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Article

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Journal of Risk and Financial Management

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Suggested Citation: Alghalith, Moawia; Lalloob, Ricardo (2012) : A general empirical model of hedging, Journal of Risk and Financial Management, ISSN 1911-8074, MDPI, Basel, Vol. 5, Iss. 1, pp. 1-19,
https://doi.org/10.3390/jrfm5010001

This Version is available at:
https://hdl.handle.net/10419/178533

A general empirical model of hedging

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ABSTRACT

In this paper, we treat output as a decision variable. Moreover, we employ a general form of basis risk. Furthermore, we relax the statistical-independence assumption between the spot price and basis risk.

1. INTRODUCTION

Futures contracts are an important and major financial instrument used by many firms/agents. They are used by firms for both risk minimization and to make speculative profits and hence it is crucial to examine hedging decisions in the futures markets.

There exist a large number of theoretical and empirical studies on the futures markets. Examples of theoretical literature include Paroush and Wolf (1992, 1989), Alghalith et al (2011), Alghalith (2009, 2006) and Mckinnon (1967) among many others. Examples of empirical literature include Paroush and Wolf (1989), Li and Vukina (1993), Alghalith (2008a) and Cita and Lien (1992), among many others.

Much of the previous literature in the futures markets assumed that the output of the firm is a constant parameter. In contrast, this paper treats output as a decision variable. More importantly, the paper employs a general form of basis risk. Furthermore, it also relaxes the statistical-independence assumption between the spot price and basis risk.

2. A REVIEW OF THE LITERATURE

2.1. Theoretical literature

Holthausen (1979) introduced a hedging and production model in the absence of basis risk. He assumed a von Neumann-Morgenstern utility function defined on profit and its goal is to maximize its expected utility of profit. This form of the utility function is used because it conveniently captures the axioms of utility. It is given by the formula:

$$Max_{x.h}\, EU(\pi) = \int_0^\infty U\,[p(x-h) + bh - c(x)]f(p)dp \qquad (1)$$

where U is the utility function, p is the market price, b is the certain forward price, x is the firm's output, h is the amount hedged, c(x) is cost function and f(p) is the firm's subjective probability distribution function. The first-order conditions are:

$$\frac{\partial EU(\pi)}{\partial x} = \int_0^\infty U'(\pi)[p - c'(x)]f(p)dp = 0 \quad (2)$$

$$\frac{\partial EU(\pi)}{\partial h} = \int_0^\infty U'(\pi)[b - p]f(p)dp = 0 \quad (3)$$

Combining both gives:

$$[b - c'(x)] \int_0^\infty U'(\pi)f(p)dp = 0 \quad (4)$$

Thus c'(x) = b as U'(π) is positive for any price. Therefore the firm produces a level of output such that the marginal cost equals the certain forward price. This is known as the separation property (the optimal output does not depend on the probability distributions and preferences; only the amount hedged depends on preferences and distributions). Hence there is separation between the financial decisions and production decisions.

The relationship between the forward price, b and the expected price E(p) can be investigated further by rewriting equation (3):

$$\int_0^\infty U'(\pi)[b - p] = EU'(\pi)E(b - p) + Cov(U'(\pi) - p) = 0 \quad (5)$$

From equation (5) we can obtain the following results. If the forward price equals the expected price the firm will hedge its entire output (h = x). If the forward price is less than the expected price, the firm will either hedge less than its entire output (h<x), or if the forward price is significantly less than the expected price, it will speculate by purchasing output in the forward markets (h < 0). If the forward price is greater than the expected price, the firm will speculate by selling forward an amount greater than its output (h > x).

Holthausen (1979) also shows through the use of the Pratt-Arrow index, defined as $r(\pi) = -U''(\pi)\Big/U'(\pi)$ and where U''(π) is the slope of the marginal utility function, the effects of increasing risk aversion on the amount of output hedged by the firm. He obtains the following

results. If the forward price is less than the expected price, the firm hedges a greater amount of its output as its level of risk aversion increases. If however the forward price is greater than the expected price, the more risk averse the firm becomes the less it will engage in speculation. The results are therefore intuitive as we will expect risk averters to prefer lower certain profits over higher expected profit.

Alghalith (2006) shows the impact of cost uncertainty on hedging. His results show that the amount hedged is greater under cost uncertainty than with cost certainty. He also shows that the difference between output and the amount hedged remains constant when cost uncertainty is introduced if the random spot price and the random input price are independent.

