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Wage inequality, labor income taxes, and the notion of social status

Ennio Bilancini and Leonardo Boncinelli

Abstract
The authors investigate the desirability of income taxes when the objective is to mitigate wasteful conspicuous consumption generated by people's status-seeking behavior. They consider the joint role of pre-tax wage inequality and of social norms determining how social status is assigned. They find that if social status is ordinal (i.e., only one's rank in the income distribution matters) then an income tax can decrease waste in conspicuous consumption only if the inequality of pre-tax wages (or earning potentials) is low enough – i.e., inequality and taxation are substitutes. Instead, if status is cardinal (i.e., also the shape of the income distribution matters) then the relationship between the inequality of pre-tax wages and the change in waste can be positive – i.e., inequality and taxation can be complements – although it is in general non-monotonic. This is because the value of social status is endogenous, potentially giving rise to a perverse self-reinforcing mechanism where more waste in conspicuous consumption induces a greater competition for status and vice versa.

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Keywords Social status; relative standing; consumption externalities; labor income; income tax; signalling; conspicuous consumption; income inequality

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1 Introduction

When people care about their relative standing in society the labor market is likely to produce inefficient outcomes (Frank, 2005, 2008). Conspicuous consumption often emerges as an instrument to signal social status (Schor, 1998), typically resulting in social waste (Howarth, 1996, 2006). Labor income taxes are a prominent instrument to mitigate these inefficiencies but their efficacy has been shown to depend on the degree of pre-tax inequality in wages or earning potentials (Ireland, 1994, 1998).[1] Moreover, the outcome of tax policies in the presence of concerns for status has been shown to crucially depend on the shared norms that determine how one's social status is assigned (Clark and Oswald, 1998; Brekke et al., 2003; Bilancini and Boncinelli, 2008, 2012). Therefore, in order to assess the efficacy of income taxes to mitigate wasteful conspicuous consumption, both the pre-tax wage inequality and the notion of social status should be considered in the analysis. To the best of our knowledge this has not been done so far. In the present paper we attempt to fill this gap.

We study a model where social status depends on relative labor income, and where agents can only observe the overall distribution of incomes and the amount of income spent on an otherwise useless conspicuous good. This induces a signalling game of conspicuous consumption where the amount of income earned plays the twofold role of generating social status and granting the purchasing power required for the signal. We stress that this feature of our model is an absolute novelty in the literature relating social status to signalling games where, typically, status is generated by an exogenously given resource (for a recent survey see Truyts, 2010). So income is desired not only for its inconspicuous value and because it allows to buy the conspicuous signal, but also because it affects the value of status itself.

The analysis is divided in two parts. In the first part, we study the consequences of a labor income tax under ordinal status – i.e., when people care only about their rank in the distribution of labor incomes. This notion of social status is widely applied in applications (see, e.g., Frank, 1985a,c; Hopkins and Kornienko, 2006, 2009; Corneo and Jeanne, 1998, 1997, 1999). Consistently with both Ireland (1998) and Corneo (2002) we show that, when status is ordinal, much depends on the pre-tax wage distribution. More precisely, while low income (low wage) people are always made better off by the introduction of a labor income tax, the implications for high income (high wage) people and social waste in conspicuous consumption depend on the degree of inequality in the wage distribution. If the wage distribution is highly unequal then waste is increased and high income people are made worse

[1] Bilancini and D'Alessandro (2012) and Heikkinen (2015) show that, in the presence of both concerns for social status and utility-enhancing social activities that require time, welfare can be increased by reducing consumption, work and growth, possibly through income taxation.

off. If the wage distribution is quite unequal then waste is decreased but high income people are still made worse off. Finally, if the wage distribution is only mildly unequal then waste is decreased substantially and high income people are made better off. The main finding here is that, when status is ordinal, labor income taxes and wage inequality are *substitutes* in mitigating wasteful conspicuous consumption.

In the second part of the paper, we analyze the consequences of a labor income tax when status is not ordinal but cardinal – i.e., when people also care about how far other people are in the distribution of incomes. Cardinal status encompasses many notions of status applied in the literature that are not ordinal, such as relative deprivation (see Runciman, 1966, for the original notion, and Stark and Taylor, 1989, for its relevance to migration), difference from mean consumption (see the seminal contribution by Duesenberry, 1949, and Harbaugh, 1996, for a more recent contribution explaining the growth-savings paradox), ratio to mean consumption (see Cooper et al., 2001, for a growth model with decreasing utility over time, and Ljungqvist and Uhlig, 2000, for a productivity-shock driven model of an economy with a procyclical optimal tax policy), and upward-looking comparisons (see Bowles and Park, 2005, and Oh et al., 2012, about Veblen effects on work hours). We find that, under cardinal status, the relationship between the inequality of pre-tax wages and the change in waste induced by an income tax is, in general, non-monotonic. One source of difficulty here is that a variety of reasonable specifications of cardinal status are possible, and not all of them share the same qualitative relationship between pre-tax wage inequality and the change in waste. To make sense out of this complexity, we provide a qualitative map of such a relationship, identifying the main cases on the basis of the relative importance of the cardinal characteristics of social status. The main finding here is that, under cardinal status, labor income taxes and wage inequality *need not be substitutes* but, actually, they *can be complements* in mitigating wasteful conspicuous consumption.

Our analysis provides a number of findings showing that the case of cardinal status is *qualitatively rather different* from the case of ordinal status. First, under cardinal status a labor income tax can be Pareto improving even if pre-tax wages are extremely unequal. Second, even in the presence of small differentials in pre-tax wage rates – a case which leads to a reduction in waste under ordinal status – the amount of waste in signalling and the total amount of work may increase. Third, since a greater signalling induces high income individuals to earn more by requiring them to work more, this outcome can potentially make low income individuals worse off – as they may fall behind rich individuals even further – notwithstanding the fact that they command a greater income and work longer hours. Fourth, under cardinal status the value of status is intrinsically endogenous – since incomes

are endogenous – so that conspicuous consumption indirectly affects the value of status by affecting the choice of how much to work – and, hence, how much income to earn – potentially giving rise to a vicious cycle: more conspicuous consumption leads to more income which in turn asks for more conspicuous consumption.

Overall, these results suggest that any policy relying on an income tax which aims at mitigating wasteful conspicuous consumption should carefully consider what is the pre-tax wage inequality and what are ruling social norms that determine status. Indeed, a policy of substantial income taxation might appear to be ineffective when pre-tax wage inequality is strong if social status is believed to be ordinal, while actually it could be very effective if status is cardinal.

The paper is organized as follows. In the next section we relate our contribution to the literature on income taxation when people have status concerns. In section 3 we describe the baseline model, providing the technical results which are required for our analysis. In section 4 we study the case of ordinal status, while in section 5 we study the case of cardinal status. Section 6 provides our conclusions and final remarks. All proofs are reported in the Appendix.

2 Income taxation under status concerns

The first contribution to investigate the desirability of a labor income tax under concerns for social status is the seminal book by Duesenberry (1949) where an entire chapter is devoted to proving that, if individuals care about the ratio between their consumption and a weighted average of others' consumption, then an income tax may be desirable also for efficiency purposes. After a period of silence, Boskin and Sheshinski (1978) were the first to tackle the issue again. Assuming that people directly care about relative consumption, they find that welfare maximization requires higher linear taxes. This result has been later generalized by Oswald (1983) to non-linear tax rules.[2] Both studies rely on a welfare function to establish optimal tax schedules, hence taking into consideration also equity issues. Such a welfarist approach has not been followed by Persson (1995) who has showed that, under assumptions similar to Boskin and Sheshinski (1978) and Oswald (1983), a linear income tax can induce a Pareto improvement. We too constrain the analysis to efficiency issues.

[2]Importantly, Oswald (1983) shows that the results of Sadka (1976) and Seade (1977) – that both the most and the least productive individual should not be taxed – are not robust to the introduction of relative concerns.

Ireland (1994, 1998) has been the first to study a model of social status signalling through conspicuous consumption. In this setup, if people care about their rank in the distribution of income, an appropriate linear taxation policy can generate a Pareto improvement. In particular, if the range of pre-tax earning capabilities is not too large, then a Pareto improving income tax exists in which the poor gain from redistribution and the rich gain from a reduction in the expenditure required to signal their status.[3] An important difference between our model and the models by Ireland (1994, 1998) is that in the latter status is assumed to depend on the distribution of the gross earning potential – i.e., individual productivity, which is exogenous to the model – while in our model status is assumed to depend on the distribution of incomes that are actually earned by individuals – which is endogenous because depends on the individual decision of how much to work. We think that if concerns for status can be legitimately thought of as hardwired, then it can be reasonable to assume that social status depends on the distribution of gross earning potentials (see Rayo and Becker, 2007; Samuelson, 2004, for a discussion on why Nature may want people to have status concerns). However, if concerns for status are thought of as instrumental, i.e., arising because status provides the means for something else (as in, e.g., Cole et al., 1992, 1998), then actually earned income seems a more appropriate status-bearing asset (see Postlewaite, 1998, for a discussion of the advantages of the instrumentalist approach).[4] A further added value of our approach is that it allows us to take into account the – possibly perverse – effects of income redistribution on status and, hence, on waste. This could not be done properly in Ireland (1994, 1998) since, in equilibrium, social status is determined by exogenous individual characteristics.[5]

Truyts (2012) provides a new argument for differential indirect taxation when consumers use consumption to communicate their status to others. In particular, the goods used for signaling only can be taxed without burden while a Ramsey rule characterizes optimal taxes when goods are used for both signaling and intrinsic consumption. An alternative policy

[3] In Ireland (1994) it is also shown that universal benefits in cash or in kind can mitigate the waste due to signalling – although things are made more complex by means-testing because of its informational value.

[4] Consider, for instance, the case where status concerns are driven by concerns for the quality of social interactions (as in Bagwell and Bernheim, 1996). Owing to the instrumental approach it must be that the quality of interactions depends positively on status because people get more benefits by interacting with high status people. If we restrict to labor income as the source of such benefits then it seems reasonable to assume that benefits depend on consumption externalities. Hence, net earned income seems a better candidate than gross earning potential as the status-bearing asset – a person with a large potential that earns nothing cannot provide benefits to peers in terms of consumption.

[5] The setup of Ireland (1998) is also applied in Ireland (2001) to study the desirability of tax progressivity in the case of quasi-linear preferences (see also Corneo, 2002, on this).

is explored by Goerke (2013): mandatory profit sharing can be Pareto-improving if labour supply is excessive due to relative consumption effects. In particular, If the rise in profit income keeps total income constant, then there is a Pareto-improving substitution effect.

Finally, one can consider our contribution as a robustness test of the basic findings on optimal labor income taxation when we allow for different notions of social status. Recently, other important robustness tests have been conducted, although along different lines of generalization, e.g., Aronsson and Johansson-Stenman (2008) study optimal non-linear income taxation when revenue can be spent on public goods.[6]

3 The model

Our model is an extension of the one developed in Bilancini and Boncinelli (2012), that in turn resembles the model in Bagwell and Bernheim (1996). The novelty here is that the status-bearing asset is labor income and, therefore, it is endogenously determined.[7] This turns out to be a non-trivial modification of the model, allowing us to study how the notion of status affects the optimality of policies regarding the taxation and redistribution of labor income.

There is a population of agents consisting of two types – one with high labor productivity, the other with low labor productivity – and whose income entirely depends on labor earnings, obtained in a competitive labor market. Hereafter, the subscript h will be used to refer to the highly productive type while the subscript l will be used to refer to the lowly productive type.[8] A fraction $\beta \in (0,1)$ of population is of l-type agents and a fraction $(1-\beta) \neq 0$ is

[6] Aronsson and Johansson-Stenman (2010) analyze how optimal income taxation changes when we also consider capital accumulation and the possibility of capital taxation. Aronsson and Johansson-Stenman (2013) consider the case where the importance of conspicuous consumption increases with leisure because it leads to greater consumption visibility. Aronsson and Johansson-Stenman (2015) deal with optimal nonlinear income taxation in an international setting, where consumers care about their relative consumption compared both with locals and people abroad.

[7] A common idea is that status depends on the current level of income or consumption, the so-called relative income hypothesis (see Clark et al., 2008, and references therein). Otherwise, social status may depend on the distribution of wealth (e.g., Robson, 1992). There may be also non-economic determinants of social status, as it is pointed out in the sociological literature, where education and occupation typically play an important role (Fershtman and Weiss, 1993; Fershtman et al., 1996). Recently, Gallice and Grillo (2018) assume that status is determined by both consumption levels and social class, the latter capturing the set of socioeconomic characteristics that affect the individual's social standing after controlling for his productivity/income.

[8] We note that the presence of just two types is by no means crucial to our results (see on this, for instance,

of h-type agents. Let w_h be the productivity of h-types and w_l the productivity of l-types, with $w_h > w_l > 0$. The time endowment is $Z > 0$ and is the same for everyone. Individuals are identical under any other respect.

Time can be allocated to either working or leisure while income can be allocated to the consumption of either a conspicuous or an inconspicuous good. The price of the inconspicuous good is normalized to 1; since the price of the conspicuous good is not going to play any relevant role, it too is normalized to 1. Leisure is indicated with z, inconspicuous consumption with c and conspicuous consumption with x. Furthermore, we posit that one's productivity, leisure and inconspicuous consumption are all unobservable to other individuals while conspicuous consumption is observable.

