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The Economics of Justice as Fairness

Abstract

In this paper we challenge the common interpretation of Rawls’ Theory of Justice as Fairness by showing that this Theory, as outlined in the Restatement (Rawls, “Justice as Fairness: a Restatement”, 2001), goes well beyond the definition of a distributive value judgment, in such a way as to embrace efficiency issues as well. A simple model is discussed to support our interpretation of the Difference Principle, by which inequalities are shown to be permitted as far as they stimulate a greater effort in education in the population, and so economic growth. To our knowledge, this is the only possibility for the inequality to be ‘bought’ by both the most-, and above all, the least-advantaged individual as suggested by the Difference Principle. Finally, by recalling the old tradition of ‘universal ex-post efficiency’, we show that a unique optimal social contract does not exist behind the veil of ignorance; more precisely, the sole set of potentially Rawls-optimal social contracts can be identified a priori, and partial justice orderings derived accordingly.

Keywords: justice, Rawls, inequality, social contract

JEL: D63, D31, J31
1 Introduction

In this paper we propose a reinterpretation of Rawls’ Theory of Justice as Fairness (hereafter Theory), by which the causality between inequality, individual decisions on education, and economic growth is modeled ‘behind the veil of ignorance’. Specifically, within our interpretation of the Theory, distributive aspects are assumed to impact on growth to the extent that the magnitude of (future) income inequalities is said to influence (ongoing) individual incentives to effort in education, and so (future) productivity in the labor market.

According to the common understanding of the Theory, an allocation is to be preferred if and only if the ‘least-advantaged’ individual is better off, independently from efficiency issues; this is the main idea usually ascribed to the maximin principle as represented by the well known Leontief preferences (Alexander 1974). In our view, however, such interpretation would not leave any room for the Rawlsian Difference Principle, by which, it is said, once education opportunities are granted to the entire population (Fair Equality of Opportunity), inequalities are admitted as far as they are to the greatest benefit of the least-advantaged. Evidently, in the absence of efficiency issues, there is no way by which the least-advantaged individual might be willing to be penalized by the introduction of inequality. As such, for any inequality to represent a benefit for the worse-off (i.e. least-advantaged), economic growth must be necessarily accounted for, that is, inequality must be stimulating growth in such a way as to make the least-advantaged, for some degree of inequality at least, more than compensated for being the worse-off. In a sense, inequality must be aimed at pro-poor growth.

In this way of thinking — as stated by Rawls’ in the Preface of the Restatement published thirty years after the Theory (Rawls 2001)1 — a revision of the common understanding of the Theory is necessary because, in our view, this Theory goes well beyond the proposal of a distributive value judgment in such a way as to embrace efficiency issues as well.2

According to our economic interpretation of Rawls’ thought, inequalities influence individual incentives to effort in education, and so the earnings capacity (e.g., wage rate) they will realize in the labor market. Most importantly, the earnings capacity is said to be co-determined by both effort in education and native talent, so that the most-advantaged individual does not necessarily correspond to the better endowed in terms of native abilities.

To better support our interpretation of the Theory, we model the Rawlsian Difference Principle through a three-stages sequential game, where knowledge available to the parties is progressively relaxed over time. Precisely, we focus on the sole Difference Principle by assuming that the Rawlsian principle of Fair

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1 "In this work I ... rectify the more serious faults in A Theory of Justice that have obscured the main ideas of justice as fairness, as I called the conception of justice presented in that book. Since I still have confidence in those ideas and think the more important difficulties can be met, I have undertaken this reformulation.” (Rawls 2001, p.xv).

2 "[I]t is not correct, I think, that maximin gives no weight to efficiency. It imposes a rule of functional contribution among inequalities; and since it applies to social arrangements that are mutually advantageous, some weight is given to efficiency” (Rawls 1974).
Equality of Opportunity holds, so that access to education is assumed to be universally granted independently from the social class of origin.

Individuals, as souls, agree on the social contract — that is, the redistribution of earnings capacity to be operated in the labor market — behind the veil of ignorance (time 0), when no information is available on either (i) preferences (i.e., propensity to effort in education), or (ii) native talent. Next, at the educational stage (time 1), individual preferences reveal and the effort decision (in education) is made under uncertainty conditions on native talent. Indeed, according to Rawls, the native talent is assumed to reveal at the working stage only (time 2), because it is not measurable ex-ante and strongly influenced by the shape of social institutions revealing ex-post only (time 2).

Solving by backward induction, we show that a unique optimal social contract cannot exist behind the veil of ignorance (time 0); as far as the individual with the higher propensity to effort in education (time 1) might be associated, ex-post, to the better or the worse endowment in terms of native talent (time 2), uncertainty conditions must be necessarily accounted for. More specifically, we determine state-contingent optimal contracts according to the notion of ‘ex-post k-efficiency’ (Harris 1978), as opportune revisited within Rawlsian framework: an allocation is said to be state-contingent optimal if there is no other allocation by which the earnings capacity of the least-advantaged can be improved in that state. Subsequently, we define the set of optimal contracts behind the veil of ignorance according to the more demanding notion of ‘universal ex-post efficiency’ (Starr 1973, Harris 1978, Hammond 1981), once again, as opportune revisited within Rawlsian framework: an allocation is said to be universally ex-post Rawls-optimal if there is no other allocation by which the earnings capacity of the least-advantaged can be improved in every state.3

The paper is organized as follows. In Section 2 we present our reinterpretation of the Theory by recalling the definition of the original position and the two Rawlsian Principles. The basic framework of our model, with the optimal decision of effort in education, is discussed in Section 3. In Section 4, the set of optimal social contracts is derived under uncertainty conditions. Concluding, the major novelties of our model, as compared to the common understanding of Rawls’ Theory, are discussed in Section 5.

2 The Theory of Justice

2.1 Contractualism in the original position

In His Theory (1971), John Bordley Rawls proposes a political conception of justice by which the stability of political institutions is to be preserved by en-
suring the overlapping consensus in the society; remarkably, the overlapping consensus is said to grant the stability of the society over time, independently of the oppressive sanctions of state power which, instead, are inevitably required in a society united on a form of utilitarianism.4

In this sense, the Theory of Justice as Fairness is usually accommodated in the old tradition of social contractualism whose best known proponents are Hobbes, Locke and Rousseau. Specifically, Rawls explores the possibility of a social contract to be agreed in the original position, or, equivalently, behind the veil of ignorance. At this stage, “individuals view themselves as potential occupants of each position in society” (Saposnik 1981) independently from individual preferences which, instead, strongly characterize Harsanyi’s (1953, 1955) impartial observer.

The focus on the original position is crucial in the Theory as, it is said, in order to permit a fair agreement (hence, the name Justice as Fairness) between free and equal persons, contractualism is required to abstract from contingencies — the particular features and circumstances of persons — which would inevitably introduce bargaining advantages jeopardizing the possibility of an overlapping consensus, and so the stability of the political institutions.5 As far as freedom from interests and desires is said to be a conditio sine qua non for any definition of justice (i.e., moral laws) to be valuable, Rawls is Kantian.6

More precisely, three conditions — fundamental for our interpretation — are said to characterize the original position, “(a) the parties do not have any knowledge of their desires and ends (except what is contained in the thin theory of the good, which supports the account of primary goods)...; (b) they do not know, and a fortiori cannot enumerate, the social circumstances in which they may find themselves or the array of techniques their society may have at its disposal; and (c) even if they could enumerate these possibilities, they have no grounds for relying on one probability distribution over them rather than another ...” (Rawls 1974, p.649).

First (a), individuals may differ from each other in terms of their individualistic preferences, or ‘ambitious’ in Rawls’ words, but these are taken as unknown behind the veil of ignorance. Second (b), individuals may also differ with respect to both social circumstances (e.g., social class of origin) and natural circumstances (e.g., native talent) but, once again, this information is not given in the original position. Most importantly, to the extent that ‘techniques

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4“A society united on a form of utilitarianism ... would likewise require the oppressive sanctions of state power to remain so” (Rawls 2001, p.34).

5“Stability results first, from the availability of principles that guarantee citizens’ fundamental interests” (Rawls 2001, p.110).

6“First, the moral law is a method of reasoning that tests an agent’s maxim, constructed out of his interests and desires in particular situations, ... And second, this method of reasoning comes prior to the definition of the moral object, and therefore, according to Kant, prescribes a truly free choice. Whereas men who act only from inclination allow their desires to define the object they seek and employ their reason only to determine the best way of achieving that object, the moral agent does the reverse, first accepting the moral law as his sole, authoritative reasoning procedure and then pursuing only those objects which this reasoning procedure dictates. ... It should now be evident that Rawls is adopting a Kantian approach to defining justice” (Hampton 1980).
at disposal of the society’ are unknown at this stage, native talent is merely potential and not measurable apart from social institutions revealing ex-post; e.g., the same native endowment may be more or less successful in the society depending on social and other contingencies.  

Third (c), the social contract is agreed under uncertainty conditions where the lack of information is radical, so that probabilities can be only defined in classical terms; i.e., since nothing makes one case more frequent than any other, each case is to be considered as equally possible. Altogether, by excluding all this information (i.e., a-b-c), it must be the case that, in the original position, no one is advantaged or disadvantaged by natural chance or social contingencies in the choice of principles, which is a conditio sine qua non for the overlapping consensus to be obtained. Notably, at this stage, individuals are supposed to decide the principles they are willing to adopt, and not the most effective means to one’s ends.

