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Volatility Spillovers and Causality of Carbon Emissions, Oil and Coal Spot and Futures for the EU and USA*

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Abstract

Recent research shows that efforts to limit climate change should focus on reducing emissions of carbon dioxide over other greenhouse gases or air pollutants. Many countries are paying substantial attention to carbon emissions to improve air quality and public health. The largest source of carbon emissions from human activities in some countries in Europe and elsewhere is from burning fossil fuels for electricity, heat, and transportation. The price of fuel influences carbon emissions, but the price of carbon emissions can also influence the price of fuel. Owing to the importance of carbon emissions and their connection to fossil fuels, and the possibility of Granger (1980) causality in spot and futures prices, returns and volatility of carbon emissions, it is not surprising that crude oil and coal have recently become a very important research topic. For the USA, daily spot and futures prices are available for crude oil and coal, but there are no daily spot or futures prices for carbon emissions. For the EU, there are no daily spot prices for coal or carbon emissions, but there are daily futures prices for crude oil, coal and carbon emissions. For this reason, daily prices will be used to analyse Granger causality and volatility spillovers in spot and futures prices of carbon emissions, crude oil, and coal. A likelihood ratio test is developed to test the multivariate conditional volatility Diagonal BEKK model, which has valid regularity conditions and asymptotic properties, against the alternative Full BEKK model, which has valid regularity conditions and asymptotic properties under the null hypothesis of zero off-diagonal elements. Dynamic hedging strategies using optimal hedge ratios will be suggested to analyse market fluctuations in the spot and futures returns and volatility of carbon emissions, crude oil and coal prices.

Keywords: Carbon emissions, Fossil fuels, Crude oil, Coal, Low carbon targets, Green energy, Spot and futures prices, Granger causality and volatility spillovers, Likelihood ration test, Diagonal BEKK, Full BEKK, Dynamic hedging.

JEL: C58, L71, O13, P28, Q42.

1. Introduction

Recent research shows that efforts to limit climate change should focus on reducing emissions of carbon dioxide over other greenhouse gases or air pollutants. Many countries are paying substantially greater attention to carbon emissions to improve air quality and public health. Carbon emissions trading programs have been established at the international, regional, national, and sub-national levels (see Figure 1).

[Insert Figure 1 here]

As can be seen from Figure 1, in a scenario of 'no carbon dioxide mitigation', global temperatures would be predicted to rise by over five degrees Celsius by 2100, but cutting emissions of methane, HFCs and black carbon would reduce this rise to around one degree Celsius. The results suggest that carbon dioxide should certainly remain central to greenhouse gas emission cuts.

[Insert Figure 2 here]

Figure 2 shows that projects and regions such as the CDM (Clean Development Mechanism), RGGI (Regional Greenhouse Gas Initiative), and EU, countries like New Zealand, Australia and South Korea, the State of California in the USA, and the Province of Quebec inn Canada, have passed and implemented programs to mitigate carbon emissions.

The programs have operated in phases, with a pilot phase from 2005 to 2007 covering the power sector and certain heavy industries, a second phase from 2008 to 2012 expanding coverage slightly, and a third phase for 2013–2020 that adds a significant range of industrial activities.

The largest source of carbon emissions from human activities in some countries in Europe and elsewhere is from burning fossil fuels for electricity, heat, and transportation. The price of fuel influences carbon emissions, but the price of carbon emissions can also influence the price of fuel.

Owing to the importance of carbon emissions and their connection to fossil fuels, and the possibility of Granger (1980) causality in spot and futures prices, returns and volatility of carbon emissions, it is not surprising that crude oil and coal have recently become a very important public policy issue, and hence also a significant research topic.

The plan of the remainder of the paper is as follows. Section 2 discusses the spot and futures data for carbon emissions, coal and oil that will be used in the empirical analysis for the EU and USA. Section 3 discusses methodological issues, including univariate and multivariate conditional volatility models, Granger causality, volatility spillovers, and optimal hedge ratios. Section 4 analyses causality in returns and volatility, as well as an interesting and novel application of the likelihood ratio test of the Diagonal BEKK model against the alternative of a Full BEKK model. Section 5 provides some concluding remarks.

2. Data

The length of the sample period for the empirical analysis was dictated by the availability of data on carbon, coal and crude oil spot and futures prices in the EU and USA. The carbon emission trading market of the EU has the longest trading period for futures prices, but not for spot prices. The USA is the leader in developing a wide

range of financial derivatives, such as futures prices, for financial, energy and commodities, but not for carbon emissions, where only spot prices are available.

Data for EU carbon emission, crude oil and coal futures are available from 1 April 2008 to 20 May 2017, and these will be analyzed in the paper. Coal spot price in the EU is available on a weekly basis. The spot prices of carbon emission and crude oil have a high correlation with the corresponding futures prices. The volume of trades in the spot market of carbon emissions is much smaller than in the futures market, as shown in Figure 3.

[Insert Figure 3 here]

Data for crude oil are available prior to 2000. However, the data for the spot prices of coal and carbon emission start from 2006/7/17 and 2008/4/1, respectively. Therefore, the data in the empirical analysis for the European Union starts from the latest date for crude oil, coal and carbon emissions, namely 2008/4/1.

Data for carbon, coal and oil spot prices from 2016/1/5 to 2017/5/20 for the USA will also be analyzed in the paper, but data for futures prices of carbon emission are not available for the USA. Spot prices for coal and crude oil start prior to 2000. However, data for carbon emission start from 2016/1/5. Consequently, the spot price data in the empirical analysis for the USA starts from the latest date for oil, coal and carbon emissions, namely 2016/1/5.

The transaction markets and units for the variables are different. EU carbon futures is the Intercontinental Exchange EU allowance, which is traded in the ICE-ICE Futures Europe Commodities market, and is expressed in Euros per metric ton. EU coal futures is ICE Rotterdam Monthly Coal Futures Contract, and is traded in the ICE-

ICE Futures Europe Commodities market. EU oil futures is the current pipeline export quality Brent blend, as supplied at Sullom Voe, is traded in the ICE-ICE Futures Europe Commodities market, and is expressed in USDs per bbl.

Carbon spot prices in the USA is given as the United States Carbon Dioxide RGGI Allowance, and is expressed in USDs per Allowance. Coal spot prices are given as the Dow Jones US Total Market Coal Index, which is expressed in USD. Oil spot returns are given as the West Texas Intermediate Cushing Crude Oil, which is expressed in USDs per bbl. All of the currency units are transformed to USD in the empirical analysis.

The endogenous variables used in the empirical analysis are daily returns, where the rate of return is obtained as the first difference in the natural logarithm of the relevant daily price data. The mnemonics $\mathrm{EUcarbon}_{\mathrm{fr}}$, $\mathrm{EUcoal}_{\mathrm{fr}}$, $\mathrm{EUoil}_{\mathrm{fr}}$ denote, respectively, the future returns of carbon emission, coal and oil in the European Union. Similarly, the mnemonics $\mathrm{UScarbon}_{\mathrm{sr}}$ $\mathrm{UScoal}_{\mathrm{sr}}$, $\mathrm{USoil}_{\mathrm{sr}}$ denote, respectively, the spot returns for carbon emission, coal and oil in the USA.

The variable sources and definitions are given in Table 1, with respect to the futures returns for the EU and spot returns for the USA, as well as their transactions markets, and descriptions of the data.

[Insert Table 1 here]

For the USA, daily spot and futures prices are available for crude oil and coal, but there are no daily spot or futures prices for carbon emissions. For the EU, there are no daily spot prices for coal or carbon emissions, but there are daily futures prices for crude oil, coal and carbon emissions.

For this reason, daily futures prices will be used to analyse Granger causality and volatility spillovers in spot and futures prices of carbon emissions, crude oil, and coal. This will be based on the Lagrange multiplier test of univariate causality in variance (strictly, causality in conditional volatility) of Hafner and Herwartz (2006), and more recently, Chang and McAleer (2017). An extension to multivariate tests of causality in conditional volatility will be a focus of the paper. A likelihood ratio test is developed to test the multivariate conditional volatility Diagonal BEKK model, which has valid regularity conditions and asymptotic properties, against the alternative Full BEKK model, which has valid regularity conditions and asymptotic properties only under the null hypothesis of zero off-diagonal elements. Dynamic hedging strategies using optimal hedge ratios will be suggested to analyse market fluctuations in the spot and futures returns and volatility of carbon emissions, crude oil and coal prices.

The descriptive statistics for the endogenous returns of the variables are given in Table 2. The highest standard deviation for the EU over the sample period is for carbon futures, followed by oil and coal futures. Similarly, the highest standard deviation for the US market is for coal spot returns, followed by carbon emission spot returns.

[Insert Table 2 here]

The returns have different degrees of skewness. The futures and spot returns of oil in the EU and US markets, and coal spot returns in the USA are skewed to the left, indicating that these series have longer left tails (extreme losses) than right tails (extreme gains). However, other returns are all skewed to the right, especially carbon emission spot return in the USA, for which the value of the skewness is high, indicating these series have more extreme gains than extreme losses.

These stylized facts should be of interest to participants in commodity markets. All of the price distributions have kurtosis that is significantly higher than 3, implying that higher probabilities of extreme market movements in either direction (gains or losses) occur in these futures markets, with greater frequency in practice than would be expected under the normal distribution.

In the EU market, the highest kurtosis is for coal futures, followed by carbon futures and oil futures. For the US market, the highest kurtosis is for carbon spot, followed by coal spot. The Jarque-Bera Lagrange multiplier statistic for normality confirm the non-normal distributions for all the returns series.

2. Methodology

Although financial returns are almost certainly stationary, the empirical analysis will commence with tests of unit roots based on ADF, DF-GLS and KPSS. This will be followed by an analysis and estimation of univariate GARCH and multivariate diagonal BEKK models (see Baba et al. (1985), Engle and Kroner (1995)), from which the conditional covariances will be used for testing co-volatility spillovers, that is, Granger causality in conditional volatility.

Despite the empirical applications of a wide range of conditional volatility models in numerous papers in empirical finance, there are theoretical problems associated with virtually all of them. The CCC (Bollerslev (1990)), VARMA-GARCH (Ling and McAlkeer (2003)), and its asymmetric counterpart, VARMA-AGARCH McAleer et al. (2009)), models have static conditional covariances and correlations, which means that accommodating volatility spillovers is not possible.

Apart from the diagonal version, the multivariate BEKK model of conditional covariances has been shown to have no regularity conditions, and hence no statistical properties (see McAleer et al. (2008) and the discussion below for further details). Therefore, spillovers can be considered only for the special case of diagonal BEKK. The multivariate DCC model of (purported) conditional correlations has been shown to have no regularity conditions, and hence no statistical properties (see Hafner and McAleer (2014) for further details).

