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**Emission taxation, green innovations and  
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# Emission taxation, green innovations and inverted-U aggregate R&D efforts in a linear state oligopoly game<sup>1</sup>

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## **Abstract**

We revisit the well known differential Cournot game with polluting emissions dating back to Bencheikroun and Long (1998), proposing a version of the model in which environmental taxation is levied on emissions rather than the environmental damage. This allows to attain strong time consistency under open-loop information, and yields two main results which can be summarized as follows: (i) to attain a fully green technology in steady state, the regulator may equivalently adopt an appropriate tax rate (for any given number of firms) or regulate market access (for any given tax rate); (ii) if the environmental damage depends on emissions only (i.e., not on industry output) then the aggregate green R&D effort takes an inverted-U shape, in accordance with Aghion *et al.* (2005), and the industry structure maximising aggregate green innovation also minimises individual and aggregate emissions.

**Keywords:** pollution, green R&D, emission taxation, differential games

**JEL Codes:** C73, H23, L13, O31, Q52

# 1 Introduction

If one takes a quick look at the static models dealing with emission taxation in oligopoly (little matters whether these models include green R&D or not), it appears that usually environmental taxation is levied on per-firm emissions rather than on the resulting (aggregate) environmental damage. The opposite instead applies instead if one examines the corresponding literature using optimal control or differential game theory.<sup>1</sup>

This poses a problem of consistency between the static and the dynamic approach to modelling the environmental impact of oligopolistic interaction on the environment and the related design of emission taxation. Moreover, judging on the basis of casual observation, the two approaches are not equally realistic. To begin with, although aggregate data on emissions may well be more readily and easily available than individual data at the single firm level, taxing a magnitude defined as the environmental damage amounts to using a quite elusive concept, as the environmental damage imputable to any single industry adds up to the cauldron of a global economic system generating global warming and other similar effects. Additionally, current rules (for instance, in the EU) require firms to explicitly declare the  $CO_2$ -equivalent emission rates of their products (e.g., cars), making these data accessible to the public and the authorities.

In view of these considerations, here we propose a differential Cournot game in which firms are being taxed in proportion to their individual emissions and react to the environmental tax rate by modifying output levels and investing in R&D for green technologies. This setup allows us to obtain several results. The first is that - taxation being linear in each firm's emission volume - the game at hand exhibits a linear state structure and therefore yields a subgame perfect equilibrium under open-loop information. The second result is that there exists a unique tax rate driving to zero the volume of emissions for any number of firms, or equivalently there exists a unique industry structure attaining the same outcome for any environmental tax

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<sup>1</sup>For exhaustive surveys of both strands of research, where these features clearly emerge, see Montero (2002b), Requate and Unold (2003), Requate (2005), Long (2010) and Lambertini (2013).

rate. The third result is that - if the environmental damage is unaffected by industry output and the tax rate is optimally set - the aggregate R&D effort at the steady state equilibrium is non-monotone in the number of firms and has an inverted-U shape, i.e., there exists a unique industry structure that maximises the collective equilibrium investment in green technologies.<sup>2</sup> This feature of the model has a clearcut connection with an ongoing discussion in the theory and empirics of the economics of innovation, which deserves to be illustrated before delving into the analysis of our specific setup.

The acquired industrial organization approach to the bearings of market power on the size and pace of technical progress can be traced back to the indirect debate between Schumpeter (1934, 1942) and Arrow (1962) on the so-called Schumpeterian hypothesis, which, in a nutshell, says that one should expect to see an inverse relationship between innovation and market power or market structure. Irrespective of the nature of innovation (either for cost reductions or for the introduction of new products), a large theoretical literature attains either Schumpeterian or Arrowian conclusion (for exhaustive accounts, see Tirole, 1988; and Reinganum, 1989).<sup>3</sup> That is, partial equilibrium theoretical IO models systematically predict a *monotone* relationship, in either direction.

