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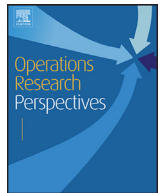
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# Safety stock placement in supply chains with demand forecast updates



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## ABSTRACT

Supply chains are exposed to many types of risks and it may not be obvious where to keep safety stocks in the supply chain to hedge against those risks, while maintaining a high customer service level. In this paper, we develop an approach to determine the safety stock levels in supply chain systems that face demand uncertainty. We model customer demand following the Martingale Model of Forecast Evolution (MMFE). An extensive body of literature discusses the safety stock placement problem in supply chains, but most studies assume independent and identically distributed demand. Our approach is based on a simulation study in which mathematical models are solved in a rolling horizon setting. It allows determining the safety stock levels at each stage of the supply chain. Based on a numerical study, we find that a big portion of the safety stocks should be placed downstream in the supply chain to achieve a high customer service level.

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## 1. Introduction

Many firms and supply chains are under the pressure to offer a high customer service level while operating efficiently with low inventory levels. At the same time, supply chains are exposed to different types of risks, such as uncertain customer demand, uncertain supply, uncertain yields, uncertain lead times, and natural and man-made disasters [17]. Several strategies have been developed to hedge against these risks, such as safety time, safety stocks or a combination of both [4].

The objective of this paper is to present an approach that allows determining safety stocks to hedge against demand uncertainty. Demand uncertainty is the risk factor that is supposed to have the biggest impact on the performance of supply chains [17]. Setting safety stocks along a supply chain has been described by Graves and Willems [7] as a strategic effort in supply chain planning that allows absorbing demand uncertainties and avoiding lost sales and backorders.

The main contributions of this paper are twofold. First, we develop an approach that determines the safety stocks in supply chains by assuming that the demand follows the Martingale Model of Forecast Evolution process. Based on a case study and an extensive simulation study, Heath and Jackson [9] shows that MMFE is a better approach to model the demand evolution. It better re-

fects the demand pattern for many products in a real-life situation, compared to modeling it as independent and identically distributed demand. Our second contribution is that we provide managerial insights about where the safety stocks should be positioned in the supply chain: at downstream stages close to the customer or upstream in the supply chain where the inventory holding cost is lower, but the response time longer.

In our approach, we assume that the supply chain is controlled by a central authority, which has full visibility on the status of the supply chain. Nowadays, many companies have implemented Advanced Planning Systems that allow full visibility of the supply chain and that assist in the coordination and decision making in the supply chain [16]. These systems use mathematical programming models to decide on the optimal quantities to be produced, given several parameters, materials and resources constraints, and the target service level [3].

The remainder of this paper is organized as follow. In the next section, we review the most relevant studies from the literature. The model formulation and the solution approach are presented in Section 3. In Section 4, we present the results of a numerical study, and in Section 5, we draw a few conclusions and present the main managerial insight from this study.

## 2. Literature review

An extensive amount of literature studies supply chains and inventory systems under uncertain demand. We refer the reader for good reviews to Axsater [1], Federgruen [6], Van Houtum et al.

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[19], and Inderfurth [10]. A large number of studies addresses the safety stock placement problem, but we will only discuss the most relevant ones.

Graves and Willems [7] discuss the so-called guaranteed-service model for setting safety stocks in a supply chain under demand uncertainty. They develop a model for determining safety stock levels in a supply chain where each stage is controlled by a basestock policy and under the assumptions that an upper bound exists for the customer demand and infinite capacity constraints. Although the objective of this paper is similar to ours, the modeling approach and assumptions are fundamentally different. Other studies that also assumed uncapacitated supply chains are Simchi-Levi and Zhao [14] and Ettl et al. [5]. Sitompul et al. [15] extend this stream of papers by considering capacity constraints. They find that safety stocks should be increased by a constant correction factor which is dependent on the capacity limitation.

