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No. 465

Should government smooth exchange rate risk?

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SHOULD GOVERNMENT SMOOTH EXCHANGE RATE RISK?

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Should Government Smooth Exchange Rate Risk?\textsuperscript{1}

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A general equilibrium model is built to explain if there are circumstances in which exchange rate risk smoothing (ERRS) policies may bring a Pareto-improvement for a indebted small open (home) economy. The model shows that this is the case when overpessimistic foreign creditors demand a large spread on the default risk-free world interest rate, whose size can be reduced by ERRS policies and, in addition, market imperfections, such as information asymmetry between foreign investors and domestic debtors, prevent home economy’s residents from internalizing all benefits and costs of the exchange rate risk reallocation into their allocative decisions.
1 Introduction

In modern times pure floating regimes are a rare phenomena. Governments tend to smooth exchange rate fluctuations to different degrees, some act in a systematic manner, others only in very extreme situations, but all intervene directly or indirectly at some point in time. In some respects, this is counter-intuitive. It is well known that exchange rate is an essential price in open economies. The movements in real exchange rates signal consumers and producers the relative scarcity of tradable goods and guarantee that the current account reacts appropriately to shocks in order to maintain international solvency\(^1\). So the question is why countries do not adopt extreme pure floating? Is there any rational justification to smooth exchange rate risk?

The public budget constraint implies that exchange rate risk smoothing (ERRS) policies amount to a reallocation of the exchange rate risk exposure across the home economy. However, if they are efficient, in the sense that they bring a Pareto-improvement for this economy, why don’t competitive markets signal the correct incentives for private agents to trade their risk exposures efficiently? We show in the paper that, under full information and perfect competitive markets, it is hard to understand the reason for intervention since the risk inherent to any source of uncertainty must be efficiently reallocated across market participants. As a consequence, Pareto-improving interventions are possible only if some market failure prevents private agents from internalizing all social benefits and costs of the risk reallocation into their allocative decisions. This question is mainly relevant for many emerging markets economies with a well developed financial market, for which the non-existence of market mechanisms can not be used as a ground for public intervention.

This paper takes seriously the questions raised above and builds a general equilibrium model to explain how market imperfections, such as information asymmetry between foreign investors and home debtors, along with other conditions, could lead ERRS policies to bring a Pareto-improvement for a small open economy. More specifically, the model shows that this may arise when the home economy is paying a spread over the default risk-free world interest rate - due to the fact that foreign investors are over pessimistic about repayment - and in addition this spread falls as a result of ERRS policies. In this case, as a result of the lower debt cost, the home economy must export less to finance its capital account deficit, increasing in this way the supply of tradable goods for the domestic market. Therefore, not only the tradable sector wealth and welfare increase, but also the nontradable sector takes advantage of a higher relative price for its

\(^{1}\)Without full price flexibility real exchange rate tend to move closely to nominal exchange rates.
Market imperfections, along with a spread whose size can be shrunk by ERRS policies, are necessary, but not sufficient, conditions for these policies to be Pareto-improving interventions, since they amount to a risk exposure reallocation across home economy’s sectors. ERRS policies lead to a Pareto-improvement only if the welfare loss of the sector having its risk exposure increased is lower than the welfare gain provided by the fall in the spread. Alternatively, these policies will not be socially optimal if they do not cause a large enough reduction in the spread to compensate the sector with a higher risk exposure. In particular, if foreign investors are not so pessimistic to demand a spread, there is no scope for Pareto improvement, since the interest rate that debtors pay is already at its lowest level.

In order ERRS policies to affect the contractual interest rate on the tradable sector’s foreign debt, it is essential that both foreign credit demand and supply curves depend on the wealth volatility of the borrowers, which in turn depends to some extent on its exposure to exchange rate shocks. A lower wealth volatility impacts not only on the default probability but also on the willingness to transfer wealth to present. The net effect on the debt cost depends on the relative strength of these effects.