The picture drastically changes as soon as one takes instead the standpoint of modern growth theory. In particular, Aghion *et al.* (2005) stress that empirical evidence shows a *non-monotone* relationship between industry concentration (or, the intensity of market competition) and aggregate R&D efforts: this takes the form of an *inverted-U curve*, at odds with all existing theoretical IO models; in the same paper, the authors provide a model yielding indeed such a concave result, and fitting the data. A thorough discussion, accompanied by an exhaustive review of the related

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<sup>2</sup>The emergence of an analogous inverted-U shaped aggregate R&D curve has been illustrated by Feichtinger *et al.* (2016) using a differential game in which the public authority regulates price and tunes the emission tax rate.

<sup>3</sup>See also Gilbert (2006), Vives (2008) and Schmutzler (2010) for add-on's on this discussion, where still the Schumpeter vs Arrow argument is unresolved.

lively debate, can be found in Aghion *et al.* (2013, 2015).

One could say that the inverted-U emerging from data says that Arrow is right for small numbers, while Schumpeter is right thereafter. Alternatively, on the same basis one could also say that neither Arrow nor Schumpeter can match reality, if our interpretation of their respective views is that “competition (resp., monopoly) outperforms monopoly (resp., competition) along the R&D dimension”. Be that as it may, there arises the need of constructing models delivering a non-monotone relationship between some form of R&D (for process, product or environmental-friendly innovations) and the number of firms in the industry.

The remainder of the paper is organised as follows. The setup is illustrated in section 2. The equilibrium analysis and the main results are laid out in section 3. Section 4 contains concluding remarks.

## 2 The model

Consider a Cournot oligopoly with a population  $n \geq 2$  of single-product homogeneous-good firms interacting over continuous time  $t \in [0, \infty)$ . At any time  $t$ , the demand function is  $p(t) = a - \sum_{i=1}^n q_i(t)$ ,  $q_i(t) \geq 0$  being the instantaneous individual output of firm  $i$ . The demand function is based on the assumption that consumers do not internalise any external effects, i.e., consumers in this market have not developed any environmental awareness. All firms use the same productive technology, described by the cost function  $C_i(t) = cq_i(t)$ . The production of the final output involves an amount of polluting emissions  $s_i(t)$  generated by the output of each firm  $i$  and evolving according to the following dynamics:

$$\dot{s}_i(t) = \frac{ds_i}{dt} = vq_i(t) - k_i(t) - z \sum_{j \neq i} k_j(t) - \delta s_i(t), \quad (1)$$

where  $\delta > 0$  is a constant decay rate and coefficient  $v \geq 0$  measures the volume of  $CO_2$ -equivalent emissions per unit of output. Variable  $k_i(t)$  is the instantaneous R&D effort of firm  $i$ , and the state equation (1) accounts for the presence of spillovers in

emission abatement, measured by parameter  $z \in [0, 1]$  (note that if  $z = 1$  the green technology is a public good). The instantaneous cost associated with the R&D activity is  $\Gamma_i(t) = wk_i^2(t)$ , with  $w > 0$ , and firm  $i$ 's emissions  $s_i(t)$  are taxed at the rate  $\tau > 0$  at every instant.<sup>4</sup> Hence, firm  $i$ 's instantaneous profits are

$$\pi_i(t) = [p(t) - c]q_i(t) - \tau s_i(t) - \Gamma_i(t), \quad (2)$$

and each firm  $i$  has to set  $q_i(t)$  and  $k_i(t)$  so as to maximise

$$\Pi_i = \int_0^\infty \{[p(t) - c]q_i(t) - \tau s_i(t) - \Gamma_i(t)\} e^{-\rho t} dt, \quad (3)$$

under the constraints posed by the state equation (1) and the initial conditions  $s_i(0) = s_{i0} > 0$ . Parameter  $\rho > 0$  represents a constant discount rate common to all firms and the policy maker.

