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Optimal Taxation in a Unionised Economy

Abstract

Unions appear to have an aversion to wage disparities among their members, leading to wage compression. This paper analyses the consequences of this for income tax policy. In a two-sector general equilibrium model we highlight the tradeoff between correcting the resource misallocation created by wage compression and the government's distributional objectives. Where the union's aversion to wage dispersion is strong, tax policy can do little to correct the distortion in the supply of trained labour, though it can come closer to achieving distributional aims. Where wage compression is less pronounced, tax policy can have significant effects on resource misallocation, at the expense of its distributional goals.

JEL-Codes: H210, H240.

Keywords: income taxation, optimal taxation, unionized economy, wage compression.

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1 Introduction

Most optimal income tax theory assumes that markets are competitive and wages are determined either by exogenous productivity, as in the Mirrlees model, or by supply and demand¹. In practice, in many countries, wage formation depends heavily on the power of trade unions and wage bargaining is important. The major part of the economics literature on trade unions assumes that firms generate profits that may accrue to the owners or be appropriated by workers through union pressure to increase wages². Typically labour is taken to be homogenous so that there is a single wage and the effect of raising the wage rate above the market clearing level is to create unemployment³. Much of the literature has therefore taken a macroeconomic perspective. The interest in tax policy in this context has mainly been motivated by concern with how it affects wage formation and consequently unemployment. The typical assumption is that unions are concerned with after-tax income and taxes will then affect how wage increases translate into changes in disposable income.

In this paper we shall address taxation in a unionised economy taking a different approach, which deviates in many respects from that just outlined. While appropriating income at the expense of firm owners may be an important concern, it is also widely acknowledged that an important objective of trade unions in countries where they are influential is to pursue wage compression, that is, to reduce the differences between gross wage rates paid to different groups of workers. There can be little doubt that the wage compression observed in the Nordic countries is to a large extent a consequence of the distributional objectives of the trade union movement. As shown for instance in Barth et al. (2015), Figure 1, the Scandinavian countries have the smallest wage dispersion among the OECD countries. Our focus in this paper will be on this wage compression aspect of a unionised economy. For this reason we assume that there is wage heterogeneity among workers, and there is a union that is averse to wage dispersion.

There is evidence that trade unions are mainly concerned with pre-tax wage dispersion⁴. We adopt this view and assume that the union has preferences solely over pre-tax wage levels and differences. Taking this point of departure, we model how the wages of union members are determined assuming that there are no profits accruing to firm owners. Since economies are not typically fully unionised we shall assume that this economy consists of a unionised sector and a non-unionised (competitive) sector. Shifting the focus entirely away from

¹See for example Allen (1982) and Lee and Saez (2008).

²Further discussion of workers' appropriation of rents can be found for instance in Geroski et al. (1996) and Jean and Nicoletti (2004).

³A few selected references are Aronsson and Sjögren (2004), Hersoug (1984), Layard et al. (1991), and Oswald (1985). An empirical paper is Lockwood et al. (2000).

⁴A source is people in The Technical Calculation Committee in Norway, which is a body establishing the factual background for the wage bargaining process. Cappelen (2018, p.8) makes the rather general statement that tax policy affects the wage growth less than was previously believed. Nor do Nymoen and Rødseth (2003) report significant effects of taxes.

the unemployment issue we shall assume that there is full employment in the economy, that is, any possible excess labour supply in the unionised sector is absorbed in the non-unionised sector. While the union is not directly influenced by taxes, worker behaviour is. In particular, we shall assume that workers are either trained or untrained and the extent to which they acquire training depends on the after-tax return to training.

Our way of modelling the economy takes us closer to the traditional optimal tax approach, where there is concern with the utility distribution among workers and the disincentive effects of taxes. However, a further departure from the traditional tax modelling is to postulate that the government and the unionised sector play a game with simultaneous moves, each taking the values of the other's choice variables as given. In other respects, the government is modelled in a conventional way. It is concerned with the welfare of all workers⁵ and sets taxes to maximise social welfare subject to a tax revenue requirement and also to constraints arising out of labour market equilibrium. In the Nash equilibrium of the game, the taxes set by the government and the wages set by the union are mutually best replies. The employment levels in the economy are then fully determined. Without real loss of generality from the point of view of the aims of this paper, we assume that the social welfare function is utilitarian.

2 The model framework

We assume that there are two sectors in the economy and two types of labour, trained and untrained. We label the sectors 0 and 1, respectively. Sector 0 uses only untrained labour, l_0 . Sector 1, which will be the one that is unionised, employs both kinds of labour with l_1 denoting untrained and l_2 trained labour. There is a population of workers/consumers of mass 1 each of whom supplies one unit of labour time inelastically. Assuming full employment,

$$l_0 = 1 - l_1 - l_2 \tag{1}$$

In order to focus sharply on the wage compression issue, we abstract entirely from bargaining over the division of any rent, which is at the core of most models addressing unionised economies in the absence of wage compression. The simplest way to do that is by assuming an economy with constant returns to scale. The production functions in the two sectors are ωl_0 and $g(l_1, l_2)$ respectively, where the latter is a standard linear homogeneous, strictly quasiconcave production function. Moreover, we assume both types of labour are required for production: $g(0, l_2) = g(l_1, 0) = 0$. We can think of both sectors' outputs being sold in the world market at fixed prices normalised at unity.

Workers are trained at a cost $v \in [0, \bar{v}]$, which depends on their exogenous innate ability, and has the cumulative distribution function $F(v)$ across the population of workers, with $F'(v) = f(v) > 0 \forall v \in [0, \bar{v}]$. Those who remain

⁵Though not of the union as such, that is, it disregards the utility of the union in formulating its social welfare function. This seems to be consistent with the usual individualistic formulation of social welfare.

untrained get a job in one of the sectors; wages adjust to achieve full employment overall.

Let c denote a worker's consumption gross of any training cost, with, in the absence of taxes, c_2 that of trained workers and c_0, c_1 the consumption levels of untrained workers in sectors 0 and 1, respectively. Assuming that workers incur the cost of training, a trained worker's consumption net of the cost of training is $c_2 - v$. A worker will then opt for training according to whether $U(c_2 - v) \gtrless U(c_1)$, where $U(\cdot)$ is a utility function, strictly concave and increasing.⁶ In the case of indifference, $v = \hat{v} = c_2 - c_1 \in [0, \bar{v}]$.

Let the wage rates in sector 1 be w_1 for untrained and w_2 for trained workers. The wage rate in the competitive non-unionised sector 0 will clearly be ω . The assumption that workers supply one unit of labour inelastically implies that $c_i = w_i, i = 1, 2$, and $\hat{v} = w_2 - w_1$ in the absence of taxes.

Given that it is desirable to have a positive number of trained workers, it is obviously optimal to train those with the lowest training cost up to \hat{v} and we assume, without further analysis, that this is the case. Then, for $\hat{v} \in [0, \bar{v}]$

$$S(\hat{v}) \equiv \int_0^{\hat{v}} f(v)dv \quad (2)$$

is the supply of trained labour in each of the economies we will be considering.

If an untrained worker faces a specific tax⁷ $t_1 \gtrless 0$ while a trained worker is taxed at t_2 then \hat{v} is defined by

$$w_2 - t_2 - \hat{v} = w_1 - t_1$$

The net wage premium is just offset by the cost of training for the marginal trained worker. The supply function for skilled labour is then

$$S(w_2 - t_2 - w_1 + t_1) \equiv \int_0^{\hat{v}} f(v)dv \quad (3)$$

with

$$\frac{\partial S}{\partial w_1} = -f(\hat{v}); \frac{\partial S}{\partial w_2} = f(\hat{v}) \quad (4)$$

$$\frac{\partial S}{\partial t_1} = f(\hat{v}); \frac{\partial S}{\partial t_2} = -f(\hat{v}) \quad (5)$$

Before proceeding to discuss models based on further assumptions about markets and unions, we establish a benchmark by characterising the first best allocation.

⁶Note we make the standard assumption in optimal tax analysis that consumers' utility functions are identical. There are no utility differences between job types.