Utility is assumed to be additive in three components measuring the individual benefits accruing from, respectively, inconspicuous consumption, leisure and status:

$$U(c, z, s) = \ln(c) + a \ln(z) + s \ , \tag{1}$$

where $a > 0$ represents the relative importance of leisure with respect to inconspicuous consumption and social status. A couple of remarks on the utility function are worth doing. First, the conspicuous good does not generate utility directly: it serves only as a signal for labor income, and hence as the means to gain status. Second, the utility from inconspicuous consumption and leisure are assumed to be logarithmic. This is done because it allows us to keep the analysis tractable and more transparent. More precisely, when utility is logarithmic, and in the absence of status-seeking effects, an income tax leads to income and substitution effects on leisure which offset each other; this makes computations easier and allows us to isolate the impact of status-seeking behavior.

The component s is assumed to depend on how individual income compares to the overall income distribution. Let ϕ be an income (cumulative) distribution on $[0, Zw_h]$ – the range of feasible incomes – and let y be an income in $[0, Zw_h]$. We write $s(\phi, y)$ for the status of an individual who is believed to possess income y when the overall distribution of incomes in the population is ϕ. If individual incomes were public information, then there would have been no gain by conspicuous consuming. However, the income of every individual is private information. So, in order to attain status, individuals engage in a signalling activity by consuming the conspicuous good x. More precisely, let $\mu(x)$ be the belief function that associates the observation of the conspicuous consumption x with a distribution ϕ of incomes and a particular income y for the sender of signal x. Status is then given by $s(\mu(x))$.[9] To rule

the model with a continuum of types in Hopkins and Kornienko, 2004).

[9]Alternatively, we might let status depend on the distribution of income net of the expenditure in sig-

out unreasonable situations, we assume that when signal x is observed, μ necessarily assigns to it an income that allows to buy x. This entails that beliefs might never provide strong out-of-equilibrium penalties, jamming the signalling. To rule this out, we also assume that an individual never finds it profitable to buy more x if this implies that he will be believed to spend his entire income on x.

In the spirit of Bilancini and Boncinelli (2008, 2012), we study how the model predictions change when different notions of status are employed. In particular, we focus on two classes of status functions which have received attention from the literature, namely ordinal status and cardinal status. When status is ordinal people have concerns only for their rank in the distribution of incomes. Therefore, $s(\phi', y') = s(\phi, y)$ if $\phi(y) = \phi'(y')$ and $\phi^-(y) = \lim_{\hat{y}\to y^-} \phi(\hat{y}) = \lim_{\hat{y}\to y'^-} \phi(\hat{y}) = \phi'^-(y')$.[10] When status is cardinal, instead, people are also interested in features of the income distribution other than from rank. For instance, under cardinal status it is likely to have $s(\phi', y) < s(\phi, y)$ when ϕ' first-order stochastically dominates ϕ over the range $[0, y)$ and both distributions are identical for higher incomes, even if the rank of an individual with income y is the same in ϕ' and ϕ. We also assume that s is bounded above, i.e., that the value of status cannot explode.[11]

Finally, a linear tax $\tau \geq 0$ is levied on income and its revenue is equally distributed to all individuals by means of a lump sum transfer T. Incomes of l-type agents and h-type agents are denoted by $y_i = (1-\tau)w_i(Z - z_i) + T$, with $i = l, h$. The hypothesis of balanced budget implies that $T = \tau\left(\beta y_l + (1-\beta)y_h\right)$, as average pre-tax and post-tax income are equal.

The decision problem of a generic individual of type i, with $i = h, l$, can be described as:

$$\max_{c,z,x} [\ln(c) + a\ln(z) + s(\mu(x))], \quad \text{s.t. } c + x \leq y_i \ . \tag{2}$$

Since the budget constraint must hold with equality, (2) can be restated as:

$$\max_{z,x} [\ln(w_i(Z - z)(1-\tau) - x + T) + a\ln(z) + s(\mu(x))] \ . \tag{3}$$

We derive the optimal leisure for given s and x, and we obtain that:[12]

nalling. Such a possibility has been explored, in a different setup, in Bilancini and Boncinelli (2012). We note that in the current model such an assumption does not represent a conceptual difficulty since, differently from Bilancini and Boncinelli (2012), the value of status is already endogenous.

[10] We use $\phi(y)$ and $\phi^-(y)$ to distinguish between, respectively, individuals with not greater income and with strictly less income.

[11] For a more detailed and formal definition of ordinal and cardinal status using income distributions see Bilancini and Boncinelli (2014).

[12] The logarithmic shape of the utility function rules out corner solutions.

$$z = \frac{a}{1+a}\left(\frac{T-x}{w_i(1-\tau)} + Z\right) . \tag{4}$$

Next step is to choose an appropriate equilibrium concept for the model. We focus on symmetric Nash equilibria in pure strategies with consistent beliefs. A vector $(z_l^*, x_l^*, z_h^*, x_h^*, \mu^*)$ is an equilibrium if and only if:

1. (z_i^*, x_i^*) maximizes utility of type i given μ^*, $i = l, h$;
2. beliefs are consistent:
   (a) if $x_l^* \neq x_h^*$ then $\mu^*(x_l^*) = (y_l^*, \phi^*)$ and $\mu^*(x_h^*) = (y_h^*, \phi^*)$,
   (b) if $x_l^* = x_h^*$ then $\mu^*(x_l^*) = \mu^*(x_h^*) = (\beta y_l^* + (1-\beta) y_h^*, \phi^*)$;

where $y_l^* = (1-\tau)w_l(Z - z_l^*) + T$, $y_h^* = (1-\tau)w_h(Z - z_h^*) + T$, and ϕ^* is the distribution where a fraction β of population earns y_l^* and a fraction $(1-\beta)$ of population earns y_h^*. To allow better readability of formulas, we set $L = s(y_l^*, \phi^*)$ and $H = s(y_h^*, \phi^*)$. Given ϕ^*, being considered to earn y_h^* is assumed to provide a higher status than being considered to earn y_l^*, namely $H > L$. We also assume that $L = s(y_l^*, \phi^*)$ and $H = s(y_h^*, \phi^*)$ are continuous in ϕ^* as long as $y_l^* \neq y_h^*$, namely that changing slightly either y_l^* or y_h^* (or both) of an entire population of types changes the value of status only slightly if incomes are different.

The above definition of equilibrium imposes only weak restrictions on out-of-equilibrium beliefs. In particular, beliefs are only required to be such that a deviation is not profitable for both l-type and h-type individuals. This great freedom in the choice of beliefs off the equilibrium path determines the existence of many pooling and separating equilibria, as in a standard signalling game. In order to get rid of this large multiplicity, and to have a unique prediction to use in comparative statics exercises, we adapt to the current setup the so-called Riley equilibrium, which is widely accepted as prominent equilibrium concept in signalling theory (see, e.g., Riley, 2001). Basically, the Riley equilibrium is the separating equilibrium where the waste in signalling is minimal. In particular, in this paper we focus on the situation where the lower income group spends nothing on signalling and the higher income group spends on signalling the minimum amount which makes a deviation not profitable for the lower income group. Unlike standard signalling models, in our setup the asset to be signalled, i.e., income, is not exogenously fixed, and either type of individuals can in principle end up with the largest asset, i.e., with the highest income. Proposition 1 establishes that a Riley equilibrium exists and that it is such that the lower income group is composed of l-type

individuals while the higher income group is composed of h-type individuals. Furthermore, Proposition 1 provides the equilibrium values of conspicuous consumption and leisure for both l-types and h-types, as well as the equilibrium lump sum transfer under balanced budget.

PROPOSITION **1.** *The Riley equilibrium exists, and at such equilibrium l-types are indifferent between following their equilibrium behavior and their best alternative that entails mimicking h-types. Furthermore, the following must hold:*

$$y_h^* > y_l^* , \tag{5}$$

$$x_l^* = 0 , \tag{6}$$

$$x_h^* = \left(1 - e^{\frac{L-H}{1+a}}\right)[w_l(1-\tau)Z + T] , \tag{7}$$

$$z_l^* = \frac{a}{1+a}\left(\frac{T}{w_l(1-\tau)} + Z\right) , \tag{8}$$

$$z_h^* = \frac{a}{1+a}\left[\frac{Te^{\frac{L-H}{1+a}} + \left(e^{\frac{L-H}{1+a}} - 1\right)(1-\tau)Zw_l}{w_h(1-\tau)} + Z\right] , \tag{9}$$

$$T = \frac{\tau(1-\tau)Z\left[\left((1-\beta)a\left(1 - e^{\frac{L-H}{1+a}}\right) + \beta\right)w_l + (1-\beta)w_h\right]}{(1+a)(1-\tau) + \tau a\left(\beta + (1-\beta)e^{\frac{L-H}{1+a}}\right)} . \tag{10}$$

For the sake of notation simplicity, from now on we will write x^* instead of x_h^*.

4 Wasteful consumption under ordinal status

We begin our analysis by considering the case where status is ordinal: H and L are fixed values that do not depend on y_l and y_h.[13] Differentiating (7), (8) and (9) with respect to τ we get:

[13] Ordinal status is widely applied in economics, starting from Frank (1985b) (see Hopkins and Kornienko, 2004; Haagsma and van Mouche, 2010; Stark, 2017, for recent contributions exploiting the ordinal properties of status).

$$\frac{\mathrm{d}x^*}{\mathrm{d}\tau} = \left(1 - e^{\frac{L-H}{1+a}}\right)\left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - w_l Z\right)\frac{1}{p}\,, \tag{11}$$

$$\frac{\mathrm{d}z_l^*}{\mathrm{d}\tau} = \frac{a}{(1+a)}\frac{1}{w_l(1-\tau)^2}\left(\frac{\mathrm{d}T}{\mathrm{d}\tau}(1-\tau) + T\right)\,. \tag{12}$$

$$\frac{\mathrm{d}z_h^*}{\mathrm{d}\tau} = \frac{a}{(1+a)}\frac{e^{\frac{L-H}{1+a}}}{w_h(1-\tau)^2}\left(\frac{\mathrm{d}T}{\mathrm{d}\tau}(1-\tau) + T\right)\,. \tag{13}$$

From (10), (11) and (13) we obtain the following preliminary results.

RESULT **1.** *A greater income tax reduces the waste in conspicuous consumption if and only if* $\mathrm{d}T/\mathrm{d}\tau < w_l Z$.

RESULT **2.** *If* $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ *at* $\tau = 0$, *then* $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ *for all* $\tau \in [0,1]$.

From result 1 we see that a greater income tax decreases total waste in conspicuous consumption if and only if the earning potential of l-types, $w_l Z$, is greater than the change in the transfer induced by the increase in τ. Note also that $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ is equivalent to saying that the inconspicuous consumption of l-types has to diminish as a result of the increase in τ.

Moreover, Result 2 implies that if the introduction of a marginal labor income tax is waste reducing, then any further increase in the tax entails a further reduction in waste. The reason is that the marginal change in the amount of income transferred from h-types to l-types is bound to be smaller than its value at $\tau = 0$. This is because, under homothetic preferences, a flat labor income tax always decreases total income and, hence, a rising tax rate can only add a decreasing amount of income to the lump sum transfer.

For the rest of this section the analysis focuses on introducing income taxation when $\tau = 0$. This greatly simplifies the analysis and, most importantly, in the light of Result 2 it allows to take a conservative perspective on waste reduction. Under $\tau = 0$ the condition $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ is satisfied if and only if

$$\frac{w_h}{w_l} < 1 + a\left(\frac{\beta}{1-\beta} + e^{\frac{L-H}{1+a}}\right) \equiv \sigma_x\,. \tag{14}$$

This shows that there is an upper bound to the degree of wage inequality for which introducing an income tax helps reducing waste.[14]

[14]Note that for $a > 0$ the right hand side of (14) is larger than unity meaning that there always exists a range of w_h/w_l such that a waste reduction is possible.

The next step is to study how the introduction of an income tax affects the equilibrium income of l-types and h-types. This is relevant in itself for obvious reasons, but for what matters here it helps to better understand the effects of the tax on individuals' utility.

RESULT **3.** *A greater income tax increases l-types' equilibrium income if and only if* $\mathrm{d}T/\mathrm{d}\tau > w_l Z$. *Moreover, a greater income tax always decreases the equilibrium income of h-types.*

From result 2 and 3 we see that an income tax decreases waste if and only if it decreases the equilibrium income of l-types. This is because a lower income makes l-types compete less fiercely for status – signalling becomes more costly for them – and, hence, it allows h-types to spend less in order to differentiate themselves from l-types. Then, from condition (14) we see that $w_h/w_l < \sigma_x$ implies that l-types' income decreases while $w_h/w_l > \sigma_x$ implies that l-types' income increases.

Result 3 also clarifies the impact of a greater tax rate on the income of h-types. The intuition is the following. When l-types' income decreases, h-types find it profitable to decrease their income as well since they experience a lower net wage and they need less conspicuous consumption to differentiate themselves from l-types. When instead the income of l-types increases, then h-types spend more on conspicuous consumption but, because of the reduced net wage, they find it optimal to reduce their inconspicuous consumption even more. Consequently, a greater tax rate always makes the rich poorer. Importantly, in the next section we will show that this result only holds under ordinal status.