The instantaneous social welfare function is

$$SW(t) = \sum_{i=1}^n \pi_i(t) + CS(t) + \tau \sum_{i=1}^n s_i(t) - D(t) \quad (4)$$

where  $CS(t) = Q^2(t)/2$  is consumer surplus and aggregate emissions  $S(t) = \sum_{i=1}^n s_i(t)$  concur with aggregate output  $Q(t) = \sum_{i=1}^n q_i(t)$  in causing the quadratic environmental damage  $D(t) = \varepsilon Q(t) + \gamma S^2(t)$ , where  $\gamma$  and  $\varepsilon$  are positive parameters.

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<sup>4</sup>A tax bill defined as a linear function of polluting emissions is commonly used in static models (see Ulph, 1996; Montero, 2002a; Chiou and Hu, 2001; and Poyago-Theotoky, 2007, *inter alia*). An alternative way of modelling emission taxation consists in assuming that the tax rate is applied to the industry-wide environmental damage (see Karp and Livernois, 1994; Benckroun and Long, 1998; 2002; and Dragone *et al.*, 2014, among many others). This is, however, highly unrealistic for several reasons. The choice we make in the present model is in line with the idea that, currently, accurate and verifiable data are indeed available at the individual firm's level (e.g., this is the case in the car industry, where the amount of carbon emissions per kilometer are declared by manufacturers on the websites).



### 3 Equilibrium analysis

Henceforth, we will omit the time argument for simplicity, whenever possible. Since the present game is a linear state one, the open-loop solution is subgame perfect (or strongly time consistent) as it yields a degenerate feedback equilibrium.<sup>5</sup> The current-value Hamiltonian of firm  $i$  is:

$$\begin{aligned}\mathcal{H}_i(\cdot) &= (p - c)q_i - \tau s_i - wk_i^2 + \lambda_{ii}\dot{s}_i + \sum_{j \neq i} \lambda_{ij}\dot{s}_j = \\ &= (\sigma - Q)q_i - rk_i^2 + \lambda_{ii}\dot{s}_i + \sum_{j \neq i} \lambda_{ij}\dot{s}_j,\end{aligned}\tag{5}$$

where  $\sigma \equiv a - c > 0$  denotes market size and  $\lambda_{ij}(t)$  is the costate variable attached by the  $i$ -th firm to the  $j$ -th state equation.

The necessary conditions (FOCs) are:

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = \sigma - 2q_i - Q_{-i} + v\lambda_{ii} = 0,\tag{6}$$

where  $Q_{-i} \equiv \sum_{j \neq i} q_j$ , and

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = -2wk_i - \lambda_{ii} - z \sum_{j \neq i} \lambda_{ij} = 0,\tag{7}$$

The adjoint equations read as follows:

$$\dot{\lambda}_{ii} = (\rho + \delta) \lambda_{ii} + \tau\tag{8}$$

and

$$\dot{\lambda}_{ij} = (\rho + \delta) \lambda_{ij}\tag{9}$$

From (9) it is apparent that the solution  $\lambda_{ij} = 0$  for all  $j \neq i$  is admissible at all times. This means that, at any instant  $t$ , firm  $i$  fully disregards the dynamics of any rival's emissions.

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<sup>5</sup>For more on the arising of strongly time consistent equilibria in differential games solved under open-loop information, see Fershtman (1987), Mehlmann (1988, ch. 4), Dockner *et al.* (2000, ch. 7) and Cellini *et al.* (2005).

