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Paternalistic Taxation of Unhealthy Food and the Intensive versus Extensive Margin of Obesity

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Abstract

This paper shows that if an individual's health costs are U-shaped in weight with a minimum at some healthy weight level and if the individual has both self control problems and rational motives for over- or underweight, the optimal paternalistic tax on unhealthy food mitigates the individual's weight problem (intensive margin), but does not induce the individual to choose healthy weight (extensive margin). Implementing healthy weight requires a further distortion (e.g. subsidy on other goods), which may render the tax on unhealthy food inferior to the option of not taxing the individual at all. In addition, with heterogeneous individuals the optimal uniform paternalistic tax may have the negative side effect of rendering otherwise healthy individuals underweight.

JEL-Codes: D030, D110, H210, I180.

Keywords: sin tax, paternalism, obesity, extensive versus intensive margin.

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1 Introduction

Obesity currently represents one of the most pestering health problems worldwide. For example, in the US more than one third (38.2%) of the population aged 15 years and over is obese, i.e. has a body mass index (BMI) larger than 30 kg/m². Similar high obesity rates can be observed in Mexico (32.4%), New Zealand (30.7%), Hungary (30.0%) and Australia (27.9%). The average obesity rate in OECD countries is 19.5% and expected to increase further over the next decades. Obesity rates among children are catching up to the rates among adults very rapidly and are also alarming. If overweight people with a BMI between 25 and 30 kg/m² are included, then even one in two adults and one in six children have weight problems in OECD countries (for all these facts see OECD, 2017). It is well known and documented that obesity and overweight are closely related to secondary disorders – like diabetes, cardiovascular diseases or even some cancers – which considerably increase mortality (WHO, 2017).

As a countermeasure against obesity and overweight, many countries have implemented taxes on unhealthy food, for instance, the soda taxes in France, Hungary, Mexico, Ireland and UK, the sugar tax in Norway and the fat tax in Denmark (WHO, 2016a). In practice, the stated aim of implementing such sin taxes on unhealthy food is to reduce the prevalence of obesity. For example, the WHO (2016b) argues that "... [e]vidence shows that a tax of 20% on sugary drinks can lead to a reduction in consumption of around 20%, thus *preventing* obesity and diabetes." (italics ours) In theory, sin taxes are usually motivated by the paternalistic approach of correcting self control problems (e.g. O'Donoghue and Rabin, 2003, 2006). People underestimate their future health costs from secondary disorders and, thus, choose unhealthy diet and weight. An individual therefore inflicts an externality on itself, i.e. an 'internality', which is corrected by the optimal paternalistic sin tax on unhealthy food.

Our paper shows that there is a gap between the stated aim of taxes on unhealthy food in practice and the paternalistic foundation in the economic literature. We show that the optimal paternalistic tax on unhealthy food is effective at the *intensive margin*, i.e. it mitigates weight problems of individuals, but not at the *extensive margin*, i.e. it does not induce individuals to choose healthy weight. The reason is that self control problems – which should be corrected by the optimal paternalistic tax – influence only the intensive margin, but not the extensive margin. The extensive margin is purely determined by rational motives of the individuals. Implementing healthy weight – and thus overcoming also the problem at the extensive margin – requires a further,

welfare-decreasing distortion (e.g. a subsidy on other goods). This distortion may render the tax on unhealthy food inferior to the option of not taxing the individuals at all. In addition, with heterogeneous individuals, the optimal uniform paternalistic tax on unhealthy food may have the negative side effect of rendering otherwise healthy individuals underweight, that means it may have a reversed negative effect at the extensive margin. This is an important effect, too, since evidence shows that health costs increase even faster if weight falls below the healthy level than if weight increases above the healthy level (Global BMI Mortality Collaboration, 2016).

We derive these results within a theoretical model of individuals which use an exogenous income to finance the consumption of a health neutral good and an unhealthy good, e.g., fast food. The weight of an individual is proportional to unhealthy consumption and causes health costs. The model has two decisive features. First, health costs are U-shaped with a minimum at some healthy weight level. This assumption is highly relevant in the context of obesity. The Global BMI Mortality Collaboration (2016, figure 3) shows in a comprehensive meta study of over 200 studies that the all-cause mortality is U-shaped in the BMI with a minimum between 22.5 and 25 kg/m². Second, beside the self control problem individuals have a rational motive for choosing unhealthy diet und weight. More specific, the net marginal benefit of unhealthy consumption, defined as the difference between the marginal rate of substitution of unhealthy consumption and health neutral consumption and the relative fast food price, may be different from zero. As the individual sets this net marginal benefit equal to the marginal health costs, the chosen weight may deviate from healthy weight. The literature provides evidence for both self control problems and rational motives. Most striking are the papers of Cutler et al. (2003) and Courtemanche et al. (2014) which both show that the development of obesity rates can be explained by decreasing food prices and costs, yielding evidence for the rational motive, and that this impact is aggravated by hyperbolic discounting, supporting the self control hypothesis.

As benchmark, our analysis starts with the case of a representative individual in the absence of taxation. We define the tempting weight as the weight level at which the above-mentioned net marginal benefit of unhealthy food consumption becomes zero. Hence, tempting weight is the (hypothetical) weight level that would maximize the individual's utility when there would be no health costs at all. In its diet choice the individual trades off the incentive to attain this tempting weight against the need to account for health costs. Hence, it realizes a weight level between tempting and healthy weight. That is to say that the realized weight is above healthy weight (overweight) if

tempting weight is larger than healthy weight and below healthy weight (underweight) if tempting weight falls short of healthy weight. If tempting weight equals healthy weight, the individual realizes healthy weight. Importantly, whether the individual becomes over- or underweight (the extensive margin) is solely determined by the relation of tempting and healthy weight and not by the degree of self control. The self control problem only worsens over- or underweight (the intensive margin), but does not determine the direction of the individual's weight problem.

Next we determine the optimal paternalistic tax on unhealthy food. The paternalistic social planner maximizes the individual's true utility taking into account the full health costs. If revenues from the tax are redistributed back to the individual in a lump-sum manner, then the optimal tax is positive for an overweight individual and negative (subsidy) for an underweight individual. In both cases the weight problem is mitigated at the intensive margin, i.e. the overweight individual becomes less overweight and the underweight individual becomes less underweight. But the problem at the extensive margin remains, that means the overweight remains overweight and the underweight remains underweight. Hence, the optimal paternalistic tax on unhealthy food does not give the individual the right incentive to implement healthy weight. The reason is that the optimal paternalistic tax corrects only for the self control problem, but not the above-mentioned rational motive for over- or underweight.

In contrast to the often stated aim of taxes on unhealthy food in practice, the optimal paternalistic tax is therefore not enough to implement healthy weight. This raises the question how the paternalistic policy has to be adjusted in order to solve also the problem at the extensive margin. We thus consider the case where tax revenues are redistributed back by a subsidy on other goods, which is proportional to the tax. It turns out that the optimal paternalistic tax on unhealthy food then causes a larger substitution effect and thereby indeed induces the individual to choose healthy weight, independently of whether the individual is over- or underweight in the absence of taxation and independently of the degree of the self control problem. But the larger substitution effect represents an additional distortion that reduces the individual's true utility, compared to the case with lump-sum redistribution. The additional distortion may even render the tax on unhealthy food inferior to the option of not taxing the individual at all. This result occurs if the self control problem is not too pronounced.

We finally extend the analysis to heterogeneous individuals and focus on the case with two types of individuals. The types differ in the marginal utility of fast food, so the type with higher marginal utility has larger tempting weight. For the realistic case

of a uniform, type-independent tax, the optimal paternalistic tax reflects the weighted average of the individuals' self control problems. Hence, if both types are overweight, the uniform tax is positive and lower (higher) than the type-specific tax for the more (less) overweight type. It therefore still weakens the problem of the more overweight type at the intensive margin, i.e. reduces weight, and at the same time may induce the less overweight type to overcome the problem at the extensive margin, i.e. to implement healthy weight. However, if one type is overweight and the other type underweight and if the uniform tax is positive, the overweight type again reduces its problem at the intensive margin (lower weight) without solving the problem at the extensive margin (still overweight). But the underweight type actually should be subsidized and due to the positive tax this type's problem at the intensive margin grows, i.e. this type becomes more underweight. Analogously, if the second type has healthy weight in the absence of taxation, the positive tax creates a problem at the extensive margin and the type becomes underweight. Using a paternalistic tax to mitigate overweight may thus come at the cost of rendering people with healthy weight underweight.

In sum, the main contribution of our analysis to the literature is to emphasize the distinction between extensive and intensive margins when discussing fiscal policy interventions in diet choices of individuals. Ignoring this distinction may yield misleading recommendations, for instance, an inefficiently high tax burden when policy aims at reducing the prevalence of overweight and obesity at the extensive margin. Of course, our analysis provides purely qualitative insights and ignores the quantitative dimension: Are rational motives, relatively to self control problems, important enough to justify the distinction between extensive and intensive margins or are they negligible so that self control problems are the main drivers of overweight and obesity? Ikeda et al. (2010) use data from Japan and emphasize the important role of self control problems in explaining obesity rates. For the US, however, Cutler et al. (2003) argue that time saving technologies in meal preparation and, thus, cost reductions determine the development of the BMI and hyperbolic discounting aggravates this development. Using data from UK, Courtemanche et al. (2014) find that, depending on the impatience of individuals, a 1\$ increase in food prices decreases the BMI up to 0.4 points or 4 pounds and a 1 standard deviation increase in the hyperbolic discount factor reduces BMI by 0.2 units or 1.3 pounds.¹ Hence, it seems to be fair to state that the role of rational motives in diet choices is at least as important as the role of self control

¹Gruber and Köszegi (2001) provide similar mix evidence for smoking and the experimental findings in Andersen et. al. (2014) put general doubt on the predominance of non-constant discounting.

problems, even though we acknowledge the need for further empirical research.