⁷We define these taxes as simply amounts of consumption, rather than proportional tax rates. This simplifies the expressions without changing the results qualitatively. Because of the assumption of perfectly inelastic labour supplies the taxes are lump sum on earnings but of course have marginal impacts on the supply of trained labour.

3 The first best allocation

The total (value of) output in the economy is $g(l_1, l_2) + \omega l_0$. The first best optimum is then the allocation that maximises total output value *minus* the total training cost and distributes the total consumption to the agents in a way that equalises marginal social utilities in order to maximise welfare. Given the resource constraint in (1) the net output maximisation problem is

$$\max_{l_1, l_2} g\left(l_1, \int_0^{\hat{v}} f(v)dv\right) + \omega\left(1 - l_1 - \int_0^{\bar{v}} f(v)dv\right) - \int_0^{\hat{v}} v f(v)dv \quad (6)$$

The first order conditions are:⁸

$$g_1(l_1, l_2) - \omega = 0 \quad (7)$$

implying that the marginal product of untrained labour is the same in the two sectors, and

$$g_2(l_1, l_2) - \hat{v} - \omega = 0 \quad (8)$$

implying that the marginal product of trained labour net of the training cost of the marginal worker is equal to the marginal product of untrained labour.

Maximising the sum of utilities subject to total consumption being constrained by the available aggregate net output we derive the first best welfare maximum. This requires that total consumption be distributed in such a way that all agents have the same marginal utility of consumption

$$u'(c_0) = u'(c_1) = u'(c_2) \quad (9)$$

implying of course that $c_0 = c_1 = c_2$. This obviously requires that each trained worker be exactly compensated for the true amount of her training cost. For example if trained workers pay for their own training costs they receive gross payments $c(v) = c_2 + v$, $\forall v \in [0, \hat{v}]$. Alternatively the "planner" could pay all training costs directly.⁹ If all trained workers are given the same gross consumption out of which they pay their own training costs obviously marginal utilities are lower the lower is a trained worker's v and the allocation is not first best on distributional grounds. Note that in this allocation trained workers with $v < \hat{v}$ receive no rents from their lower training costs.

⁸Partial derivatives in subscripts.

⁹One could imagine a state training programme to which workers are admitted on the basis of an infallible test of ability.

4 The unionised economy

We now analyse an economy where in sector 1 a union sets wage rates in an otherwise competitive market. We assume that all firms in this market are identical and so we can work in terms of a representative price-taking firm. Equilibrium in the market for trained labour in this sector requires

$$l_2 = S(w_2 - t_2 - w_1 + t_1) \quad (10)$$

while cost minimising behaviour requires that the marginal rate of technical substitution is equated to relative factor prices,

$$\frac{g_1(l_1, l_2)}{g_2(l_1, l_2)} = \frac{w_1}{w_2} \quad (11)$$

The constant returns to scale assumption implies the existence of the unit cost function $C(w_1, w_2)$ which is independent of the scale of output. Furthermore the factor mix in this sector is determined by

$$\frac{C_1(w_1, w_2)}{C_2(w_1, w_2)} = \frac{l_1^0}{l_2^0} = m\left(\frac{w_2}{w_1}\right) \quad (12)$$

where l_i^0 is the amount of labour of type $i = 1, 2$ required to produce one unit of output. At given wages, the output expansion path is characterised by

$$l_1 = m\left(\frac{w_2}{w_1}\right) l_2 \quad (13)$$

which determines l_1 for given l_2 . Note that $m'(\cdot) > 0$, an increase in the relative price of trained labour increases the untrained labour intensity of production and conversely.

Non-negative profit equilibrium requires that the unit cost be no greater than the output price:

$$C(w_1, w_2) \leq 1 \quad (14)$$

Thus, given wage rates determined by the union, and tax rates determined by the planner, equilibrium employment in this sector is given by (10) and (13), and employment levels lie on the efficient expansion path given by (12).

The case we are interested in is one in which the union is averse to the wage dispersion in the unionised sector. The absence of any rents, which results from assuming constant returns to scale, allows us to focus on this wage compression issue by abstracting entirely from bargaining over the division of rents. In choosing wage rates the union maximises

$$U = u_1(w_2) - ku_2(w_2 - w_1)$$

where $k \geq 0$ is a measure of the relative strength of the union's aversion to wage inequality.¹⁰ The utility functions have the properties: $u'_1 > 0$, $u''_1 < 0$, $u'_2 > 0$,

¹⁰We include k for purposes of analysing the effects of variations in the strength of the union's aversion to wage inequality on its choices.

$u_2'' > 0$, $u_2(0) = 0$, and $u_2(\cdot)$ is defined on the domain $\{(w_1, w_2); w_2 \geq w_1 \geq \omega\}$. The union would like trained workers to have a high wage but it also dislikes wage dispersion. Where the wage dispersion is already large the benefit from a further increase in w_2 may be outweighed by the disbenefit of additional wage dispersion. We note that there is an asymmetry in the objective function in favour of w_2 . Reasons for this could be that trained workers are more influential in the union or that it is generally considered fair that there should be some wage compensation for acquiring training.

Since the union realises it cannot drive firms out of business, it maximises subject to the break even constraint $C(w_1, w_2) \leq 1$. We examine explicitly the conditions under which $w_1 > \omega$ so that untrained workers in the unionised sector are made better off than in the market economy. To do this we impose the constraint $w_1 \geq \omega$ on the union's problem, which is therefore

$$\max_{w_i} U \quad \text{s.t.} \quad C(w_1, w_2) \leq 1, \quad w_1 \geq \omega \quad (15)$$

From the first order conditions for this problem¹¹ we have:

$$ku_2'(w_2 - w_1) = \gamma l_1^0 - \nu \quad (16)$$

$$u_1'(w_2) - ku_2'(w_2 - w_1) = \gamma l_2^0 \quad (17)$$

together with the constraints. Note that these conditions are independent of employment levels l_1, l_2 as long as they lie on the expansion path. Thus for example a change in l_2 caused by a change in a tax rate that changes the supply of trained labour simply leads to a movement along the expansion path and the union does not change its desired wage rates.

Suppose $k = 0$. Then we avoid a contradiction if and only if $\nu > 0$, in which case $w_1 = \omega$. Given $u_1'(w_2) > 0$ the break even constraint certainly binds, $\gamma = u_1'(w_2)/l_2^0 > 0$.

If now $k > 0$ we can show¹² that as long as its value lies below a reasonable upper bound, we have $\partial w_1 / \partial k > 0$, $\partial w_2 / \partial k < 0$, so that we have $w_1 > \omega$ and so $\nu = 0$. Thus from (16) we must have $w_2 > w_1 > \omega$ in the equilibrium.

Finally, if we assume that in the equilibrium for the total employment levels in the unionised sector conditions (10) and (13) hold, we have that at any given wage ratio

$$\frac{\partial l_1}{\partial k} = m \left(\frac{w_2}{w_1} \right) \frac{\partial l_2}{\partial k} < 0 \quad (18)$$

$$\frac{\partial l_2}{\partial k} = f(\hat{v}) \left[\frac{\partial w_2}{\partial k} - \frac{\partial w_1}{\partial k} \right] < 0 \quad (19)$$

Thus we can state

Proposition 1 *On the assumptions of the model and subject to a reasonable upper bound, the stronger the union's aversion to wage difference $w_2 - w_1$ the*

¹¹ See Appendix A for a derivation and proofs of the following propositions.

¹² See Appendix A.

lower is w_2 and the higher is w_1 , with $w_2 > w_1 > \omega$, and employment of both types of worker is lower than where the union is indifferent to wage compression.