Paroush and Wolf (1989) examined the effect of the basis risk on the firm's optimal output and hedge. They showed that the existence of basis risk reduces the optimal output.

The previous studies that dealt with basis risk, including Alghalith (2009), Li and Vukina (1998), Lence (1995), Paroush and Wolf (1989, 1992), Anderson and Danthine (1983), and Newbery and Stiglitz (1981), did not show the impact of the basis risk on the hedge position of the firm (agent). Also, they did not link the hedge position to the structure of the futures price (unbiased, upward biased, or downward biased). Moreover, the vast majority of these studies relied on restrictive assumptions. For example, Paroush and Wolf (1989, 1992) assumed constant absolute risk aversion and statistical independence between the spot price risk and basis risk. Moreover, the previous studies assumed a specific form of the basis risk.

Alghalith (2010) extended the analysis of basis risk on the hedge position. He found that the presence of basis risk does not change the hedge position of the firm. More specifically he concludes that the agent will over-hedge, under-hedge or full-hedge by the same degree and under the same circumstances as in the absence of basis risk. His results are obtained without assuming independence between price risk and basis risk and without restricting the utility function, the distributions and the form of the basis risk.

In the case where there is uncertainty in both output and price, Alghalith (2008c) shows the effect of the introduction of these on the optimal output and hedge. He shows that the amount hedged by the firm will change in proportion to the change in output. In other words, the

difference between the hedged amount and output under certainty will be maintained with the introduction of output uncertainty.

One model that is of particular interest is the mean-variance model as described by Mckinnon (1967). In general, a firm will engage in hedging for two motives: speculation and risk minimization (pure hedging). The mean variance model provides a framework whereby we can decompose these two components. The firm's profit function is given by:

$$\tilde{\pi} = \tilde{p}y - \left(b - \tilde{f}\right)h \quad (1)$$

Where $\tilde{p}$ is the spot price, y is output and it is assumed fixed and thus there is no cost function, b is the non-random futures price, $\tilde{f}$ is the random futures price and h is the amount hedged. If we divide by y we obtain:

$$\frac{\tilde{\pi}}{y} = \tilde{p} - \left(b - \tilde{f}\right)H \quad (2)$$

where H = h/y is the hedge ratio. The firm maximizes the utility of profit written as:

$$Max_h \left(E\tilde{\pi} - \alpha Var(\tilde{\pi})\right) \quad (3)$$

where $E\tilde{\pi}$ is the expected profit, $Var(\tilde{\pi})$ is the variance of profit and $\alpha > 0$ is a measure of the level of risk aversion. The variance minimization is based on the pure hedge motive, while the expected value maximization represents the speculative motive.

We can divide by y again to get:

$$Max_H \left(\frac{E\tilde{\pi}}{y} - \alpha Var\left(\frac{\tilde{\pi}}{y}\right)\right) = Max_H \left(E\tilde{p} - \left(b - E\tilde{f}\right)H - \propto [\sigma_p^2 + H^2\sigma_f^2 - 2H\sigma_{pf}]\right) \quad (4)$$

where $E\tilde{p}$ is the expected market price and $E\tilde{f}$ is the expected futures price, σ_p^2 is the variance of the market price, σ_f^2 is the variance of the futures price and σ_{pf} is covariance of the spot and futures prices. Solving for H we get:

$$H^* = \frac{\sigma_{pf}}{\sigma_f^2} + \left(b - E\tilde{f}\right)/2\alpha\sigma_f^2 \quad (5)$$

where $\alpha > 0$ is a measure of the level of risk aversion.

The term $\frac{\sigma_{pf}}{\sigma_{\tilde{f}}^2}$ is called the pure hedge ratio and is the component associated with risk minimization. A stronger positive correlation between the futures price and the spot price yields a higher optimal hedge ratio. However a higher variance of the futures price yields a lower optimal hedge ratio. The second term $(b - E\tilde{f})/2\alpha\sigma_f^2$ represents the speculative component of the hedge ratio. Breaking this term down further, we see that the higher the current futures price, the more the firm will speculate. Also, the higher the level of risk aversion of the firm and the variance of the futures price the less the firm speculates.