The analysis of univariate and multivariate conditional volatility models below is a summary of what has been presented in the literature (see, for example, Caporin and McAleer (2012)), although a comprehensive discussion of the Full and Diagonal BEKK models is not available in any published source. In particular, application of the likelihood ratio test of the Diagonal BEKK model as the mull hypothesis against the alternative hypothesis of a Full BEKK model does not seem to have been considered in the literature.

The first step in estimating multivariate models is to obtain the standardized residuals from the conditional mean returns shocks. For this reason, the most widely-used univariate conditional volatility model, namely GARCH, will be presented briefly, followed by the two most widely estimated multivariate conditional covariance models, namely the Diagonal and Full BEKK models.

2.1 Univariate Conditional Volatility

Consider the conditional mean of financial returns, as follows:

$$y_t = E(y_t|I_{t-1}) + \varepsilon_t, \quad (1)$$

where the financial returns, $y_t = \Delta log P_t$, represent the log-difference in financial commodity or agricultural prices, P_t, I_{t-1} is the information set at time *t-1*, and ε_t is a conditionally heteroskedastic error term, or returns shock. In order to derive conditional volatility specifications, it is necessary to specify the stochastic processes underlying the returns shocks, ε_t. The most popular univariate conditional volatility model, GARCH model, is discussed below.

Now consider the random coefficient AR(1) process underlying the return shocks, ε_t:

$$\varepsilon_t = \phi_t \varepsilon_{t-1} + \eta_t \tag{2}$$

where

$\phi_t \sim iid(0, \alpha)$, $\alpha \geq 0$,

$\eta_t \sim iid(0, \omega)$, $\omega \geq 0$,

$\eta_t = \varepsilon_t / \sqrt{h_t}$ is the standardized residual, with h_t defined below.

Tsay (1987) derived the ARCH (1) model of Engle (1982) from equation (2) as:

$$h_t \equiv E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 \tag{3}$$

where h_t represents conditional volatility, and I_{t-1} is the information set available at time *t-1*. A lagged dependent variable, h_{t-1}, is typically added to equation (3) to improve the sample fit:

$$h_t \equiv E(\varepsilon_t^2 | I_{t-1}) = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{4}$$

From the specification of equation (2), it is clear that both ω and α should be positive as they are the unconditional variances of two different stochastic processes.

Given the non-normality of the returns shocks, the Quasi-Maximum Likelihood Estimators (QMLE) of the parameters have been shown to be consistent and asymptotically normal in several papers. For example, Ling and McAleer (2003) showed that the QMLE for a generalized ARCH(*p*,*q*) (or GARCH(*p*,*q*)) is consistent if the second moment is finite. A sufficient condition for the QMLE of GARCH(1,1) in equation (4) to be consistent and asymptotically normal is $\alpha + \beta < 1$.

In general, the proofs of the asymptotic properties follow from the fact that GARCH can be derived from a random coefficient autoregressive process. McAleer et al. (2008) give a general proof of asymptotic normality for multivariate models that are based on proving that the regularity conditions satisfy the conditions given in Jeantheau (1998) for consistency, and the conditions given in Theorem 4.1.3 in Amemiya (1985) for asymptotic normality.

2.2 Multivariate Conditional Volatility

The multivariate extension of the univariate ARCH and GARCH models is given in Baba et al. (1985) and Engle and Kroner (1995). In order to establish volatility spillovers in a multivariate framework, it is useful to define the multivariate extension of the relationship between the returns shocks and the standardized residuals, that is, $\eta_t = \varepsilon_t/\sqrt{h_t}$.

The multivariate extension of equation (1), namely $y_t = E(y_t|I_{t-1}) + \varepsilon_t$, can remain unchanged by assuming that the three components are now $m \times 1$ vectors, where m is the number of financial assets. The multivariate definition of the relationship between ε_t and η_t is given as:

$$\varepsilon_t = D_t^{1/2}\eta_t, \tag{5}$$

where $D_t = diag(h_{1t}, h_{2t}, \dots, h_{mt})$ is a diagonal matrix comprising the univariate conditional volatilities.

Define the conditional covariance matrix of ε_t as Q_t. As the $m \times 1$ vector, η_t, is assumed to be *iid* for all m elements, the conditional correlation matrix of ε_t, which is equivalent to the conditional correlation matrix of η_t, is given by Γ_t. Therefore, the conditional expectation of (5) is defined as:

$$Q_t = D_t^{1/2}\Gamma_t D_t^{1/2}. \tag{6}$$

Equivalently, the conditional correlation matrix, Γ_t, can be defined as:

$$\Gamma_t = D_t^{-1/2}Q_t D_t^{-1/2}. \tag{7}$$

Equation (6) is useful if a model of Γ_t is available for purposes of estimating Q_t, whereas (7) is useful if a model of Q_t is available for purposes of estimating Γ_t.

Equation (6) is convenient for a discussion of volatility spillover effects, while both equations (6) and (7) are instructive for a discussion of asymptotic properties. As the elements of D_t are consistent and asymptotically normal, the consistency of Q_t in (6) depends on consistent estimation of Γ_t, whereas the consistency of Γ_t in (7) depends on consistent estimation of Q_t. As both Q_t and Γ_t are products of matrices, with inverses in (7), neither the QMLE of Q_t nor Γ_t will be asymptotically normal based on the definitions given in equations (6) and (7).

2.3 Diagonal BEKK

The Diagonal BEKK model can be derived from a vector random coefficient autoregressive process of order one, which is the multivariate extension of the univariate process given in equation (2):

$$\varepsilon_t = \Phi_t \varepsilon_{t-1} + \eta_t, \tag{8}$$

where

ε_t and η_t are $m \times 1$ vectors,

Φ_t is an $m \times m$ matrix of random coefficients,

$\Phi_t \sim iid(0, A)$, A is positive definite,

$\eta_t \sim iid(0, C)$, C is an $m \times m$ matrix.

Vectorization of a full matrix A to *vec* A can have dimension as high as $m^2 \times m^2$, whereas vectorization of a symmetric matrix A to *vech* A can have a smaller dimension of $m(m+1)/2 \times m(m+1)/2$.

In a case where A is a diagonal matrix, with $a_{ii} > 0$ for all $i = 1,\ldots,m$ and $|b_{jj}| < 1$ for all $j = 1,\ldots,m$, so that A has dimension $m \times m$, McAleer et al. (2008) showed that the multivariate extension of GARCH(1,1) from equation (8) is given as the Diagonal BEKK model, namely:

$$Q_t = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B', \tag{9}$$

where A and B are both diagonal matrices, though the last term in equation (9) need not come from an underlying stochastic process. The diagonality of the positive definite matrix A is essential for matrix multiplication as $\varepsilon_{t-1}\varepsilon'_{t-1}$ is an $m \times m$

matrix; otherwise equation (9) could not be derived from the vector random coefficient autoregressive process in equation (8).

2.4 Full, Triangular and Hadamard BEKK

The full BEKK model in Baba et al. (1985) and Engle and Kroner (1995), who do not derive the model from an underlying stochastic process, is presented as:

$$Q_t = CC' + A\varepsilon_{t-1}\varepsilon'_{t-1}A' + BQ_{t-1}B', \tag{10}$$

except that *A* and (possibly) *B* in equation (10) are now both full matrices, rather than the diagonal matrices that were derived in equation (9) using the stochastic process in equation (8). The full BEKK model can be replaced by the triangular or Hadamard (element-by-element multiplication) BEKK models, with similar problems of identification and (lack of) existence.

A fundamental technical problem is that the full, triangular and Hadamard BEKK models cannot be derived from any known underlying stochastic processes, which means there are no regularity conditions (except by assumption) for checking the internal consistency of the alternative models, and consequently no valid asymptotic properties of the QMLE of the associated parameters (except by assumption).

Moreover, as the number of parameters in a full BEKK model can be as much as *3m(m+1)/2,* the "curse of dimensionality" will be likely to arise, which means that convergence of the estimation algorithm can become problematic and less reliable when there is a large number of parameters to be estimated.

As a matter of empirical fact, estimation of the full BEKK can be problematic even when *m* is as low as 5 financial assets. Such computational difficulties do not arise for the Diagonal BEKK model. Convergence of the estimation algorithm is more likely when the number of commodities is less than 4, though this is nevertheless problematic in terms of interpretation.

Therefore, in the empirical analysis, in order to investigate volatility spillover effects, the solution is to use the Diagonal BEKK model for estimation. A likelihood ratio test is developed to test the multivariate conditional volatility Diagonal BEKK model in equation (9) (where *A* and *B* are both diagonal matrices), which has valid regularity conditions and asymptotic properties, against the alternative Full BEKK model in equation (10) (where *A* and *B* in are now both full matrices), which has valid regularity conditions and asymptotic properties only under the null hypothesis of zero off-diagonal elements. The likelihood ratio test of the null Diagonal BEKK model against the alternative of the Full BEKK model does not yet seem to have been presented in the literature.

2.5 Granger Causality, Volatility Spillovers, and Optimal Hedge Ratios

McAleer et al. (2008) showed that the QMLE of the parameters of the Diagonal BEKK model were consistent and asymptotically normal, so that standard statistical inference on testing hypotheses is valid. Moreover, as Q_t in (9) can be estimated consistently, Γ_t in equation (7) can also be estimated consistently.

The Diagonal BEKK model is given as equation (9), where the matrices *A* and *B* are given as:

$$A = \begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_{mm} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & b_{mm} \end{bmatrix} \tag{11}$$

The Diagonal BEKK model permits a test of Co-volatility Spillover effects, which is the effect of a shock in commodity *j* at *t*-1 on the subsequent co-volatility between *j* and another commodity at *t*. Given the Diagonal BEKK model, as expressed in equations (9) and (10), the subsequent co-volatility must only be between commodities *j* and *i* at time *t*.

Chang et al. (2015) define Full and Partial Volatility and Covolatility Spillovers in the context of Diagonal and Full BEKK models. Volatility spillovers are defined as the delayed effect of a returns shock in one asset on the subsequent volatility or covolatility in another asset. Therefore, a model relating Q_t to returns shocks is essential, and this will be addressed in the following sub-section. Spillovers can be defined in terms of full volatility spillovers and full covolatility spillovers, as well as partial covolatility spillovers, as follows, for $i, j, k = 1,...,m$:

(1) **Full volatility spillovers:** $\partial Q_{iit} / \partial \varepsilon_{kt-1},\ k \neq i$; (12)

(2) **Full covolatility spillovers:** $\partial Q_{ijt} / \partial \varepsilon_{kt-1},\ i \neq j,\ k \neq i, j$; (13)

(3) **Partial covolatility spillovers:** $\partial Q_{ijt} / \partial \varepsilon_{kt-1},\ i \neq j,\ k = either\ i\ or\ j$. (14)

Full volatility spillovers occur when the returns shock from financial asset *k* affects the volatility of a different financial asset *i*.

Full covolatility spillovers occur when the returns shock from financial asset *k* affects the covolatility between two different financial assets, *i* and *j*.