We now turn our attention to individuals' utility. Differentiating utility functions at equilibrium with respect to τ we obtain

$$\frac{\mathrm{d}U_l}{\mathrm{d}\tau}\Big|_{\tau=0} = \frac{(1+a)}{w_l Z}\frac{\mathrm{d}T}{\mathrm{d}\tau} - 1 \,, \tag{15}$$

$$\frac{\mathrm{d}U_h}{\mathrm{d}\tau}\Big|_{\tau=0} = -1 + \frac{e^{\frac{L-H}{1+a}}(1+a)\dfrac{\mathrm{d}T}{\mathrm{d}\tau}}{w_h Z - \left(1 - e^{\frac{L-H}{1+a}}\right) w_l Z} \,. \tag{16}$$

By imposing (15) and (16) to be positive we get the following inequalities, respectively

$$\frac{w_h}{w_l} > 1 - a\left(1 - e^{\frac{L-H}{1+a}}\right) \equiv \sigma_l \,, \tag{17}$$

$$\frac{w_h}{w_l} < 1 + \frac{e^{\frac{L-H}{1+a}}(1-\beta)a\left(1 - e^{\frac{L-H}{1+a}}\right)}{1 - e^{\frac{L-H}{1+a}}(1-\beta)} \equiv \sigma_h \,. \tag{18}$$

By combining conditions (14), (17) and (18) we obtain the following:[15]

PROPOSITION **2.** *The introduction of a marginal income tax whose revenue is evenly distributed makes l-types better off. Moreover, it generates:*

i) less waste and a higher utility for h-types, if $w_h/w_l < \sigma_h$*;*

ii) less waste and a lower utility for h-types, if $\sigma_h < w_h/w_l < \sigma_x$*;*

iii) greater waste and a lower utility for h-types, if $w_h/w_l > \sigma_x$*.*

The proof of the Proposition can be found in the Appendix – it substantially consists of demonstrating that $\sigma_h < \sigma_x$. Figure 1 shows the three relevant intervals of the wage distribution.

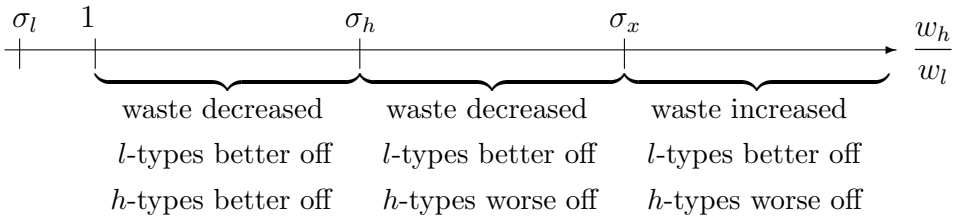

Figure 1: The effects of a marginal increase in τ as a function of w_h/w_l.

Further insights can be obtained by looking at how σ_x and σ_h vary in response to changes in the exogenous parameters of the model, i.e., a, β, $L - H$, and Z. From (14), (17) and (18) we immediately see that Z plays no role at all. The reason is that types have identical endowments and homothetic preferences – changes in Z only have scale effects which leave σ_x and σ_h unaffected. So, we could normalize Z (e.g., by setting $Z = 1$), as we did for the prices of the conspicuous good, without any substantial loss of generality. However, as it will become clear in the next section, this holds for ordinal status but not for cardinal status, so we prefer not to normalize Z to better compare results in the two cases.[16]

A larger status differential $H - L$, i.e., a larger net benefit of being considered rich instead of poor, induces a smaller σ_x. This means that waste reduction is obtained for a smaller

[15]Note that for $a > 0$ inequality (17) is always satisfied as the right hand side is strictly smaller than one. Moreover, for $a > 0$ the right hand side of (18) is strictly greater than one as the second term is positive. This implies that there is a range of wage distributions where a marginal increase of τ makes everyone strictly better off.

[16]By inspection of (19), i.e., the counterpart of (11) under cardinal status, we see that Z does play a role beyond scale effects.

range of wage distributions. The intuition here is that a larger status differential implies that the rich have a larger optimal expenditure in signalling which, in turn, implies that they have a larger optimal labor income; hence, a marginal labor income tax transfers more money from the rich to the poor, making it more likely that the poor become rich enough to force the rich to spend more in signalling in order to differentiate from them.

Less obviously, the impact of a greater $H-L$ on σ_h is non-monotonic. More precisely, it is negative for $H-L<-\ln(1-\sqrt{\beta})(1+a)$ and positive for $H-L>-\ln(1-\sqrt{\beta})(1+a)$.[17] This is because, besides the positive effect described for σ_x which is increasing in $H-L$, there is also a constant negative effect: a greater $H-L$ makes h-types work more and, hence, being taxed more. For small values of $H-L$ this latter effect dominates.

Finally, the impact of a greater preference for leisure a is positive on σ_x and ambiguous on σ_h. A greater a makes both l-types and h-types work less, and hence earn less. As a result a marginal tax transfers less money from the rich to the poor, making it less likely that the poor become rich enough to force the rich to spend more in signalling in order to differentiate from them. This explains why σ_x increases. A further effect of smaller earnings is that, depending on the relative change in incomes, the poor may find it relatively more or less attractive to engage in social competition for status through conspicuous consumption. If the poor find it more attractive then a marginal tax will make the rich save less on signalling since the poor are now more costly to discourage. In this case, the direction of change in σ_h is ambiguous. If, instead, the poor find it less attractive to engage in social competition, then σ_h increases.

5 Wasteful consumption under cardinal status

We now consider the case where status is cardinal, that is, both H and L depend on the equilibrium incomes y_l^* and y_h^*, which in turn implies that H and L depend on τ. Let L_{y_l}, L_{y_h},H_{y_l} and H_{y_h} denote the derivatives of L and H with respect to y_l^* and y_h^*.[18] Let us also assume, as it seems reasonable, that $L_{y_l}\geq 0$, $L_{y_h}\leq 0$, $H_{y_l}\leq 0$ and $H_{y_h}\geq 0$ and that they are all bounded.[19]

Our main point here is that, under cardinal status, the introduction of a labor income tax has an additional consequence which is otherwise absent under ordinal status: the prize

[17] The cutoff value can be obtained by differentiating σ_h with respect to $H-L$.

[18] We implicitly assume that s is such that L and H are differentiable.

[19] See Definition 1 (concerns for status) and 3 (cardinal concerns) in Bilancini and Boncinelli (2014) for more details.

of the social competition – i.e., the value of status itself – may change. This in turn affects how the income tax impacts on wasteful conspicuous consumption.

By differentiating (7) with respect to τ and by opportunely rearranging terms (again, we conduct the analysis at $\tau = 0$):

$$\underbrace{\left(\frac{1}{w_l Z} - ae^{\frac{L-H}{1+a}}\frac{H_{y_h}-L_{y_h}}{(1+a)^2}\right)}_{\text{Cardinal status indirect effect}}\frac{\mathrm{d}x^*}{\mathrm{d}\tau} = \underbrace{\left(1-e^{\frac{L-H}{1+a}}\right)\frac{1-\beta}{(1+a)^2}\left[\frac{w_h}{w_l}-\sigma_x\right]}_{\text{Ordinal status effect, } <0 \Leftrightarrow \frac{w_h}{w_l}<\sigma_x} + \tag{19}$$
$$+\underbrace{Z[w_l\lambda(H_{y_l}-L_{y_l}, H_{y_h}-L_{y_h}) - w_h\eta(H_{y_l}-L_{y_l}, H_{y_h}-L_{y_h})]}_{\text{Cardinal status direct effect}, <0 \Leftrightarrow \frac{w_h}{w_l}>\frac{\lambda}{\eta}}.$$

where λ and η are functions that summarize how changes in the status prize – brought about by changes in incomes – directly affect the change in conspicuous consumption through wage inequality. More precisely, λ and η measure the sensitivity to, respectively, w_l and w_h of that part of the change in conspicuous consumption which is induced by a change in the status prize $H - L$ because of a change in y_h and y_l. The detailed specification of λ and η can be found in the Appendix (see the proof of Result 4).

By inspecting (19) we see that, besides the effect already seen in the case of ordinal status – represented by the first term of the right hand side, which we call *ordinal status effect* – there are two additional effects which exist because of cardinal status. One is what we call *cardinal status direct effect* and is represented by the second term of the right hand side of (19). It accounts for the impact of τ on the status prize $H - L$ through the effect on net incomes. Note that both sign and magnitude of this direct effect depend on how relative wages compare to the ratio between functions λ and η, which in turn depend on both L_{y_l}–H_{y_l} and L_{y_h}–H_{y_h}, i.e., on how the status prize is affected by a change in y_l and y_h. The following result summarizes how the cardinal direct effect behaves.

Result **4.** *The cardinal status direct effect at $\tau = 0$ is decreasing in w_h and increasing in w_l, becoming negative if and only if $w_h/w_l > \lambda/\eta$. Moreover, we have that:*

- *if* $\dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l} < \dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h}$ *then* λ/η *is increasing in* $\dfrac{H_{y_h}-L_{y_h}}{|H_{y_l}-L_{y_l}|}$;
- *if* $\dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l} = \dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h}$ *then* λ/η *is constant;*
- *if* $\dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l} > \dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h}$ *then* λ/η *is decreasing in* $\dfrac{H_{y_h}-L_{y_h}}{|H_{y_l}-L_{y_l}|}$.

Already from equation (19) one can see that wage inequality negatively affects waste through the cardinal direct effect. This is in sharp contrast with the positive impact of wage inequality through the ordinal status effect. Indeed, under cardinal status a greater inequality increases the status prize and, hence, increases wasteful consumption which in turn makes the income tax more effective.

Moreover, from Result 4 we understand under what circumstances the critical threshold λ/η depends positively or negatively on the relative sensitivity of the status prize $H - L$ to incomes. It turns out that what the crucial issue is whether the marginal transfer $\mathrm{d}T/\mathrm{d}\tau$ is more sensitive to w_h or w_h: if the marginal transfer grows more (less) in w_h than in w_l, then concerns for social status that give a relative greater importance to getting a high income have the effect of increasing (decreasing) the degree of wage inequality required for the direct cardinal effect to be negative. Intuitively, if the marginal transfer grows more in w_h than in w_l, then the net effect of a greater wage inequality is to increase the marginal transfer, with the result that total wasteful consumption by h-types is more likely to increase.

More can be said on the sign of the cardinal direct effect if we impose an extra bit of structure on how the status prize $H - L$ reacts to changes in incomes:

RESULT **5.** *If* $\left(\dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l} - \dfrac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h}\right)[(H_{y_h} - L_{y_h}) + (H_{y_l} - L_{y_l})] \leq 0$ *then the cardinal status direct effect is negative.*

From Result 5 we see that, to obtain a negative cardinal direct effect, it is enough to have that the marginal transfer is not increasing in wage inequality when the status prize is more sensitive to high incomes or, equivalently, to have that the marginal transfer is not decreasing in wage inequality when the status prize is more sensitive to low incomes. The intuition for this result goes along the same lines described for Result 4, with the addition that the stated condition ensures that $\lambda/\eta \leq 1$. Also, Result 5 implies that if income affects the status prize symmetrically, i.e., $H_{y_l} - L_{y_l} = |H_{y_h} - L_{y_h}|$, then the cardinal status direct effect at $\tau = 0$ is always negative. This leads to the following conclusion: if status concerns are such that the social pain felt by an l-type for an increase in y_h^* is the same as the one felt for a decrease in y_l^*, then a marginal tax reduces waste via the cardinal direct effect.

The second cardinal effect is represented by the coefficient of $\mathrm{d}x^*/\mathrm{d}\tau$ appearing on the left hand side of (19), and we call it *cardinal status indirect effect*. It is indirect in the sense that it accounts for the change in $H - L$ generated by the variation of y_h^* which, in turn, is generated by the change in x^* in the first place. The intuition is the following. Because of the increase in τ, the amount of conspicuous consumption which makes l-types indifferent between being considered rich and being considered poor also changes. This in turn affects

the choice of h-types about how much to work and, hence, their income. As a result the status prize $H-L$ also changes and this feedbacks on the amount of conspicuous consumption x^* which makes l-types indifferent between being considered rich and being considered poor. Note that neither the change in x^* nor the change in y_h^* does alter the equilibrium choice of l-types, as in equilibrium their conspicuous consumption is nil. This explains why the coefficient representing the cardinal indirect effect contains the term $L_{y_h}-H_{y_h}$ but not the term $L_{y_l}-H_{y_l}$.