2.2 Empirical literature

Futures contracts have been in practice, applied to more than just commodities such as gold and gas, they have made their way into a wide number of markets. Futures contracts have been developed for such things as the weather, in the form of the *degree day index* based on temperature which is traded on the Chicago Mercantile Exchange (CME) and for financial assets. Financial futures usually take the form of short-term interest rate futures, long-term interest rate futures and stock index futures and are traded on many exchanges. (Bailey 2005)

Turning to the empirical literature we can examine how the theories of futures contracts have been applied and some of the statistical models that have been used. Lien and Tse (2002) point out that the conventional method of statistically obtaining the optimal hedge ratio, which they call the minimum-variance hedge ratio, involves running an ordinary least squares model of spot prices on futures prices: $p_t = \propto + \beta f_t + \varepsilon_t$, where the estimate of the coefficient β gives the optimal hedge ratio. However, it is noted by Lien and Tse (2002) that the problem with this model "is its dependence on the unconditional second moments, whereas the true minimum-variance hedge ratio is based on conditional second moments."

Bell and Krasker (1986) tried to solve the problem with the conventional model by adjusting the regression coefficients so as to be functions of the information available. However in empirical works the functional forms of these adjusted coefficients are unknown and it is left to the researchers to decide on the model specification (Lien and Tse 2002). Cita and Lien (1992) empirically examined the wheat futures market using this model, "allowing both the intercept

and slope to be linear functions of the historical spot and futures returns."(Lien and Tse 2002) They found this method to be more efficient than the conventional method.

Others have modified the approach, Myers and Thompson (1989) suggested using "a large number of lagged price changes as information variables" (Lien and Tse 2002), in the regression model. Fama and French (1987), however, argued that the basis has predictive power for spot returns and thus advocates the use of this as information variables. Viswanath (1993) also advocates the use of the basis but argued the spot and futures prices converge at the maturity date. Therefore the spot returns would adjust to the basis and thus he takes this into account in his version of the regression model. Castelino (1992) follows a different approach; in his approach "the futures price follows a random walk while the spot price adjusts to the lagged basis." (Lien and Tse 2002)

Empirically, it is common to find that both spot and futures prices data series typically contain a unit root. According to Engle and Granger (1987) the existence of a unit root leads to the possible existence of a cointegration relationship. Lien and Tse (2002) showed the error-correction model based on the spot and futures prices and used them to derive a model that can be estimated using OLS and takes into consideration this cointegrating relationship between the spot and futures prices. The model they derived based on Myers and Thompson (1989) is:

$$\Delta p_t = \propto_p + \lambda \Delta f_t + \sum_{i=1}^{m} \beta_{pi} \Delta p_{t-i} + \sum_{j=1}^{n} \gamma_{pj} \Delta f_{t-j} + \theta_p z_{t-1} + \varepsilon_{pt}$$

where z_{t-1} is a stationary linear combination of p_t and f_t with the differenced values of the spot and futures price being indicative of their returns. The OLS estimate of λ is the estimated minimum-variance hedge ratio. It is noted by (Lien and Tse 2002) that "Chou, Denis and Lee (1996) applied this approach to the Hang Seng Index and found that the hedging performance improved over the conventional OLS method."

Sowell (1992), Cheung and Lai (1993) and Dueker and Startz (1998) considered that there may be a fractional cointegration relationship between two variables of the same order. Lien and Tse (1999) attempted to derive the minimum-variance hedge ratio using this type of approach. In this case z_{t-1} will be integrated of order $d < 1$ while p_t and f_t carry a unit root. Lien and Tse (1999)

applied this approach to the Nikkei Stock Average and found that while the data supports the existence of fractional cointegration, "it does not lead to any improvement in the hedging performance." (Lien and Tse 2002)

Balke and Fomby (1997) considered what is known as a threshold cointegration model. In this model the spot and futures prices adhere to differing cointegrating patterns based on whether the basis is greater than or lower than a certain threshold level. According to Lien and Tse (2002), "Dwyer, Locke and Yu (1996) found that the basis of the S&P 500 can be well described by a threshold autoregressive model. While Gao and Wang (1999) established a similar result for the S&P 500 futures price." However according to Lien and Tse (2002), while the literature on the topic has grown, its' implications "on the minimum-variance hedge ratio and on the hedging performance have not been investigated."

The previous discussion assumes that the optimal hedge is constant over time however; "Bera, Garcia and Roh (1997) considered the hedge ratio to be time varying and more specifically, following a random walk" (Lien and Tse 2002). However, they found that this approach failed to improve the hedge performance because they did not consider that the hedge ratio will vary over time because the conditional moments of the spot and futures returns are varying over time. However, since empirical works suggests the existence of time varying volatility, "most researchers adopt the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) framework"(Lien and Tse 2002).