Partial covolatility spillovers occur when the returns shock from financial asset *k* affects the covolatility between two financial assets, *i* and *j*, one of which can be asset *k*

When $m = 2$, only spillovers (1) and (3) are possible as full covolatility spillovers depend on the existence of a third financial asset.

This leads to the definition of a Co-volatility Spillover Effect as:

$$\frac{\partial H_{ij,t}}{\partial \varepsilon_{j,t-1}} = a_{ii} \times a_{jj} \times \varepsilon_{i,t-1},\ i \neq j.$$

As $a_{ii} > 0$ for all i, a test of the co-volatility spillover effect is given as a test of the null hypothesis:

$$H_0 : a_{ii} a_{jj} = 0,$$

which is a test of the significance of the estimate of $a_{ii} a_{jj}$ in the following co-volatility spillover effect, as $\varepsilon_{i,t-1} \neq 0$:

$$\frac{\partial H_{ij,t}}{\partial \varepsilon_{j,t-1}} = a_{ii} a_{jj} \varepsilon_{i,t-1},\ i \neq j.$$

If H_0 is rejected against the alternative hypothesis, $H_1 : a_{ii} a_{jj} \neq 0$, there is a spillover from the returns shock of commodity *j* at *t*-1 to the co-volatility between commodities *i* and *j* at *t* that depends only on the returns shock of commodity *i* at *t*-1. It should be

emphasized that the returns shock of commodity *j* at *t*-1 does not affect the co-volatility spillover of commodity *j* on the co-volatility between commodities *i* and *j* at *t*. Moreover, spillovers can and do vary for each observation *t*-1, so that the empirical results average co-volatility spillovers will be presented, based on the average return shocks over the sample period.

Granger (1980) causality is based on the following vector AR (VAR) models:

$$x(t) = a_0 + a_1 x(t-1) + \cdots + a_m x(t-m) + b_1 y(t-1) + \cdots + b_n y(t-n) + u(t), \quad (15)$$

$$y(t) = c_0 + c_1 y(t-1) + \cdots + c_n y(t-n) + d_1 x(t-1) + \cdots + d_m x(t-m) + v(t)\,. \quad (16)$$

The null hypothesis of Granger non-causality of $y(t-1)$ on $x(t)$ is based on testing:

$$H_0:\ b_i = 0 \text{ for all } i=1,\cdots,n$$

in equation (12), while the null hypothesis of Granger non-causality of $x(t)$ on $y(t-1)$ is based on testing:

$$H_0:\ d_i = 0 \text{ for all } i=1,\cdots,m$$

in equation (13).

For the multivariate conditional mean returns equation:

$$y_{it} = E(y_{it}|I_{t-1}) + \varepsilon_{it}\ ,\ i = 1,2,\cdots,m, \quad (17)$$

the bivariate random coefficient autoregressive process for ε_{it} is given as:

$$\varepsilon_{it} = \phi_{it}\varepsilon_{it-1} + \phi_{jt}\varepsilon_{jt-1} + \eta_{it},\ i \neq j, \tag{18}$$

where

$\phi_{it} \sim iid(0, \alpha_i),\ \alpha_i \geq 0,$

$\phi_{jt} \sim iid(0, \alpha_j),\ \alpha_j \geq 0,$

$\eta_{it} \sim iid(0, \omega_i),\ \omega_i \geq 0\ ,$

$\eta_{it} = \varepsilon_{it}/\sqrt{h_{it}}$ is the standardized residual,

h_{it} is the conditional volatility obtained by setting $\phi_{jt} = 0$ in bivariate equation (15):

$$\varepsilon_{it} = \phi_{it}\varepsilon_{it-1} + \eta_{it},$$

$$E(\varepsilon_{it}^2 | I_{t-1}) \equiv h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2.$$

Adding another commodity, as in the bivariate equation (15), gives:

$$\varepsilon_{it} = \phi_{it}\varepsilon_{it-1} + \phi_{jt}\varepsilon_{jt-1} + \eta_{it},\ i \neq j,$$

$$E(\varepsilon_{it}^2 | I_{t-1}) \equiv h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \alpha_j \varepsilon_{jt-1}^2,$$

while adding first-order lags of h_{it} and h_{jt} gives:

$$h_{it} = \omega_i + \alpha_i \varepsilon_{it-1}^2 + \alpha_j \varepsilon_{jt-1}^2 + \beta_i h_{it-1} + \beta_j h_{jt-1},$$

where

$$\alpha_i \geq 0, \alpha_j \geq 0, \beta_i \in (-1, 1), \beta_j \in (-1, 1).$$

The null hypothesis of non-causality in volatility is given as a test of:

$$H_0: \alpha_j = \beta_j = 0.$$

Based on the empirical results, dynamic hedging strategies using optimal hedge ratios will be suggested to analyse market fluctuations in the spot and futures returns and volatility of carbon emissions, crude oil and coal prices.

Using the hedge ratio: $R_{H,t} = R_{S,t} - \gamma_t R_{F,t}$ and its variance, namely:

$$var(R_{H,t}|\Omega_{t-1}) = var(R_{S,t}|\Omega_{t-1}) - 2\gamma_t cov(R_{S,t}, R_{F,t}|\Omega_{t-1}) + \gamma_t^2 \, var(R_{F,t}|\Omega_{t-1}),$$

the optimal hedge ratio is given as:

$$\gamma_t|\Omega_{t-1} = cov(R_{S,t}, R_{F,t}|\Omega_{t-1})/var(R_{F,t}|\Omega_{t-1}).$$

An extension of the recent research on realized matrix-exponential stochastic volatility with asymmetry, long memory and spillovers, in Asai, Chang and McAleer (2017), to multivariate conditional volatility models, especially the use of the matrix-exponential transformation to ensure a positive definite covariance matrix, will enable a significant extension of the univariate Granger causality tests to be extended to multivariate Granger causality tests. This would be a novel extension of the paper.

3. Unit Root Tests

In order to evaluate the characteristics of the data, we investigate whether shocks to a series are temporary or permanent in nature. We will use the ADF test (Dickey and Fuller, 1979, 1982; Said and Dickey, 1984), DF-GLS test (Elliott et al., 1996), and KPSS test (Kwiatkowski et al., 1992) to test for unit roots in the individual returns series. The ADF and DF-GLS tests are designed to test for the null hypothesis of a unit root, while the KPSS test is used for the null hypothesis of stationarity.

In Table 3, based on the ADF test results, the large negative values in all cases indicate rejection of the null hypothesis of unit roots at the 1% level. Based on the KPSS test, the small positive values in all cases do not reject the null hypothesis of stationary at the 1% level. For the DF-GLS test, the futures returns of carbon emissions and of coal in the EU, and the spot returns of carbon emissions in the USA, reject the null hypothesis of unit roots at the 1% level. However, the results of the coal and oil spot returns do not reject the null hypothesis. It should be noted that, for the USA, a relatively small sample size of 310 observations is used.

[Insert Table 3 here]

4. Granger Causality and Spillovers in Returns and Volatilities

Table 4 reports the results for the Granger (1980) causality and spillover tests in returns and volatilities. There is no evidence of bidirectional Granger causality between carbon and coal futures for the EU. However, oil futures in the EU has a causal effect on carbon emissions futures in the EU. For the USA, carbon emissions spot has a causal effect on coal spot as well as on oil spot.

[Insert Table 4 here]

Estimates of the DBEKK and Full BEKK models for EU Carbon, Coal, and Oil Futures returns are given in Table 5. The estimates of the weighting coefficients, A(1,1), are similar for the two models, but the estimates of the weighting coefficients A(2,2) and A(3,3) are different for the two models. Similar comments apply to the estimates of the matrix stability coefficients, B(1,1), B(2,2), and B(3,43), respectively.

Given the differences in two of the three weighting coefficients in A in Table 5, it is not particularly surprising that the likelihood ratio test in Table 6 of the null hypothesis, DBEKK, against the alternative hypothesis, Full BEKK, leads to rejection of the null hypothesis that the off-diagonal elements of A are zero. The calculated chi-squared statistic with 6 degrees of freedom, at 34.42, is greater than the critical value of 16.81 at the 1% level. Therefore, DNBEKK is rejected, but Full BEKK is not appropriate as it is valid only under the null hypothesis of zero off-diagonal coefficients for the weighting matrix A. In short, the Diagonal BEKK model is rejected, but the full BEKK model is not an appropriate replacement.

[Insert Tables 5 and 6 here]

Estimates of the DBEKK and Full BEKK models for US Carbon, Coal, and Oil Spot returns are given in Table 7. The estimates of the three weighting coefficients, A(1,1), A(2,2) and A(3,3), are reasonably similar for the two models, as are the estimates of the stability coefficients B(1,1) and B(2,2), though the estimates of B(3,3) are different for the two models.

In view of the similarities in the estimates of the three weighting coefficients in A in Table 7, the likelihood ratio test in Table 8 of the null hypothesis, DBEKK, against the alternative hypothesis, Full BEKK, leads to a less strong rejection of the null

hypothesis that the off-diagonal elements of A are zero, as compared with the outcome in Table 6. The calculated chi-squared statistic with 6 degrees of freedom, at 22.18, is greater than the critical value of 16.81 at the 1% level. Therefore, DNBEKK is rejected but, as in the previous case, Full BEKK is not appropriate as it is valid only under the null hypothesis of zero off-diagonal coefficients for the weighting matrix A. In short, the Diagonal BEKK model is rejected, but the full BEKK model is not an appropriate replacement.

[Insert Tables 7 and 8 here]

In light of the discussion based on equations (14), partial co-volatility spillovers with DBEKK are presented in Table 9. Based on the estimates of the weighting matrix A, 6 of the 8 partial co-volatility spillovers are negative, which means that a shock in one of carbon emission, coal or oil will have a one-period delayed negative impact on the conditional correlation between itself and one of the other two commodities. Two of the 8 partial co-volatility spillovers are positive, so an opposite effect will be observed.

[Insert Table 9 here]

Given the discussion based on equations (12) – (13), full co-volatility spillovers with DBEKK are presented in Table 10. Based on the estimates of the weighting matrix A, 2 of the 6 full co-volatility spillovers are negative, which means that a shock in one of carbon emission, coal or oil will have a one-period delayed negative impact on the conditional correlation between two of the other commodities. Two of the 6 full co-volatility spillovers are positive, so an opposite effect will be observed, while 2 of the 6 full co-volatility spillovers are zero, in which case there will be no spillovers. The results for full co-volatility spillovers in Table 10 are not as clear or helpful as in the

case of the partial co-volatility spillovers in Table 9, as the estimates of the off-diagonal elements in the weighting matrix A are not especially large.

[Insert Table 10 here]

The unconditional and conditional volatility of carbon, coal, and oil futures returns for the EU are shown in Figure 4, while the unconditional and conditional volatility of carbon, coal, and oil spot returns for the USA are shown in Figure 5. Both figures show there is a significant difference between the conditional and unconditional volatilities.

