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Allocation of partially renewable resources: Concept, models and applications

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Manuskripte
aus den
Instituten für Betriebswirtschaftslehre
der Universität Kiel

No. 455

**Allocation of Partially Renewable Resources
- Concept, Models and Applications**

Schirmer, Drexl

September 1997

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Abstract: Enterprises facing pressure to reduce costs and to make better use of existing staff or equipment can, at least in part, meet this pressure by more efficient and intelligent planning; methods for this task lie at the heart of management science and operations research. Successful application of such methods depends on unambiguous model formulations; these in turn call for expressive modelling concepts, which allow to capture a wide range of requirements appearing in real-world problems. Throughout the last decade, one particularly successful modelling concept in this regard has been the one of scarce resources. We generalize the classical resource concepts of nonrenewable and renewable resources to partially renewable ones. Using the resource-constrained project scheduling problem as a vehicle, this paper intends to demonstrate different ways to make use of the new concept, in terms of model formulations as well as of practical applications.

Keywords: PARTIALLY RENEWABLE RESOURCES; PROJECT MANAGEMENT / SCHEDULING; MODELLING LOGICAL RELATIONS/PRACTICAL APPLICATIONS

1. Introduction

Enterprises are and have been facing mounting pressures to exercise reductions in the costs arising from producing their goods or services and to make better use of existing staff or equipment. It is well-known from practical experience that this pressure can be met, at least in part, by more efficient and intelligent planning; methods for this task lie at the heart of management science and operations research. Successful application of these methods, however, depends to a large degree on the ability to unambiguously and efficiently model the relevant specifics of the problems tackled (Drexl et al. 1995). This ability, in turn, calls for expressive modelling concepts, which allow to capture a wide range of requirements appearing in real-world problems.

Throughout the last decade, one particularly successful modelling concept in this regard has been the one of scarce resources (Blazewicz et al. 1983; Blazewicz et al. 1986). Indeed, the scheduling of scarce resources to individual activities (or jobs) over time is a recurring problem for managers at all hierarchical levels. In scheduling theory, resources are classified according to their availability; usually, two types are distinguished, viz. nonrenewable and renewable resources. We examine the new type of partially renewable resources and address the generalization of the classical resource-constrained project scheduling problem (RCPSp) to the case of partially renewable resources (RCPSp/PI). Our choice of this problem as the vehicle of study is motivated by the fact that, although resource scheduling is used in a number of research areas, project scheduling is and has been the most active one of these. While solution methods for the RCPSp/PI are discussed elsewhere (Böttcher et al. 1996), this paper intends to demonstrate different ways to make use of the new concept, in terms of model formulations as well as of logical and practical applications.

Accordingly, the remainder of this work is organized as follows. In Section 2 we explicate the RCPSp/PI and investigate its positioning in the context of project scheduling as well as its complexity. In Section 3 we suggest several alternative model formulations and examine under

which circumstances one might be more appropriate than the other. In Section 4 a variety of requirements expressible in terms of partially renewable resources is described; this list is complemented by citing real-world problems where these requirements do arise. Some final remarks in Section 5 conclude the contribution.

2. Partially Renewable Resources

In the sequel, we will examine the resource-constrained project scheduling problem under partially renewable resources (RCPSP/PI), discussing several interesting properties in detail. In Section 2.1, we explain the ideas underlying the concept of partially renewable resources, while a formal presentation of the problem is presented in Section 2.2. In Section 2.3 we prove that partially renewable resources generalize traditional resource concepts; in Section 2.4 we show the RCPSP/PI to be NP-equivalent in the strong sense. In Section 2.5 finally, we address some properties specific to the new resource type.

2.1. Motivation and Concept

To motivate the concept of partially renewable resources, let us consider a (rather small) example from the field of personnel scheduling. Acme Burgers, Inc. is a catering company facing the task to assign staff to weekly schedules. As the planning is done on the basis of half days, each day is divided into an early and a late shift. A corresponding model for this situation may comprise a planning horizon of 14 periods, each period representing one shift.

For the sake of simplicity, let us assume that Acme possesses a workforce of merely three. Each of these holds a different form of labor contract, one being a full-time employee, one a part-time employee, and one a freelancer. Both the part-time employee and the freelancer are available on working days and weekends alike. As their assignments can be distributed arbitrarily over the week, they may be represented in terms of two renewable resources with a capacity of one. The full-time employee, in contrast, is working on weekdays only, so he could be represented in terms of a renewable resource with a time-varying per-period capacity of one for all working day shifts and zero capacity for the weekend shifts. Now, let us also assume that contractual or legislative regulations limit the weekly workload to four days for the full-time, two-and-a-half days for the part-time employee, and five days for the freelancer. To account for this, the model could be augmented by additional nonrenewable resources for the employees, with a capacity of eight, five, and ten periods, respectively.

In this fashion, a variety of different restrictions concerning the availability of resources such as labor can be expressed using the classical resource concepts only. Add, however, on top of this a union agreement stipulating that no employee, be it full- or part-time, may work on Saturday *and* Sunday such that at least one weekend day must be off; no such restriction pertains to the freelancer. This requirement can be formulated with neither of the classical resource types because these are unable to define capacity limitations holding for several periods in total, i.e. for subsets of periods. It is this deficiency that can be overcome by partially renewable resources since they allow to define capacity limitations over arbitrary subsets of periods. In general, partially renewable resources may be characterized by the following assumptions:

- All activities have to be processed within a number of T periods.
- There is a number S of so-called period subsets Π_s ($1 \leq s \leq S$) each of which is a subset of the set of all periods.
- Processing the activities requires the presence of one or several partially renewable resources p ; the demand for resource p entailed by activity j is given by k_{jp} .
- For each resource p ($1 \leq p \leq P$) and each period subset Π_s ($1 \leq s \leq S$), the amount which is available over all periods of that subset is limited by K_{ps} .

Now the above example can be couched as follows:

- We have a number of $T = 14$ shifts.
- There are three partially renewable resources representing the full-time employee ($p = 1$), the part-time employee ($p = 2$), and the freelancer ($p = 3$).
- The 19 period subsets Π_s employed are defined as follows: The first 14 period subsets allow to formulate the capacity limitations over the individual shift, while the next subset comprises all shifts of the week and thus allows to represent limitations over the whole planning horizon. The remaining subsets are made up of all possible combinations of a Saturday with a Sunday shift (cf. Figure 1 where the shifts comprised in a subset are shaded).
- The resource capacities are set as shown in Table 1. To ensure that staff is scheduled no more than once per shift, $k_{jp} = 1$ is used for each assignment j and $K_{ps} = 1$ ($1 \leq p \leq 3$; $1 \leq s \leq 14$), except for $K_{1s} = 0$ ($11 \leq s \leq 14$) which prevent the full-time employee from working on the weekend at all. The maximum workloads per week are guaranteed by $K_{p,15}$. Finally, $K_{ps} = 1$ ($1 \leq p \leq 2$; $16 \leq s \leq 19$) implies that no employee can be scheduled to any Sunday shift when he is assigned to at least one Saturday shift and vice versa.