Bollerslev, Engle and Woodridge (1988) suggested the use of a form of the GARCH known as the general VECH-GARCH, which says that the conditional variance of the spot returns are affected by its own history and the history of the squared innovations in the spot returns. The conditional variance of the futures returns and the covariance of the spot and futures also follow a similar structure. Lien and Tse (2002) noted that this model failed to ensure that the conditional variance-covariance matrix of the spot and futures returns to be positive semi-definite, this shortcoming was also found empirically by Lien and Lou (1994) for the foreign currency markets. Bollerslev (1990) specified a model known as the constant-correlation GARCH (CC-GARCH) to deal with this problem. However as Lien and Tse (2002) noted the "conventional hedge strategies perform as well as or better than the GARCH strategies."

The Stochastic Volatility model is an alternative model specification used to capture time varying second moments (Lien and Tse 2002). This model captures the concept that changes in price are caused by information arrival (Andersen 1996). Heynen and Kat (1994) found that this model outperformed GARCH and exponential GARCH models in forecasting stock indices. Lien (1999) theoretically analyzes the optimal hedge using this model.

It is noted that many of the empirical studies done on futures contracts have dealt mainly with one source of uncertainty. However, there is a limited amount of work that considers multiple sources of uncertainty. This is due to the difficulty in deriving estimating equations that consider multiple sources of risk. Also, the correlations among the different risks are another major problem. The statistical independence assumption is usually used to simplify estimation but in some cases it is unrealistic and thus must be relaxed.

Alghalith (2008d) has addressed these difficulties and provides empirical researchers a way to derive simple estimation procedures with multiples sources of risk and ways to relax the independence assumption. The profit function is:

$$\pi = pq + (g - p)h - c(y, w) \tag{1}$$

Where p is the spot price, q= y+ θη, η is the random component of output and y is the non-random output, h is the hedge, g is the non-random forward price, c(y,w) is the cost function and w is the input factors. The firm maximizes the expected utility of profit with respect to y and h:

$$V(\bar{p}, \sigma, \theta, w, g, B) = EU(p(y^* + \theta\eta) + (g - p)h^* - c(y^*, w) + B) \tag{2}$$

where V is the indirect utility function, y^* and h^* are the optimal values of y and h respectively, $\bar{p}$ is the expected spot price, σ is the standard deviation of the spot price and B is a shift parameter. Alghalith (2005) shows that the final estimating equations are:

$$y^* - h^* = \frac{V_{\bar{p}}(A) + \sum_i V_{\bar{p}i}\widetilde{w}_i + V_{\bar{p}g}\tilde{g} + V_{\bar{p}\bar{p}}\tilde{p} + V_{\bar{p}\sigma}\tilde{\sigma} + V_{\bar{p}\theta}\tilde{\theta} - \beta\theta(\bar{p} + \sigma\delta)}{1 + V_{B\bar{p}}\tilde{p} + \sum_i V_{Bi}\widetilde{w}_i + V_{Bg}\tilde{g} + V_{B\sigma}\tilde{\sigma} + V_{B\theta}\tilde{\theta}} \tag{3}$$

$$h^* = \frac{V_{\bar{p}}(A) + \sum_i V_{gi}\widetilde{w}_i + V_{gg}\tilde{g} + V_{g\bar{p}}\tilde{p} + V_{g\sigma}\tilde{\sigma} + V_{g\theta}\tilde{\theta}}{1 + V_{B\bar{p}}\tilde{p} + \sum_i V_{Bi}\widetilde{w}_i + V_{Bg}\tilde{g} + V_{B\sigma}\tilde{\sigma} + V_{B\theta}\tilde{\theta}} \tag{4}$$

where A = $(\hat{p}, \hat{\sigma}, \hat{\theta}, \hat{w}, \hat{g}, \hat{B})$ and the tildes in the above equation denote deviations from the point of approximation. The equations allow for the computation of important comparative statics with respect to the optimal hedge h^*.

