy in that the real variables  $\tilde{Y}$  and  $\tilde{W}$  are determined by the first 2 equations (32)-(33), while the nominal variables  $\pi$  and  $R$  are determined by the other 2 equations (34)-(35) once the real variables are determined.

In general, since the monetary policy rule is regime dependent, so would be the steady-state equilibrium variables. However, such regime dependence of the steady-state equilibrium renders it difficult to log-linearize the equilibrium conditions around a particular steady state. We would like to avoid this situation and focus on a steady state that is independent of regimes. This purpose can be achieved by appropriate choice of  $\kappa(s)$ . Specifically, we set  $\kappa(s) = \left[ \frac{\lambda}{\beta} \pi^* \tilde{Y}^{-\phi_y(s)} \right]^{1-\rho(s)}$ , where  $\tilde{Y}$  can be solved from the “real part” of the equilibrium system (i.e., (32)-(33)). With  $\kappa(s)$  so chosen, we obtain a unique steady-state value for inflation and the nominal interest rate. These are given by

$$\pi = \pi^*, \quad R = \frac{\lambda}{\beta} \pi^*. \quad (36)$$

**IV.2. Equilibrium Dynamics.** We now study the log-linearized system of equilibrium conditions around the deterministic steady state described above. We focus here on the key equations that characterize equilibrium dynamics and relegate derivations of these equations to an Appendix.

The log-linearized optimal pricing equation is given by

$$\begin{aligned} \hat{\pi}_t - \gamma(s_{t-1})\hat{\pi}_{t-1} &= \beta\psi_1(s_t, s_{t-1})E_t(\hat{\pi}_{t+1} - \gamma(s_t)\hat{\pi}_t) \\ &+ \psi_2(s_{t-1}) \left[ \frac{\xi+1}{\alpha} \tilde{y}_t + \frac{b}{\lambda-b} (\tilde{y}_t - \tilde{y}_{t-1} + \hat{\nu}_t) \right] + \psi_2(s_{t-1})\hat{\mu}_{wt}, \end{aligned} \quad (37)$$

where

$$\psi_1(s_t, s_{t-1}) = \frac{\bar{\eta}}{\eta(s_{t-1})} \frac{1 - \eta(s_{t-1})}{1 - \eta(s_t)}, \quad \psi_2(s_{t-1}) = \frac{(1 - \beta\bar{\eta})(1 - \eta(s_{t-1}))}{\eta(s_{t-1})} \frac{1}{1 + \theta_p(1 - \alpha)/\alpha},$$

and  $\bar{\eta}$  is the Ergodic mean of the random variable  $\eta(s_t)$ . Here,  $\hat{\pi}_t$  denotes the inflation rate,  $\tilde{y}_t$  denotes the output gap,  $\hat{\nu}_t$  denotes the productivity shock, and  $\hat{\mu}_{wt}$  denotes the cost-push shock.

Equation (37) here generalizes the standard Phillips curve by introducing partial indexation and, more importantly, regime-dependent frequencies of price adjustments and inflation indexation. In the special case where  $\eta_t = \bar{\eta}$  and  $\gamma_t = \gamma$  for all  $t$ , this equation reduces to a standard Phillips curve relation with partial indexation, such as the one in Woodford (2003) and Giannoni and Woodford (2003) (augmented with habit formation). If we further impose that  $\gamma = 0$  and  $b = 0$ , so that there is no indexation and no habit formation, then (37) collapses to the pure forward-looking Phillips-curve relation with the real marginal cost represented by an output gap. In general, as the frequency of price adjustments (measured by  $1 - \eta_t$ ) and the degree of inflation indexation (measured by  $\gamma_t$ ) are regime dependent, the Phillips curve relation in (37) needs to take into account that both  $\eta_t$  and  $\gamma_t$  are random variables. More strikingly, the Phillips curve relation here is no longer linear! The non-linearity poses challenge for computation, an issue that we will address below.

The log-linearized intertemporal Euler equation is given by

$$\begin{aligned} E_t \tilde{y}_{t+1} - \frac{\lambda + b}{\lambda} \tilde{y}_t + \frac{b}{\lambda} \tilde{y}_{t-1} = \\ \left(1 - \frac{b}{\lambda}\right) \left(\hat{R}_t - E_t \hat{\pi}_{t+1}\right) + \left(\frac{b}{\lambda} - \rho_\nu\right) \hat{\nu}_t - \frac{(\lambda - b)(1 - \rho_a)}{\lambda} \hat{a}_t, \end{aligned} \quad (38)$$

where  $\hat{R}_t = \log(R_t/R)$  denotes the nominal interest rate. Evidently, in the special case with no habit formation (i.e.,  $b = 0$ ), equation (38) collapses to the standard intertemporal Euler equation that relates expected output growth to the real interest rate.

Finally, the log-linearized interest rate rule is given by

$$\hat{R}_t = \rho_r(s_t) \hat{R}_{t-1} + (1 - \rho_r(s_t)) [\phi_\pi(s_t) \hat{\pi}_t + \phi_y(s_t) \tilde{y}_t] + \varepsilon_{rt}. \quad (39)$$

## V. PARAMETERIZATION

The parameters for our regime-switching structural model include (i) “deep” parameters that are constant across policy regimes and (ii) regime-dependent parameters. Throughout this paper, we consider only  $h = 2$  policy regimes. The deep parameters include  $\beta$ , the subjective discount factor;  $b$ , the habit parameter;  $\xi$ , the inverse Frisch elasticity of labor supply;  $\alpha$ , the elasticity of output with respect to labor;  $\theta_p$ , the elasticity of substitution between differentiated goods;  $\mu_w$  and  $\rho_w$ , the mean and the

first-order autocorrelation of the cost-push shock process;  $\lambda$ , the trend growth rate of productivity;  $\rho_a$  and  $\rho_\nu$ , the AR(1) coefficients of the preference shock and of the productivity growth processes; and  $\sigma_r$ ,  $\sigma_a$ ,  $\sigma_w$ , and  $\sigma_\nu$ , the standard deviation of the monetary policy shock, the preference shock, the cost-push shock, and the technology shock. The regime-dependent parameters include policy parameters  $\rho_r$ ,  $\phi_\pi$ , and  $\phi_y$  and the stickiness and indexation parameters  $\eta$  and  $\gamma$ .

The values of the parameters that we use in this paper are summarized in Table 1. These parameter values correspond to a quarterly model. We set  $\lambda = 1.005$  so that the average annual growth rate of per capital GDP is 2%. We set  $\beta = 0.9952$  so that, given the value of  $\lambda$ , the average annual real interest rate (equal to  $\lambda/\beta$ ) is 4%. Following the literature, we set  $b = 0.75$ , which is in the range considered by Michele Boldrin and Fisher (2001). The parameter  $\xi$  corresponds to the inverse Frisch elasticity of labor supply, which, according to most micro-studies, is small (Pencavel, 1986). We set  $\xi = 2$ , corresponding to a Frisch elasticity of 0.5. We set  $\alpha = 0.7$ , corresponding to a labor income share of 70%. The parameter  $\theta_p$  determines the steady-state markup. Some studies suggest that the value-added markup is about 1.05 when factor utilization rates are controlled for; without such a correction, it is higher at about 1.12 (Basu and Fernald, 2002). Some other studies suggest an even higher value-added markup of about 1.2 (with no correction for factor utilization) (Rotemberg and Woodford, 1997). In light of these studies, we set  $\theta_p = 10$  so that the steady-state markup is 1.1. For the parameters governing the shock processes, we set  $\rho_a = 0.9$ ,  $\rho_\nu = 0.2$ ,  $\rho_w = 0.9$ ,  $\sigma_a = 0.25$ ,  $\sigma_r = 0.2$ ,  $\sigma_w = 0.4$ , and  $\sigma_\nu = 0.2$ .