In our model the spread is paid because foreign investors are relatively more pessimistic than home debtors about the ability of the latter to repay. For example, consider the particular case in which debtors have incentive to repay in all states of nature, but foreign investors do not believe that repayment will occur in the worst states and hence they require a spread. In this case, it is easy to see that the higher debt cost necessarily leads to a welfare loss since debtors will repay for sure with or without spread. Therefore, any public intervention capable of reducing this spread, such as ERRS policies, may bring a Pareto-improvement. In this sense, an important question is whether heterogeneous beliefs about default probability is an empirically relevant assumption, or better, in which circumstances this is more likely to be observed. As suggested by Calvo and Mendoza (2000a,b), this assumption seems to be consistent with the fact of that financial globalization in a context of institutional constraints, such as short-selling restrictions, reduces the incentives for market participants to collect costly country-specific information, so that informational-based herd behavior is more likely to occur in international financial market. This in turn promotes and exacerbates contagion in financial crisis experienced by emerging market economies, in that foreign investors get overpessimistic about economies not fundamentally related to the ones triggering the crisis.

Even in the favorable case for ERRS policies, an important question is still to be answered.
Given that home country’s residents could trade privately their risk exposure, why do they fail to internalize the welfare effect of a lower debt cost into their allocative decisions? The model assumes that foreign investors are imperfectly informed about the individual portfolio composition of each debtor. More specifically, only the aggregate exchange rate risk exposure of each home economy’s sector can be directly observed by foreign investors, so that they are not able to monitor the individual risk exposure of each debtor separately. As a result, if an individual debtor decides to buy more hedge against exchange rate shocks, she is not able to take full advantage of the impact of this decision on the spread she pays, since her sector as a whole can free ride on her. In this sense, the benefit in terms of a lower spread provided by a higher hedging position turns out to be a rival and non-excludable ”good”, which allows our model to be identified as a particular case of congestion game. In fact, as debtors do not take into account all social benefits and costs of their decisions, the amount of risk exposure reallocated across sectors in competitive markets is below the socially optimal level. The informational friction that gives rise the market failure above can also be supported by the fact that financial globalization, under institutional constraints that limit the use of costly information, tends to reduce the incentives for foreign investor to collect costly information.

The paper does not conclude that ERRS policies will always be Pareto-efficient. On the contrary, we show that there are more cases where the opposite result occurs. The purpose is to distinguish the circumstances under which ERRS policies could be socially justifiable. In this sense, we conclude that they are more likely to occur when foreign investors are very pessimistic about the home economy’s performance and hence about its ability to repay. The reason is that, as foreign investors realize a high default probability, they require a large increase in the contractual interest rate in order to provide additional credit to the home economy. Conversely, they offer a large reduction in the spread if the debt is reduced. This means that the foreign credit’s supply curve is little responsive to the contractual interest rate, so that the effect of ERRS policies on this rate turns out to be very strong.

The paper is organized as follows. Section 2 describes the main hypothesis of the model. Section 3 solves the general equilibrium solution. Section 4 derives and interprets the welfare effects of ERRS policies. Section 5 concludes.
2 Description of the Model

This section describes the central aspects of the economy that we model to explain the main issues discussed above.

2.1 World economy Consider a nonmonetary, small open economy, which lasts for two periods: $t = 0, 1$. We call this economy and the rest of the world as home country and foreign country respectively, indexed by $j = H, F$. The home country comprises a tradable and a nontradable sector, indexed by $i = T, NT$. Each sector has a very large number of individuals, which are identical in all aspects. Individuals can differ across sectors. Whenever we refer to a sector, we have in mind its representative agent. Foreign country’s residents are risk-neutral, whereas home country’s ones are risk-averse. We assume rational expectations and that home country’s sectors share the same information set. There are no artificial barriers to the international flow of goods and capital. The subscript $t$ indicates that a variable is known at period $t$.

2.2 Shocks on the home country There is no production. At period $t$ ($t = 0, 1$), the sector $T$ ($NT$) is endowed with an exogenous amount of a single tradable (nontradable) good, denoted by $y_T^t$ ($y_{NT}^t$), which can vary across periods. Given the purpose of the paper, the unanticipated shocks introduced into the model must be able to explain, to a large extent, the empirically observed exchange rate volatility. In this sense, as discussed in more detail below, the law of one price implies that the shocks impacting on both nominal and real exchange rates have in common the fact that they change the supply of the tradable good for the home country. These shocks can have either a domestic origin, such as technological shifts in the tradable sector’s productivity, or an external origin, such as changes in the world price of the commodities or in the flow of foreign direct investment\(^2\). No matter the origin, the effect of these shocks on the tradable good’s domestic availability and hence on the wealth of both home country’s sectors can be proxied in the model by the effect of shocks to the sector $T$’s endowment. With this purpose, we assume that $y_T^t$ has an uniform distribution, given by