What is known behind the veil of ignorance, instead, is the object of the social contract, i.e. primary goods. Indeed, individuals are assumed to agree on the identification of primary goods which, according to Rawls, consist of those things citizens need, as free and equal persons, in order to have ‘command’ over exchangeable means for satisfying human needs and interests, and which have not to be confused with things it is simply rational to want or desire, or to prefer or even to crave. In this sense, individual preferences (or ‘ambitions’) are assumed to capture the instrumental value of goods (i.e., ‘command over resources’) more than their intrinsic value. In this perspective, for instance, income and wealth are said to belong to the set of primary goods to the extent that they are fundamental to implement a political conception of the person as free and equal, endowed with the moral powers, and capable of being a fully cooperating member of the society.  

Within the Restatement (Rawls 2001), most of the emphasis is posed on the lifetime earnings capacity (‘lifetime income prospect’ in Rawls’ words), which is intended as the synthetic measure, or index, quantifying the primary goods an individual may have access to when the working age is achieved. Most importantly, the earnings capacity is a potential value which is defined up to the entire time endowment in the labor market, leisure included, of each individual.  

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7 “[T]he conceptions of the good that individuals form depend in part on their natural abilities and the way in which these are shaped and realized by social and other contingencies” (Rawls 1975, p.552). For an extensive discussion on the non measurability of native talent behind the veil of ignorance, see Rawls’ (1974) reply to Alexander and Musgrave.

8 “I note some possible misinterpretations of primary goods that may lead one to overemphasize their individualistic bias. First: a comment about wealth ... wealth consists of (legal) command over exchangeable means for satisfying human needs and interests ... For whatever form they take, natural resources and the means of production, and the rights to control them, as well as rights to services, are wealth” Rawls (1975, p.540).

9 “In elaborating justice as fairness we assume that all citizens are normal and fully cooperating members of society ... and so willing to work and to do their part in sharing the burdens of social life, provided of course the terms of cooperation are seen as fair. But how is this assumption expressed in the difference principle? ... Are the least advantaged, then, those who live on welfare and surf all day off Malibu? This question can be handled in two ways: one is to assume that everyone works a standard working day; the other is to include in the index of primary goods a certain amount of leisure time ... Surfers must somehow support
is crucial because it automatically implies that the sole inequalities originating from different wage rates, not income, matter in that, by virtue of background procedural justice, the ex-post distribution of income — as obtained according to the two principles discussed in the next Section — is to be regarded as just (or, at least, not unjust) whatever this distribution turns out to be.

Differences in citizens’ earnings capacity is said to be influenced by such things as their social class of origin, their native endowments, their ambitions (e.g., propensity to effort in education), their opportunities for education, and their good or ill fortune over the course of life.

Most importantly — once opportunity for education is universally granted independently from the social class of origin (Rawlsian Fair Equality of Opportunity) — when the working age is achieved, individuals are endowed with some earnings capacity depending on their wage rate which, in turn, is determined by their ‘native talent’ and on their effort in education at the previous stage, that is, on how much they have cooperated “by training and educating their native endowments putting them to work within a fair system of social cooperation” (Rawls 2001, p.68).

In the absence of any scheme of social cooperation (i.e., redistribution), the earnings capacity obtained at the working age corresponds to the ‘realized talent’, which is measurable as opposed to the ‘native talent’ that is not.

Diversely, when a scheme of social cooperation is agreed behind the veil of ignorance, the earnings capacity of each individual is not necessarily anchored to its own realized talent any longer; in a cooperative system, the scheme of wages is expected to re-allocate among its own members the overall amount of talent realized in the society as a whole. In this scenario, the wage rate of each individual in the labor market is inevitably determined by a mix of its own realized talent and the one realized by others, in a way that embodies some redistribution from the most to the least-advantaged (as identified in terms of lifetime earnings capacity).

2.2 The two principles of justice

Given the very basic set up characterizing the original position, Rawls suggests two principles which, in His view, would make differences in lifetime earnings capacity legitimate and consistent with the idea of free and equal citizenship in a society seen as a fair system of cooperation: the principle of Liberty and the principle of Equality.

By the former, it is said, “[e]ach person has an equal right to the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for all” (Rawls 1974, p.639). By the latter, “[s]ocial and economic inequalities are to meet two conditions: they must be (a) to the greatest expected benefit of the least-advantaged (the maximin criterion); and (b) attached to offices and positions open to all under conditions of fair equality of opportunity” (Rawls 1974, p.639).

themselves. Of course, if leisure time is included in the index, society must make sure that opportunities for fruitful work are generally available” (Rawls 2001, p.179).
The Liberty principle is said to have a priority on Equality, meaning that the former cannot be violated in the name of the latter. Such a priority is crucial for any attempt to formalize Rawls’ thought, because it automatically implies that equality cannot be pursued through progressive taxation as this would violate the Liberty principle. More precisely, the redistribution of wealth and income can be admitted exclusively to prevent excessive concentrations of property and wealth, especially those likely to lead to political domination, as they would threaten the political liberties, i.e. the basic liberties safeguarded by the first principle. Indeed, even if the Rawlsian social contract is implemented, excessive concentrations may still come out from bequests and inheritance, as well as from separate and seemingly fair agreements between individuals, which would inevitably jeopardize the overlapping consensus.10

The second principle, the Equality principle, embodies two different criteria, respectively, (a) the ‘Difference Principle’ and (b) the principle of ‘Fair Equality of Opportunity’, where the latter is said to have a priority on the former; more specifically, once educational possibilities are granted for all members of the society, the social contract is to be designed in such a way as to permit the sole inequalities benefitting the least-advantaged. Remarkably, a criterion is ‘defined’ behind the veil of ignorance by which the least-advantaged is ‘identified’ ex-post only, that is, once the ‘realized talent’ has revealed.

To the extent that political institutions are supposed to neutralize different opportunities for education (and, given that good or ill fortune is normally distributed), it must be the case that, under fair equality of opportunity, citizen’s differences of lifetime earnings capacity originate from different talent and/or ambitions (e.g., propensity to effort in education).

As a consequence — and to our knowledge this aspect has not been properly emphasized in the common understanding of Rawls’ thought — worse endowments in terms of native talent do not necessarily imply lower lifetime earnings capacity (i.e. least-advantaged), because native endowments must first be realized through effort in education, which belongs to the private sphere of individual decisions.11 Most importantly, to the extent that individual decisions matter, the social contract — as defined behind the veil of ignorance — is not to be intended as merely redistributive but, also, the mechanism-design by which incentives to effort in education are determined.

In what follows, we model our interpretation of the Theory in such a way as to account for the implications of inequalities on individual decisions of effort

10“The progressive principle of taxation might not be applied to wealth and income for the purposes of raising funds (releasing resources to government), but solely to prevent accumulations of wealth that are judged to be inimical to background justice, for example, to the fair value of the political liberties and to fair equality of opportunity. It is possible that there need be no progressive income taxation at all” (Rawls 2001, p.161).

11“Even supposing that the least-advantaged ... include many individuals born into the least-favored social class of origin, and many of the least (naturally) endowed and many who experience more bad luck and misfortune, nevertheless those attributes do not define the least advantaged. Rather, it happens that there may be a tendency for such features to characterize many who belong to that group” (Rawls 2001, p.59).
According to the Difference Principle, inequalities are legitimate to the extent that they induce growth which is benefitting the least-advantaged; notably, to the extent that some inequalities may induce growth that is penalizing the least-advantaged, not all ‘growth-enhancing’ inequalities are admissible. This poses a precise limit on the maximum inequality admissible in the society which, in a way, evokes the ideal of pro-poor growth.

3 The Model: Basic Framework

In this section we discuss a simple analytical framework by which the Rawlsian theory - as revisited in the previous Section - is formalized. More precisely, the Difference Principle is modeled once, according to Rawls, conditions of Fair Equality of Opportunity are taken for granted for the entire population, meaning that, educational opportunities are assumed to have already been equalized for all individuals, so that the social class of origin can be omitted.

Given a population of two individuals, let $\theta_i, \theta_j \in \mathbb{R}^+$ be the native talent of the $i$th and $j$th individual respectively. According to Rawls theory, native endowments can be inferred ex-post only, in that this value is not measurable in itself ex-ante, and highly dependent on the design of social institutions revealing ex-post. As such, given a population of two individuals who are assumed to differ in terms of native talent, with $\theta_H > \theta_L$, two different states of the world are to be considered. Depending on social institutions, either $\theta_i = \theta_H$ and $\theta_j = \theta_L$, or $\theta_i = \theta_L$ and $\theta_j = \theta_H$. Most importantly, as we observed in the previous Section, behind the veil of ignorance the probability is intended in classical terms, so that the two states are equally probable.

Given the native talent, let $\Theta$ be the realized talent where, for the sake of simplicity, we assume

$$\Theta_i = e^i \theta_i$$

$$\Theta_j = e^j \theta_j$$

with $e$ indicating effort in education. For our purposes, $\Theta$ is assumed to indicate the money-value of the realized talent obtained at the working age, which might be though as a sort of individual productivity determined by education decisions and native talent.

Let $\ell$ be the individual lifetime earnings capacity indicating the primary goods an individual may potentially have access to. As we said in the previous Section, the lifetime earnings capacity is a potential value defined up to the entire time endowment, that is the same for all individuals by definition; formally, $\ell^i = \Theta^i T$ and $\ell^j = \Theta^j T$ with $T$ indicating the time endowment. Without loss of generality, the time endowment is normalized to $T = 1$, so that $\ell^i$ measures equivalently the lifetime earnings capacity and the wage rate of the $i$th individual.