Our paper links two important strands of the theoretical literature. First, the literature on the paternalistic approach of self control problems in order to justify taxes on unhealthy food. The basic argument is made by O'Donoghue and Rabin (2003, 2006). Extensions can be found, for example, in Haavio and Kotakorpi (2011, 2016), Cremer et al. (2012, 2016). In contrast to our analysis, all these studies assume monotonically increasing instead of U-shaped health costs. Hence, their approaches do not contain the distinction between extensive and intensive margin and cannot address the question whether the optimal paternalistic tax implements healthy weight. Second, there is a literature on rational obesity which explains over-/underweight with rational motives instead of self control problems. The initial paper is by Levy (2002). Extensions can be found in, e.g., Dragone (2009) and Dragone and Savorelli (2012). In contrast to our analysis, this literature ignores taxation and, thus, also cannot address the question whether healthy weight is obtained by a tax on unhealthy food.² We combine both strands of literature by taking the issue of taxation in the presence of self control problems from the first strand and the rational motive for weight problems from the second strand. By doing so, we are the first to investigate the implications of paternalistic taxation in the presence of extensive and intensive margins. The analysis builds on our earlier paper Kalamov and Runkel (2017) where we use a general, full dynamic self control model with sophisticated or naive individuals in order to identify the distinction between extensive and intensive margins in self control problems. In fact, the benchmark case without taxation in the present paper is a special case of this general model. But in Kalamov and Runkel (2017) we also ignore taxation issues and therefore cannot derive the results obtained in the present paper.

In the next section, we introduce the basic framework. Section 3 considers optimal paternalistic taxation in the case of a representative individual, before we turn to the case with heterogeneous individuals in Section 4. Section 5 concludes.

2 Basic Framework

Model assumptions. We start by considering a representative individual. The individual consumes an unhealthy good in quantity x and another, health neutral good in quantity z . We refer to the unhealthy good as junk or fast food. Since we consider

²The studies on sin taxation build on the hyperbolic discounting framework of Laibson (1997), while the rational obesity literature is related to rational addiction (Becker and Murphy, 1988).

a one shot model, the consumption of junk food is proportional to weight. Hence, x can be interpreted as both junk food consumption and weight of the individual.

The individual's utility from the two goods is given by $V(z, x)$. The utility function $V(z, x)$ is twice continuously differentiable and satisfies $V_{kk} < 0 < V_k$ for $k = z, x$. Moreover, it is assumed to be concave in (z, x) . Junk food consumption harms the health of the individual. The associated health costs are represented by the twice continuously differentiable function $C(x)$. We assume $C(x^H) = 0$ and $C_x(x) \leq 0$ if and only if $x \leq x^H$, where x^H is the healthy level of junk food or, equivalently, healthy weight. The second derivative of the health cost function is $C_{xx}(x) > 0$. Hence, health costs of junk food consumption are U-shaped with a minimum at healthy weight x^H where health costs vanish. As already mentioned in the Introduction, this shape of $C(x)$ is consistent with empirical evidence (Global BMI Mortality Collaboration, 2016).

The true net utility of the individual equals utility from consumption less health costs of junk food. Formally, it can be written as

$$u = V(z, x) - C(x). \quad (1)$$

In deciding on optimal consumption, the individual suffers from a self control problem. Hence, it takes into account only a fraction β of its health costs. The decision utility of the individual therefore reads

$$\hat{u} = V(z, x) - \beta C(x). \quad (2)$$

Throughout our formal analysis we assume $\beta \in [0, 1[$. Decision utility (2) then differs from true utility (1) since the individual underestimates health costs. For comparison purposes only, we sometimes refer to the case $\beta = 1$ where true utility and decision utility coincide and the individual does not face a self control problem.

Without loss of generality, the market prices of both goods are normalized to one. Consumption of junk food is taxed at rate τ whereas consumption of the other good is subsidized at rate σ . For simplicity, we refer to τ as a fat tax, even though it may also be a sugar or soda tax. The individual has a given income e and receives a lump-sum transfer ℓ from the government. The budget constraint of the individual is

$$(1 + \tau)x + (1 - \sigma)z = e + \ell. \quad (3)$$

Hence, expenditures for junk food and the other good have to be financed from income and the lump-sum transfer. The relative junk food price amounts to $(1 + \tau)/(1 - \sigma)$.³

³Note that our model is a one shot framework where the individual consumes only once. It can be

Consumption decision of the individual. The individual chooses consumption x and z in order to maximize its decision utility \hat{u} defined in (2) subject to the budget constraint (3), taking as given income e as well as the lump-sum transfer ℓ and the policy instruments τ and σ . Solving (3) with respect to z , inserting into (2) and setting the derivative with respect to x equal to zero gives the first-order condition

$$V_x(z^*, x^*) - \frac{1 + \tau}{1 - \sigma} V_z(z^*, x^*) = \beta C_x(x^*), \quad (4)$$

where the star indicates the solution to the individual's maximization problem. Equation (4) states that the individual chooses junk food consumption such that the net marginal utility (LHS) just equals the perceived marginal health costs (RHS). Together with (3) it determines the individual's junk food demand x^* as a function of the self control parameter β and the policy variables τ , σ and ℓ .

An important property of the individual's decision can be derived when we consider the impact of the self control parameter on junk food demand. Totally differentiating (4) and taking into account that z^* has to satisfy the budget constraint (3) yields

$$\frac{\partial x^*}{\partial \beta} = \frac{C_x}{V_{xx} + (1 + \tau)^2 V_{zz} / (1 - \sigma)^2 - 2(1 + \tau) V_{zx} / (1 - \sigma) - \beta C_{xx}} \begin{matrix} \leq \\ \geq \end{matrix} 0 \Leftrightarrow x^* \begin{matrix} \geq \\ \leq \end{matrix} x^H. \quad (5)$$

Note that the denominator in (5) has to be negative due to the second-order condition of utility maximization. Equation (5) states that a more severe self control problem (a lower β) increases junk food consumption and weight when the individual is already overweight and reduces junk food consumption and weight when the individual is already underweight. There is no effect of the self control parameter on junk food if the individual has healthy weight. These insights from (5) have the important implication that the self control parameter β affects the intensive margin but not the extensive margin. Hence, the self control problem influences the extent of the weight problem but does not determine whether the individual becomes over- or underweight.

Solution in the absence of any policy. As a benchmark, we first characterize the individual's consumption decision in the absence of any policy. Setting $\tau \equiv \sigma \equiv \ell \equiv 0$

shown that our basic insights are also obtained in a full dynamic model like the one used in Kalamov and Runkel (2017), where a sophisticated or naive individual with self control problems consumes in every period and where the relation of the individual's weight and fast food is modeled by a standard capital equation of motion. In fact, our simplified model is obtained as limited case of such an elaborate approach if we assume that weight fully decays in each period.

and denoting the individual's junk food consumption in the absence of taxation by $x^* = x^o$, the first-order condition in (4) can be rewritten as

$$N(x^o) = V_x(e - x^o, x^o) - V_z(e - x^o, x^o) = \beta C_x(x^o), \quad (6)$$

where $N(x) := V_x(e - x, x) - V_z(e - x, x)$ denotes the net marginal utility of junk food in the absence of any policy. If we define x^T by $N(x^T) = 0$, then x^T gives the individual's junk food consumption in the (hypothetical) situation without any policy intervention and without health costs. In the following x^T is called the tempting junk food consumption or, equivalently, the tempting weight. We can then prove the following result (all proofs are relegated to the Appendix).

Proposition 1. *Assume $\tau \equiv \sigma \equiv \ell \equiv 0$. The individual's junk food demand x^o is then characterized by the following properties:*

- (a) *If $x^H < x^T$, then $x^H < x^o < x^T$ (overweight).*
- (b) *If $x^T < x^H$, then $x^T < x^o < x^H$ (underweight).*
- (c) *If $x^T = x^H$, then $x^T = x^o = x^H$ (healthy weight).*

Proposition 1 shows that in the absence of any policy intervention the individual's junk food demand x^o lies between the tempting level x^T and the healthy level x^H . If healthy consumption x^H falls short of tempting consumption x^T , the individual faces a trade-off between eating more in order to reach tempting weight x^T and eating less in order to ensure healthy weight x^H . The solution to this trade-off is a weight level x^o between the two extremes. The individual is then overweight ($x^o > x^H$), as shown in part (a) of Proposition 1. If tempting consumption x^T is lower than healthy consumption x^H , the individual balances the incentive to eat less for tempting weight x^T and the incentive to eat more for healthy weight x^H . The individual ends up with underweight ($x^o < x^H$), according to Proposition 1 (b). Part (c) of Proposition 1 contains the knife edge case in which tempting weight x^T and healthy weight x^H just coincide. The individual then does not face a trade-off and chooses healthy weight ($x^o = x^H$).

The insights from Proposition 1 confirm the conclusion which we already draw from equation (5): The extensive margin of the individual's self control problem is not influenced by the self control problem. The proposition holds for all values of β , even for the case $\beta = 1$ where the individual does not face a self control problem. Instead, whether the individual becomes over- or underweight solely depends on the relation between tempting junk food consumption x^T and healthy junk food consumption x^H . The individual will deviate from healthy weight as long as both consumption levels

are not equal and, thus, the net marginal utility of junk food consumption is not zero. Since this holds independently of the parameter β , the individual's decision at the extensive margin is determined by a rational motive and not by self control problems.