By mutually excluding such assignments the requirement of at least one weekend day off is met. Note that setting $K_{1p} = 1$ ($16 \leq p \leq 19$) is redundant since $K_{1p} = 0$ ($11 \leq p \leq 14$).

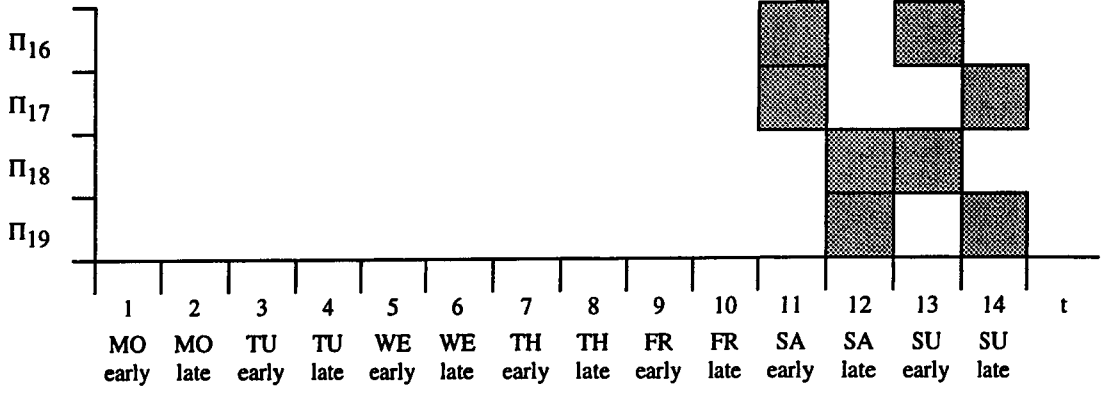


Figure 1: All Combinations of Saturday and Sunday Shifts

It is easy to see that the limitations of the partially renewable resource over the first 14 period subsets are equivalent to the limitations of the before-mentioned renewable resource while period subset 15 serves the purpose of a nonrenewable resource. Only the last four subsets are actually related to a partially renewable resource in the strict sense. We will show later on that renewable and nonrenewable resources can generally be expressed in terms of partially renewable ones.

Subset Index s	Period Subset Π_s	Capacity			Resource Type
		K_{1s}	K_{2s}	K_{3s}	
1	{1}	1	1	1	renewable
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
10	{10}	1	1	1	renewable
11	{11}	0	1	1	renewable
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
14	{14}	0	1	1	renewable
15	{1,...,14}	8	5	10	nonrenewable
16	{11, 13}	1	1	2	partially renewable
17	{11, 14}	1	1	2	partially renewable
18	{12, 13}	1	1	2	partially renewable
19	{12, 14}	1	1	2	partially renewable

Table 1: Exemplary Period Subsets and Capacities

2.2. Problem Setting

Apart from the above assumptions which pertain to partially renewable resources, we employ the usual parameters of the RCPSp. All problem parameters are summarized (in alphabetical order) in Table 2.

W.l.o.g. the parameters P and S as well as all d_j are assumed to be nonnegative integers, J and T to be positive integers, k_{jp} and K_{ps} to be integers. These restrictions entail no loss of generality since they are equivalent to allowing rational numbers, i.e. fractions, and multiplying them with the smallest common multiple of their denominators. Let denote $J = \{1, \dots, J\}$ the set of all activities and $T = \{1, \dots, T\}$ the set of all periods. Finally, all Π_s are assumed to be subsets of the set T .

The goal is to find an assignment of periods to all activities (a *schedule*) that ensures for each partially renewable resource p and each period subset Π_s that the total consumption of p by all activities performed in that period subset does not exceed the total capacity of p within Π_s , respects the partial order \angle , and minimizes the total project length.

Model Parameter	Definition
d_j	Non-preemptable duration of activity j
J	Number of activities, indexed by j
k_{jp}	Per-period consumption of partially renewable resource p required to perform activity j
K_{ps}	Total availability of partially renewable resource p over all periods of Π_s ($1 \leq s \leq S$)
P	Number of partially renewable resources, indexed by p
Π_s	Specific period subset ($1 \leq s \leq S$)
S	Number of period subsets, indexed by s
T	Number of periods, indexed by t
\angle	Partial order on the activities, representing precedence relations

Table 2: Problem Parameters of the RCPSp/ Π

To simplify matters, it is assumed w.l.o.g. that activity 1 is the unique first and activity J the unique last activity w.r.t. \angle , i.e. $1 \angle j$ and $j \angle J$ ($2 \leq j \leq J-1$). The fictitious activities 1 and J are called *dummy activities*, meaning that $d_1 = d_J = 0$ and $k_{1p} = k_{Jp} = 0$ ($1 \leq p \leq P$). We also follow common practice by deriving some additional parameters from the above problem parameters; though not necessary prerequisites, they usually allow to reduce the number of variables within some of the constraints. First, let denote P_j ($1 \leq j \leq J$) the set of all *immediate*

predecessors of activity j w.r.t. \angle . Second, for each activity j ($1 \leq j \leq J$) earliest finish times EFT_j and latest finish times LFT_j may be calculated by traditional forward and backward recursion, the latter starting at time T .

Using the above parameters, a schedule can be represented by integer variables y_j ($1 \leq j \leq J$) denoting the period in which activity j is completed. In addition, we denote by A_t ($1 \leq t \leq T$) the set of all activities which are active (cp. Talbot and Patterson 1978) in period t and thus may consume scarce resources.

Then the RCSP/PI can be described in the following way:

Minimize

$$Z(y) = y_J \quad (1)$$

subject to

$$y_i \leq y_j - d_j \quad (2 \leq j \leq J; i \angle j) \quad (2)$$

$$\sum_{t \in \Pi_s} \sum_{j \in A_t} k_{jp} \leq K_{ps} \quad (1 \leq p \leq P; 1 \leq s \leq S) \quad (3)$$

$$y_j \in \{EFT_j, \dots, LFT_j\} \quad (1 \leq j \leq J) \quad (4)$$

Minimization of the objective function (1) enforces the earliest possible completion of the last activity J and thus leads to the minimal schedule length. The precedence constraints (2) guarantee that the precedence order is respected while the capacity constraints (3) limit the total resource consumption of each partially renewable resource in each period subset to the available amount.

This formulation of the RCSP/PI fully and correctly describes the properties of an optimal solution. We should emphasize, however, that this formulation bears a substantial deficiency: It cannot be tackled with standard solvers for combinatorial optimization problems, such as LINDO, OSL, or CPLEX. This is due to the fact that determining the sets A_t for the optimal solution in advance, which is clearly a necessary prerequisite for the application of any standard solver, is impossible to do since these can be calculated only from the optimal schedule. In other words, the A_t can be known no earlier than the optimal solution such that formulating this model in a way suited for standard solvers would require prior determination of the optimum¹.