$$
y_T^t \sim U \left[ \mu_j - \eta , \mu_j + \eta \right], \mu_j > \eta > 0 . \tag{1}
$$

where the subscript in $\mu_j$ allows for heterogeneous beliefs across countries with respect to the sector $T$’s expected endowment. This fact will imply that the countries may disagree on the\(^2\)As to the external shocks, this occurs because international transference of resources occurs with tradable goods.
sector $T$’s ability to repay its foreign debt. As explained above, shocks to $y_{11}^T$ are the relevant source of uncertainty in the economy. For this reason, nothing is lost if $y_{11}^{NT}$ is assumed non-stochastic and strictly positive in both periods. It follows from (1) that $E_0,y_{11}^T = \mu_j (j = H, F)$ and $VAR_0[y_{11}^T] = \frac{1}{3}(1-2\eta)^2$. As shown along the paper, the possibility for $\mu_H > \mu_F$ is of most interest, since this will allow ERRS policies to bring a Pareto-improvement under some circumstances. But how could we provide theoretical and empirical support for this fact?

As argued by Calvo and Mendoza (2000a,b), financial globalization could reduce the incentives for foreign investors to collect country-specific information. This would occur if institutional constraints such as limits on short positions kept investors from taking full advantage of costly information, while portfolio diversification continued to be an attractive investment strategy even without full information\(^3\). In the context of this model, these informational frictions could explain why $\mu_F < \mu_H$. To see this, suppose the home country rests initially on an equilibrium with $\mu_H = \mu_F$ and next foreign investors receive a bad sign about its fundamentals. Suppose also that this sign is false and that home country’s residents know this but cannot release credible information for some adverse selection or moral hazard-related reason. Just as a reference, it is worth considering first what occurs if foreign investors act on their own account and pay the cost to know whether the sign is true or not. In this case, their expectations on the sector $T$’s productivity do not change, so that $\mu_F$ gets unaltered. On the other hand, suppose that the informational frictions above lead at least a significant fraction of the foreign investors not to have incentive to collect information on the sign. In this case, they could assign a positive probability to the event of that the sign is true and hence revise downwards their expectations. Once the sign is actually false by assumption, this implies that they would become overpessimistic about home country’s fundamentals, so that $\mu_F < \mu_H^4$. We suggest there reasons why this

\(^3\)Obviously, this results depends on that sovereign securities’ returns are less than perfectly correlated.

\(^4\)Earlier work on the welfare effects of overoptimism and overpessimism is Svensson and Persson (1983). They build a two-period model very similar to ours, in which (1) agents smooth consumption over time, (2) period-2 income is uncertain and (3) the economy is keynesian at period 1, in the sense that rigidities in prices and wages lead the output to be demand determined. Next, they show that overoptimism on the future income can have a positive net welfare effect because: (1) it has a first order positive welfare as it expands period 1- income and reduces the unemployment and (2) although the expectational error introduces a misallocation of consumption over time, this effect is of second order if the economy is only marginally overoptimism. The conclusion is that overoptimism introduces a distortion that ameliorate the allocative inefficiency caused by price rigidities. The same could also be said about overpessimism if there was overemployment at period 1.

As noticed along our paper, the contrasts and the similarities between the two models are evident. For instance: without overpessimism, markets assure allocative efficiency here in our model. Therefore, overpessimism causes a
could occur, both related to the destabilizing role of herd behavior in financial markets\textsuperscript{5}. The first one is that, as informational frictions do exist, international credit market is likely to be divided into informed and uninformed investors. In addition to use their limited information set, uninformed investors form their expectations by observing the actions of informed investors. However, informed investors are not able to trustfully signal whether their action are induced by changes in home country’s fundamentals or by factors relevant only for themselves. In this case, a shock unrelated to home country’s fundamentals could lead informed investors to take an action that would be wrongly interpreted by uninformed investors as a bad signal about home country’s economy\textsuperscript{6}. The second reason is also a history of herding, but it assumes that all investors are evenly imperfect informed, although they have different information sets. In this case, suppose that just a small fraction of the market perceives a rumor as enough credible to induce a defensive reaction against home country’s securities. If all other investors bring this action into their information set, this could trigger a domino effect on the larger group, leading it to herd on the smaller one, so that the market a whole would end up revising downwards its expectation on the home country’s fundamentals. Both reasons above are examples of information-based herding. However, as explained in Bikhchandani and Sharma (2001), besides being motivated by lack of information, herding also can occur if the compensation scheme of fund managers depends on their performances relative to other similar professionals or to a benchmark, so that imitation is rewarded. In this case, the institutional features of the asset management business would distort manager’s incentives towards mimicing the market behavior\textsuperscript{7}.