Other relevant studies that used a simulation approach to determine safety stock levels are Jung et al. [11], Jung et al. [12], and Boulaksil et al. [3]. Jung et al. [12] propose a simulation-based approach to determine the safety stocks in a chemical process supply chain. Jung et al., (2008) extend this work by including capacity constraints. Boulaksil et al. [3] develop simulation based optimization approach to determine the safety stocks for a multi-stage supply chain. The safety stock levels were determined based on a simulation approach in which the planning model was solved in a rolling horizon setting.

All these studies have in common that they assume that the customer demand is independent and identically distributed, which may not be a realistic assumption in many business contexts [9]. Evolving forecasts and demand patterns may be more realistic in a real-life setting. A few papers have considered such demand patterns, but with a strong focus on improving the forecasting method or on inventory planning.

Heath and Jackson [9] introduce MMFE as a modeling technique for evolving demand forecasts and compare it with traditional forecasting methods. The authors find that MMFE outperforms the traditional forecasting methods in terms of forecast accuracy and it results in lower total supply chain cost. Güllü [8] studies a two-echelon supply chain that consists of a central depot and multiple retailers, under forecast evolution. He obtains the system-wide order-up-to level and the expected system cost under the forecast evolution model, that he compares with the order-up-to level and the expected system cost under a standard demand model. The standard demand model results in higher order-up-to levels and higher system costs. Similar results have been obtained by Toktay and Wein [18]. Yücer [21] builds on Heath and Jackson's work to model the evolution of forecasts in a two-stage production-distribution system by using stationary, normally distributed demand with an autoregressive order-1 structure (AR-1). Using a series of simulations, the results demonstrate that his production-distribution model yields significantly better results when using MMFE demand forecasts compared to moving average or exponential smoothing.

Many more papers use the MMFE as a demand forecasting model for other purposes (see e.g. [20]). However, none of the papers determined the safety stock levels by using the MMFE demand forecast evolution, which is the objective of this paper.

### 3. Model formulation

We consider a supply chain with several stages, but customer demand can only be satisfied from the most downstream stage and we assume that all unsatisfied demand is backordered. The supply chain is controlled by a centralized planning system, such

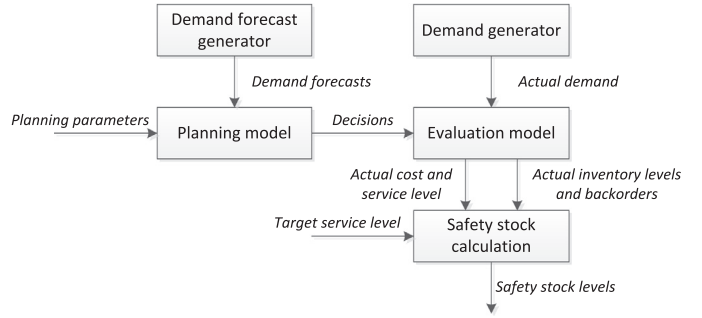


Fig. 1. A schematic overview of the safety stock placement approach.

as an Advanced Planning System that has full visibility and supports the decision making in the supply chain. The planning problem is formulated by mathematical programming principles and assumes a planning horizon of  $T$  discrete time periods. Each time period  $t$ , the planning model is solved, given several input parameters, status information about inventory levels and backorders, and the demand forecasts for each period of the planning horizon. The solution of the planning model for time period  $t=1$  are implemented, which represent production and inventory decisions for the current period. Decisions variables for future time periods represent only planned decisions. After the decisions are made, the actual demand gets revealed based on which the actual cost and customer service level are determined for the current time period. Then, the planning horizon is shifted by one period and a new planning problem arises that needs to be solved following the same approach. Each time when the horizon is shifted, the demand forecasts are updated following the MMFE method. The updates are due to updated information and circumstances. By repeating this cycle very often, we are able to derive the distribution of the inventory levels and backorders, which allows us to determine the required safety stock levels to achieve the target service level.

Our solution approach to determine the safety stock levels consists of a number of steps that are shown in Fig. 1.

#### 3.1. Demand (forecast) generator

In this section, we describe the model that generates the demand forecasts that are input to the planning model. We apply the approach of Heath and Jackson [9] who propose a general technique called the Martingale Model of Forecast Evolution (MMFE). At the beginning of time period  $t$ , the demand forecasts for the coming  $T$  time periods become available, where  $T$  is the planning horizon. Kindly note that we assume that customer demand can only be satisfied from the most downstream stage. Hence, the demand (forecast) generator only generates demand (forecasts) for the most downstream stage.