For the regime-dependent parameters, we consider two monetary policy regimes. The first regime, called the bad regime, corresponds to the Mitchell-Burns policy, which does not take a strong stance against inflation fluctuations. The second regime, called the good regime, corresponds to the Volcker-Greenspan-Bernanke regime under which price stability is a primary goal. Based on the estimates obtained by Clarida, Galí, and Gertler (2000), we set  $\rho_{r1} = 0.68$ ,  $\rho_{r2} = 0.79$ ;  $\phi_{\pi1} = 0.83$ ,  $\phi_{\pi2} = 2.15$ ; and  $\phi_{y1} = 0.27$ ,  $\phi_{y2} = 0.93$ . These values of policy parameters are consistent with the estimates obtained by Lubik and Schorfheide (2004). As discussed widely in the literature, the bad regime tends to be destabilizing the economy and can lead to large fluctuations in inflation and output. In this regime, we assume that firms adjust prices more frequently. For the firms that cannot optimize prices, they are more likely to choose inflation indexation under the bad regime than under the good regime. Consequently, we set  $\eta_1 = 0.66$  and  $\eta_2 = 0.75$ , so that price contracts last on average for 3

quarters under the bad regime and 4 quarters under the good regime; we set  $\gamma_1 = 1$  and  $\gamma_2 = 0$ , so that there is full indexation under the bad regime and no indexation under the good regime.

The literature suggests a wide range of values for  $\eta$ . The work by Eichenbaum and Fisher (2007) suggests that, in a standard Calvo model with mobile capital, the estimated value of  $\eta$  based on postwar US data can be as high as 0.85; although a lower value in the neighborhood of 0.66 can be obtained if capital inputs are firm specific. CEE (2005) also obtain an estimate of  $\eta = 0.66$ . The survey by Taylor (1999) suggests a value of  $\eta = 0.75$ , while the study by Bils and Klenow (2004) based on disaggregate consumer price data suggests more frequent price changes, with half of prices lasting 5.5 months or less. Our parameterized value of  $\eta$  lies within the range of these empirical studies. The relatively longer duration of price contracts under the good regime, as we have assumed, is also consistent with the finding by Lubik and Schorfheide (2004) that price stickiness has increased in the post-1982 period.

For the parameters in the transition matrix  $Q$ , we set  $q_{11} = 0.9$  and  $q_{22} = 0.9$  (and accordingly,  $q_{21} = 0.1$  and  $q_{12} = 0.1$ ). In our quantitative analysis, we experiment with other values of transition probabilities to check the sensitivity of our results to these parameters.

## VI. SOLVING THE REGIME-SWITCHING STRUCTURAL MODEL

Our model has two non-standard features that pose a challenge for computation. First, since we consider both the bad regime and the good regime of monetary policy, our parameterization allows for equilibrium indeterminacy. Second, since we allow some key parameter to vary with the monetary policy regime, the equilibrium system is in general non-linear when the policy regime follows a stochastic Markov switching process. To solve our regime-switching model, we use the generalized MSV approach developed by Farmer, Waggoner, and Zha (2006), which utilizes the conical VAR form of Sims (2002).

Since the parameters in the equilibrium system (in particular, those in the Phillips curve relation (37)) depend on regimes in period  $t$  and  $t - 1$ , it is useful to define an “composite regime” that includes all possible realizations of regimes in periods  $t$  and  $t - 1$ . Denote by  $\tilde{s}_t$  the composite regime. Then we have

$$\tilde{s}_t = \{s_t, s_{t-1}\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

Accordingly, the transition matrix for the composite regime is given by

$$\tilde{Q}_{4 \times 4} = \begin{bmatrix} q_{11} & q_{11} & 0 & 0 \\ 0 & 0 & q_{12} & q_{12} \\ q_{21} & q_{21} & 0 & 0 \\ 0 & 0 & q_{22} & q_{22} \end{bmatrix},$$

where the  $q_{ij}$ 's are elements in the  $Q_{2 \times 2}$  matrix.

We use the following notations:

- $n$  = number of all variables (including expectation terms) for each regime, as in the Gensys setup
- $m$  = number of fundamental shocks
- $h$  = number of policy regimes
- $h^*$  = number of shock regimes
- $n_1$  = number of equations in each regime
- $n_2$  = number of expectation errors
- $n_3$  = number of fixed-point equations
- $\tilde{Q} = h \times h$  matrix of transition matrix, whose elements sum up to 1 in each column

In our model, we have  $n = 8$ ,  $m = 4$ ,  $h = 4$ ,  $h^* = 1$ ,  $n_1 = 6$ ,  $n_2 = 2$ ,  $n_3 = n_2(h - 1) = 6$ .

We can now rewrite the equilibrium conditions described in (37) - (39) and the shock process in (18)-(28) in a compact form

$$\underset{n_1 \times n}{A_{\tilde{s}_t}} \underset{n \times 1}{x_t} = \underset{n_1 \times n}{B_{\tilde{s}_t}} \underset{n \times 1}{x_{t-1}} + \underset{n_1 \times m}{\Psi} \underset{m \times 1}{\varepsilon_t}, \quad (40)$$

where

$$x_t = [\hat{\pi}_t, \tilde{y}_t, \hat{R}_t, \hat{a}_t, \hat{\mu}_{wt}, \hat{\nu}_t, E_t \hat{\pi}_{t+1}, E_t \tilde{y}_{t+1}]'$$

is a  $8 \times 1$  vector of variables to be solved and

$$\varepsilon_t = [\varepsilon_{rt}, \varepsilon_{at}, \varepsilon_{wt}, \varepsilon_{vt}]'$$

is a  $4 \times 1$  vector of shocks.

The coefficient matrices  $A_{\tilde{s}_t}$  and  $B_{\tilde{s}_t}$  in (40) involve parameters that are possibly regime-dependent. To fix notations, we make the following definition:

$$\begin{aligned} \gamma_1(\tilde{s}_t) &= \gamma(s_{t-1}), & \gamma_0(\tilde{s}_t) &= \gamma(s_t), \\ \psi_1(\tilde{s}_t) &= \psi_1(s_t, s_{t-1}), & \psi_2(\tilde{s}_t) &= \psi_2(s_{t-1}), \\ \rho_r(\tilde{s}_t) &= \rho_r(s_t), & \phi_\pi(\tilde{s}_t) &= \phi_\pi(s_t), & \phi_y(\tilde{s}_t) &= \phi_y(s_t), \end{aligned}$$

Since  $\gamma_0(\tilde{s}_t)$ ,  $\gamma_1(\tilde{s}_t)$ ,  $\psi_2(\tilde{s}_t)$ , and the policy parameters  $\rho_r(\tilde{s}_t)$ ,  $\phi_\pi(\tilde{s}_t)$ , and  $\phi_y(\tilde{s}_t)$  are all functions of the regime in a given period and thus do not involve regimes across 2 periods, they have the following properties:

$$\begin{aligned}\gamma_0(\tilde{s}_t = 1) &= \gamma_0(\tilde{s}_t = 2), & \gamma_0(\tilde{s}_t = 3) &= \gamma_0(\tilde{s}_t = 4), \\ \gamma_1(\tilde{s}_t = 1) &= \gamma_1(\tilde{s}_t = 3), & \gamma_1(\tilde{s}_t = 2) &= \gamma_1(\tilde{s}_t = 4), \\ \psi_2(\tilde{s}_t = 1) &= \psi_2(\tilde{s}_t = 3), & \psi_2(\tilde{s}_t = 2) &= \psi_2(\tilde{s}_t = 4), \\ \rho_r(\tilde{s}_t = 1) &= \rho_r(\tilde{s}_t = 2), & \rho_r(\tilde{s}_t = 3) &= \rho_r(\tilde{s}_t = 4), \\ \rho_\pi(\tilde{s}_t = 1) &= \rho_\pi(\tilde{s}_t = 2), & \rho_\pi(\tilde{s}_t = 3) &= \rho_\pi(\tilde{s}_t = 4), \\ \rho_y(\tilde{s}_t = 1) &= \rho_y(\tilde{s}_t = 2), & \rho_y(\tilde{s}_t = 3) &= \rho_y(\tilde{s}_t = 4),\end{aligned}$$

We now fill in the matrices  $A_{\tilde{s}_t}$ ,  $B_{\tilde{s}_t}$ , and  $\Psi$  using the equilibrium conditions.