[Insert Figures 4 and 5 here]

The conditional co-volatility correlations for carbon, coal, and oil futures returns for the EU are shown in Figure 6, while the conditional co-volatility correlations for carbon, coal, and oil spot returns for the USA are shown in Figure 7. Both figures show there are substantial differences in the correlations of conditional co-volatility across the two markets and time periods for carbon, coal, and oil futures returns.

[Insert Figures 6 and 7 here]

The optimal hedge ratios for carbon, coal, and oil futures returns for the EU, and optimal hedge ratios for carbon, coal, and oil spot returns for the USA, are given in Figures 8 and 9, respectively. The hedge ratios show how the covariances in returns between two assets changes relative to the variance of the hedging instrument. Both figures show there is substantial variation in the optimal hedge ratios, so that the futures and spot prices of carbon emissions, coal and oil should be considered

contemporaneously and simultaneously in a portfolio that links the prices, returns and volatilities of carbon emissions to the use of fossil fuels.

[Insert Figures 8 and 9 here]

Finally, Figure 10 shows the optimal hedge ratios for carbon futures returns for the EU and both coal and oil spot returns for the USA. In all case, the optimal hedge rations vary substantially, which suggests that it would be sensible to use both markets to hedge carbon emission futures returns in the EU against both coal and oil spot price returns in the USA.

5. Concluding Remarks

The paper discussed recent research that showed efforts to limit climate change have been focusing on the reduction of carbon dioxide emissions over other greenhouse gases or air pollutants. Many countries have paid great attention to carbon emissions in order to improve air quality and public health. The largest source of carbon emissions from human activities in many countries in Europe and around the world has been from burning fossil fuels. The prices of both fuel and carbon emissions can and do have simultaneous and contemporaneous effects on each other.

Owing to the importance of carbon emissions and their interconnection to the prices, financial returns and associated volatilities of fossil fuels, and the possibility of Granger causality in spot and futures prices, returns and volatility of carbon emissions, it is not surprising that crude oil and coal, and their interactions with carbon emission prices, returns and volatility, have recently become very important for public policy and an associated research topic.

For the USA, daily spot and futures prices are available for crude oil and coal, but there are no daily spot or futures prices for carbon emissions. For the EU, there are no daily spot prices for coal or carbon emissions, but there are daily futures prices for crude oil, coal and carbon emissions. For this reason, daily prices were used to analyse Granger causality and volatility spillovers in spot and futures prices of carbon emissions, crude oil, and coal.

A likelihood ratio test was developed to test the multivariate conditional volatility Diagonal BEKK model, which has valid regularity conditions and asymptotic properties, against the alternative Full BEKK model, which has valid regularity conditions and asymptotic properties under the null hypothesis of zero off-diagonal elements. In short, Full BEKK has no desirable mathematical or statistical properties, except either under the null hypothesis of zero off-diagonal elements of the weighting matrix, or simply by assumption.

Dynamic hedging strategies using optimal hedge ratios were suggested to analyse market fluctuations in the spot and futures returns and volatility of carbon emissions, crude oil and coal prices. It was suggested that the futures and spot prices of carbon emissions, coal and oil should be considered contemporaneously and simultaneously in a portfolio that links the prices, returns and volatilities of carbon emissions to the use of fossil fuels, and that it would be sensible to use both markets to hedge carbon emission futures returns in the EU against both coal and oil spot price returns in the USA.

Table 1

Data Sources and Definitions

Variable name	Definitions	Transaction market	Description
$\mathbf{EUcarbon_{fr}}$	EU carbon futures return	ICE-ICE Futures Europe Commodities	ICE EUA Futures Contract EUR/MT
$\mathbf{EUcoal_{fr}}$	EU coal futures return	ICE-ICE Futures Europe Commodities	ICE Rotterdam Monthly Coal Futures Contract USD/MT
$\mathbf{EUoil_{fr}}$	EU oil futures return	ICE-ICE Futures Europe Commodities	Current pipeline export quality Brent blend as supplied at Sullom Voe USD/bbl
$\mathbf{UScarbon_{sr}}$	US carbon spot return	over the counter	United States Carbon Dioxide RGGI Allowance USD/Allowance
$\mathbf{UScoal_{sr}}$	US coal spot return	over the counter	Dow Jones US Total Market Coal Index USD
$\mathbf{USoil_{sr}}$	US oil spot return	over the counter	West Texas Intermediate Cushing Crude Oil USD/bbl

Note: ICE is the Intercontinental Exchange; EUA is the EU allowance; MT is metric ton; RGGI (Regional Greenhouse Gas Initiative) is a CO2 cap-and-trade emissions trading program comprised of ten New England and Mid-Atlantic States that will commence in 2009 and aims to reduce emissions from the power sector. RGGI will be the first government mandated CO2 emissions trading program in USA

Table 2

Descriptive Statistics

2 April 2008 – 19 May 2017 for EU

6 January 2016 – 19 May 2017 for USA

Variable	Mean	Median	Max	Min	SD	Skewness	Kurtosis	Jarque-Bera
$\mathbf{EUcarbon_{fr}}$	-0.078	-0.038	24.561	-42.457	3.349	-0.708	17.624	21434.2
$\mathbf{EUcoal_{fr}}$	-0.022	0	17.419	-22.859	1.599	-1.268	44.924	175155.8
$\mathbf{EUoil_{fr}}$	-0.026	-0.015	12.707	-10.946	2.246	0.054	6.522	1232.8
$\mathbf{UScarbon_{sr}}$	-0.248	0	13.937	-36.446	2.986	-5.236	66.269	61346.8
$\mathbf{UScoal_{sr}}$	0.177	0.104	17.458	-14.183	4.041	0.047	5.343	81.99
$\mathbf{USoil_{sr}}$	0.094	0.037	11.621	-8.763	2.712	0.431	4.690	53.69

Table 3

Unit Root Tests

2 April 2008 – 19 May 2017 for EU

6 January 2016 – 19 May 2017 for USA

Variables	ADF	DF-GLS	KPSS
$\mathbf{EUcarbon_{fr}}$	-37.79*	-3.09*	0.05*
$\mathbf{EUcoal_{fr}}$	-35.48*	-10.34*	0.12*
$\mathbf{EUoil_{fr}}$	-51.97*	-1.53	0.10*
$\mathbf{UScarbon_{sr}}$	-10.64*	-1.46	0.06*
$\mathbf{UScoal_{sr}}$	-19.30*	-0.43	0.18*
$\mathbf{USoil_{sr}}$	-20.96*	-0.78	0.07*

Notes: * denotes the null hypothesis of a unit root is rejected at 1%.

Table 4

Granger Causality Test for Returns

2 April 2008 – 19 May 2017 for EU

6 January 2016 – 19 May 2017 for USA

Variables		Lags	Outcome		Null hypothesis			
					A does not cause B		B does not cause A	
A	B				F-test	p-value	F-test	p-value
$EUcarbon_{fr}$	$EUcoal_{fr}$	1	EUcarbon	←	$EUcoal_{fr}$	0.6190	0.4315	5.7112	0.0169
$EUcarbon_{fr}$	$EUoil_{fr}$	1	$EUcarbon_{fr}$	←	$EUoil_{fr}$	0.2337	0.6289	4.1837	0.0409
$UScarbon_{sr}$	$UScoal_{sr}$	1	$UScarbon_{sr}$	→	$UScoal_{sr}$	4.6809	0.0312	0.9142	0.3397
$UScarbon_{sr}$	$USoil_{sr}$	1	$UScarbon_{sr}$	→	$USoil_{sr}$	5.1310	0.0241	0.0075	0.9313

Table 5

DBEKK and Full BEKK for EU Carbon, Coal, and Oil Futures

2 April 2008 – 19 May 2017

DBEKK	C			A			B		
$CARBON_{fr}$	0.379***	0.024**	0.128***	0.311***			0.947***		
	(0.055)	(0.010)	(0.024)	(0.025)			(0.009)		
$COAL_{fr}$		0.088***	0.022		0.118***			0.991***	
		(0.010)	(0.075)		(0.007)			(0.001)	
OIL_{fr}			0.000			-0.205***			-0.977***
			(0.077)			(0.013)			(0.003)

Full BEKK	C			A			B		
$CARBON_{fr}$	0.435***	-0.067*	0.077	0.331***	-0.014***	0.007	0.936***	0.009	-0.005
	(0.055)	(0.038)	(0.072)	(0.023)	(0.004)	(0.006)	(0.009)	(0.007)	(0.010)
$COAL_{fr}$		0.000	0.000	0.037	-0.086***	0.120***	0.274***	0.737***	1.110***
		(0.068)	(0.103)	(0.029)	(0.011)	(0.017)	(0.036))	(0.015)	(0.023)
OIL_{fr}			-0.000	-0.104***	-0.032**	-0.168***	-0189***	-0.052***	0.054***
			(0.101)	(0.026)	(0.013)	(0.010)	(0.024)	(0.011)	(0.015)

Notes : 1. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$, $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

2. Standard errors are in parentheses, ** denotes significant at 1%, * denotes significant at 5%.

Table 6

LR Test of DBEKK and Full BEKK for EU Futures

2 April 2008 – 19 May 2017

Log-likelihood for DBEKK	-14,815.88
Log-likelihood for Full BEKK	-14,798.72
LR test statistic with 6 df	34.32
Critical value at 1% with 6 df	16.81

Table 7

DBEKK and Full BEKK for US Carbon, Coal, and Oil Spot

6 January 2016 – 19 May 2017

DBEKK	C			A			B		
$CARBON_{sr}$	0.854***	-0.276	0.129	0.707***			0.757***		
	(0.105)	(0.294)	(0.332)	(0.073)			(0.038)		
$COAL_{sr}$		0.256	0.299*		-0.199***			0.972***	
		(0.314)	(0.154)		(0.034)			(0.008)	
OIL_{sr}			0.000			-0.222***			-0.964***
			(1.029)			(0.0035)			(0.010)

Full BEKK	C			A			B		
$CARBON_{sr}$	0.772***	0.119	0.685***	0.632***	-0.023	-0.077	0.791***	0.004	-0.034
	(0.092)	(0.606)	(0.178)	(0.054)	(0.089)	(0.064)	(0.025)	(0.112)	(0.063)
$COAL_{sr}$		0.000	0.000	0.002	-0.320***	0.036	-0.042	0.900***	0.578***
		(0.528)	(0.715)	(0.033)	(0.058)	(0.041)	(0.046)	(0.056)	(0.044)
OIL_{sr}			0.000	-0.028	-0.072	-0.252***	0.010	-1.267***	0.140*
			(0.721)	(0.049)	(0.092)	(0.060)	(0.080)	(0.074)	(0.082)

Notes : 1. $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$, $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$