Let  $\vec{d}_t$  denote the forecast vector  $\vec{d}_t = (d_{t,t+1}, d_{t,t+2}, \dots, d_{t,t+T})$ . In this vector,  $d_{t,t+s}$  denotes the demand forecast made in time period  $t$  for the demand in time period  $t+s$ . We assume that for all the further periods, the demand forecast is equal to the mean demand  $\mu$ . At the end of period  $t$ ,  $D_t$  becomes available, which is the demand realization in time period  $t$ . The demand and demand forecasts evolve from one time period to the next according to an additive evolution model. Given  $\vec{d}_t$ , the forecasts get updated by:

$$\begin{aligned}
 d_{t+1,t+2} &= d_{t,t+2} + \varepsilon_{t+1,t+2} \\
 d_{t+1,t+3} &= d_{t,t+3} + \varepsilon_{t+1,t+3} \\
 &\dots \\
 d_{t+1,t+T+1} &= \mu + \varepsilon_{t+1,t+T+1}
 \end{aligned}$$

For any  $t \geq 1$ ,  $\vec{\varepsilon}_t = (\varepsilon_{t,t+1}, \dots, \varepsilon_{t,t+T})$  is the vector that represents the evolution of demand and forecasts from  $t-1$  to  $t$ . Consequently, demand of period  $t+1$  is found by adding an error term to the most recent forecasts  $d_{t,t+1}$ , i.e.,  $D_{t+1} = d_{t,t+1} + \varepsilon_{t+1,t+1}$ . We assume that  $\{\vec{\varepsilon}_t, t \geq 1\}$  forms a stationary and independent sequence with zero mean and which follows the normal distribution. We refer the reader to Heath and Jackson [9] for a justification of these assumptions. The main idea of this approach is that as we get closer to the demand period  $t$ , by updating the demand forecasts for that period successively (through obtaining new information in each period), we reduce the standard deviation of the forecast errors. Please note that by iterating the evolution equations sufficiently many times, we obtain:

$$\begin{aligned} d_{t,t+k} &= d_{t-1,t+k} + \varepsilon_{t,t+k} = d_{t-2,t+k} + \varepsilon_{t-1,t+k} + \varepsilon_{t,t+k} \\ &= \dots = \mu + \sum_{i=1}^{T-k} \varepsilon_{t-i,t+k} \end{aligned}$$

In many real-life situations, demand forecasts for a certain product are obtained for a number of periods ahead (the planning horizon) by using statistical methods and by considering several factors, such as prices of products from the competitors, and expert judgments [2]. These forecasts are updated from one period to the next period based on new information received by the company.

### 3.2. Planning model

We assume a supply chain with  $J$  stages, where stage  $j=1$  is the most downstream stage. The planning model determines the optimal material and resource quantities to release in the whole supply chain for the planning horizon  $T$  with the objective to minimize the total inventory holding and backorder cost in the supply chain. The demand forecasts are input to the planning model, like other planning parameters, such as the lead times, fixed batch sizes, and capacity levels. We develop the planning model according to the mathematical programming principles to allow the model being implemented in Advanced Planning Systems. We keep the planning model as simple as possible, as our objective is to determine the safety stock levels.

$\alpha^j$	unit inventory holding cost at stage $j$
$\beta^j$	unit backorder cost at stage $j$
$\delta^{j-1,j}$	bill-of-materials factor from stage $j-1$ to stage $j$
$\hat{I}_t^j$	planned inventory level in time period $t$ at stage $j$
$\hat{B}_t^j$	planned backorder quantity in time period $t$ at stage $j$
$P_t^j$	production quantity in time period $t$ at stage $j$
$F_t^j$	frozen production quantity in time period $t$ at stage $j$
$A_t^j$	planned production quantity in time period $t$ at stage $j$
$R_t^j$	replenishment quantity in time period $t$ at stage $j$
$C^j$	capacity level at stage $j$
$\hat{\Pi}$	planned total supply chain cost
$L^j$	Lead time of stage $j$

$$\text{Min } \hat{\Pi} = \sum_{j=1}^J \left( \alpha^j \sum_{t=1}^T \hat{I}_t^j \right) + \sum_{j=1}^J \left( \beta^j \sum_{t=1}^T \hat{B}_t^j \right)$$

s.t.