$$\begin{aligned}A_{\tilde{s}_t} &=_{6 \times 8} \begin{bmatrix} -[1 + \beta\psi_1(\tilde{s}_t)\gamma_0(\tilde{s}_t)] & \psi_2(\tilde{s}_t) \left[ \frac{1+\xi}{\alpha} + \frac{b}{\lambda-b} \right] & 0 & 0 & \psi_2(\tilde{s}_t) & \frac{\psi_2(\tilde{s}_t)b}{\lambda-b} & \beta\psi_1(\tilde{s}_t) & 0 \\ 0 & -\frac{\lambda+b}{\lambda} & -\frac{\lambda-b}{\lambda} & \frac{(\lambda-b)(1-\rho_a)}{\lambda} & 0 & \frac{\rho_\nu\lambda-b}{\lambda} & \frac{\lambda-b}{\lambda} & 1 \\ -(1-\rho(\tilde{s}_t))\phi_\pi(\tilde{s}_t) & -(1-\rho(\tilde{s}_t))\phi_y(\tilde{s}_t) & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ B_{\tilde{s}_t} &=_{6 \times 8} \begin{bmatrix} -\gamma_1(\tilde{s}_t) & \psi_2(\tilde{s}_t)\frac{b}{\lambda-b} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b}{\lambda} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho(\tilde{s}_t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_a & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_w & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_\nu & 0 & 0 \end{bmatrix}, \\ \Psi &=_{6 \times 4} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \sigma_r & 0 & 0 & 0 \\ 0 & \sigma_a & 0 & 0 \\ 0 & 0 & \sigma_w & 0 \\ 0 & 0 & 0 & \sigma_\nu \end{bmatrix},\end{aligned}$$



Following Farmer, Waggoner, and Zha (2006), we can expand the system under each regime, described above, into an expanded linear system to obtain the MSV solution. Appendix B describes the detail of how to form this expanded system.

## VII. QUANTITATIVE ANALYSIS

Since monetary policy regime has switched a number of times through the U.S. history, a regime-switching DSGE model of the type studied in this paper is a natural starting point for quantitative analysis. In this section we use the parameterization discussed in Section V to answer the following questions pertinent to changes in monetary policy. How important is the effect of expected regime switches? How do such effects affect the impact of policy changes on the macroeconomy? For this purpose, we compare the equilibrium implications of two versions of our model, one in which agents naively believe that the existing policy regime will persist indefinitely and one in which agents take into account probabilistic switches in future policy regime. Within each version of the model we also study two scenarios, both with stochastic regime shifts in policy, but in one scenario we impose that the parameters  $\eta$  and  $\gamma$  that govern firms' pricing behaviors do not vary with policy regimes and in the other we relax this imposition. For the "constant-regime" model

**VII.1. Asymmetric expectation effects.** To gauge the importance the expectation effects of changes in policy regimes, we compare the dynamic behaviors of macroeconomic variables in our regime-switching model with those in the "constant-regime" version of the model in which agents naively assume that the current regime would last indefinitely.

We begin by examining the case with regime switches in policy but with constant  $\eta$  and  $\gamma$ . Figure 1 displays the impulse responses of inflation, output, the nominal interest rate, expected inflation, expected output, and the real marginal cost under the bad regime. At the top of the graphs, "MP" stands for a monetary policy shock, "Demand" stands for a preference shock, "Cost-push" stands for a cost-push shock, and "Tech" stands for a technology shock. Within each graph, two sets of impulse responses are plotted. One corresponds to the version of the model where agents assume the current regime will last indefinitely (the solid line) and the other corresponds to the baseline version of our model where agents take regime-switching into account in forming expectations (the dashed line). The difference between these two sets of impulse responses captures the expectation effects of regime shifts in policy. As shown in Figure 1, when agents expect the policy to shift from the bad regime to the good regime with

a non-trivial probability, the dynamic responses of all variables, and particularly those following a demand shock or a cost-push shock, are substantially dampened. Even with a modest probability of 10% of switching to the good regime, the dampening effects are quite large.<sup>4</sup> If the bad regime is less persistent, so that it is more likely to switch to the good regime, the expectation effects of regime switching can be further magnified.

To obtain a quantitative measure of the expectation effect of regime shifts in policy, we simulate time series of inflation, output, and the interest rate and compute the first-order autocorrelations (persistence) and unconditional standard deviations (volatility).<sup>5</sup> Table 2 reports these results. Comparing Panels A and B in the table reveals that, under the bad regime, expectations of a possible switch to the good regime in the future help dampen the macroeconomic fluctuations substantially: the volatility falls from 0.15 to 0.004 for inflation, from 0.008 to 0.003 for output, and from 0.01 to 0.004 for the nominal interest rate. In comparison, expectations of regime switching do not have large effects on the persistence of the macroeconomic variables. Our result suggests that a purely backward-looking model may likely contribute changes in volatility to those in shock variances rather than changes in monetary policy regime.

Figure 2 display the impulse responses under the good regime in the case with changes in policy regimes but with constant  $\eta$  and  $\gamma$ . Although expectations of a possible switch to the bad regime make the responses slightly more volatile, the model ignoring the expectation effects nonetheless approximates the regime-switching model very well. The lack of expectation effects under the good regime is also evident by comparing the results across Panels A and B in Table 2 under the good regime scenario. This result is consistent with the view that monetary policy is more effective in an environment with a low inflation target (Bernanke and Mishkin, 1997; Mishkin, 2004).

We now turn to the case with both the price-stickiness parameter  $\eta$  and the inflation-indexation parameter  $\gamma$  varying with policy regimes. As we have argued, these parameters are likely to change with policy regime, especially when we consider a potentially large change in policy. Figures 3 and 4 display the impulse responses of macroeconomic variables under the bad and the good regimes. Similar to the case with constant  $\eta$  and  $\gamma$ , the effects of expecting the policy to switch from the bad regime to the good regime (captured by the differences between the solid and dashed lines in Figure 3) appear

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<sup>4</sup>Asymmetric expectation effects remain to be strong even if we set  $q_{11} = 0.98$  and  $q_{22} = 1.0$ , the probabilities that might be viewed as being more in line with the empirical evidence.

<sup>5</sup>We simulate 2500 periods and discard the first 500 observations to avoid dependence of the results on initial conditions. Increasing the number of periods in the simulation produces no visible change in the results.

large, but the effects of expecting the policy to switch from the good regime to the bad regime (Figure 4) seem small. The simulated persistence and volatility of inflation, output, and the nominal interest rate, as reported in Table 3, confirm that expectations of regime switches can substantially reduce macroeconomic volatility under the bad regime, but have small effects under the good regime. Such a small effect holds even when we set  $q_{22} = 0.7$ .

**VII.2. The Great Moderation.** The results discussed in Section VII.1 show that expectations about changes in future monetary policy can play an important role in affecting the dynamics of macroeconomic variables. Since these expectation effects can significantly dampen the macroeconomic volatility under the bad policy regime, the following questions naturally arise. Are there significant differences in macroeconomic volatility across the bad and good regimes? What role do changes in firms' pricing behavior play when we allow the relevant parameters  $\eta$  and  $\gamma$  to vary with policy regimes?

These questions are important because the volatility of both inflation and output in the U.S. economy has declined substantially since the 1980s. This kind of reduction in macroeconomic volatility is dubbed the “Great Moderation” (Stock and Watson, 2003)). Although what may have caused the Great Moderation is still under debate, there is a broad consensus that monetary policy played a large role in achieving lower inflation variability (e.g., Bernanke (2004)). Since output volatility and inflation volatility have moved together in the last thirty years, both in the United States and in other industrial economies (e.g., Blanchard and Simon (2001)), Bernanke (2004) suggests that monetary policy may have also played a nontrivial role in moderating output variability as well.

Figure 5 displays the impulse responses of macroeconomic variables in the model with switching policy regimes, where we impose the assumption that the price-stickiness parameter  $\eta$  and the inflation-indexation parameter  $\gamma$  do not vary with policy regime. The figure shows that, as monetary policy switches from the bad regime (the solid line) to the good regime (the dashed line), the responses of inflation to each of the three shocks are visibly dampened. The responses of output and the nominal interest rate do not appear to change much across the two regimes. To measure how much of the volatility is reduced for each variable, Panel B in Table 2 shows that when monetary policy switches from the bad regime to the good regime, the volatility of inflation is substantially lowered (from 0.0037 to 0.0011, a reduction of about 70%). This finding is consistent with the view that monetary policy has played an important

role in achieving inflation stability. However, going from the bad regime to the good regime does not lead to much reduction in the volatilities of output and the nominal interest rate. Output volatility falls from 0.0028 to 0.0022, a less than 22% reduction; and interest-rate volatility falls from 0.0037 to 0.0025, a reduction of about 33%.