Since the cardinal indirect effect is never greater than unity – recall that $L_{y_h}-H_{y_h} \leq 0$ – one could suggest to interpret it as the reciprocal of a sort of waste multiplier. However, and this is somewhat surprising, the cardinal indirect effect can take both positive and negative values, and in particular it can be lower than -1.[20] If the effect is positive, then it acts indeed as a proper multiplier: it magnifies the impact of an increase in τ. Therefore, when the sum of the ordinal effect and the direct cardinal effect is negative (positive), then the indirect cardinal effect multiplies waste reduction (increase). If, instead, the cardinal indirect effect is negative, then it may either magnify or lessen the change in x^* and, in addition, it reverts its direction of change. The reason behind this perhaps counterintuitive outcome is that a first change in x^* triggers further changes in x^* that go in the opposite direction and that more than offset the first one. For instance, we might have that an increase in τ has the direct effect of making the status prize less attractive and conspicuous consumption more costly but, because it makes the incomes of l-types and h-types more similar, it requires a greater conspicuous consumption for l-types to be indifferent between being considered rich and being considered poor; this, in turn, forces h-types to work more and hence increases both their income and the status prize of being considered rich; if the cardinal status effect is negative it means that this latter effect dominates leading to an overall increase in x^*. We stress that the result crucially depends on the interaction of two characteristics of our signalling model, namely the cardinality of status and the endogeneity of the status-bearing

[20] We abstract from the case where $(1+a)^2 = -e^{\frac{L-H}{1+a}}(L_{y_h}-H_{y_h})aw_lZ$ and therefore $\mathrm{d}x^*/\mathrm{d}\tau$ cannot be determined (the hypotheses of the Implicit Function Theorem are not met). Intuitively, a small variation of x^* is not sufficient to re-establish equilibrium conditions since it induces behaviors which in turn require a further and almost identical variation of x^*.

asset.[21,22]

In conclusion, under cardinal status we have two additional effects of τ on x^* that can drastically change – with respect to the case of ordinal status – the range of w_h/w_l for which waste decreases. The following proposition summarizes the possible cases:

PROPOSITION **3.** *The introduction of a marginal income tax whose revenue is evenly distributed leads:*

i) to a waste increase (decrease) for w_h/w_l sufficiently high and to a waste decrease (increase) otherwise, whenever $\lambda/\eta \leq 1$ and the cardinal status indirect effect is positive (negative);

ii) to a waste increase (decrease) for w_h/w_l sufficiently high, to a waste decrease (increase) for w_h/w_l in an intermediate range, and either to a waste decrease or to a waste increase for w_h/w_l sufficiently low, whenever $\sigma_x > \lambda/\eta > 1$ and the cardinal status indirect effect is positive (negative);

iii) to a waste increase (decrease) for w_h/w_l sufficiently high, either to a waste decrease or to a waste increase for w_h/w_l in an intermediate range, and to a waste increase (decrease) for w_h/w_l sufficiently low, whenever $\lambda/\eta > \sigma_x > 1$ and the cardinal status indirect effect is positive (negative).

To better illustrate cases i)-iii) described in Proposition 3, we provide a graphical representation for each of them. An example of case i) is represented in Figure 2, an example of case ii) is represented in Figure 3, while an example of case iii) is represented in Figure 4. Cases i)-iii) give rise to different relationships between wage inequality and change in waste, but in all cases such a relationship turns out to be non-monotonic. This can be seen in Figures 2, 3, and 4 by looking at the sign of $\mathrm{d}x^*/\mathrm{d}\tau$ along a counterclockwise path – i.e., a path of increasing wage inequality – that begins from a point where w_h/w_l is almost 1 and the cardinal status indirect effect is negative.

Furthermore, in the light of Result 4, we can conclude that both the relative sensitivity of the status prize to incomes and the sensitivity of the marginal transfer to wages play a

[21] In Bilancini and Boncinelli (2012) we do not observe such an effect because the status-bearing asset is exogenous.

[22] In the case of a negative cardinal status indirect effect, the feedback process might diverge. However, given that both labor supply and conspicuous consumption are bounded quantities and that both leisure and consumption are essential, divergence must be considered as unlikely unless also utility from status diverges, which can be regarded as a rather exceptional case.

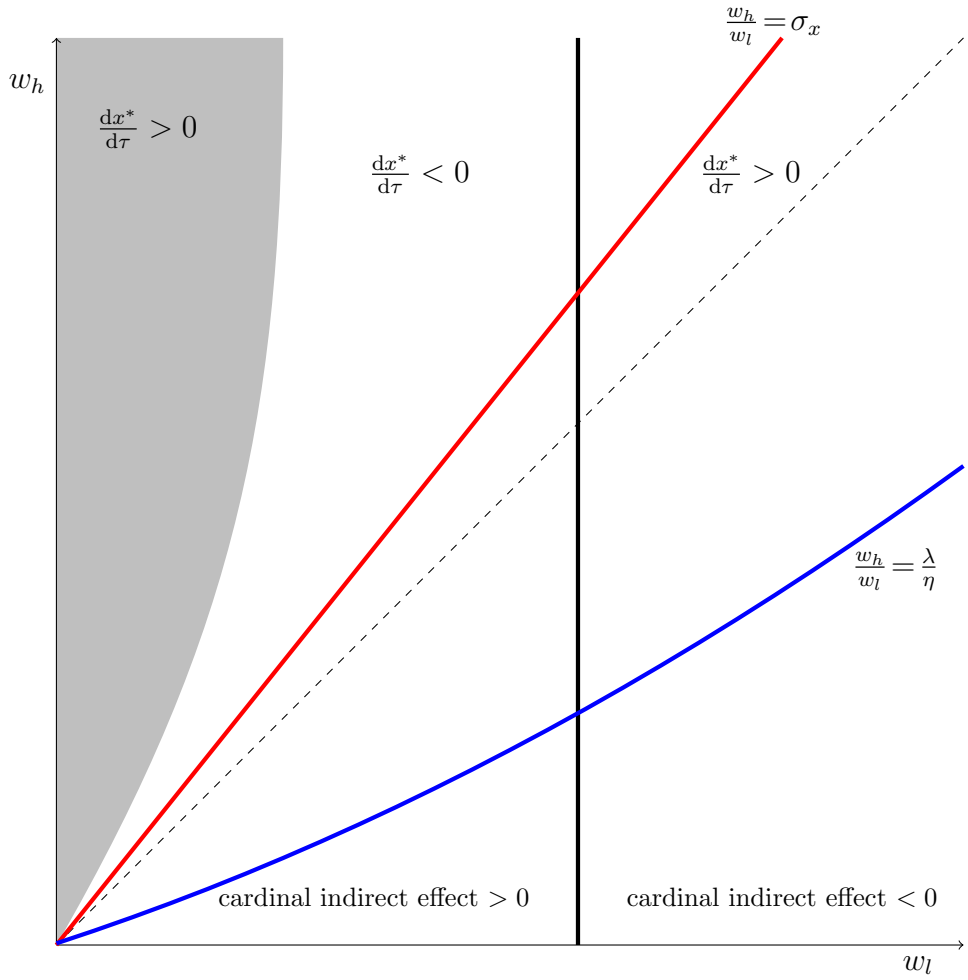

Figure 2: A case where the cardinal status direct effect is negative for every w_h/w_l, as implied by Result 5. The ordinal status effect is positive and when $w_h/w_l > \sigma_x$, and for sufficiently high values of w_h/w_l more than offsets the cardinal status direct effect (the shaded area on the left). The sign of $\mathrm{d}x^*/\mathrm{d}\tau$ is obtained taking also into account the sign of cardinal status indirect effect.

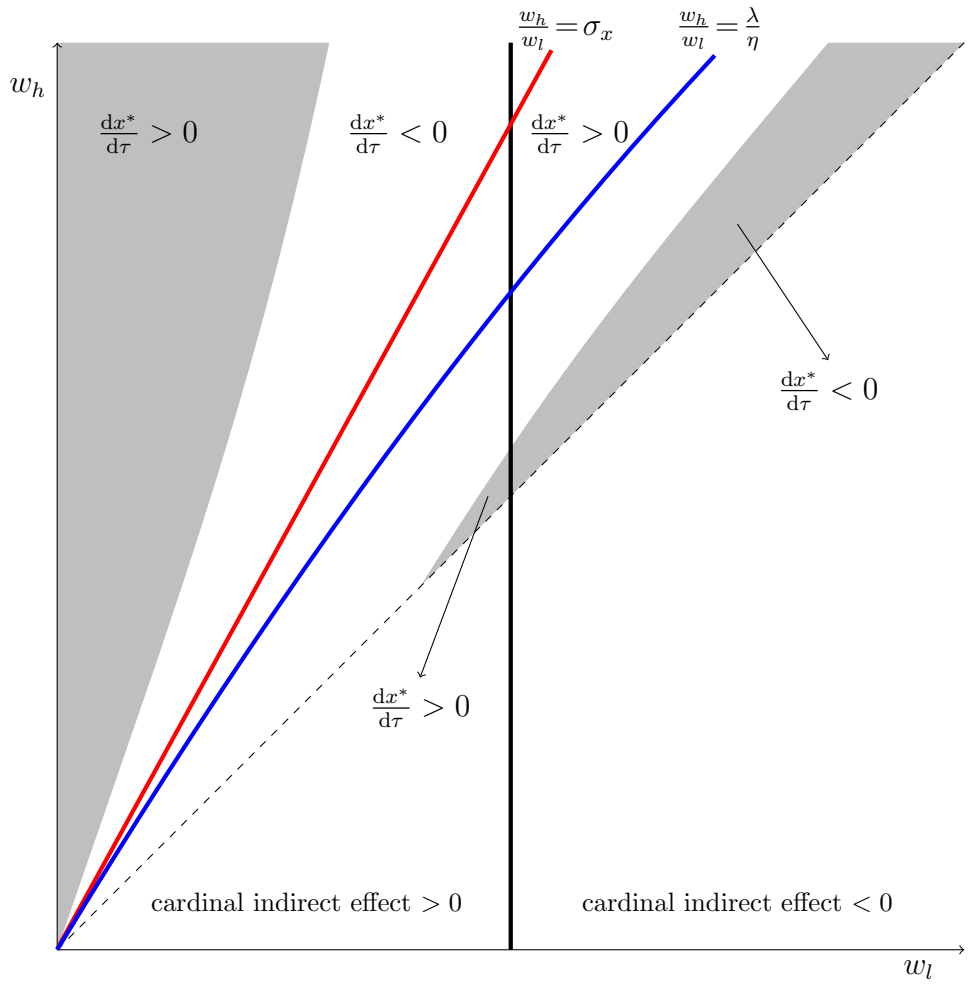

Figure 3: A case where the cardinal status direct effect is negative only for $w_h/w_l > \lambda/\eta$, as implied by Result 4. The ordinal status effect is positive and when $w_h/w_l > \sigma_x > \lambda/\eta$, so when $\sigma_x > w_h/w_l > \lambda/\eta$ the sum of the cardinal status direct effect and the ordinal status effect is negative. For a sufficiently high value of w_h/w_l the positive ordinal status effect dominates (the shaded area on the left). For sufficiently low values of w_h/w_l the positive cardinal status direct effect dominates, although this can happen only if w_l is not too small (the shaded area on the right). The sign of $\mathrm{d}x^*/\mathrm{d}\tau$ is obtained taking also into account the sign of cardinal status indirect effect.

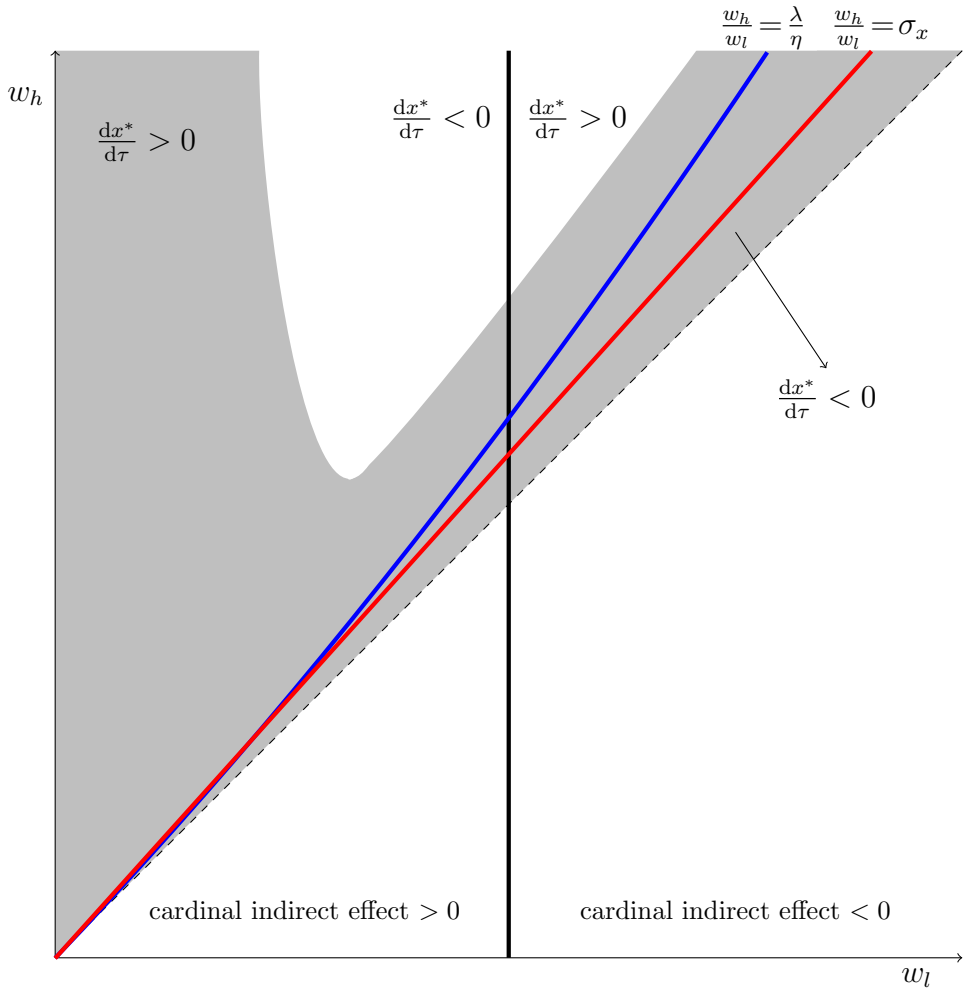

Figure 4: A case where the cardinal status direct effect is negative only for $w_h/w_l > \lambda/\eta > \sigma_x$, as implied by Result 4. The ordinal status effect is positive for $w_h/w_l > \sigma_x$, so when $\lambda/\eta > w_h/w_l > \sigma_x$ the sum of the cardinal status direct effect and the ordinal status effect is positive. For a sufficiently high value of w_h/w_l the positive ordinal status effect dominates, while for a sufficiently low value of w_h/w_l the positive cardinal status direct effect dominates. In either case the sum of the ordinal status effect and the cardinal status direct effect is positive (the shaded area). The sign of $\mathrm{d}x^*/\mathrm{d}\tau$ is obtained taking also into account the sign of cardinal status indirect effect.

crucial role to establish which of cases i)-iii) actually arises. In particular, if the marginal transfer is more (less) sensitive to w_h than to w_l, then a greater (smaller) relative sensitivity of the status prize to the income of h-types increases the likelihood of case iii) with respect to case ii) and the likelihood of case ii) with respect to case i). Note that, in the light of Result 5, we see that when incomes affect the status prizes symmetrically only case i) is feasible. Therefore, to the extent that one considers the typical situation to be characterized by such a symmetry, Result 5 and Proposition 3 together imply that case i) is the typical situation.