Proposition 2 *Since in the unionised economy with $k > 0$, $g_1(l_1, l_2) = w_1 > \omega$, output and employment of both types of worker in the unionised sector are lower than in the first best allocation.*

These conclusions are quite intuitive. First, if $k = 0$ the union cares only about the trained workers' wage, and so it sets the untrained workers' wage at its lowest feasible value and maximises that of the trained workers, subject to the break even constraint. With $k > 0$, increasing k raises the wage of the untrained workers, reduces that of the trained workers and shifts employment levels down. The fall in w_2/w_1 causes substitution of trained labour for untrained, but the reduced net return to training reduces the supply of trained workers and so total employment and output in the unionised sector must fall (recall that the firm is indifferent between output levels that break even). The displaced workers find jobs in the non unionised sector at wage rate ω . Given that firms in the unionised sector minimise cost, the condition in the first best allocation that $g_1(l_1, l_2) = \omega$ is violated, with underemployment of resources in the unionised sector.

Thought of diagrammatically, in the (w_1, w_2) -plane the break even constraint can be represented by a line with slope $-l_1^0/l_2^0$, and (16) and (17) simply represent a tangency solution given by

$$\frac{ku'_2(.)}{u'_1(.) - ku'_2(.)} = \frac{1}{[u'_1(.) / ku'_2(.)] - 1} = \frac{l_1^0}{l_2^0} \quad (20)$$

This establishes that we must have $k < [u'_1(w_2)/u'_2(w_2 - w_1)]$, otherwise indifference curves in this plane are upward sloping. Then holding (w_1, w_2) at the equilibrium point an increase in k increases the marginal rate of substitution and so implies a new equilibrium with higher w_1 and lower w_2 . This then reduces aggregate employment in this sector as just described.

Having characterised the equilibrium of the unionised sector and its main comparative statics we now turn to the analysis of taxation.¹³

5 Optimal Taxation with Wage Compression

As in the non-unionised economy, the government is utilitarian, and it has at its disposal taxes which determine the consumption levels of each type of worker. The central question concerns the type of tax system that is feasible. It would be possible to take what we might call a *Pigovian* approach: the government could

¹³Note that if the union also cared about the size of its membership $l_1 + l_2$ there would be an effect tending to counteract the process just described. For simplicity however we exclude this possibility in order to focus on the effects of the desire for wage compression.

design taxes explicitly to correct the resource misallocation in the unionised sector. For example, simply by choosing appropriate taxes on trained and untrained labour in the unionised sector to increase employment of trained and reduce that of untrained labour it could undo the resource misallocation created by the union's compression of gross wages. In reality however, given a complex multi-sectoral economy with many unionised and non-unionised sectors and different unions with possibly different preferences, such a Pigovian approach does not seem very feasible or relevant. The real income tax system covers a large range of wage types and sectors, and so could not be finely calibrated to deal with the wage compression issue in any one sector or industry alone. A more plausible policy would in that case be to use a subsidy on training costs, or a subsidised training system, to correct the tendency of wage compression to distort the supply of trained workers.

We ask a more general question: given a tax system that has to apply to the whole economy, with tax rates that are differentiated not by worker type as such but by income levels, how is the optimal design of such taxes affected by the existence of a unionised sector in which the union has a preference for compression of gross wages? This is equivalent to placing second best constraints on the government's feasible choice of tax instruments.

We compare two possible two-bracket¹⁴ piecewise linear tax systems,¹⁵ in which a tax t_1 is applied to relatively lower incomes and a tax t_2 to relatively higher incomes. Untrained workers in the non-unionised sector have the lowest gross income and trained workers in the unionised sector have the highest. Untrained workers' wages in the latter sector lie somewhere in between, to an extent dependent on the union's preference for wage compression.

That suggests that we should compare two alternative tax systems. In one, t_1 applies to untrained workers in the non-unionised sector and t_2 to untrained as well as trained workers in the unionised sector. We refer to this as *pooling by sector*. In the other, t_1 applies to untrained workers in both the non-unionised and unionised sectors and t_2 to trained workers in the latter. This is *pooling by training*. We could think of pooling by sector as arising when the difference between w_1 and ω is relatively large, and pooling by training when the difference is relatively small. In other words, pooling by sector is more likely to arise when the unions aversion to wage differentials is relatively high and pooling by training when it is relatively low. Since we are looking for the government's best response to the wage choices of the union, we take w_1, w_2 as given throughout the analysis.

¹⁴Allowing three possible taxes, one for each wage type, comes very close to assuming a Pigovian tax problem. It allows the government to equalise marginal utilities and net incomes across all three wage types, though it cannot do anything about the intramarginal rents earned by trained workers with $v < \hat{v}^*$. Restricting the tax system to two brackets does we believe more closely replicate the constraints on the tax instruments available to the government in a real economy in a simple way.

¹⁵A linear (single bracket) tax would be even more constrained but there is no *a priori* reason why this should be adopted.

5.1 Pooling by sector

Given w_1, w_2 , the government's choices of specific taxes t_1 in the non-unionised sector and t_2 in the unionised sector imply that it cannot influence directly the value of $\hat{v} = w_2 - t_2 - w_1 - t_1 = w_2 - w_1$. We do not assume that the government can levy taxes on trained workers conditioned on their values of v , nor that a Mirrlees-type optimal nonlinear tax on trained workers is feasible. The earlier conditions (10) and (13) imply the equilibrium employment levels:

$$l_0 = 1 - (l_1 + l_2) \quad (21)$$

$$l_1 = m\left(\frac{w_2}{w_1}\right) l_2 \quad (22)$$

$$l_2 = \int_0^{w_2 - w_1} dF(v)$$

We write the government's social welfare function¹⁶ as:

$$\begin{aligned} \mathcal{W} = & [1 - (l_1 + l_2)]U(\omega - t_1) + l_1 U(w_1 - t_2) \\ & + \int_0^{w_2 - w_1} U(w_2 - v - t_2) dF(v) \end{aligned} \quad (23)$$

with w_1, w_2 given. The government's tax revenue constraint is:

$$t_1[1 - (l_1 + l_2)] + t_2\left(m\left(\frac{w_2}{w_1}\right) + 1\right) \int_0^{\hat{v}} dF(v) \geq 0 \quad (24)$$

We assume that the tax revenue requirement is zero, which is a useful simplification without any real loss of generality.¹⁷ Note also that we need to introduce formally the condition $w_1 - t_2 \geq \omega - t_1$ since this must hold if the firms in the unionised sector are to be able to hire the untrained workers they want at the Nash equilibrium. It is clear from this that under pooling by sector the government cannot influence the difference between gross wages of trained and untrained workers in the unionised sector. We summarise this as:

Proposition 3 *When the wage difference between trained and untrained workers in the unionised sector is relatively small, so that both groups of workers are in the upper tax bracket, the general income tax system cannot affect relative wage rates in this sector and so a Pigovian tax/subsidy policy, or, more realistically, subsidisation or public provision of training programmes, would be necessary to correct the under-supply of trained labour resulting from the wage compression.*

From the first order conditions (see Appendix B) we obtain the following:

¹⁶Note that we assume trained workers have to pay their own training cost.

¹⁷With a positive revenue requirement both taxes are likely to be positive so the propositions here translate into statements about the relative levels of the taxes.

Proposition 4 *In the Nash equilibrium the government chooses tax rates $t_1^* < 0 < t_2^*$ to equalise marginal utilities and incomes of all untrained workers and the marginal trained workers, $w_2 - \hat{v} - t_2^* = w_1 - t_2^* = \omega - t_1$.*

By imposing a tax on the better paid workers of both types in the unionised sector and paying a subsidy to those in the non-unionised sector¹⁸ the government can bring net incomes of all but the intramarginal trained workers in the unionised sector into alignment, without affecting the differential between trained and untrained workers in the unionised sector and therefore the ratio l_1/l_2 , which is fixed by the gross wage rates. As just pointed out, it does however lack an instrument for changing the relative net wages of trained and untrained workers in the unionised sector so cannot reduce the distortion in trained worker supply created there by wage compression. Even though at the margin incomes are equalised as just shown, employment in the unionised sector remains unchanged because the demand for trained labour and the differential with untrained labour is unchanged. Even though they have lost their rents from being in the unionised sector, the untrained workers there have no incentive to switch to the non-unionised sector.