As we have discussed, changes in monetary policy may affect firms' price-setting behavior. To examine the consequence of allowing firms' behavior to respond to changes in policy regimes, we now consider the scenario in which the price-stickiness parameters  $\eta$  and the inflation-indexation parameter  $\gamma$  both vary with policy regime. Figure 6 reports the impulse responses in this scenario. Compared to the case with constant  $\eta$  and  $\gamma$  (see Figure 5), allowing firms' behavior to vary with policy regime helps dampen the responses of output and the nominal interest rate, in addition to dampening the response of inflation. Table 3 (Panel B) shows that, as the policy switches from the bad regime to the good regime, not only inflation becomes more stable, but also the volatilities of output and the nominal interest rate are both reduced substantially. In particular, the output volatility falls by about a half (from 0.0030 to 0.0013) and the interest-rate volatility falls by about 63% (from 0.0041 to 0.0015). These findings lends support to the view that monetary policy may have played an important role in the Great Moderation.<sup>6</sup>

**VII.3. Endogenous propagation of shocks.** If we turn off the persistence parameters in all the shocks and make them i.i.d., there is not much of the expectation effect. To see if endogenous propagation mechanisms can give rise to the expectation effect, we increase the strategic complementarity in price setting and thereby increase the endogenous propagation of shocks to make the equilibrium inflation and output more persistent, it can be shown that there are important expectation effects of regime switches even under iid shocks.

In particular, when we change the parameters so that  $\theta_p = 21$  (corresponding to a 5% steady-state markup, in line with the empirical findings by Basu and Fernald (2002)) and  $\eta = 0.75$  (4 quarters of average duration of price contracts), the expectation effect in the bad regime become significant, especially for the impulse responses to a monetary policy shock. These new parameter values imply a smaller value of  $\psi_2$  in equation (37) and thus a stronger strategic complementarity in firms' price-setting decisions in the sense of V.V. Chari and McGrattan (2000), Huang and Liu (2001, 2002), and Dotsey and King (2006). With a stronger strategic complementarity, as shown by

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<sup>6</sup>The Great Moderation is stronger when we set  $q_{11} = 0.98$  and  $q_{22} = 1.0$ , the probabilities that might be viewed as being more in line with the empirical evidence.

these authors, inflation and output dynamics are more persistent. As such, there are important asymmetric expectation effects of regime switches that are caused entirely by the endogenous propagation mechanisms.

## VIII. CONCLUSION

We have studied a standard DSGE model where monetary policy follows a Markov switching process between two distinct regimes: a bad regime under which the policy responds weakly to fluctuations in inflation and a good regime under which the price stability is a top priority. We have shown that (1) because macroeconomic dynamics are nonlinear functions of the underlying model parameters, the expectation effect of regime switches in monetary policy is asymmetric across regimes and (2) by allowing firms' pricing behavior to vary with policy regime, the volatility of both inflation and output can be significantly reduced when policy switches out of the bad regime into the good regime.

Since the expectation effect can be quantitatively important under the bad policy regime, it should *not* be ignored in the DSGE model that aims at assessing the impact of a regime change in historical monetary policy. In the good policy regime, on the other hand, the expectation effect of future policy change is quantitatively insignificant. This asymmetric finding offers an explanation of why the post-1980 monetary policy in the U.S. has been successful in reducing the volatility of both inflation and output, despite agents' disbelief that the good policy will last forever (Goodfriend and King, 2005).

Because our structural model is able to generate the Great Moderation in both inflation and output, we hope that our quantitative findings help motivate researchers to take up a challenging task of estimating such a model to a long sample that covers different policy regimes and structural breaks.

## APPENDIX A. PROOFS OF PROPOSITIONS

**A.1. Proof of Proposition 1.** We solve the model (7) by the method of undetermined coefficients. Given the solution form  $\hat{\pi}_t = \alpha_{s_t} z_t$  for  $s_t \in \{1, 2\}$ , (7) implies that

$$\begin{aligned}\phi_1 \alpha_1 z_t &= q_{11} \alpha_1 \rho z_t + q_{21} \alpha_2 \rho z_t + \gamma \rho z_t, \\ \phi_2 \alpha_2 z_t &= q_{12} \alpha_1 \rho z_t + q_{22} \alpha_2 \rho z_t + \gamma \rho z_t,\end{aligned}$$

where we have used the relation  $E_t z_{t+1} = \rho z_t$ . Matching the coefficients on  $z_t$ , we obtain

$$\phi_1 \alpha_1 = q_{11} \alpha_1 \rho + q_{21} \alpha_2 \rho + \gamma \rho, \quad (\text{A1})$$

$$\phi_2 \alpha_2 = q_{12} \alpha_1 \rho + q_{22} \alpha_2 \rho + \gamma \rho. \quad (\text{A2})$$

It follows that the solution  $[\alpha_1, \alpha_2]'$  is given by the expression in (8).

**A.2. Proof of Proposition 2.** Denote by  $\alpha = [\alpha_1, \alpha_2]'$  and  $C = \gamma \rho [1, 1]'$ . The MSV solution in (8) can be rewritten as

$$\alpha = A^{-1} C,$$

Since  $A$  is positive definite,  $\alpha_1$  and  $\alpha_2$  are both positive.

To establish the first inequality in (9), we impose the relation  $q_{11} = 1 - q_{21}$  and differentiate (A1) and (A2) with respect to  $q_{21}$  to obtain

$$\begin{aligned} \phi_1 \frac{\partial \alpha_1}{\partial q_{21}} &= q_{11} \rho \frac{\partial \alpha_1}{\partial q_{21}} + (\alpha_2 - \alpha_1) \rho + q_{21} \rho \frac{\partial \alpha_2}{\partial q_{21}} \\ \phi_2 \frac{\partial \alpha_2}{\partial q_{21}} &= q_{12} \rho \frac{\partial \alpha_1}{\partial q_{21}} + q_{22} \rho \frac{\partial \alpha_2}{\partial q_{21}}. \end{aligned}$$

With appropriate substitutions, we get

$$\frac{\partial \alpha_1}{\partial q_{21}} = \frac{\gamma \rho^2 (\phi_2 - q_{22} \rho) (\phi_1 - \phi_2)}{\det(A)^2} < 0,$$

where the inequality follows from the assumption that  $\phi_1 < 1 < \phi_2$ . Similarly, we can show that

$$\frac{\partial \alpha_2}{\partial q_{12}} = \frac{\gamma \rho^2 (\phi_1 - q_{11} \rho) (\phi_2 - \phi_1)}{\det(A)^2}.$$

Since  $A$  is assumed to be positive definite, we have  $\det(A) > 0$  so that

$$\phi_1 - q_{11} \rho > \frac{q_{21} q_{12} \rho^2}{\phi_2 - q_{22} \rho} > 0.$$

This inequality, along with the assumption that  $\phi_2 > \phi_1$ , implies that  $\frac{\partial \alpha_2}{\partial q_{12}} > 0$ .

**A.3. Proof of Proposition 3.** Given the solution form  $\hat{\pi}_t = \bar{\alpha}_j z_t$ , we have  $E_t \hat{\pi}_{t+1} = \bar{\alpha}_j \rho z_t$  and (11) is a result from matching the coefficients of  $z_t$ .

**A.4. Proof of Proposition 4.** The solution for the regime-switching model (8) can be rewritten as

$$\alpha_j = \frac{q_{ij}\rho + \phi_i - q_{ii}\rho}{\det(A)}, \quad i, j \in \{1, 2\}, \quad i \neq j.$$

Using the solution for the constant regime model in (11), we have

$$\begin{aligned} \frac{\bar{\alpha}_1 - \alpha_1}{\alpha_2 - \bar{\alpha}_2} &= \frac{\frac{1}{\phi_1 - \rho} - \frac{q_{21}\rho + \phi_2 - q_{22}\rho}{\det(A)}}{\frac{q_{12}\rho + \phi_1 - q_{11}\rho}{\det(A)} - \frac{1}{\phi_2 - \rho}} \\ &= \frac{\phi_2 - \rho}{\phi_1 - \rho} \frac{\det(A) - (\phi_1 - \rho)(q_{21}\rho + \phi_2 - q_{22}\rho)}{(\phi_2 - \rho)(q_{12}\rho + \phi_1 - q_{11}\rho) - \det(A)} \\ &= \frac{\phi_2 - \rho}{\phi_1 - \rho} \frac{1 - q_{11}}{1 - q_{22}}. \end{aligned}$$

The desired inequality in (12) follows from the assumptions that  $q_{11} = q_{22}$  and  $\phi_2 > \phi_1$ .

