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Article

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A note on the sojourn time distribution of an M/G/1 queue with a single working vacation and vacation interruption

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ABSTRACT

We study the paper of Gao and Liu (2013). In the paper, the authors pay little attention to the customer's sojourn time in their model although both the queue length and the busy cycle are seriously analyzed. The objective of this note is to derive the sojourn time distribution of Gao and Liu's model without considering Bernoulli schedule. A simple numerical example is also given to demonstrate our result.

1. Introduction

For the past twenty years, a server vacation queue has been widely investigated because it can be characterized by utilizing the idle time of the server to do other work, such as maintenance, machine repair, or just taking a break. In practice, many manufacturing and computing systems are modeled as the server vacation queue [1,2].

In the classical server vacation queue, a server stops working during a vacation. Consider, however, that a system may be equipped with a substitute server which works at a different (probably lower) service rate while the main server leaves for a vacation. Such a system is called a working vacation queue. The concept of a working vacation is firstly introduced by Servi and Finn [3] to analyze a reconfigurable wavelength-division multiplexing optical access network.

The working vacation is typically divided into two policies: multiple working vacations (MWV) and single working vacation (SWV). The MWV policy operates as follows. The main server starts a vacation if the system becomes empty. During a vacation, the substitute server provides low-rate services to customers. If the main server returns from a vacation finding no customers waiting, it takes another vacation. Otherwise, it ends the vacation and changes the service rate to the regular rate. For more details on MWV policy, readers may refer to [4–10].

Meanwhile, under the SWV policy, the server takes only one vacation if the system becomes empty. Thus, if the main server returns from a vacation finding no customers waiting, it waits until a customer arrives without taking another vacation. Otherwise, it changes the service rate to the regular rate. SWV can be regarded as a post-processing time during which the server works at a lower service rate rather than stops working. For example, an agent in a call center is required to do additional tasks after speaking with a customer. The agent may provide service to the next customer at a low rate while performing additional tasks. In another case, suppose a machine with a policy in which it does maintenance after one run of production while functioning at a lower rate. Gao and Liu [11] investigated an M/G/1/SWV queue. Chae et al. [12] discussed a GI/M/1/SWV queue and a discrete-time GI/Geo/1/SWV queue. Banik [13] extended the system to a finite buffer GI/M/1/N/SWV queue. Lately, Xu et al. [14] studied the discrete-time Geo/G/1/SWV queue.

We often encounter the situation that the server can stop the vacation once some indices of the system, such as the number of customers, achieve the certain value during a vacation. In many real life congestion situations, urgent events occur during a vacation and the server must come back to work rather than continuing to take the residual vacation. For example, if the number of customers exceeds the special value during a vacation and the server continues to take the vacation, it leads to large cost of waiting customers. Therefore, vacation interruption is more reasonable to the server vacation queues. Vacation interruption was introduced by Li and Tian [15,16] and it was applied to an M/G/1/MWV queue [17], an M/G/1/MWV queue with retrials [18], a MAP/G/1/MWV queue [19], an M/M/1 queue with balking and impatient customers [20], and a discrete-time GI^X/Geo/1/N queue [21].

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Gao and Liu [22] recently introduced a single working vacation and Bernoulli-schedule vacation interruption policy into an M/G/1 queue. The server enters a single vacation when there are no customers and he/she can take service at a lower rate in the vacation period. On the other hand, the server can stop or continue the vacation according to the following policy: if there are customers in the queue at the instant of a service completion, the server resumes a normal service period with probability p (i.e., the vacation is interrupted) or continues the vacation with probability $1-p$. Gao and Liu [22] obtained the probability generating function (PGF) of the stationary queue length at various epochs and analyzed the busy cycle of the model. However, they paid little attention to the sojourn time distribution in their work. The objective of this note is to present the explicit Laplace–Stieltjes transform (LST) of the sojourn time distribution of Gao and Liu's model [22] with setting $p=1$ (i.e., the vacation is always interrupted). This allows us to analyze the sojourn time distribution more simply and this note can be a reference to derive the sojourn time distribution of an original model.