Proposition 3 also tells us that the sign of the cardinal status indirect effect crucially affects the direction of change in waste. In the general case we cannot say much about the sign of the cardinal status indirect effect, if not that both a greater wage for l-types and a greater sensitivity of the status prize to the income of h-types tend to turn the effect negative. In one case of interest, however, we can pin down more precise conditions that characterize a negative cardinal status indirect effect. This case is when the status prize $H - L$ only depends on the income gap $y_h^* - y_l^*$, a specification of status concerns that is quite common in the economic literature on status (e.g. Clark and Oswald, 1998; Cooper et al., 2001; Goerke and Hillesheim, 2013). Note that, under such a specification, the status prize is affected symmetrically by changes in y_h and y_l, implying that we are in case i) of Proposition 3. Moreover, the following proposition holds:

PROPOSITION **4.** *Let both L and H depend on the income gap $(y_h - y_l)$ only. Then, the introduction of a marginal income tax whose revenue is evenly distributed leads to a negative cardinal status indirect effect if and only if it increases the income gap. Moreover, in such a case the waste necessarily increases.*

Proposition 4 clarifies one important implication of cardinal status when the status prize depends on income differences only. A marginal tax on labor income can increase the equilibrium income gap only if waste increases. In other words, a greater waste is a prerequisite for a greater tax rate to increase post-tax income inequality. Hence, in order for the income tax to be socially efficient it must not increase post-tax inequality as measured by the income gap.

Before turning our attention to individuals' utility, one further difference with the case of ordinal status is worth mentioning. Under cardinal status $\mathrm{d}T/\mathrm{d}\tau$ is not granted anymore to be decreasing in τ. Actually, we have the following result:

RESULT **6.** *If $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ at $\tau = 0$ and $\mathrm{d}(H - L)/\mathrm{d}\tau \leq 0$, then $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ for all $\tau \in [0, 1]$.*

In words, if the impact of a greater τ on $H-L$ is positive, then people are induced to work more and, hence, the marginal transfer increases in τ. Nevertheless, we continue to focus on the case of $\tau=0$. The reason is that, besides providing a better analytical tractability, at $\tau=0$ we can have a more neat comparison with the results obtained under ordinal status.[23] However, from Result 6, and more in general from the fact that cardinal effects may be large and of either sign, we see that assuming $\tau=0$ no longer entails a conservative perspective on waste reduction.

Finally, we turn to the effects of the introduction of a marginal income tax on individuals' utility. To assess this issue we have to consider both the effects on utility due to the change in waste and the effects on utility due to the change in H and L. Combining such effects gives rise to a large variety of cases. Instead of providing a case-based analysis, we prefer to focus on the possibility of obtaining a Pareto improvement, emphasizing the differences with the case of ordinal status. Differentiating utility functions at equilibrium with respect to τ we get the counterparts of (15) and (16) under cardinal status:

$$\left.\frac{\mathrm{d}U_l}{\mathrm{d}\tau}\right|_{\tau=0} = \frac{(1+a)}{w_l Z}\frac{\mathrm{d}T}{\mathrm{d}\tau} - 1 + \frac{\mathrm{d}L}{\mathrm{d}\tau}\,, \tag{20}$$

$$\left.\frac{\mathrm{d}U_h}{\mathrm{d}\tau}\right|_{\tau=0} = -1 + \frac{e^{\frac{L-H}{1+a}}\left((1+a)\dfrac{\mathrm{d}T}{\mathrm{d}\tau} - w_l Z\dfrac{\mathrm{d}(H-L)}{\mathrm{d}\tau}\right)}{w_h Z - \left(1-e^{\frac{L-H}{1+a}}\right) w_l Z} + \frac{\mathrm{d}H}{\mathrm{d}\tau}\,. \tag{21}$$

By manipulating (20) and (21) we get that utility increases when, respectively:

$$\underbrace{\frac{w_h}{w_l} - \left[1 - a\left(1-e^{\frac{L-H}{1+a}}\right)\right]}_{\text{Ordinal status effect, }>0} + \underbrace{\left(L_{y_l}\frac{\mathrm{d}y_l^*}{\mathrm{d}\tau} + L_{y_h}\frac{\mathrm{d}y_h^*}{\mathrm{d}\tau}\right)\frac{1}{(1-\beta)}}_{\text{Cardinal status effect of } L} > 0\,, \tag{22}$$

$$\underbrace{\left[\left((1-\beta)\,a\left(1-e^{\frac{L-H}{1+a}}\right)+\beta\right)e^{\frac{L-H}{1+a}} + \left(1-e^{\frac{L-H}{1+a}}\right)\right] - \frac{w_h}{w_l}\left(1-e^{\frac{L-H}{1+a}}\,(1-\beta)\right)}_{\text{Ordinal status effect, }>0 \,\Leftrightarrow\, \frac{w_h}{w_l}<\sigma_h} +$$

$$+\underbrace{\left(L_{y_l}\frac{\mathrm{d}y_l^*}{\mathrm{d}\tau} + L_{y_h}\frac{\mathrm{d}y_h^*}{\mathrm{d}\tau}\right)e^{\frac{L-H}{1+a}}}_{\text{Cardinal status effect of } L} + \underbrace{\left(H_{y_l}\frac{\mathrm{d}y_l^*}{\mathrm{d}\tau} + H_{y_h}\frac{\mathrm{d}y_h^*}{\mathrm{d}\tau}\right)\left(\frac{w_h}{w_l}-1\right)}_{\text{Cardinal status effect of } H} > 0\,. \tag{23}$$

[23] Note that in $\tau=0$ the marginal transfer $\mathrm{d}T/\mathrm{d}\tau$ is the same as under ordinal status (see the proof of Result 6).

From (22) we see that the impact of τ on y_l^* and y_h^*, and hence on L, can increase or decrease the threshold value of w_h/w_l for which l-types are made better off. In particular, differently from what seen for ordinal status, under cardinal status the introduction of a labor income tax may make l-types worse off: if L decreases enough to offset the positive ordinal status effect, then l-types' utility decreases. Notably, this may happen even if the equilibrium income of l-types increases. Indeed, a higher tax rate may increase the expenditure in signalling by h-types, and because of this it may induce h-types to work more and hence obtain a higher income, which in turn can reduce l-types' social status to an extent that more than offsets the benefits of their higher income.

From (23) we see that also the threshold value of w_h/w_l for which h-types are made better off depends on how τ affects the equilibrium incomes and the status prize. However, in this case both the change in H and the change in L matter, and the reason is that $H - L$ affects the equilibrium amount of conspicuous consumption x^*. In particular, we see that the new threshold is given by the sum of σ_h – which is got by imposing that the first term in (23) is greater than zero – and the net cardinal effects of L and H. The cardinal effect of L has the same sign of $L_{y_l}(\mathrm{d}y_l^*/\mathrm{d}\tau) + L_{y_h}(\mathrm{d}y_h^*/\mathrm{d}\tau)$ meaning that a rise in the status prize of being considered poor positively affects the utility of h-types. The reason is that a greater L makes l-types less inclined to compete for being considered rich and, therefore, it allows h-types to spend less on conspicuous consumption. On the contrary, a change in H has two effects which counteract each other. On the one side, an increase of H raises the equilibrium utility of h-types directly. On the other side, however, it increases the social prize of being considered rich and therefore, in equilibrium, it makes h-types spend more on wasteful conspicuous consumption in order to discourage l-types from emulation. As (23) reveals, the former effect always prevails, and the cardinal effect of H comes out to be of the same sign of $H_{y_l}(\mathrm{d}y_l^*/\mathrm{d}\tau) + H_{y_h}(\mathrm{d}y_h^*/\mathrm{d}\tau)$.

Together inequalities (22) and (23) give the necessary and sufficient conditions for the introduction of a marginal income tax to generate a Pareto improvement. The following proposition reports an important implication of such conditions.

PROPOSITION **5.** *For any value of $w_h/w_l > 1$, there exist differentiable functions $H(y_h, y_l)$ and $L(y_h, y_l)$, with $L_{y_l} \geq 0$, $L_{y_h} \leq 0$, $H_{y_l} \leq 0$ and $H_{y_h} \geq 0$, such that the introduction of a marginal labor income tax whose revenue is evenly distributed induces both a reduction in waste and a strict Pareto improvement.*

The proof of Proposition 5 is given in the Appendix. Here we just provide the intuition of the result. Fix w_h/w_l. If the ordinal effect is already pushing towards a waste reduction

and a Pareto improvement then it suffices to have the cardinal effect weak enough not to offset the ordinal effects. If, instead, the ordinal effect pushes towards a waste increase and lower utility for h-types, then we can think of a cardinal definition of status such that the status of being considered rich, H, is not very much sensitive to the income of l-types and h-types while the status of being considered poor, L, is sensitive enough to induce a large change in L but not so much to change the sign of the cardinal status indirect effect. Under such a definition of status we have that taxing labor income and evenly redistributing the tax revenue makes l-types better off: l-types consume more inconspicuous goods, their status increases – as L increases – and they enjoy more leisure. Moreover, l-types find it less profitable to engage in social competition because the status prize, $H - L$, is now smaller. This decreases the amount of conspicuous consumption that h-types must use to separate themselves from l-types. Therefore, h-types can be made better off: h-types lose at most a little in terms of their status – because H does not change much – while they certainly increase both their inconspicuous consumption and their leisure due to the reduced competition for status – i.e., x^* decreases. This case is by no means exceptional. For instance, definitions of social status based on relative deprivation and upward-looking comparisons do have similar characteristics.

6 Conclusions

In this paper we have investigated the impact of labor income taxes when agents can signal their relative standing by spending on a conspicuous good. We have assumed that the tax revenue is redistributed by means of lump sum transfers and that status depends on the distribution of net incomes. Our main result is the characterization of how the desirability of a labor income tax depends on the definition of social status.

We contributed in two ways to the literature on income taxation under status concerns. In the first place, our results suggests that under ordinal status the introduction of a labor income tax is desirable only if the distribution of pre-tax wages is not too unequal. This confirms the results obtained in a different setup by Ireland (1998) and Corneo (2002). In addition, we have characterized two relevant thresholds of inequality in pre-tax wage. If inequality is below the lowest threshold – i.e, pre-tax wages are quite close – then the introduction of a linear labor income tax reduces waste in conspicuous consumption and makes everybody better off. If inequality is between the two thresholds – i.e., pre-tax wages are neither too close or too distant – then the tax reduces waste, makes low income individuals better off but high income individual worse off. Finally, if inequality is above highest thresh-

old – i.e., pre-tax wages are quite distant – then the tax increases waste, making again low income individuals better off and high income individual worse off. So we understand that, when status is ordinal, the inequality of pre-tax wages and the taxation of labor income are substitutes with respect to the objective of mitigating losses due to status-seeking behavior.

In the second place, we have analyzed the effects of taxing and redistributing labor income under cardinal status, providing a number of novel findings which show that the results obtained for ordinal status need not hold for cardinal status. First, under cardinal status it is neither true that lowly productive individuals are always made better off by the introduction of a labor income tax, nor that a greater inequality in pre-tax wage rates necessarily makes waste reduction less likely. In particular, under cardinal status a labor income tax could be Pareto improving even if pre-tax wage rates are extremely unequal, while it could increase waste even when pre-tax wage rates are very similar. Indeed, we found that under cardinal status the relationship between the inequality of pre-tax wages and the impact of the tax on waste is non-monotonic, following non-trivial patterns. So, we have opted to describe how such a relationship is linked to the relative importance of the cardinal characteristics of social status. Furthermore, under cardinal status a self-reinforcing mechanism can arise: more conspicuous consumption leads to work more in order to earna greater income, which in turn increases the status prize and, hence, asks for more conspicuous consumption. The reason for this is that the value social status becomes endogenous under cardinal status, since the status prize depends on labor incomes. Thus, the introduction of a labor income tax might move the economy towards vicious equilibria sustained by the fact that a high conspicuous consumption requires a high income that in turn makes the status of being considered rich highly valuable (with respect to the status of being considered poor) and, hence, it makes conspicuous consumption worth its spending. Overall, these findings suggest that, under cardinal status, labor income taxes and wage inequality need not be substitutes – actually, they might well be complements – in mitigating the inefficiencies of status-seeking behavior.