Alghalith (2006) applies this procedure to U.S. corn farmers and showed the following. An increase in the price/output risk reduces the optimal output. Also, an increase in price risk causes an increase in the optimal hedge, with the hedge ratio also increasing. Also an increase in the output riskiness caused the optimal hedge to decrease. An increase in the expected spot price causes the optimal hedge to decrease which results in the hedge ratio falling as output increases. Increases in the forward price also increase the optimal hedge and hedge ratio.

3. THE ESTIMATING EQUATIONS

The risk averse firm maximizes the expected utility of the profit with respect to the hedge:

$$Max_h \, Eu(\tilde{\pi})$$

The profit function is given by:

$$\tilde{\pi} = \tilde{p}q + \left(b - \tilde{f}\right)h,$$

where u is the utility function, q is the quantity of the asset, h is the hedged quantity, $\tilde{p}$ is the random spot (market) price, b is the current non-random futures price and $\tilde{f}$ is the random future futures price.

Given that:

$$\tilde{p} = \bar{p} + \sigma\tilde{\varepsilon}$$

where $\bar{p}$ is the expected spot price and $\sigma\tilde{\varepsilon}$ is the random component of the spot price.

Substituting this into the profit function, we obtain:

$$\tilde{\pi} = \gamma\bar{p}q + \sigma\tilde{\varepsilon}q + \left(b - \tilde{f}\right)h + a$$

where γ is a shift parameter initially equal to 1 and a is also a shift parameter initially equal to 0.

Letting V be indirect utility function, we obtain

$$V(\bar{p}, \sigma, \delta, b, \rho, a, \gamma) = Eu(\tilde{\pi}^*) = Eu\left(\gamma\bar{p}q + \sigma\tilde{\varepsilon}q + \left(b - \tilde{f}(\tilde{\eta})\right)h^* + a\right), \quad (1)$$

where σ is the volatility of the spot price, δ is a measure of basis risk , $\rho = Cov(\tilde{p}, \tilde{f})$ and the superscript * denotes the optimal value.[1] Taking the derivative of both sides of (1) with respect to γ and a, respectively, yields:

$$V_\gamma(.) = \bar{p}qEu'(.),$$

$$V_a(.) = Eu'(.),$$

where the subscripts denote partial derivatives. Thus:

$$q = \frac{V_\gamma(.)}{V_a(.)\bar{p}} \quad (2)$$

we used a second-order Taylor's expansion of the indirect utility around the arbitrary point of expansion $\psi = \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{b}, \hat{a}, \hat{\gamma})(\hat{p},$, then we partially differentiated the expansion with respect to γ and a , respectively, and obtained:

$$V_\gamma(.) \approx V_\gamma(\psi) + V_{\gamma\gamma}(\psi) + V_{\gamma\bar{p}}(\psi)(\bar{p} - \hat{p}) + V_{\gamma\sigma}(\psi)(\sigma - \hat{\sigma}) + V_{\gamma\delta}(\psi)\left(\delta - \hat{\delta}\right) + V_{\gamma\rho}(\psi)(\rho - \hat{\rho}) + V_{\gamma b}(\psi)\left(b - \hat{b}\right)$$

We set γ and a equal to their initial value and we let $V_\gamma(\psi) + V_{\gamma\gamma}(\psi) = V_\alpha(\psi)$. Thus:

$$V_\gamma(.) \approx V_\alpha(\psi) + V_{\gamma\bar{p}}(\psi)(\bar{p} - \hat{p}) + V_{\gamma\sigma}(\psi)(\sigma - \hat{\sigma}) + V_{\gamma\delta}(\psi)\left(\delta - \hat{\delta}\right) + V_{\gamma\rho}(\psi)(\rho - \hat{\rho}) + V_{\gamma b}(\psi)\left(b - \hat{b}\right) \quad (3)$$

$$V_a(.) \approx V_a(\psi) + V_{a\bar{p}}(\psi)(\bar{p} - \hat{p}) + V_{a\sigma}(\psi)(\sigma - \hat{\sigma}) + V_{a\delta}(\psi)\left(\delta - \hat{\delta}\right) + V_{a\rho}(\psi)(\rho - \hat{\rho}) + V_{ab}(\psi)\left(b - \hat{b}\right) \quad (4)$$

Substituting (3) and (4) into (2) and suppressing the notations, we obtain:

$$q = \frac{V_\alpha + V_{\gamma\bar{p}}\breve{p} + V_{\gamma\sigma}\breve{\sigma} + V_{\gamma b}\breve{b} + V_{\gamma\rho}\breve{\rho} + V_{\gamma\delta}\breve{\delta}}{(1 + V_{a\bar{p}}\breve{p} + V_{a\sigma}\breve{\sigma} + V_{ab}\breve{b} + V_{a\rho}\breve{\rho} + V_{a\delta}\breve{\delta})\bar{p}} \quad (5)$$

where the superscripts ˘ denote deviations from the point of approximation. In addition, we normalized $V_a(\psi)$ to one since equation (5) is homogenous of degree zero in all the parameters.