¹ Talbot and Patterson (1978) as well as Drexel et al. (1995) refer to this kind of formulation as a conceptual model.

2.3. Relation to Other Resource Concepts

This section positions partially renewable resources in the context of traditional resource types. Let denote N (R) the number of nonrenewable (renewable) resources.

2.3.1. Nonrenewable Resources

A nonrenewable resource n ($1 \leq n \leq N$) with capacity K_n can be expressed in terms of a partially renewable resource p where $\Pi_p = \{1, \dots, T\}$ and $K_p = K_n$. In this case the above constraints (3) translate to the well-known nonrenewable resource constraints

$$\sum_{j=2}^{J-1} k_{jn} \sum_{t=1}^T x_{jt} \leq K_n \quad (1 \leq n \leq N) \quad (5)$$

2.3.2. Renewable Resources

A renewable resource r ($1 \leq r \leq R$) with capacity K_r can be expressed in terms of T partially renewable resources p where $\Pi_p = \{p\}$ and $K_p = K_r$ ($1 \leq p \leq T$). In this case the above constraints (3) translate to the well-known renewable resource constraints

$$\sum_{j=2}^{J-1} k_{jr} \sum_{q=t}^{t+d_j-1} x_{jq} \leq K_r \quad (1 \leq r \leq R; 1 \leq t \leq T) \quad (6)$$

2.3.3. Renewable Resources with Time-Varying Capacities

The canonical extension of fixed to time-varying period capacities leads to capacity profiles of renewable resources². A renewable resource r ($1 \leq r \leq R$) with time-varying capacities K_{rt} ($1 \leq t \leq T$) can be expressed in terms of T partially renewable resources p where $\Pi_p = \{p\}$ and $K_p = K_{rp}$ ($1 \leq p \leq T$). In this case, the above constraints (3) translate to the corresponding renewable resource constraints

$$\sum_{j=2}^{J-1} k_{jr} \sum_{q=t}^{t+d_j-1} x_{jq} \leq K_{rt} \quad (1 \leq r \leq R; 1 \leq t \leq T) \quad (7)$$

As a consequence, when modelling resource-constrained scheduling problems it suffices to restrict oneself to using partially renewable resources only since all other types can be expressed

² Cf. Bartusch et al. (1988); Drexl, Grünwald (1993). Of course, this extension is not applicable to nonrenewable resources, since these are capacitated only in total.

in terms of them. Hence, this resource type may be seen as unifying since it combines the expressive power of the traditional resource types.

2.3.4. *Partially Renewable Resources of Icmeli and Rom*

Recently, Icmeli and Rom (1996) proposed a new modelling concept for the RCPSP, using integer variables to denote the completion times of the activities j . In contrast to traditional models, which use an equidistant discretization into 'small buckets' of one period length each, they define equidistant milestones on the time line, thus dividing it into fixed length 'large buckets' of several periods length. Large bucket models generally use less variables since they consider temporal information on a more aggregate level than small bucket ones. To account for the fact that most activities are completed between two milestones, completion times are expressed as convex linear combinations of the nearest two milestones. Precedence and resource constraints are then formulated in terms of these convex combinations. Resources are assumed to be allocated to the intervals between adjacent milestones: Within each interval they are regarded as nonrenewable but they renew at the beginning of each interval. The authors refer to these resources as partially renewable ones.

Though bearing the same name, this concept substantially differs from partially renewable resources as we understand them. Establishing equidistant milestones essentially defines macro periods, as opposed to the micro periods in which e.g. activity durations are measured. In order to avoid addressing these micro periods explicitly, the above convex combinations are used; the only explicit temporal unit is the macro period between two milestones. Consequentially, the partially renewable resources of Icmeli and Rom are in effect traditional renewable resources as they renew at the beginning of each (macro) period. In other words, the innovation in their approach lies not in a new resource concept but in aggregating periods in order to reduce the computational effort of standard solvers which in turn necessitates expressing all events within these (macro) periods as convex combinations of their respective boundaries.

2.4. *Complexity Results*

The RCPSP/ Π belongs to the class of strongly NP-equivalent problems. The first step in showing this is to prove the corresponding feasibility problem RCPSP/ Π^{feas} to be strongly NP-complete. The proof relies upon the *3-partition problem* (3PP) which is well-known to be strongly NP-complete (Garey and Johnson 1975):

3-PARTITION (3PP) Given $L \in \mathbb{N}$, $S = \{1, \dots, 3 \cdot L\}$, $B \in \mathbb{N}$, and $\mu_j \in \mathbb{N}$ ($1 \leq j \leq 3 \cdot L$) where

$$B/4 < \mu_j < B/2 \quad (1 \leq j \leq 3 \cdot L) \quad (8)$$

$$\text{and } \sum_{j \in S} \mu_j = L \cdot B \quad (9)$$

is there a partition of S into L disjoint subsets S_i ($1 \leq i \leq L$) such that

$$\sum_{j \in S_i} \mu_j = B \quad (1 \leq i \leq L) \quad (10)$$

holds true? Note that the conditions on the μ_j ($1 \leq j \leq 3 \cdot L$) imply that $|S_i| = 3$ ($1 \leq i \leq L$). Under the usual, standard encoding (STD) of instances (Schirmer 1995), the magnitude and length of an instance I are $\text{MAX}_{\text{STD}}(I) \approx O(B)$ and $\text{LNG}_{\text{STD}}(I) \approx O(L \cdot \log B)$.

Theorem 1: The feasibility variant of the RCPSP/ Π is **NP**-complete in the strong sense.

Proof: (RCPSP/ $\Pi^{\text{feas}} \in \text{NP}$) Let $J \in \mathbb{N}$, $d_1 = d_J = 0$, $d_j \in \mathbb{N}$ ($2 \leq j \leq J-1$), $P \in \mathbb{N}$, $k_{jp} \in \mathbb{N}_0$ ($1 \leq j \leq J$; $1 \leq p \leq P$), $S \in \mathbb{N}$, $K_{ps} \in \mathbb{N}$ ($1 \leq p \leq P$; $1 \leq s \leq S$), $T \in \mathbb{N}$, $\Pi_{ps} \subseteq \{1, \dots, T\}$, and \angle a partial order on $\{1, \dots, J\}$ constitute an arbitrary instance I of RCPSP/ Π^{feas} . Letting denote $K^{\max} = \max\{K_{ps} \mid 1 \leq p \leq P; 1 \leq s \leq S\}$, the following bounds are straightforward (even though tighter bounds could be derived for the d_j , these will be sufficient for our purposes):

$$d_j \leq T \quad (1 \leq j \leq J) \quad (11)$$

$$k_{jp} \leq K^{\max} \quad (1 \leq j \leq J; 1 \leq p \leq P) \quad (12)$$

$$K_{ps} \leq K^{\max} \quad (1 \leq p \leq P; 1 \leq s \leq S) \quad (13)$$

$$|\angle| \leq J^2 \quad (14)$$

Thus, $\text{MAX}_{\text{STD}}(I) \approx O(K^{\max})$ and $\text{LNG}_{\text{STD}}(I) \approx O(J^2 \cdot P \cdot S \cdot T \cdot \log K^{\max})$. Since a certificate of the RCPSP/ Π^{feas} consists of one value - not larger than T - for each of the J decision variables, reading a certificate has a time complexity of $O(J \cdot \log T)$. Evaluating the constraints requires one addition and one comparison for each of the at most $J(J-1)/2$ precedence constraints (2) and at most $J \cdot T$ additions and one comparison for each of the $P \cdot S$ resource constraints (3), totalling $O(J^2 + J \cdot P \cdot S \cdot T) \approx O(J^2 \cdot P \cdot S \cdot T)$ operations; assuming each operation to take constant time, also the complexity of the evaluation is $O(J^2 \cdot P \cdot S \cdot T)$. So, it can be verified in polynomial time that the schedule satisfies the constraints.