5.2 Pooling by training

We now have that t_1 is the tax on untrained workers, whether in the unionised or non-unionised sectors, and t_2 that on trained workers. The government's problem is now

$$\max_{t_1, t_2} \mathcal{W} \text{ s.t. } t_1 [1 - \int_0^{w_2 - t_2 - w_1 + t_1} dF(v)] + t_2 \int_0^{w_2 - t_2 - w_1 + t_1} dF(v) \geq 0 \quad (25)$$

with the further constraint $w_1 - t_1 \geq \omega - t_1$. From this we see that the income tax system cannot affect the net wage difference between unionised and non-unionised untrained workers, so that the constraint is nonbinding given that the union sets $w_1 > \omega$. On the other hand the government now has instruments to influence the mix of trained and untrained workers in the unionised sector.

From the first order conditions for this problem (see Appendix C) we have

Proposition 5 *In the Nash equilibrium the government chooses tax rates $t_1^* \geq t_2^*$, to satisfy the two conditions:*

$$\lambda^* = U'(\omega - t_1^*)l_0^* + U'(w_1 - t_1^*)l_1^* + \int_0^{w_2 - t_2^* - w_1 + t_1^*} U'(w_2 - v - t_2^*)dF(v) \quad (26)$$

$$t_2^* - t_1^* = \frac{1}{\lambda^* f(\hat{v})} \left\{ \int_0^{w_2 - t_2^* - w_1 + t_1^*} [\lambda^* - U'(w_2 - v - t_2^*)dF(v)] \right. \\ \left. - (1 + m(\cdot))f(\hat{v})[U(w_1 - t_1^*) - U(\omega - t_1^*)] \right\} \quad (27)$$

¹⁸With a sufficiently high positive tax revenue requirement both tax rates would be positive but the result would still apply to their relative values. This is of course the realistic case.

The first of these conditions shows that the marginal utilities cannot be equalised, with for $v < \hat{v}$,

$$U'(w_2 - v - t_2^*) < U'(w_2 - \hat{v} - t_2^*) = U'(w_1 - t_1^*) < U'(\omega - t_1^*). \quad (28)$$

The second condition shows that it is possible to have a case in which the government pays a subsidy to trained workers and taxes untrained workers, even though this worsens the income distribution, that is, $t_1^* > 0 > t_2^*$. This is more likely the greater the density of trained workers at the margin, $f(\hat{v}^*)$, the greater the distortion in supply of trained workers arising from wage compression, as expressed by $m(\cdot)$, the greater the difference between the utilities of untrained workers in the unionised and non-unionised sectors respectively, and the higher the marginal utility of trained workers relative to the average, or equivalently, using (26), the lower the sums of marginal utilities of untrained workers, $U'(\omega - t_1^*)l_0^* + U'(w_1 - t_1^*)l_1^*$.

The intuition is straightforward: the government would like to reduce income inequality and also the distortion in supply of trained workers, given the available tax instruments. Subsidising untrained workers and taxing trained workers now does nothing to reduce the inequality between untrained workers in the two sectors, reduces the gap between trained and untrained workers in the unionised sector, but worsens labour supply distortion. It could still be optimal if the density of trained workers at the margin is sufficiently small, since this reduces the gain from increasing the training incentive, and also if the difference between incomes of untrained workers in the two sectors is small, because this lowers the loss of utility to the untrained workers in the unionised sector, who are displaced when the supply of trained workers in that sector is reduced. Although in the converse case subsidising trained and taxing untrained workers is obviously regressive, if the density of trained workers at the margin is sufficiently high, subsidising trained workers has a large impact on their supply and so is more successful in mitigating the effects of wage compression. A large gap between wages of untrained workers in the two sectors also reduces the attractiveness of subsidising untrained workers and so strengthens the net gain from reducing the labour supply distortion.

6 Comparative statics

We can think of two events inducing changes in wages in the economy. One is the introduction of a union setting wages that deviate from the equilibrium wages in a market economy. Another is a change in the union's preferences expressed by k in our model. An interesting question is how optimal taxes set by the government respond to wage changes in either tax regime that we have addressed. We shall consider an increase of w_1 with w_2 adjusting to restore the break even condition $c(w_1, w_2) = 1$. We provide the derivation of the comparative static results in Appendix D. In the interest of tractability we assume that there is a uniform distribution of training cost. This assumption implies that the wage derivative of labour supply (S') is constant.

6.1 Pooling by sector.

The wage changes will diminish the wage dispersion in the unionised sector but this dispersion is not of direct concern when setting t_2 , since t_2 does not redistribute within the unionised sector. However, disposable income will fall for the worst-off agents as t_1 must increase while ω is fixed. The reason is that the increase in w_1 will enhance the number of low-tax payers and reduce the number of high-tax payers so that t_1 must be raised to restore budget balance. As a consequence marginal utility increases for workers in the market sector and they benefit more in terms of utility from the redistribution considered at the optimum. A reinforcing effect is that untrained workers in the unionised sector are made better off due to the increase in w_1 (assuming $1 - \frac{dt_1}{dw_1} > 0$), U'_1 diminishes, and so does the utility loss from increasing t_2 . A counteracting effect is that trained workers are made worse off, U'_2 rises, and so does the utility loss induced by a larger t_2 . The standard assumption is that $-U''$ is decreasing in income implying that $-U''(\omega - t_1) > U''(w_1 - t_2) > U''(w_2 - v - t_2)$. Where $-U''$ is sufficiently large for lower incomes and decreasing in income the equity case for further redistributive taxation is strengthened.

But even if there is an equity case for more extensive redistribution via the tax system less redistribution may be achievable by changing taxes so that further differentiation of taxes may not be worthwhile. It is important to note that the equity case for redistribution and the opportunity for achieving redistribution are not independent. To recognise this, neglect for the moment the opposing effects above on workers in the unionised sector and make use of the formal analysis in the Appendix. The following expression is helpful:

$$\begin{aligned} & (U'_0 - U'_1) \frac{\partial}{\partial w_1} (l_1 + l_2) - U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} = \\ & \left[-U''(\omega - t_1) (l_1 + l_2) \frac{1}{l_0} - (U'_0 - U'_1) \right] \times \\ & \left\{ \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(\frac{l_1}{l_2} + 1 \right) - m' \left(\frac{w_2}{w_1} \right) \frac{\partial \left(\frac{w_2}{w_1} \right)}{\partial w_1} \right\} \end{aligned} \quad (29)$$

The terms in curly brackets express the decline in the number of workers in the unionised sector due to three effects. Increasing w_1 and lowering w_2 discourages the supply of trained workers and also the number of untrained workers in the unionised sector through complementarity and adjustment of the factor mix. The downsizing of employment in the unionised sector pushes up t_1 and increases the marginal utility of workers in the market sector as described above ($-U'' > 0$) but diminishes the achievable redistributive gain (due to $U'_0 - U'_1 > 0$) since there will be fewer workers to tax for the benefit of a larger number of beneficiaries. The larger are S' (the supply responsiveness of l_2) and m' (the substitution response to a change in relative wages) the smaller is the opportunity for achieving redistribution but also the stronger is the equity case

for redistribution. We see that whether the effect of the term in curly brackets is in favour of a larger or smaller t_2 depends on the sign of the bracketed term.

6.2 Pooling by training

The tax perturbation considered at the optimum (increasing t_2 and lowering t_1) redistributes income between trained workers in the unionised sector and untrained workers employed both in the unionised and in the market sector. The wage changes (increasing w_1 and lowering w_2) redistribute income between trained and untrained workers in the unionised sector and weakens the case for deploying taxes for this purpose. However, workers in the market sector are made worse off when t_1 is raised to balance the government budget. Where marginal utility is particularly sensitive to income changes for the worst off agents there is an equity argument for increasing t_2 and lowering t_1 in the interest of the workers in the market sector. The wage changes will also affect the sectoral allocation of labour both directly and indirectly. By themselves the wage changes will reallocate more workers to the market sector where they benefit from the tax perturbation. The wage changes will also weaken the tendency of the tax perturbation to push workers out of the unionised sector and into the market sector where they are worse off. Where all these factors are strong, it is conceivable that there is a case for a more redistributive tax policy (increasing t_2 and lowering t_1) in response to the wage changes under consideration. However there are counteracting effects that may reverse this outcome. The wage changes will diminish the tax relief accruing to the low-income workers and also enhance the utility loss incurred by the workers being pushed into the market sector.