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12 “Thus the principles of social justice are macro and not necessarily micro principles” (Rawls 1974).
In the absence of redistribution, the lifetime earnings capacity (or, equivalently, the wage rate) is fully determined by the realized talent, so that $\ell_i = \Theta_i$ and $\ell_j = \Theta_j$. Differently, when redistribution is allowed behind the veil of ignorance, given the budget constraint $(\ell_i + \ell_j) = (\Theta_i + \Theta_j)$, the lifetime earnings capacity of each individual is not anchored any longer to the corresponding realized talent.

In what follows, we focus on the case of a linear redistributive system, so that

$$\ell_i = \alpha + (1 - \beta)\Theta_i$$
$$\ell_j = \alpha + (1 - \beta)\Theta_j$$

(2)

where $\alpha > 0$ and $\beta \in [0, 1]$ identifies the scheme of wages (or, equivalently, scheme of social cooperation). Remarkably, to the extent that the budget constraint is required to hold, i.e. $(\ell_i + \ell_j) = (\Theta_i + \Theta_j)$, from (2) it must be the case that $\alpha = (\beta/2)(\Theta_i + \Theta_j)$, so that (2) can be equivalently rewritten as

$$\ell_i = \frac{\beta}{2} \Theta_j + \left(1 - \frac{\beta}{2}\right) \Theta_i$$
$$\ell_j = \frac{\beta}{2} \Theta_i + \left(1 - \frac{\beta}{2}\right) \Theta_j$$

(3)

where the higher is $\beta$, the greater is the contribution to the $i$th lifetime earnings capacity of the $j$th realized talent, and vice versa. Evidently, if $\beta = 0$ then the earnings capacity corresponds to the realized talent for each individual, whereas if $\beta = 1$ then the earnings capacity of each individual is equally distributed and equally determined by the realized talent of the members of the society.

Also, it is worth observing that the redistribution originating from the scheme of wages is ordering-preserving by construction, in that the identification of the least-advantaged individual, i.e. the sign of $(\ell_i - \ell_j) = (1 - \beta)(\Theta_i - \Theta_j)$, is independent of $\beta \in [0, 1]$.

Given our definitions of the realized talent and the lifetime earnings capacity (or, wage rate), let’s turn to the timing of the game. At time 0 (original position), the two (groups of) individuals define the scheme of wages — redistributing lifetime earnings capacity according to (3) — behind the veil of ignorance, i.e., without any information about their native talent and preferences. At time 1 (educational stage), individual preferences reveal for both individuals; at this stage, each individual is supposed to choose effort in education in such a way as to maximize the expected lifetime earnings capacity, given the scheme of wages proclaimed at the previous stage. Specifically, the expected lifetime earnings capacity is defined with respect to native talent which, according to Rawls, reveals at the working age only. At time 2 (working stage), the two individuals are assumed to differ from each other in terms of the mutually-exclusive native talents, $\theta_H > \theta_L$. As such, two different states of the world are to be considered at time 2; either the $i$th individual is associated to $\theta_H$ whereas the $j$th individual is of the type $\theta_L$, or vice versa. Most importantly, at time 1, the
probability is intended in classical terms, so that, as for preferences at time 0, the two states of the world are taken as equally probable.

Thus, the optimal scheme of wages can be defined by backward induction, in that the optimal social contract agreed at time 0 is expected to account for individual decisions on effort in education at stage 1 which, in turn, account for expectations on talent revealing at time 2. Notably, as far as preferences and talent reveal at different stages, it must be the case that individual responsibility for effort in education is contemplated behind the veil of ignorance, and, in line with Rawls’ idea, preferences and talent are not equally handled when determining the optimal social contract.

3.1 Educational stage

In this Section, we assume that individuals act rationally by choosing effort in education in such a way as to maximize their objective function, as defined in terms of utility. In contrast with the tradition of welfare-consequentialism, the notion of utility is merely indicative in this framework, as it is intended to measure command over resources, that is, the instrumental value of primary goods “that are generally necessary to enable citizens adequately to develop and fully exercise ... their determinate conceptions of the good” (Rawls 2001, p.57). Evidently, this is not to be confused with the intrinsic value of goods (e.g., happiness, betterness) that is characterizing the utilitarian tradition.

We consider a quasi-linear utility function which depends on (i) the expected lifetime earnings capacity,\(^{13}\) and (ii) the dis-utility\(^ {14}\) from effort in education.\(^ {15}\)

Let \(a^i, a^j \in [0,1]\) indicate the relative contribution of the lifetime earnings capacity to the overall utility of the \(i\)th and the \(j\)th individual respectively. We assume that individuals differ from each other in terms of such mutually-exclusive relative contribution, which we will refer to as “propensity to effort in education”. Thus, given \(a_H > a_L\), once the \(i\)th preferences have revealed, it must be the case that the \(j\)th preferences can be inferred by the \(i\)th individual, and vice versa (i.e., complete information). To simplify the formalization, we hypothesize \(a^i = a_H\) (so, \(a^j = a_L\)), as the opposite case implies perfectly symmetric definitions. As a result, the utilities of the two individuals are defined as follows

\[
U^i = a^i \left[ E(\ell^i) \right] + (1 - a^i) \left[ 1 - (e^i)^2 \right]
\]

\[
U^j = a^j \left[ E(\ell^j) \right] + (1 - a^j) \left[ 1 - (e^j)^2 \right] \quad (4)
\]

\(^{13}\)According to Rawls (2001, p.59), “... the inequalities to which the difference principle applies are differences in citizens’ (reasonable) expectations of primary goods over a complete life. These expectations are their life-prospects.”

\(^ {14}\)Alternatively, the dis-utility may be formalized as a resource cost for education included in the budget constraint of the utility maximization (Phelps 1973).

\(^ {15}\)Alternatively, the utility function might be defined with respect to the share of, not the level of, lifetime earnings capacity of each individual. This assumption would allow for strategic interactions between individual decisions on effort (i.e., Nash equilibrium) in such a way as to embody the open competition for better positions in the distribution of lifetime earnings capacity. However, while enriching the basic framework, strategic interactions would not alter the main results of this paper, so that we opted for keeping it as simple as possible.
where \((e^2)\) is the dis-utility from effort, which is assumed to be increasing and convex, whereas \(E(\ell)\) is the expected lifetime earnings capacity. Specifically, to the extent that possible states are taken as equally probable, the expected lifetime earnings capacity of the two individuals is defined with respect to the (mutually-exclusive) native talents revealing at time 2, i.e. \(\theta_H > \theta_L\), as follows,

\[
E(\ell^i) = \frac{1}{2} \ell_H^i(e^i, e^j, \theta^H, \theta^L, \beta) + \frac{1}{2} \ell_L^i(e^i, e^j, \theta^H, \theta^L, \beta) \\
E(\ell^j) = \frac{1}{2} \ell_L^j(e^i, e^j, \theta^H, \theta^L, \beta) + \frac{1}{2} \ell_H^j(e^i, e^j, \theta^H, \theta^L, \beta)
\]  
(5)

where \(\ell_k^i(\cdot), \ell_k^j(\cdot), k = H, L\), are, respectively, the \(i\)th and the \(j\)th state-contingent lifetime earnings capacities, as obtained by replacing (1) in (3) with \(\theta^i, \theta^j = \theta_k, k = H, L\) and \(\theta^i \neq \theta^j\).

Therefore, the optimal decisions of effort in education associated to each propensity to effort are

\[
e^*_H = \frac{a^H(2 - \beta)(\theta_H + \theta_L)}{8(1 - a^H)} \\
e^*_L = \frac{a^L(2 - \beta)(\theta_H + \theta_L)}{8(1 - a^L)}
\]  
(6)

Not surprisingly, optimal effort in education is decreasing with \(\beta\) for both preference types in that, from (1) and (2), if \(\beta\) increases, then the contribution of its own realized talent to its own lifetime earnings capacity decreases.

In addition, to the extent that optimal effort is chosen at time 1 depending on the expected lifetime earnings capacity which, in turn, is defined with respect to native talents revealing at time 2, it must be the case that \(\theta_H\) and \(\theta_L\) are both influencing the optimal decision of effort of each preference type.

Finally, as regards the comparison between the two preference types, the individual with a higher propensity to effort will always opt for a greater effort at time 1. Notice that, as far as the individual with the higher propensity to effort at time 1 does not necessarily correspond to the individual with the greater lifetime earnings capacity at time 2, i.e. the donor at time 2 according to (3), there is no incentive for the better preference type to conceal its propensity to effort (Musgrave 1974), i.e. to miss the opportunity to realize its native talent at time 1.

4 Original position

Given the optimal effort each preference type is willing to exert, the optimal scheme of wages \((\beta)\) can be defined by solving backward, i.e., behind the veil of ignorance.

Let the \(i\)th individual be the one endowed with the higher propensity to effort. To the extent that the two individuals are assumed to differ from each other in terms of native talents, i.e. \(\theta_H\) and \(\theta_L\) with \(\theta_H > \theta_L\), two different
states of the world are to be considered: either (i) the native talent of the
ith individual (with higher propensity to effort) reveals of type \( \theta_H \) (implying
j’s \( \theta_L \)-type) which we refer to as ‘concordant-state’, or (ii) the native talent
of the ith individual reveals of type \( \theta_L \) (implying j’s \( \theta_H \)-type) which we refer
to as ‘discordant-state’. As such, in the concordant-state the ith individual is
the “most-advantaged”, whereas the other individual is the “least-advantaged”. Differently, in the discordant-state, the least-advantaged cannot be identified a
priori as the individual with a better propensity to effort is the penalized one
in terms of native talent, and vice versa.