(3PP \propto RCPSP/ Π^{feas}): Let $L \in \mathbb{N}$, $S = \{1, \dots, 3 \cdot L\}$, $B \in \mathbb{N}$, $\mu_j \in \mathbb{N}$ ($1 \leq j \leq 3 \cdot L$) constitute an arbitrary instance of 3PP. Then construct an instance of RCPSP/ Π^{feas} as follows: $J = 3 \cdot L$, $d_j = 1$ ($1 \leq j \leq J$), $P = 1$ (we thus omit its index), $k_j = \mu_j$ ($1 \leq j \leq J$), $S = T = L$, $\Pi_s = \{s\}$ ($1 \leq s \leq S$), $K_s = B$ ($1 \leq s \leq S$), and \angle empty (cf. Figure 2).

$K_s = B$	$k_1 = \mu_1$	\vdots	\vdots	\vdots	\vdots	$K_s/4 < k_j < K_s/2$ $B/4 < \mu_j < B/2$
	$k_2 = \mu_2$	\vdots	$k_j = \mu_j$	\vdots	$k_{J-1} = \mu_{3L-1}$	
	\vdots	\vdots	\vdots	\vdots	$k_J = \mu_{3L}$	
	$\underbrace{\hspace{1cm}}_{S_1}$	\dots	$\underbrace{\hspace{1cm}}_{S_i}$	\dots	$\underbrace{\hspace{1cm}}_{S_L}$	

Figure 2: 3PP Transforms to RCPSP/ Π^{feas}

To prove this transformation as pseudo-polynomial, let us first argue that there exists a feasible schedule for the constructed instance of RCPSP/ Π^{feas} iff there is a partition of S as described above. Assuming that there exists such a partition of S , we can feasibly schedule all activities in S_i ($1 \leq i \leq L$) to period i . Since the S_i form a partition of S , this procedure covers all activities, hence the domain constraints (4) are met. Due to \angle empty there are no precedence constraints (2). Finally, (10) translates to

$$\sum_{t \in \Pi_s} \sum_{j \in A_t} k_j = K_s \quad (1 \leq s \leq S) \quad (15)$$

so the schedule also satisfies the resource constraints (3). Conversely, if such a feasible schedule exists, a partition of S as above can be obtained by putting together in S_i ($1 \leq i \leq L$) all elements j ($1 \leq j \leq 3 \cdot L$) scheduled to period i . Due to (4) this yields a partition, and due to (3) also the desired property (10) holds. The transformation can be performed in pseudo-polynomial, even in polynomial time since each of the $O(L)$ assignments takes $O(\log B)$ time. As all values of the 3PP-instance form part of the derived instance, the transformed instance will not be of categorially smaller length. Finally, the largest number occurring in the constructed instance, B , is polynomial in length and magnitude of the 3PP-instance since B is also the largest number in the 3PP-instance. So, the above transformation from 3PP to RCPSP/ Π^{feas} is indeed pseudo-polynomial which implies the strong NP-completeness of RCPSP/ Π^{feas} . ■

Building upon this result, it is easy (although lengthy) to show that the RCPSP/ Π is strongly NP-hard as well as strongly NP-easy by exhibiting pseudo-polynomial reductions from the RCPSP/ Π^{feas} to the RCPSP/ Π and vice versa. We therefore state the following theorem without proof; details of the proof technique involved are described in Schirmer (1995), a similar proof for the RCPSP can be found in Schirmer (1996).

Theorem 2 The RCPSP/ Π is strongly NP-equivalent.

2.5. *Special Properties*

Nonrenewable resources n are capacitated over the complete planning horizon, therefore the exact time when an activity j starts has no bearing on the total activity demand k_{jn} incurred. The capacity of renewable resources r , in contrast, is limited for each period separately, while the total activity demand of j for r is $k_{jr} \cdot d_j$. Hence the feasibility of a schedule depends on the periods to which the activities are assigned: If two activities overlap in a period, they may overload a resource in that period, requiring to delay one activity until the other terminates, freeing capacities again. Large capacities allow for more parallel execution of activities while small capacities entail more sequential execution. In this sense, the total demand for and thus the remaining capacities of nonrenewable resources are independent from the corresponding schedule. For renewable resources, however, only the individual activity demand k_{jr} and the total activity demand $k_{jr} \cdot d_j$ are schedule-independent whereas the total demand for a specific resource r in a period t and thus the corresponding remaining capacities are schedule-dependent.

Partially renewable resources are capacitated over their respective period subsets only; in all other periods of the planning horizon there are no limitations. Also in the presence of partially renewable resources the feasibility of a schedule is determined by the position of the activities on the time axis: Two activities overlapping within a period subset may require more than the capacity of the corresponding resource; thus scarce resources may delay certain activities into uncapacitated periods, outside the period subsets. Consequentially, although the total activity demand in principle is $k_{jp} \cdot d_j$, only that part which falls into capacitated periods, i.e. within a period subset, is relevant for algorithmic purposes. Hence, attention is restricted from the total activity demand to the relevant demand RD_{jpt} of activity j for partially renewable resource p when started in period t .