$$\hat{I}_t^j - \hat{B}_t^j = \hat{I}_{t-1}^j - \hat{B}_{t-1}^j + R_t^j - d_t^j$$

$$R_t^j = P_{t-L^j}^j$$

$$P_t^j = (1 - \gamma)F_t^j + \gamma A_t^j$$

$$P_t^j \leq C^j$$

$$d_t^j = \delta^{j-1,j} \cdot P_t^{j-1}, \quad \forall j \geq 2$$

$$\gamma = \begin{cases} 0 & \text{if } t \leq L \\ 1 & \text{otherwise} \end{cases}$$

$$\hat{I}_t^j, \hat{B}_t^j, A_t^j \geq 0$$

In the planning model, the planning horizon is divided into the *frozen horizon* and the *decision horizon*. In the decision horizon, the planning model decides on  $A_t^j$ , which are production quantities to become available after  $L$  time periods, while the frozen horizon contains  $F_t^j$  that remain unchanged. Once the planning problem is solved and  $A_t^j$  are determined for  $[t+L+1, t+T]$ , the planning horizon is shifted by one period. When shifting the horizon,  $A_{t+L+1}^j$  will become  $F_{t+L}^j$  after shifting the horizon.

### 3.3. Evaluation model

After the planning model is solved, the actual demand  $D_t$  gets revealed, based on which we can determine the actual inventory level, the actual backorder quantity, the actual total supply chain cost  $\Pi$ .

$I_t^j$	actual inventory level in time period $t$ at stage $j$
$B_t^j$	actual backorder quantity in time period $t$ at stage $j$
$D_t^j$	actual demand quantity in time period $t$
$\Pi$	actual total supply chain cost
$SL^j$	actual service level offered at stage $j$

$$I_t^j = (I_{t-1}^j - B_{t-1}^j + R_t^j - D_t^j)^+$$

$$B_t^j = (B_{t-1}^j - I_{t-1}^j - R_t^j + D_t^j)^+$$

$$\Pi = \sum_{j=1}^J \left( \alpha^j \sum_{t=1}^T I_t^j \right) + \sum_{j=1}^J \left( \beta^j \sum_{t=1}^T B_t^j \right)$$

After many simulation runs, we will be able to determine the actual service level offered, which is defined as the fraction of demand satisfied immediately from stock

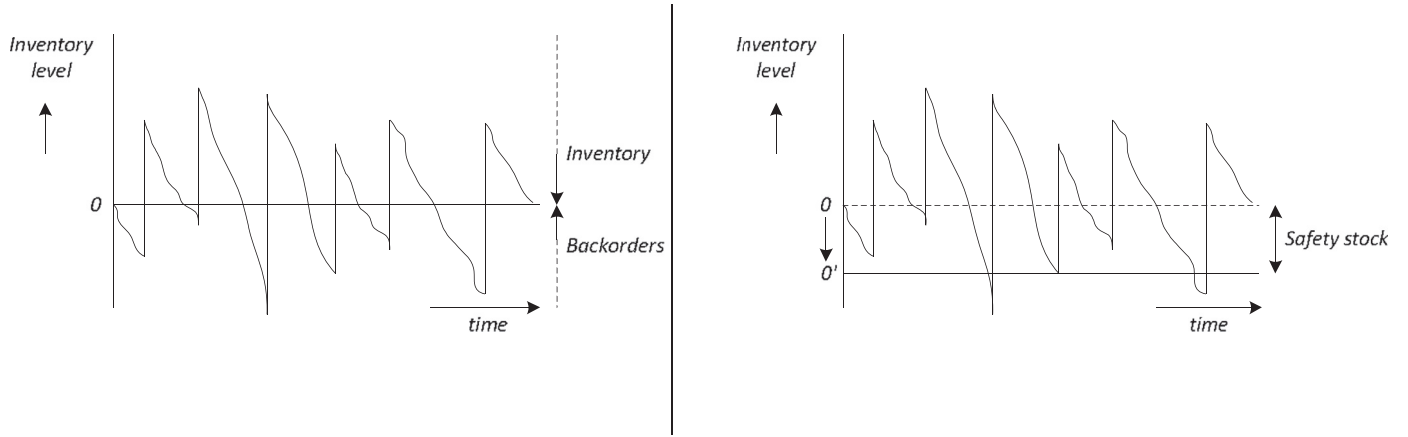
$$SL_j = 1 - \sum_t \frac{B_t^j}{D_t^j}.$$