Using  $\lambda_{ij} = 0$  and imposing symmetry on states and controls, i.e.  $s_i = s_j = S$ ,  $k_i = k_j = k$ ,  $q_i = q_j = q$  and consequently  $\lambda_{ii} = \lambda_{jj} = \lambda$  for all  $i \neq j$ , we proceed to use (7) to derive the control equation for the green R&D effort  $k$ , as follows:

$$\dot{k} = -\frac{\dot{\lambda}}{2w} = -\frac{(\rho + \delta)\lambda + \tau}{2w} \quad (10)$$

which, noting - again from (7) - that  $\lambda = -2wk$ , can be rewritten as

$$\dot{k} = \frac{2w(\rho + \delta)k - \tau}{2w} \quad (11)$$

The optimal output associated with the Cournot-Nash equilibrium ( $CN$ ) at any time  $t$  can instead be directly obtained by solving FOC (6):

$$q^{CN} = \frac{\sigma - 2vwk}{n + 1} \quad (12)$$

which obviously collapses onto the static Cournot-Nash output any green R&D effort being absent.

We may now characterise the steady state of the system. Imposing stationarity on (11) yields

$$k^{ss} = \frac{\tau}{2w(\rho + \delta)} \quad (13)$$

where superscript  $ss$  stands for *steady state*. The above expression establishes our first result:

**Lemma 1** *For any given  $\tau > 0$ , the individual and aggregate green R&D efforts in steady state are positive. Moreover, the aggregate R&D effort is monotonically increasing in the number of firms.*

In particular, the second part of the above Lemma says that, since the aggregate equilibrium expenditure  $K^{ss} = n\tau / [2w(\rho + \delta)]$  is linearly increasing in the number of firms, the present model seems to possess an Arrovian flavour. We will come back to this important aspect in the remainder.

Now observe that the steady state individual output is

$$q^{ss} = \frac{\sigma - 2vwk^{ss}}{n+1} \quad (14)$$

which is lower than the static Cournot-Nash output, and strictly positive provided that

$$v \in \left(0, \frac{\sigma(\rho + \delta)}{\tau}\right). \quad (15)$$

Substituting  $(k^{ss}, q^{ss})$  into the state equation (1) and imposing stationarity, we obtain

$$s^{ss} = \max \left\{ \frac{2\sigma vw(\rho + \delta) - \tau[2v^2w + (n+1)(1 + z(n-1))]}{2\delta w(n+1)(\rho + \delta)}, 0 \right\}. \quad (16)$$

The following result applies:

**Proposition 2** *The steady state  $(s^{ss}, q^{ss}, k^{ss})$  is a saddle point.*

**Proof.** Given that the optimal output can be identified at any time in a quasi-static way, the state-control system solely describes the dynamics of  $(s^{ss}, k^{ss})$ , and after imposing the symmetry conditions  $k_i = k$  and  $s_i = s$  for all  $i$ , it can be written as follows:

$$\begin{aligned} \dot{s} &= \frac{v(\sigma - 2vwk)}{n+1} - [1 + z(n-1)]k - \delta s \\ \dot{k} &= \frac{2w(\rho + \delta)k - \tau}{2w} \end{aligned} \quad (17)$$

The stability properties of the above system can be assessed via the trace and determinant of the following  $2 \times 2$  Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial \dot{s}}{\partial s} & \frac{\partial \dot{s}}{\partial k} \\ \frac{\partial \dot{k}}{\partial s} & \frac{\partial \dot{k}}{\partial k} \end{bmatrix} = \begin{bmatrix} -\delta & -1 - \frac{2v^2w}{n+1} - z(n-1) \\ 0 & \delta + \rho \end{bmatrix} \quad (18)$$

The trace is  $\mathcal{T}(J) = \rho > 0$  while the determinant is  $\Delta(J) = -\delta(\delta + \rho) < 0$ , therefore the steady state equilibrium is a saddle point. ■