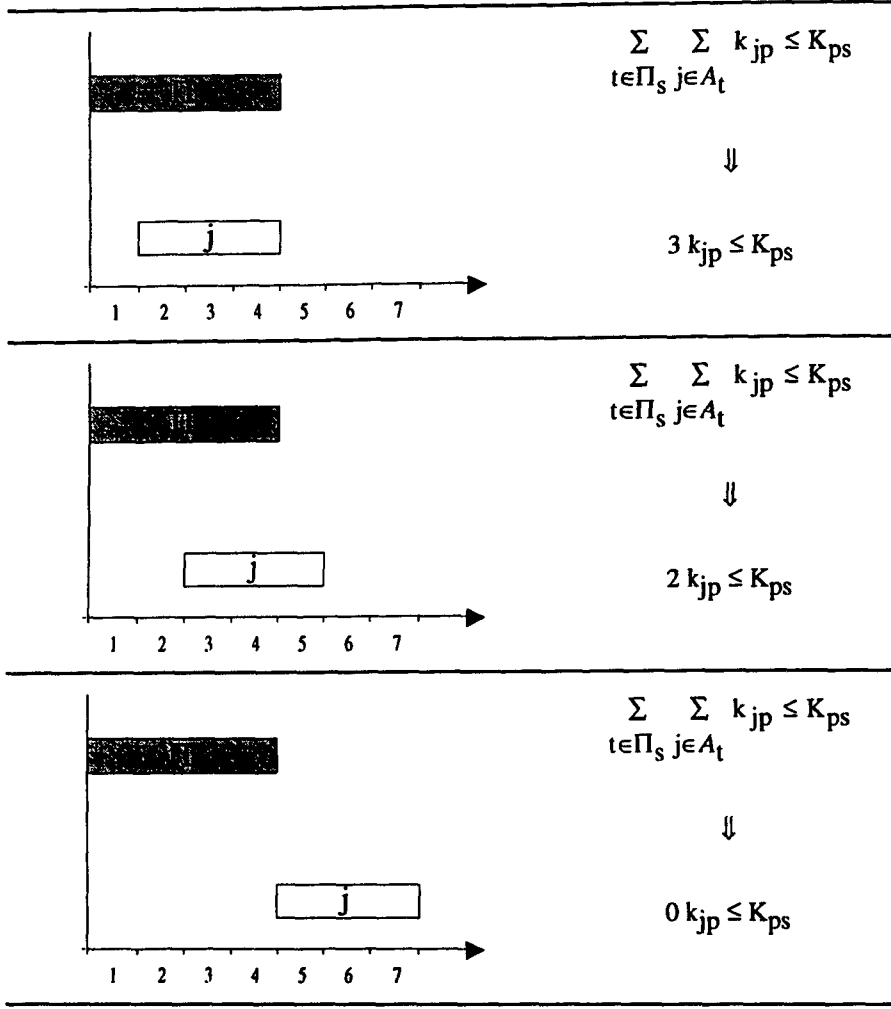


Figure 3: Total Activity Demand for Partially Renewable Resources

For notational brevity let Q_{jt} denote those periods of T in which (non-dummy) activity j would be active were it started in period t , i.e.

$$Q_{jt} = \{t, \dots, t+d_j-1\} \quad (2 \leq j \leq J-1; 1 \leq t \leq T) \quad (16)$$

Then, the relevant demand can be calculated from

$$RD_{jpt} = k_{jp} \cdot |Q_{jt} \cap \Pi_s| \quad (2 \leq j \leq J-1; 1 \leq p \leq P; EST_j \leq t \leq LST_j; 1 \leq s \leq S) \quad (17)$$

as soon as earliest and latest start times EST_j and LST_j are determined in the usual fashion for each activity j . In this regard, relevant demand RD_{jpt} and thus remaining capacities are schedule-dependent although the individual activity demand k_{jp} is schedule-independent. An example is shown in Figure 3 where scheduling activity j to different periods results in different relevant demand for a partially renewable resource, depending on whether all, some, or none of the active periods fall into the corresponding period subset.

A comprehensive overview of resource demands and remaining capacities³ for the different resource types is provided in Table 3, where ST_j denotes the starting period of activity j .

Resource Type	Nonrenewable	Renewable	Partially Renewable
Individual Activity Demand	k_{jn} <i>schedule-independent</i>	k_{jr} <i>schedule-independent</i>	k_{jp} <i>schedule-independent</i>
Total Activity Demand	k_{jn} <i>schedule-independent</i>	$k_{jr} \cdot d_j$ <i>schedule-independent</i>	$RD_{jpt} = k_{jp} \cdot Q_{jt} \cap \Pi_s $ <i>schedule-dependent</i>
Total Demand	$\sum_{j=1}^J k_{jn}$ <i>schedule-independent</i>	$\sum_{j \in A_t} k_{jr}$ <i>schedule-dependent</i>	$\sum_{j=1}^J RD_{j,p,ST_j}$ <i>schedule-dependent</i>
Remaining Capacities	$RK_n = K_n - \sum_{j=1}^J k_{jn}$ <i>schedule-independent</i>	$RK_r = K_r - \sum_{j \in A_t} k_{jr}$ <i>schedule-dependent</i>	$RK_{ps} = K_{ps} - \sum_{t \in \Pi_s} \sum_{j \in A_t} k_{jp}$ <i>schedule-dependent</i>

Table 3: Resource Concepts

3. Model Formulations

In this section, we cast the RCPSP/ Π in terms of three different models. In contrast to the above problem formulation, these are binary optimization models which can be solved by standard solvers - at least for smaller instances. The models, which all are equivalent in the sense that they describe the same sets of feasible and optimal solutions⁴, are presented in Sections 3.1 through 3.3. Section 3.4 addresses the differences between the models and their respective suitability for different situations.

³ Note that equivalently the remaining capacity of a partially renewable resource p can also be calculated from

$$RK_{ps} = K_{ps} - \sum_{t \in \Pi_s} \sum_{j \in A_t} k_{jp}$$

as done implicitly in capacity constraints (3) of the problem setting.

⁴ A formal definition of the equivalence of models is discussed in Brüggenmann (1995, pp. 112-117). We omit the proofs as the equivalence of the models is obvious.

3.1. Simple Model

This model is a straightforward representation of the above problem setting. Using the conceptualization of Table 2, the assignment of activities to periods can be represented by binary variables x_{jt} ($1 \leq j \leq J$; $0 \leq t \leq T$) where

$$x_{jt} = \begin{cases} 1 & \text{if activity } j \text{ is completed in period } t \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

allows to formulate a binary optimization model - using the general framework given in Pritsker, Watters, and Wolfe (1969) - as follows:

Minimize

$$Z(x) = \sum_{t=EFT_J}^{LFT_J} t x_{Jt} \quad (19)$$

subject to

$$\sum_{t=EFT_j}^{LFT_j} x_{jt} = 1 \quad (1 \leq j \leq J) \quad (20)$$

$$\sum_{t=EFT_i}^{LFT_i} t x_{it} \leq \sum_{t=EFT_j}^{LFT_j} (t - d_j) x_{jt} \quad (2 \leq j \leq J; i \in P_j) \quad (21)$$

$$\sum_{j=2}^{J-1} k_{jp} \sum_{t \in \Pi_s} \sum_{q \in Q_{jt}} x_{jq} \leq K_{ps} \quad (1 \leq p \leq P; 1 \leq s \leq S) \quad (22)$$

$$x_{jt} \in \{0, 1\} \quad (1 \leq j \leq J; 0 \leq t \leq T) \quad (23)$$

Minimization of the objective function (19) enforces the earliest possible completion of the last activity J and thus leads to the minimal schedule length. The activity completion constraints (20) guarantee that each activity is executed once and completed within the interval $[EFT_j, LFT_j]$. The precedence constraints (21) represent the precedence order. The capacity constraints (22) limit the total resource consumption of each partially renewable resource in each period subset to the available amount.