In the concordant-state, let \( \Theta_{HH} \) (resp. \( \Theta_{LL} \)) be the realized ability of
the ith (resp. jth) individual with higher (resp. lower) propensity to effort
and better (resp. worse) native talent, as obtained by replacing in (1) the
optimal effort from (6) with \( e_i = e_H, e_j = e_L, \theta_i = \theta_H, \theta_j = \theta_L \). Clearly,
\( \Theta_{HH} > \Theta_{LL} \). As such, let \( \ell_{HH} \) and \( \ell_{LL} \) be the state-contingent lifetime earnings
capacity obtained from \( \Theta_{HH} \) and \( \Theta_{LL} \) by implementing the scheme of wages in
(3) where, as observed in the previous Section, \( \Theta_{HH} > \Theta_{LL} \) implies \( \ell_{HH} > \ell_{LL} \)
(and vice versa). According to Rawls, if the discordant-state occurs, then the
least-advantaged individual is the LL-type, i.e., the individual with the worst
endowment in terms of both talent and propensity to effort.

In the discordant-state, let \( \Theta_{HL} \) (resp. \( \Theta_{LH} \)) be the realized ability of
the ith (resp. jth) individual with higher (resp. lower) propensity to effort
and worse (resp. better) native talent, as obtained by replacing in (1) the
optimal effort from (6) with \( e_i = e_H, e_j = e_L, \theta_i = \theta_L, \theta_j = \theta_H \). Also, let \( \ell_{HL} \) and \( \ell_{LH} \)
be the state-contingent lifetime earnings capacity obtained from \( \Theta_{HL} \) and \( \Theta_{LH} \)
as before. Here, the least-advantaged individual may be either the one endowed
with the lower propensity to effort but better native talent, i.e. \( \Theta_{HL} > \Theta_{LH} \),
or the one endowed with higher propensity to effort but worse native talent, i.e.
\( \Theta_{HL} < \Theta_{LH} \). Formally, in the discordant-state, the least-advantaged individual
is identified by the following (equivalence) condition.

\[
\Theta_{HL} \trianglerighteq \Theta_{LH} \iff a_H (1 - a_L) \theta_L \trianglerighteq a_L (1 - a_H) \theta_H
\]  

Condition (7) ‘defines’ the least-advantaged position behind the veil of igno-
rance when the discordant-state occurs, but, most importantly, it does not
‘identify’ the least-advantaged individual, whose identity will come to knowledge
at time 2 only.

Since two different and equally probable states (i.e., the concordant and
the discordant one) must be accounted for, the \( \beta^* \) that maximizes the lifetime
earnings capacity of the least-advantaged is inevitably state-contingent.

In Section 4.1, the two state-contingent optimal contracts, i.e. for the con-
cordant and discordant-state, are determined; each of them implies a distribution
of lifetime earnings capacity at time 2. In Section 4.2, given the state-contingent
distributions of lifetime earnings capacity, the (overall) optimal contract is
determined under uncertainty conditions, which is done by evoking the notion of
4.1 State-contingent optimal contracts

According to the definition of ‘ex-post k-efficiency’ (Harris 1978), an allocation is said to be efficient in state \( k \) if there is no feasible allocation such that, in state \( k \), the utility of an individual is increased without worsening the utility of another individual. Within the Rawlsian framework, the original notion of ‘ex-post k-efficiency’ must be revisited in such a way as to account for the sole lifetime earnings capacity of the least-advantaged. Specifically, according to the Difference Principle, for each state, the two optimal (state-contingent) contracts \( \beta_1^* \) and \( \beta_2^* \) are determined by maximizing, respectively, the lifetime earnings capacity of the least-advantaged individual, i.e., \( \ell_{LL} \) in the concordant state, either \( \ell_{LH} \) or \( \ell_{HL} \) in the discordant case depending on condition (7).

It is worth observing that any variation of the scheme of wages (\( \beta \)) generates two different effects on the two lifetime earnings capacities. On the one hand, according to (3), any increase of \( \beta \) implies a redistribution in terms of realized talent from the most to the least-advantaged type, meaning that \( \beta \) is a redistributive parameter (direct effect). On the other hand, \( \beta \) acts as a sort of wage-premium determining the dis-incentive to effort; specifically, from (6), if \( \beta \) increases then the relative contribution of the \( i \)th (\( j \)th) realized talent to its own lifetime income decreases, so that any individual is less willing to make high effort in education (indirect effect). In this sense, a dis-incentive effect is to be considered too.

Evidently, the redistributive and the dis-incentive effect are both reducing the lifetime earnings capacity of the most-advantaged, whereas a trade-off occurs in the case of the least-advantaged individual. For the latter, if the dis-incentive effect is dominating for all \( \beta \)'s, then the lifetime earnings capacity is strictly decreasing in \( \beta \), so that redistribution is never desirable \((\beta^* = 0)\). Differently, if the redistributive effect dominates, for some \( \beta \) at least, the dis-incentive effect of the least-advantaged, then the lifetime earnings capacity is increasing in \( \beta \) over this range, so that redistribution is desirable \((\beta^* > 0)\). These aspects are formalized for each state (i.e., concordant and discordant) in the two following Propositions.

Proposition 4.1 (Optimal social contract: concordant-state).

In the concordant-state,

\[ \text{\( \diamond \) if } (a_H \theta_H)(1 - a_L) \leq 2(a_L \theta_L)(1 - a_H) \text{ } \forall a_H, a_L, \theta_H, \theta_L, \text{ then the lifetime earnings capacity of the least-advantaged is strictly decreasing with respect to } \beta \in [0, 1], \text{ therefore } \beta^*_1 = 0 ; \]

\[ \text{\( \diamond \) if } (a_H \theta_H)(1 - a_L) > 2(a_L \theta_L)(1 - a_H) \text{ } \forall a_H, a_L, \theta_H, \theta_L, \text{ then the lifetime earnings capacity of the least-advantaged is } \cap\text{-shaped with respect to } \beta \in [0, 1], \text{ and } \]

\[ \beta^*_1 = \frac{a_H(a_L - 1)\theta_H - 2a_Ha_L\theta_L + 2a_L\theta_L}{a_H(\theta_H - a_Ha_L\theta_L + a_L\theta_L)} \]

Proof. See Appendix A.1.

\[ \Box \]
Differently, in the discordant-state, two different possibilities must be considered because the individual with the higher propensity to effort and worse native talent might be either the most- or the least-advantaged depending on condition (7). Specifically, we denote by $\beta^*_21$ the optimal contract when the individual with the better native talent is the least-advantaged, and by $\beta^*_22$ the optimal contract when the individual with the worse native talent is the least-advantaged.

**Proposition 4.2 (Optimal social contracts: discordant-state).**

In the discordant-state, given $a_H\theta_L(1-a_L) > a_L\theta_H(1-a_H)$,

1. if $a_H\theta_L(1-a_L) \leq 2a_L\theta_H(1-a_H) \forall a_H, a_L, \theta_H, \theta_L$, then the lifetime earnings capacity of the least-advantaged is strictly decreasing with respect to $\beta \in [0,1]$, therefore $\beta^*_21 = 0$;
2. if $a_H\theta_L(1-a_L) > 2a_L\theta_H(1-a_H) \forall a_H, a_L, \theta_H, \theta_L$, then the lifetime earnings capacity of the least-advantaged is $\cap$-shaped with respect to $\beta \in [0,1]$, and
   
   $$\beta^*_21 = \frac{2(a_H - 1)a_L\theta_H - a_H a_L\theta_L + a_H\theta_L}{(a_H - 1)a_L\theta_H - a_H a_L\theta_L + a_H\theta_L}$$

In the discordant-state, given $a_H\theta_L(1-a_L) < a_L\theta_H(1-a_H)$,

1. if $a_L\theta_H(1-a_H) \leq 2a_H\theta_L(1-a_L) \forall a_H, a_L, \theta_H, \theta_L$, then the lifetime earnings capacity of the least-advantaged is strictly decreasing with respect to $\beta \in [0,1]$, therefore $\beta^*_22 = 0$;
2. if $a_L\theta_H(1-a_H) > 2a_H\theta_L(1-a_L) \forall a_H, a_L, \theta_H, \theta_L$, then the lifetime earnings capacity of the least-advantaged is $\cap$-shaped with respect to $\beta \in [0,1]$, and
   
   $$\beta^*_22 = \frac{(a_H - 1)a_L\theta_H - 2a_H a_L\theta_L + 2a_H\theta_L}{(a_H - 1)a_L\theta_H - a_H a_L\theta_L + a_H\theta_L}$$

**Proof.** See Appendix A.2. \(\square\)

Propositions 4.1 and 4.2 show that, for redistribution to be desirable behind the veil of ignorance (i.e., $\beta^* > 0$), the lifetime earnings capacity of the least-advantaged must be $\cap$-shaped.