Now note that  $s^{ss} > 0$  for all

$$\tau < \tau_s \equiv \frac{2\sigma vw(\rho + \delta)}{2v^2w + (n+1)[1 + z(n-1)]} > 0 \quad (19)$$

which reveals that any tax rate at least equal to  $\tau_s$  drives the individual and collective volume of polluting emissions to zero in steady state, irrespective of industry structure. Equivalently, taking  $\tau > 0$  in such a way that green R&D activities do take place, one easily verifies that  $s^{ss} = 0$  for all

$$n \geq n_s \equiv \max \left\{ 1, \frac{-\tau + \sqrt{\tau [\tau + 4z(2\sigma vw(\rho + \delta) - \tau(1 + 2v^2w - z))]} }{2\tau z} \right\}. \quad (20)$$

This implies:

**Lemma 3** *A regulator may attain a fully green technology at the steady state in two ways: either by fixing  $\tau \geq \tau_s$  for any given industry structure, or by regulating market access in such a way that  $n \geq n_s$  for any given tax rate  $\tau > 0$ .*

To this regard, it is worth noting that the above Lemma (in particular if read in terms of the industry structure driving  $s^{ss}$  to zero for any given tax rate  $\tau$  on emissions), identifies  $n_s$  as the optimal number of firms in the commons, where the concept of ‘commons’ has to be interpreted as the volume of polluting emissions (or the size of the negative externality generated by them,  $S^2$ ) rather than, as is traditionally the case in the extant literature dating back to Gordon (1954) and Hardin (1968), a common resource pool being overexploited. In view of this analogy, we may ask ourselves whether an optimal number of firms can be identified in this setup, in relation to either the minimization of the volume of polluting emissions or the maximization of social welfare, net of the environmental damage.<sup>6</sup>

However, ‘green’ here means  $s^{ss} = 0$ , but the overall environmental damage  $D^{ss} = \varepsilon n q^{ss}$  is still strictly positive. Alternatively, the authority may tune  $\tau$  so as to minimise

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<sup>6</sup>In adopting this viewpoint, we broadly follow a path opened by Cornes and Sandler (1983), Cornes *et al.* (1986), Mason *et al.* (1988) and Mason and Polasky (1997), where the exploitation of natural resources in oligopoly is considered.

$D^{ss} = \varepsilon n q^{ss} + \gamma (n s^{ss})^2$ . The resulting tax rate is:

$$\tau_D \equiv \frac{2vw(\rho + \delta) [n\sigma(n+1)[1+z(n-1)]\gamma + w(n+1)\delta^2\varepsilon]}{n[2v^2w + (n+1)(1+z(n-1))]\gamma} \quad (21)$$

At  $\tau_D$ , we have that the overall environmental damage  $D$  is strictly positive unless  $\varepsilon = 0$ . A related - and intuitive - result can be outlined by comparing (19) and (21):

**Lemma 4**  $\tau_D > \tau_s$  for all  $\varepsilon > 0$ .

That is, if industry output contributes to the environmental damage, the tax rate minimising  $D^{ss}$  strictly exceeds the tax rate driving steady state emissions  $s^{ss}$  to zero.

The case in which  $\varepsilon = 0$  and the environmental damage coincides with the square of aggregate polluting emissions lends itself to the analysis of the bearings of industry structure on the aggregate level of green R&D in steady state. If indeed  $\varepsilon = 0$ , and  $\tau = \tau_s = \tau_D$ , the industry green effort at equilibrium is

$$K^{ss}(\tau_D)|_{\varepsilon=0} = nk^{ss}(\tau_D)|_{\varepsilon=0} = \frac{\sigma v n}{2v^2w + (n+1)[1+z(n-1)]} \quad (22)$$

with

$$\frac{\partial K^{ss}(\tau_D)|_{\varepsilon=0}}{\partial n} = \frac{\sigma v [1 + 2v^2w - z(n^2 + 1)]}{[2v^2w + (n+1)(1+z(n-1))]^2} \quad (23)$$

The above expression is nil in correspondence of<sup>7</sup>

$$n_K = \frac{\sqrt{1 + 2v^2w - z}}{z} \geq 2 \quad \forall z \in \left[0, \frac{1 + 2v^2w}{5}\right) \quad (24)$$

which implies the following:

**Proposition 5** *If (i)  $\varepsilon = 0$ ; (ii)  $\tau = \tau_D$ ; and (iii) the spillover level characterising firms' green R&D activities is sufficiently low, the aggregate R&D effort at the steady state equilibrium exhibits an inverted-U shape, reaching its maximum at*

$$n_K = \sqrt{\frac{1 + 2v^2w - z}{z}}.$$

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<sup>7</sup>It can be easily checked that

$$\frac{\partial^2 K^{ss}(\tau_D)|_{\varepsilon=0}}{\partial n^2} < 0$$

at  $n = n_K$ . Hence,  $n_K$  indeed maximises  $K^{ss}(\tau_D)$ .

The value of  $K^{ss}(\tau_D)|_{\varepsilon=0}$  in  $n = n_K$  is

$$K^{ss}(\tau_D, n_K)|_{\varepsilon=0} = \frac{\sigma v}{1 + 2\sqrt{z(1 + 2v^2w - z)}}. \quad (25)$$

The above Proposition illustrates a case in which the aggregate innovation incentives of an industry being subject (and reacting) to environmental regulation take the form of an inverted-U curve with a single peak at some  $n > 1$  (as in Aghion *et al.*, 2005, 2013). This finding - interesting in itself as it reveals the presence of an inverted-U shaped aggregate R&D curve - has a relevant consequence, which can be spelled out as follows. The sign of  $n_s - n_K$  is the sign of<sup>8</sup>

$$nz - \sqrt{z(1 + 2v^2w - z)} \quad (26)$$

This establishes that when  $n = n_K$  the expression in (26) is nil and therefore indeed  $n_s = n_K$ , which implies our final result:

**Proposition 6** *If  $\varepsilon = 0$  and  $\tau = \tau_s = \tau_D$ , the number of firms which drives down to zero the volume of individual and aggregate polluting emissions coincides with the number of firms at which the aggregate green R&D curve reaches its unique maximum.*

The above Proposition can be reformulated in alternative but equivalent terms by saying that a public authority in charge of regulating this industry faces no dilemma or tradeoff between the price effect and the external effect when it comes to simultaneously tailoring the pressure of environmental taxation and market access in order to maximise the effectiveness of green R&D on one side and minimise emissions on the other, as - provided aggregate output has no bearing on the environmental impact of these firms - there exists a unique pair  $(n_s = n_K, \tau_D = \tau_s)$  allowing the policy maker to get two eggs in one basket.

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<sup>8</sup>To obtain (26), one has just to plug  $\varepsilon = 0$  and  $\tau = \tau_s = \tau_D$  in

$$n_s = \frac{-\tau + \sqrt{\tau[\tau + 4z(2\sigma vw(\rho + \delta) - \tau(1 + 2v^2w - z))]} }{2\tau z}$$

and then simplify the resulting expression for  $n_s - n_K$ .

## 4 Concluding remarks

We have modified the dynamic Cournot game with environmental effects whose first formulation can be found in Benchekroun and Long (1998), supposing that a public authority adopts a linear taxation scheme by imposing an exogenous tax rate on the individual volume of polluting emissions, rather than taxing each firm in proportion to the environmental damage caused by aggregate emissions.

This construction ensures the presence of strong time consistency under open-loop strategies, a feature which in itself makes the model more easily tractable. As for the economic insight, our modelling choice delivers two main policy conclusions. The first is that to attain a fully green technology in steady state, the regulator is indifferent between adopting an appropriate tax rate (which is uniquely defined for any given number of firms) or regulating entry by identifying the optimal number of firms admitted to the industry (which is also uniquely defined for any given tax rate). The second is that, if the environmental damage depends on emissions only, then the aggregate investment takes in green innovations exhibits an inverted-U shaped curve, and, under the optimal tax rate, the number of firms maximising aggregate R&D coincides with the number of firms driving to zero aggregate emissions.

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