The remainder of this note is organized as follows. In Section 2, we present some notations and preliminary results. In Section 3, the main result on the sojourn time distribution is derived. Finally, numerical experiments are conducted to investigate the influence of the mean length of a vacation on the mean sojourn time.

2. Preliminaries

Customers arrive at a single server queue, according to a Poisson process at a rate λ. The system under our study is governed by a first-in first-out (FIFO) service discipline. The main server takes only one vacation each time when the system becomes empty and the substitute server provides services at a lower speed to the customers during a vacation. If, at the instant of completing the service, there are customers in the system during a vacation, the system returns to the normal working level (i.e., vacation interruption occurs). Otherwise, the main server continues the vacation. Meanwhile, if there are no customers when a vacation ends, the main server waits until a customer arrives at the system without taking another vacation. Otherwise, the system switches to the normal working level. We assume that the service interrupted at the end of a vacation is lost and it is restarted with a different distribution at the beginning of the following normal service period. Let S_1 denote the service times during the normal service period. S_1 has the probability density function (PDF) $s_1(x)$ and LST $S_1^*(\theta)$. Let S_0 denote the service times during the vacation period. S_0 has the PDF $s_0(x)$ and LST $S_0^*(\theta)$. The length of a vacation, denoted by V, is exponentially distributed with the rate ν. We assume that the interarrival times of customers, service times, and vacation times are mutually independent.

Our system is represented by a Markov process. Let $N(t)$ be the number of customers in the system at t. Let $\xi(t)$ be the system state at t and is defined as follows:

$$\xi(t)=\begin{cases}0, & \text{The system is in a working vacation period at } t,\\ 1, & \text{The system is in a normal service period at } t.\end{cases}$$

$\{N(t),\ \xi(t),\ S_{R,i}(t),\ t\geq 0\}$, for $i\in\{0,\ 1\}$, is then the Markov process, where the supplementary variables $S_{R,i}(t)$ denote the remaining service time at t when $\xi(t)=i$. To establish the system equations we define the following limiting probabilities:

$$P_{n,i}=\lim_{t\to\infty}\Pr\{N(t)=n,\ \xi(t)=i\},\quad n\geq 0,$$

$$P_{n,i}(x)dx=\lim_{t\to\infty}\Pr\{N(t)=n,\ \xi(t)=i,\ x<S_{R,i}(t)<x+dx\},\quad n\geq 1.$$

With the above probabilities, we set up the system equations at the steady state as follows:

$$(\lambda+\nu)P_{0,0}=P_{1,0}(0)+P_{1,1}(0), \tag{1}$$

$$-\frac{d}{dx}P_{1,0}(x)=\lambda P_{0,0}s_0(x)-(\lambda+\nu)P_{1,0}(x), \tag{2}$$

$$-\frac{d}{dx}P_{n,0}(x)=\lambda P_{n-1,0}(x)-(\lambda+\nu)P_{n,0}(x),\quad n\geq 2, \tag{3}$$

$$\lambda P_{0,1}=\nu P_{0,0}, \tag{4}$$

$$-\frac{d}{dx}P_{1,1}(x)=P_{2,0}(0)s_1(x)+\nu P_{1,0}s_1(x)+\lambda P_{0,1}s_1(x)-\lambda P_{1,1}(x)+P_{2,1}(0)s_1(x), \tag{5}$$

$$-\frac{d}{dx}P_{n,1}(x)=P_{n+1,0}(0)s_1(x)+\nu P_{n,0}s_1(x)+\lambda P_{n-1,1}(x)-\lambda P_{n,1}(x)+P_{n+1,1}(0)s_1(x),\quad n\geq 2. \tag{6}$$