### 3.4. Safety stock calculation

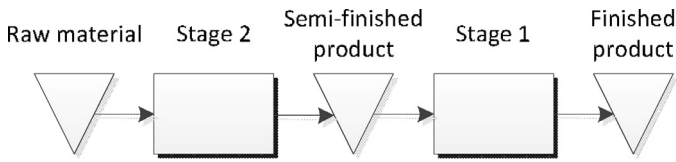
After having determined  $SL_j$ , it might be that it is below the target service level  $SL_j^*$ . In that case, the ratio  $\sum_t \frac{B_t^j}{D_t^j}$  was obviously too high, which means that the amount of backorders was too high. To find the maximum amount of backorders that satisfies the target service level, we adjust the numerator of the function  $SL_j$  by adding an amount to it that is equal to the safety stock  $SS_j$ . The safety stock amount that is added is:

$$SS_j = \begin{cases} \sum_t B_t^j - (1 - SL_j) \sum_t D_t^j & \text{if } SL_j < SL_j^* \\ 0 & \text{otherwise} \end{cases}$$

The idea of the adjustment procedure is shown in Fig. 2. The left-sided figure shows the inventory development without any safety stocks. In that figure, we notice that the target service level may not be achieved due to the large amount of backorders. The right-sided figure in Fig. 2 shows the inventory development after having added safety stocks. This approach is similar to the



**Fig. 2.** The left-sided figure shows the inventory development over time, the right-sided shows how the safety stocks shift the inventory function such that the amount of backorders is decreased.



**Fig. 3.** Simulated two-stage supply chain.

safety stock adjustment procedure as presented by Kohler-Gudum and De Kok [13].

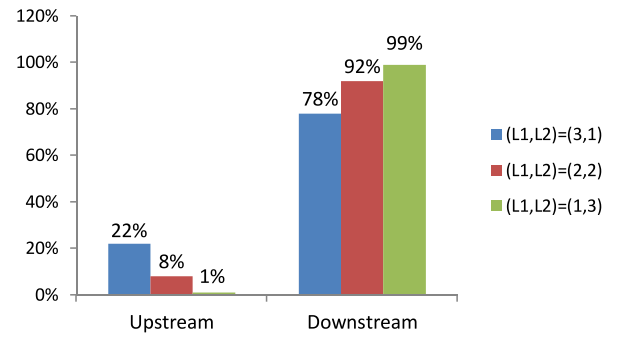
#### 4. Numerical study

In this section, we present the results of a numerical study that we conducted by simulating the approach that is presented in the previous section. The simulation study was meant to test our approach and to provide some managerial insights about the safety stock placement in supply chains.

For the simulation study, we consider a two-stage supply chain, as shown in Fig. 3. We assume that the supply of raw materials is sufficient and we are interested in determining the safety stock levels of the semi-finished products and the finished products. Stages 1 and 2 produce the finished goods and semi-finished goods respectively. We assume that an Advanced Planning and Scheduling system is used to assist the decision making in the supply chain. The simulation study was designed in a way that it reflects many real-life situations much as possible [3].