3.2. Refined Model

The refined model differs from the simple one in that for each resource only those period subsets are considered over which it is actually limited. Hence, for each resource p S_p denotes the number of corresponding period subsets. If each resource is capacitated over all period subsets, then the total number of subsets will remain the same. If, on the other hand, some resources are actually capacitated only over some of the period subsets then less information needs to be stored since uncapacitated resource-period subset combinations are not considered.

The changed parameters are summarized (in alphabetical order) in Table 4.

Model Parameter	Definition
K_{ps}	Total availability of partially renewable resource p over all periods of Π_{ps} ($1 \leq s \leq S_p$)
Π_{ps}	Specific period subset associated with resource p ($1 \leq s \leq S_p$)
S_p	Number of period subsets associated with resource p , indexed by s

Table 4: Changed Parameters of the Refined RCPSP/ Π -Model

Finally, for notational brevity let F_j denote those periods of T in which (non-dummy) activity j could be finished, i.e.

$$F_j = \{EFT_j, \dots, LFT_j\} \quad (2 \leq j \leq J-1) \quad (24)$$

Now, the resource constraints (22) transform to

$$\sum_{j=2}^{J-1} k_{jp} \sum_{t \in \Pi_{ps}} \sum_{q \in Q_{jt} \cap F_j} x_{jq} \leq K_{ps} \quad (1 \leq p \leq P; 1 \leq s \leq S_p) \quad (25)$$

3.3. Normalized Model

In order to avoid the tedious addressing of period subsets via a resource *and* a period subset index which is necessary in the refined model, the distinction between the different resources with which period subsets are associated is dropped here. This can be done by replacing each combination of a resource and a period subset (note that several resources may be limited over the same subset) by one separate, "new" resource capacitated over that subset. This process is called normalization (Böttcher 1995, p. 27). Afterwards, each normalized resource is defined

over exactly one period subset such that the number of subsets equals the number of resources. If no two resources are capacitated over the same period subset, then the resulting number of subsets will remain the same. If, on the other hand, some resources share the same period subsets then the total number of subsets to be considered will actually reduce. In any case, doing so allows to omit the parameter S_p as well as the resource index p from Π_{ps} . However, the intuitivity of resources reduces since they do no longer correspond to real-world entities: One resource does not necessarily represent one employee.

The changed parameters are summarized (in alphabetical order) in Table 5.

Model Parameter	Definition
K_p	Total availability of partially renewable resource p over all periods of Π_p
Π_p	The period subset associated with resource p

Table 5: Changed Parameters of the Normalized RCPSP/ Π -Model

Now, the resource constraints (22) transform to

$$\sum_{j=2}^{J-1} k_{jp} \sum_{t \in \Pi_p} \sum_{q \in Q_{jt} \cap F_j} x_{jq} \leq K_p \quad (1 \leq p \leq P) \quad (26)$$

3.4. Assessment

In order to assess the suitability of the different models for different situations, consider the example summarized in Table 6 where two resources are differently capacitated over one common and two different period subsets.

Resource	Period Subset	Capacity
1	{3, 4}	7
	{4, 5}	4
2	{4, 5}	2
	{5, 6}	3

Table 6: Example on RCPSP/ Π -Models

Representing this (part of a) planning situation in terms of the different period subset and capacity types presented leads to the instantiations shown in Figure 4.

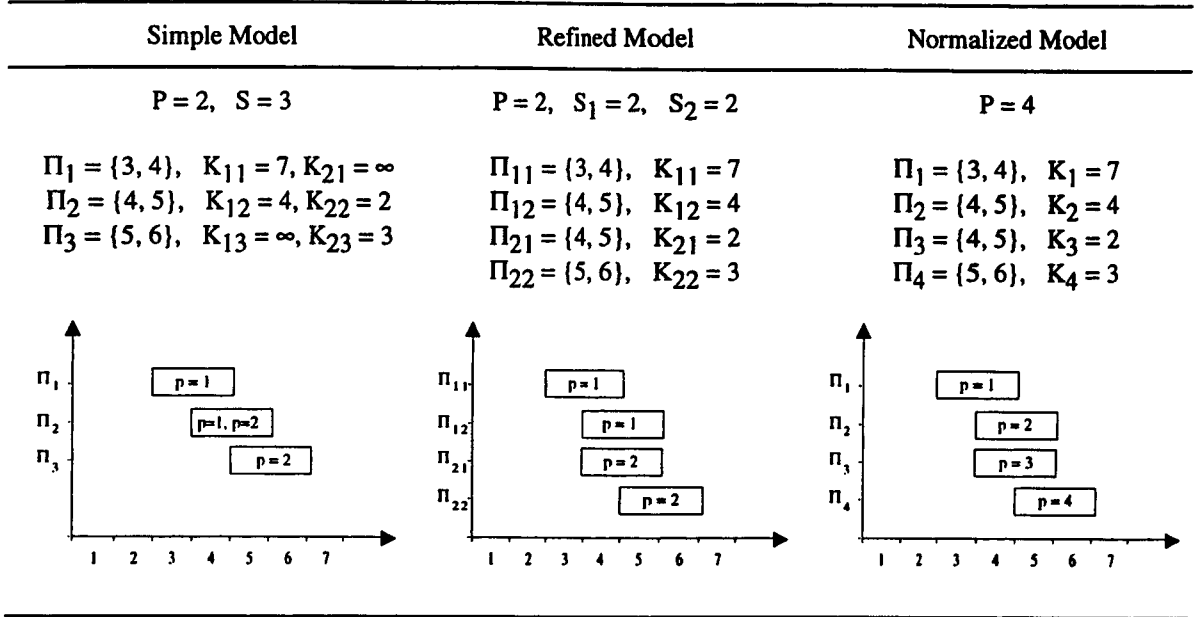


Figure 4: Exemplary Instantiations of the RCPSP/ Π -Models

It is evident that the capacity parameters of the simple model require redundant information to be stored for uncapacitated combinations of period subsets and resources. In addition, the model needs two indices (p, s). The interpretation of the resources, on the other hand, is straightforward since they correspond directly to whatever real-world objects they represent. The situation is somewhat reversed for the normalized model. No redundant capacity values need to be stored. Also, any formal treatment of the model requires only one index (p). Yet, the interpretation of the resources is less than straightforward because each normalized resource corresponds to one specific combination of resource and period subset in the simple model and thus in the real world. The refined model provides a compromise between the above models. It maintains an intuitive interpretation of resources, avoids redundant value storing; these advantages come at the price of having to use two indices (p, s).

Consequently, the simple model is more appropriate if most resources are capacitated over the same period subsets (which obviates the need for redundancies) whereas the refined model is more suitable whenever most resources are capacitated over different, dedicated subsets. Due to the lack of a second index, the normalized model is particularly suited for formal purposes; we will therefore use the normalized model in the sequel.