7 Conclusions

Unions appear to have a marked aversion to wage disparities among different groups of workers among their members, which leads to a tendency towards wage compression in their wage negotiations. This paper has analysed the consequences of this observation for tax policy. We set up a simple two-sector general equilibrium model in which firms earn no rents and a competitive non-unionised sector absorbs any workers displaced by wage-setting in the unionised sector. This therefore rules out the issues that have been analysed in depth in the existing literature, which have been concerned with unions' rent appropriation and macroeconomic effects on inflation and employment, in order to focus on the distortions to the supply of trained labour arising out of compression in the gross wage rates of trained and untrained workers. The government and union are engaged in a one-shot non-cooperative game in which the union chooses gross wage rates and the government chooses tax rates in a marginal rate-progressive income tax system. On grounds of realism, we do not assume that the government can choose Pigovian taxes/subsidies that directly offset the effects of wage compression, but rather assume it has to work through a general

income tax system that applies to all incomes in the economy, those of unionised and non-unionised workers alike. However, we do point out circumstances under which publicly provided or at least subsidised training programmes could play a role in offsetting the effects of wage compression.

An important behavioural parameter is the strength of the union's aversion to wage dispersion. Where this is strong, trained and untrained workers' incomes are likely to be relatively close and both could fall in the upper income bracket of a progressive tax system, with untrained non-unionised workers' incomes in the lower bracket. In this case, tax policy can do little to correct the distortion in the supply of trained labour created by the wage compression - this is where subsidisation of training could play a role. However the government can pursue its income-redistributional objectives by taxing workers in the non-unionised sector - the lower bracket - relatively less heavily,¹⁹ at the cost of further distorting the resource allocation by reducing the incentives to incur training costs.

On the other hand, where wage compression is less pronounced and the gap between wage rates of untrained workers in the unionised and non-unionised sectors is relatively narrow, all these workers are in the lower income tax bracket. In that case, tax policy can have significant effects on resource distortion, possibly even to the extent of taxing trained workers relatively less heavily than the untrained,²⁰ at the expense obviously of its distributional goals. In this way, the analysis clarifies the effects of the trade off between distributional objectives and correction of distortions in resource allocation on the design of a general income tax system.

8 Appendix A

The union's Lagrange function is

$$\mathcal{L} = u_1(w_2) - ku_2(w_2 - w_1) + \gamma(1 - C(w_1, w_2)) + \nu(w_1 - \omega) \quad (30)$$

FOCs are

$$\frac{\partial \mathcal{L}}{\partial w_1} = ku'_2(w_2 - w_1) - \gamma C_1(w_1, w_2) + \nu = 0 \quad (31)$$

$$\frac{\partial \mathcal{L}}{\partial w_2} = u'_1(w_2) - ku'_2(w_2 - w_1) - \gamma C_2(w_1, w_2) = 0 \quad (32)$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = 1 - C(w_1, w_2) \geq 0; \gamma \geq 0; \gamma \frac{\partial \mathcal{L}}{\partial \gamma} = 0 \quad (33)$$

$$\frac{\partial \mathcal{L}}{\partial \nu} = w_1 - \omega \geq 0; \nu \geq 0; \nu \frac{\partial \mathcal{L}}{\partial \nu} = 0 \quad (34)$$

¹⁹Or, in the case of pure redistribution with a zero net revenue requirement, setting a negative tax in this bracket.

²⁰That is, in the case of pure redistribution possibly even setting a negative tax rate on the higher incomes of trained workers.

Suppose first that $k = 0$, so that the union has no aversion to wage inequality. Then from (29) since $C_1 > 0$ we must have $\nu > 0$ and so $w_1 = \omega$. With $C_2 > 0$ we must have $\gamma = u'_1(w_2)/C_2 > 0$ and so $C(w_1, w_2) = 1$, the break even constraint binds.

Now suppose that $k > 0$ and also that, as we would expect, $w_1 > \omega$ so that $\nu = 0$. From (31) we must have that $u'_1(w_2) > ku'_2(w_2 - w_1)$, which places a bound on how averse to wage dispersion the union can be. We carry out the comparative statics analysis of this equilibrium with respect to k by excluding ν from (30) and dropping the second constraint since it is nonbinding.

Define the determinant

$$D_0 \equiv \begin{vmatrix} -ku''_2 - \gamma C_{11} & ku''_2 - \gamma C_{12} & -C_1 \\ ku''_2 - \gamma C_{21} & u''_1 - ku''_2 - \gamma C_{22} & -C_2 \\ -C_1 & -C_2 & 0 \end{vmatrix} > 0 \quad (35)$$

where the sign follows from the second order condition. Standard analysis gives the derivatives:

$$\frac{\partial w_1}{\partial k} = u'_2[C_2^2 + C_1 C_2]/D_0 > 0 \quad (36)$$

$$\frac{\partial w_2}{\partial k} = -u'_2[C_2^2 + C_1 C_2]/D_0 < 0 \quad (37)$$

9 Appendix B: Pooling by Sector

In this case the government's Lagrange function is:

$$\begin{aligned} \mathcal{L} = & l_0 U(\omega - t_1) + l_1 U(w_1 - t_2) + \int_0^{w_2 - w_1} U(w_2 - v - t_2) dF(v) \\ & + \lambda \{t_1 l_0 + t_2(l_1 + l_2)\} + \mu(w_1 - t_2 - \omega + t_1) \end{aligned} \quad (38)$$

FOCs are

$$\frac{\partial \mathcal{L}}{\partial t_1} = [\lambda^* - U'(\omega - t_1^*)]l_0^* + \mu^* = 0 \quad (39)$$

$$\frac{\partial \mathcal{L}}{\partial t_2} = [\lambda^* - U'(w_1 - t_2^*)]l_1^* + \lambda^* l_2^* - \int_0^{w_2 - w_1} U'(w_2 - v - t_2^*) dF(v) - \mu^* = 0 \quad (40)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = t_1(1 - l_1 - l_2) + t_2(l_1 + l_2) = 0 \quad (41)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = w_1 - t_2 - \omega + t_1 \geq 0; \mu^* \geq 0; \mu^* \frac{\partial \mathcal{L}}{\partial \mu} = 0 \quad (42)$$

We prove below that $\mu^* > 0$, and so $w_1 - t_2 = \omega - t_1$. Then, using the government budget constraint (40) we have that

$$t_2^* > 0 > t_1^* \quad (43)$$

To prove $\mu^* > 0$, suppose to the contrary that $w_1 - t_2 > \omega - t_1$ and so $\mu^* = 0$. Then summing (38) and (39) gives:

$$\lambda^* = U'(\omega - t_1^*)l_0^* + U'(w_1 - t_2^*)l_1^* + \int_0^{w_2 - w_1} U'(w_2 - v - t_2^*)dF(v) > 0 \quad (44)$$

while with $\mu_1^* = 0$ (38) implies

$$\lambda^* = U'(\omega - t_1^*) \quad (45)$$

and (39) implies

$$\lambda^*(l_1^* + l_2^*) = U'(w_1 - t_2^*)l_1^* + \int_0^{w_2 - w_1} U'(w_2 - v - t_2^*)dF(v) \quad (46)$$

Noting that

$$U'(w_2 - \hat{v} - t_2^*) > U'(w_2 - v - t_2^*) \quad \forall v < \hat{v} \quad (47)$$

integrating both sides gives

$$U'(w_2 - \hat{v} - t_2^*)l_2^* > \int_0^{w_2 - w_1} U'(w_2 - v - t_2^*)dF(v) \quad (48)$$

and so

$$\lambda^*(l_1^* + l_2^*) < U'(w_1 - t_2^*)l_1^* + U'(w_2 - \hat{v} - t_2^*)l_2^* = U'(w_1 - t_2^*)(l_1^* + l_2^*) \quad (49)$$

Thus we have

$$U'(w_1 - t_2^*) > \lambda^* = U'(\omega - t_1^*) \quad (50)$$

contradicting the assumption that $w_1 - t_2^* > \omega - t_1^*$.

