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Measures of the Output Gap in the Eurozone: An Empirical Assessment of Selected Methods

Abstract
The paper discusses some widely used methods for estimating output gaps based on aggregated data for the eurozone. Though these methods exhibit some common features, an empirical comparison demonstrates that the various techniques differ substantially. In particular, the correlation of output gaps calculated with different methods is generally low, the methods imply different turning points, and the estimated level of the output gap differs greatly. Moreover, tests suggest that some of the methods commonly used have only limited information content for inflation forecasting in the euro-zone. Conclusions for business cycle analysis and economic policy are offered.

Ansätze zur Schätzung des Output Gaps in der Euro-Zone: Ein empirischer Vergleich ausgewählter Methoden

Zusammenfassung

JEL-classification: E32
Keywords: Output Gap, European Monetary Union, Business Cycles

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1. Introduction

Although the concepts of potential GDP and Output Gaps are widely used in macroeconomics, the calculated numbers trace back to very different concepts and theories. Therefore, the numbers have fundamentally different policy implications. For example, the output gap might be relevant for the question, if and to what extent unemployment can be attributed to a lack of overall demand (c.f. e.g. Solow 2000). Moreover, the literature devoted to so-called Taylor rules has proved the relevance of this concept for monetary policy (Taylor 2000). With the introduction of the as a new economic entity it is therefore of particular relevance to learn about the output gap within this so-called „Euroland“.

An output gap is defined as the difference between – unobservable -potential and actual GDP. Therefore, the precise understanding of the meaning of the word output gap depends on the definition of potential GDP. In his seminal paper, Okun (1962) defined potential GDP as the answer to the question „How much output can the economy produce under conditions of full employment?“ (Okun 1962: 145). Moreover, he emphasized that potential GDP is a short-run concept which takes „most of the facts about the economy (...) as they exist: technological knowledge, the capital stock, neutral recourses, the skill and the education of the labor force are all data, rather than variables.“ (Okun 1962: 147). However, the understanding of potential GDP has changed during the last decades. A more recent definition is given by de Masi (1997) who defined potential GDP as „the maximum output an economy can sustain without generating a rise in inflation“ (De Masi 1997: 40).

Different measurements of Output Gaps trace back to competing general interpretations of economic fluctuations. Broadly speaking, one can distinguish a „trend deviation“ interpretation of changes in overall production and a „gap closing“ view on cyclical phenomena (de Long 2000: 84). The first viewpoint assumes that business cycles are fluctuations around a long-run trend. The main purpose of a trend-cycle decomposition is in that case to identify the cycle as the succession of some recurrent economic fluctuations (Burns and Mitchell, 1946). In contrast, the probably more traditional view interprets business cycles as a decline below some level of potential output. Though both views seem to be quite similar their policy implications are very different. On the one hand, the interpretation of business cycles as trend deviations view restricts the role of stabilization policy. Policy measures cannot increase the level of output
systematically. Thus, no first order welfare gains are possible. The only thing stabilization policy can do in such a framework is to reduce the variance of output around the trend. If, on the other hand, economic policy is “gap closing” welfare gains are possible simply because the level of production and real income might be higher after a policy measure. In the first case, some automatic “mean reversion” forces preclude a long lasting divergence between the trend and the effective output, whereas in the second case persistent output gaps cannot be ruled out. Although economic interpretation may differ, it is however important to recall that any trend/potential output - cycle decomposition relies upon a theoretical model, either explicit or implicit (Micolet 1999, Fayolle 1996).

This point can be illustrated with the interpretation of economic fluctuations. Modern macroeconomics sees economic fluctuations as a result of a number of different shocks. From this perspective the question arises what shocks should be taken into account by a measure of potential GDP or the output gap. One extreme side of the possible spectrum is presented by early version of the real business cycle school (Boschen and Mills 1990). In their view all fluctuations of real GDP should be seen as fluctuations of potential GDP. Given this line of argumentation, there is no such thing as an output gap. Note, however, that regardless of the mentioned argument trend deviations can occur even in this type of models, because the models are stochastic version of neoclassical growth models. Therefore, real output may differ from its trend due to random productivity shocks.

In principle, all long-lasting shocks should determine potential GDP and all transitory shocks should enter the output gap. A wide range of models attribute long lasting shocks to the supply side of the economy, whereas transitory shocks are seen as business cycle fluctuations. If this view is correct, monetary and spending shocks should define the output gap and supply shocks should define potential GDP. However, “shock hunting” is more an art than a science. Thus, the attribution of long-lasting disturbances to the supply side is not undisputed (consider for instance the interpretation of the increase in unemployment in Europe or the effect of demand shortages on technical progress). Moreover, the emphasis on shocks is considered by some authors as to some extend overdone. For example, Zarnowitz stresses the importance of endogenous factors for explaining the American business cycle in the nineties (Zarnowitz 1999).
A second serious problem regarding the measurement of output gaps is the time horizon for which the estimation is done. This is of particular relevance for policy makers. For example, a short run measure of the output gap may indicate inflationary pressures. However, if the monetary authorities assume that investment will pick up and, thus, potential GDP will increase faster than before, it would be unnecessary to increase interest rates. For instance, if for some reason an impulse is given to the economy firms will invest more. This will, in turn, lead to an increased capacity utilization and inflationary pressures in the short run. However, in the medium run additional investment may lead to higher potential GDP as well. In other words, if no specific constraint on capital accumulation is identified (e.g. a low level of profitability), the potential output is not constrained by capital on the medium-long run and the only effective constraint is labor. Different interpretations of the equilibrium unemployment rate are also available, referring to different time horizons (Richardson et al. 2000). This short-term NAIRU is defined as the equilibrium unemployment rate in the absence of temporary supply shocks. The short-run NAIRU, in contrast, is simply the unemployment rate consistent with a stabilizing the inflation rate from one time period to another. Furthermore, the long-run equilibrium rate of unemployment may be used. This concept is more in line with natural rate models. Here, the NAIRU refers to a steady state. It is therefore possible to distinguish several definitions of the output gap when considering the question whether or not monetary policy should react to the inflationary pressures. For example, structural reforms in the labor market can lower the NAIRU and bring it nearer to the natural rate. Also, inflationary pressure may accelerate because of transitory effects of import prices on the short run NAIRU without endangering the long-term potential growth. A related question is the problem whether or not the potential GDP growth may have changed recently due to the so-called „new economy“ effect (see ECB 2000 for a critical discussion of this hypothesis). All in all, one may define a medium output gap – in contrast to the more conventional short-run measure – a concept for which the capital stock is an endogenous variable. Then, other production factors – for example the availability of skilled labor or technological knowledge – are limiting production.

From discussed problems it follows that the criteria to evaluate estimates of the output gap differs strongly depending on the purpose of the concrete aim of the analyses and the theoretical underpinning of the discussion. One may distinguish three possible goals of the estimation of output gaps: (i) the analysis of cyclical fluctuations, that is the measurement of endogenous variations of the economic activity, or, according to the more dominant view, the cumulative
impulsion-propagation effect of some exogenous shocks (ii) the evaluation of the differences between the change of actual GDP and of potential growth and (iii) the discussion of the adequacy of economic policy measures.

With regard to the first point it is possible to translate the requirements of a reasonable measure of cyclical fluctuations into statistical requirements. For example, one may argue that fluctuations should be persistent and that the cyclical component of overall output should be stationary. These criteria are in line with the interpretation of the business cycle as the succession of some recurrent - stationary - and significant - persistent - economic fluctuations. Based on these criteria it is possible to evaluate several statistical methods. However, seen from the perspective of a structural model of the economy the main disadvantage of these approaches is the lack of economic interpretation and prospective view. In particular, the methods are silent about any possible difference between overall demand and supply in the sense of the “gap closing” view of economic policy. Therefore, one is unable – based on the statistical methods – to evaluate the adequacy of economic policy. Last but not least, it seems important to consider that the true picture of the cycle – and inflationary pressures - is probably more complete when using many indicators than when using one single “output-gap” measure (Svensson 1999).