The simulation procedure is as follows. In each time period  $t$ , the following steps are conducted:

1. The demand forecast generator generates  $\vec{d}_t = (d_{t,t+1}, d_{t,t+2}, \dots, d_{t,t+T})$ . For the first run, we assume that  $\vec{d}_t = \mu$ . In next runs, it gets updated based on the equations presented in Section 3.1.
2. The status information of the supply chain  $(I_{t-1}^1, I_{t-1}^2, B_{t-1}^1, B_{t-1}^2, F_t^1, F_{t+1}^1, \dots, F_{t+L}^1, F_t^2, F_{t+1}^2, \dots, F_{t+T}^2)$  is extracted from the database.
3. The planning model is solved, and the decisions are implemented.
4.  $D_t$  is revealed by the demand generator, and the evaluation model is solved, based on which  $(I_t^1, I_t^2, B_t^1, B_t^2)$  are determined and stored in the database.
5. The planning horizon is shifted by one period, and  $t + 1$  becomes period  $t$ . Go to step 1.



**Fig. 4.** The impact of the lead time structure on the safety stock allocation.

**Table 1**

Values used in the simulation study.

$\sigma_\varepsilon$	{20;100;200}
$\sigma_\varepsilon/\mu$	{0.2;1;2}
$L^j$	{1,2,3}
$\beta^1$	{2,10,50}
$\beta^2$	{1,5,25}

These 5 steps are repeated  $N$  times to determine the distribution of  $I_t^1, B_t^1$  and  $I_t^2, B_t^2$ , based on which  $SS_1$  and  $SS_2$  are determined that allow to achieve  $SL_1^*$  and  $SL_2^*$ . In the simulation study,  $N = 300$  runs, and the number of replications 3. The warm-up period was considered 10 periods. Hence, the results of the first 10 periods were not taken into consideration. All simulation runs were conducted on a desktop computer with a Pentium 4, 3.6 Ghz processor and 2 Gb RAM. For the numerical study, we assume  $\mu = 100$ ,  $\alpha_1 = 1$ , and  $\alpha_2 = 0.5$ , and  $T = L^1 + L^2 + 1$ . The other parameters were varying according to Table 1.

The results of all experiments can be found in Appendix A. From the results, we notice that in all runs, more safety stocks are placed at the downstream stage than at the upstream stage. We find that the two parameters that determine the safety stock allocation in the supply chain are: lead time structure and the level of demand uncertainty. Fig. 4 shows how the lead time structure affects the safety stock allocation in the supply chain. We notice that for all scenarios where  $(L_1, L_2) = (1, 3)$ , the average safety stock placement is 78% at the downstream stage and 22% at the upstream stage. When increasing  $L_1$ , it results in a shift of almost all safety stocks to downstream in the supply chain. This means that if the most downstream stage has a short lead time, some safety



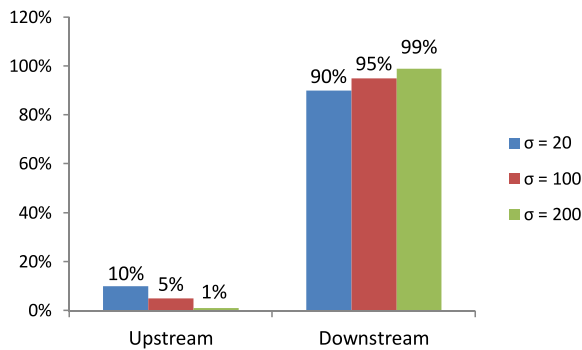


Fig. 5. The impact of the level of demand uncertainty on the safety stock allocation.

stocks can be placed upstream in the supply chain. Otherwise, all safety stocks should be placed at the most downstream stage in the supply chain.

Fig. 5 shows how the level of demand uncertainty affects the safety stock allocation in the supply chain. When demand uncertainty is low ( $\sigma_\epsilon = 20$ ), on average, about 10% of the safety stock is placed upstream in the supply chain. When the level of demand uncertainty increases, this decreases to an average of about 5%.