From the above equations, we obtain intermediate results which are used to express the sojourn time distribution. Let us define the following LST and joint transform for $i\in\{0,\ 1\}$:

$$P_{n,i}^*(\theta)=\int_0^\infty e^{-\theta x}P_{n,i}(x)dx;\qquad P_i^*(z,\theta)=\sum_{n=1}^\infty P_{n,i}^*(\theta)z^n.$$

Note that $P_i^*(z,\theta)$ is the joint transform of the queue length and the remaining time of an ongoing service at arbitrary epochs. From the PASTA property [23], $P_i^*(z,\theta)$ is stochastically equivalent to the joint transform of the queue length and the remaining time of an ongoing service at arrival epochs. Since the derivation of $P_i^*(z,\theta)$ is a routine, we just present results (see Appendix for its derivation). We have Eqs. (7)–(10) which are given in Box I.

3. Main results: sojourn time distribution

In this section, we express the LST of the FIFO sojourn time (i.e., the waiting time plus the service time) of a test customer (TC) in terms of (7)–(10). A TC's arrival may belong to one of the following cases:

Case 1. A TC arriving during the working vacation period finds the server idle. Then, the TC's service is immediately started at a lower service rate.

Case 2. A TC arriving during the normal service period finds the server idle. Then, the TC's service is immediately started at a normal service rate.

Case 3. A TC arriving during the working vacation period finds that the server is busy.

Case 4. A TC arriving during the normal service period finds that the server is busy.

Let W_i denote the sojourn time of a TC that arrives in *Case* i, $i=1,\ 2,\ 3,\ 4$, and define $W_i^*(\theta)=\Pr\{\text{Case } i\}\,E\left[e^{-\theta W_i}\middle|\ \text{Case } i\right]$.

In *Case* 1, an arriving TC is immediately provided with service at a lower rate. If the vacation service time is shorter than the remaining vacation time, the TC's service will be completed at a lower rate. Otherwise, the working vacation ends and the server must provide the TC with new service at a normal rate. Thus, the TC's sojourn time is the remaining vacation time plus the normal service time. Note that the remaining time of the ongoing

$$P_{0,0} = \frac{\lambda\nu(1-\lambda E[S_1])}{\lambda^2+\lambda\nu+\nu^2-\lambda^2(1+\nu E[S_1])S_0^*(\nu)}, \tag{7}$$

$$P_{0,1} = \frac{\nu^2(1-\lambda E[S_1])}{\lambda^2+\lambda\nu+\nu^2-\lambda^2(1+\nu E[S_1])S_0^*(\nu)}, \tag{8}$$

$$P_0^*(z,\theta) = \frac{\lambda P_{0,0} z\left(S_0^*(\nu+\lambda-\lambda z)-S_0^*(\theta)\right)}{\theta-\nu-\lambda+\lambda z}, \tag{9}$$

$$P_1^*(z,\theta) = \frac{P_{0,0}z(1-z)\left(S_1^*(\lambda-\lambda z)-S_1^*(\theta)\right)\left[\lambda(\lambda+\nu)\left(1-S_0^*(\nu+\lambda-\lambda z)\right)+\nu(\nu+\lambda-\lambda z)\right]}{(\theta-\lambda+\lambda z)(\nu+\lambda-\lambda z)\left(S_1^*(\lambda-\lambda z)-z\right)} \tag{10}$$

Box I.

vacation is stochastically equivalent to a new vacation time due to the memoryless property of the exponential vacation time. Considering this, we have

$$\begin{aligned} W_1^*(\theta) &= P_{0,0}\left(\Pr\{S_0<V\}E\left[e^{-\theta S_0}\middle|S_0<V\right]\right. \\ &\quad \left.+\Pr\{S_0\geq V\}E\left[e^{-\theta(V+S_1)}\middle|S_0\geq V\right]\right) \\ &= P_{0,0}\left(\Pr\{S_0<V\}E\left[e^{-\theta S_0}\middle|S_0<V\right]\right. \\ &\quad \left.+\Pr\{S_0\geq V\}S_1^*(\theta)E\left[e^{-\theta V}\middle|S_0\geq V\right]\right) \\ &= P_{0,0}\left[S_0^*(\theta+\nu)+\frac{\nu S_1^*(\theta)\left(1-S_0^*(\theta+\nu)\right)}{\nu+\theta}\right]. \end{aligned} \tag{11}$$