Basically, when the lifetime earnings capacity of the least-advantaged is $\cap$-shaped, redistribution is desirable at $\beta = 0$, so that $\beta$ is increased. However, when $\beta$ increases, the dis-incentive effect becomes stronger for both the least- and the most-advantaged individuals, so that the cake to be redistributed is reduced, and the redistributive effect jeopardized; evidently, the optimal state-contingent contract is obtained when the dis-incentive and redistributive effects perfectly compensate to each other at the margin for the least-advantaged.

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16Notice that, if $a_H(1-a_L)\theta_L = a_L(1-a_H)\theta_H$, then the two individuals are equally endowed in terms of realized talent (7), so that there is no inequality in terms of lifetime earnings capacity, but the two individuals may differ in terms of preferences and native talent; as far as, according to our interpretation of Rawls’ Theory, the sole inequalities in terms of the (endogenous) realized, not native, talent matter, this case is irrelevant.
Most importantly, it is worth observing that, once the optimal (state-contingent) social contract has been achieved, any additional increase in redistribution would not ameliorate the distribution of earnings capacity, proving that legitimate inequalities are clearly permitted in the Theory of Justice as Fairness.

On the other way around, this explains why, starting from a perfectly egalitarian social contract ($\beta = 1$), any marginal increase of inequality (i.e., diminishing $\beta$) induces higher effort of both individuals in such a way as to enhance their lifetime earnings capacity; most importantly, for the least-advantaged, the effect of the minor redistribution is initially more than compensated by the increasing cake due to greater incentive to effort for both, the most and the least-advantaged. As such, a marginal decrease of $\beta$ from $\beta = 1$ generates Pareto improvements (and so, economic growth) which are bought by both individuals. Subsequently, once the break-even point is achieved, for any additional increase of inequality, the redistributive effect becomes dominating for the least-advantaged individual, so that its lifetime earnings capacity decreases. From now on, any additional increase of inequality — even if growth enhancing — is not bought by the least-advantaged individual in that, growth is not of the pro-poor kind.

Finally, from the comparison between optimality conditions in Propositions 4.1 and 4.2, it is worth observing that, as shown in Appendix (A.2), (i) $\beta_1^* > \beta_{21}^*$ and (ii) $\beta_1^* > \beta_{22}^*$, meaning that, the greater is inequality of realized talent originating from endowments (preferences and native abilities), the more redistribution is expected to characterize the social contract.

4.2 Optimal contract under uncertainty conditions

In the previous Section, two state-contingent optimal contracts, for the concordant and the discordant-state respectively, have been identified. Specifically, it can be shown that, for any $a_H, a_L, \theta_H, \theta_L \in [0, 1]$, (i) $\beta_1^* = \beta_{21}^*$ if and only if $\theta_H = \theta_L$, whereas (ii) $\beta_1^* = \beta_{22}^*$ if and only if $a_H = a_L$. Intuitively, to the extent that $\beta_1^*$ and $\beta_{21}^*$ are both obtained when the least-advantaged corresponds to the individual with the lowest propensity to effort, it must be the case that the difference can be originating from the heterogeneity of native talent only. Similarly, when considering $\beta_1^*$ and $\beta_{22}^*$, the least-advantaged is characterized by the worse native talent, but different preferences.

As such, unless valid motivations are adduced by which one or the other state is neglected on a priori grounds, the optimal contract(s), which we denote by $\beta^*$, is to be defined under uncertainty conditions.

According to the existing literature (Starr 1973; Harris 1978; Hammond 1981), different approaches can be used to define efficiency under uncertainty conditions. Even if the debate between different optimality conditions in the presence of uncertainty conditions is not the object of our analysis, let’s recall the distinction made between ‘ex-ante efficiency’ and ‘universal ex-post efficiency’.

By the former, an allocation is said to be ex-ante efficient if there is no feasible allocation so that the expected utility (e.g., von Neumann-Morgenstern) of an individual can be enhanced without worsening the expected utility of another
individual. Differently, by the latter, an allocation is said to be universally ex-post efficient if there is no feasible allocation such that, for each possible state, the utility of an individual is increased without worsening the utility of another individual.

Consequently, by virtue of ex-ante efficiency, an ‘ex-ante Pareto improvement’ occurs if all individuals are indifferent, and at least one individual strictly prefers allocation \( x \) as compared to \( y \) in terms of expected utility. Instead, an ‘universal ex-post Pareto improvement’ is obtained when all individuals are indifferent in each state, and at least one individual in one state is better-off in \( x \) as compared to \( y \). Evidently, the universal ex-post approach is much more demanding than the ex-ante approach; however, the universal ex-post approach is the only one ensuring ex-post consistency of efficiency orderings, meaning that, if an allocation is strictly preferred under uncertainty conditions, then the same allocation is still preferred once the information has revealed.

Coming back to our model, to the extent that both individuals have access to the same (empty) information set at time 0 (i.e., behind the veil of ignorance), the ‘ex-ante efficiency’ approach would be a non-starter for egalitarianism, as both individuals would be clearly associated to the same expected lifetime earnings capacity as defined with respect to the four equally-probable and mutually-exclusive possible states (i.e., \( \ell_{HH}, \ell_{HL}, \ell_{LH}, \ell_{LL} \)).

The universal ex-post approach is definitely to be preferred for our purposes. By the latter, (state-contingent) lifetime earnings capacities are not aggregated across different states at the individual level. Instead, an ordering among different schemes is defined by comparing state-contingent distributions of lifetime earnings capacity with different degrees of inequality, which is the very scope of the Rawlsian ‘Difference Principle’.\(^{17}\)

In what follows, universal ex-post efficiency is implemented to characterize the optimal contract(s) under uncertainty conditions. Consistently with the Rawlsian framework, dominance conditions are applied to the distribution of lifetime earnings, not utilities.\(^{18}\) Two different formalizations of universal ex-post efficiency are considered. First, we implement the standard idea of ‘universal ex-post Pareto-efficiency’, by which optimality is defined by accounting for the lifetime earnings capacity of both, the most- and the least-advantaged individuals in the concordant and discordant-state. Next, since the bulk of the Theory of Justice as Fairness is aimed at improving the sole condition of the least-advantaged individual (maximin), ‘universal ex-post Rawls-efficiency’ is defined by focusing exclusively on the least-advantaged individual in the two states.

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\(^{17}\) Consider a situation in which an impending climate change will alter the distribution of well-being on Earth. Suppose that only two scenarios are considered possible. In one scenario, the extreme latitudes gain and the low latitudes suffer, whereas the reverse occurs in the other scenario (Fleurbaey 2010). Even if ex-post egalitarianism is inevitably jeopardized, the same climate change would be harmless in terms of expected utilities.

\(^{18}\) Under standard symmetry assumptions, if the utility (increasing) of each individual depends on its lifetime earnings only, then Pareto efficiency is equivalently defined with respect to the distributions of incomes and utilities (Amiel and Cowell 1994).
As it will be clearer in what follows, when moving from certainty to uncertainty conditions, universal ex-post Pareto-efficiency does not alter the nature of the Pareto dominance criterion, which is a partial ordering independently from uncertainty. On the contrary, the introduction of uncertainty sensibly modifies the Rawls criterion, which is a complete ordering under certainty conditions (in that \( \beta \) is uniquely defined in each state), but a partial ordering when uncertainty is accounted for. Most importantly, the set of optimal contracts, as obtained in terms of universal ex-post Rawls-efficiency, is shown to be a subset of the universally ex-post Pareto-optimal contracts, which makes the notion of Rawls-efficiency more suitable for policy purposes.

4.2.1 Universal ex-post Pareto-efficiency

From the previous Section, let’s consider the relationship between the lifetime earnings capacities of the two individuals in each of the two states, which we will refer to as the ‘state-contingent Rawls-efficiency frontiers’, where the \( j \)th individual is assumed to be the least-advantaged in both the concordant and the discordant-state (so, \( \beta_2^*_{21} \) and \( \beta_2^*_{22} \) are indicated by \( \beta_2^* \) hereafter).

To the extent that the lifetime earnings capacity of the most-advantaged is strictly decreasing with respect to \( \beta \), the lifetime earnings capacity of the least-advantaged can be defined with respect to the lifetime earnings capacity of the most-advantaged (see Appendix A.3). This is plotted in Fig. 1, where \( \beta \) is decreasing along the x-axis by construction. Evidently, if the lifetime earnings capacity of the least-advantaged individual is \( \cap \)-shaped with respect to \( \beta \), then the (state-contingent) Rawls-efficiency frontier must be \( \cap \)-shaped as well.\(^{19}\)

Differently, if the lifetime earnings capacity of the least-advantaged is strictly decreasing with \( \beta \), then the corresponding frontier must be positively sloped (Fig. 1). More precisely, to the extent that \( \beta_1^* \geq \beta_2^* \geq 0 \), if \( \beta_1^* = 0 \) then \( \beta_2^* = 0 \), not vice versa; equivalently, if the lifetime earnings capacity of the least-advantaged in the concordant-state is strictly decreasing with respect to \( \beta \), then it must be strictly decreasing in the discordant-state as well.

Since the sole index of primary goods — lifetime earnings capacity — matters in the Rawlsian framework, rank-dominance criteria apply (Saposnik 1981, Amiel and Cowell 1994), so that universal ex-post Pareto-efficiency is defined as follows.

**Definition 4.3 (Universal ex-post Pareto-efficiency).**

An allocation is said to be universally ex-post Pareto-optimal if there is no other feasible allocation by which the lifetime earnings capacity of one individual cannot be increased without worsening the lifetime earnings capacity of the other individual both in the concordant and the discordant-state.