## APPENDIX B. THE EXPANDED MODEL

To solve the model described in (40), we stack all variables under each regime and form an expanded model

$$\underset{32 \times 32}{A} \underset{32 \times 1}{X_t} = \underset{32 \times 32}{B} \underset{32 \times 1}{X_{t-1}} + \underset{32 \times 48}{\Gamma_u} \underset{48 \times 1}{u_t} + \underset{32 \times 2}{\Gamma_\eta} \underset{2 \times 1}{\eta_t}, \quad (\text{A3})$$

where

$$\underset{32 \times 1}{X_t} = \begin{bmatrix} \underset{8 \times 1}{x_{1,t}} \\ \vdots \\ \underset{8 \times 1}{x_{4,t}} \end{bmatrix} \equiv \begin{bmatrix} \underset{8 \times 1}{\boldsymbol{\nu}\{\tilde{s}_t = 1\} x_t} \\ \vdots \\ \underset{8 \times 1}{\boldsymbol{\nu}\{\tilde{s}_t = 4\} x_t} \end{bmatrix},$$

$$\underset{32 \times 32}{A} = \begin{bmatrix} \underbrace{\text{diag}(A_1, \dots, A_h)}_{24 \times 32} \\ \underbrace{2 \text{ expectation errors}}_{2 \times 32} \\ \underbrace{6 \text{ fixed-point equations}}_{6 \times 32} \end{bmatrix},$$

$$= \left[ \begin{array}{c} \underbrace{\text{diag}(A_1, \dots, A_h)}_{24 \times 32} \\ \underbrace{\begin{bmatrix} \mathbf{I}_2 & \mathbf{O}_{2 \times 6} & \vdots & \cdots & \vdots & \mathbf{I}_2 & \mathbf{O}_{2 \times 6} \end{bmatrix}}_{2 \times 32} \\ \underbrace{\begin{bmatrix} \mathbf{O}_{2 \times 8} & \Phi(\tilde{s} = 2)_{2 \times 7} & \mathbf{O}_{2 \times 8} & \mathbf{O}_{2 \times 8} \\ \mathbf{O}_{2 \times 8} & \mathbf{O}_{2 \times 8} & \Phi(\tilde{s} = 3)_{2 \times 8} & \mathbf{O}_{2 \times 8} \\ \mathbf{O}_{2 \times 8} & \mathbf{O}_{2 \times 8} & \mathbf{O}_{2 \times 8} & \Phi(\tilde{s} = 4)_{2 \times 8} \end{bmatrix}}_{6 \times 32} \end{array} \right]$$

$$\begin{aligned} B_{32 \times 32} &= \left[ \begin{array}{c} \underbrace{\text{diag}(B_1, \dots, B_4)(\tilde{Q} \otimes \mathbf{I}_8)}_{24 \times 32} \\ \underbrace{2 \text{ expectation errors}}_{2 \times 32} \\ \underbrace{\mathbf{O}_{6 \times 32}}_{6 \times 32} \end{array} \right] \\ &= \left[ \begin{array}{c} \underbrace{\begin{bmatrix} q_{11}B_1 & \cdots & q_{14}B_1 \\ \vdots \\ q_{41}B_4 & \cdots & q_{44}B_4 \end{bmatrix}}_{24 \times 32} \\ \underbrace{\begin{bmatrix} \mathbf{O}_{2 \times 6} & \mathbf{I}_2 & \vdots & \cdots & \vdots & \mathbf{O}_{2 \times 6} & \mathbf{I}_2 \end{bmatrix}}_{2 \times 32} \\ \underbrace{\mathbf{O}_{6 \times 32}}_{6 \times 32} \end{array} \right], \end{aligned}$$

$$\Gamma_u = \left[ \begin{array}{c|c} \mathbf{I}_{24} & \mathbf{I}_{24} \\ \hline \mathbf{O}_{8 \times 24} & \mathbf{O}_{8 \times 24} \end{array} \right], \quad u_t = \left[ \begin{array}{c} S_{s_t} \quad X_{t-1} \\ \hline \mathcal{E}_t \end{array} \right],$$

$$\begin{aligned} S_{s_t} &= \begin{bmatrix} (\boldsymbol{\nu}\{\tilde{s}_t = 1\} - \tilde{q}_{11})B_1 & \cdots & (\boldsymbol{\nu}\{\tilde{s}_t = 1\} - \tilde{q}_{14})B_1 \\ \vdots & & \vdots \\ (\boldsymbol{\nu}\{\tilde{s}_t = 4\} - \tilde{q}_{41})B_4 & \cdots & (\boldsymbol{\nu}\{\tilde{s}_t = 4\} - \tilde{q}_{44})B_4 \end{bmatrix} \\ &\equiv \text{diag}(B_1, \dots, B_4)[(\mathbf{e}_{\tilde{s}_t}\mathbf{1}'_4 - \tilde{Q}) \otimes \mathbf{I}_8], \end{aligned}$$

$$\mathbf{e}_{s_t} = \begin{bmatrix} \boldsymbol{\nu}\{\tilde{s}_t = 1\} \\ \vdots \\ \boldsymbol{\nu}\{\tilde{s}_t = 4\} \end{bmatrix}, \quad \mathbf{1}_4 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix},$$



$$\begin{aligned}
\mathcal{E}_t &= \underbrace{\begin{bmatrix} \Psi & & \mathbf{O} \\ & \dots & \\ \mathbf{O} & & \Psi \end{bmatrix}}_{24 \times 16} \underbrace{\begin{bmatrix} \boldsymbol{\nu}\{\tilde{s}_t = 1\}\varepsilon_t \\ \vdots \\ \boldsymbol{\nu}\{\tilde{s}_t = 4\}\varepsilon_t \end{bmatrix}}_{16 \times 1}, \\
\Gamma_\eta &= \begin{bmatrix} \mathbf{O}_{24 \times 2} \\ \mathbf{I}_2 \\ \mathbf{O}_{6 \times 2} \end{bmatrix}.
\end{aligned}$$

TABLE 1. Parameter values

Deep parameters					
Preferences:	$\beta = 0.9952$	$\xi = 2$	$b = 0.75$		
Technologies:	$\alpha = 0.7$	$\lambda = 1.005$	$\theta_p = 10$		
Aggregate Shocks:					
<i>Persistence:</i>	$\rho_a = 0.9$	$\rho_w = 0.9$	$\rho_\nu = 0.2$		
<i>Standard Dev.:</i>	$\sigma_r = 0.2$	$\sigma_a = 0.25$	$\sigma_w = 0.4$	$\sigma_\nu = 0.2$	
Regime transition prob.:	$q_{11} = 0.9$	$q_{22} = 0.9$			

Regime dependent parameters					
Regime	$\rho_r$	$\phi_\pi$	$\phi_y$	$\eta$	$\gamma$
Dovish regime:	0.68	0.83	0.27	0.66	1
Hawkish regime:	0.79	2.15	0.93	0.75	0

TABLE 2. Model with regime switches in policy only: persistence and volatility

<i>A. Ignoring Expectation Effects</i>						
	Volatility			Persistence		
	Inflation	Output	Interest rate	Inflation	Output	Interest rate
Dovish regime	0.0148	0.0080	0.0099	0.9692	0.8691	0.9616
Hawkish regime	0.0007	0.0019	0.0019	0.9539	0.8774	0.7811

<i>B. Accounting for Expectation Effects</i>						
	Volatility			Persistence		
	Inflation	Output	Interest rate	Inflation	Output	Interest rate
Dovish regime	0.0037	0.0028	0.0037	0.9684	0.8431	0.8960
Hawkish regime	0.0011	0.0022	0.0025	0.9531	0.8848	0.8500