Since, in *Case* 2, a TC arriving during the normal service period sees no customers, the TC is immediately provided with service at a normal rate. Therefore, we have

$$W_2^*(\theta) = P_{0,1}\times S_1^*(\theta). \tag{12}$$

In *Case* 3, we use the total probability theorem to obtain $W_3^*(\theta)$. If the remaining vacation service time is shorter than the remaining vacation time, after completing a service during the vacation period, vacation interruption occurs and the server must come back to work. Hence, the TC's sojourn time is the sum of the normal service times of the customers who are in the queue, plus the remaining vacation service time of the customer in service. On the other hand, if the remaining vacation service time is longer than the remaining vacation time, the ongoing vacation service is not completed and the server changes the service rate to the normal rate at the instant of the vacation completion. Therefore, the TC's sojourn time is the sum of the normal service times of the customers who are in the system, plus the remaining vacation time. Putting these all together, we have

$$\begin{aligned} W_3^*(\theta) &= \sum_{n=1}^{\infty}\int_{x=0}^{\infty}P_{n,0}(x)\Pr\left\{S_{R,0}<V\middle|S_{R,0}=x\right\} \\ &\quad\times\left(S_1^*(\theta)\right)^n e^{-\theta x}dx+\sum_{n=1}^{\infty}\int_{x=0}^{\infty}P_{n,0}(x) \\ &\quad\times\int_{y=0}^{x}\Pr\left\{V=y\middle|S_{R,0}=x\right\}dy\left(S_1^*(\theta)\right)^{n+1}e^{-\theta y}dx \\ &= \sum_{n=1}^{\infty}\int_{x=0}^{\infty}P_{n,0}(x)\left(S_1^*(\theta)\right)^n e^{-(\theta+\nu)x}dx \\ &\quad+\sum_{n=1}^{\infty}\int_{x=0}^{\infty}P_{n,0}(x)\frac{\nu\left(S_1^*(\theta)\right)^{n+1}}{\theta+\nu}\left(1-e^{-(\theta+\nu)x}\right)dx \\ &= \frac{\theta+\nu-\nu S_1^*(\theta)}{\theta+\nu}\sum_{n=1}^{\infty}P_{n,0}^*(\theta+\nu)\left(S_1^*(\theta)\right)^n \\ &\quad+\frac{\nu S_1^*(\theta)}{\theta+\nu}\sum_{n=1}^{\infty}P_{n,0}\left(S_1^*(\theta)\right)^n \\ &= \frac{\theta+\nu-\nu S_1^*(\theta)}{\theta+\nu}P_0^*\left(S_1^*(\theta),\ \theta+\nu\right) \\ &\quad+\frac{\nu S_1^*(\theta)}{\theta+\nu}P_0^*\left(S_1^*(\theta),0\right). \end{aligned} \tag{13}$$

Finally, in *Case* 4, our system behaves as a standard M/G/1 queue while the server is busy. Hence, the TC's sojourn time is the sum of the normal service times of the customers who are in the queue, plus the remaining normal service time of the customer in service. This leads to

$$\begin{aligned} W_4^*(\theta) &= \sum_{n=1}^{\infty}\int_{x=0}^{\infty}P_{n,1}(x)\left(S_1^*(\theta)\right)^n e^{-\theta x}dx \\ &= \sum_{n=1}^{\infty}P_{n,1}^*(\theta)\left(S_1^*(\theta)\right)^n \\ &= P_1^*\left(S_1^*(\theta),\theta\right). \end{aligned} \tag{14}$$