Formally, let \( \beta^A \in [0,1] \) be the contract whose corresponding state-contingent distributions of lifetime earnings capacities are \( \bar{\ell}_1^A = \{\ell_{1H}^A, \ell_{1L}^A\} = \{\ell_{1L}, \ell_{1H}\} \)

\(^{19}\)In Fig 1, the maximum lifetime earnings capacity of the least-advantaged appears greater in the discordant case as compared to the concordant one; however this is not generalizable as the opposite result may occur as well.
Figure 1: Rawls-efficiency frontiers

(concordant-states), and $\ell_A^H = \{\ell_H^A, \ell_H^A\} = \{\ell_{HL}^A, \ell_{HL}^A\}$ (discordant-states), with the equivalence conditions holding by symmetry. As such, $\beta^A$ is the universally ex-post optimal scheme if there is no $\beta^B \neq \beta^A$ such that, together, (i) $\bar{\ell}_B^1$ is a Pareto improvement of $\bar{\ell}_A^1$, and (ii) $\bar{\ell}_B^2$ is a Pareto improvement of $\bar{\ell}_A^2$.

**Proposition 4.4 (Universally ex-post Pareto-optimality).**

The set of universally ex-post Pareto-optimal social contract is:

- $\exists \beta^* = 0$, if the lifetime earnings capacity of the least-advantaged individual is strictly decreasing with respect to $\beta$ in the concordant-state;
- $0 \leq \beta^* \leq \beta_1^*$, if the lifetime earnings capacity of the least-advantaged individual is $\cap$-shaped in the concordant-state, whatever the discordant-state.

**Proof.** Straightforward from Appendix A.1, A.2, and A.3.

Proposition 4.4 highlights that, if the Rawls-efficiency frontier is strictly increasing in the concordant-state (which implies a strictly increasing frontier in the discordant-state as well), then $\beta^* = 0$; that is, by reducing $\beta$ (i.e., by moving to the right on the $x$-axis in Fig. 1), it must be the case that both individuals are made better off, whatever the state, until $\beta^* = 0$ is attained.

Instead, if the frontier is strictly increasing in the discordant-state only (so, $\cap$-shaped in the concordant-state), then, by reducing $\beta$, individuals are made better off in both states until $\beta_1^*$ is achieved; this is sufficient to exclude optimality of the $\beta$’s in the interval $[\beta_1^*, 1]$. On the contrary, once $\beta_1^*$ is achieved, by moving further to the right on the $x$-axis, i.e. increasing the lifetime earnings
capacity of the most-advantaged, it must be the case that there exists at least one state, that is the concordant-state, by which the least-advantaged individual is made worse off. To the extent that universal ex-post Pareto improvements are not attainable any longer, all $\beta$’s in $[0, \beta^*_1]$ are universally ex-post optimal.

Finally, if the two Rawls-efficiency frontiers are both $\cap$-shaped like in Fig. 1, all social contracts such that $\beta^*_1 < \beta < 1$ (left-side in Fig. 1) cannot be optimal in that, as before, the lifetime earnings capacity of both individuals can be increased by switching to $\beta^*_1$. Instead, for all $\beta$’s such that $0 < \beta < \beta^*_1$ (right-side in Fig. 1), optimality holds as there are no alternative schemes by which an universal ex-post Pareto improvement can be attained; by reducing $\beta$ from $\beta^*_1$, i.e. increasing the lifetime earnings capacity of the most-advantaged, there exists at least one state — that is the concordant state — by which the least-advantaged individual is made worse off.

4.2.2 Universal ex-post Rawls-efficiency

Universal ex-post Pareto-optimality is supposed to account for the lifetime earnings of both, the most- and the least-advantaged individual, in a way that resembles the idea of Pareto-dominance. However, as for Propositions 4.1 and 4.2, one may argue that the bulk of the Theory of Justice as Fairness is aimed at improving the sole condition of the least-advantaged individual (maximin). In this sense, universal ex-post Pareto-optimality, as defined in Proposition 4.4, may be weakened according to the maximin principle by focusing exclusively on the least-advantaged individual as follows.

**Definition 4.5 (Universal ex-post Rawls-efficiency).**

An allocation is said to be universally ex-post Rawls-optimal if there is no other feasible allocation by which the lifetime earnings capacity of the least-advantaged individual is increased in one state without reducing in the other state.

In this view, the definition of the optimal social contract becomes less stringent as compared to the standard universal ex-post Pareto-efficiency. The following Proposition identifies, according to Definition 4.5, the intervals the optimal scheme must belong to, depending on the shape of the (state-contingent) Rawls-efficiency frontier.

**Proposition 4.6 (Universally ex-post Rawls-optimality.).**

The set of universally ex-post Rawls-optimal social contract is:

- $\beta^* = 0$, if the lifetime income prospect of the least-advantaged individual is strictly decreasing with respect to $\beta$ in the concordant-state;
- $0 \leq \beta^* \leq \beta^*_1$, if the lifetime income prospect of the least-advantaged individual is strictly decreasing with respect to $\beta$ in the discordant-state but $\cap$-shaped in the concordant-state;
- $\beta^*_2 \leq \beta^* \leq \beta^*_1$, if the lifetime income prospect of the least-advantaged individual is $\cap$-shaped in both the concordant and the discordant-state.
Proof. Straightforward from Appendix A.1, A.2, and A.3.

Although the $\beta^* = 0$ solution is the same as in Proposition 4.4, the other possibility (i.e., $\beta^* > 0$) is now more articulated in that, two different possibilities are to be conceived. More precisely, if the Rawls-efficiency frontier is $\cap$-shaped in the concordant state, and strictly increasing in the discordant-state, then it must be the case that all contracts such that $\beta \in [\beta_1^*, 1]$ can be ameliorated according to Definition 4.5 by opting for $\beta_1^*$. Moving further to the right from $\beta = \beta_1^*$, to the extent that the frontier is $\cap$-shaped in the concordant-state, there is no alternative contract by which the lifetime earnings capacity of the least-advantaged is increased independently from the state; this is similar to the result obtained in Proposition 4.4.

If both frontiers are $\cap$-shaped, then contracts in the interval $\beta \in [\beta_1^*, 1]$, as before, cannot be optimal. However, in contrast with Proposition 4.4, the rest of the contracts are not necessarily optimal any longer because, for all contracts such that $\beta > \beta_2^*$, the lifetime earnings capacity of the least-advantaged increases independently from the state. Consequentially, the sole contracts such that $\beta \in [\beta_2^*, \beta_1^*]$ are universally ex-post Rawls-optimal. Evidently, as compared to Proposition 4.4, universally ex-post Rawls-optimal social contracts are a subset of the more general universal ex-post case.

From Definition 4.5, partial justice orderings\(^{20}\) can be derived accordingly. Formally, let $\ell_j^L(B)$, $\ell_j^H(B)$, $\ell_j^L(A)$ and $\ell_j^H(A)$ be the state-contingent lifetime earnings capacity of the $(j\text{th})$ least-advantaged individual as obtained when the contracts $\beta^A$ and $\beta^B$ are considered, with the subscripts LL and LH referring to the concordant and the discordant-state respectively. Also, let $\beta^B \succ \beta^A$ indicate that $\beta^B$ is strictly preferred to $\beta^A$, with $\sim$ indicating the symmetric component of the justice ordering, whereas $\beta^B \parallel \beta^A$ signifies that $\beta^B$ and $\beta^A$ are non-comparable.

According to Definition 4.5, $\ell_j^L(B) \gtrless \ell_j^L(A)$, $\ell_j^H(B) \gtrless \ell_j^H(A) \iff \beta^B \gtrsim \beta^A$; $\beta^B \parallel \beta^A$ otherwise.

Basically, for an ‘universal ex-post Rawls improvement’ to occur, a contract must be enhancing the lifetime earnings capacity of the least-advantaged in both, the concordant and the discordant-state.

Two observations are required concerning, respectively, the relation between Rawls improvements and Rawls-optimality, and the comparison between Rawls improvements and Pareto improvements. First, the optimality of a contract does not imply that this is to be preferred to a non-optimal one; indeed, universal ex-post Rawls-optimality is neither a necessary, nor a sufficient condition for the universal ex-post Rawls improvement to occur.\(^{21}\)

\(^{20}\)Rawls expressly refers to justice orderings, not individual or social welfare ones, where different levels of justice are said to “represent how claims to goods cooperatively produced are to be shared among those who produced them, and they reflect an idea of reciprocity” (Rawls 2001, p.62).

\(^{21}\)Clearly, it is not necessary because $\beta^B \succ \beta^A$ may occur even if $\beta^B, \beta^A \not\in [\beta_2^*, \beta_1^*]$. In addition, sufficiency does not hold because the optimality of $\beta^B$ (i.e., $\beta^B \in [\beta_2^*, \beta_1^*]$) and
Second, and most importantly, when considering optimal contracts, universal ex-post Rawls-efficiency is shown to imply universal ex-post Pareto-efficiency, not vice versa. However, as regards universal ex-post Pareto and Rawls improvements, the two criteria are shown to be equivalent if, and only if, the attention is restricted to the sole contracts ensuring economic growth \((\beta^*_1, 1)\); specifically, universal ex-post Pareto improvements imply growth, whereas universal ex-post Rawls improvements might be obtained in the presence of negative growth as well \((0, \beta^*_2)\). \(^{22}\)

5 Concluding Remarks: What’s New?

In the existing literature, Rawls’ Theory is usually evoked to underpin infinite aversion to inequality in social welfare analysis. Starting from Alexander (1973), the Rawls’ maximin criterion has been usually represented by Leontief preferences to rank utility distributions originating from a fixed (exogenous) amount of resources (e.g., income).