2. Standard errors are in parentheses, ** denotes significant at 1%, * denotes significant at 5%.

Table 8

LR Test of DBEKK and Full BEKK for US Spot

6 January 2016 – 19 May 2017

Log-likelihood for DBEKK	-2,499.27
Log-likelihood for Full BEKK	-2,488.18
LR test statistic with 6 df	22.18
Critical value at 1% with 6 df	16.81

Table 9

Partial Co-volatility Spillovers with DBEKK for EU and USA

2 April 2008 – 19 May 2017 for EU

6 January 2016 – 19 May 2017 for USA

Market	$(\frac{\partial H_{ij,t}}{\partial \varepsilon_{k,t-1}})$		Average Co-volatility Spillovers		
EU	j=k=$coal_{fr}$,	i=$carbon_{fr}$	-0.001	=	-0.030*0.311*0.118
	j=k=$carbon_{fr}$,	i=$coal_{fr}$	0.001	=	0.026*0.311*0.118
	j=k=oil_{fr},	i=$carbon_{fr}$	0.002	=	-0.030*0.311*-0.205
	j=k=$carbon_{fr}$,	i=oil_{fr}	0.001	=	-0.023*0.311*-0.205
USA	j=k=$coal_{sr}$,	i=$carbon_{sr}$	0.020	=	-0.140*0.707*-0.199
	j=k=$carbon_{sr}$,	i=$coal_{sr}$	-0.002	=	0.012*0.707*-0.199
	j=k=oil_{sr},	i=$carbon_{sr}$	0.022	=	-0.140*0.707*-0.222
	j=k=$carbon_{sr}$,	i=oil_{sr}	0.003	=	-0.022*0.707*-0.222

Note: Co-volatility Spillovers: $\frac{\partial H_{ij,t}}{\partial \varepsilon_{k,t-1}} = a_{ii}a_{jj}\varepsilon_{i,t-1}$.

Table 10

Full Co-volatility Spillovers with Full BEKK for EU and USA

2 April 2008 – 19 May 2017 for EU

6 January 2016 – 19 May 2017 for USA

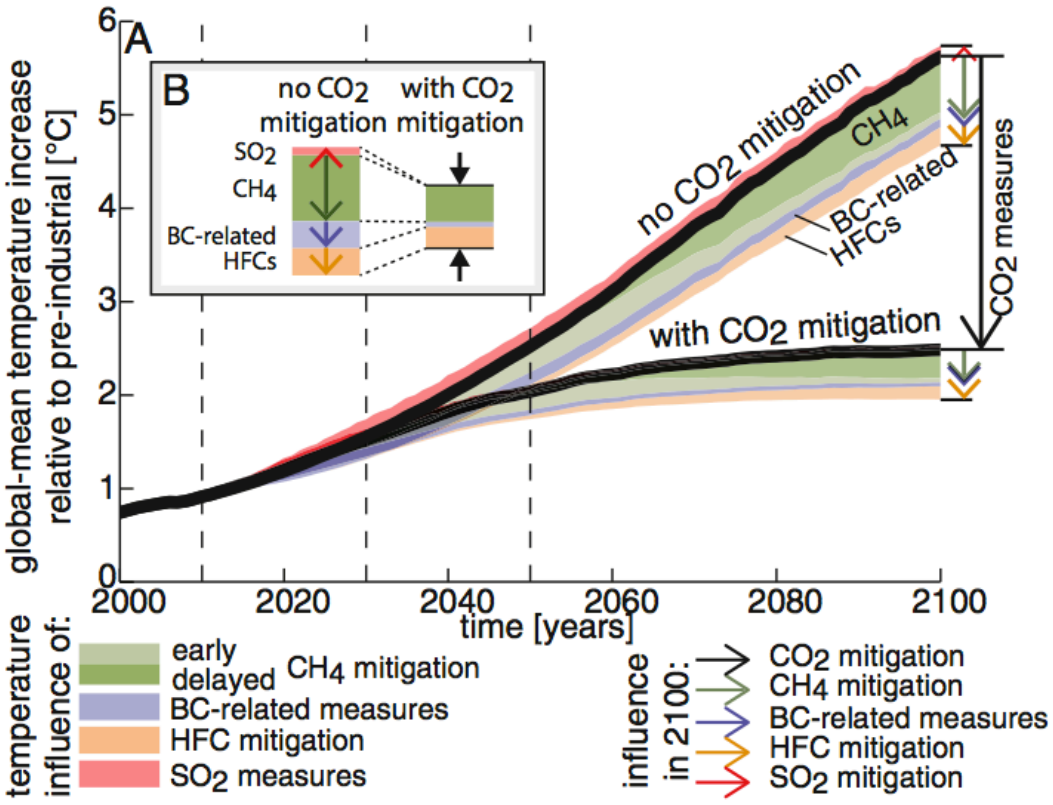
Market		$(\frac{\partial H_{ij,t}}{\partial \varepsilon_{k,t-1}})$		Co-volatility Spillovers
EU	$j=coal_{fr}$,	$i=carbon_{fr}$	$k=oil_{fr}$	-0.001
	$j=oil_{fr}$,	$i=carbon_{fr}$	$k=coal_{fr}$,	0
	$j=coal_{fr}$,	$i=oil_{fr}$	$k=carbon_{fr}$	0.001
USA	$j=coal_{sr}$,	$i=carbon_{sr}$	$k=oil_{sr}$	-0.002
	$j=oil_{sr}$,	$i=carbon_{sr}$	$k=coal_{sr}$	0.004
	$j=coal_{sr}$,	$i=oil_{sr}$	$k=carbon_{sr}$	0

Note: Co-volatility Spillovers: $\frac{\partial H_{ij,t}}{\partial \varepsilon_{k,t-1}} = a_{ii}a_{jk}\varepsilon_{i,t-1} + a_{ij}a_{jk}\varepsilon_{j,t-1} + a_{ik}a_{ji}\varepsilon_{i,t-1} + a_{ik}a_{jj}\varepsilon_{j,t-1} + 2a_{ik}a_{jk}\varepsilon_{k,t-1}$.

Figure 1

Global Mean Temperatures

With and Without Carbon Dioxide Mitigation



Source: Rogelj et al. (2014).