TABLE 3. Model with regime switches in both policy and firms' behavior:  
persistence and volatility

<i>A. Ignoring Expectation Effects</i>						
	Volatility			Persistence		
	Inflation	Output	Interest rate	Inflation	Output	Interest rate
Dovish regime:	0.0148	0.0080	0.0099	0.9692	0.8691	0.9616
Hawkish regime:	0.0004	0.0014	0.0014	0.8447	0.8218	0.6752

<i>B. Accounting for Expectation Effects</i>						
	Volatility			Persistence		
	Inflation	Output	Interest rate	Inflation	Output	Interest rate
Dovish regime:	0.0044	0.0030	0.0041	0.9702	0.8545	0.9107
Hawkish regime:	0.0004	0.0013	0.0015	0.8349	0.8013	0.7243

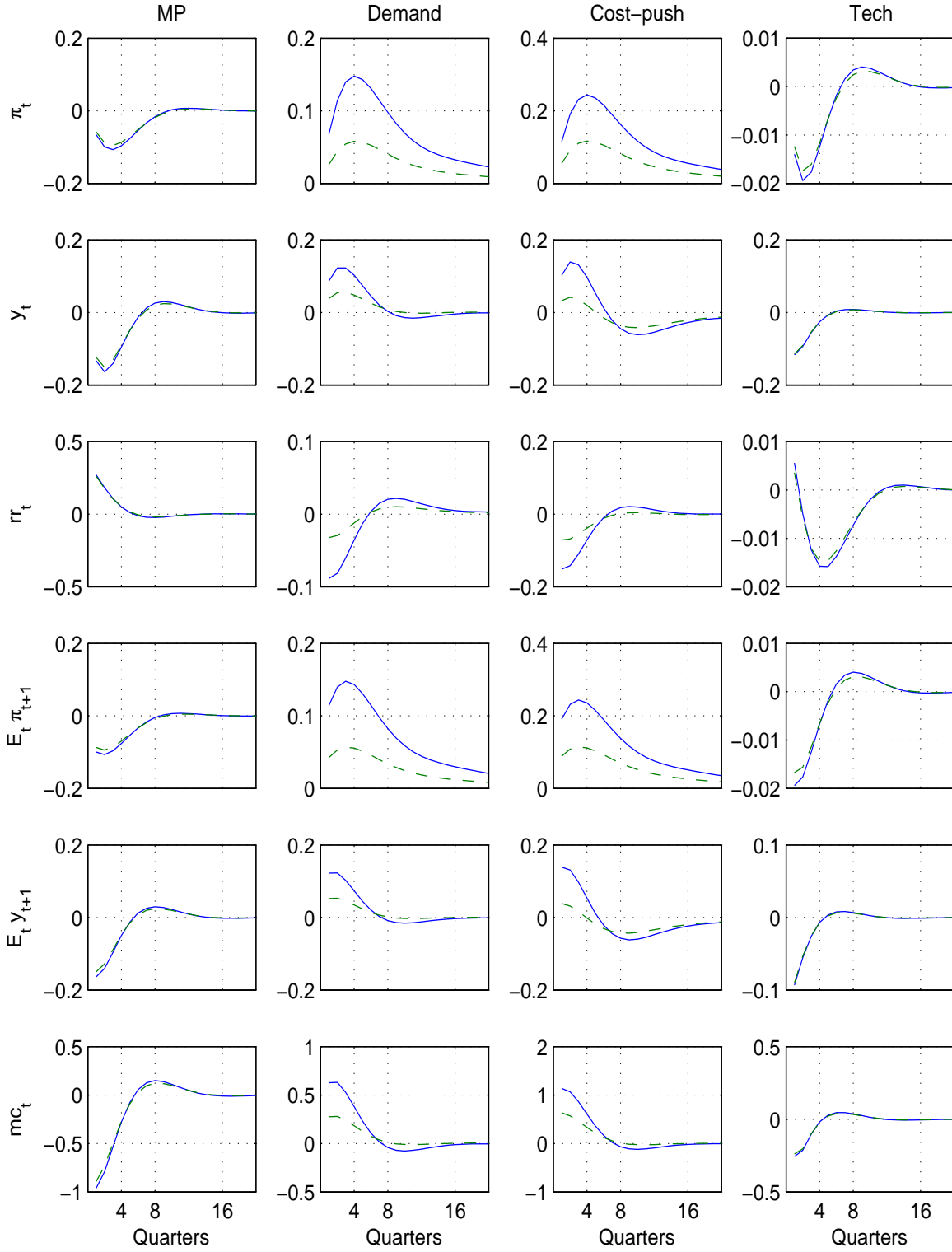


FIGURE 1. Regime switching in policy only: impulse responses under the bad policy regime. The solid line represents the responses from the model that ignores regime shifts in future policy. The dashed line represents the responses from our regime-switching model.

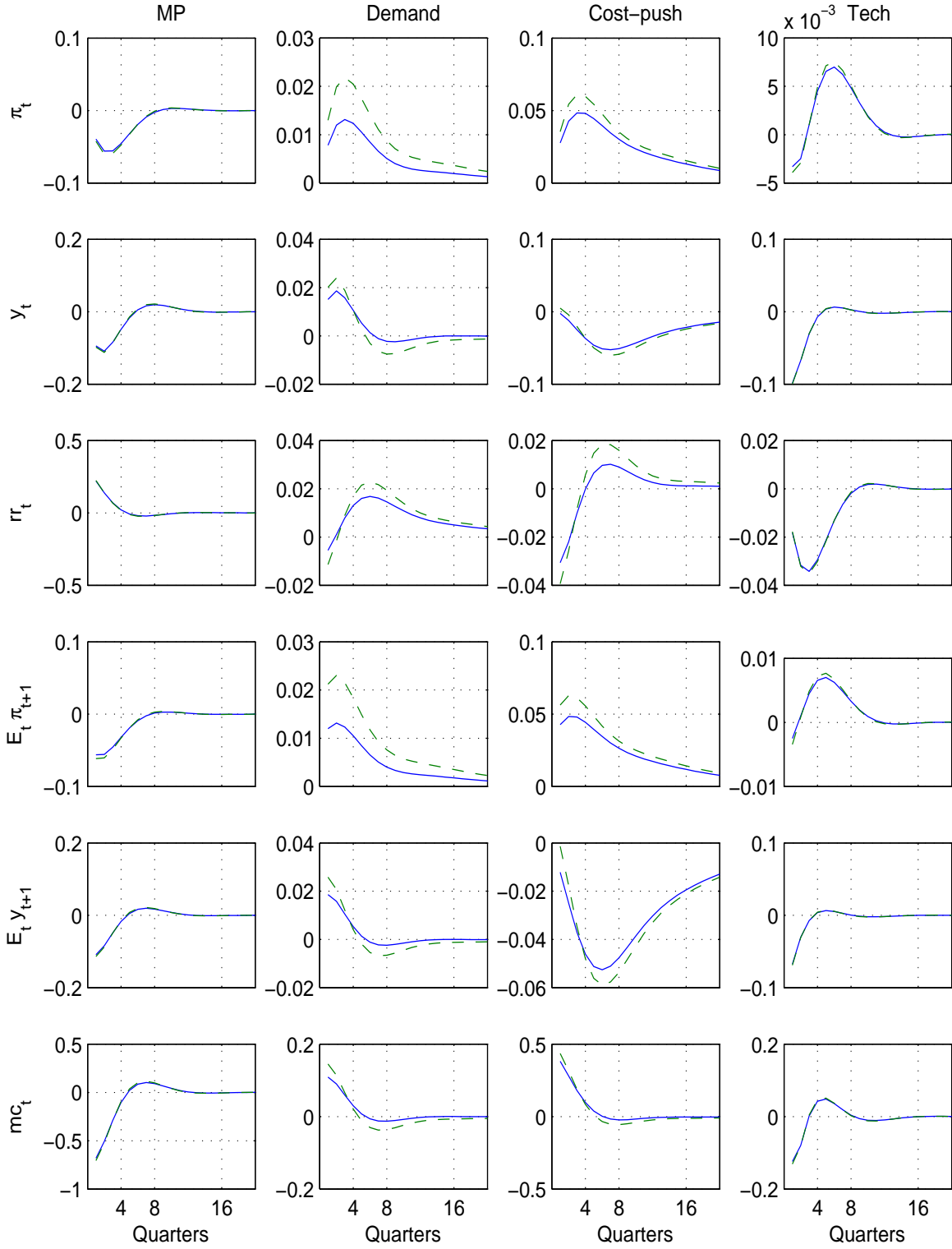


FIGURE 2. Regime switching in policy only: impulse responses under the good policy regime. The solid line represents the responses from the model that ignores regime shifts in future policy. The dashed line represents the responses from our regime-switching model.