In this paper, according to Rawls’ Theory, the sole inequalities of primary goods, not utility, are considered. Specifically, we refer to the (state-contingent) distributions of lifetime earnings capacity, with the latter indicating the index of primary goods associated to each individual at the working age. In addition, the lifetime earnings capacity is assumed to be co-determined by both native talent and effort in education where, differently from the common understanding of Rawls’ Theory, effort is assumed to be endogenously determined from individual preferences (or, ambitions) characterizing (i) the propensity to, or (ii) the cost of, effort in education. As such, in our model, preferences capture the instrumental value of lifetime earnings capacity (i.e., command over resources), and not its intrinsic value in terms of some notion of betterness.

In addition, to the extent that the overall time endowment — equal for all by definition — is said to be a primary good on its own, in our framework, inequalities in terms of lifetime earnings capacity are independent from the leisure/effort decision in the labor market, meaning that, in contrast with the old tradition of welfare-consequentialism and according to background procedural justice, effective income distributions are totally irrelevant within the Rawlsian perspective we propose.

the non-optimality of \(\beta^A\) (i.e., \(\beta^A \not\in [\beta^*_2, \beta^*_1]\)) do not necessarily imply \(\beta^B \succ \beta^A\); e.g., let’s suppose that (i) \(\beta^B \in [\beta^*_2, \beta^*_1]\) and (ii) \(\beta^A \in [0, \beta^*_2]\). By (i) and (ii), it must be the case that \(\ell^*_L(B) > \ell^*_L(A)\), meaning that, in the concordant-state, the lifetime earnings capacity of the least-advantaged is higher when \(\beta^B\) is implemented. However, if \(\ell^*_L(H)(B) < \ell^*_L(H)(A)\), then \(\beta^A\) is to be preferred in the discordant-state. To the extent that the two schemes of wages are differently ranked depending from the state, by definition of ‘universal ex-post Rawls improvement’, it must be the case that \(\beta^B\) and \(\beta^A\) are not comparable (i.e., \(\beta^B || \beta^A\)) in the case above.

\(^{22}\)Quoting Rawls (2001, p.63), “A further feature of the difference principle is that it does not require continual economic growth over generations to maximize upward indefinitely the expectations of the least advantaged (assessed in terms of income and wealth). That would not be a reasonable conception of justice”.

21
Remarkably, according to our interpretation of the Theory, we assume that information on effort in education and native talent are progressively relaxed over time. With this purpose in mind, we consider a three-stages sequential timing consisting of the original position (or, veil of ignorance), the educational stage, and the working stage. Since preferences reveal at the educational stage, whereas native talent, as influenced by the shape of social institutions, reveals at the working stage only, it must be the case that, by backward induction, responsibility for individual decisions is automatically accounted for when determining the optimal social contract behind the veil of ignorance.

In addition, as far as native talent reveals ex-post, the identification of the least-advantaged is possible at the working stage only. This implies that, in our model, the least-advantaged does not necessarily correspond to the individual with the worse native talent, because the better endowment in terms of native talent might be more than compensated by the worse endowment in terms of propensity to effort. To the extent that the better (worse) endowed in terms of native talent might be either the better, or the worse endowed in terms of propensity to effort, two different states of the world, i.e., the concordant and the discordant-state, must be accounted for. As such, we model Rawls’ Theory under uncertainty conditions.

In social welfare analysis, the presence of uncertainty conditions is known to characterize Harsanyi’s (1953, 1955) Impartial Observer as well. However, we argue that the two frameworks strongly differ from each other with respect to their ultimate end. Rawls’ veil of ignorance is aimed at the definition of an agreement (social contractualism) between free and equal persons concerning the identification of legitimate inequalities, whereas Harsanyi’s ignorance is used to obtain an impartial definition of social welfare in terms of betterness. As such, behind the veil of ignorance, Rawls’ souls are supposed to assess inequalities by viewing themselves as potential occupants of each position in a distribution, independently from the identity and preferences of each individual (Saposnik 1981), whereas Harsanyi’s “impersonality requires that the observer have an equal chance of being put in the place of any individual member of the society, with regard not only to his objective social (and economic) conditions, but also to his subjective attitudes and tastes” (Mongin 2001). Last but not least, to the extent that inequality, not social welfare, is indicated as the object of Rawls’ Theory, the uncertainty behind the Rawlsian veil of ignorance does not concern exclusively the individual position within a distribution, but, mostly, the possibility of alternative distributions (i.e. concordant and discordant-state) with different degrees of inequality. In this scenario, the notion of expected (von Neumann-Morgenstern) utility, which is essential in Harsanyi’s Theory, is a non-starting for Rawlsian uncertainty, as it would obscure the inequality of state-contingent distributions which, instead, is captured by the notion of universal ex-post efficiency.

Given the uncertainty conditions above, we draw a separating line between state-contingent and overall optimality of the social contract (respectively, Sec-

\footnote{For details on this distinction see Hampton (1980).}
tion 4.1 and 4.2). Within each state, the state-contingent contract yields two effects, the dis-incentive and the redistributive effect, which are shown to be conflicting to each other for the least-advantaged individual. As such, redistribution, within each state (concordant or discordant), is found to be desirable if and only if there exists at least a state-contingent contract such that the redistributive effect over-compensates the dis-incentive effect for the least-advantaged. If this is the case, then redistribution is desirable until the reduction of the cake — induced by the dis-incentive effect on both the most- and the least-advantaged — is not so strong to jeopardize the redistributive effect. In this sense, we suggest that Rawls’ contribution goes well beyond distributive justice in such a way as to strain into the existing literature on the equity-efficiency trade-off, where the distribution of effort is not taken as given (Mirrlees 1971, Phelps 1973, Stiglitz 1987); specifically, in line with Phelps (1973), redistribution is permitted until the income/utility of the least-advantaged individual is maximized.

As a major departure from this literature, we introduce a dynamic setting where the social contract is decided behind the veil of ignorance, whereas optimal effort in education is decided at the next stage, when the native talent has not revealed yet. This difference is substantial. In our model, the identity of the least-advantaged is unknown at the time of the social contract, so that the maximin principle can be unanimously ‘agreed’ by both, the most- and the least-advantaged individuals behind the veil of ignorance. Differently, in the standard literature on the equity-efficiency trade-off, as far as the identity of the least-advantaged is known at the time of the contract, the maximin can be only ‘imposed’ to the most-advantaged. As such, we also offer a normative framework by which (social) contractualism behind the veil of ignorance is rigorously formalized in such a way as to ensure the stability of political institutions.\textsuperscript{24}

Finally, as compared to the common understanding of Rawls’ Theory, we emphasize that Rawls’ contractualism automatically involves uncertainty conditions in such a way as to jeopardize the possibility of a complete ordering in terms of social justice. More specifically, if redistribution is desirable in the concordant-state only, then Rawls optimality implies Pareto optimality, and vice versa. Differently, when redistribution is desirable in both, the concordant and the discordant-state, Rawls optimality implies Pareto optimality, not the converse. In this sense, we show that the set of Rawls-optimal contracts is a subset of Pareto-optimal ones, so that, in our view, ‘Justice as Fairness’ may represent a reasonable starting-gate for the refinement of Pareto-optimality. In addition, as regards Pareto and Rawls improvements, it is worth observing that, while Pareto improvements necessarily imply economic growth, Rawls improve-

\textsuperscript{24}As compared to Phelps (1973), where individuals are assumed to differ from each other with respect to native talent only, in our model individuals also differ in terms of preferences. As such, in our model the possibility of a discordant-state automatically implies uncertainty with respect to the identification of the least-advantaged, whereas this possibility is not conceived in the existing literature. Also, in Phelps (1973), taxation applies to the effective income realized in the labor market which is defined as a function of native talent and effort in education. In our model, instead, native talent and effort in education determine the earnings capacity, i.e. the wage rate, of the individual, which, to the extent that the entire time endowment is regarded as primary good, corresponds to potential, not effective, income.
ments may be conceived in the presence of negative growth as well. However, when redistribution is desirable in the concordant-state only, Rawls and Pareto improvements are shown to be equivalent to each other.

\footnote{Notably, in our model Pareto improvements are conceived in the weak form only; as such, the well know conflict between strong Pareto improvements and the maximin principle is irrelevant in our framework.}
Appendix

A.1: Proof of Proposition 4.1

By replacing (1) and (6) into (3), the lifetime earnings capacity in the concordant-state of the HH- and LL-type are, respectively,

\[\ell_{HH} = -\frac{(\beta - 2)(\theta_H + \theta_L)(a_H(a_L - 1)(\beta - 2)\theta_H - a_Ha_L\beta\theta_L + a_L\beta\theta_L)}{16(a_H - 1)(a_L - 1)}\]

\[\ell_{LL} = \frac{1}{16} (\beta - 2)(\theta_H + \theta_L) \left( \frac{a_H\beta\theta_H}{a_H - 1} - \frac{a_L(\beta - 2)\theta_L}{a_L - 1} \right)\]

where \(\ell_{HH} > \ell_{LL}\) by construction. From the first-order condition

\[\frac{\partial \ell_{HH}}{\partial \beta} = 0 \rightarrow \beta = \frac{2a_H\theta_H(1 - a_L) - a_L\theta_L(1 - a_H)}{a_H\theta_H(1 - a_L) - a_L\theta_L(1 - a_H)} > 1\]

In addition,

\[\frac{\partial^2 \ell_{HH}}{\partial^2 \beta} = -\frac{(\theta_H + \theta_L)(a_H(a_L - 1)\theta_H - a_Ha_L\theta_L + a_L\theta_L)}{8(a_H - 1)(a_L - 1)} > 0\]

This proves that \(\beta > 1\) is a minimum, so that \(\ell_{HH}\) is always decreasing in \(\beta \in [0, 1]\).