Our findings are relevant, we believe, for at least two reasons. The first is that they provide an argument in favor of the claim that, in models with status concerns, the applied definition of status need to be well founded, as much depends on it (Bilancini and Boncinelli, 2008). In this regard, our findings not only suggest that inequality does not need to be a substitute for redistribution (Bilancini and Boncinelli, 2012), but they also indicate that if status is cardinal then its value can become endogenous and can give rise to effects on both incomes and waste that are unforeseeable if one sticks to a model with ordinal status. This seems particularly relevant in the light of Bilancini and Boncinelli (2014), who show that

the presence of information asymmetries in matching markets are conducive to concerns for cardinal status (see also the models in Hopkins, 2012; Bhaskar and Hopkins, 2016), and of Bilancini and Boncinelli (2018), who show that cardinal status naturally arises when rewards of today's competition for status represent endowments of tomorrow's competition for status (see Hopkins, 2008; Hopkins and Kornienko, 2010, for a discussion on the role of endowments and rewards).

The second reason is more specific to the issue of the optimal tax policy. In the light of our results on cardinal status, we can conclude that the degree of pre-tax wage inequality does not imply much *per se* about the desirability of a labor income tax. In particular, under some specifications of cardinal status a greater wage inequality may ask for a greater taxation and redistribution whereas under some other specifications it may ask for exactly the opposite. However, we are not in a situation where "everything goes". As indicated by our Propositions 3 and 4, to know what is better we need to know the actual shape of status concerns. In our opinion, this asks for conducting an adequate research on the way social status is computed and evaluated by people.

References

Aronsson, T. and O. Johansson-Stenman (2008). When the Joneses' consumption hurts: Optimal public good provision and nonlinear income taxation. *Journal of Public Economics 92*(5), 986–997.

Aronsson, T. and O. Johansson-Stenman (2010). Positional concerns in an OLG model: Optimal labor and capital income taxation. *International Economic Review 51*(4), 1071–1095.

Aronsson, T. and O. Johansson-Stenman (2013). Veblen's theory of the leisure class revisited: Implications for optimal income taxation. *Social Choice and Welfare 41*(3), 551–578.

Aronsson, T. and O. Johansson-Stenman (2015). Keeping up with the joneses, the smiths and the tanakas: On international tax coordination and social comparisons. *Journal of Public Economics 131*, 71–86.

Bagwell, L. S. and B. D. Bernheim (1996). Veblen effects in a theory of conspicuous consumption. *American Economic Review* (86), 349–373.

Bhaskar, V. and E. Hopkins (2016). Marriage as a rat race: Noisy premarital investments with assortative matching. *Journal of Political Economy 124*(4), 992–1045.

Bilancini, E. and L. Boncinelli (2008). Ordinal vs cardinal status: Two examples. *Economics Letters* (101), 17–19.

Bilancini, E. and L. Boncinelli (2012). Redistribution and the notion of social status. *Journal of Public Economics 96*(9), 651–657.

Bilancini, E. and L. Boncinelli (2014). Instrumental cardinal concerns for social status in two-sided matching with non-transferable utility. *European Economic Review 67*, 174–189.

Bilancini, E. and L. Boncinelli (2018). When today's rewards are tomorrow's endowments: The effects of inequality on status competition. *Mimeo*.

Bilancini, E. and S. D'Alessandro (2012). Long-run welfare under externalities in consumption, leisure, and production: A case for happy degrowth vs. unhappy growth. *Ecological Economics 84*, 194–205.

Boskin, M. J. and E. Sheshinski (1978). Optimal redistributive taxation when individual welfare depends upon relative income. *Quarterly Journal of Economics 92*, 589–601.

Bowles, S. and Y. Park (2005). Emulation inequality and work hours: Was thorsten veblen right? *The Economic Journal* (115), F397–F412.

Brekke, K. A., R. B. Howarth, and K. Nyborg (2003). Status-seeking and material affluence: evaluating the hirsch hypothesis. *Ecological Economics 45*(1), 29–39.

Clark, A., P. Frijters, and M. Shields (2008). "Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and Other Puzzles". *Journal of Economic Literature* (46), 99–144.

Clark, A. and R. J. Oswald (1998). Comparison-concave utility and following behavior in social and economic settings. *Journal of Public Economics* (70), 133–155.

Cole, H., G. Mailath, and A. Postlewaite (1992). Social norms, savings behavior, and growth. *Journal of Political Economy* (100), 1092–1125.

Cole, H., G. Mailath, and A. Postlewaite (1998). Class systems and enforcement of social norms. *Journal of Public Economics* (70), 5–35.

Cooper, B., C. Garcia-Penalosa, and P. Funck (2001). Status effect and negative utility growth. *Economic Journal* (111), 642–665.

Cooper, B., C. Garcia-Penalosa, and P. Funk (2001). Status effects and negative utility growth. *The Economic Journal 111*(473), 642–665.

Corneo, G. (2002). The efficient side of progressive income taxation. *European Economic Review* (46), 1359–1368.

Corneo, G. and O. Jeanne (1997). "Conspicuous Consumption, Snobbism and Conformism". *Journal of Public Economics* (66), 55–71.

Corneo, G. and O. Jeanne (1998). Social organization, status and saving behavior. *Journal of Public Economics* (70), 37–51.

Corneo, G. and O. Jeanne (1999). Pecuniary emulation, inequality and growth. *European Economic Review* (43), 1665–1678.

Duesenberry, J. S. (1949). *Income, Savings and the Theory of Consumer Behaviour*. Cambridge, MA: Harvard University Press.

Fershtman, C., K. M. Murphy, and Y. Weiss (1996). "Social Status, Education and Growth". *Journal of Political Economy* (104), 108–132.

Fershtman, C. and Y. Weiss (1993). "Social Status, Culture and Economic Performance". *Economic Journal* (103), 946–959.

Frank, R. (2005). Positional externalities cause large and preventable welfare losses. *American Economic Review 95*(2), 137–141.

Frank, R. (2008). Should public policy respond to positional externalities? *Journal of Public Economics 92*(8), 1777–1786.

Frank, R. H. (1985a). *Choosing the Right Pond: Human Behavior and the Quest for Status*. New York: Oxford University Press.

Frank, R. H. (1985b). The demand for unobservable and other nonpositional goods. *The American Economic Review 75*(1), 101–116.

Frank, R. H. (1985c). "The Demand for Unobservable and Other Positional Goods". *American Economic Review* (75), 101–116.

Gallice, A. and E. Grillo (2018). Economic and social-class voting in a model of redistribution with social concerns. *Carlo Alberto Notebooks 448*.

Goerke, L. (2013). Profit sharing and relative consumption. *Economics Letters 118*(1), 167–169.

Goerke, L. and I. Hillesheim (2013). Relative consumption, working time, and trade unions. *Labour Economics 24*(0), 170–179.

Haagsma, R. and P. van Mouche (2010). Equilibrium social hierarchies: A non-cooperative ordinal status game. *The BE Journal of Theoretical Economics 10*(1).

Harbaugh, R. (1996). "Falling Behind the Joneses: Relative Consumption and the Growth-Savings Paradox". *Economic Letters* (53), 297–304.

Heikkinen, T. (2015). (de) growth and welfare in an equilibrium model with heterogeneous consumers. *Ecological Economics 116*, 330–340.

Hopkins, E. (2008). "Inequality, Happiness and Relative Concerns: What Actually is Their Relationship?". *Journal of Economic Inequality* (6), 351–372.

Hopkins, E. (2012). Job market signaling of relative position, or becker married to spence. *Journal of the European Economic Association 10*(2), 290–322.

Hopkins, E. and T. Kornienko (2004). Running to keep in the same place: Consumer choice as game of status. *American Economic Review* (94), 1085–1107.

Hopkins, E. and T. Kornienko (2006). "Inequality and Growth in the Presence of Competition for Status". *Economic Letters* (93), 291–296.

Hopkins, E. and T. Kornienko (2009). Status, affluence, and inequality: Rank-based comparisons in games of status. *Games and Economic Behavior 67*(2), 552–568.

Hopkins, E. and T. Kornienko (2010). Which inequality? the inequality of endowments versus the inequality of rewards. *American Economic Journal: Microeconomics 2*(3), 106–137.

Howarth, R. B. (1996). Status effects and environmental externalities. *Ecological Economics 16*(1), 25–34.

Howarth, R. B. (2006). Optimal environmental taxes under relative consumption effects. *Ecological Economics 58*(1), 209–219.

Ireland, N. (1994). On limiting the market for status signals. *Journal of Public Economics* (53), 91–110.

Ireland, N. (1998). Status seeking, income taxation and efficiency. *Journal of Public Economics* (70), 99–113.

Ireland, N. (2001). Optimal income tax in the presence of status effects. *Journal of Public Economics* (81), 193–212.

Ljungqvist, L. and H. Uhlig (2000). Tax policy and aggregate demand management under catching up with the joneses. *American Economic Review 90*(3), 356–366.

Oh, S.-Y., Y. Park, and S. Bowles (2012). Veblen effects, political representation, and the reduction in working time over the 20th century. *Journal of Economic Behavior & Organization 83*(2), 218–242.

Oswald, A. (1983). Altruism, jealousy and the theory of optimal non-linear taxation. *Journal of Public Economics 20*, 77–87.

Persson, M. (1995). Why are taxes so high in egalitarian societies? *Scandinavian Journal of Economics 97*, 569–580.

Postlewaite, A. (1998). The social basis of interdependent preferences. *European Economic Review* (42), 779–800.

Rayo, L. and G. Becker (2007). Evolutionary efficiency and happiness. *Journal of Political Economy* (115), 302–337.

Riley, J. G. (2001). Silver signals: Twenty-five years of screening and signalling. *Journal of Economic Literature* (39), 432–478.

Robson, A. (1992). "Status, the Distribution of Wealth, Private and Social Attitudes to Risk". *Econometrica* (60), 837–857.

Runciman, W. G. (1966). *Relative Deprivation and Social Justice*. London: Routledge and Kegan Paul.

Sadka, E. (1976). On income distribution, incentive effects and optimal taxation. *Review of Economic Studies 43*, 261–267.

Samuelson, L. (2004). Information-based relative consumption effects. *Econometrica* (72), 93–118.

Schor, J. B. (1998). *The Overspent American: Upscaling, Downshifting and the New Consumer*. New York: Basic Book.

Seade, J. K. (1977). On the shape of optimal tax schedules. *Journal of Public Economics 7*, 203–235.

Stark, O. (2017). Migration when social preferences are ordinal: Steady-state population distribution and social welfare. *Economica 84*(336), 647–666.

Stark, O. and J. E. Taylor (1989). Relative deprivation and international migration oded stark. *Demography 26*(1), 1–14.

Truyts, T. (2010). Social status in economic theory. *Journal of Economic Surveys 24*(1), 137–169.

Truyts, T. (2012). Signaling and indirect taxation. *Journal of Public Economics 96*(3), 331–340.

APPENDIX

A Proofs

A.1 Proof of Result 1

The result is immediately got from (11) by noticing that $e^{\frac{L-H}{1+a}} < 1$ for $L < H$.

A.2 Proof of Result 2

We take the first derivative of T with respect to τ and we obtain that

$$\frac{\mathrm{d}T}{\mathrm{d}\tau} = \frac{ZK\left[(1-2\tau)E - \tau(1-\tau)E'\right]}{E^2}\,, \tag{24}$$

where

$$K \equiv \left((1-\beta)a\left(1-e^{\frac{L-H}{1+a}}\right)+\beta\right)w_l + (1-\beta)w_h\,, \tag{25}$$

$$E \equiv (1+a)(1-\tau) + \tau a\left(\beta + (1-\beta)e^{\frac{L-H}{1+a}}\right)\,, \tag{26}$$

$$E' \equiv \frac{\mathrm{d}E}{\mathrm{d}\tau}\,. \tag{27}$$

We take the second derivative of T with respect to τ and we obtain that

$$\frac{\mathrm{d}^2T}{\mathrm{d}\tau^2} = \frac{2ZK}{E^3}(E'\tau - E)(E + E' - E'\tau)\,. \tag{28}$$

Note that (28) is non-positive if $a\left(\beta(1-\beta)e^{\frac{L-H}{1+a}}\right) \geq 0$, which is always satisfied.