In the following we will discuss several criteria for output gaps, which refer to different requirements to such a number. For instance, the persistence of the gap or the analyses of the turning points refer to business cycle analysis. On the other hand, the volatility of implied potential GDP measures may tell something on the shocks assumed to be part of potential GDP. Last, we will discuss the information contend of gap measure with regard to inflation since this point is important with regard to all mentioned expectations to output gap measures.

Before implementing any method to estimate the output gap some assumptions have to be made. For the purpose of this paper we assume the existence of a common European business cycle. Though this notion is nothing less then self-evident, it can be justified for two reasons. First, several empirical analyses have shown that there is indeed a common cycle across the member countries of the European monetary union (see Artis et al. 1999, Bai et al.1997, Blake et. al 2001). Second, the discussion of aggregated data for Euroland is of particular relevance for economic policy since the European central bank has to decide over a „one fits all“ interest rate
(see Gerlach and Smets 1999). Therefore, the average situation is of interest for monetary authorities. In this paper, we use aggregated data for the time series needed to estimate output gaps for the eurozone. In particular, the data calculated by Fagan et al. (2001) are used for the time period from 1980 to 1990. From 1991 onwards, the official data provided by EUROSTAT are employed. Of course, this aggregation procedure implies a lot of judgment and is debatable.

2. Methods to Estimate Output Gaps: A Birds Eye View

This multiplicity of requirements has led to a wide range of different approaches. Therefore, the literature on estimating output gaps and related concepts is very large and has been growing quickly in recent years (see Dupasquier et al. (1999), Claus et al. (2000), Apel and Jansson (1999), Donde and Saggar (1999) and Boone (2000), Mc Morrow and Roeger (2001) for surveys). At a first glance, one can distinguish four groups of methods: direct measures of the cycle from survey data, non-structural (i.e. statistical) methods, theory-based methods and multivariate methods. Figure 1 gives an overview of the competing approaches.

Figure 1: Methods to Estimate Output Gaps

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1 Beyer et al (2001) provide a detailed discussion of problems related to the aggregation of pan-European data.
3. Direct Measures of the Output Gap: Survey Data

For short time horizons, production technology is fixed and inputs are complementary. Supply can be limited by the capital stock or by the available working force. If production is constrained by capital, it is possible to calculate the potential growth and the production gap by using business survey data. Though the concrete questions in the surveys differ across the Eurozone, the European Commission (2000) provides a time series for industrial capacity utilization. Thus, potential output equals effective output ($Y_t$) plus the gap between the available capacities and a level coherent with the absence of tensions on the goods market.

\[ [3.1] \quad Y_t^* = \frac{CAP_t^*}{CAP_t} Y_t \]

Where $CAP$ is the utilization rate, $Y_t^*$ represents potential output and $CAP_t^*$ denotes the degree of capacity utilization coherent with the absence of tensions on the goods market.\(^2\)

However, the limits of this approach are numerous. First, degrees of capacity utilization are only available for the business-manufacturing sector. Industrial production is more variable than the utilization rate, which leads to an unlikely high variability for the production capacity. Second, survey data are subjective by definition. Hence, it is impossible to determine the level of the utilization rate coherent with the absence of tensions on the goods market $CAP_t^*$. In most applied research this variable is considered to equal the average level of the utilization rate over the investigation period. Third, due to the short-term time horizon, the influence of investment is not considered. In the medium run, potential output may increase due to high investment rates. All in all, the method seems more appropriate for measuring the evolution rather than the level of the output gap. However, survey data are the only non-estimated direct obtained data in this field and should therefore be considered seriously, at least when it comes to the determination of business cycle turning points, as industrial production is the most volatile part of overall output.

\[^2\] Note, however, that potential GDP calculated by equation 3.1 is by no means the maximum possible production.

This section deals with the evaluation of so-called non-structural univariate measures of the business cycle. Broadly speaking, this phrase includes all methods that are based on some statistical procedure rather than referring explicitly on an economic theory (Cogley 1997). The distinction between structural and non-structural approaches is less clear than it sounds. On the one hand, some of the so-called theory based methods, such as the one factor production function approach, often turn out to be more or less a trend extraction method. Moreover, some of the theory-based methods use trends or filters as inputs for estimation. The interest in non-structural methods is partly motivated by the fact that they require less information than theory-based methods. For example, they can be applied in cases, where only a single time series is available. This might be of relevance for the Euro-zone since there is still a lack of data on the aggregate level. Moreover, the methods can be implemented to model any time series of interest. This allows for a discussion of the cyclical behavior of all parts of the economy, i.e. different types of expenditures and different sectors. Non-structural measures might therefore be used for a discussion of stylized facts of the business cycle. Additionally, some univariate methods force the obtained time series representing the output gap to be stationary.

Nevertheless, non-structural univariate methods have also several serious shortcomings. First, Quah (1992) makes a rather fundamental point and argues that it is impossible to disentangle the relative importance of demand and supply shocks in an univariate framework. Second, there is no explicit link between any economic measure and medium term economic growth as measured by the trend component. Hence, it is not possible to give any substantial economic advice to policy maker’s questions about how to improve trend growth. Third, the possibility of a persistent output gap is ruled out by assumption rather than based on any empirical result. A fourth problematic point is that non-structural measures need some additional judgment on the true nature of the business cycle. The latter is normally not undisputed among researchers. For example, normalization or the choice of a smoothing parameter is necessary. This gives some room for ambiguity and ad-hoc assumptions. Thus, two researchers using the same method will not necessarily end up with the same estimate of the output gap (Le Bihan et al. 1997). Last, the underlying understanding of the business cycle is quite restrictive. In particular, it is implicitly assumed that business cycles have more or less the same duration and that they are symmetric. Both assumption, however, are problematic as they are, for example, in contrast the so-called
classical definition of the business cycle tracing back to the NBER tradition (see e.g. Artis et al. 1997).

4.1 Linear De-trending

Linear de-trending might be seen as a benchmark for estimating trend and cycle. If we let $y_t$ denote the log of real GDP at time $t$, then the estimation of potential GDP is based on the simple OLS-regression:

$$y_t = \beta_0 + \beta_1 t + u_t$$

The fit of this equation gives an estimate of potential GDP and the residual $u_t$ is the estimated output gap. Since we have a logarithmic specification, the estimate $\hat{\beta}_1$ gives the average trend growth over the period under investigation. The estimation implies some normalization since the residuals have zero mean.

4.2 Phase Average De-trending

Since a stable linear deterministic trend function is very unlikely to be stable over time, a common alternative is a segmented trend model, which is a linear trend framework allowing for at least one structural break. We need an assumption at which point in time the structural break occurs. We apply a method to search for a possible structural break in the trend of real GDP (Kim 1997, Zivot and Andrews 1992). The following test equation for a unit root is used:

$$\Delta y_t = \mu + \beta t + \gamma D_t + \alpha y_{t-1} + \sum_{i=1}^j c_i \Delta y_{t-i} + e_t$$

where $y$ denotes the time series under investigation, $D$ a dummy variable and $t$ the deterministic trend. The standard Dickey-Fuller-test is calculated for alternative breaking points.

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Another method very popular to estimate potential GDP among practitioners is peak-to-peak de-trending. The underlying argument seems, at a first glance, straightforward. The maximum observed production in the past is counted as the potential GDP with maximum capacity utilization. However, since the economy has (hopefully) some trend growth, the method requires additional information, namely some a priori information on the dating of the business cycle. Moreover, the method is particularly problematic with respect to the actual data. If GDP increases fast, it is not clear whether this is a positive trend deviation or a higher trend growth. Last, but not least one should mention that the method ignores most of the data points of the time series and, therefore, the vast majority of the information available. All in all, we consider this method not appropriate at all and will not present results obtained by it.
points. When the absolute value of the test statistic (Zivot-Andrews-statistic) reaches its maximum, a structural break is identified. This procedure follows the argument that an I(1)-variable can (and should) be decomposed in an I(1)-trend- here deterministic - component and an I(0)-cyclical-component. The test procedure suggests a structural break in the deterministic component of Euroland’s GDP in the second quarter of 1992. Thus, the phase average detrending method has used two trends: one up to this date, another following this event.