Let W denote the unconditional sojourn time of a TC. Combining (11)–(14), we have $W^*(\theta)$ given by

$$\begin{aligned} W^*(\theta) &= \frac{W_1^*(\theta)+W_2^*(\theta)+W_3^*(\theta)+W_4^*(\theta)}{W_1^*(0)+W_2^*(0)+W_3^*(0)+W_4^*(0)} \\ &= \frac{P_{0,0}\left\{\lambda(\theta-\lambda)S_0^*(\theta+\nu)\left(\theta+\nu-\nu S_1^*(\theta)\right)+\theta S_1^*(\theta)\left[\lambda^2+\nu(\theta+\lambda+\nu)\right]\right\}}{\lambda(\nu+\theta)\left(\theta-\lambda+\lambda S_1^*(\theta)\right)}. \end{aligned} \tag{15}$$

Remark 1. Now that we are dealing with the Poisson arrival process queuing system, the PASTA property [23] is employed to derive $W_i^*(\theta)$.

4. Numerical examples

In this section, we first present some numerical examples to investigate the influence of the mean vacation time on the mean sojourn time. In all cases, customer arrivals are generated according to a Poisson process at a rate of 0.75. The normal service time distributions are assumed to follow one of the three distributions: exponential in Fig. 1, hyperexponential in Fig. 2, and Erlang in Fig. 3. Specifically, we use *Exp*(1) in Fig. 1. The density function used in Fig. 2 is $p\lambda_1 e^{-\lambda_1 t}+(1-p)\lambda_2 e^{-\lambda_2 t}$, where $p=0.25$, $\lambda_1=1.5$, and $\lambda_2=0.9$. In Fig. 3, we use Erlang (3, 3). That is, the normal service time is the sum of three independent exponential random variables having a common rate 3. On the other hand, we use the exponential distribution for the working vacation service time: we commonly use the *Exp*(0.25) in all the figures.

The vertical axis of each figure represents the mean sojourn time, $E[W]$, and the horizontal axis shows mean vacation time, ν^{-1}.

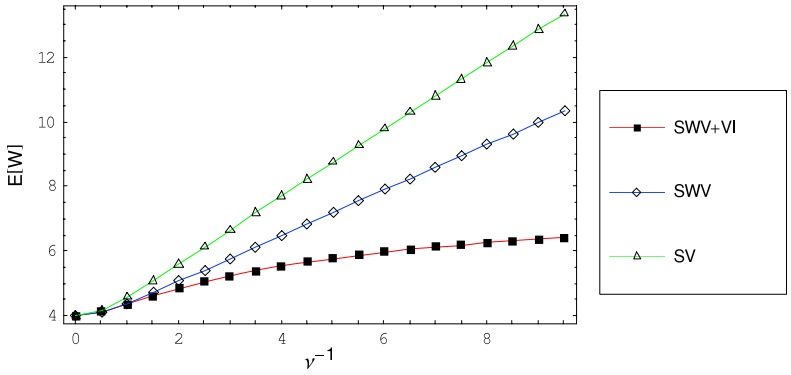

Fig. 1. Mean sojourn time of the M/M/1 over the mean length of a vacation.

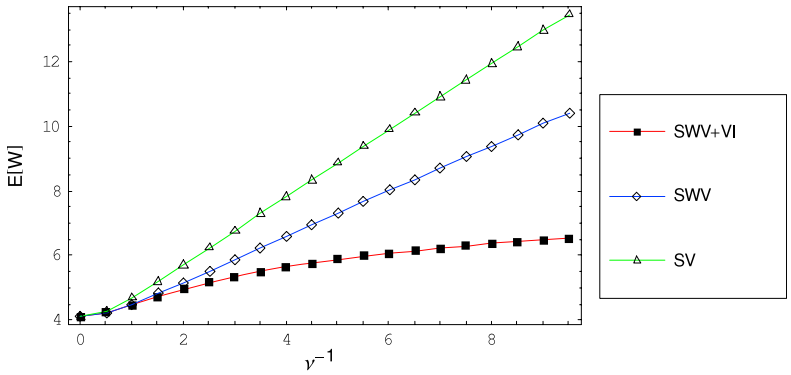

Fig. 2. Mean sojourn time of the $M/H_2/1$ over the mean length of a vacation.