3 Optimal Policy With Representative Consumer

Next we turn to the analysis of the optimal paternalistic policy and first stick to the assumption of a representative individual. We consider two policies which differ in the way how revenues from the fat tax are redistributed back to the individual.

Redistribution by lump-sum transfer. The first policy considers redistribution by the lump-sum transfer ℓ . Hence, we set $\ell = \tau x$ and $\sigma \equiv 0$. Taking into account this policy in the private budget constraint (3), the first-order condition (4) of the individual's maximization problem can be rewritten as

$$V_x(e - x^*, x^*) - (1 + \tau)V_z(e - x^*, x^*) = \beta C_x(x^*). \quad (7)$$

This condition determines the individual's junk food demand x^* as a function of the fat tax rate τ . We denote this function by $x^* = X^\ell(\tau)$, where the superscript ℓ indicates the case with lump-sum redistribution of tax revenues.

In order to determine the optimal fat tax, consider a social planner who chooses the tax rate such as to maximize utility of the individual, taking into account the reaction of the individual to tax rate changes. We focus on the paternalistic approach meaning that the social planner maximizes true utility u from (1) instead of decision utility \hat{u} from (2). The maximization problem of the social planner can be written as

$$\max_{\tau} u = V[e - X^\ell(\tau), X^\ell(\tau)] - C[X^\ell(\tau)].$$

The first-order condition reads

$$\frac{du}{d\tau} = [V_x(\cdot) - V_z(\cdot) - C_x(\cdot)]X_\tau^\ell = 0. \quad (8)$$

Using the first-order condition (7) of the individual's maximization problem in order to replace V_x in (8) by $(1 + \tau)V_z + \beta C_x$ and solving for τ gives

$$\tau^\ell = (1 - \beta) \frac{C_x[X^\ell(\tau^\ell)]}{V_z[e - X^\ell(\tau^\ell), X^\ell(\tau^\ell)]}, \quad (9)$$

which is an implicit equation determining the optimal paternalistic fat tax rate τ^ℓ in case redistribution is done by the lump-sum transfer ℓ . Inserting the optimal tax rate (9) into the individual's first-order condition (7) implies

$$N[X^\ell(\tau^\ell)] = V_x[e - X^\ell(\tau^\ell), X^\ell(\tau^\ell)] - V_z[e - X^\ell(\tau^\ell), X^\ell(\tau^\ell)] = C_x[X^\ell(\tau^\ell)]. \quad (10)$$

This equation determines junk food demand of the individual when the paternalistic fat tax is optimally chosen and tax revenues are redistributed back lump-sum.

With the help of equations (9)–(10) we can prove the following result.

Proposition 2. *Assume $\sigma \equiv 0$ and $\ell = \tau x$. The optimal paternalistic fat tax rate τ^ℓ is then given by equation (9) and the corresponding junk food demand $X^\ell(\tau^\ell)$ is determined by equation (10). The following statements hold:*

- (a) *If $x^H < x^T$, then $\tau^\ell > 0$ and $x^H < X^\ell(\tau^\ell) < x^o < x^T$ (overweight).*
- (b) *If $x^T < x^H$, then $\tau^\ell < 0$ and $x^T < x^o < X^\ell(\tau^\ell) < x^H$ (underweight).*
- (c) *If $x^T = x^H$, then $\tau^\ell = 0$ and $x^T = x^o = X^\ell(\tau^\ell) = x^H$ (healthy weight).*

Proposition 2 (a) [(b)] shows that the optimal paternalistic fat tax is positive [negative], if the individual is overweight [underweight] in the absence of any policy intervention. Hence, an overweight individual is taxed for its junk food consumption while an underweight individual receives a junk food subsidy. The fat tax (fat subsidy) therefore reduces (increases) the individual's junk food consumption such that the individual becomes less overweight (underweight), compared to the situation without any policy intervention characterized in Proposition 1. An individual with a healthy weight is not taxed at all and, thus, realizes the same junk food consumption and the same weight as without taxation, as shown in part (c) of Proposition 2.

Even though the optimal paternalistic policy mitigates the weight problem of the individual, the most important insight from Proposition 2 is that the optimal paternalistic policy does not completely eliminate the individual's weight problem. The overweight individual still remains overweight ($x^o > X^\ell > x^H$) and the underweight individual still remains underweight ($x^o < X^\ell < x^H$). Hence, with lump-sum redistribution of fat tax revenues, there is no non-zero paternalistic tax rate $\tau^\ell \neq 0$ that induces the individual to implement healthy weight. Put differently, the optimal paternalistic fat tax mitigates the weight problem at the intensive margin, but does not solve the problem at the extensive margin. This observation is in stark contrast to the stated aim of policy makers using fat and sugar taxes. The intuition is that the optimal

paternalistic policy solely aims at correcting the self control problem, i.e. internalizing the internality that the individual inflicts on itself. As explained above, however, the individual deviates from healthy weight not because of self control problems but because of rationale motives; and the rational motives are not corrected for by the paternalistic approach. This story is also confirmed by the fact that in the absence of self control problems ($\beta = 1$) the optimal paternalistic tax in (9) is always zero, independent of whether the individual is over- or underweight without taxation.

Redistribution by subsidy on the other good. What has to be done in order to overcome also the problem at the extensive margin of the individual's weight decision? In order to answer this question, we assume redistribution is now by the subsidy on the other good instead of the lump-sum transfer. Set $\ell \equiv 0$ and $\sigma = \alpha\tau$ with $\alpha = x^H/(e - x^H)$, i.e. the subsidy rate σ is assumed to be in fixed proportion to the tax rate τ . As we will see below, the factor α is set such that the public budget $\tau x = \sigma z$ still holds. Inserting this policy together with the private budget constraint (3) into (4), the first-order condition of the individual's utility maximization can be written as

$$V_x \left[\frac{e}{1 - \alpha\tau} - \frac{1 + \tau}{1 - \alpha\tau} x^*, x^* \right] - \frac{1 + \tau}{1 - \alpha\tau} V_z \left[\frac{e}{1 - \alpha\tau} - \frac{1 + \tau}{1 - \alpha\tau} x^*, x^* \right] = \beta C_x(x^*). \quad (11)$$

This equation again determines the individual's junk food demand x^* as a function of the tax τ , as in case of lump-sum redistribution. Since we now have another redistribution mode, however, the functional relation is different and denoted by $\tau^* = X^s(\tau)$.

The maximization problem of the paternalistic social planner becomes

$$\max_{\tau} u = V \left[\frac{e}{1 - \alpha\tau} - \frac{1 + \tau}{1 - \alpha\tau} X^s(\tau), X^s(\tau) \right] - C[X^s(\tau)].$$

The first-order condition reads

$$\frac{du}{d\tau} = \left[V_x(\cdot) - \frac{1 + \tau}{1 - \alpha\tau} V_z(\cdot) - C_x(\cdot) \right] X_{\tau}^s(\cdot) + \frac{\alpha e - (1 + \alpha)X^s(\cdot)}{(1 - \alpha\tau)^2} V_z(\cdot) = 0. \quad (12)$$

This equation determines the optimal paternalistic fat tax rate with subsidy redistribution, which we denote by τ^s . Taking into account $\alpha = x^H/(e - x^H)$ and using (11) in order to replace $V_x - (1 + \tau)V_z/(1 - \alpha\tau)$ by βC_x , we can rewrite (12) as

$$-\frac{(1 - \beta)C_x[X^s(\tau^s)]}{e - x^H} X_{\tau}^s(\tau^s) + \frac{e[x^H - X^s(\tau^s)]}{[e - (1 + \tau^s)x^H]^2} V_z(\cdot) = 0. \quad (13)$$

This equation is satisfied if the optimal tax rate τ^s is such that the corresponding junk food consumption $X^s(\tau^s)$ just equals healthy consumption x^H , since for $X^s(\tau^s) = x^H$

we have $C_x[X^s(\tau^s)] = 0$ and both fractions in (13) become zero. Hence, under the optimal policy with subsidy redistribution the individual always realizes healthy weight.

It remains to determine the optimal tax and subsidy rates. Inserting $x^* = X^s(\tau^s) = x^H$ together with $\alpha = x^H/(e - x^H)$ into equation (11), taking into account $C_x(x^H) = 0$ and solving for the tax and subsidy rates yields

$$\tau^s = \frac{(e - x^H)[V_x(e - x^H, x^H) - V_z(e - x^H, x^H)]}{(e - x^H)V_z(e - x^H, x^H) + x^H V_x(e - x^H, x^H)}, \quad (14)$$

$$\sigma^s = \frac{x^H[V_x(e - x^H, x^H) - V_z(e - x^H, x^H)]}{(e - x^H)V_z(e - x^H, x^H) + x^H V_x(e - x^H, x^H)}. \quad (15)$$

Note that with these policy instruments, the private budget constraint (3) implies $z = e - x^H =: z^H$. Moreover, we obtain $\tau^s x^H = \sigma^s(e - x^H) = \sigma^s z^H$, so the public budget constraint is satisfied under the optimal policy.

With these insights we get the following proposition.