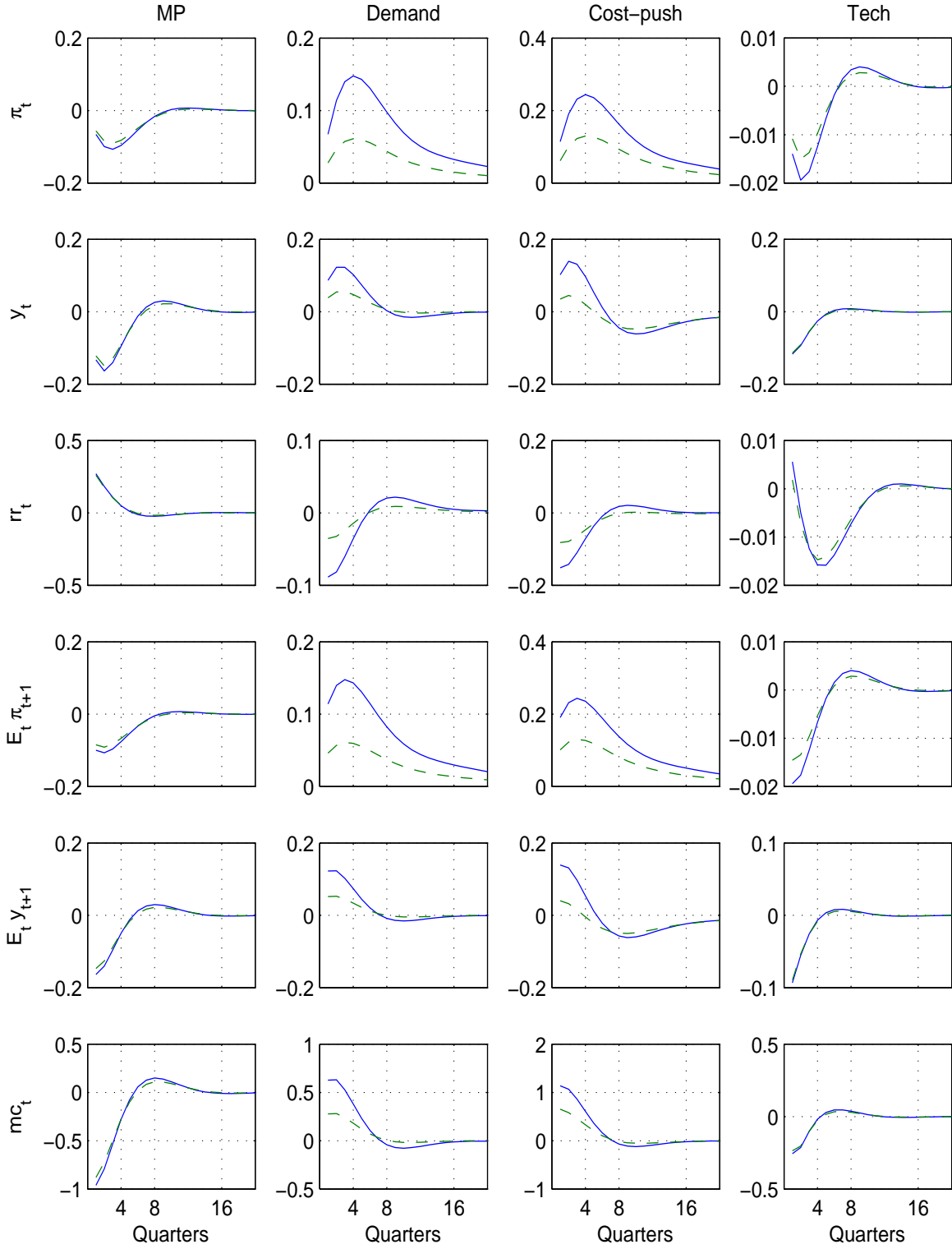


FIGURE 3. Regime switching in both policy and firms' behavior: impulse responses under the bad policy regime. The solid line represents the responses from the model that ignores regime shifts in future policy. The dashed line represents the responses from our regime-switching model.

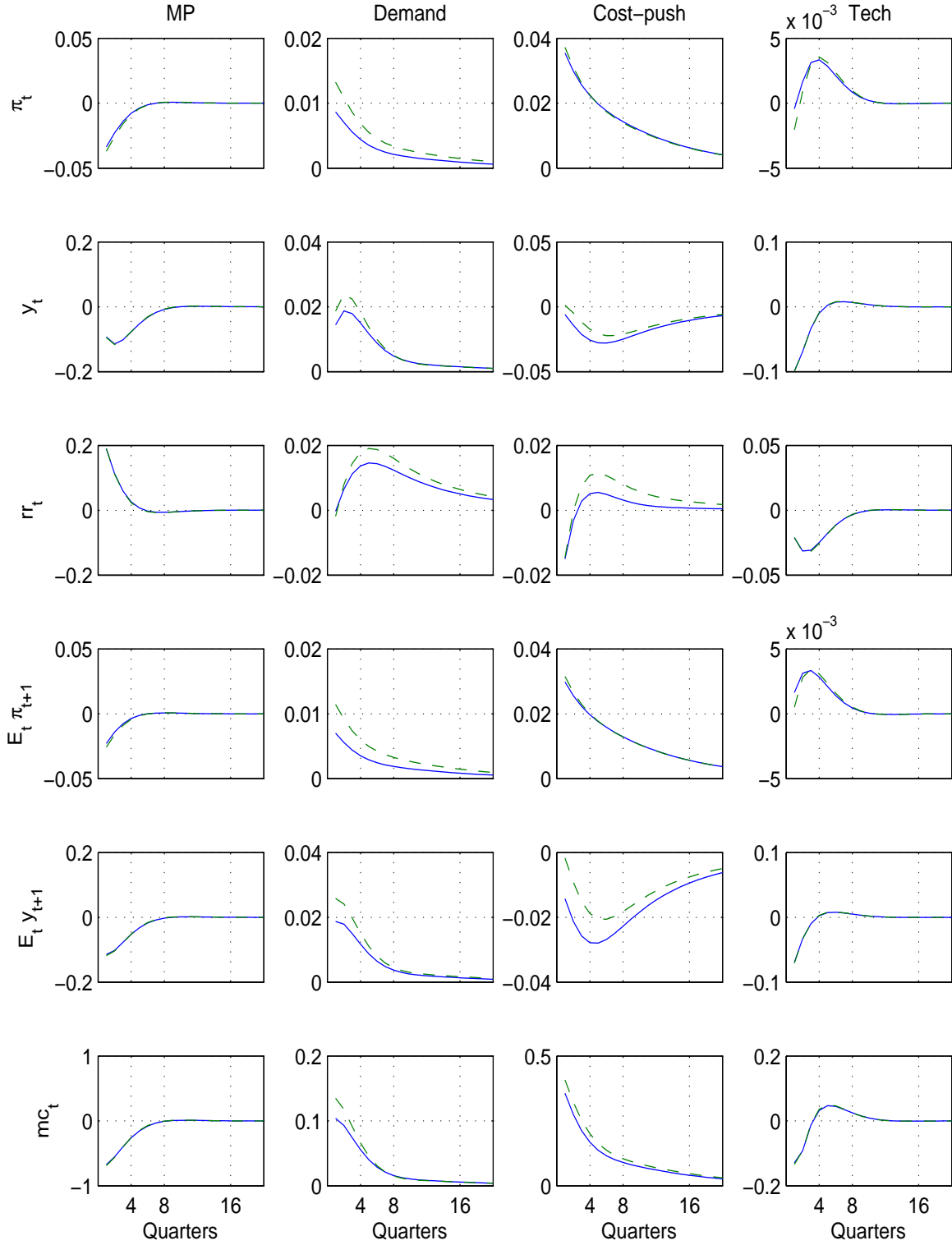


FIGURE 4. Regime switching in both policy and firms' behavior: impulse responses under the good policy regime. The solid line represents the responses from the model that ignores regime shifts in future policy. The dashed line represents the responses from our regime-switching model.