A.3 Proof of Result 3

The equilibrium income of l-types is

$$y_l^* = w_l(Z - z_l^*)(1-\tau) + T = \frac{Zw_l(1-\tau)+T}{1+a}\,. \tag{29}$$

Taking the derivative with respect to τ we get that $\mathrm{d}y_l^*/\mathrm{d}\tau > 0$ if and only if $\mathrm{d}T/\mathrm{d}\tau > Zw_l$. Moreover, the equilibrium income of h-types is

$$y_h^* = w_h(Z - z_h^*)(1-\tau) + T = \frac{Z w_h (1-\tau)}{1+a} - \frac{a}{1+a}\left[T e^{\frac{L-H}{1+a}} - \left(1 - e^{\frac{L-H}{1+a}}\right)(1-\tau) Z w_l\right] + T \,. \tag{30}$$

Taking the derivative with respect to τ we get that $\mathrm{d}y_h^*/\mathrm{d}\tau > 0$ if and only if

$$w_h Z - \frac{\mathrm{d}T}{\mathrm{d}\tau} < \left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - w_l Z\right) a \left(1 - e^{\frac{L-H}{1+a}}\right) \,. \tag{31}$$

We note that for $\tau = 0$ the above inequality does not hold. Furthermore, since $\mathrm{d}^2T/\mathrm{d}\tau^2 < 0$ (as shown in proof of Result 2), we conclude that inequality (31) never holds for $\tau \in [0, 1]$.

A.4 Proof of Result 4

Functions λ and η are defined as follows:

$$\begin{aligned}
\lambda(H_{y_l} - L_{y_l}, H_{y_h} - L_{y_h}) &= \frac{e^{\frac{L-H}{1+a}}}{(1+a)^3}(H_{y_l} - L_{y_l})\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta - (1+a)\right] + \\
&\quad + \frac{e^{\frac{L-H}{1+a}}}{(1+a)^3}(H_{y_h} - L_{y_h})\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta\right] > 0 \,, \qquad (32) \\
\eta(H_{y_l} - L_{y_l}, H_{y_h} - L_{y_h}) &= \frac{e^{\frac{L-H}{1+a}}}{(1+a)^3}(H_{y_l} - L_{y_l})\left[(1-\beta) - (1+a)\right] + \\
&\quad + \frac{e^{\frac{L-H}{1+a}}}{(1+a)^3}(H_{y_h} - L_{y_h})(1-\beta) > 0 \,. \qquad (33)
\end{aligned}$$

From (32) and (33) we have that λ/η is equal to:

$$\frac{(H_{y_l} - L_{y_l})\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta - (1+a)\right] + (H_{y_h} - L_{y_h})\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta\right]}{(H_{y_l} - L_{y_l})[(1-\beta) - (1+a)] + (H_{y_h} - L_{y_h})(1-\beta)} \,. \tag{34}$$

Dividing both the numerator and the denominator of (34) by $|H_{y_l} - L_{y_l}|$ we get:

$$\frac{-\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta - (1+a)\right] + \frac{H_{y_h} - L_{y_h}}{|H_{y_l} - L_{y_l}|}\left[(1-\beta)a(1 - e^{\frac{L-H}{1+a}}) + \beta\right]}{-[(1-\beta) - (1+a)] + \frac{H_{y_h} - L_{y_h}}{|H_{y_l} - L_{y_l}|}(1-\beta)} \,. \tag{35}$$

First, note that in $\tau = 0$ we have:

$$\begin{aligned}\frac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l} &= \frac{(1-\beta)a(1-e^{\frac{L-H}{1+a}})+\beta}{1+a}\,,\\ \frac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h} &= \frac{(1-\beta)}{1+a}\,.\end{aligned}$$

so that $\mathrm{d}T/\mathrm{d}\tau\mathrm{d}w_l$ is greater, equal, or smaller than $\mathrm{d}T/\mathrm{d}\tau\mathrm{d}w_h$ if and only if $(1-\beta)a(1-e^{\frac{L-H}{1+a}})+\beta$ is greater, equal, or smaller than $(1-\beta)$.

Second, taking the derivative of (35) with respect to $(H_{y_h}-L_{y_h})/(|H_{y_l}-L_{y_l}|)$ we see that it is positive if and only if $(1-\beta)a(1-e^{\frac{L-H}{1+a}})+\beta > 1-\beta$.

Finally, for the case $(1-\beta)a(1-e^{\frac{L-H}{1+a}})+\beta = (1-\beta)$ we have that the first term of the numerator and the first term of the denominator of (35) are identical, so that λ/η is constant in $(H_{y_h}-L_{y_h})/(|H_{y_l}-L_{y_l}|)$.

A.5 Proof of Result 5

From (19) we have that the cardinal status direct effect is negative if and only if $w_l\lambda < w_h\eta$. Since $w_l < w_h$, a sufficient condition for a negative cardinal status direct effects is that:

$$\begin{aligned}&\lambda(H_{y_l}-L_{y_l}, H_{y_h}-L_{y_h}) \leq \eta(H_{y_l}-L_{y_l}, H_{y_h}-L_{y_h}) \Leftrightarrow\\ &\Leftrightarrow (H_{y_l}-L_{y_l})\left[\left((1-\beta)a(1-e^{\frac{L-H}{1+a}})-1\right)+\beta\right] \leq -(H_{y_h}-L_{y_h})\left[\left((1-\beta)a(1-e^{\frac{L-H}{1+a}})-1\right)+\beta\right] \Leftrightarrow\\ &\Leftrightarrow [(H_{y_l}-L_{y_l})+(H_{y_h}-L_{y_h})]\left[\left((1-\beta)a(1-e^{\frac{L-H}{1+a}})-1\right)+\beta\right] \leq 0\,. \qquad (36)\end{aligned}$$

Finally, note that:

$$\frac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_l}-\frac{\mathrm{d}^2T}{\mathrm{d}\tau\mathrm{d}w_h} = \left((1-\beta)a(1-e^{\frac{L-H}{1+a}})-1\right)+\beta\,. \qquad (37)$$

A.6 Proof of Result 6

We take the first derivative of T with respect to τ in the case of cardinal status, and we obtain that

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}\tau} &= \frac{ZK\left[(1-2\tau)E-\tau(1-\tau)E'\right)}{E^2}+\\ &+ \frac{\mathrm{d}(H-L)}{\mathrm{d}\tau}\frac{a}{1+a}\tau(1-\beta)y_l e^{\frac{L-H}{1+a}}\,. \qquad (38)\end{aligned}$$

From the proof of Result 2 we know that the first term of the right hand side is decreasing in τ. Moreover, the second term of the right hand side is equal to 0 at $\tau = 0$. Therefore, if $\mathrm{d}(H - L)/\mathrm{d}\tau \leq 0$ and $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ at $\tau = 0$, then $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ for all $\tau \in [0, 1]$.

A.7 Proof of Proposition 1

Consider a Riley equilibrium. Since types separate, $x_l \neq x_h$. We now prove that this implies that $y_h^* > y_l^*$. If $y_h^* = y_l^*$, then there would be no interest in signalling, and $x_l = 0 = x_h$, against the hypothesis of $x_l \neq x_h$. Suppose then that $y_l^* > y_h^*$. In equilibrium l-type individuals must find it not profitable to deviate from x_l to x_h, therefore

$$\ln\left(\frac{w_l(1-\tau)Z}{1+a} - \frac{a}{1+a}(T - x_l^*) - x_l^* + T\right) + a\ln\left(\frac{a}{1+a}\left(\frac{T - x_l^*}{w_l(1-\tau)} + Z\right)\right) + L \geq$$
$$\geq \ln\left(\frac{w_l(1-\tau)Z}{1+a} - \frac{a}{1+a}(T - x_h^*) - x_h^* + T\right) + a\ln\left(\frac{a}{1+a}\left(\frac{T - x_h^*}{w_l(1-\tau)} + Z\right)\right) + H\,. \quad (39)$$

We will now prove that h-type individuals must strictly prefer choosing x_l^* than x_h^*, and so in Riley equilibrium it cannot be that $y_l^* > y_h^*$. First note that if $x_l^* < x_h^*$ then it is immediate to conclude that h-type individuals strictly prefer x_l^* to x_h^*. Hence, suppose $x_l^* > x_h^*$. We take the derivative with respect of w_l – evaluated at a generic w – of both the left hand side and right hand side of the above inequality, and we easily establish the following inequality:

$$\frac{(1-\tau)Z}{w(1-\tau)Z + T - ax_l} - \frac{a}{w + Zw(1-\tau)} > \frac{(1-\tau)Z}{w(1-\tau)Z + T - ax_h} - \frac{a}{w + Zw(1-\tau)}\,,$$

which implies, together with (39), that h-type individuals strictly gain passing from x_h^* to x_l^*. Therefore, it must be that $y_h^* > y_l^*$. We also observe that $y_h^* - y_l^*$ is bounded away from zero for all values of x_l^* and x_h^* such that $x_h^* \geq x_l^*$.

We now show that a Riley equilibrium exists. Consider a profile of strategies that is parameterized with respect to x_h, i.e., to the signal of h-type individuals. In particular, we set $x_l = 0$, i.e., the low types spend nothing on signalling, and we let $x_h > 0$ free to vary, but with types that separate from each other; suppose also that z_l and z_h are equal to their utility maximizing levels given $x_l = 0$ and x_h. With a slight abuse of notation, we denote these levels as $z_l^*(0, x_h)$ and $z_h^*(0, x_h)$, where the functional form is obtained from equation (4). Pick out-of-equilibrium beliefs such that those individuals who deviate from the level of x prescribed by the profile are believed, with probability 1, to earn an income equal to x – which is the minimum income that allows to buy x – and, hence, get the associated status.

We now argue that we can always find x_h^* such that the profile made of $(z_l^*(0, x_h^*), 0)$ and $(z_h^*(0, x_h^*), x_h^*)$ represents the Riley equilibrium.

Given the selected beliefs, and thanks to the assumption that these beliefs are always able to discourage out-of-equilibrium increases of the signal, we can focus our attention to check that l-types do not find it profitable to imitate the h-types, and viceversa.

Let us start from l-types. We observe that, for x_h that is low enough, l-types finds it profitable to buy the same amount of signal of the rich. At the same time, by the fact that the utility from status is bounded we can conclude the l-types finds it unprofitable to imitate the rich when x_h is large enough. Finally, by exploiting the continuity of the functions involved we can affirm that there exists a minimum level of signal, that we call x_h^*, which makes l-types indifferent between imitating the rich or not. In particular, at that level we have that expression (39) is satisfied as equality.

We now consider h-types. Since $w_h > w_l$, it is easy to check that h-types strictly prefer to spend x_h^* on signalling and be recognized as rich, than save on signalling and be recognized as poor.

We observe that, by construction, the profile that we have just shown to be an equilibrium is the one where expenditure in signaling is minimized and, therefore, it is the Riley equilibrium.

We now turn our attention to the conditions that must hold in the Riley equilibrium. Condition (6) holds by construction. For the other equilibrium conditions, recall that l-types must necessarily be indifferent between imitating h-types and not imitating them, implying that the following must hold:

$$\begin{aligned} &\ln\left(\frac{w_l(1-\tau)Z+T}{1+a}\right)+a\ln\left(\frac{a}{1+a}\left(\frac{T}{w_l(1-\tau)}+Z\right)\right)+L= \\ &=\ln\left(\frac{w_l(1-\tau)Z+T-x_h^*}{1+a}\right)+a\ln\left(\frac{a}{1+a}\left(\frac{T-x_h^*}{w_l(1-\tau)}+Z\right)\right)+H\ . \end{aligned} \tag{40}$$

Thanks to the log-specification, from (40) we can easily derive (7). Inserting (6) and (7) in (4) and exploiting equilibrium conditions, we obtain (8) and (9). Finally, we substitute (8) and (9) into the definition of balanced budget transfer T, and we obtain (10).

A.8 Proof of Proposition 2

We prove that $\sigma_x > \sigma_h$, and then the proposition follows from the inequalities (18), (17) and (14) established in the text. By using (18) and (14), the inequality $\sigma_x > \sigma_h$ can be written, after some simplifications, as

$$\left(\frac{1}{1-\beta}-\left(1-e^{\frac{L-H}{1+a}}\right)\right)>\frac{e^{\frac{L-H}{1+a}}(1-\beta)\left(1-e^{\frac{L-H}{1+a}}\right)}{1-e^{\frac{L-H}{1+a}}(1-\beta)}\,,$$

which gives

$$\frac{\beta}{(1-\beta)\left[1-(1-\beta)e^{\frac{L-H}{1+a}}\right]}>0\,.$$

A.9 Proof of Proposition 3

Consider the sum of the ordinal status effect and the cardinal status direct effect:

$$\Sigma(w_l,w_h)=\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}\left(\frac{w_h}{w_l}-\sigma_x\right)+Z(w_l\lambda-w_h\eta)\,.$$

The sign of $\Sigma(w_l,w_h)$ concords with the sign of the following second degree polynomial in w_l and w_h:

$$Z\lambda w_l^2-\left[\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}\sigma_x+Z\eta w_h\right]w_l+\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}w_h\,. \tag{41}$$

Equating expression (41) to zero we obtain a second degree equation in w_l whose solutions are functions of w_h (and other parameters). Solving for w_l we obtain the following solutions:

$$w_l^-(w_h)=\frac{\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}\sigma_x+Z\eta w_h-\sqrt{\Delta}}{2Z\lambda}\,, \tag{42}$$

$$w_l^+(w_h)=\frac{\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}\sigma_x+Z\eta w_h+\sqrt{\Delta}}{2Z\lambda}\,, \tag{43}$$

$$\Delta=Z^2\eta^2w_h^2+2\left[\sigma_x\eta-2\lambda\right]\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}Zw_h+\left[\left(1-e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{(1+a)^2}\right]^2\sigma_x^2\,.$$

Whenever we have $\Delta>0$, three cases are possible: (a) $\Sigma(w_l,w_h)>0$ for $w_l<w_l^-(w_h)$, (b) $\Sigma(w_l,w_h)\leq 0$ for $w_l^-(w_h)\leq w_l\leq w_l^+(w_h)$, and (c) $\Sigma(w_l,w_h)>0$ for $w_l>w_l^+(w_h)$. Note that, if $w_l^-(w_h)$ exists, then $w_l^-(w_h)>0$, implying that case (a) is feasible. However, since $w_l<w_h$, the feasibility of $w_l>w_l^-(w_h)$ is not warranted, so that case (b) and case (c) are not always feasible.