4.3 Robust Trend Estimation
One major shortcoming of the estimation of a linear trend model is that it is obviously overly simplistic. Nobody assumes seriously that such a simple function is indeed a good approximation of the data generating process. Therefore, Coe and McDermott (1997) suggest use of non-parametric estimates of the trend function. "The aim of a non-parametric regression estimation (...) is to approximate an unknown trend function arbitrarily closely, given a large enough sample" (Coe and McDermott 1997: 76). When these estimators are used it is not necessary to specify the functional form of the trend function. However, one has to assume that the "trend has an adequate number of derivates so that it is smooth" (Coe and McDermott 1997: 76) relative to the gap. In this paper we will use the same choices as Coe and McDermott.⁴

4.4 Hodrick-Prescott Filter
The Hodrick-Prescott (HP) filter (Hodrick and Prescott 1997) has probably become the most popular way of de-trending economic time series in the last recent years. This is mainly due to the fact that it can be very easily calculated and implemented in virtually any econometric software package. If y denotes real GDP, the filter is defined as

\[
\hat{m}(x) = \frac{1}{T} \sum_{i=1}^{T} K_h(x - t / T) y_i
\]

\[
= \frac{1}{T} \sum_{i=1}^{T} K_h(x - t / T)
\]

where \(K_u = h^{-1} K(u/h)\), \(h\) is the bandwidth parameter and \(T\) the sample size. The bandwidth parameter gives the size of the data window used in regression. Again, we follow the setting used by Coe and McDermott (1997) and make use of an Epanechnikov kernel.

⁴ This procedure is simplified compared to the original idea in two ways (Kim 1996: 72): First, we do not allow for a structural break in the constant, but only in the trend variable. Second, after one structural break is identified, the procedure is not applied again. Both restrictions reflect the fact that the time series is very short.

⁵ They take the Nadaraya-Watson-Estimator. For the time trend it takes the form:
with $y_t^*$ as the smooth component which gives the estimate of potential GDP in this context. A HP-filter is more or less a "moving average for snobs" (Kuttner 1994). Broadly speaking the procedure described in [4.3] contains two commands: (i) minimize the distance between the actual and the trend value of the time series and (ii) minimize the change of the trend value. Obviously, the commands contradict each other. Therefore, a weight has to be given to both aims. This is done by choosing the factor $\lambda$. For quarterly data, a smoothing factor of 1600 has become somewhat like an „industrial standard“. Though this assumption can be justified, the arbitrary choice of the smoothing parameter is one of the mayor criticisms of the filter.

The HP-filter has been controversial in the literature. It has been argued in favor of the filter, that an output gap calculated with an HP-filter is a stationary time series even if the original series is I(1) or even integrated of a higher degree (Cogley and Nason 1995). Moreover, if the filter is applied to artificial data taken from a calibrated model where the “true” data generating process is known it provides a good (although not the best) approximation of the cycle (Cogley 1997). The HP-filter has also some serious shortcomings. First, it is completely mechanistic. It has no explicit foundation in any economic theory. Second, the results hinge on the arbitrary choice of the smoothing parameter. Third, the end-of-sample problem limits the practical usefulness of the filter (Razzak 1997). Fourth, a long lasting negative (or positive) output gap is ruled out a priori by the HP-filter. If one believes, for example, that actual GDP has drifted away from its potential path for, say, a decade or more, the filter will not show this development as a negative output gap but as a lower growth of potential GDP. More

\[
[4.3] \quad \min \sum_{t=1}^{T} (y_t - y_t^*)^2 + \lambda \sum_{t=2}^{T-1} \left((y_{t+1}^* - y_t^*) - (y_t^* - y_{t-1}^*)\right)^2
\]

In their original paper Hodrick and Prescott argue "a five percent cyclical component is moderately large as is a one-eighth of one percent change in the rate of growth in a quarter" (Hodrick and Prescott 1997: 4). This leads to \[ \left( \frac{5}{1/8} \right)^2 = 1600 \]. Some studies discuss the appropriate setting of the smoothing parameter. Ravn and Uhlig (1997) recommend "the fourth power in the change of the frequency of observations". This will lead to a value of 6.25 for annual data rather than 100 in the original paper of Hodrick and Prescott. Baxter and King (1995) argue, that a smoothing parameter of 10 will do the same trend cycle decomposition as using 1600 for quarterly data.

see Canova (1998), Harvey and Jaeger (1993) for arguments against the filter and Burnside (1998) for a defense of its use.

For example, Baxter and King (1995) find that it takes additional data for three years or twelve quarters to make sure that the actual Output gap makes sense.
specifically, the filter applied with the usual smoothing parameters removes changes in real
GDP shorter than approximately three years and longer than 20 years. If the true business cycle
lasts between 2 and 32 quarters this setting can be justified as a good approximation of an
almost ideal filter (Baxter and King 1995).

On the one hand, one might argue that the definition of business cycles implies some judgment
on the frequency of the fluctuations to be counted. One the other hand, one might assume that
real GDP can indeed drive away from its potential for a longer time period. For example,
Barrel and Sefton (1995) argue that the US cycle in the 1980's lasted for around 8-10 years and
therefore longer than the HP-filter with the "industrial standard" setting of $\lambda = 1600$ will accept
as a cyclical phenomena. Fifth, the HP-filter forces the business cycle to be symmetric, that is,
it assumes expansions and contractions to be of the same length on average (Psa radarkis and
Sola (1997), but note Sichel (1993)). Sixth, the filter will lead to nonsense results if there are
statistical breaks in the time series (Razzak and Dennis (1995)). For example, if one would
filter German GDP without any additional calculation, the growth of potential GDP would
increase in the late eighties sharply because of reunification.\footnote{Some of the shortcomings have lead to a discussion in the literature to solve some of the problems. For example, Razzak (1997) recommends a recursive calculation of the filter. He argues, that this procedure will lead to a true filter rather than to a smoother like the HP-filter is in his eyes.} This list of arguments for and
against the filter is surely not complete. We will discuss some other aspects when we will turn
to the evaluations of output gap measures.

4.5 The Band-pass-filter
Another recent contribution to the discussion of the appropriate measure of the cyclical
component of real GDP and other macroeconomic time series is the band pass filter developed
by Baxter and King (1995). The reasoning behind this filter comes from spectral analysis. The
basic idea is that one can define business cycles as fluctuations of a certain frequency. A
standard setting is for example to count fluctuations longer than six quarters and shorter than 32
quarters as cycles.\footnote{Baxter and King call this the Burns/Mitchell setting of the filter.} Fluctuations with a higher frequency are normally seen as irregular or seasonal, fluctuations with a lower frequency are seen as "trend" or potential GDP in this case.
Given a judgment on the true length of the business cycle one can define an optimal band/pass
filter that will exclude all fluctuations from real GDP. However, one serious practical shortcoming remains: the filter is calculated by a moving average and, thus, has no values for the most recent quarters. In fact, if one follows the standard setting as suggested by Baxter and King (1995) and translates the filter into a two-sided twelve quarter moving average three years are lost in the analysis of the recent business cycle situation.\footnote{However, in the following empirical analysis we make use of the RATS-procedure written by A. Taylor. This procedure adds artificial data at start and end of the series using AR backcasts and forecasts. This renders it possible to provide actual data for trend and cycle.}

4.6 Unobserved Component Method

The methods described so far share a main disadvantage, that is, they postulate a priori the nature of the trend. The econometrics of time series, however, underlines that the properties of the time series should determine the detrending method. Choosing an inappropriate procedure can lead to spurious econometric results and, thus, the autocorrelation function and the apparent cyclical properties of the time series may occur as an artifact (Henin, 1989). Stochastic detrending methods, on the contrary, have the advantage to rely on a precise specification of the process generating both cycle and trend. Hence, they take explicitly into account the link between past growth and the level of the output gap, as the stochastic nature of the trend reflects the permanent effect of the shocks driving the economy. Moreover, the use of stochastic shocks in the output gap allows to define eventually new initial conditions and may, therefore, fit better with the true nature of business cycles (Fayolle, 2000).