[1] Previous literature assumed the following specification $\tilde{f} = \beta\tilde{p} + \delta\tilde{\eta} = \beta(\bar{p} + \sigma\tilde{\varepsilon}) + \delta\tilde{\eta}, E\tilde{\varepsilon}\tilde{\eta} = 0$, where $\tilde{\eta}$ is basis risk, $\tilde{\varepsilon}$ is price risk, β is a parameter

The partial derivatives in (5) are the parameters to be estimated; while $\check{p}, \check{\sigma}, \check{b}, \check{\rho}, \check{\delta}$ and q are observed data.

Similarly, to derive expression for h* we take the derivative of both sides of (1) with respect to *b* and *a*, respectively, which yields:

$$V_b(.) = h^* Eu'(.),$$
$$V_a(.) = Eu'(.),$$

Thus

$$h^* = \frac{V_b(.)}{V_a(.)} \tag{6}$$

Again, we use a second-order Taylor's expansion of the indirect utility around the arbitrary point of expansion $= (\hat{p}, \hat{\sigma}, \hat{\delta}, \hat{\rho}, \hat{b}, \hat{a})$, then we partially differentiated the expansion with respect to *b* and *a* , respectively, and obtained

$$V_b(.) \approx V_b(\psi) + V_{b\bar{p}}(\psi)(\bar{p} - \hat{p}) + V_{b\sigma}(\psi)(\sigma - \hat{\sigma}) + V_{b\delta}(\psi)\left(\delta - \hat{\delta}\right) + V_{b\rho}(\psi)(\rho - \hat{\rho}) + V_{bb}(\psi)\left(b - \hat{b}\right) \tag{7}$$

$$V_a(.) \approx V_a(\psi) + V_{a\bar{p}}(\psi)(\bar{p} - \hat{p}) + V_{a\sigma}(\psi)(\sigma - \hat{\sigma}) + V_{a\delta}(\psi)\left(\delta - \hat{\delta}\right) + V_{a\rho}(\psi)(\rho - \hat{\rho}) + V_{ab}(\psi)\left(b - \hat{b}\right) \tag{8}$$

Substituting (7) and (8) into (6) and suppressing the notations, we obtain

$$h^* = \frac{V_b + V_{b\bar{p}}\check{p} + V_{b\sigma}\check{\sigma} + V_{bb}\check{b} + V_{b\rho}\check{\rho} + V_{b\delta}\check{\delta}}{1 + V_{a\bar{p}}\check{p} + V_{a\sigma}\check{\sigma} + V_{ab}\check{b} + V_{a\rho}\check{\rho} + V_{a\delta}\check{\delta}} \tag{9}$$

Again we normalized $V_a(\psi)$ to one since equation (9) is homogenous of degree zero in all the parameters.

Equations (5) and (9) provide a convenient way to derive comparative statics results. For example, the impact of a marginal change in expected price on total output is obtained by simply differentiating both sides of (5) with respect to $\bar{p}$:

$$\frac{\partial q}{\partial \bar{p}} = \frac{V_{\gamma\bar{p}} D - (\bar{p} + V_{a\bar{p}}\bar{p})N}{D^2},$$

where D and N are the denominator and the numerator of (5), respectively.

At the point of expansion N = V_α and D = $\bar{p}$; thus

$$\frac{\partial q}{\partial \bar{p}} = \frac{V_{\gamma\bar{p}}\bar{p} - (\bar{p} + V_{a\bar{p}}\bar{p})V_\alpha}{\bar{p}^2}.$$

Similarly,

$$\frac{\partial q}{\partial \sigma} = \frac{V_{\gamma\sigma}\bar{p} - V_{a\sigma}V_\alpha}{\bar{p}^2},$$

$$\frac{\partial q}{\partial b} = \frac{V_{\gamma b}\bar{p} - V_{ab}V_\alpha}{\bar{p}^2},$$

$$\frac{\partial q}{\partial \rho} = \frac{V_{\gamma\rho}\bar{p} - V_{a\rho}V_\alpha}{\bar{p}^2},$$

$$\frac{\partial q}{\partial \delta} = \frac{V_{\gamma\delta}\bar{p} - V_{a\delta}V_\alpha}{\bar{p}^2}.$$