Proposition 3. *Assume $\ell \equiv 0$ and $\sigma = \alpha\tau$ with $\alpha = x^H/(e - x^H)$. The optimal paternalistic fat tax rate τ^s and the other good subsidy rate σ^s are then given by equations (14) and (15), respectively. The individual always attains healthy weight $x^* = X^s(\tau^s) = x^H$. Moreover, the following statements hold:*

- (a) *If $x^H < x^T$, then $\tau^s > 0$ and $\sigma^s > 0$.*
- (b) *If $x^T < x^H$, then $\tau^s < 0$ and $\sigma^s < 0$.*
- (c) *If $x^T = x^H$, then $\tau^s = 0$ and $\sigma^s = 0$.*

According to Proposition 3, the social planner may use an optimal paternalistic fat tax τ^s to induce healthy weight, provided the revenues are redistributed back to the individual not in a lump-sum way, but by a subsidy on the other good, σ^s , that is proportional to the fat tax and revenue neutral. If in the absence of taxation the individual is overweight ($x^H < x^T$), the optimal tax and subsidy are both positive according to Proposition 3 (a). For an underweight individual ($x^T < x^H$) both policy instruments are negative, resulting in a fat subsidy and tax on the other good, as shown in Proposition 3 (b). For the knife edge case of an individual with healthy weight ($x^T = x^H$) both instruments are zero, as can be seen in Proposition 3 (c).

Hence, in contrast to the paternalistic fat tax with lump-sum redistribution characterized in Proposition 2, the paternalistic fat tax with subsidy redistribution characterized in Proposition 3 is suitable to overcome the problem at the extensive margin

and to implement healthy weight of individuals. Intuitively, the reason is that the policy with subsidy redistribution has a stronger substitution effect than the policy with lump-sum redistribution. The relative junk food price equals $p = (1+\tau)/(1-\sigma)$. Under lump-sum redistribution, we have $\sigma = 0$ and $p = 1 + \tau$. Under subsidy redistribution, in contrast, $\sigma = \alpha\tau$ and $p = (1 + \tau)/(1 - \alpha\tau)$. Hence, the impact of the fat tax on the junk food price, $dp/d\tau$, is larger under subsidy redistribution than under lump-sum redistribution. Since income effects are not present (all revenues are redistributed back to the individual), it follows that the effect of the tax rate τ on junk food demand and weight is larger under subsidy redistribution than under lump-sum redistribution.

But how does the policy with subsidy redistribution perform in terms of true welfare? In order to answer this question, define

$$u^o := V(e - x^o, x^o) - C(x^o), \quad (16)$$

$$u^\ell := V[e - X^\ell(\tau^\ell), X^\ell(\tau^\ell)] - C[X^\ell(\tau^\ell)], \quad (17)$$

$$u^s := V[e - X^s(\tau^s), X^s(\tau^s)] - C[X^s(\tau^s)] = V(e - x^H, x^H). \quad (18)$$

Equation (16) gives the individual's true utility in the absence of taxation, while (17) and (18) reflect the individual's true utility under the optimal fat tax with lump-sum and subsidy redistribution, respectively. If tempting weight equals healthy weight ($x^T = x^H$), we obtain $x^o = X^\ell(\tau^\ell) = x^H$ and, thus, $u^o = u^\ell = u^s$. True utility is then always the same, independently of the tax policy. The rationale is obvious since in this knife edge case the individual always realizes healthy weight. More interesting is the welfare comparison if tempting weight and healthy weight deviate. We then obtain

Proposition 4. *If $x^T \neq x^H$, then the individual's true utility satisfies*

- (a) $u^s < u^\ell$.
- (b) $u^s \begin{smallmatrix} \leq \\ \geq \end{smallmatrix} u^o$ iff $\beta \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \bar{\beta}$ with $\bar{\beta} \in [0, 1[$.

Proposition 4 (a) shows that true utility of the individual is lower when redistribution is by the subsidy instead of the lump-sum transfer. Intuitively, the optimal paternalistic tax with lump-sum redistribution (Proposition 2) distorts the individual's junk food demand in order to correct the self control problem and the associated internalities. It mitigates the weight problem of the individual at the intensive margin, but does not implement healthy weight at the extensive margin. The optimal tax with subsidy redistribution (Proposition 3) goes a step further and also targets at the extensive margin. It distorts the individual's junk food demand not only to correct the internalities

and to mitigate the weight problem, but to bring the individual to its healthy weight. Hence, the policy with subsidy redistribution needs a larger distortion and, thus, leaves the individual with a lower well-being than the policy with lump-sum distribution.⁴

Part (b) of Proposition 4 may worsen the picture even more. Trying to overcome the weight problem at the extensive margin and thereby to induce healthy weight with the help of a paternalistic tax on unhealthy food may be inferior to not taxing the individual at all and accepting the individual's self control problem. This result is obtained if the self control parameter β is not too low and, thus, the self control problem is not too severe. The rationale is straightforward. If the self control problem is moderate, then the individual's loss in true utility from incomplete self control is small relative to the loss in true utility when the individual is forced to implement healthy weight. The individual should then not be induced to choose healthy weight. Put differently, paternalistically forcing the individual to overcome its problem at the extensive margin may be welfare-reducing in terms of true utility. Admittedly, for β small enough this result is reversed, since the distortion due to self control problem is relatively large. Whether β is larger or lower than $\bar{\beta}$ is in the end an empirical question. However, our result in part (b) of Proposition 4 at least shows that policy makers should be careful in inducing individuals to choose healthy weight.

4 Optimal Policy With Heterogeneous Consumers

Next turn to the case with heterogeneous individuals. In order to highlight the most interesting implications, we focus on the special case with only two types of individuals.

Consumption decision of individuals. As stated by O'Donoghue and Rabin (2006), in the context of fat taxes differences in the preferences for junk food and in the self control parameter seem to be most interesting. In our analysis, the self control parameter does not determine whether an individual is over- or underweight. Hence, variations in this parameter only result in different degrees of under- or overweight, but do not produce a situation where some individuals are overweight and the others are underweight or have healthy weight, which is the most interesting situation for our purposes.

⁴Note that we cannot do better by setting τ and σ independently, instead of assuming $\sigma = \alpha\tau$. Also using all three instruments τ , σ and ℓ does not solve the problem. In both cases, it can be shown that, as long as the public budget constraint is satisfied, we obtain exactly the same solution as for the policy with lump-sum redistribution characterized in Proposition 2.

We therefore still assume that individuals have the same self control parameter, but differ in the preferences for junk food consumption. There are two types of individuals denoted by $i = 1, 2$. The number of type i individuals is n_i with the normalization that the total number of individuals is $n_1 + n_2 = 1$. To ease the analysis we will focus on the specific case of a quasi-linear utility function $V(z_i, x_i) = z_i + W(x_i, \theta_i)$ with $0 < \theta_1 < \theta_2$, $W_{xx} < 0 < W_x$ and $W_{x\theta} > 0$. Hence, type 2 individuals have a stronger preference for junk food consumption than type 1 individuals, since the marginal utility from junk food consumption is increasing in the preference parameter θ_i .

The focus in the analysis of heterogeneous individuals is on lump-sum redistribution ($\sigma = 0$). For comparison purpose only, we allow the fat tax and the lump-sum payment to differ between types of individuals. The main results will be derived, however, under the assumption that all individuals have to pay the same tax rate and obtain the same lump-sum transfer. Formally, τ_i and ℓ_i are the fat tax imposed on and the lump-sum transfer received by type i individuals. Later we assume $\tau_i = \tau$ and $\ell_i = \ell$ for $i = 1, 2$. The decision of a type i individual is described by the maximization problem

$$\max_{x_i} \hat{u}_i = e + \ell_i - (1 + \tau_i)x_i + W(x_i, \theta_i) - \beta C(x_i),$$

where we have used the budget constraint $(1 + \tau_i)x_i + z_i = e + \ell_i$ in order to replace z_i . The first-order condition of type i 's utility maximization reads

$$W_x(x_i^*, \theta_i) - 1 - \tau_i = \beta C_x(x_i^*). \quad (19)$$

This condition has the same interpretation as in the case of a representative individual. It determines junk food consumption of a type i individual as a function of the tax rate and the preference parameter, i.e. $x_i^* = X(\tau_i, \theta_i)$. Totally differentiating gives $X_\tau(\cdot) = 1/(W_{xx} - \beta C_{xx}) < 0$ and $X_\theta(\cdot) = -W_{x\theta}/(W_{xx} - \beta C_{xx}) > 0$. An increase in the fat tax thus reduces junk food consumption and, for a given fat tax, junk food consumption of type 1 will be lower than junk food consumption of type 2 since $\theta_1 < \theta_2$.

Solution in the absence of any policy. First, consider again the case without taxation, i.e. $\tau_i \equiv \ell_i \equiv 0$. Let $x_i^* = X(0, \theta_i) =: x_i^o$ be junk food demand of type i in the absence of taxation. The first-order condition (19) can then be rewritten as

$$M(x_i^o, \theta_i) = W_x(x_i^o, \theta_i) - 1 = \beta C_x(x_i^o), \quad (20)$$

with $M(x_i, \theta_i) := W_x(x_i, \theta_i) - 1$. The function M represents the net marginal utility of fast food in the absence of taxation and plays the same role as the function N in the case

of a representative individual. We obtain $M_x(\cdot) = W_{xx}(\cdot) < 0$ and $M_\theta(\cdot) = W_{x\theta} > 0$, so $M(\cdot)$ is decreasing in x_i and increasing in θ_i . The tempting weight of type i individuals is denoted by x_i^T and implicitly defined by $M(x_i^T, \theta_i) = 0$. Due to $\theta_1 < \theta_2$ and $M_\theta > 0$, it follows $x_1^T < x_2^T$, so type 1 individuals have lower tempting junk food consumption than type 2 individuals. It is then straightforward to prove

Proposition 5. *Proposition 1 applies to both types of individuals, except that we have to replace x^T by x_i^T and x^o by x_i^o for $i = 1, 2$. Moreover, we obtain $x_1^o < x_2^o$.*

Whether an individual of type i is overweight or underweight (extensive margin) depends solely on the relation between tempting weight x_i^T and healthy weight x^H , whereas the self control parameter β determines the extent of the individual's weight problem (intensive margin). This is the same result with the same intuition as we obtained in Proposition 1 for the representative individual. In Proposition 5 we obtain the additional insight that, in the absence of taxation, type 1 individuals consume less junk food than type 2 individuals. The reason is that type 1 individuals have a lower preference for junk food. Note that this result allows situations where some individuals are overweight (if $x^H < x_i^T$) and others are underweight or have a healthy weight ($x_j^T \leq x^H$ for $j \neq i$). This will be the most interesting case in the subsequent analysis.