The unobservable component (UC) method rests on the assumption that both potential GDP and the output gap are unobservable and, hence, statistical techniques have to be used to decompose a time series into these components (de Brouwer 1998). For example, output $y_t$ can be decomposed into a permanent ($y_t^p$) a transitory ($z_t$) component, and an irregular error:

\begin{equation}
[4.4] \quad y_t = y_t^p + z_t + \epsilon_t
\end{equation}

where $\epsilon_t$ is white noise.
The permanent component can be seen as an estimate of potential GDP whereas the transitory component is an estimate of the output gap. Permanent output is a local linear one for which both the level and the slope are random walks specified as follows:

\[ y^p_t = \mu_{t-1} + y^p_{t-1} + \eta_t \]  

\[ \mu_t = \mu_{t-1} + \zeta_t \]

where $\eta_t$ and $\zeta_t$ are orthogonal white noises with variance $\sigma^2_\eta$ and $\sigma^2_\zeta$ respectively. The output gap is assumed to follow an ARMA(p,q) stationary process.

The general trend definition outlined above encompasses a wide range of possibilities (e.g. deterministic trend, random walk with drift, moving average) (Fayolle, Micolet and Trequattrini, 1999). In particular, it is possible to explicitly model breaks in time series as, for example, the German reunification. These models can be written in state space form and hence analyzed using Kalman filter techniques and estimated using Maximum Likelihood estimators. The method has been applied frequently (see Funke (1998) using German data or Fayolle (1996) for French data) and has given reasonable estimates for both output gap and potential GDP. The approach also highlights a limitation of the HP-filter mentioned above. In particular, the HP filter is an optimal filter only, if the potential output obeys a random walk in which the drift term also follows a random walk, and the output gap is a white noise (King and Rebello 1993).

5. **Structural or Theory Based Measures of the Output Gap**

Structural methods rely on a specific economic theory. In contrast to the non-structural methods discussed so far, they assume a certain economic theory to be correct. One can distinct two broad groups of structural methods. One the on hand it is possible to rely on multivariate statistical methods with theoretical assumptions in so-called structural VARs (SVARs). One the other hand structural methods can be based on an aggregate production function. In principle,

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12 In our empirical work below we apply a quite simple approach assuming potential GDP to follow a random walk with drift and the Output gap to be represented by a AR(2) process.
the use of these methods allows for more persistent estimates of the output gap since most of the underlying theories treat trend and cycle independently. Approaches based on production functions try to unearth the nature of constraints that limit output (for example labor, capital, global factor productivity). Therefore, they require an analysis of the nature and the transmission of the disequilibria.

A key problem in implementing structural models, especially production functions, is the lack of information available. For example, there are statistical requirements like, for instance, the need of appropriate capital stock data. More importantly, information about the correct theory of the economy is necessary. A broad consensus of economic theory, which can be used as an undisputed point of departure, can hardly be identified. Moreover, for some key variables, theory-based models depend on non-observable variables, too. For example, approaches based on a production function often need an estimation of the NAIRU as an input. Thus, the statistical methods have been used even in a so-called theory-based approach.

5.1 Okun’s Law
The oldest structural approach to estimate potential GDP relies on Okun’s (1962) seminal paper. The method assumes that working force is the limiting factor of production. Therefore, the unemployment rate is an indicator of the output gap. Consider, for example, the relation:

\[
(U_t - U_t^*) = \alpha (Y_t - Y_t^*)
\]

where \( U \) represents the unemployment rate and \( \frac{1}{\alpha} \) is the so-called Okun coefficient. Thus, the equation relates cyclical unemployment to cyclical component of real GDP. In his seminal contribution Okun (1962) assumes a coefficient about 3, that is one-percentage point increase in the output gap is indicated by a 0.3 percentage point decline of the cyclical unemployment rate. However, neither the equilibrium unemployment rate nor potential output can be observed directly. The estimation of the output gap then depends on an exogenous natural unemployment

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13 In general, such data are not available for the Euro-zone. Either one cannot obtain data for each member country nor are these data comparable. However, Bolt and van Els (2000) present estimates for each member country of EMU. Unfortunately, they are not explicit on the data sources. Moreover, note that a more sophisticated capital stock orientated approaches requires information not only on the level of the capital stock, but also on his age structure (Görzig 2000).
rate. Okun (1962) has suggested a 4% unemployment rate to determine a potential output since this is the average post-war level for the United States.

This relationship can be criticized for several reasons. First, Okun estimates the $\alpha$ coefficient based on an estimation of the relation between the unemployment evolution and production. This method is similar to the estimation of a reduced form of an employment equation and of a labor supply equation, which takes into account both the response of labor supply to unemployment and the adjustment of employment to production. Thus, an often-found low value of the coefficient can come from a lagged adjustment of employment to production. In the medium term the coefficient should be close to 0.7 or 0.8 since employment will grow to maintain productivity on its long-term trend. Second, the relation assumes unemployment to be stationary around a level and, thus, variations of unemployment to be of purely cyclical nature. Though this might be a fair guess with respect to the US, in the European case, the unemployment rate is often found to be non-stationary.

5.2 Production Function approaches: OECD and European Commission Estimates

The use of production functions to determine potential GDP and the output gap requires a lot of information: an assumption on the production technology, the estimation of equilibrium employment, information on the level of capital stock and of total factor productivity are needed. As production factors are not substitutable in the short run (that is the production technology is a of a so-called putty-clay technology) the use of a Cobb-Douglas function may appropriate to evaluate the level of the potential output and the output gap. However, the Cobb-Douglas production function is frequently used in applied research, since it is very easy to interpret and implement. But, when considered on an empirical basis, the Cobb-Douglas function is often rejected by the data (e.g. Baudchon et al 1997). As regards the OECD estimation of potential GDP the – labor augmenting - technological progress is considered to follow an exogenous trend. The approach taken by European Commission (McMorrow and Röger, 2001), links the technological progress to the current and past investment activities within a vintage model.

The main problem encountered when implementing a production function approach is that an estimation of an equilibrium rate of unemployment, for example a NAIRU, is needed. A pure
structural estimation would involve a system of equations explaining wage and price setting behavior. However, this has been rarely done in the literature. Several problems make it a very difficult task to implement such an approach. First, there is considerable disagreement about the appropriate structural model to be used (Richardson et al. 2000). For instance, supply shocks have only a transitory effect on the NAIRU when using a traditional Phillips curve specification, whereas they have permanent effects when using a wage setting-price setting approach (Sterdyniak et al. 1997). Second, many measurement problems arise with respect to factors supposed to enter the theoretical model. Moreover, estimations are often very sensitive to minor specification changes. Third, recent studies suggest that the impact of shocks and institutions on the NAIRU is rather complex (Passet and Jestaz, 1998, Conseil d’Analyse Economique 2000) and should be better analyzed in cross country- rather than in time series analysis (Blanchard and Wolfers 1999). As a consequence of these problems most of the recent estimations of the NAIRU use a reduced form Phillips curve approach, where the rate of change of the nominal prices \( \pi_t \) is proportional to the level of intensity of use of labor \( U_t^* \) (represents the NAIRU) and supply shocks \( z_t \) (Gordon, 1997, Staiger, Stock and Watson 1997).

\[
[5.2.] \quad \pi_t = a(L)\pi_{t-1} + b(L)(U_t - U_t^*) + c(L)z_t + e_t
\]

While this specification is coherent with different theoretical frameworks (IMF 1998, Roberts 1997), it does not determine the equilibrium rate. Thus, an additional estimation of the equilibrium rate of unemployment is needed. Until recently, the OECD has estimated a NAWRU\(^{14}\) based on reduced wage inflation Phillips curve without supply shocks. The estimated NAWRU was in a first step considered to be constant, and derived in a second step from the estimation of the first difference of a wage inflation equation (Elmeskov and Mc Farlan 1993, Giorno et al 1995, Giorno and Suyker 1997). With the development of multivariate filtering methods (see below), the OECD has developed new estimation procedures for the NAIRU based on the equation [5.2.], within a multivariate unobservable component model estimated where the NAIRU is assumed to follow a random walk. (Richardson et al 2000, Boone 2000). Such models can be written in state-space form and, therefore, analyzed by Kalman filter techniques and estimated using Maximum Likelihood

\(^{14}\) Non Accelerating Rate of Wages Unemployment Rate.
estimators. This approach has, compared to structural equations, the disadvantage to depend to a large extent on the assumptions made on the process generating the NAIRU.