As regards the least-advantaged,

\[\frac{\partial \ell_{LL}}{\partial \beta} = 0 \rightarrow \beta = \frac{a_H\theta_H(1 - a_L) - 2a_L\theta_L(1 - a_H)}{a_H\theta_H(1 - a_L) - a_L\theta_L(1 - a_H)} \tag{8}\]

where it can be shown that \(\beta \in [0, 1]\) if \(\frac{a_H\theta_H}{a_H - a_L} \geq \frac{2(a_L\theta_L)}{1 - a_L}\), otherwise \(\beta < 0\). In addition,

\[\frac{\partial^2 \ell_{LL}}{\partial^2 \beta} = \frac{(\theta_H + \theta_L)(a_H(a_L - 1)\theta_H - a_Ha_L\theta_L + a_L\theta_L)}{8(a_H - 1)(a_L - 1)} < 0\]

This proves that, if \(\frac{a_H\theta_H}{a_H - a_L} \leq \frac{2(a_L\theta_L)}{1 - a_L}\), then \(\beta\) is a negative maximum, which implies that \(\ell_{LL}\) is always decreasing in \(\beta \in [0, 1]\). Otherwise, \(\frac{a_H\theta_H}{a_H - a_L} > \frac{2(a_L\theta_L)}{1 - a_L}\), so that \(\ell_{LL}\) is \(\cap\)-shaped in \(\beta \in [0, 1]\), with \(\beta^*_L\) identified by (8).

A.2: Proof of Proposition 4.2

By replacing (1) and (6) into (3), the lifetime earnings capacity in the discordant-state of the HL- and LH-type are, respectively,

\[\ell_{HL} = \frac{1}{16} (\beta - 2)(\theta_H + \theta_L) \left( \frac{a_H\beta\theta_H}{a_H - 1} - \frac{a_H(\beta - 2)\theta_L}{a_H - 1} \right)\]

\[\ell_{LH} = -\frac{(\beta - 2)(\theta_H + \theta_L)((a_H - 1)a_L(\beta - 2)\theta_H - a_Ha_L\beta\theta_L + a_H\beta\theta_L)}{16(a_H - 1)(a_L - 1)}\]

where, depending on conditions (7), two different cases must be considered, i.e., either \(\ell_{HL} > \ell_{LH}\) or \(\ell_{LH} > \ell_{HL}\).
In the first scenario with \( \ell_{HL} > \ell_{LH} \), from the first-order condition
\[
\frac{\partial \ell_{HL}}{\partial \beta} = 0 \rightarrow \beta = \frac{(a_H - 1)a_L \theta_H - 2a_H a_L \theta_L + 2a_H \theta_L}{(a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L} > 1
\]
In addition,
\[
\frac{\partial^2 \ell_{HL}}{\partial^2 \beta} = \frac{(\theta_H + \theta_L)((a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L)}{8(a_H - 1)(a_L - 1)} > 0
\]
This proves that \( \beta > 1 \) is a minimum, so that \( \ell_{HL} \) is always decreasing in \( \beta \in [0, 1] \).

As regards the least-advantaged,
\[
\frac{\partial \ell_{LH}}{\partial \beta} = 0 \rightarrow \beta = \frac{2(a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L}{(a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L} \tag{9}
\]
where it can be shown that \( \beta \in [0, 1] \) iff \( \frac{a_H \theta_L}{1-a_H} > \frac{2(a_H \theta_H)}{1-a_L} \), otherwise \( \beta < 0 \). In addition,
\[
\frac{\partial^2 \ell_{LH}}{\partial^2 \beta} = -\frac{(\theta_H + \theta_L)((a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L)}{8(a_H - 1)(a_L - 1)} < 0
\]
This proves that, if \( \frac{a_H \theta_L}{1-a_H} \leq \frac{2(a_H \theta_H)}{1-a_L} \), then \( \beta \) is a negative maximum, which implies that \( \ell_{LH} \) is always decreasing in \( \beta \in [0, 1] \). Otherwise, \( \frac{a_H \theta_L}{1-a_H} > \frac{2(a_H \theta_H)}{1-a_L} \), so that \( \ell_{LH} \) is \( \cap \)-shaped in \( \beta \in [0, 1] \), and \( \beta^{21}_{\cap} \) is identified by (9).

Let \( \beta^*_1 \) be the optimal state-contingent contract in the concordant state when \( \ell_{HL} \) is \( \cap \)-shaped in \( \beta \in [0, 1] \), and let \( \beta^*_2 \) be the optimal state-contingent contract in the discordant state when \( \ell_{LH} \) is \( \cap \)-shaped in \( \beta \in [0, 1] \), then
\[
\beta^*_1 - \beta^*_2 = \frac{a_H a_L (1-a_H)(1-a_L)(\theta_H^2 - \theta_L^2)}{(a_L(1-a_H)(\theta_H - \theta_L))(a_H \theta_H (1-a_L) - a_L \theta_L(1-a_H))}
\]
which can be shown to be always positive, i.e., \( \beta^*_1 > \beta^*_2 \); \forall a_H, a_L, \theta_H, \theta_L.

In the second scenario with \( \ell_{HL} < \ell_{LH} \), from the first- and second-order conditions above, it must be the case that \( \ell_{LH} \) is always decreasing in \( \beta \in [0, 1] \), whereas \( \ell_{HL} \) is \( \cap \)-shaped in \( \beta \in [0, 1] \) for \( \frac{a_H \theta_L}{1-a_H} > \frac{2(a_H \theta_H)}{1-a_L} \) with
\[
\frac{\partial \ell_{HL}}{\partial \beta} = 0 \rightarrow \beta = \frac{(a_H - 1)a_L \theta_H - 2a_H a_L \theta_L + 2a_H \theta_L}{(a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L} \tag{10}
\]
otherwise it is always decreasing in \( \beta \in [0, 1] \). Let \( \beta^*_{22} \), as indicated in (10), be the optimal state-contingent contract in the discordant state when \( \ell_{HL} \) is \( \cap \)-shaped in \( \beta \in [0, 1] \), then
\[
\beta^*_1 - \beta^*_{22} = \frac{\theta_H \theta_L ((a_H - a_L)(a_H + a_L - 2a_H a_L))}{((a_H - 1)a_L \theta_H - a_H a_L \theta_L + a_H \theta_L)(a_H(a_L - 1)\theta_H - a_H a_L \theta_L + a_H \theta_L)}
\]
which is positive when \( a_H(1 - 2a_L) > -a_L \). Clearly, this condition is always satisfied for all \( a_L \in [0, 1/2] \). In addition, for all \( a_L \in [1/2, 1] \), it holds if \( a_H < \frac{a_L}{2a_L - 1} \), where \( a_L \geq 2a_L - 1 \forall a_L \leq 1 \) implies that \( a_H \leq 1 \leq \frac{a_L}{2a_L - 1} \). This proves that \( a_H(1 - 2a_L) > -a_L \) is always satisfied \( \forall a_L < 1 \), so that \( \beta_1^* > \beta_2^* \forall a_H, a_L, \theta_H, \theta_L \).

**A.3: State-contingent Rawls-efficiency frontier**

In what follows, we construct the Rawls-efficiency frontier for the concordant-state. For the sake of brevity, the same procedure is omitted for the discordant state.

Recall from Appendix A.1 that \( \ell_{HH} \) is strictly decreasing in \( \beta \in [0, 1] \). By considering its inverse function, and by replacing \( \beta(\ell_{HH}) \) in the lifetime earnings capacity of the least advantaged, the Rawls-efficiency frontier is obtained for the concordant-state, i.e.,

\[
\ell_{LL} = \frac{(a_L (\Gamma[\cdot] + \theta_H \theta_L + \theta_L^2) - \Gamma[\cdot])}{16(1 - a_L)(\theta_H + \theta_L)(a_H \theta_H (1 - a_L) - a_L \theta_L (1 - a_H))}
\]

\[
(a_H (a_L \Gamma[\cdot] - \Gamma[\cdot]) + 2(a_L - 1)\theta_H^2 + (3a_L - 2)\theta_H \theta_L + a_L \theta_L^2) +
\]

\[
- a_L \Gamma[\cdot] + \Gamma[\cdot] - a_L \theta_H \theta_L - a_L \theta_L^2
\]

where

\[
\Gamma[a_H, a_L, \theta_H, \theta_L, \ell_{HH}] = 
\sqrt{(\theta_H + \theta_L)(a_H - 1)(\theta_H + \theta_L)a_L^2 \theta_L^2 - 16 \ell_{HH}(a_L - 1)(a_L \theta_L (1 - a_H) + a_H \theta_H (a_L - 1)))}
\]

\[
(a_H - 1)(a_L - 1)^2
\]

**References**