Type-specific policy as benchmark. Consider now the case in which each type faces a specific tax rate τ_i and a lump-sum transfer $\ell_i = \tau_i X(\tau_i, \theta_i)$.⁵ The paternalistic social planner maximizes the sum of true utilities of all consumers, i.e. $w := n_1 u_1 + n_2 u_2$ where $u_i = e + \ell_i - (1 + \tau_i)x_i + W(x_i, \theta_i) - C(x_i)$. Using $x_i = x_i^* = X(\tau_i, \theta_i)$ and $\ell_i = \tau_i X(\tau_i, \theta_i)$, the maximization problem of the social planner reads

$$\max_{\tau_1, \tau_2} w = \sum_{i=1}^2 n_i u_i = \sum_{i=1}^2 n_i \left\{ e - X(\tau_i, \theta_i) + W[X(\tau_i, \theta_i), \theta_i] - C[X(\tau_i, \theta_i)] \right\}.$$

The first-order conditions are

$$\frac{\partial w}{\partial \tau_i} = n_i \left\{ W_x[X(\tau_i, \theta_i), \theta_i] - 1 - C_x[X(\tau_i, \theta_i)] \right\} X_\tau^i = 0, \quad i = 1, 2. \quad (21)$$

Replacing $W_x(\cdot) - 1$ from (19) by $\tau_i + \beta C_x(\cdot)$ and solving with respect to τ_i gives the optimal paternalistic fat tax rate

$$\tau_i^\ell = (1 - \beta) C_x[X(\tau_i^\ell, \theta_i)], \quad i = 1, 2. \quad (22)$$

⁵Due to the quasi-linear utility, it makes no difference if we assume that each individual obtains the same lump-sum transfer, as long as all revenues are redistributed back to individuals.

Inserting back into the first-order condition (19) gives

$$M[X(\tau_i^\ell, \theta_i), \theta_i] = W_x[X(\tau_i^\ell, \theta_i), \theta_i] - 1 = C_x[X(\tau_i^\ell, \theta_i)], \quad (23)$$

From (22) and (23) we obtain

Proposition 6. *Proposition 2 applies to both types of individuals, except that we have to replace x^T by x_i^T and $X^\ell(\tau^\ell)$ by $X(\tau_i^\ell, \theta_i)$ for $i = 1, 2$. Moreover, we obtain $X(\tau_1^\ell, \theta_1) < X(\tau_2^\ell, \theta_2)$ and $\tau_1^\ell < \tau_2^\ell$.*

As in the case of a representative individual, type-specific fat taxes with lump-sum redistribution mitigate the weight problems of both types at the intensive margin, but do not implement healthy weight at the extensive margin. The additional insight from Proposition 6 is that even with taxation type 1 individuals consume less junk food than type 2 individuals and, therefore, face a lower tax rate. If both types are overweight, this means that type 2 individuals are taxed heavier than type 1 individuals ($0 < \tau_1^\ell < \tau_2^\ell$). If both types are underweight, type 1 individuals receive a higher subsidy than type two individuals ($\tau_1^\ell < \tau_2^\ell < 0$). If type 1 individuals are underweight and type 2 individuals are overweight, type 1 is subsidized while type 2 is taxed ($\tau_1^\ell < 0 < \tau_2^\ell$).

Uniform fat taxes. A type-specific policy is unrealistic due to distributional and informational reasons. We therefore now consider the more realistic case that each individual faces the same tax rate τ and obtains the same lump-sum transfer ℓ . As each individual takes the policy instruments τ and ℓ as given, the junk food consumption of a type i individual is still characterized by the first-order condition (19) and now given by $x_i^* = X(\tau, \theta_i)$. The common lump-sum transfer equally distributes total tax revenues over all individuals, i.e. $\ell = \tau[n_1X(\tau, \theta_1) + n_2X(\tau, \theta_2)]$. Taking into account this expression, the social planner's maximization problem now reads

$$\max_{\tau} w = \sum_{i=1}^2 n_i u_i = \sum_{i=1}^2 n_i \left\{ e - X(\tau, \theta_i) + W[X(\tau, \theta_i), \theta_i] - C[X(\tau, \theta_i)] \right\}.$$

Due to the quasi-linear utility function, this objective is the same as in the case with the type-specific policy, except that now all individuals face the same tax rate τ . The first-order condition to paternalistic welfare maximization can be written as

$$\frac{\partial w}{\partial \tau} = \sum_{i=1}^2 n_i \left\{ W_x[X(\tau, \theta_i), \theta_i] - 1 - C_x[X(\tau, \theta_i)] \right\} X_\tau^i. \quad (24)$$

Using the first-order condition (19) in order to replace $W_x(\cdot) - 1$ by $\tau + \beta C_x(\cdot)$ and solving for τ gives the optimal paternalistic fat tax rate

$$\tau^\ell = (1 - \beta) \frac{n_1 X_\tau^1 C_x[X(\tau^\ell, \theta_1)] + n_2 X_\tau^2 C_x[X(\tau^\ell, \theta_2)]}{n_1 X_\tau^1 + n_2 X_\tau^2}. \quad (25)$$

Inserting this tax rate back into the consumers' first-order condition (19) gives

$$\begin{aligned} M[X(\tau^\ell, \theta_i)] &= W_x[X(\tau^\ell, \theta_i)] - 1 \\ &= C_x[X(\tau^\ell, \theta_i)] - \frac{(1 - \beta)n_j X_\tau^j}{n_i X_\tau^i + n_j X_\tau^j} \left\{ C_x[X(\tau^\ell, \theta_i)] - C_x[X(\tau^\ell, \theta_j)] \right\}, \end{aligned} \quad (26)$$

for $i, j = 1, 2$ and $i \neq j$. This a system of two equations determining the junk food consumption levels with uniform taxation, i.e. $X(\tau^\ell, \theta_1)$ and $X(\tau^\ell, \theta_2)$.

The analysis of (25) and (26) is now more complex than in the case with type-specific taxes, since many constellations are possible under heterogeneity. The following proposition therefore focuses on the most important insights only.

Proposition 7. *The optimal uniform paternalistic fat tax τ^ℓ with lump-sum redistribution is given by (25). The associated levels of junk food consumption, $X(\tau^\ell, \theta_1)$ and $X(\tau^\ell, \theta_2)$, are determined by (26) and satisfy $X(\tau^\ell, \theta_1) < X(\tau^\ell, \theta_2)$. Moreover, the optimal uniform paternalistic policy is characterized by the following properties:*

- (a) *Suppose $x^H < x_1^T < x_2^T$. Then $\tau^\ell > 0$. If τ^ℓ is such that $X(\tau^\ell, \theta_1) = x^H$, then we obtain $x^H < X(\tau_2^\ell, \theta_2) < X(\tau^\ell, \theta_2) < x_2^o$.*
- (b) *Suppose $x_1^T \leq x^H < x_2^T$. Then, $X(\tau^\ell, \theta_1) \neq x^H$ and $X(\tau^\ell, \theta_2) \neq x^H$.*
- (c) *Suppose $x_1^T \leq x^H < x_2^T$ and $\tau^\ell > 0$. Then, we obtain $X(\tau^\ell, \theta_1) < x_1^o \leq X(\tau_1^\ell, \theta_1) \leq x^H$ and $x^H < X(\tau_2^\ell, \theta_2) < X(\tau^\ell, \theta_2) < x_2^o$.*
- (d) *Suppose $x_1^T < x^H \leq x_2^T$ and $\tau^\ell < 0$. Then, we obtain $x_1^o < X(\tau^\ell, \theta_1) < X(\tau_1^\ell, \theta_1) < x^H$ and $x^H \leq X(\tau_2^\ell, \theta_2) \leq x_2^o < X(\tau^\ell, \theta_2)$.*

Part (a) of Proposition 7 seems to give a slightly more optimistic answer to the question whether the aim of realizing healthy weight with fat taxes in practice is consistent with the paternalistic foundation of fat taxes in the literature. If both types of individuals are overweight ($x^H < x_1^T < x_2^T$), the optimal uniform tax is positive ($\tau^\ell > 0$) and it may be possible that the less overweight individuals (type 1) attain healthy weight [$X(\tau^\ell, \theta_1) = x^H$] and the more overweight individuals (type 2) reduce their weight compared to the situation without taxation [$X(\tau^\ell, \theta_2) < x_2^o$], even though they do not attain healthy weight [$x^H < X(\tau^\ell, \theta_2)$]. Put differently, the optimal uniform fat tax may

mitigate the weight problem of the more overweight individuals at the intensive margin and, at the same time, may solve the weight problem of the less overweight individuals at the extensive margin. The intuition of this insight is that, due to averaging of self control problems in the optimal uniform tax (25), the less overweight individuals (type 1) face a higher tax burden than in the case of type-specific taxes and, thus, reduce their junk food consumption further. If this effect is strong enough, type 1 individuals may even attain healthy weight $[X(\tau^\ell, \theta_1) = x^H]$. The more overweight individuals (type 2) bear a lower tax burden and consume more than in case of type-specific taxes $[X(\tau^\ell, \theta_2) < X(\tau^\ell, \theta_2)]$, indeed, but the tax rate is positive and, thus, they consume less and have lower weight than in the absence of taxation $[X(\tau^\ell, \theta_2) < x_2^o]$.