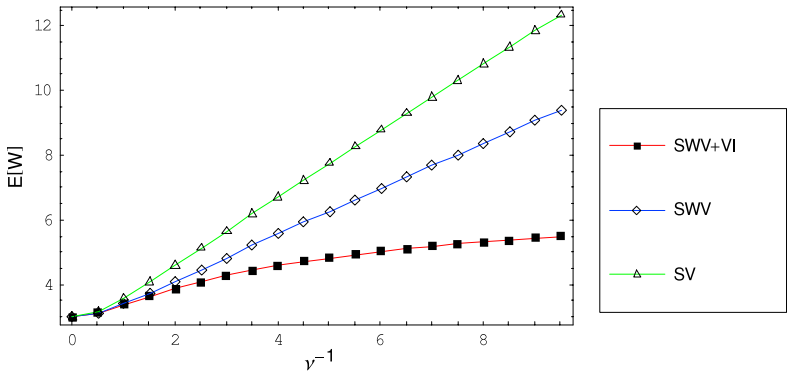

Fig. 3. Mean sojourn time of the $M/E_3/1$ over the mean length of a vacation.

In each figure, we compare the mean sojourn time of three types of vacation policies: the single vacation (SV), the single working vacation (SWV), and the single vacation and vacation interruption (SWV + VI).

As shown in all the figures, $E[W]$ increases with an increase of ν^{-1}, and when ν^{-1} approaches 0, $E[W]$ will arrive at a fixed value, i.e., our system is reduced to the standard M/G/1 queue. Furthermore, as expected, the SWV + VI policy outperforms both the SV policy and the SWV policy. This is explained by the following: the longer the mean vacation time is, the higher the probability that vacation interruption occurs becomes. In other words, a long vacation time leads the server to come back to a normal working level more frequently. As a result, more customers are taken up for service at a normal service rate.

5. Conclusion

In this note, we studied sojourn time distribution of Gao and Liu's queuing model [22] without considering Bernoulli schedule. Using the intermediate results in the process of solving system equations, we easily derive the explicit LST of the sojourn time distribution of our model.

Appendix. Derivation of $P_i^*(z, \theta)$

First we take the LSTs of (2), (3), (5), and (6) in Section 2 and get

$$-\left(\theta P_{1,0}^*(\theta) - P_{1,0}(0)\right) = \lambda P_{0,0} S_0^*(\theta) - (\lambda + \nu) P_{1,0}^*(\theta), \tag{A.1}$$

$$-\left(\theta P_{n,0}^*(\theta) - P_{n,0}(0)\right) = \lambda P_{n-1,0}^*(\theta) - (\lambda + \nu) P_{n,0}^*(\theta), \quad n \geq 2, \tag{A.2}$$

$$-\left(\theta P_{1,1}^*(\theta) - P_{1,1}(0)\right) = P_{2,0}(0) S_1^*(\theta) + \nu P_{1,0} S_1^*(\theta) + \lambda P_{0,1} S_1^*(\theta) - \lambda P_{1,1}^*(\theta) + P_{2,1}(0) S_1^*(\theta), \tag{A.3}$$

$$-\left(\theta P_{n,1}^*(\theta) - P_{n,1}(0)\right) = P_{n+1,0}(0) S_1^*(\theta) + \nu P_{n,0} S_1^*(\theta) + \lambda P_{n-1,1}^*(\theta) - \lambda P_{n,1}^*(\theta) + P_{n+1,1}(0) S_1^*(\theta), \quad n \geq 2. \tag{A.4}$$

$$P_1^*(z,\theta)=\frac{P_{0,0}z(1-z)\left(S_1^*(\lambda-\lambda z)-S_1^*(\theta)\right)\left[\lambda(\lambda+\nu)\left(1-S_0^*(\nu+\lambda-\lambda z)\right)+\nu(\nu+\lambda-\lambda z)\right]}{(\theta-\lambda+\lambda z)(\nu+\lambda-\lambda z)\left(S_1^*(\lambda-\lambda z)-z\right)} \tag{A.10}$$