Thus we have $w_2 - \hat{v} - t_2^* = w_1 - t_2^* = \omega - t_1^*$.

10 Appendix C: Pooling by Training

The government's Lagrange function is now:

$$\begin{aligned} \mathcal{L} = & [1 - (l_1 + l_2)]U(\omega - t_1) + l_1U(w_1 - t_1) \\ & + \int_0^{w_2 - t_2 - w_1 + t_1} U(w_2 - v - t_2)dF(v) + \lambda(t_1(l_0 + l_1) + t_2l_2) \end{aligned} \quad (51)$$

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial t_1} = [\lambda^* - U'(\omega - t_1^*)]l_0 + [\lambda^* - U'(w_1 - t_1^*)]l_1$$

$$+f(\hat{v})\{U(w_2 - \hat{v} - t_2^*) + mU(w_1 - t_1^*) - (1+m)U(\omega - t_1^*) + \lambda^*(t_2^* - t_1^*)\} = 0 \quad (52)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial t_2} &= \int_0^{w_2 - t_2^* - w_1 + t_1^*} [\lambda^* - U'(w_2 - v - t_2^*)] dF(v) \\ -f(\hat{v}^*)\{U(w_2 - \hat{v}^* - t_2^*) - U(\omega - t_1^*) + m(\cdot)[U(w_1 - t_1^*) - U(\omega - t_1^*)] + \lambda^*(t_2^* - t_1^*)\} &= 0 \end{aligned} \quad (53)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = t_1^*(l_0 + l_1) + t_2^*l_2 = 0 \quad (54)$$

Recalling that $w_1 > \omega$, it is clear that the tax system cannot equalise after tax incomes of untrained workers in the unionised sector with those in the non-unionised sector. The best it can do in distributional terms, using the sum of (51) and (52), is to equate the average marginal utility with the shadow price of government revenue:

$$\lambda^* = U'(\omega - t_1^*)l_0^* + U'(w_1 - t_1^*)l_1^* + \int_0^{w_2 - t_2^* - w_1 + t_1^*} U'(w_2 - v - t_2^*) dF(v) \quad (55)$$

Of course the level of employment adjusts to equalise the wage net of tax and training cost of the marginal trained worker with that of the untrained workers in the unionised sector, $w_2 - \hat{v} - t_2^* = w_1 - t_1^*$.

The interesting possibility in this case, is, as discussed further in the text of the paper, that under reasonable conditions we could have the result $t_2^* < 0 < t_1^*$.

11 Appendix D Comparative Statics

We shall consider an increase of w_1 with w_2 adjusting to restore the break even condition $c(w_1, w_2) = 1$. To facilitate the comparative statics we shall employ a more direct approach to the tax optimisation than the Lagrange method adopted in most of the paper. As above, we distinguish between pooling by sector and pooling by training.

Pooling by sector.

When trained and untrained labour in the unionised sector face the same tax $\hat{v} = w_2 - w_1$. When wages are fixed \hat{v} and $S(\hat{v})$ are determined, and l_1 and l_0 follow. The tax revenue constraint is

$$l_0 t_1 + (l_1 + l_2) t_2 = 0$$

and it follows that

$$\frac{dt_1}{dt_2} = -\frac{(l_1 + l_2)}{l_0}$$

We can then model the welfare maximising tax setting as the choice of t_2 that maximises

$$\mathcal{W} = l_0 U(\omega - t_1) + l_1 U(w_1 - t_2) + \int_0^{\hat{v}} U(w_2 - v - t_2) dF(v)$$

$$\frac{d\mathcal{W}}{dt_2} = -l_0 U'(\omega - t_1) \frac{dt_1}{dt_2} - l_1 U'(w_1 - t_2) - \int_0^{\hat{v}} U'(w_2 - v - t_2) dF(v) = 0$$

Inserting the expression for $\frac{dt_1}{dt_2}$ yields

$$l_0 U'(\omega - t_1) \frac{l_1 + l_2}{l_0} - l_1 U'(w_1 - t_2) - \int_0^{\hat{v}} U'(w_2 - v - t_2) dF(v) = 0$$

The second order condition is $\frac{d^2 \mathcal{W}}{dt_2^2} < 0$.

We can write the tax revenue constraint as

$$t_1 + (t_2 - t_1) \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S(w_2 - w_1) = 0$$

We can consider t_1 as an implicitly defined function of t_2 and w_1 , where w_2 is a function of w_1 . We find that

$$\frac{\partial t_1}{\partial w_1} + (t_2 - t_1) \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(-\frac{l_1}{l_2} - 1 \right) + (t_2 - t_1) m' \left(\frac{w_2}{w_1} \right) \frac{\partial \left(\frac{w_2}{w_1} \right)}{\partial w_1} l_2 - \frac{\partial t_1}{\partial w_1} (l_1 + l_2) = 0$$

$$\frac{\partial t_1}{\partial w_1} = (t_2 - t_1) \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(\frac{l_1}{l_2} + 1 \right) \frac{1}{l_0} - (t_2 - t_1) m' \left(\frac{w_2}{w_1} \right) \frac{\partial \left(\frac{w_2}{w_1} \right)}{\partial w_1} \frac{1}{l_0} > 0$$

We note that $\frac{\partial t_1}{\partial w_1}$ is larger the larger are S' (the supply responsiveness of l_2) and m' (the substitution response to relative wages).

$$\begin{aligned} \frac{\partial}{\partial w_1} (l_1 + l_2) &= \frac{\partial}{\partial w_1} \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S(w_2 - w_1) \\ &= \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(-\frac{l_1}{l_2} - 1 \right) + l_2 m' \left(\frac{w_2}{w_1} \right) \frac{\partial \left(\frac{w_2}{w_1} \right)}{\partial w_1} < 0 \\ \frac{\partial}{\partial w_1} l_0 &= -\frac{\partial}{\partial w_1} (l_1 + l_2) > 0 \end{aligned}$$

We carry out comparative statics by differentiating the first order condition.

$$\frac{d^2 \mathcal{W}}{dt_2^2} \frac{dt_2}{dw_1} + \frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2} = 0$$

$$\frac{dt_2}{dw_1} = \frac{\frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2}}{-\frac{d^2\mathcal{W}}{dt_2^2}}$$

We see that the sign of $\frac{dt_2}{dw_1}$ is determined by the sign of $\frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2}$.

$$\begin{aligned} \frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2} &= U'(\omega - t_1) \frac{\partial}{\partial w_1} (l_1 + l_2) - U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} - l_1 U''(w_1 - t_2) - U'(w_1 - t_2) \frac{\partial l_1}{\partial w_1} \\ &\quad - \int_0^{\hat{v}} U''(w_2 - v - t_2) \left(-\frac{l_1}{l_2}\right) dF(v) - U'(w_2 - \hat{v} - t_2) S' \left(-\frac{l_1}{l_2} - 1\right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2} &= U'(\omega - t_1) \frac{\partial}{\partial w_1} (l_1 + l_2) - U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} - l_1 U''(w_1 - t_2) - U'(w_1 - t_2) \frac{\partial l_1}{\partial w_1} \\ &\quad - \int_0^{\hat{v}} U''(w_2 - v - t_2) \left(-\frac{l_1}{l_2}\right) dF(v) - U'(w_2 - \hat{v} - t_2) \frac{\partial l_2}{\partial w_1} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2} &= (U'_0 - U'_1) \frac{\partial}{\partial w_1} (l_1 + l_2) - U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} - l_1 U''(w_1 - t_2) \\ &\quad - \int_0^{\hat{v}} U''(w_2 - v - t_2) \left(-\frac{l_1}{l_2}\right) dF(v) \\ &= 0 \end{aligned}$$

Now let us consider and interpret each term in the expression above. We recall that our starting point is an optimum characterised by the condition that a small revenue-neutral tax perturbation increasing t_2 and lowering t_1 leaves welfare unchanged.