Figure 2

Implementation of Programs to Mitigate Carbon Emissions

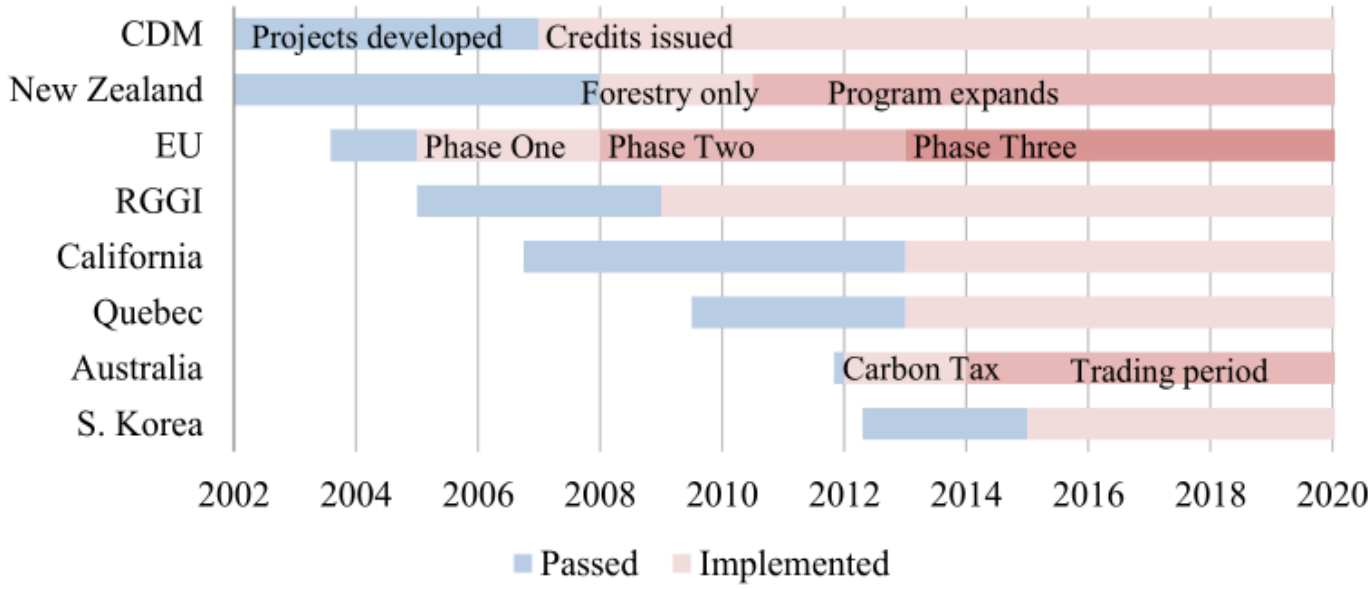


Figure 3

Carbon Futures and Spot Volumes for EU

10 December 2012 – 19 May 2017

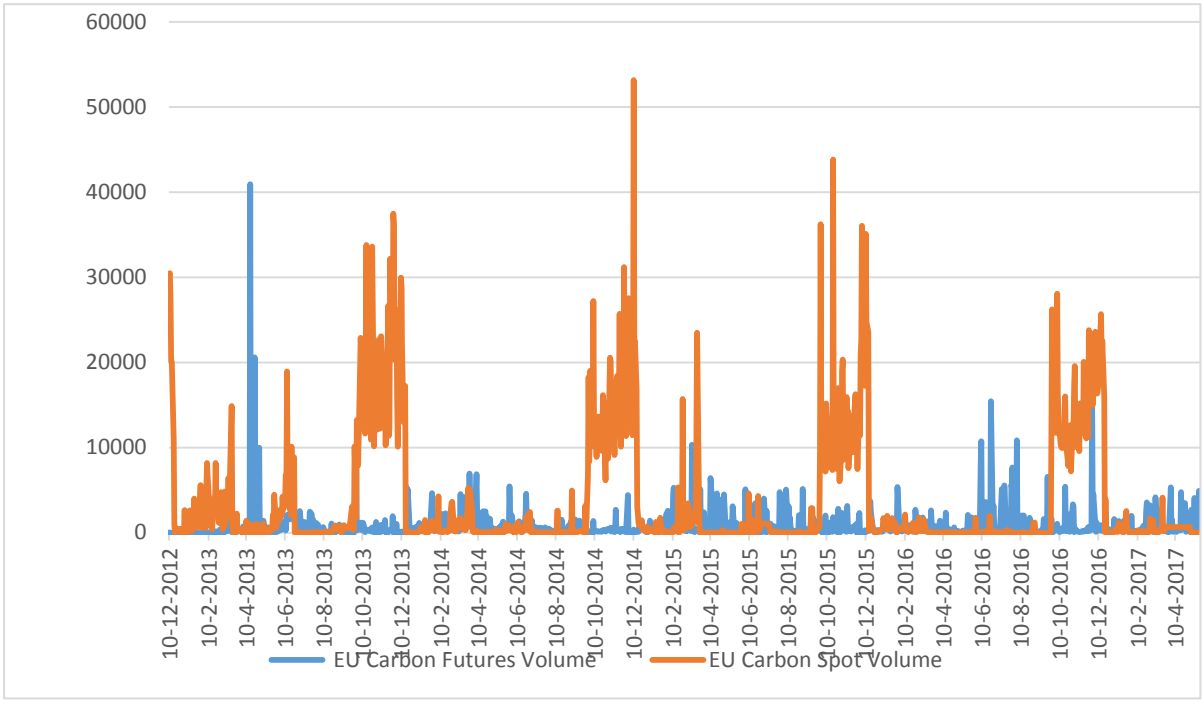


Figure 4

Unconditional and Conditional Volatility of Carbon, Coal, Oil Futures for EU

2 April 2008 - 19 May 2017

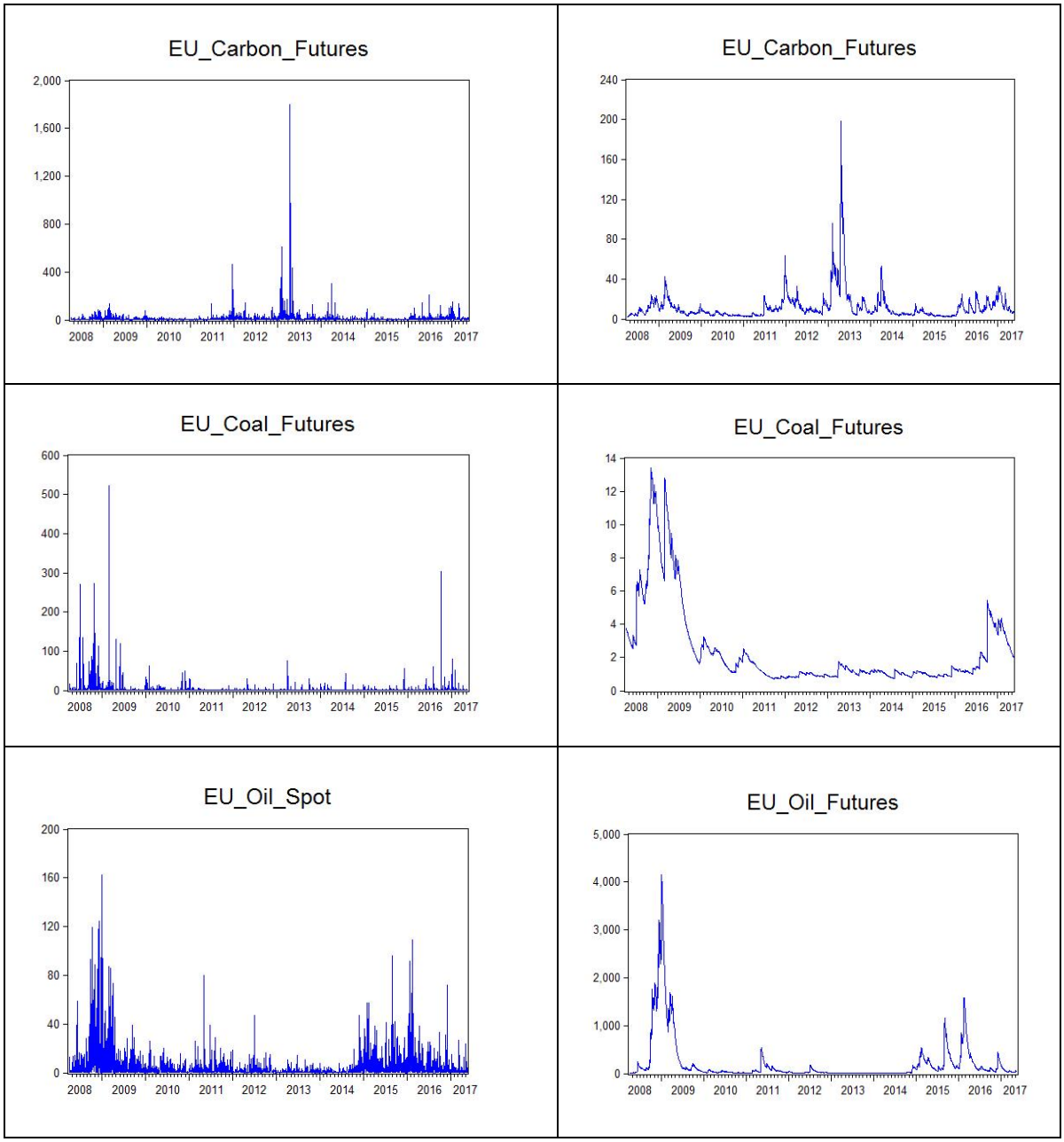


Figure 5

Unconditional and Conditional Volatility of Carbon, Coal, Oil Spot for USA

6 January 2016 – 19 May 2017

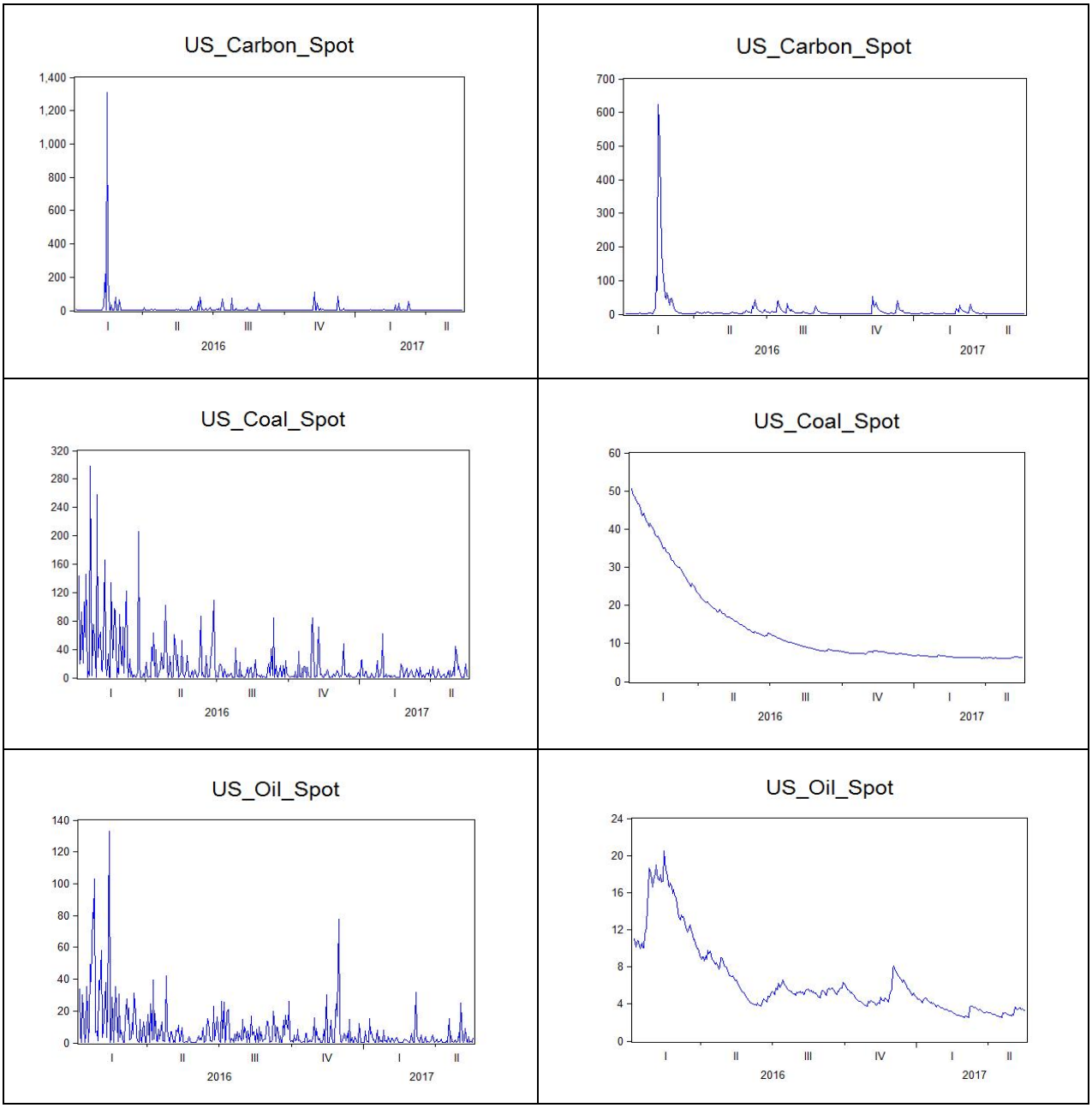


Figure 6

Conditional Co-volatility and Correlation for Carbon, Coal, Oil Futures for EU

2 April 2008 - 18 May 2017

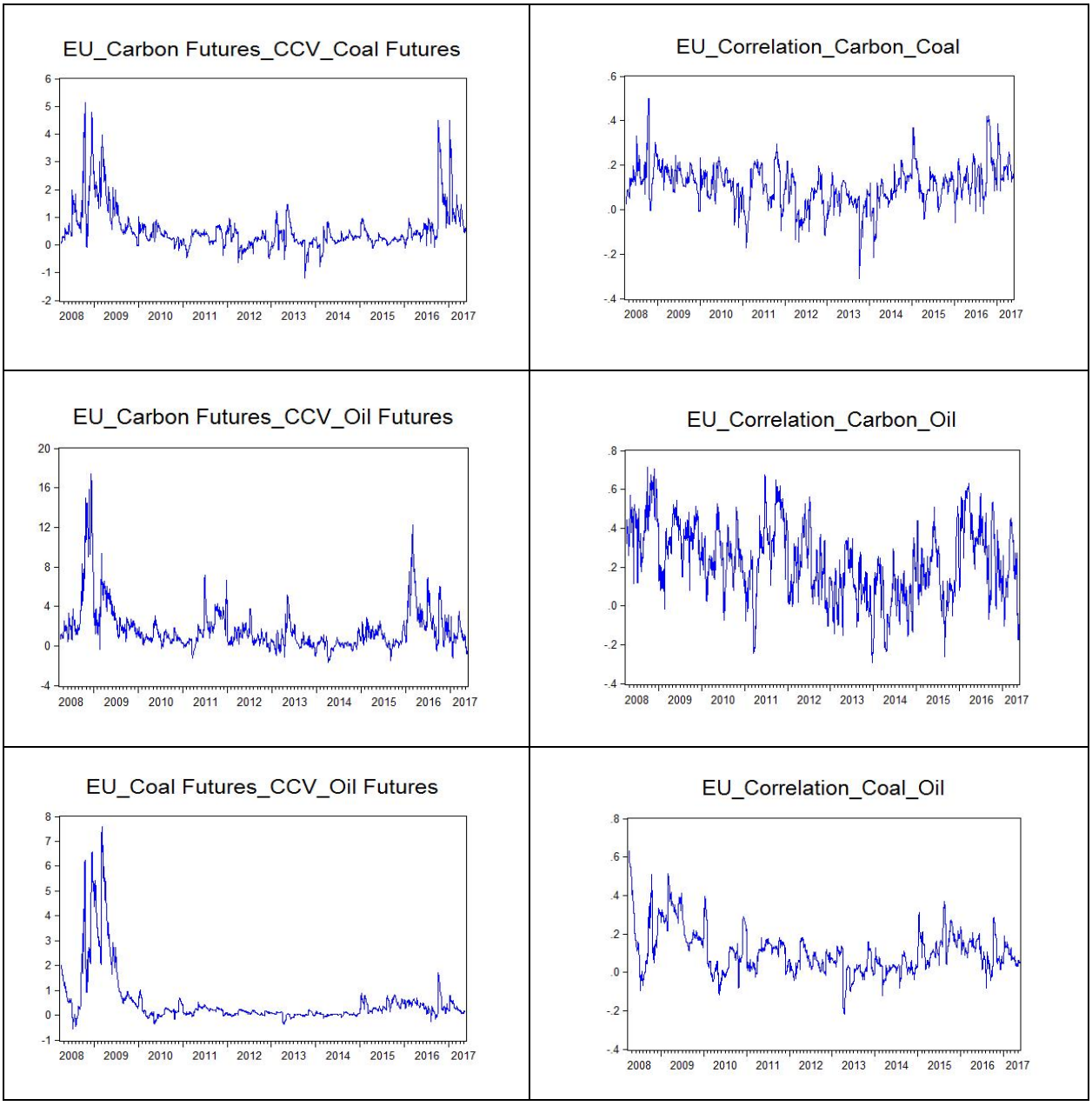


Figure 7

Conditional Co-volatility and Correlation for Carbon, Coal, Oil Spot for USA

6 January 2016 – 18 May 2017

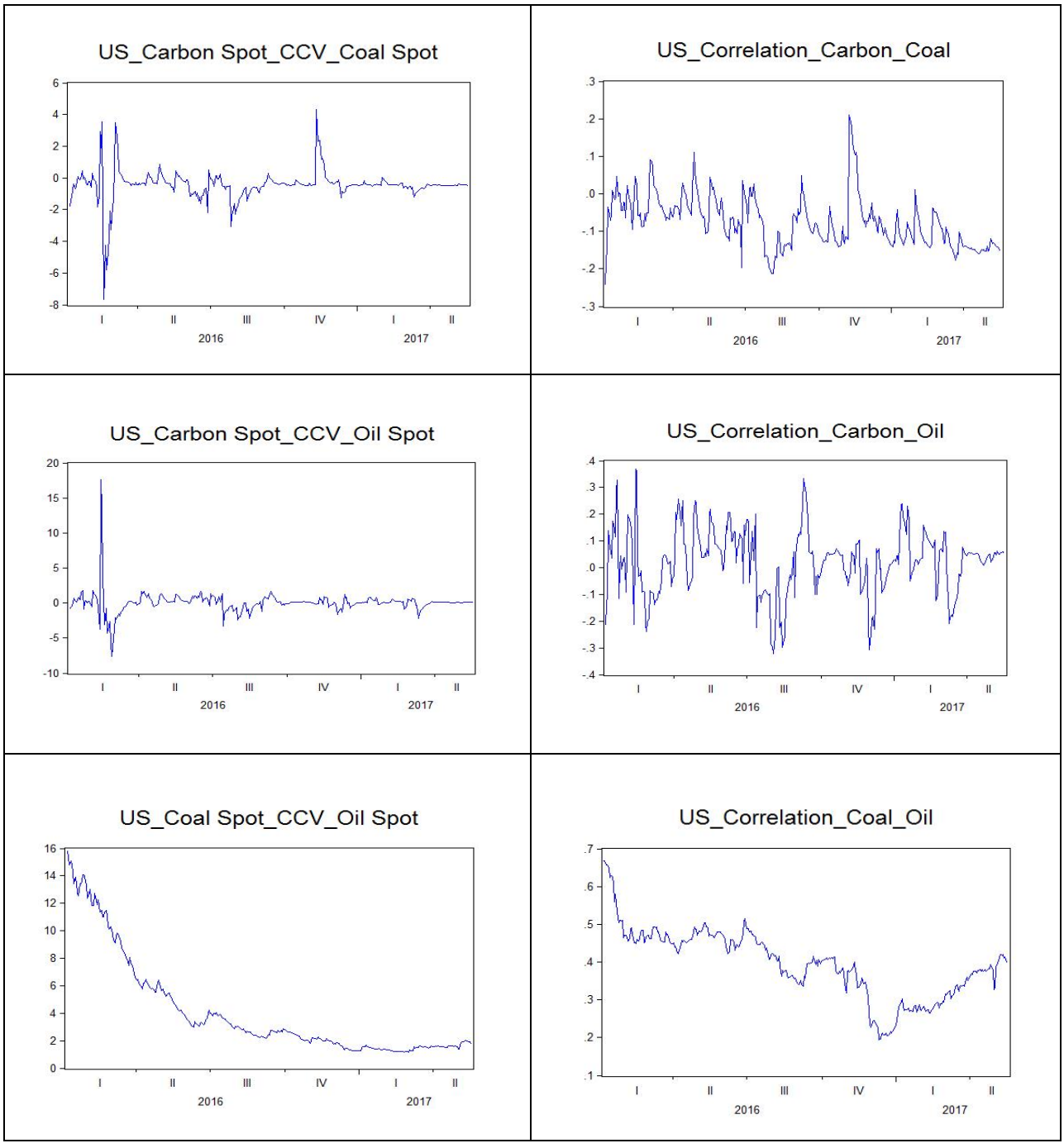


Figure 8

Optimal Hedge Ratios for Carbon, Coal, Oil Futures for EU

2 April 2008 - 19 May 2017

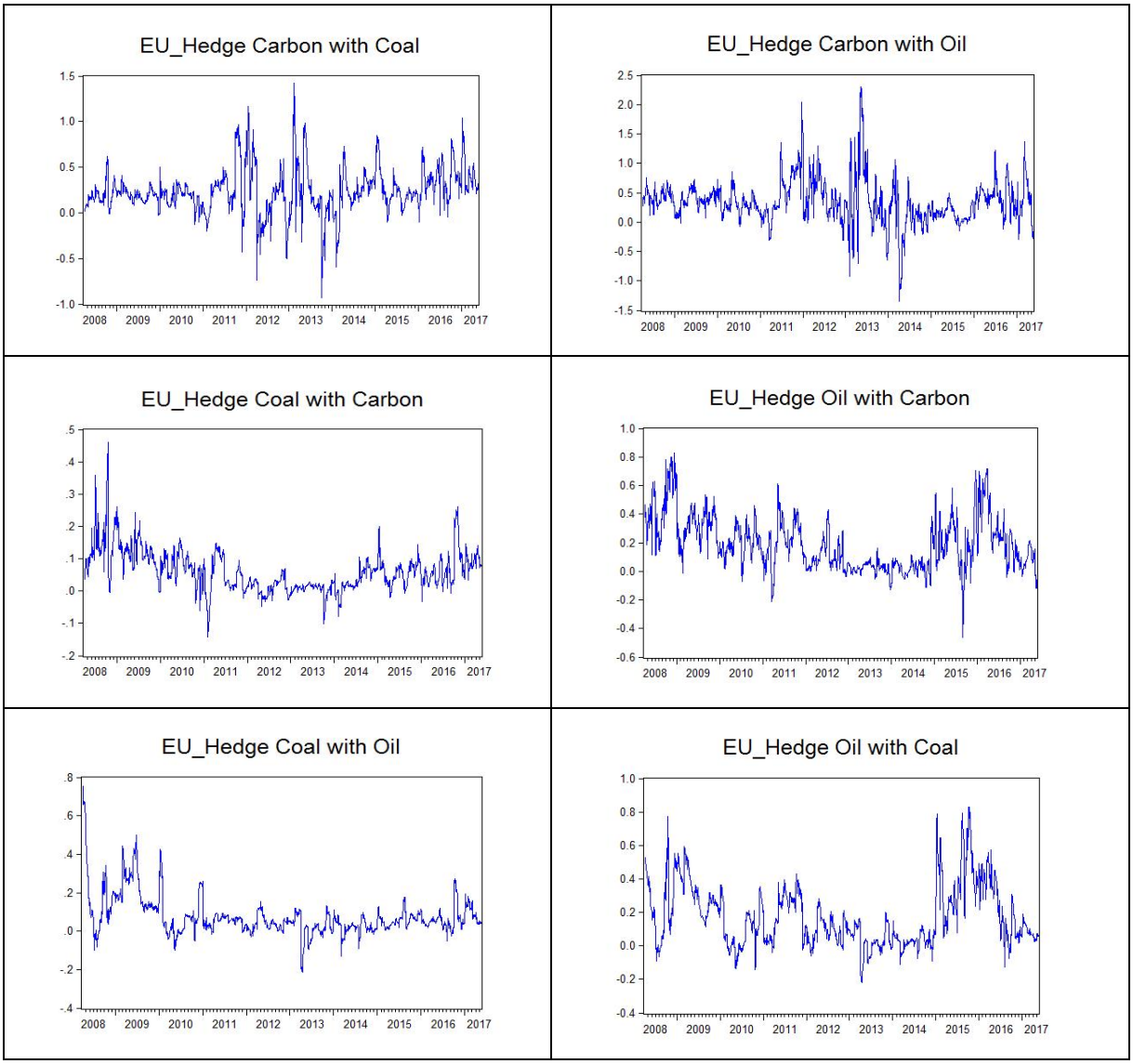


Figure 9

Optimal Hedge Ratios for Carbon, Coal, Oil Spot for USA

6 January 2016 – 18 May 2017

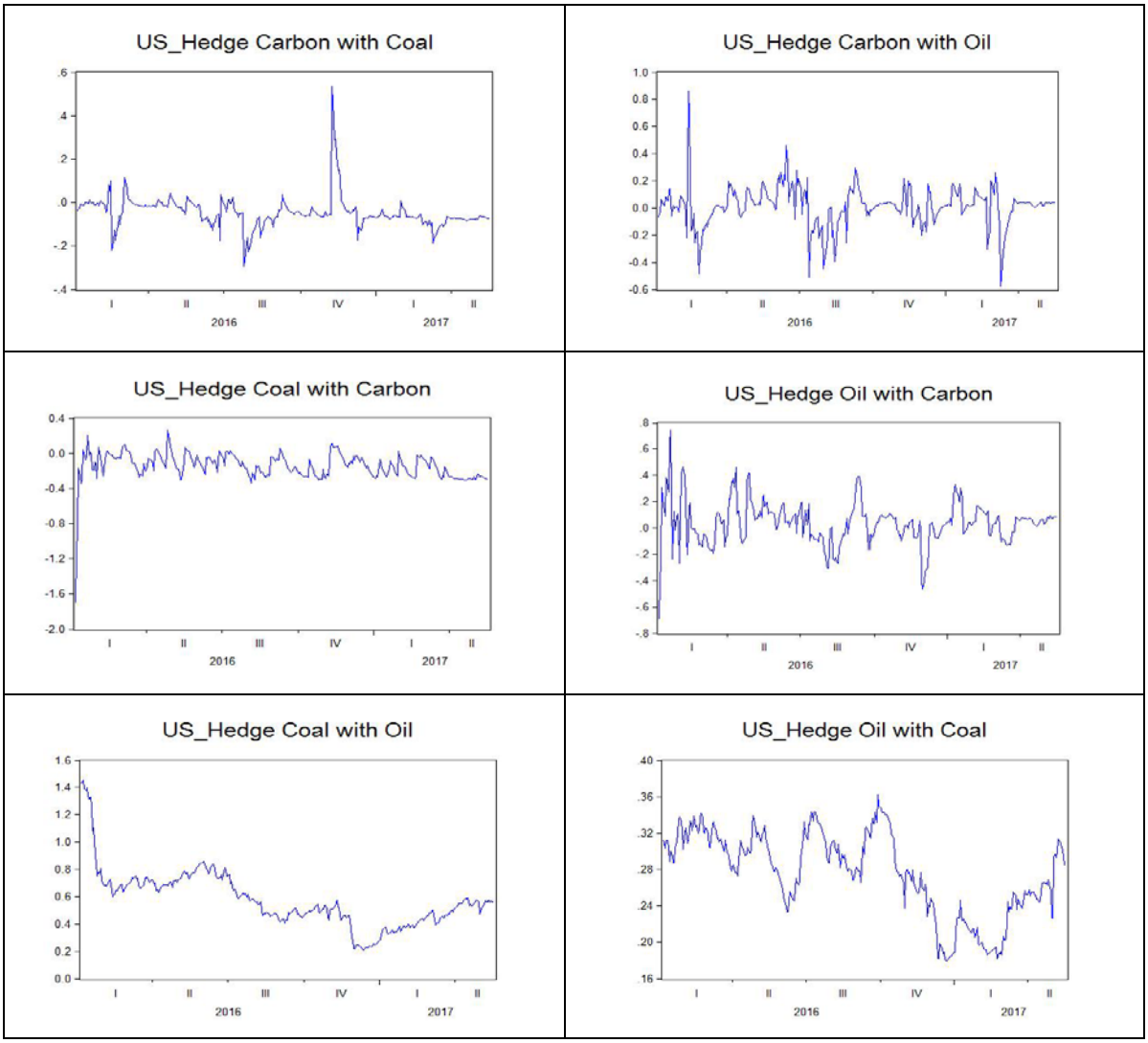
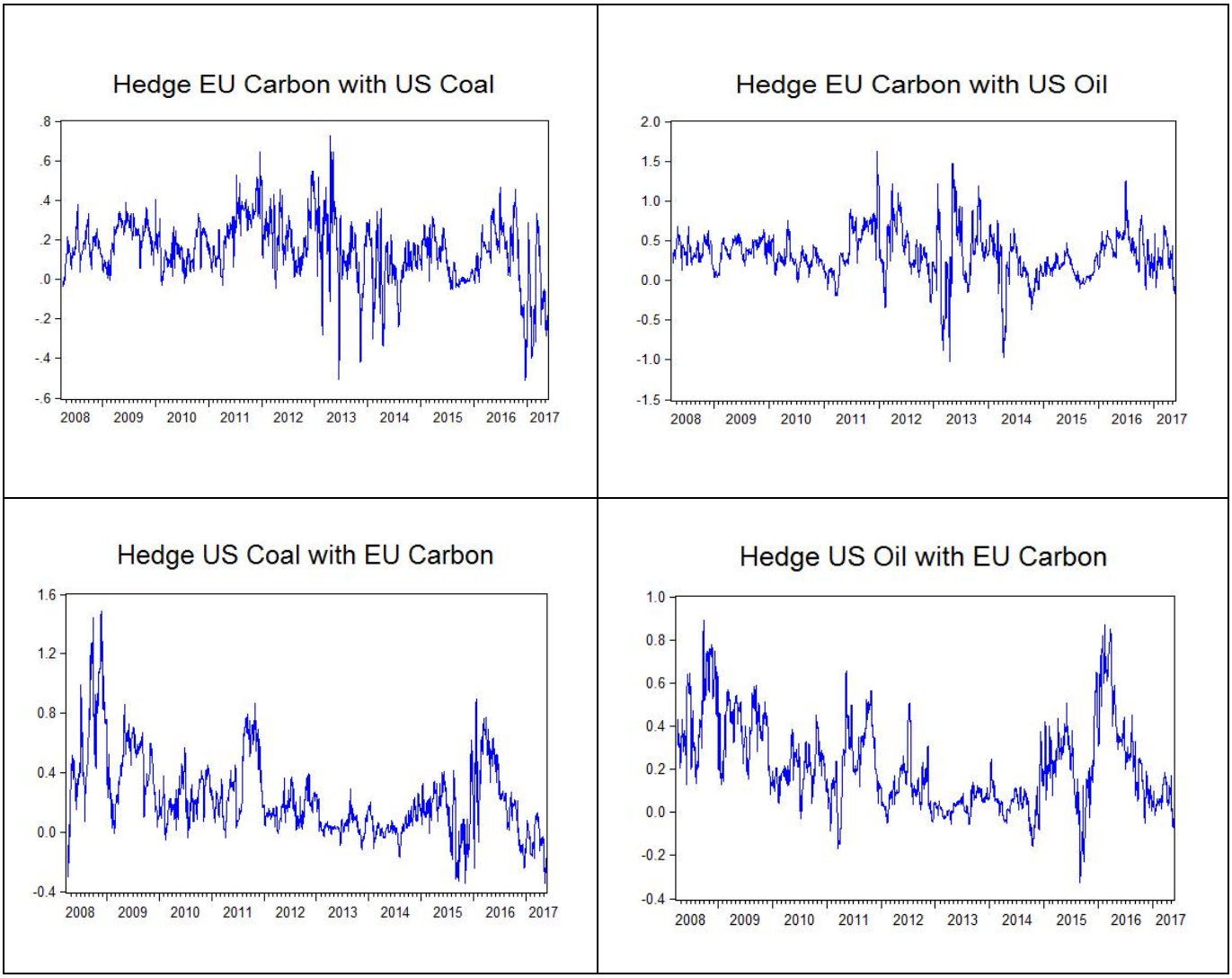


Figure 10

Optimal Hedge Ratios for Carbon Futures of EU and Coal, Oil Spot of USA

2 April 2008 - 18 May 2017



References

Asai, M., C.-L. Chang and M. McAleer (2017), Realized matrix-exponential stochastic volatility with asymmetry, long memory and spillovers, to appear in Journal of Econometrics.

Amemiya, T. (1985), Advanced Econometrics, Harvard University Press, Cambridge, MA, USA.

Baba, Y., R.F. Engle, D. Kraft and K.F. Kroner (1985), Multivariate simultaneous generalized ARCH, Unpublished manuscript, Department of Economics, University of California, San Diego, CA, USA.