Box II.

$$\begin{aligned}P(z)&=P_{0,0}+P_{0,1}+P_0^*(z,0)+P_1^*(z,0)\\&=\frac{P_{0,0}(\nu+\lambda)}{\lambda}+\frac{P_{0,0}z\left[\lambda\left(1-S_0^*(\nu+\lambda-\lambda z)\right)\left\{\lambda(1-z)+\nu\left(1-S_1^*(\lambda-\lambda z)\right)\right\}+\nu(\nu+\lambda-\lambda z)\left(1-S_1^*(\lambda-\lambda z)\right)\right]}{\lambda(\nu+\lambda-\lambda z)\left(S_1^*(\lambda-\lambda z)-z\right)}\end{aligned} \tag{A.11}$$

Box III.

Multiplying (A.1) through (A.4) by z^n and then summing over all possible values of n, together with (1) and (4) in Section 2, we gain

$$(\theta-\nu-\lambda+\lambda z)P_0^*(z,\theta)=P_0(z,0)-\lambda P_{0,0}zS_0^*(\theta), \tag{A.5}$$

$$\begin{aligned}&(\theta-\lambda+\lambda z)P_1^*(z,\theta)\\&\quad=z^{-1}P_1(z,0)\left(z-S_1^*(\theta)\right)-S_1^*(\theta)\\&\qquad\times\left[z^{-1}P_0(z,0)+\nu P_{0,0}z+\nu P_0^*(z,0)-(\lambda+\nu)P_{0,0}\right],\end{aligned} \tag{A.6}$$

where $P_i(z,0)=\sum_{n=1}^{\infty}P_{n,i}(0)z^n$ for $i\in\{0,\ 1\}$. Inserting $\theta=\nu+\lambda-\lambda z$ into (A.5) and then simplifying it, we have

$$P_0(z,0)=\lambda P_{0,0}zS_0^*(\nu+\lambda-\lambda z). \tag{A.7}$$

Similarly, substituting $\theta=\lambda-\lambda z$ into (A.6) and simplifying it using (A.7), we obtain

$$P_1(z,0)=\frac{zS_1^*(\lambda-\lambda z)\left[\lambda P_{0,0}\left(1-S_0^*(\nu+\lambda-\lambda z)\right)+\nu P_{0,0}(1-z)-\nu P_0^*(z,0)\right]}{S_1^*(\lambda-\lambda z)-z}. \tag{A.8}$$

Incorporating (A.7) back into (A.5) then yields

$$P_0^*(z,\theta)=\frac{\lambda P_{0,0}z\left(S_0^*(\nu+\lambda-\lambda z)-S_0^*(\theta)\right)}{\theta-\nu-\lambda+\lambda z}. \tag{A.9}$$

With (A.7)–(A.9), (A.6) can be rewritten as Eq. (A.10) which is given in Box II.

Remark A.1. The PGF of the system size at arbitrary epochs, denoted by $P(z)$, is given by Eq. (A.11) in Box III where $P_{0,0}=\frac{\lambda\nu(1-\lambda E[S_1])}{\lambda^2+\lambda\nu+\nu^2-\lambda^2(1+\nu E[S_1])S_0^*(\nu)}$, which can be determined by the normalizing condition, $P_{0,0}+P_{0,1}+P_0^*(1,0)+P_1^*(1,0)=1$.

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