\textsuperscript{5}Herding occurs when investors are influenced into reversing a planned decision after observing the actions of other investors.

\textsuperscript{6}For example, an event like Russian default in 1998 could lead the big players in emerging countries securities’ markets to make large margin calls, which could be interpreted by other investors as bad news about the performance of Latin American and East Asian economies, which are not fundamentally related to Russian economy.

\textsuperscript{7}The vehicles suggested above for overpessimism are consistent to the well documented empirical evidence of that financial globalization has exacerbated contagion in financial crisis experienced by emerging market economies. Contagion occurs when an emerging economy, without having its own economic fundamentals substantially changed, is affected adversely by an irrational defensive reaction of international financial market participants to economic turbulences in another emerging economy.
2.3 ERRS policies  As explained in more detail below, shocks to $y^T_1$ impact on relative prices and hence they have a widespread effect on the home country’s economy. In this sense, they give rise to a macroeconomic risk to which the wealth and the welfare of both home country’s sectors are exposed. With the purpose of smoothing the risk exposure across sectors, the home country’s government transfers $\left|\left(\mu_H - y^T_1\right)h_0\right|$ units of the tradable good for the sector $T$ ($NT$) at $t = 1$ if and only if $\left(\mu_H - y^T_1\right)h_0$ is positive (negative), where $h_0$ is a policy parameter determined exogenously by the government. At this same period, the public budget constraint implies that the government must receive this same amount from the other sector. Given the simple structure of the model, ERRS policies consist in setting $h_0 \neq 0$. Obviously, no policy is implemented when $h = 0$. In section 4, we examine the welfare effects of a change in $h_0$ around $h_0 = 0^9$. This comparative statics allows us to determine whether or not ERRS policies bring a Pareto-improvement for the home country. The size and the sign of the parameter $h_0$ summarize all information on the ERRS policy. Compared to $h_0 = 0$, the sector $T$’s wealth volatility decreases (increases) with a positive (negative) $h_0$ as this sector receives a positive transference when an adverse shock hits its endowment ($y^T_1 < \mu_H$). Moreover, just the opposite effect occurs with the sector $NT$’s wealth volatility, since the relative price of this sector’s endowment is positively related to $y^T_1$. Therefore, the sector $T$’s wealth has its exposure to exchange rate risk decreased (increased) when $h_0 > 0$ ($< 0$), while the reverse occurs with the sector $NT$’s wealth.

As intervention in the model aims to reallocate exchange rate risk across sectors, how is this risk related to shocks to $y^T_1$? From the law of one price, the real exchange rate reflects the relative price of the tradable good. This fact has two consequences: (1) the primary sources of real exchange rate volatility are shocks to the domestic supply of both tradable and nontradable goods and (2) changes in the real exchange rate impact on the real value of assets and liabilities held by both home country’s sectors. In short, shocks to any sector impact on the real exchange rate.

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8The public budget must be balanced at $t = 1$ because the home country expires in this period.

9We just consider ERRS policies such that $|h_0| < \kappa < 1$, where $\kappa$ is very small. This is also discussed in the section 4.

10This is because, when $h_0 > 0$ ($< 0$), the sector $NT$ will transfer (receive) resources to (from) the government when its wealth is reduced by an adverse shock to $y^T_1$, which reduces the relative price of its own endowment.

11In true, the home country’s relative price of the tradable good is the product between the real exchange rate and the foreign country’s relative price of the tradable good. For sake of simplicity, let’s suppose that the latter is constant.