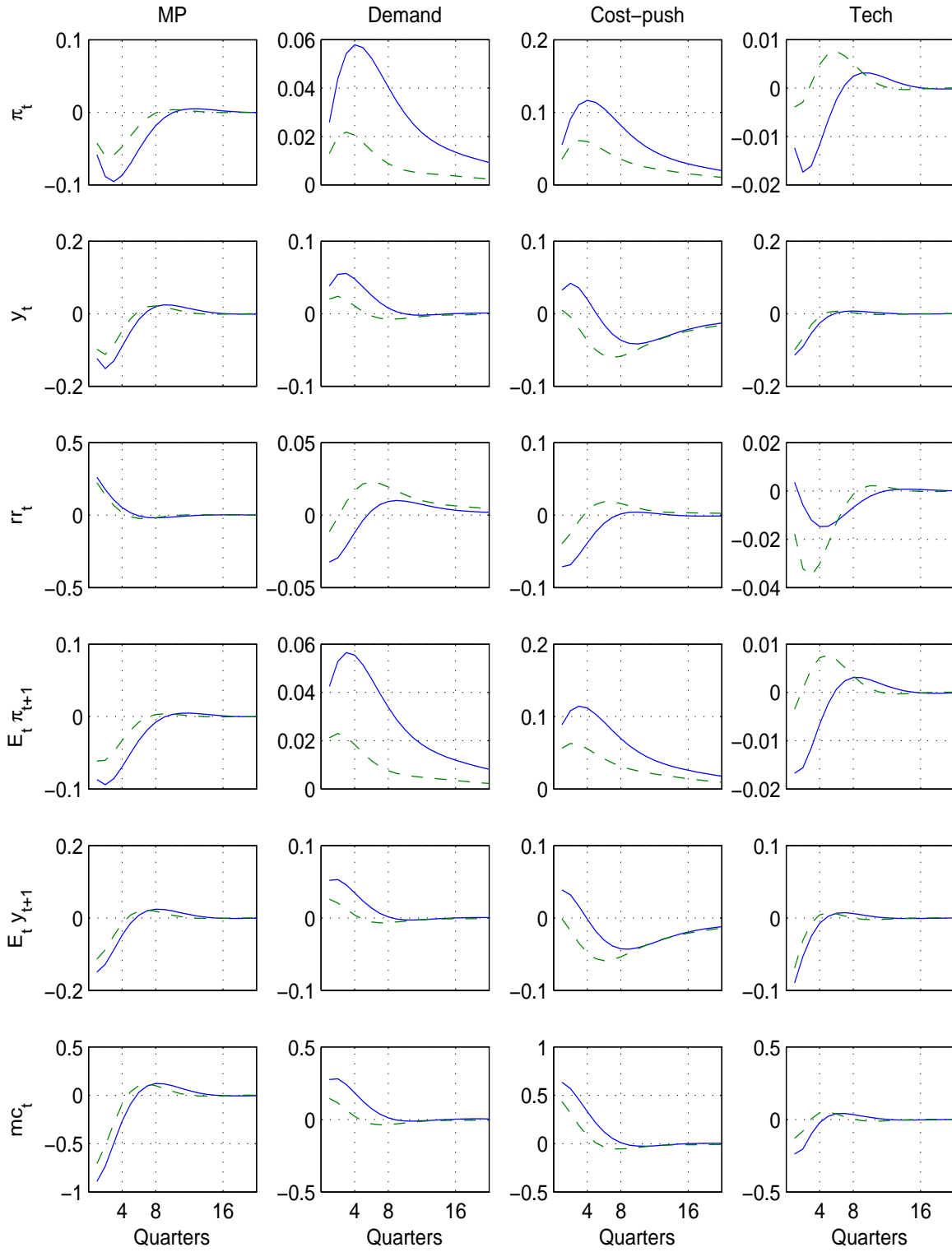


FIGURE 5. Impulse responses in the regime-switching model with policy regime changing only. The solid line represents the responses under the bad policy regime; the dashed line represents the responses under the good policy regime.



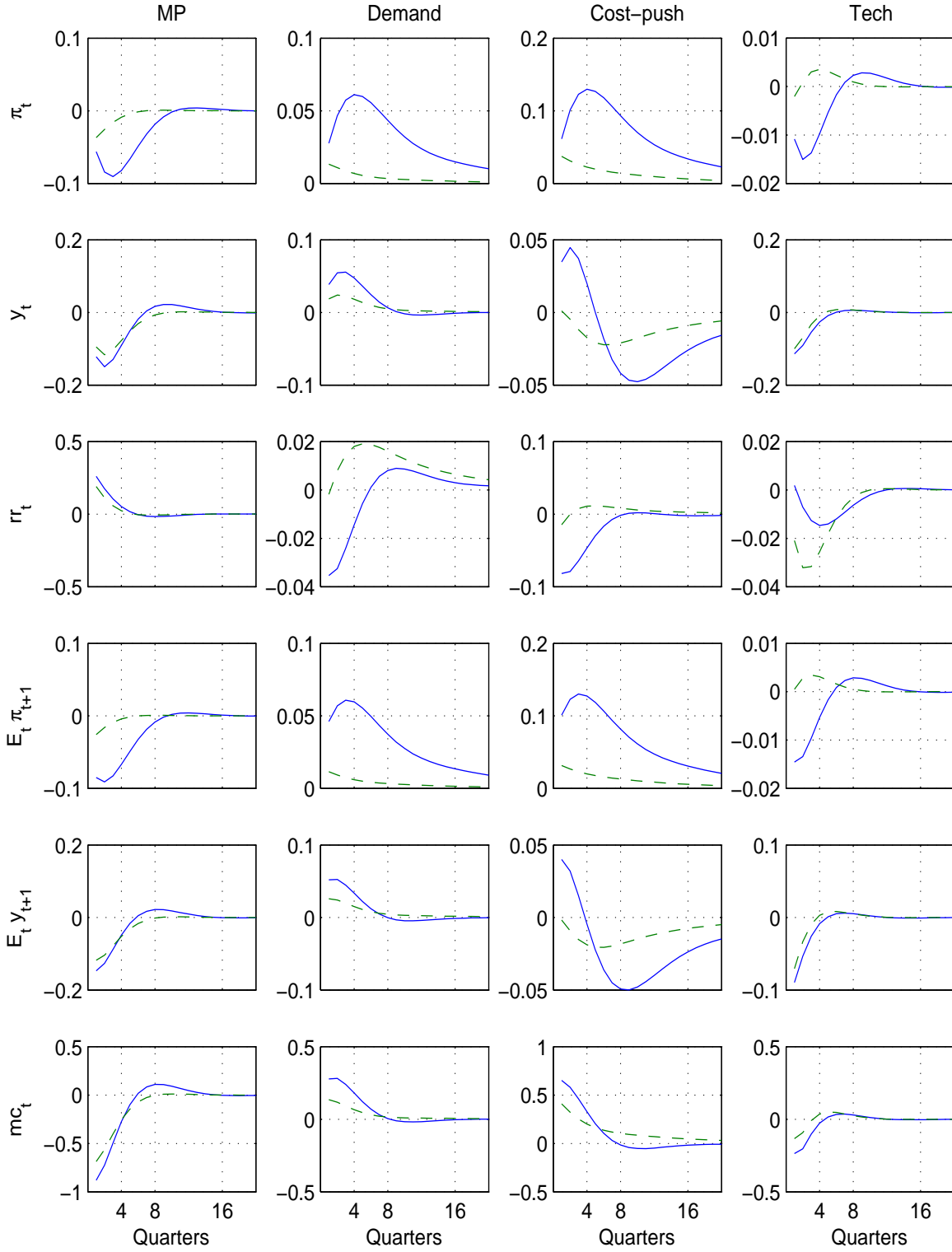


FIGURE 6. Impulse responses in the regime-switching model with changes in both policy regime and firms' behavior. The solid line represents the responses under the bad policy regime; the dashed line represents the responses under the good policy regime.

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