Bollerslev, T. (1986), Generalized autoregressive conditional heteroscedasticity, Journal of Econometrics, 31(3), 307-327.

Bollerslev, T. (1990), Modelling the coherence in short-run nominal exchange rate: A multivariate generalized ARCH approach, Review of Economics and Statistics, 72, 498-505.

Bollerslev, T., R.F. Engle, and J.M. Wooldridge (1988), A capital asset pricing model with time varying covariance, Journal of Political Economy, 96(1), 116-131.

Caporin, M. and M. McAleer (2012), Do we really need both BEKK and DCC? A tale of two multivariate GARCH models, Journal of Economic Surveys, 26(4), 736-751.

Chang, C.-L., Y.-Y. Li and M. McAleer (2015), Volatility Spillovers Between Energy and Agricultural Markets: A Critical Appraisal of Theory and Practice, Tinbergen Institute Discussion Paper 15-077/III, Tinbergen Institute.

Chang, C.-L. and M. McAleer (2017), A simple test for causality in volatility, Econometrics, 5(1:15), 1-5.

Chang, C.-L., M. McAleer, and R. Tansuchat (2011), Crude oil hedging strategies using dynamic multivariate GARCH, Energy Economics, 33(5), 912-923.

Dickey D.A. and W.A. Fuller (1979), Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association, 74(366), 427-431.

Dickey, D.A. and W.A. Fuller (1981), Likelihood ratio statistics for autoregressive

time series with a unit root, Econometrica, 49(4), 1057-1072.

Elliott, G., T.J. Rothenberg and J.H. Stock (1996), Efficient tests for an autoregressive unit root, Econometrica, 813-836.

Engle, R.F. (1982), Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, Econometrica, 50(4), 987-1007.

Engle, R.F. and K.F. Kroner (1995), Multivariate simultaneous generalized ARCH, Econometric Theory, 11(1), 122-150.