12A real depreciation of the home currency increases the real value of the sector T’s endowment and reduces the real value of the sector NT’s endowment. In addition, if the foreign liabilities of home country’s sectors are denominated in foreign currency, a higher real exchange rate has a adverse effects on their wealth.
rate and this in turn impacts on the both sectors’ wealth. Besides, shocks to different sectors have opposite effects on the real exchange rate. Adverse shocks to nontradable good’s supply raise domestic prices without changing nominal exchange rate, causing a real appreciation of the home currency. On the opposite way, adverse shocks to tradables good’s supply raise the nominal exchange rate at a rate above the inflation rate, causing a real depreciation of the home currency\textsuperscript{13}. It is important to note that nominal exchange rate volatility is driven just by shocks to the tradable sector, since shocks to the nontradable sector affect only the nominal aggregate price index. Hence, since intervention, as described above, is designed just to smooth across sectors the exposure to shocks to the domestic availability of the tradable good, which are the disturbances hitting the nominal exchange rate, the model works as an adequate framework to examine ERRS policies implemented through an intervention in the nominal exchange rate market.

However, if shocks to both sectors impact on the real exchange rate, why does the model focus only on the real exchange rate volatility driven by shocks to the tradable sector? Why not to analyze the working and the effects of ERRS policies designed to smooth the exposure to shocks to the nontradable sector, which also affect the real exchange rate through changes in the nominal aggregate price index? Two empirical evidences lead us to limit our analysis to shocks to the tradable sector. First, even after floating exchange rate regimes were introduced, policymakers in some emerging economies have continued to intervene in the nominal exchange rate markets sporadically. In fact, faced with strong pressures pushing spot exchange rate up, monetary authorities in some of these countries provide the market with a long position on a dollar-indexed asset (bond or derivative security)\textsuperscript{14}. As the government holds the short position, this is clearly a ERRS policy\textsuperscript{15}. As seen above, since nominal exchange rate risk volatility is

\textsuperscript{13}This occurs because an adverse shock to the tradable sector has two effects on the home country’s prices: (1) as long as foreign country’s nominal prices remain unaltered, the law of one price implies that the nominal exchange rate depreciation is proportionally equal to the increase in the tradable good’s nominal price and (2) the home country’s aggregate price index, which is a weighted average of the nominal tradable and nontradable prices, increases proportionally less that the tradable good’s nominal price.

\textsuperscript{14}Examples in Latin America are the issues of dollar-indexed Treasury bonds in Brazil, Chile and Colombia. This way of indirect intervention in exchange rate market allows us to relate ERRS policies to the literature on the optimal public debt indexation and denomination. Examples of this literature are Gale (1990), Goldfajn (1995,1998) and Missale (1997).

\textsuperscript{15}Is this only fear of floating or there is a welfare argument behind these policies? As a ground for intervention, it is argued that the high pass-through of these economies makes it essential to avoid the deleterious effects of the excessive exchange rate volatility on the internal and external equilibrium.
driven by shocks to the tradable sector, we have thus a good reason to focus on this source of risk. Second, many of the emerging economies referred above were successful in achieving price stabilization in the recent past, so that real exchange rate swings are now related basically to nominal exchange rate moves\textsuperscript{16}. Thus, real exchange rate risk in these countries is expected to be determined to large extent by the exposure to shocks to the tradable sector.

Now it is easier to understand why money is not introduced into the model. One can wrongly interpret ERRS policies just as an instrument for reallocation of the nominal exchange rate risk exposure and therefore feel uncomfortable with a nonmonetary approach to this issue. However, which sort of risk is actually smoothed when ERRS policies are implemented by an intervention in the nominal exchange rate market? In true, for the reasons cited above, intervention in this model aims to smooth the portion of the real exchange rate risk induced by shocks to the tradable sector, which are the disturbances that give rise to the nominal exchange rate volatility\textsuperscript{17}.

ERRS policies must not be implemented just by an intervention in the nominal exchange rate market. As an alternative policy, we could imagine that the home country’s government concede a subsidy to the sector it wishes to protect whenever the wealth of this sector falls in consequence of a shock to the real exchange rate. Of course that the government budget constraint would necessarily force the other sector to bear the increase in public expenses when shocks arise, so that this policy also causes a risk exposure reallocation across sectors. In short, all that is necessary is some kind of public intervention through which the government is able to compensate one of the sectors - at the expense of the other one - whenever a shock to real exchange rate reduces its wealth. Therefore, the way as intervention is described at the beginning of this section should be seen only as the result - in terms of transference of resources - of the institutional mechanism set by the government.