The European Commission estimates a time varying NAWRU based on a standard – non linear - estimation of the following equation:

\[ [5.2.] \dot{w}_t = a(U'(TD_t, r_t, tax_t, tfp_t) - U_t) + b(L)\dot{w}_{t-1} \]

Where \( b(1) = 1 \), TD is a constant or a deterministic trend, \( r \) is the ex post real long term interest rate, tax is a comprehensive tax measure, tfp is the growth rate of real trend total factor productivity (Mac Morrow and Roeger, 2000, 2001). The equation refers explicitly to a bargaining model, where tax rates influence the reservation wage, total factor productivity and interest rate the labor demand. This specification allows for a partly structural explanation of the NAIRU, but misses the impact of many supply shocks.\(^{15}\)

5.3 Long-run Restriction Models
Structural VAR models go back to a seminal paper by Blanchard and Quah (1989). The underlying theory for the estimation of potential GDP is an aggregate supply and demand model and the assumption that nominal shocks are neutral in the long run. For example, Funke (1997) uses a bivariate vector autoregressive (VAR) model including the log of output and the inflation rate to model the German output gap. The starting point of the analysis is a bivariate VAR model including the change in real GDP (\( \Delta Y \)) and the change in the price level (\( \Delta P \)). The lag length of the VAR is determined on the basis of information criteria. The moving average representation of the underlying structural model can be written as:

\[ \Delta Y_t = \pi_t + \sum_{i=1}^{\infty} \phi_i \Delta Y_{t-i} + \sum_{i=1}^{\infty} \theta_i \Delta P_{t-i} + \epsilon_t \]

\(^{15}\) Used in conjunction with a potential labor force, the NAIRU provides the potential level of employment. Most estimations found in the literature make no specific assumptions on the response of labor force participation to changes in unemployment and use, thus, a trend labor force as a proxy for the potential labor force. However, this may lead to incorrect estimations of potential employment. See (Plane 2001, for recent estimations of labor force response to unemployment variations.)
\[
\begin{pmatrix}
\Delta \ln Y_t \\
\Delta \ln P_t
\end{pmatrix}
= \sum_{i=0}^{\infty} L^i A_i \varepsilon_t
\]  

[5.6]

where \( A \) is a polynomial matrix, \( L \) the lag operator and \( \varepsilon_t \) are white noise residuals capturing supply and demand shocks. To identify the structural disturbances driving the system, a long-run neutrality restriction is imposed. To be more specific, it is assumed that the impact of a change in the inflation rate on the change in real output is zero in the long-run. More technically, the matrix of long-run multipliers \( A(1) \) is forced to be upper triangular:

\[
\sum_{i=0}^{\infty} A_{1,i} = 0
\]

Moreover, to achieve the necessary number of restrictions to identify the structural residuals from the disturbances of an unrestricted VAR, the variances of the two shocks \( \varepsilon_t \) are normalized to unity.

Probably the most striking advantage of the SVAR approach for estimating potential GDP and the output gap is that it provides a strong critique of univariate de-trending methods and helps to understand the main disadvantages of univariate filters. Within the baseline bivariate SVAR approach the development of real GDP growth can be decomposed into the following components:

- The deterministic component of the model,
- Supply shocks,
- Demands shocks or, in more sophisticated models, any other nominal shocks.

The output gap within this framework is given by the fraction of GDP movements explained by nominal shocks. In other words, potential GDP is given by the deterministic component of the model and by the impact of supply shocks. This distinction makes clear why univariate de-trending methods may be misleading in certain situations. Suppose a major positive supply shock hits the economy. Any univariate filter will take this as an increase of the output gap. However, this is not in line with economic reasoning since, by definition, supply shocks should
not enter into the output gap. This line of argumentation can be illustrated by figure 5.1. The exhibit compares an output gap calculated with a simple deterministic trend with a SVAR gap obtained from a system taking into account both demand and supply shocks. It turns out that the deterministic trend implicitly interprets the entire increase of production after the opening of Eastern Europe at the beginning of the 90’s as a demand shock. Thus, this method indicates very serious inflationary pressures. In contrast, the role of supply shocks is much more important in the SVAR approach. Consequently, the output gap is much lower. For example, the SVAR gap considers the development of 1992/93 as a serious recession whereas the simple trend indicates just a normalization of the gap. Hence, not taking into account the full picture of the shocks driving the economy implies the risk to indicate inflationary pressures that, in fact, never have existed.

Figure 5.1: Supply Shocks and Output Gaps - An Illustration

As a possible limit of these approach one should keep in mind that the SVAR approach assumes that long-lasting shocks can be attributed to supply and transitory shocks can be seen a demand shocks.
6. Recent Developments in Estimating Potential GDP: Multivariate Methods

Because both non-structural and structural methods have been criticized in the literature in recent years, alternative approaches combining both lines of research have been widely discussed (see Dupasquier et al. (1999) for a survey). In the following, some of these methods will be discussed.\footnote{16}{Additional approaches may include the method advocated by Cochrane (1994) (see Schumacher (2000) for a related approach applied to European data).}

6.1 Multivariate Beveridge Nelson Decomposition

The multivariate Beveridge-Nelson Decomposition suggests use of the information contained in the co-movement of a number of economic time series to estimate output gaps (Barrel and Sefton 1995: 69). For example, a change in output correlated with a change in employment would indicate a supply side shock and, therefore, a change in potential GDP. In contrast, if the change in output is correlated with the change in consumption, a demand shock is more likely. The multivariate Beveridge Nelson decomposition defines potential GDP as the level of output that is reached after all transitory dynamics have worked themselves out (Dupasquier et al. 1999: 582). An application of this technique with respect to Euroland is provide by Schumacher (1999). In his model long-run restrictions are delivered from a multi-country macro model. To make a long story short, the underlying assumption is that the co-movement of Euroland's output with the output of other regions or countries (Japan, U.S.) defines the equilibrium relation of the system. The results show a reasonable statistical fit of the model. Moreover, the implied time series of the output gap makes sense economically.

6.2 The Multivariate HP-Filter

The main shortcoming of the non-structural methods is that they do not refer explicitly to economic theory. Hence, a number of authors have tried to combine structural equations and non-structural measures of the business cycle. A recently discussed approach is the multivariate Hodrick-Prescott Filter by Laxton and Tetlow (1992). The aim of this method is to add economic information to the filter. This information can come from known economic relationships as well as from indicators of capacity utilization. Consider, for example, the following equations (see Conway and Hunt 1997):

\[ \pi_t = \pi_t^e + A(L)(y_t - y_t^*) + \epsilon_{\pi,t} \]
This equation gives an augmented Philips-curve relationship. The actual inflation rate \( \pi \) depends on inflation expectations \( \left( \pi^e \right) \) and the current and lagged output gap.

\[ u_t = nairut - B(L)(y_t - y_t^*) + \varepsilon_{u,t} \]

This equation shows an Okun-relationship: the current unemployment rate depends on the (exogenous) NAIRU and the current and lagged output gap.

\[ cu_t = cu^T_t + C(L)(y_t - y_t^*) + \varepsilon_{cu,t} \]

This equation exploits available information on the capacity utilization \( (cu) \) from survey data. The residuals of these equations can be taken into account in minimizing the following loss equation:

\[ L = \sum_{t=1}^{T} (y_t - y_t^*) + \lambda \sum_{t=2}^{T} \left( y_{t+1}^* + y_t^* - y_t^* - y_t - 1 \right)^2 + \sum_{t=1}^{T} \beta_t \varepsilon_{\pi,t}^2 + \sum_{t=1}^{T} \mu_t \varepsilon_{U,t}^2 + \sum_{t=1}^{T} \psi_t \varepsilon_{cu,t}^2 \]

Given this equation, serious computational problems arise. Although it is generally possible to estimate all the coefficients of the model, the majority of the related literature assumes the weights of the influence of the structural equations in the filter to be known. For example, Haltmaier (1996) uses a weight of the inflation parameter of 400.