Unfortunately, this optimistic picture vanishes when we turn to the more realistic case in which some individuals may already have healthy weight or are even underweight. As stated in part (b) of Proposition 7, if type 2 individuals are overweight ($x^H < x_2^T$), but type 1 individuals have healthy weight ($x_1^T = x^H$) or are even underweight ($x_1^T < x^H$), then under the optimal uniform fat tax no type realizes healthy weight at the extensive margin $[X(\tau^\ell, \theta_1) \neq x^H \text{ and } X(\tau^\ell, \theta_2) \neq x^H]$. This is true independently of whether the optimal paternalistic tax rate τ^ℓ is positive or negative. The intuition is as follows. Actually, in order to attain healthy weight, overweight individuals (type 2) have to be taxed heavier and underweight individuals (type 1) have to be subsidized more than in case of a type-specific policy. However, the uniform tax reflects the weighted average of the self control problems. Hence, the tax burden on overweight individuals and the subsidy benefits for underweight individuals are reduced (instead of increased), implying that no type attains healthy weight.

For the realistic constellation that a part of the population is underweight or has healthy weight, Proposition 7 (c) and (d) even work out some unwarranted reversed effects of the optimal uniform fat tax at the extensive and intensive margins. Part (c) considers the case where the optimal tax is positive ($\tau^\ell > 0$). Then, overweight individuals become less overweight than without taxation $[X(\tau^\ell, \theta_2) < x_2^o]$, indeed. But if type 1 individuals are underweight ($x_1^T < x^H$), the positive fat tax aggravates this type's weight problem at the intensive margin $[X(\tau^\ell, \theta_1) < x_1^o < x^H]$ and if type 1 individuals have healthy weight ($x_1^T = x^H$), the positive fat tax even creates a problem at the extensive margin and the otherwise healthy type 1 becomes underweight $[X(\tau^\ell, \theta_1) < x_1^o = x^H]$. The reason is that type 1 individuals actually have to be subsidized (if underweight) or not taxed at all (if healthy weight), but the self control problem of the overweight type 2 renders the uniform tax positive and gives type 1

the incentive to consume less junk food and to realize lower weight. Hence, tackling overweight is at the costs of individuals with healthy weight or underweight.

An even more unintended effect is obtained in part (d) of Proposition 7. Here, type 1 individuals are underweight ($x_1^T < x^H$), type 2 individuals are overweight ($x^H < x_2^T$) or have healthy weight ($x^H = x_2^T$) and the tax rate is negative ($\tau^\ell < 0$). In this case, the underweight problem of type 1 individuals is mitigated at the intensive margin [$x_1^o < X(\tau^\ell, \theta_1)$], but type 2 individuals become (even more) overweight [$x^H \leq x_2^o < X(\tau^\ell, \theta_2)$], since they are now subsidized and consume more junk food. Of course, this case seems to be less relevant in practice since it requires that underweight is more severe than overweight and renders the optimal paternalistic fat tax rate negative.

An example. In order to prove that the cases considered in Proposition 7 (a)–(d) are not empty, we consider an example. Assume $W(x_i, \theta_i) = (1 + \theta_i)x_i - x_i^2/2$ and $C(x_i) = \gamma(x_i - x^H)^2/2$ with $\gamma > 0$. Note that we have chosen the parameters such that x_i^T , determined by $M(x_i^T, \theta_i) = W_x(x_i^T, \theta_i) - 1 = \theta_i - x_i^T = 0$, just equals $x_i^T = \theta_i$. From (19) we obtain $X(\tau, \theta_i) = (\theta_i - \tau + \beta\gamma x^H)/(1 + \beta\gamma)$. Using this in (22) and (25), it is straightforward to derive the optimal type-specific and uniform tax rates as

$$\tau_i^\ell = \frac{\gamma(1 - \beta)}{1 + \gamma}(\theta_i - x^H) \gtrless 0 \Leftrightarrow x_i^T = \theta_i \gtrless x^H, \quad i = 1, 2, \quad (27)$$

$$\tau^\ell = \frac{\gamma(1 - \beta)}{1 + \gamma}(n_1\theta_1 + n_2\theta_2 - x^H) \gtrless 0 \Leftrightarrow n_1x_1^T + n_2x_2^T = n_1\theta_1 + n_2\theta_2 \gtrless x^H. \quad (28)$$

Note that the sign of the type-specific taxes depends on the relation between healthy weight x^H and type-specific tempting weight x_i^T , whereas the sign of the uniform tax depends on the relation between healthy weight x^H and average tempting weight $n_1x_1^T + n_2x_2^T$. The reason is that the type-specific taxes reflect the self control problem of the respective type, whereas the uniform tax reflects the average self control problem of both individuals. Inserting τ_i^ℓ and τ^ℓ into $X(\tau, \theta_i)$ we obtain the associated junk-food consumption and weight levels

$$X(0, \theta_i) = \frac{\theta_i + \beta\gamma x^H}{1 + \beta\gamma} \gtrless x^H \Leftrightarrow x_i^T = \theta_i \gtrless x^H, \quad (29)$$

$$X(\tau_i^\ell, \theta_i) = \frac{\theta_i + \gamma x^H}{1 + \gamma} \gtrless x^H \Leftrightarrow x_i^T = \theta_i \gtrless x^H, \quad (30)$$

$$X(\tau^\ell, \theta_i) = \frac{\theta_i + \gamma x^H}{1 + \gamma} - \frac{\gamma n_j(1 - \beta)(\theta_j - \theta_i)}{(1 + \gamma)(1 + \beta\gamma)}, \quad (31)$$

for $i, j = 1, 2$ and $i \neq j$. In contrast to $X(0, \theta_i)$, $X(\tau_i^\ell, \theta_i)$ does not contain the self control parameter β , indicating that the type-specific taxes τ_i^ℓ correct the self control

problem of the individuals. $X(\tau^\ell, \theta_i)$ equals $X(\tau_i^\ell, \theta_i)$ corrected by a term containing $\theta_j - \theta_i$, which reflects averaging of self control problems in the uniform tax τ^ℓ .

From equation (31) we obtain $X(\tau^\ell, \theta_i) = x^H$ if

$$x_i^T - x^H = \theta_i - x^H = \gamma n_j (1 - \beta) (\theta_j - \theta_i) / (1 + \beta \gamma). \quad (32)$$

For $i = 1$ and $j = 2$ this equation specifies the parameter constellations for which Proposition 7 (a) holds. Moreover, for $x_1^T \leq x^H$ and $x_2^T > x^H$, equation (32) is neither satisfied for $i = 1$ and $j = 2$ nor for $i = 2$ and $j = 1$, since $\theta_2 > \theta_1$. This presents an example for part (b) of Proposition 7. Next note from (28) that $\tau^\ell > 0$ if and only if $n_1 \theta_1 + n_2 \theta_2 > x^H$. From (29) and (31) it is straightforward to show, that this latter condition ensures $X(\tau^\ell, \theta_1) < x_1^0 = X(0, \theta_1)$, which is the most important property in part (c) of Proposition 7. Finally, from (28) we know that $\tau^\ell < 0$ if and only if $n_1 \theta_1 + n_2 \theta_2 < x^H$, which by (29) and (31) in turn implies $X(\tau^\ell, \theta_2) > x_2^0 = X(0, \theta_2)$. This confirms the most important insight from part (d) of Proposition 7.

5 Conclusion

Our analysis shows that the often stated aim of taxes on unhealthy food in practice is not necessarily covered by the paternalistic foundation of such taxes in the literature. For a representative individual, the optimal paternalistic tax on unhealthy food internalizes the externality that the individual inflicts on itself by its self control problem, but it leaves uncorrected rationale motives for unhealthy weight. Hence, the tax mitigates weight problems at the intensive margin, but leaves unsolved the weight problem at the extensive margin. Targeting also the extensive margin and, thus, inducing the individual to choose healthy weight, is more costly in terms of the individual's true welfare, since it requires a further, welfare-reducing distortion. Depending on the degree of the self control problem, we show that the additional distortion may be large enough to render taxation of unhealthy food inferior to non-taxation. With heterogeneous individuals, the optimal paternalistic tax may even have reversed side effects at the extensive and intensive margins. If the tax is positive, individuals which have healthy weight in the absence of taxation may become underweight and individuals that are already underweight in the absence of taxation become even more underweight.