$(U'_0 - U'_1) \frac{\partial}{\partial w_1} (l_1 + l_2) < 0$: The rise in t_2 will raise more revenue to be transferred from better-off agents in the unionised sector to worse-off agents in the market sector, and more the larger is the number of workers in the unionised sector. This number will shrink in response to an increase in w_1 that makes the acquisition of training relatively less attractive. Less redistribution is achieved, and the partial effect is to weaken the case for increasing t_2 .

$-U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} > 0$: As the increase in w_1 will increase the number of low-tax payers and reduce the number of high-tax payers t_1 must be raised to restore budget balance. Then workers in the market sector are made worse off. As a consequence their marginal utility increases and they benefit more in terms of utility from the redistribution considered at the optimum. The partial effect is to strengthen the case for increasing t_2 .

$-l_1 U''(w_1 - t_2) > 0$: When w_1 increases untrained workers in the unionised sector become better off and their marginal utility falls. They will incur a

smaller utility loss due to an increase in t_2 than before. The partial effect is to strengthen the case or increasing t_2 .

$-\int_0^{\hat{v}} U''(w_2 - v - t_2) \left(-\frac{l_1}{l_2}\right) dF(v) < 0$: When w_1 increases and w_2 decreases trained workers become worse off and their marginal utility rises. They will incur a larger utility loss due to an increase in t_2 than before. The partial effect is to weaken the case or increasing t_2 at the margin.

We see that there are ambiguous effects on the optimal tax policy. The standard assumption is that $-U''$ is decreasing in income implying that $-U''(\omega - t_1) > -U''(w_1 - t_2) > -U''(w_2 - v - t_2)$. Where $-U''$ is sufficiently large for lower incomes and decreasing in income the equity case for further redistributive taxation is strengthened. This finding may be somewhat counterintuitive since the wage dispersion has diminished. The reason for this result the disposable income has fallen for the worst-off agents and a larger t_2 does not redistribute within the unionised sector. But even if there is an equity case for more extensive redistribution via the tax system less redistribution may be achievable by changing taxes so that further differentiation of taxes may not be worthwhile. It is important to note that the equity case for redistribution and the opportunity for achieving redistribution are not independent.

The larger are S' (the supply responsiveness of l_2) and m' (the substitution response relative wages) the smaller is the opportunity for achieving redistribution but also the stronger is the equity case for redistribution as the worst off agent is made worse off.

Considering the first and second term in () we find

$$(U'_0 - U'_1) \frac{\partial}{\partial w_1} (l_1 + l_2) - U''(\omega - t_1) (l_1 + l_2) \frac{\partial t_1}{\partial w_1} = \left[-U''(\omega - t_1) (l_1 + l_2) (t_2 - t_1) \frac{1}{l_0} - (U'_0 - U'_1) \right] \left\{ \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(\frac{l_1}{l_2} + 1 \right) - m' \left(\frac{w_2}{w_1} \right) \frac{\partial \left(\frac{w_2}{w_1} \right)}{\partial w_1} \right\}$$

We see that whether the effect of the term in curly brackets is in favour of a larger or smaller t_2 depends on the sign of the bracketed term. Then consider the pooling by training.

Pooling by training

A balanced budget for the government requires

$$(1 - S(w_2 - t_2 - w_1 + t_1)) t_1 + t_2 S(w_2 - t_2 - w_1 + t_1) = 0$$

Also in this context we can treat t_1 as an implicitly defined function of t_2 and w_1 , with w_2 being a function of w_1 due to the break even constraint. Differentiating wrt w_1 we get

$$\frac{dt_1}{dw_1} = \frac{-S'(t_2 - t_1)}{1 - S + (t_2 - t_1) S'} \left(\frac{dw_2}{dw_1} - 1 \right)$$

which is equivalent to

$$\frac{dt_1}{dw_1} = \frac{S'(t_2 - t_1)}{1 - S + (t_2 - t_1) S'} \left(\frac{l_1}{l_2} + 1 \right)$$

Differentiating wrt t_2

$$(1 - S) \frac{\partial t_1}{\partial t_2} + S + (t_2 - t_1) S' \left(-1 + \frac{\partial t_1}{\partial t_2} \right) = 0$$

$$-\frac{\partial t_1}{\partial t_2} = \frac{l_2 - (t_2 - t_1) S'}{1 - l_2 + (t_2 - t_1) S'}$$

which is the tax relief (or additional benefit) received by untrained workers made possible by the increase in t_2 . Assuming that either tax can raise extra tax revenue, $\frac{\partial t_1}{\partial t_2} < 0$. Rearranging terms, we can write

$$-\frac{\partial t_1}{\partial t_2} = \frac{l_2}{1 - l_2} - \frac{(t_2 - t_1) S'}{(1 - l_2)(1 - l_2 + (t_2 - t_1) S')}$$

where we can interpret $\frac{(t_2 - t_1) S'}{(1 - l_2)(1 - l_2 + (t_2 - t_1) S')}$ as a cost of redistribution due to the discouragement of training induced by the tax perturbation. We assume that $\frac{d}{dw_1} (w_1 - t_1) = 1 - \frac{dt_1}{dw_1} > 0$. $\frac{dl_2}{dw_1} = S' \left(-1 - \frac{l_1}{l_2} + \frac{dt_1}{dw_1} \right) < 0$.

$$\frac{d}{dw_1} \frac{l_2}{1 - l_2} < 0$$

$$\frac{dl_1}{dw_1} = l_2 m' \frac{d\left(\frac{w_2}{w_1}\right)}{dw_1} + m \frac{dl_2}{dw_1} < 0$$

$$l_0 = 1 - \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S(w_2 - t_2 - w_1 + t_1)$$

$$\frac{dl_0}{dw_1} > 0$$

$$\frac{dl_0}{dt_2} = - \left(1 + m \left(\frac{w_2}{w_1} \right) \right) S' \left(-1 + \frac{\partial t_1}{\partial t_2} \right) > 0$$

We note that a larger w_1 will have ambiguous effects on $-\frac{\partial t_1}{\partial t_2} = \frac{l_2 - (t_2 - t_1) S'}{1 - l_2 + (t_2 - t_1) S'}$. The induced reduction in l_2 will diminish the tax relief while the increase in t_1 will have the opposite effect since the tax wedge $t_2 - t_1$ will shrink. We find that

$$\frac{d}{dw_1} \left(-\frac{\partial t_1}{\partial t_2} \right) = \frac{\frac{dl_2}{dw_1} + S' \frac{dt_1}{dw_1}}{(1 - l_2 + (t_2 - t_1) S')^2}$$

Assuming that the labour supply effect of a larger w_1 and lower w_2 (the former term) dominates the effect via t_1 (the latter term) the tax relief diminishes.

$$\frac{d}{dw_1} \frac{dl_0}{dt_2} = \left(1 + m \left(\frac{w_2}{w_1}\right)\right) S' \frac{d}{dw_1} \left(-\frac{\partial t_1}{\partial t_2}\right) - S' \left(-1 + \frac{\partial t_1}{\partial t_2}\right) m' \frac{d\left(\frac{w_2}{w_1}\right)}{dw_1} < 0$$

When the tax relief accruing to untrained workers will shrink acquisition of training becomes more attractive and there will be fewer untrained workers flowing into the market sector. A further effect is that a larger w_1 will reduce the ratio l_1/l_2 so that a lowering of l_2 induced by t_2 will have a smaller effect on l_1 and more untrained workers will remain in the unionised sector rather than flow into the market sector.