### 6.3 Multivariate Unobserved Component Models

The UC method discussed for the univariate case in the previous section can be extended into a multivariate approach taking into account additional equations. For example, Gerlach and Smets (1999) estimate the following model:

\[ y_t = y^p_t + z_t \]

\[ y^p_{t+1} = \mu^x + y^p_t + \varepsilon^y_t \]

\[ \pi_{t+1} = \alpha(L)\pi_t + \beta z_t + \varepsilon^x_{t+1} \]

\[ z_{t+1} = \phi_1 z_t + \phi_2 z_{t-1} + \lambda (i_t - \pi_t) + \varepsilon^z_{t+1} \]
Where potential output is assumed to follow a random walk, where the equation [6.7] links inflation to the lagged output gap and lagged inflation rates. Moreover, equation [6.9] is a reduced form aggregate demand equation relates the output gap to its own lags and the real interest rate.

Gerlach and Smets (1999) provide results using this technique with respect to Euroland. All in all, they conclude that their model fits the data of the European currency area quite well and produces reasonable output gaps. Generally, multivariate UC models can be extended in various directions. For example Flaig and Ploetscher (2000) suggest adding an equation describing the development of survey data on capacity utilization and report that the estimation of the output gap becomes much more efficient.

7. **Empirical Assessment of Selected Methods**

Given the large number of possible estimates of potential GDP and the output gap, the question arises whether one can establish empirical criteria to evaluate competing methods to estimate the output gap. We discuss these problems by comparing the results of the estimates and a brief analysis whether they share some common stylized facts. Second, we take a look at the autocorrelation function of the cyclical component to figure out whether the estimates lead to an average length of the fluctuations that match the usual definition of the phenomena „business cycle“. Third, we analyze the volatility of both the cyclical and the growth component. If the traditional view on the business cycle, rather than the real business cycle story, were correct, then one would expect quite a smooth measure of potential GDP. Another way to deal with this question is to discuss the predictive power of the output gap with respect to inflation. The underlying argument here is that from a theoretical point of view the gap is a measure for the excess supply or demand in the aggregated goods market. Hence, a positive output gap should correspond to increasing prices (or inflation) and a negative output gap should lead to declining prices or inflation rates.
7.1. Turning Points, Autocorrelation Function of Cyclical Component and Average Duration of the Cycle

An estimate of the output gap should show at least some cyclical behavior. Hence, its autocorrelation function should become negative at any specific lag. Some of the methods discussed above use this idea to define the trend/cycle composition. Fluctuations with a very high frequency might be seen as irregular or seasonal while fluctuations with a very low frequency are often considered trends. Hence, it seems natural to take a look at the autocorrelation function of the above estimates to evaluate whether they imply a reasonable picture of the cycle. Figure 7.1 gives the auto-correlation function of some selected measures of the output gap. It turns out that, very broadly speaking, the non-structural measures imply a relatively short cycle, whereas the structural measures tend to leave space for very persistent effects within the gap. This also holds for the output gap based on a linear trend function. The shortest cycle is suggested by survey data.

Figure 7.1: Autocorrelation Functions of the Output Gaps

Next, we turn to the implied business cycle turning points. The business cycles identified here refer to the growth cycle concept mentioned above. The following method has been used to identify the cycles (Fouet, 1993): a peak is the latest positive output gap preceding a decrease, a trough is the lowest output gap just before an increase of the time series. In the case that there
are two points in time showing the same level of the output gap, the earlier point has been chosen as a turning point.

- Insert Figure 7.2 here -

As can be seen from table 7.2 and figure 7.2, the methods tell different stories concerning Europe’s business cycle. Apparently, methods with a strong influence of the deterministic trend component tend to imply only few completed “major” cycles. On the contrary, more flexible de-trending methods show a lot more fluctuations, sometime coming near to white noise. Thus, the choice of the method is not unimportant, in particular for practitioners in the field of business cycle analyses. This point is well illustrated by the identification of minor cycles in the nineties, especially those associated with the Asian and Russian crisis by some methods. In contrast, other approaches (e.g. output gaps based on robust trend and segmented trend models) hardly identify the slowdown of 1998 as a growth recession, since the implied trend growth is rather low. Very volatile output gaps, in particular the one founded on robust trend estimation, make it moreover difficult to identify turning points. Of particular interest are the turning points of the gap based on survey data. Since these data are original data and do not rely on an estimation they may be seen as a benchmark for the turning point analysis. Unfortunately, none of the other methods reproduces the turning points implied by survey data accurately. However, industry does represent the whole economy. For example, manufacturing may well be more sensitive to external shocks than, say, the services sector. Thus, whether or not the turning points implied by manufacturing are a reasonable benchmark depends on the nature of shocks buffering the economy.

Auto correlation functions and turning points both illustrate different possible assessments of the level of the output gap in the nineties. On the one hand, the OECD output gap and the HP filter identify roughly the same business cycle turning points. On the other hand, the OECD output gap identifies a persistent under-utilization of production factors in the nineties, whereas the HP filter gives the picture of a slowdown of potential GDP growth.

7.2 Correlation of Output Gaps Calculated with Different Methods

The analysis so far has emphasized the differences between the methods. However, it might be argued that the choice of the concrete method to estimate the gap is of limited importance
because the similarities of the gap estimates might be large. This section reveals that this is not case, even if the simple trend extraction methods are considered. First, as can be seen from table 7.1, the correlation of the gap series is rather small in some cases.

### Table 7.1 Correlations of Selected Gap-variables for Euroland (1985:1 to 2000:2)

<table>
<thead>
<tr>
<th>Method</th>
<th>Survey</th>
<th>Linear</th>
<th>Segmented</th>
<th>Robust</th>
<th>HP filter</th>
<th>BP filter</th>
<th>UC</th>
<th>OECD</th>
<th>SVAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey data</td>
<td>1.00</td>
<td>0.56</td>
<td>0.78</td>
<td>0.62</td>
<td>0.76</td>
<td>0.77</td>
<td>0.59</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>Linear trend</td>
<td>0.56</td>
<td>1.00</td>
<td>0.72</td>
<td>0.46</td>
<td>0.79</td>
<td>0.78</td>
<td>0.97</td>
<td>0.85</td>
<td>0.42</td>
</tr>
<tr>
<td>Segmented trend</td>
<td>0.78</td>
<td>0.72</td>
<td>1.00</td>
<td>0.88</td>
<td>0.90</td>
<td>0.79</td>
<td>0.68</td>
<td>0.76</td>
<td>0.70</td>
</tr>
<tr>
<td>Robust trend</td>
<td>0.62</td>
<td>0.46</td>
<td>0.88</td>
<td>1.00</td>
<td>0.83</td>
<td>0.64</td>
<td>0.43</td>
<td>0.51</td>
<td>0.66</td>
</tr>
<tr>
<td>HP(1600)-filter</td>
<td>0.76</td>
<td>0.79</td>
<td>0.90</td>
<td>0.83</td>
<td>1.00</td>
<td>0.87</td>
<td>0.76</td>
<td>0.80</td>
<td>0.70</td>
</tr>
<tr>
<td>BP(6,32)-filter</td>
<td>0.77</td>
<td>0.78</td>
<td>0.79</td>
<td>0.64</td>
<td>0.87</td>
<td>1.00</td>
<td>0.79</td>
<td>0.85</td>
<td>0.76</td>
</tr>
<tr>
<td>UC estimation</td>
<td>0.59</td>
<td>0.97</td>
<td>0.68</td>
<td>0.43</td>
<td>0.76</td>
<td>0.79</td>
<td>1.00</td>
<td>0.92</td>
<td>0.51</td>
</tr>
<tr>
<td>OECD Estimation</td>
<td>0.79</td>
<td>0.85</td>
<td>0.76</td>
<td>0.51</td>
<td>0.80</td>
<td>0.85</td>
<td>0.92</td>
<td>1.00</td>
<td>0.70</td>
</tr>
<tr>
<td>SVAR-Gap</td>
<td>0.75</td>
<td>0.42</td>
<td>0.70</td>
<td>0.66</td>
<td>0.70</td>
<td>0.76</td>
<td>0.51</td>
<td>0.70</td>
<td>1.00</td>
</tr>
</tbody>
</table>

In particular, the correlation of the linear trend with the survey-based and the SVAR-founded methods is quite low. Obviously, this is due to the fact that, in contrast to other methods, the SVAR method identifies a cycle in the 1983-1987 period. Both the HP-filter and the BP-filter show fair, though not very strong correlations with the other methods.