Using self control problems of individuals as justification of sin taxes, in general, and taxes on unhealthy food, in particular, is controversial among economists, because this kind of argument is based on paternalistic grounds. However, our results should

not be understood as a further argument against the paternalistic approach. Instead, it is more a critique on the aim many politicians pursue with taxes on unhealthy food in practice: Even if we accept the paternalistic approach, it does not imply that taxes on unhealthy food should be set such that individuals realize healthy weight. A part of the deviation from healthy weight is caused by rational motives and, from an economic point of view, should not be corrected by governmental intervention. Only the part of over-/underweight that goes back to self control problems has to be eliminated. Put differently, taxes on unhealthy food should correct the individual's weight problem at the intensive margin, but not at the extensive margin. As discussed in the Introduction, the underlying distinction between self control problems and rational motives in diet choice seems to be relevant not only from a theoretical-qualitative but also from an empirical-quantitative point of view. Of course, further empirical work is needed. There are already a number of articles that empirically investigate taxes on unhealthy food. See Griffith et al. (2017) for a recent review. To the best of our knowledge, however, the implications of the distinction between extensive and intensive margins for the optimal tax rate have not yet been addressed in the literature. This is an important task that is beyond the scope of the present paper and therefore left for future research.

Appendix

Proof of Proposition 1. Concavity of V implies $V_{xx}(\cdot)V_{zz}(\cdot) - [V_{zx}(\cdot)]^2 > 0$ or, equivalently, $V_{xx}(\cdot) < [V_{zx}(\cdot)]^2/V_{zz}(\cdot)$. From the definition of $N(x)$ we then obtain $N_x(\cdot) = V_{xx}(\cdot) + V_{zz}(\cdot) - 2V_{zx}(\cdot) < [V_{zx}(\cdot) - V_{zz}(\cdot)]^2/V_{zz}(\cdot) < 0$. Hence, the function $N(x)$ is a decreasing function with a zero at $x = x^T$. Due to $C_{xx}(x) > 0$, the functions $C_x(x)$ and $\beta C_x(x)$ are both increasing functions with a zero at $x = x^H$. Hence, if $x^H < x^T$, then the intersection between $N(x)$ and $\beta C_x(x)$, as required for the first-order condition (6), lies in the interval $]x^H, x^T[$ where $C_x(x) > 0$. Hence, $x^H < x^o < x^T$, as stated in (a) of Proposition 1. The proofs of parts (b) and (c) are perfectly analogous.

Proof of Proposition 2. Remember that $N(x)$ is decreasing with a zero at $x = x^T$ whereas $C_x(x)$ and $\beta C_x(x)$ are both increasing with a zero at $x = x^H$. Using (10), the property that $X^\ell(\tau^\ell)$ always lies between x^T and x^H is proven by the same steps as we show in Proposition 1 that x^o lies between x^T and x^H . In order to show the relation between x^o and $X^\ell(\tau^\ell)$, note that the former is determined by (6) whereas the latter is determined by (10). While the RHS of (6) equals $\beta C_x(\cdot)$, the RHS of (10) only contains

$C_x(\cdot)$. For $x > x^H$ ($x < x^H$) the function $\beta C_x(x)$ is below (above) the function $C_x(x)$ since $\beta < 1$. Hence, if $x^H < x^T$, the intersection of $N(x)$ with $C_x(x)$, as required for the determination of $X^\ell(\tau^\ell)$, lies to the left of the intersection of $N(x)$ with $\beta C_x(x)$, as required for the determination of x^o . It follows $X^\ell(\tau^\ell) < x^o$, as stated in part (a) of Proposition 2. This argument is reversed if $x^T < x^H$, proving the statement in part (b). For $x^T = x^H$, all three functions $N(x)$, $\beta C_x(x)$ and $C_x(x)$ intersect at the same point on the x -axis implying $x^o = X^\ell(\tau^\ell)$, as stated in part (c) of Proposition 2.

It remains to prove the sign of the optimal tax rate τ^ℓ . If $x^H < x^T$, we know $X^\ell(\tau^\ell) > x^H$ and, thus, $C_x[X^\ell(\tau^\ell)] > 0$. Inserting into (9) yields $\tau^\ell > 0$, as stated in part (a) of Proposition 2. Conversely, for $x^T < x^H$ we obtain $X^\ell(\tau^\ell) < x^H$ and therefore $C_x[X^\ell(\tau^\ell)] < 0$ and $\tau^\ell < 0$, while $x^T = x^H$ implies $C_x[X^\ell(\tau^\ell)] = 0$ and $\tau^\ell = 0$. This shows part (b) and part (c) and completes the proof of Proposition 2.

Proof of Proposition 3. It only remains to prove parts (a)–(c). The sign of τ^s and σ^s in (14) and (15) are equal to the sign of $N(x^H) = V_x(e - x^H, x^H) - V_z(e - x^H, x^H)$. Since $N'(x) < 0$ and $N(x^T) = 0$, it is straightforward that $x^H < x^T$ implies $N(x^H) > 0$ and, thus, $\tau^s > 0$ and $\sigma^s > 0$. In contrast, $x^T < x^H$ yields $N(x^H) < 0$ and $\tau^s < 0$ and $\sigma^s < 0$, whereas for $x^T = x^H$ we obtain $N(x^H) = 0$ and $\tau^s = \sigma^s = 0$.

Proof of Proposition 4. True utility under subsidy redistribution is u^s from (18). Now suppose we have the same junk food demand $x = x^H$, the same other good consumption $z = e - x^H$ and the same tax revenues $\tau^s x^H$ as in case of subsidy redistribution, but redistribution is by the lump-sum transfer $\ell = \tau^s x^H$ and the subsidy is $\sigma = 0$. The private budget $(1 + \tau^s)x + z = e + \ell$ is then still satisfied and true utility again equals u^s . But for $x^T \neq x^H$ we know from Proposition 2 that the policy $(\tau, \ell, \sigma) = (\tau^s, \tau^s x^H, 0)$ does not maximize true utility when redistribution is lump-sum; for $x^T \neq x^H$ the utility maximum for lump-sum redistribution is obtained under the policy $(\tau, \ell, \sigma) = (\tau^\ell, \tau^\ell X^\ell(\tau^\ell), 0)$, with $\tau^\ell \neq \tau^s$ as well as $X^\ell(\tau^\ell) \neq x^H$, and equals u^ℓ given by (17). It follows $u^s < u^\ell$ as stated in part (a) of Proposition 4.

In order to prove part (b) consider first the limiting case $\beta = 1$. From (6), (10) and $\beta = 1$ it follows $X^\ell(0) = x^o$ and equation (16) and (17) imply $u^\ell = u^o$. This is the maximum of true utility u . For $x^T \neq x^H$, however, Proposition 1 implies that $x^o \neq x^H$ so that true utility under healthy weight is lower than its maximum, i.e. $u^s < u^o$. Hence, we have proven part (b) of Proposition 4 for the extreme case $\beta = 1$. Equation (4) together with the assumption that V and C are twice continuously differentiable

imply that x^* , and therefore x^o and $X^\ell(\tau^\ell)$, as well as true utility are continuous in β . Hence, if we reduce β slightly below one, we can still apply the same arguments as for $\beta = 1$ and also obtain $u^s < u^o$. This is true until the self control parameter falls below a certain threshold $\bar{\beta} \in [0, 1[$, which completes the proof of part (b).

Proof of Proposition 5. The proof of Proposition 1 can immediately be applied to each type of individuals, except that we have to replace N by M . It remains to show $x_1^o < x_2^o$. This follows from $x_i^o := X(0, \theta_i)$, $X_\theta(\cdot) > 0$ and $\theta_1 < \theta_2$.

Proof of Proposition 6. The proof of Proposition 2 can immediately be applied to each type of individuals, except that we have to replace N by M . The relation $X(\tau_1^\ell, \theta_1) < X(\tau_2^\ell, \theta_2)$ follows from $x_1^T < x_2^T$, $M_x < 0$ and $M_\theta > 0$, since the intersection between M and C_x for type 1 individuals lies to the left of the intersection between M and C_x for type 2 individuals, independently of the relation between x_i^T and x^H . The inequality $X(\tau_1^\ell, \theta_1) < X(\tau_2^\ell, \theta_2)$ implies $C_x^1 < C_x^2$ and, thus, $\tau_1^\ell < \tau_2^\ell$ by (22).

Proof of Proposition 7. $X(\tau^\ell, \theta_1) < X(\tau^\ell, \theta_2)$ follows from $\theta_1 < \theta_2$ and $X_\theta > 0$. In order to prove part (a), notice first that $X(\tau^\ell, \theta_1) = x^H$ implies $C_x[X(\tau^\ell, \theta_1)] = 0$. From $x^H = X(\tau^\ell, \theta_1) < X(\tau^\ell, \theta_2)$ follows $C_x[X(\tau^\ell, \theta_2)] > 0$ and, thus, $\tau^\ell > 0$ from (25). Defining $\chi_i := n_i X_\tau^i / (n_i X_\tau^i + n_j X_\tau^j) \in]0, 1[$, equation (26) for $i = 2$ can be written as

$$M[X(\tau^\ell, \theta_2), \theta_2] = [1 - (1 - \beta)\chi_1]C_x[X(\tau^\ell, \theta_2)].$$

$X(\tau_2^\ell, \theta_2)$ is determined by (23) for $i = 2$ or, equivalently,

$$M[X(\tau_2^\ell, \theta_2), \theta_2] = C_x[X(\tau_2^\ell, \theta_2)].$$

Since $M_x(x, \theta) < 0$ for all x and $[1 - (1 - \beta)\chi_1]C_x(x) < C_x(x)$ for all $x > x^H$, we have $X(\tau^\ell, \theta_2) > X(\tau_2^\ell, \theta_2)$. Due to $X_\tau < 0$ and $\tau^\ell > 0$, it follows $X(\tau^\ell, \theta_2) < x_2^o$.