## 5. Conclusions

In this paper, we presented an approach that optimizes the safety stock placement in a supply chain under demand uncertainty. We modeled the customer demand following the Martingale Model of Forecast Evolution (MMFE) method, which may better reflect the customer demand evolution for many products compared to when assuming it being independently and identically distributed. Our approach is a simulation study in which the supply chain planning model is frequently solved in a rolling horizon setting. We assume that all unsatisfied demand is backordered. By doing so, the distribution of the actual inventory and back-order levels can be derived, which allows the determination of the safety stock levels such that target customer service levels are achieved.

We also conducted a numerical study. We find in our experiments that almost all safety stocks are placed downstream in the supply chain, close to the final customer, especially when the demand uncertainty is high. Safety stocks are only shifted to upstream stages in the supply chain if the downstream stage has a relatively short lead time and customer demand is less uncertain. These insights might be counter intuitive to many managers who believe that safety stocks should be placed upstream in the supply chain, as storing the products at upstream stages is cheaper than at downstream stages.

## Appendix A. Summary of the simulation results

	Input parameters					Stage 1				Stage 2					
	$\sigma_\epsilon$	$L^1$	$L^2$	$\beta^1$	$\beta^2$	$\bar{B}^1$	$\bar{I}^1$	$SL^1$	$SS_1$	$\bar{B}^2$	$\bar{I}^2$	$SL^2$	$SS_2$	$\Pi$	
1	20	1	3	2	1	37	294	0.72	30	27	157	0.81	20	474	
2	20	1	3	10	5	41	231	0.79	31	29	964	0	28	1268	
3	20	1	3	50	25	291	114	0	278	35	1150	0	34	16,146	

(continued on next page)

	Input parameters					Stage 1				Stage 2					
	$\sigma_\epsilon$	$L^1$	$L^2$	$\beta^1$	$\beta^2$	$\bar{B}^1$	$\bar{I}^1$	$SL^1$	$SS_1$	$\bar{B}^2$	$\bar{I}^2$	$SL^2$	$SS_2$	$\Pi$	
4	20	2	2	2	1	186	483	0.84	128	5	10	0.97	0	866	
5	20	2	2	10	5	557	180	0	543	3	145	0.97	0	28,112	
6	20	2	2	50	25	713	63	0	698	6	73	0.97	0	35,888	
7	20	3	1	2	1	34	684	0.60	30	3	23	0.97	0	776	
8	20	3	1	10	5	108	357	0.12	102	5	15	0.96	0	1468	
9	20	3	1	50	25	15	425	0.78	12	1	17	0.99	0	1254	
10	100	1	3	2	1	1133	649	0	1088	98	1582	0.85	65	3805	
11	100	1	3	10	5	945	249	0.26	881	84	4041	0.25	78	12,146	
12	100	1	3	50	25	152	1001	0.59	134	76	3206	0.30	71	10,302	
13	100	2	2	2	1	2006	677	0	1941	4	480	0.99	0	4933	
14	100	2	2	10	5	1779	241	0	1720	8	515	0.99	0	18,332	
15	100	2	2	50	25	2682	524	0	2617	3	2156	0.99	0	135,783	
16	100	3	1	2	1	2689	1839	0	2622	11	20	0.99	0	7237	
17	100	3	1	10	5	95	3589	0.76	75	5	13	0.99	0	4572	
18	100	3	1	50	25	1505	284	0	1463	4	34	0.99	0	75,679	
19	200	1	3	2	1	870	1349	0.31	807	251	2818	0.79	192	4750	
20	200	1	3	10	5	193	2160	0.78	149	45	5516	0.80	34	7077	
21	200	1	3	50	25	1253	699	0.27	1167	207	5297	0.07	196	71,153	
22	200	2	2	2	1	6129	1950	0	5988	5	55	0.99	0	14,241	
23	200	2	2	10	5	5908	621	0	5768	9	1077	0.99	0	60,286	
24	200	2	2	50	25	2307	3157	0	2227	3	1821	0.99	0	119,514	
25	200	3	1	2	1	4936	1531	0	4823	143	59	0.94	30	11,577	
26	200	3	1	10	5	29	6856	0.98	0	27	73	0.99	0	7315	
27	200	3	1	50	25	1903	8782	0.66	1625	15	532	0.99	0	104,873	

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