We can model the welfare maximising tax setting as the choice of t_2 that maximises

$$\mathcal{W} = l_0 U(\omega - t_1) + l_1 U(w_1 - t_1) + \int_0^{\hat{v}} U(w_2 - v - t_2) dF(v)$$

$$\begin{aligned} \frac{d\mathcal{W}}{dt_2} &= l_0 U'(\omega - t_1) \left(-\frac{\partial t_1}{\partial t_2}\right) + l_1 U'(w_1 - t_1) \left(-\frac{\partial t_1}{\partial t_2}\right) - \int_0^{\hat{v}} U'(w_2 - v - t_2) dF(v) \\ &\quad + U_0 \frac{dl_0}{dt_2} + U_1 \frac{dl_1}{dt_2} + U(w_2 - \hat{v} - t_2) \frac{dl_2}{dt_2} = 0 \end{aligned}$$

$$\begin{aligned} \frac{d\mathcal{W}}{dt_2} &= (1 - l_1 - l_2) U'(\omega - t_1) \left(-\frac{\partial t_1}{\partial t_2}\right) + l_1 U'(w_1 - t_1) \left(-\frac{\partial t_1}{\partial t_2}\right) - \int_0^{\hat{v}} U'(w_2 - v - t_2) dF(v) \\ &\quad + (U_0 - U_1) \frac{dl_0}{dt_2} = 0 \end{aligned}$$

The second order condition is $\frac{d^2 \mathcal{W}}{dt_2^2} < 0$. We carry out comparative statics by differentiating the first order condition.

$$\begin{aligned} \frac{d^2 \mathcal{W}}{dt_2^2} \frac{dt_2}{dw_1} + \frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2} &= 0 \\ \frac{dt_2}{dw_1} &= \frac{\frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2}}{-\frac{d^2 \mathcal{W}}{dt_2^2}} \end{aligned}$$

Recalling the second order condition, we see that the sign of $\frac{dt_2}{dw_1}$ is determined by the sign of $\frac{\partial}{\partial w_1} \frac{d\mathcal{W}}{dt_2}$.

$$\begin{aligned}
\frac{d}{dw_1} \frac{d\mathcal{W}}{dt_2} = & [l_0 U'_0 + l_1 U'_1] \frac{d}{dw_1} \left(-\frac{\partial t_1}{\partial t_2} \right) \\
& - l_0 U''_0 \frac{\partial t_1}{\partial w_1} \\
& + l_1 U''_1 (w_1 - t_1) \left(-\frac{\partial t_1}{\partial t_2} \right) \left(1 - \frac{\partial t_1}{\partial w_1} \right) \\
& - \int_0^{\hat{v}} U''(w_2 - v - t_2) dF(v) \frac{\partial w_2}{\partial w_1} \\
& + U'_0 \left(-\frac{\partial t_1}{\partial t_2} \right) \left(-\frac{dl_1}{dw_1} - \frac{dl_2}{dw_1} \right) \\
& + U'_1 \left(-\frac{\partial t_1}{\partial t_2} \right) \frac{dl_1}{dw_1} \\
& - U'_1 \frac{dl_2}{dw_1} \\
& + (U_0 - U_1) \frac{d}{dw_1} \frac{dl_0}{dt_2} \\
& + \left(U'_0 \left(-\frac{dt_1}{dw_1} \right) - U'_1 \left(1 - \frac{dt_1}{dw_1} \right) \right) \frac{dl_0}{dt_2}
\end{aligned}$$

We characterise the tax optimum by the condition that a small revenue-neutral tax perturbation increasing t_2 and lowering t_1 leaves welfare unchanged. This marginal tax reform involves a trade off between a number of effects that will be affected by a change in wages that may tilt the tax setting in favour of a larger or smaller t_2 . In order to limit the number of cases to be considered we shall assume that we have an optimum where $t_2 > t_1$. We can then examine the various terms derived above.

$$[l_0 U'_0 + l_1 U'_1] \frac{d}{dw_1} \left(-\frac{\partial t_1}{\partial t_2} \right) :$$

The tax perturbation considered at the optimum redistributes income from trained workers to untrained workers who benefit from a tax relief $\left(-\frac{\partial t_1}{\partial t_2} \right)$ made possible by the increase in t_2 . As considered above, the wage changes will diminish the tax relief. The partial effect is to weaken the case for setting a larger t_2 .

$$-l_0 U''_0 \left(-\frac{dt_1}{dw_1} \right) \frac{\partial t_1}{\partial t_2} :$$

The revenue-maintaining increase in t_1 will lower the income of type 0 workers and increase the marginal utility they get from the tax relief. This strengthens the case for a larger t_2 .

$$-S(w_2 - t_2 - w_1 + t_1) \frac{l_1}{l_2} U''_1 \left(1 - \frac{dt_1}{dw_1} \right) \frac{\partial t_1}{\partial t_2} :$$

The wage changes make type 1 workers better off (assuming $1 - \frac{dt_1}{dw_1} > 0$) and diminishes their marginal benefit from the tax relief. The partial effect is to weaken the case for a larger t_2 .

$$- \int_0^{\hat{v}} U''(w_2 - v - t_2) f(v) dv \frac{dw_2}{dw_1} :$$

The lower income for trained workers will enhance their marginal utility of income and increase the utility loss they incur due to an increase in t_2 . The partial effect is to weaken the case for increasing t_2 .

$$[U'_0 - U'_1] \left(-\frac{\partial t_1}{\partial t_2} \right) \left(-\frac{dl_1}{dw_1} \right) :$$

A larger w_1 will make it less attractive to employ untrained workers in the unionised sector. More untrained workers will flow from the unionised sector into the market sector and receive a larger utility gain from the tax relief (since $U'_0 > U'_1$), strengthening the case for a larger t_2 .

$$-U'_0 \left(-\frac{\partial t_1}{\partial t_2} \right) \left(-\frac{dl_2}{dw_1} \right) :$$

There will be fewer trained workers and therefore more untrained workers ending up in the market sector where they benefit from the tax relief, strengthening the case for a larger t_2

$$-U'_1 \frac{dl_2}{dw_1} :$$

There will be fewer trained workers who are hit by the increase in t_2 making it less harmful to raise t_2 .

$$(U_0 - U_1) \frac{d}{dw_1} \frac{dl_0}{dt_2} > 0$$

As observed above, the change in the wage structure will diminish the number of workers being shifted to the market sector where they experience a loss of utility. The smaller harmful effect makes an increase in t_2 more desirable.

$$\left(U'_0 \left(-\frac{dt_1}{dw_1} \right) - U'_1 \left(1 - \frac{dt_1}{dw_1} \right) \right) \frac{dl_0}{dt_2}$$

The utility loss of moving workers into the market sector will increase since type 1 workers are made better off and type 0 workers worse off. The partial effect is to weaken the case for increasing t_2 .

References

- [1] Allen, F. (1982). Optimal linear taxation with general equilibrium effects on wages. *Journal of Public Economics* 17, 135-143
- [2] Aronsson, T. and T. Sjögren (2004). Is the Optimal Labor Income Tax Progressive in a Unionized Economy? *Scandinavian Journal of Economics* 106, 661-675
- [3] Barth, E., K.O. Moene, and F. Willumsen (2014). The Scandinavian model – an interpretation. *Journal of Public Economics* 117, 60-72.
- [4] Cappelen, Å. (2018). Income policy and The Technical Calculation Committee for income settlements (in Norwegian: Inntektspolitikken og Det tekniske beregningsutvalget for inntektsoppgjørene), *Samfunnsøkonomen* 132, 4-12.

- [5] Lockwood, B., T. Sløk, T. and T. Tranæs (2000). Progressive Taxation and Wage Setting: Some Evidence for Denmark, *Scandinavian Journal of Economics* 102, 707 – 723.
- [6] Geroski, P., P. Gregg and J. van Reenen (1996). Market imperfections and employment. OECD Economic Studies No 26.1996/1.
- [7] Layard, R., S. Nickell and R. Jackman, (1991). Unemployment. Oxford University Press, Oxford. Hersoug, T. (1984). Union wage responses to tax changes, *Oxford Economic Papers* 36, 37- 51.
- [8] Lee, D. and E. Saez (2008). Optimal minimum wage policy in competitive labor markets, NBER working paper 14320.
- [9] Jean, S. and G. Nicoletti (2004). Regulation and Wage Premia. *CEPII working paper*. 2004.
- [10] Nymoen, R. and A. Rødseth (2003). Explaining unemployment: some lessons from Nordic wage formation. *Labour Economics* 10, 1-29.
- [11] Oswald, A.J. (1985). The Economic Theory of Trade Unions: An Introductory Survey. The *Scandinavian Journal of Economics* 87, 160-193.