The proof of part (b) is by contradiction. Suppose that at least for one $i \in \{1, 2\}$ we have $X(\tau^\ell, \theta_i) = x^H$. Consider first the case $X(\tau^\ell, \theta_2) = x^H$. We then obtain $C_x[X(\tau^\ell, \theta_2)] = 0$ and (26) for $i = 2$ can be rewritten as

$$M(x^H, \theta_2) = (1 - \beta)\chi_1 C_x[X(\tau^\ell, \theta_1)],$$

which gives the value of $M(x, \theta_2)$ at $x = x^H$. Since $M_x(x, \theta_2) < 0$ for all x and $x^H < x_2^T$, we have $M(x^H, \theta_2) > 0$ and, thus, by the above equation $C_x[X(\tau^\ell, \theta_1)] > 0$. It follows $X(\tau^\ell, \theta_1) > x^H = X(\tau^\ell, \theta_2)$, a contradiction to our general result $X(\tau^\ell, \theta_1) < X(\tau^\ell, \theta_2)$.

A similar contradiction can be obtained for the second case where $X(\tau^\ell, \theta_1) = x^H$. We then obtain $C_x[X(\tau^\ell, \theta_1)] = 0$ and (26) for $i = 1$ becomes

$$M(x^H, \theta_1) = (1 - \beta)\chi_2 C_x[X(\tau^\ell, \theta_2)],$$

which gives $M(x, \theta_1)$ at $x = x^H$. Due to $M_x(x, \theta_1) < 0$ for all x and $x_1^T \leq x^H$ it follows $M(x^H, \theta_1) \leq 0$ and, thus, by the above equation $C_x[X(\tau^\ell, \theta_2)] \leq 0$. This in turn implies $X(\tau^\ell, \theta_2) \leq x^H = X(\tau^\ell, \theta_1)$, a contradiction to our result $X(\tau^\ell, \theta_1) < X(\tau^\ell, \theta_2)$.

In order to prove part (c), note first that $x_1^T \leq x^H$ implies $\tau_1^\ell \leq 0$ and $x_1^o \leq X(\tau_1^\ell, \theta_1) \leq x^H$ by Proposition 5 and Proposition 6. Due to $X_\tau < 0$ and our assumption $\tau^\ell > 0$, we obtain $X(\tau^\ell, \theta_1) < x_1^o \leq X(\tau_1^\ell, \theta_1) \leq x^H$, as stated in part (c). $X(\tau^\ell, \theta_1) < x^H$ implies $C_x[X(\tau^\ell, \theta_1)] < 0$. This in turn yields $C_x[X(\tau^\ell, \theta_2)] > 0$, since otherwise we cannot have $\tau^\ell > 0$, see equation (25). It follows $X(\tau^\ell, \theta_2) > x^H$. Moreover, $X(\tau^\ell, \theta_2)$ is determined by (26) for $i = 2$, which can be written as

$$M[X(\tau^\ell, \theta_2), \theta_2] = [1 - (1 - \beta)\chi_1]C_x[X(\tau^\ell, \theta_2)] + (1 - \beta)\chi_1 C_x[X(\tau^\ell, \theta_1)].$$

Remember that $X(\tau_2^\ell, \theta_2)$ is determined by (23) for $i = 2$, i.e.

$$M[X(\tau_2^\ell, \theta_2), \theta_2] = C_x[X(\tau_2^\ell, \theta_2)].$$

Since $M_x(x, \theta) < 0$ for all x and since $C_x[X(\tau^\ell, \theta_1)] < 0$ implies $[1 - (1 - \beta)\chi_1]C_x(x) + (1 - \beta)\chi_1 C_x[X(\tau^\ell, \theta_1)] < C_x(x)$ for all x , we obtain $X(\tau^\ell, \theta_2) > X(\tau_2^\ell, \theta_2)$. Since $X_\tau < 0$ and $\tau^\ell > 0$, it follows $X(\tau^\ell, \theta_2) < x_2^o$, which completes the proof of part (c).

The proof of part (d) is similar. $x^H \leq x_2^T$ implies $\tau_2^\ell \geq 0$ and $x^H \leq X(\tau_2^\ell, \theta_2) \leq x_2^o$ by Proposition 5 and Proposition 6. Since $X_\tau < 0$ and $\tau^\ell < 0$ by assumption, we obtain $x^H \leq X(\tau_2^\ell, \theta_2) \leq x_2^o < X(\tau^\ell, \theta_2)$, as stated in part (d). $x^H < X(\tau^\ell, \theta_2)$ gives $C_x[X(\tau^\ell, \theta_2)] > 0$. $\tau^\ell < 0$ then requires $C_x[X(\tau^\ell, \theta_1)] < 0$ and, thus, $X(\tau^\ell, \theta_1) < x^H$. Moreover, $X(\tau^\ell, \theta_1)$ is determined by (26) for $i = 1$, which can be written as

$$M[X(\tau^\ell, \theta_1), \theta_1] = [1 - (1 - \beta)\chi_2]C_x[X(\tau^\ell, \theta_1)] + (1 - \beta)\chi_2 C_x[X(\tau^\ell, \theta_2)].$$

$X(\tau_1^\ell, \theta_1)$ is determined by (23) for $i = 1$ or, equivalently,

$$M[X(\tau_1^\ell, \theta_1), \theta_1] = C_x[X(\tau_1^\ell, \theta_1)].$$

For all $x < x^H$ we obtain $C_x(x) < 0$ and thereby $[1 - (1 - \beta)\chi_2]C_x(x) > C_x(x)$ as well as $[1 - (1 - \beta)\chi_2]C_x(x) + (1 - \beta)\chi_2 C_x[X(\tau^\ell, \theta_2)] > C_x(x)$, since $C_x[X(\tau^\ell, \theta_2)] > 0$. Hence, the above two equations imply $X(\tau^\ell, \theta_1) < X(\tau_1^\ell, \theta_1)$ since $M_x(x, \theta) < 0$ for all x . Due to $X_\tau < 0$ and $\tau^\ell < 0$ it finally follows $x_1^o < X(\tau^\ell, \theta_1)$.

References

- Andersen, S., Harrison, G.W., Lau, M.I. and E.E. Rutström (2014), 'Discounting Behavior: A Reconsideration', *European Economics Review* 71, 15-33.
- Becker, G.S. and K.M. Murphy (1988), 'A Theory of Rational Addiction', *Journal of Political Economy* 96, 675-700.
- Courtemanche, C., Heutel, G. and P. McAlvanah (2014). 'Impatience, Incentives and Obesity', *Economic Journal* 125, 1-31.
- Cremer, H., De Donder, P., Maldonado, D. and P. Pestieau (2012), 'Taxing Sin Goods and Subsidizing Health Care', *Scandinavian Journal of Economics* 114, 101-123
- Cremer, H., Goulao, C. and K. Roeder (2016), 'Earmarking and the Political Support of Fat Taxes', *Journal of Health Economics* 50, 258-267.
- Cutler, D., Glaeser, E. and J. Shapiro (2003), 'Why Have Americans Become More Obese?' *Journal of Economic Perspectives* 17, 93-118.
- Dragone, D. (2009), 'A Rational Eating Model of Binges, Diets and Obesity', *Journal of Health Economics* 28, 799-804.
- Dragone, D. and L. Savorelli (2012), 'Thinness and Obesity: A Model of Food Consumption, Health Concerns, and Social Pressure', *Journal of Health Economics* 31, 243-256.
- Global BMI Mortality Collaboration (2016), 'Body-Mass Index and All-Cause Mortality: Individual-Participant-Data Meta-Analysis of 239 Prospective Studies in Four Continents', *Lancet* 388, 776-786.
- Griffith, R., O'Connell, M. and K. Smith (2017), 'Corrective Taxation and Internalities from Food Consumption', *CESifo Economic Studies*, 1-14.
- Gruber, J. and B. Köszegi (2001), Is Addiction 'Rational'? Theory and Evidence, *Quarterly Journal of Economics* 116, 1261-1303.
- Haavio, M. and K. Kotakorpi (2011), 'The Political Economy of Sin Taxes', *European Economic Review* 55, 575-594.

- Haavio, M. and K. Kotakorpi (2016), 'Sin Licenses Revisited', *Journal of Public Economics* 44, 40-51.
- Ikedo, S., Kang, M.I. and F. Ohtake (2010), 'Hyperbolic Discounting, the Sign Effect, and the Body Mass Index ', *Journal of Health Economics* 29, 268-284.
- Kalamov, Z. and M. Runkel (2017), 'Self Control Problems With Intensive and Extensive Margins', *mimeo*.
- Laibson, D. (1997), 'Hyperbolic Discounting and Golden Eggs', *Quarterly Journal of Economics* 112, 443-477.
- Levy, A. (2002), 'Rational Eating: Can it Lead to Overweightness or Underweightness?', *Journal of Health Economics* 21, 887-899.
- O'Donoghue, T. and M. Rabin (2003), 'Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes', *American Economic Review* 93, 186-191.
- O'Donoghue, T. and M. Rabin (2006), 'Optimal Sin Taxes', *Journal of Public Economics* 90, 1825-1849.
- OECD (2017), *Obesity Update 2017*, <https://www.oecd.org/els/health-systems/Obesity-Update-2017.pdf> (download November 2017).
- WHO (2016a), 'Putting Taxes into the Diet Equation', *Bulletin World Health Organization* 94, 239-240.
- WHO (2016b), 'Taxes on Sugary Drinks: Why Do It?', <http://apps.who.int/iris/bitstream/10665/250303/1/WHO-NMH-PND-16.5-eng.pdf> (download November 2017).
- WHO (2017), *Obesity and Overweight: Fact Sheet*, <http://www.who.int/mediacentre/factsheets/fs311/en/> (download November 2017).