To shed further light on the similarities between the approaches, we will make use of the so-called concordance statistic (Scott 2000, Scacciavillani and Swagel 1999). The test statistic takes the form:

\[
C_{ij} = T^{-1} \left\{ \sum (S_{i,t} \cdot S_{j,t} + (1-S_{i,t}) \cdot (1-S_{j,t})) \right\}
\]

\[
S_{i,j} = \begin{cases} 1 & \text{if} \quad \text{Gap}_{i,j} > 0 \\ 0 & \text{else} \end{cases}
\]
The statistic will give the value 1 if both gap measures have the same sign for a certain time period. In contrast, it will be zero if the sign of both measures always alternates. Thus, based on a null hypothesis of the flip of a coin on the sign of the gaps, the test statistic will be centered around 0.5. Table 7.3 shows the results of this task.

**Table 7.2: Implied Business Cycle Turning Points for Selected Models**

<table>
<thead>
<tr>
<th>Method</th>
<th>Turning Points</th>
</tr>
</thead>
</table>

P denotes business cycle peak, T denotes business cycle trough.

It turns out that there is no pair of methods for which the application of the concordance statistic reveals a value of one. Thus, no two methods tell exactly the same story of the business cycle in Euroland. However, all statistics are well above 0.5. This indicates that the methods do not contradict each other.
Table 7.3: Concordance Statistic for the Output Gap Measures (1985 I to 2000 II)

<table>
<thead>
<tr>
<th>Method</th>
<th>Linear Trend</th>
<th>Segmented Trend</th>
<th>HP-Filter</th>
<th>BP-Filter</th>
<th>Survey Data</th>
<th>UC Estimation</th>
<th>OECD Estimation</th>
<th>SVAR-Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Trend</td>
<td>1.00</td>
<td>0.74</td>
<td>0.69</td>
<td>0.81</td>
<td>0.84</td>
<td>0.73</td>
<td>0.92</td>
<td>0.89</td>
</tr>
<tr>
<td>Segmented Trend Model</td>
<td>(-)</td>
<td>1.00</td>
<td>0.85</td>
<td>0.87</td>
<td>0.77</td>
<td>0.85</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>Robust de-trending</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.89</td>
<td>0.76</td>
<td>0.84</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>HP-Filter</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.71</td>
<td>0.82</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>BP-Filter</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.79</td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Survey Data</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.71</td>
<td>0.74</td>
<td>0.71</td>
</tr>
<tr>
<td>UC Estimation</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.94</td>
<td>0.81</td>
</tr>
<tr>
<td>OECD Estimation</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
<td>0.84</td>
</tr>
<tr>
<td>SVAR-Gap</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

7.3 Stability of the Estimates

From a practitioner’s point of view, one of the most important features of a useful estimate of an output gap is the stability of the estimates during real time. Orphanides and van Noorden (1999) discuss the problems of estimating the current - that is end-of-sample - output gap with respect to US data. They conclude that the ex post revisions of the output gap series had been of the same order of magnitude as the output gap itself. Moreover, the revision turned out to be most important around business cycle turning points. As regards the sources of the revisions, the authors argue that ex-post revisions of the underlying data were not the predominant source of changes in the output gap estimates. Rather, the main problem was caused by the difficulties encountered in estimating the actual rate of trend growth.

The underlying problem is also illustrated by figure 7.2, which shows the output gap for the first quarter of 1997. The gaps are estimated either by the HP-filter or by linear de-trending. Step by step, additional information is included, that is additional quarters are included in the sample on which the estimation has been based. The results give a clear warning against rather mechanistic filtering. The HP-filter shows almost no output gap if only information up the first quarter of 1997 is included. If the estimation is based on information available in the 2000 the output gap is about -1 percent, a large value for a gap obtained from a HP-filter. Of course, one
may add predicted values before filtering, although this will only help if the forecasts are correct.

Figure 7.3: The output gap in 1997:1 calculated based on different Samples

<table>
<thead>
<tr>
<th>97:01</th>
<th>97:03</th>
<th>98:01</th>
<th>98:03</th>
<th>99:01</th>
<th>99:03</th>
<th>00:01</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.4</td>
<td>-2.0</td>
<td>-1.6</td>
<td>-1.2</td>
<td>-0.8</td>
<td>-0.4</td>
<td>0.0</td>
</tr>
</tbody>
</table>

7.4 Volatility of Trend and Cycle Component

Table 7.4 comprises a set of descriptive statistics on the gap and potential GDP series. It turns out that there is some trade-off with respect to the volatility of gaps and growth of potential GDP. The higher the standard deviation of the gap variable, the smoother is the series of potential GDP. The extreme case is, of course, linear de-trending which assumes a constant rate of growth over the selected sample. Generally, economic theory suggests that potential GDP should be less volatile than actual output.¹⁷ Both output gap and trend occur to be very volatile in case of the calculations based on survey data, which illustrates the problems of

¹⁷ As already mentioned, however, this notion is not undisputed. Theories suggesting a dominant role of technology shocks for the business cycle, for example, might provide a justification for a volatile potential GDP time series (e.g. Boschen and Mills 1990).
method mentioned above. Although potential GDP developed based on the structural method is also sensitive to cyclical factors such as capital accumulation, its volatility remains lower than the one of the majority of the statistical methods. In contrast, the introduction of supply and demand shocks in the determination of the trend and the output-gap in the SVAR method leads to a relatively high volatility of the trend.

**Table 7.4** Descriptive Statistics for the Gap-variables for Euroland (1985:1 to 2000:2)

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
<th>Actual growth of potential GDP</th>
<th>Standard deviation of growth rate of potential GDP</th>
<th>Unit root test for output gap(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey data</td>
<td>0.00</td>
<td>0.12</td>
<td>2.56</td>
<td>1.36</td>
<td>2.12</td>
<td>-2.15**</td>
</tr>
<tr>
<td>Linear trend</td>
<td>0.16</td>
<td>-0.38</td>
<td>1.87</td>
<td>2.21</td>
<td>0</td>
<td>-2.31**</td>
</tr>
<tr>
<td>Segmented trend</td>
<td>-0.41</td>
<td>-0.81</td>
<td>1.49</td>
<td>2.48/1.70</td>
<td>≈0</td>
<td>-1.96**</td>
</tr>
<tr>
<td>Robust trend</td>
<td>0.15</td>
<td>0.04</td>
<td>0.97</td>
<td>1.35</td>
<td>0.69</td>
<td>-1.30</td>
</tr>
<tr>
<td>HP(1600)-filter</td>
<td>-0.02</td>
<td>-0.18</td>
<td>0.83</td>
<td>2.46</td>
<td>0.56</td>
<td>-2.55**</td>
</tr>
<tr>
<td>BP(6,32)-filter</td>
<td>-0.04</td>
<td>-0.18</td>
<td>0.75</td>
<td>3.00</td>
<td>0.82</td>
<td>-2.08**</td>
</tr>
<tr>
<td>UC estimation</td>
<td>0.31</td>
<td>-0.40</td>
<td>1.39</td>
<td>2.33</td>
<td>0.47</td>
<td>-1.93*</td>
</tr>
<tr>
<td>OECD Estimation</td>
<td>-0.51</td>
<td>-0.97</td>
<td>1.39</td>
<td>2.45</td>
<td>0.44</td>
<td>-2.06**</td>
</tr>
<tr>
<td>SVAR-Gap</td>
<td>-0.36</td>
<td>-0.47</td>
<td>1.03</td>
<td>2.17</td>
<td>0.95</td>
<td>-2.21**</td>
</tr>
</tbody>
</table>

\(^a\)ADF-test. Test specification: no constant, no deterministic trend, four lags included. — *** (**, *) denotes rejection at the 1 (5, 10) percent level.