Granger, C.W.J. (1980), Testing for causality: A personal viewpoint, Journal of Economic Dynamics and Control, 2, 329-352.

Hafner, C.M. and H. Herwartz (2006), A Lagrange multiplier test for causality in variance, Economics Letters, 93(1), 137-141.

Hafner, C. and M. McAleer (2014), A one line derivation of DCC: Application of a vector random coefficient moving average process, Tinbergen Institute Discussion Paper 14-087, The Netherlands.

Jeantheau, T. (1998), Strong consistency of estimators for multivariate ARCH models, Econometric Theory, 14(1), 70-86.

Kwiatkowski, D., P.C.B. Phillips, P Schmidt, and Y Shin, (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, Journal of Econometrics, 54(1-3), 159-178.

Ling, S. and M. McAleer (2003), Asymptotic theory for a vector ARMA-GARCH model, Econometric Theory, 19(2), 280-310.

McAleer, M., F. Chan, S. Hoti and O. Lieberman (2008), Generalized autoregressive conditional correlation, Econometric Theory, 24(6), 1554-1583.

McAleer, M. and C. Hafner (2014), A one line derivation of EGARCH, Econometrics, 2, 92–97.

McAleer, M., S. Hoti and F. Chan (2009), Structure and asymptotic theory for multivariate asymmetric conditional volatility, Econometric Reviews, 28, 422-440.

Rogelj, J., M. Meinshausen, J. Sedláček, and R. Knutti. (2014), Implications of potentially lower climate sensitivity on climate projections and policy,

Environmental Research Letters, 9(3), 3-10.

Said, S.E. and D.A. Dickey (1984), Testing for unit roots in autoregressive-moving average models of unknown order, Biometrika, 71 (3), 599-607.

Tsay, R.S. (1987), Conditional heteroscedastic time series models, Journal of the American Statistical Association, 82(398), 590-604.