Similarly, if we look at the impact of a marginal change in expected price on the optimal hedge is obtained by simply differentiating both sides of (9) with respect to $\bar{p}$:

$$\frac{\partial h^*}{\partial \bar{p}} = \frac{V_{b\bar{p}}D - V_{a\bar{p}}N}{D^2},$$

where D and N are the denominator and the numerator of (5), respectively.
At the point of expansion N = V_b and D = 1; thus

$$\frac{\partial h^*}{\partial \bar{p}} = V_{b\bar{p}} - V_{a\bar{p}}V_b.$$

Similarly,

$$\frac{\partial h^*}{\partial \sigma} = V_{b\sigma} - V_{a\sigma}V_b,$$

$$\frac{\partial h^*}{\partial b} = V_{bb} - V_{ab}V_b,$$

$$\frac{\partial h^*}{\partial \rho} = V_{b\rho} - V_{a\rho}V_b,$$

$$\frac{\partial h^*}{\partial \delta} = V_{b\delta} - V_{a\delta}V_b.$$

4. THE RESULTS AND CONCLUSION

We implemented the model using data pertaining to the U.S. natural gas futures. The data was taken from the Energy Information Administration (EIA). The data series include the Henry Hub spot price, the Henry Hub futures price and the quantity of natural gas and the hedged quantity. The data is monthly and spans the period March 2000 to March 2010.

We used the methods in Alghalith (2008b) to generate data series for the moments $\bar{p}, \sigma, \delta, \rho$. Similarly, we generated data series for δ using the following formula:

$$\delta_t = \sqrt{\sum_{j=1}^{3} (f_{t-j} - p_{t-j})^2}.$$

We used a non-linear least squares regression to estimate (5) and (9) and estimates of the coefficients were obtained (see tables 1 and 2). The estimated coefficients were then used to compute the comparative statics. The results appear in Tables 4 and 5.

The results are, in general, intuitive. Most evidently, the optimal hedge increases with price risk but decreases with basis risk. Very importantly, these quantitative results can be used by firms and agents in the industry as convenient formulas in adjusting their hedge in response to a change in price/risk.

Table 1. Estimation Results for Output

COEFFICIENTS	VALUE
$V_\alpha(\psi)$	11.8536
$V_{\gamma\bar{p}}$	15.9304
$V_{\gamma\sigma}$	26.6798

$V_{\gamma b}$	108.8839
$V_{\gamma \rho}$	58.7484
$V_{\gamma \delta}$	-17.2698
$V_{a\bar{p}}$	-0.0046
$V_{a\sigma}$	-0.0078
V_{ab}	-0.032
$V_{a\rho}$	-0.0171
$V_{a\delta}$	0.005

Table 2. Estimation Results of Optimal Hedge

COEFFICIENTS	VALUE
$V_a(\psi)$	15007.36
$V_{b\bar{p}}$	7259.104
$V_{b\sigma}$	96295.03

V_{bb}	49363.17
$V_{b\rho}$	-31145.14
$V_{b\delta}$	27600.54
$V_{a\bar{p}}$	-92.7672
$V_{a\sigma}$	0.0316
V_{ab}	13.3465
$V_{a\rho}$	-0.0087
$V_{a\delta}$	0.0142

Table 3. Comparative Statics Results for Output

Comparative static	**Result**
$\frac{\partial q}{\partial \bar{p}}$	295.959
$\frac{\partial q}{\partial \sigma}$	2382.6718
$\frac{\partial q}{\partial b}$	9813.2718

$\frac{\partial q}{\partial \rho}$	5246.5692
$\frac{\partial q}{\partial \delta}$	-1542.2923

Table 4. Comparative Statics Results for Optimal Hedge

Comparative static	Result
$\frac{\partial h}{\partial \bar{p}}$	1,399,449.42
$\frac{\partial h}{\partial \sigma}$	95,821.0676
$\frac{\partial h}{\partial b}$	-150,933.0105
$\frac{\partial h}{\partial \rho}$	-31,014.0057
$\frac{\partial h}{\partial \delta}$	-27387.1504

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