There are also substantial differences with respect to estimates of the recent level of the output gap. These differences illustrate the possible divergent economic interpretation entailed in the methods. All non-structural methods lead to the judgment that the recent output gap is positive with the exception that the linear trend model points to a closed gap. In contrast, the structural models based on a production function and a NAIRU show a somewhat negative output gap, though it is closes at the end of the sample. The SVAR gap has been positive recently. The table also illustrates the problem of the persistence of the output gap. Tests on
non-stationarity lead to the conclusion that - with one exception - the estimated output gaps are stationary.

7.5 Information Content with Regard to Inflation

Another empirical criterion for evaluating estimates of the output gap is whether or not they contain information with regard to inflation (Astley and Yates 1999, Heimonen and Pehkonen 1998, Cerra and Saxena (2000), Claus 2000). The underlying argument is that the output gap is an indicator of excess demand or supply in the aggregated goods market. Thus, if excess demand increases, inflationary pressures should also increase. To analyze this aspect, we calculate the correlation coefficient of each output gap time series with current inflation and the inflation rate four quarters ahead to capture possible leads of the gap series. Moreover we estimate a simple inflation equation:

\[ \Delta \pi_t = \alpha_0 + \sum_{j=1}^{4} \alpha_{uj} \Delta \pi_{t-j} + \sum_{j=0}^{4} \alpha_{2j} \text{gap}_{t-j} + u_t \]

and test the hypothesis $\alpha_{2i} = 0$. If this cannot be rejected, then there is no information content with respect to inflation in the gap series. One can also view equation [7.1] as one half of a test on Granger non-causality. A good estimate of the output gap should Granger-cause inflation. A more theory-orientated view might consider the equation as a very simple version of an expectations-augmented Phillips curve (Scott 2000).

Table 7.5 presents the results of the analysis. In general, the methods perform very poorly in this context. In the estimations presented, the lag length of both the lagged endogenous variables and the respective gap variables have been set equal to 4 quarters. A noteworthy exception is the SVAR gap. A reason for this finding is that the SVAR gap already uses the information on the inflation rate in the estimation process for the output gap.
Table 7.5: Testing the Information Content of Selected GAP-variables with respect to the Change of the Inflation Rate in Euroland

<table>
<thead>
<tr>
<th>Method</th>
<th>Inflation Model</th>
<th>First Difference Model</th>
<th>Deviation from objective model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear De-trending</td>
<td>1.33</td>
<td>1.48</td>
<td>3.21**</td>
</tr>
<tr>
<td>Segmented Trend Model</td>
<td>1.70</td>
<td>1.78</td>
<td>2.48*</td>
</tr>
<tr>
<td>Robust Trend Estimation</td>
<td>1.56</td>
<td>1.41</td>
<td>2.63*</td>
</tr>
<tr>
<td>HP-filter</td>
<td>1.54</td>
<td>1.02</td>
<td>2.09*</td>
</tr>
<tr>
<td>BP-filter</td>
<td>1.17</td>
<td>0.90</td>
<td>2.03*</td>
</tr>
<tr>
<td>Survey Data</td>
<td>0.64</td>
<td>0.74</td>
<td>0.62</td>
</tr>
<tr>
<td>UC estimation</td>
<td>0.41</td>
<td>0.24</td>
<td>1.43</td>
</tr>
<tr>
<td>OECD Estimation</td>
<td>0.58</td>
<td>0.82</td>
<td>1.60</td>
</tr>
<tr>
<td>SVAR Estimation</td>
<td>2.99**</td>
<td>4.19***</td>
<td>2.22*</td>
</tr>
</tbody>
</table>

*** (**, *) denotes rejection of the hypothesis at the 1 (5, 10) percent level.

Although the performance of the gap variables is not impressive at all, some caution should be taken before drawing any wide-reaching conclusions based on these results. First, the estimations are generally not very robust against specification changes. For example, choosing shorter lags with regard to the gap variables leads to the result of significant information content in the IMF and OECD estimates. Second, our results are in variance to the results of e.g. Claus (2000) or Heimonen and Pehkonen (1998) who report a significant information content of some prominent output gap measures for inflation using data for individual countries. Thus, it may well be that the insignificant results are specific for aggregated data for the Euro-zone. A third related point is that the inflation rate in Euroland experienced a strong downward trend during the investigation period. This might reflect a change in the inflation target of the European central banks. Thus, we have also estimated a model, which takes into account the implicit target of these banks. As can be seen from the third column of table 7.5 the results generally improve, leading to significant results in some cases. In one way, this method supposes that the cost (in terms of accessible output) of the reduction of the price target of the

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18 We are grateful to Jan Gottschalk who has provided us the data of the implicit target of the central banks. These data have been obtained by applied a slightly modified version of the approach by Gerlach and Svensson (2000)
central bank in an environment with rigidities is already integrated in the output gap calculation. Fourth, other methods to investigate the information content might be necessary. For example, out-of-sample tests or vector autoregressive models could be used (Claus 2000). However, even if one takes into account the shortcomings of the present estimations the performance of the popular gap variables is still disappointing and strengthens the demand for more sophisticated models.

8. Conclusions for Economic Policy in Euroland
The discussion above has revealed that there is large uncertainty on the "true" output gap in Euroland in more than one respect. Following the ECB (2000) several forms of uncertainty can be distinguished. First, there is uncertainty within a given method since every estimation is a point estimate and confidence intervals should be included. Normally, this confidence band is rather wide and a zero output gap cannot be ruled out. We illustrate this point by plotting the output gap based on a simple unobserved component model mentioned above.

Second, the amount of the gap varies with the method used. The difference between competing approaches is as large as 3 percentage points or more. A third aspect of uncertainty is the time span used in the analysis. Some of the methods are quite sensitive with respect to this point. All in all, the actual output gap is far from exactly known. Hence, the repercussion this fact on may have on monetary policy has to be discussed. Smets (1997) analyses the optimal response of monetary policy in a macro model with an uncertain output gap. He argues that within a Taylor-rule-type of monetary reaction function monetary policy makers should react less to the current output gap than to inflation in a world with uncertainty. Furthermore, he points out that this line of argumentation may help to explain why observed short term interest rates are normally much less volatile as the respective Taylor rule interest rate. However, Drew and Hunt (1998) use a large structural model of the New Zealand economy for stochastic simulations to evaluate the response of economic performance to competing monetary rules. They conclude that rules that take into account the uncertainty with regard to the output gap make no big difference to rules without such a feature. Thus, the optimal responses to output gap uncertainty is still an open question.
However, it might well be that an investment in research on this topic will lead to a higher pay off than calculating additional measures of the output gap would. The preceding discussion, however, at least has tried to show that the choice of a specific method can be made with the help of different criteria, depending on the concrete goal of the research.
References


Bolt, W. and P.J.A. van Els (2000), Output Gap and Inflation in the EU. *DNB Staff Reports* No. 44. Amsterdam.


Figure 7.2: Selected Output gap Measures for the Euro-zone

Based on Survey Data

Based on a Linear Trend

Based on Segmented Trend Model

Based on Robust Trend Estimation

Based on a Hodrick/Prescott (1600) filter

Based on a Band/Pass (6, 32) filter

Based on an Unobserved Component Model

Based on OECD Estimations

Based on a SVAR