Whenever we have $\Delta\leq 0$, the only possible case is $\Sigma(w_l,w_h)>0$. Direct calculation shows that the minimum value of Δ as function of w_h is attained for:

$$\tilde{w}_h = 2\left(2\frac{\lambda}{\eta} - \sigma_x\right)\left(1 - e^{\frac{L-H}{1+a}}\right)\frac{(1-\beta)}{Z\eta(1+a)^2}$$

Evaluating Δ at $\tilde{w}_h$ we see, after some algebra, that it is positive if and only if $\sigma_x > \lambda/\eta$, which therefore is a sufficient condition for both $\Delta > 0$ and the existence of at least case (a).

We are now ready to prove claims i), ii), and iii), in turn.

Let $\lambda/\eta \leq 1$. Since $\sigma_x > 1$, we have that $\lambda/\eta < \sigma_x$ which implies that $\Delta > 0$ and therefore at least case (a) exists. Moreover, by Result 4, the cardinal status direct effect is negative for every w_h/w_l. Since the ordinal status effect is negative for $w_l > \sigma_x w_h$, we also have that $\Sigma(w_l, w_h) < 0$ for $w_l > \sigma_x w_h$, implying that case (b) exists. By the same token, case (c) must be impossible.

Let $\sigma_x > \lambda/\eta > 1$. The first inequality implies that $\Delta > 0$ and therefore at least case (a) exists. Moreover, from Result 4 follows that the cardinal status direct effect is negative for every $w_l < w_h\eta/\lambda$. Since the ordinal status effect is negative for $w_l > w_h/\sigma_x$, we have that $\Sigma(w_l, w_h) < 0$ for $w_h/\sigma_x < w_l < w_h\eta/\lambda$, implying that case (b) exists. Case (c) might be feasible or not, so that for $w_l > w_h/\sigma_x$ we can have $\Sigma(w_l, w_h)$ of either sign.

Let $\lambda/\eta > \sigma_x > 1$. The first inequality implies that we can have $\Delta \leq 0$. If this is the case then $\Sigma(w_l, w_h) > 0$. If, instead, $\Delta > 0$ then at least case (a) exists. Again from Result 4, we know that the cardinal status direct effect is positive for $w_l > w_h\eta/\lambda$. Since the ordinal status effect is positive for $w_l < w_h/\sigma_x$, we have that $\Sigma(w_l, w_h) > 0$ for $w_h\eta/\lambda < w_l < \sigma_x w_h$, which can fall in case (a) or case (c). For the range $w_h\eta/\lambda < w_l < \sigma_x w_h$ to fall in case (a) it must hold that $w_h/\sigma_x > w_l^-(w_h)$. Direct calculation shows that $w_h/\sigma_x > w_l^-(w_h)$ if and only if $\lambda/\eta < \sigma_x$, implying that the range $w_h\eta/\lambda < w_l < \sigma_x w_h$ falls in case (c), so that also case (b) is feasible, meaning that we have $\Sigma(w_l, w_h) < 0$ for intermediate values of w_h/w_l.

Finally, note that for any given value of w_l, Z, $H - L$, and a, we can have the cardinal status indirect effect either positive or negative, depending on the value of $H_{y_h} - L_{y_l}$.

A.10 Proof of Proposition 4

We want to show that:

$$H_{y_h} - L_{y_h} > \frac{(1+a)^2}{aw_l Z e^{\frac{L-H}{1+a}}} \;\Leftrightarrow\; \frac{\mathrm{d}\left(y_h - y_l\right)}{\mathrm{d}\tau} > 0 \;\Rightarrow\; \frac{\mathrm{d}x^*}{\mathrm{d}\tau} > 0 \tag{44}$$

From (29) and (30) we get

$$y_h^* - y_l^* = \frac{Z(w_h - w_l)(1-\tau)}{1+a} + \frac{a}{1+a}\left[T\left(1 - e^{\frac{L-H}{1+a}}\right) + \left(1 - e^{\frac{L-H}{1+a}}\right)(1-\tau)Zw_l\right]. \tag{45}$$

Differentiating (45) with respect to τ at $\tau = 0$ we get

$$\frac{\mathrm{d}\left(y_h^* - y_l^*\right)}{\mathrm{d}\tau} = -\frac{Z(w_h - w_l)}{1+a} + \frac{a}{1+a}\left[\frac{\mathrm{d}T}{\mathrm{d}\tau}\left(1 - e^{\frac{L-H}{1+a}}\right) - \left(1 - e^{\frac{L-H}{1+a}} + \frac{\mathrm{d}\left(e^{\frac{L-H}{1+a}}\right)}{\mathrm{d}\tau}\right) Z w_l\right] . \tag{46}$$

Since both L and H depend on $(y_h - y_l)$ we get that $L_{y_h} = -L_{y_l}$ and $H_{y_h} = -H_{y_l}$. This implies that

$$\frac{\mathrm{d}\left(e^{\frac{L-H}{1+a}}\right)}{\mathrm{d}\tau} = \frac{e^{\frac{L-H}{1+a}}}{(1+a)}\left(L_{y_h} - H_{y_h}\right)\frac{\mathrm{d}(y_h - y_l)}{\mathrm{d}y_l} . \tag{47}$$

Plugging (47) in (46) and assuming that the indirect cardinal effect is different from zero, we can solve for $\mathrm{d}(y_h - y_l)/\mathrm{d}\tau$ as follows:

$$\frac{\mathrm{d}\left(y_h^* - y_l^*\right)}{\mathrm{d}\tau} = \frac{(1+a)\left[Z(w_l - w_h) + a\left(1 - e^{\frac{L-H}{1+a}}\right)\left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - Z w_l\right)\right]}{(1+a)^2 + Z a w_l (L_{y_h} - H_{y_h}) e^{\frac{L-H}{1+a}}} . \tag{48}$$

Considering the value of $\mathrm{d}T/\mathrm{d}\tau$ at $\tau = 0$ (see proof of Result 2) it can be shown that the numerator of (48) is negative if and only if

$$w_h\left[\frac{a\left(1 - e^{\frac{L-H}{1+a}}\right)(1-\beta)}{1+a} - 1\right] + \\ + w_l\left[1 - a\left(1 - e^{\frac{L-H}{1+a}}\right) + a\frac{\left(1 - e^{\frac{L-H}{1+a}}\right)\left((1-\beta)a\left(1 - e^{\frac{L-H}{1+a}}\right) + \beta\right)}{1+a}\right] < 0 . \tag{49}$$

The coefficient of w_h is negative while the coefficient of w_l might be either negative or positive. It follows that if (49) holds for $w_h = w_l$ then it holds for any $w_h > w_l$. Imposing $w_h = w_l$ we get that inequality (49) holds if and only if $(1-\beta)\left(1 - e^{\frac{L-H}{1+a}}\right) < 1$ which is always the case. Therefore, the numerator of (48) is negative. From this follows the equivalence result in (44).

The remaining part of the Proposition can be proved by noting that

$$y_h^* - y_l^* = \frac{Z w_h(1-\tau) + T + a x^*}{1+a} - \frac{Z w_l(1-\tau) + T}{1+a} , \tag{50}$$

from which, differentianting with respect to τ at $\tau = 0$, we get

$$\frac{\mathrm{d}\left(y_h^* - y_l^*\right)}{\mathrm{d}\tau} = \frac{Z(w_l - w_h)}{1+a} + \frac{a}{1+a}\frac{\mathrm{d}x^*}{\mathrm{d}\tau} \tag{51}$$

Since the first term of (51) is negative, if expression (51) is positive then it must be that $\mathrm{d}x^*/\mathrm{d}\tau$ is positive.

A.11 Proof of Proposition 5

We want to show that, for any given value of $w_h/w_l > 1$, we can find an array of values for H, L, H_{y_h}, H_{y_l}, L_{y_h}, L_{y_l} such that:

(i) $H > L$, $H_{y_h} \geq 0$, $H_{y_l} \leq 0$, $L_{y_h} \leq 0$, $L_{y_l} \geq 0$;

(ii) $\dfrac{\mathrm{d}x^*}{\mathrm{d}\tau} < 0$ at $\tau = 0$;

(iii) $\dfrac{\mathrm{d}U_l}{\mathrm{d}\tau} > 0$ at $\tau = 0$;

(iv) $\dfrac{\mathrm{d}U_h}{\mathrm{d}\tau} > 0$ at $\tau = 0$.

Fix $w_h/w_l > 1$ and suppose that $H_{y_l} = L_{y_h} = 0$. Then, equation (19), (22) and (23) can be rewritten as, respectively:

$$\left(1 - e^{\frac{L-H}{1+a}}\frac{H_{y_h} a w_l Z}{(1+a)^2}\right)\frac{\mathrm{d}x^*}{\mathrm{d}\tau} = \left(1 - e^{\frac{L-H}{1+a}}\right)\left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - w_l Z\right) + \\ -\frac{w_l Z e^{\frac{L-H}{1+a}}}{(1+a)^2}\left[L_{y_l}\left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - w_l Z\right) - H_{y_h}\left(\frac{\mathrm{d}T}{\mathrm{d}\tau} - w_h Z\right)\right], \tag{52}$$

$$\frac{w_h}{w_l} - \left[1 - a\left(1 - e^{\frac{L-H}{1+a}}\right)\right] + L_{y_l}\frac{\mathrm{d}y_l^*}{\mathrm{d}\tau}\frac{1}{(1-\beta)} > 0\,, \tag{53}$$

$$Z\left[\left((1-\beta)\,a\left(1 - e^{\frac{L-H}{1+a}}\right)e^{\frac{L-H}{1+a}} + \beta\right) + \left(1 - e^{\frac{L-H}{1+a}}\right)\right] - \frac{w_h}{w_l}Z\left(1 - e^{\frac{L-H}{1+a}}(1-\beta)\right) + \\ + L_{y_l}\frac{\mathrm{d}y_l^*}{\mathrm{d}\tau}Z e^{\frac{L-H}{1+a}} + H_{y_h}\frac{\mathrm{d}y_h^*}{\mathrm{d}\tau}\left(\frac{w_h}{w_l} - 1\right)Z > 0\,. \tag{54}$$

Consider then the case where $\mathrm{d}T/\mathrm{d}\tau < w_l Z$ – i.e., the ordinal effect on waste is negative – which implies that $\mathrm{d}y_l/\mathrm{d}\tau < 0$, that $\mathrm{d}T/\mathrm{d}\tau < w_h Z$, and that the first two terms of

the right-hand side of (54) sum up to a positive amount. Then by setting $L_{y_l} = 0$ and $H_{y_h} < (1+a)^2 / \left(Z w_l e^{\frac{L-H}{1+a}}\right)$ we get that inequality (53) and (54) are satisfied and that $\mathrm{d}x^*/\mathrm{d}\tau < 0$. Note that this holds for any value of H and L such that $H > L$.

Consider now the case where $\mathrm{d}T/\mathrm{d}\tau > w_l Z$ – i.e., the ordinal effect on waste is positive – which implies that $\mathrm{d}y_l/\mathrm{d}\tau > 0$ and that the first two terms of the left-hand side of (54) sum up to a negative amount. Then by setting $H_{y_h} = 0$ we get that inequality (53) is satisfied while the negativity of $\mathrm{d}x^*/\mathrm{d}\tau$ and the positivity of the left-hand side of (54) are obtained, respectively, if and only if

$$L_{y_l} > \frac{\left(1 - e^{\frac{L-H}{1+a}}\right)(1+a)^2}{w_l Z e^{\frac{L-H}{1+a}}}\,, \tag{55}$$

$$L_{y_l} > \frac{(1+a)}{w_l Z e^{\frac{L-H}{1+a}}} \frac{w_l\left[\left((1-\beta)\,a\left(1 - e^{\frac{L-H}{1+a}}\right) + \beta\right) + \left(1 - e^{\frac{L-H}{1+a}}\right)\right] - w_h\left(1 - e^{\frac{L-H}{1+a}}\,(1-\beta)\right)}{w_h(1-\beta) - w_l\left(1 + a - (1-\beta)a\left(1 - e^{\frac{L-H}{1+a}}\right) - \beta\right)}\,. \tag{56}$$

For given values of H and L the right-hand sides of (55) and (56) are finite numbers. Therefore, for such values, there exists $L_{y_l} > 0$ such that both (55) and (56) are satisfied.

The proof concludes by noting that for any given array of values for H, L, H_{y_h}, H_{y_l}, L_{y_h}, L_{y_l} such that $H > L$, $H_{y_h} \geq 0$, $H_{y_l} \leq 0$, $L_{y_h} \leq 0$, $L_{y_l} \geq 0$, we can always find differentiable functions $H(y_h, y_l)$ and $L(y_h, y_l)$ that are consistent with such an array.