4. Modelling Capabilities

In this section we provide some suggestions on how partially renewable resource constraints can be employed. In an interesting example on partially renewable resources, Böttcher (1995, pp. 22-26) uses them to formulate the famous eight-queens-problem of chess, the basic idea being to transform logical conditions on the positioning of the queens into equivalent resource constraints. Taking up this idea, we systematically explore the expressive power of the new resource concept. To account for the wide range of possible applications, we start from a theoretical point of view: We briefly cover the interpretation of logical relations in Section 4.1 and then, in Section 4.2, we exhibit a number of logical relations which can be couched in terms of partially renewable resources. To underscore their practical relevance, we offer a number of exemplary real-world applications of such relations in Section 4.3.

4.1. Interpreting Logical Relations

Before starting to demonstrate how to formulate logical relations by partially renewable resources, let us first of all motivate the recourse to logical (or boolean) relations. Probably most obvious is the correspondence between boolean (having domain {"true", "false"}) variables used in propositional logic and binary (having domain {0, 1}) ones. Yet also the correspondence between binary and integer variables, both widely used in combinatorial optimization, is in many cases straightforward. To see this, recall that attached to all of these is usually a specific meaning or interpretation, indicating what situation is represented by a specific value of a specific variable. Then binary variables may simply provide a dissected representation of a situation otherwise represented by integer variables.

For the RCPSP/II, for example, any $x_{jt} = 1$ stands for an activity j being completed in a period t , the same situation modeled by setting $y_j = t$. Keeping this in mind, it is straightforward to translate the interpretation (computer science here prefers the term semantics) of logical relations to (binary as well as integer) decision variables and thus to the situations represented. Consider, for instance, the conjunction of several boolean variables which holds if all the variables share the value "true"; in other words, the relation simply requires several variables to hold true. In terms of project scheduling, where we essentially deal with activities being either processed or not in certain periods, the analogous interpretation is that activity j is processed in period t whenever the binary decision variable x_{jt} possesses the value one in a period t' or the integer decision variable y_j equals t' where $t \leq t' \leq t+d_j-1$. It is in this way that we can interpret logical relations on decision variables as representing requirements on the processing of activities.

4.2. Modelling Logical Relations

4.2.1. Logical Identity

Logical identity is a unary boolean relation which holds if the corresponding variable holds true. We thus interpret identity to require a particular job to be processed in some specified periods; then, the restriction to the special case of one period is obvious.

Now, let denote $T' = \{t_1, \dots, t_n\}$ a contiguous set of periods and $j \in J$ an activity, having duration $d_j \leq n$, which shall be processed within the set T' . This requirement can be represented in terms of a partially renewable resource p where

$$\Pi_p = T \setminus T', K_p = 0, k_{jp} = 1 \quad (\text{any } k_{jp} > K_p \text{ suffices}) \quad (27)$$

Using negative values for resource demand and capacities, an even more compact formulation can be obtained from

$$\Pi_p = T', K_p = -1, k_{jp} = -1 \quad (\text{any } k_{jp} \leq K_p < 0 \text{ suffices}) \quad (28)$$

4.2.2. Negation (NOT)

In contrast to the above identity, negation (also known by its operator NOT) is a unary boolean relation holding true if the corresponding variable does not hold. We thus interpret negation to prevent a particular job from being active in specified periods; again, the restriction to the special case of one period is obvious.

Let denote $T' \subseteq T$ a set of periods and $j \in J$ an activity which must not be processed within any of the periods in T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = 0, k_{jp} = 1 \quad (\text{any } k_{jp} > K_p \text{ suffices}) \quad (29)$$

4.2.3. Conjunction (AND)

Conjunction (also known as AND) is an n -ary ($n \geq 2$) relation holding if all corresponding variables possess the value "true". We thus interpret conjunction as stipulating several jobs to be processed in some specified periods.

Let denote $T' = \{t_1, \dots, t_n\}$ a contiguous set of periods and $J' \subseteq J$ a set of activities, having duration $d_j \leq n$ each, all of which are to be processed within T' . Then define a partially renewable resource p where either

$$\Pi_p = T \setminus T', K_p = 0, k_{jp} = 1 \quad (j \in J') \quad (\text{any } k_{jp} > K_p \text{ suffices}) \quad (30)$$

or, using J' to abbreviate $|J'|$,

$$\Pi_p = T', K_p = -J', k_{jp} = -1 \ (j \in J') \quad (\text{any } k_{jp} \leq K_p / J' < 0 \text{ suffices}) \quad (31)$$

4.2.4. Disjunction (OR)

Vice versa to the above, disjunction (also known by its operator OR) is an n -ary ($n \geq 2$) relation that holds if at least one of the corresponding variables holds true. We may regard disjunction as requiring some of a given set of jobs to be processed in certain periods.

Let denote $T' = \{t_1, \dots, t_n\}$ a contiguous set of periods and $J' \subseteq J$ a set of activities, having duration $d_j \leq n$ each, at least one of which must be processed within T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = -1, k_{jp} = -1 \ (j \in J') \quad (\text{any } k_{jp} = K_p < 0 \text{ suffices}) \quad (32)$$

4.2.5. Negated Conjunction (NAND)

Negated conjunction (also known by its operator NAND, short for NOT AND) is an n -ary (for $n \geq 2$) relation holding if not all corresponding variables possess the value "true". We may consider this relation as ensuring that at least one activity is prevented from being scheduled within some specified periods.

Let denote $T' = \{t_1, \dots, t_n\}$ a contiguous set of periods and $J' \subseteq J$ a set of at least two activities, having duration $d_j \leq n$ each, not all of which may be processed within T' . Then, using J' to abbreviate $|J'|$, define a partially renewable resource p where

$$\Pi_p = T', K_p = J'-1, k_{jp} = 1 \ (j \in J') \quad (\text{any } k_{jp} \cdot J'-1 = K_p \text{ suffices}) \quad (33)$$

4.2.6. Negated Disjunction (NOR)

Negated disjunction (also known by its operator NOR, short for NOT OR) holds whenever none of the corresponding variables holds. Hence, this relation can be seen as excluding all jobs from being processed in parallel.

Let denote $T' \subseteq T$ a set of periods and $J' \subseteq J$ a set of activities, none of which may be processed within any of the periods in T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = 0, k_{jp} = 1 \ (j \in J') \quad (\text{any } k_{jp} > K_p \text{ suffices}) \quad (34)$$

4.2.7. Exclusive Disjunction (XOR)

Exclusive disjunction (also known by its operator XOR, short for EXCLUSIVE OR) holds whenever exactly one of the corresponding variables holds. Therefore, it allows only one of several alternative jobs to be active within certain periods.

Let denote $T' = \{t_1, \dots, t_n\}$ a contiguous set of periods and $J' \subseteq J$ a set of activities, having duration $d_j \leq n$ each, exactly one of which is to be processed within T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = -1, k_{jp} = -1 \ (j \in J') \quad (\text{any } k_{jp} = K_p < 0 \text{ suffices}) \quad (35)$$

4.3. *Modelling Practical Applications*

4.3.1. *Prescribed Assignments*

In certain applications, one or several activities are to be executed in specific periods. Consider the field of school timetabling: Some lessons may only take place in specific periods or sets of periods because e.g. a municipal swimming pool or a community sports ground may be used by a given school only in the early morning hours (Alvarez-Valdés et al. 1996).

This is a situation where logical identity can be applied.

4.3.2. *Forbidden Assignments*

Assume now the opposite situation: One or several activities are to be excluded from being processed in some specific periods; any schedule violating this requirement will be infeasible. Such a situation arises e.g. within the aviation industry, in the context of flight simulator scheduling. The availability of certain ground-based flight simulators used for cockpit crew training and evaluation purposes may be temporally restricted because, in order to generate additional revenue, certain training periods are sold to other airlines (Leoff 1996).

These requirements call for negation of the corresponding assignments.

4.3.3. *Maximum Quotas*

In the field of technical training in the aviation industry, it is common especially for smaller airlines to send no more than a certain number of technicians to external training courses; this quota serves to avoid undue shortages in the workforce of departments such as maintenance which are critical for operations. In addition, an airline may want to restrict training of its employees to a specific quarter of the year. Training companies, on the other side, can be found to keep, as long as practicable, at least one instructor for each topic currently taught unassigned. In this way, an unforeseen absence (e.g. due to illness) of a "scarce" instructor does not imply that a complete course has to be canceled (Haase et al. 1997). All these requirements can be expressed in terms of maximum quotas. Similar restrictions may be found in enterprises where demand for certain products or services follows predictable patterns.

If at least one activity, which here represents an employee, is to remain unscheduled within some contiguous set $T' = \{t_1, \dots, t_n\}$ of periods, negated conjunction can be used as described

above. In a more general setting, where the number of jobs not to be scheduled within T' is greater than one, let m denote the maximum number of jobs to be processed within T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = m, k_{jp} = 1 \ (j \in J) \quad (\text{any } k_{jp} \cdot m = K_p \text{ suffices}) \quad (36)$$

4.3.4. Minimum Quotas

A variety of regulations govern the field of flight crew scheduling: Pilots may have to log minimum numbers of flying hours, landings, and approaches under certain flight regimes (e.g. daytime, nighttime, instrument conditions) within certain intervals in order to maintain their piloting privileges (FAR Part 61.57⁵). Civilian airline pilots must also undergo specific check flights at least once every half year. Within the tactical crew training program, military pilots have to complete, in addition to the above, minimum numbers of different training sorties (e.g. air-to-air combat, air-to-ground combat, or search-and-rescue missions) to maintain their "combat ready" status (Luftflottenkommando 1991, Chapter 19).

If at least one activity is to be scheduled within a given contiguous set $T' = \{t_1, \dots, t_n\}$ of periods, disjunction can be used as described above. In a more general setting, where the number of jobs to be scheduled within T' is greater than one, let m denote the minimum number of jobs to be processed within T' . Then define a partially renewable resource p where

$$\Pi_p = T', K_p = -m, k_{jp} = -1 \ (j \in J) \quad (\text{any } k_{jp} \cdot m = K_p < 0 \text{ suffices}) \quad (37)$$

4.3.5. Calendarization

Consider a situation where the durations of some activities depend on the number of free days such as weekends and holidays taking place during their execution (e.g. the time required to erect a scaffolding) while others are not at all sensitive to free days (e.g. the time required for a concrete layer to harden). In other words, the processing of certain activities may be interrupted by non-workable days, even though in principle activities are assumed to be nonpre-emptable. Thus, rather than being a constant, the duration of some activities may be a function of the specific period in which they begin.

A common approach when scheduling activities to real-world dates is to introduce a calendar, specifying which days are weekends, holidays, and other non-workable days (Jüngen, Kowalczyk 1995). The problem of calendarization is then handled in two successive steps: First, a schedule is determined by an appropriate algorithm. Second, the model-related start dates of the schedule are transformed into real-world ones by mapping them onto the calendar, consid-

⁵ FAR stands for Federal Aviation Regulations, the legal code which governs civil aviation in the United States.

ering only working days. Assume, for instance, a simple working calendar where all days except of the weekends are working days. An activity that takes four days and begins on a Wednesday would thus end on the following Monday. This procedure, however, has the major drawback that, since all activities are mapped in the same way to subsequent working days, the condition that some of them may not be interrupted by non-working days could be violated. Zhan (1992) proposes a polynomial exact algorithm overcoming this deficiency, even in the presence of maximum time-lags. His algorithm solves only the uncapacitated case and is not applicable in the presence of scarce resources, though.

Using partially renewable resources, the steps of scheduling and calendarization can be taken simultaneously. To achieve this, one has again to interpret time as a resource, measured in terms of periods. Let denote $T' \subseteq T$ a set of periods, $j \in J$ an activity not to be interrupted by any of the periods in T' whereas another activity $j' \in J$ may be interrupted by them. Then let p denote a partially renewable resource where

$$\Pi_p = T', K_p = 0, k_{jp} = 1, k_{j'p} = 0 \quad (\text{any } k_{jp} > K_p \text{ and } k_{j'p} \leq K_p \text{ suffices}) \quad (38)$$

4.4. Assessment

As has been demonstrated, a number of logical relations on binary variables can be expressed in terms of partially renewable resources. Using a straightforward interpretation of these relations, a variety of practically relevant requirements on feasible solutions can be modeled which allows to represent real-world problem settings in a convenient and elegant manner.

Two notes on related approaches seem in place. First, identity and negation as interpreted above can also be expressed in terms of renewable resources in the presence of either time-varying capacities or time windows (Bartusch et al. 1988), although doing so increases the amount of capacity data, and thus the length of the instances, by a factor $O(T)$ as compared to partially renewable resources. Second, certain logical relations between binary variables can also be formulated directly as constraints (e.g. Hillier, Lieberman 1990, pp. 458-460). Yet, all these relations hold true throughout the whole planning horizon; as partially renewable resources can be defined for arbitrary period subsets, our approach also allows to express logical relations which are valid for certain subsets of periods only. Restricting such constraints to certain periods only is possible but mandates the use of time-indexed binary variables whereas partially renewable resources allow to formulate logical relations also for purely integer models (as demonstrated by the above model (1) - (4)).

Finally, let us point out the limits of this approach. It is well-known that all relations in propositional logic can be built by induction, using only negation and either conjunction or disjunction, as long as these relations can be applied iteratively. Using combinatorial constraints, however, to represent logical relations, fails to meet this prerequisite. Here, composition of

relations is always strictly conjunctive since the feasibility of a solution is tantamount to satisfying all of the requirements. Therefore, in order to express the full range of logical relations, one would have to apply, in addition to conjunction, negation to composite requirements, which is impossible.

5. Summary

In this paper, we examined the concept of partially renewable resources which, apart from unifying and extending the well-known concepts of renewable and nonrenewable resources, allows to formulate a variety of requirements relevant in practical applications. The expressive power of this concept thus opens up a compact and elegant way of devising mathematical models of real-world planning situations.

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