We allow for private risk exposure reallocation by introducing a market for hedging into the home country. More specifically, at $t = 0$, the sectors can trade among them a forward contract-type security that pays off $\left( \mu_H - f_0 y^T_0 \right)$ units of the tradable good at $t = 1$, where $f_0$ is the market-determined premium of this contract. Note that this contract requires no disbursement at $t = 0$. We denote by $q^i_0$ the sector $i$’s hedging position acquired in this market, which can

\textsuperscript{16}In this sense, Brazil and Chile are notorious examples of inflation targeting experiences in emerging economies.

\textsuperscript{17}Another reason to build a nonmonetary model is that we are just concerned with the welfare effects of exchange rate shocks transmitted through changes in the relative prices of the tradable and the nontradable goods. We do not address, for instance, the welfare effects of these shocks induced by the higher volatility of the inflation rate, whose analysis does require money.
be a long \((q^i_t > 0)\) one or a short \((q^i_t < 0)\) one. This means that, given a position equal to \(q^i_0\), the sector \(i\) will receive (pays) \(\left| (\mu_H - f_0Y^iT) q^i_0 \right|\) units of the tradable good at \(t = 1\) if and only if \( (\mu_H - f_0Y^iT) q^i_0 \) is positive (negative). Moreover, the equilibrium level of \(f_0\) is such that the domestic market for hedging clears, so that \(q^T_0 + q^{NT}_0 = 0\).

2.4 Competitive international capital market  At \(t = 0\), the sector \(i\) \((i = T, NT)\) can concede or receive loans from the foreign country, which are promised to be repaid at \(t = 1\). The sector \(i\)'s net foreign debt at \(t = 0\), denoted by \(d^i_0\), is the net amount of loans borrowed by this sector in this period\(^{18}\), which are denominated in tradable goods\(^{19}\). When \(d^i_0 > 0\) \((d^i_0 < 0)\), we say that the sector \(i\) is a debtor (creditor) of the foreign country. Therefore, the sector \(i\) transfers wealth from \(t = 1\) to \(t = 0\) when \(d^i_0 > 0\), while the reverse occurs when \(d^i_0 < 0\).

The sector \(i\) may have incentive to default when it is a foreign debtor\(^{20}\). The penalties for default cause a loss of utility (desutility) given by \(\varepsilon^i \geq 0\)\(^{21}\). As default is possible, the contractual (promised) interest rate on the foreign loans borrowed by the sector \(i\), denoted by \(g^i_0\), may be higher than the default risk-free world interest rate, denoted by \(r_0\). Both \(g^i_0\) and \(r_0\) are quoted in tradable goods. Moreover, as default probability may differ across sectors\(^{22}\), it is possible that \(g^T_0 \neq g^{NT}_0\).

As explained in subsection 2.2, financial globalization under institutional constraints could weaken the incentives for foreign investors to collect costly country-specific information. If this claim is valid for information on home country’s fundamentals, which encompasses the relevant macroeconomic and financial aggregate variables, so should be it for the same type of information concerning individual economic units. The idea is that, in general, the more disaggregated the

\(^{18}\)When \(d^i_0 < 0\), the sector \(i\) is a creditor of the foreign country.

\(^{19}\)This assumption amounts to say that foreign debt is denominated mostly in foreign currency, according to the "original sin" argument raised by Eichengreen and Hausmann (1999). To understand this claim, note that, as long as foreign country’s prices are constant, the law of one price implies that the effect of shocks to exchange rate on the real value of a foreign debt denominated in tradable goods is the same as that on a foreign debt denominated in foreign currency.

\(^{20}\)We assume that foreign country’s residents never default when \(d^i_0 < 0\). However, we can say in advance that this assumption is irrelevant because, given the purpose of the model, we will be interested only in general equilibrium solutions such that the home country’s sectors are indebted with the foreign country.

\(^{21}\)Penalties for default at \(t = 1\) can not be derived endogenously in the model because the world economy ends in this period. Therefore, we simply assume that such costs are exogenous. In the model, we assume that \(\varepsilon^i\) results from some kind of punishment that reduce the debtor’s welfare without impacting directly on its consumption.

\(^{22}\)For instance, this fact will occur when \(\varepsilon^T \neq \varepsilon^{NT}\).
information is, the harder its availability is. In view of this fact, we assume that foreign investors are imperfectly informed on the individual portfolio of each sector i’s debtor, which among other things determines her default probability. More specifically, only the aggregate foreign debt and the aggregate hedging position of each sector can be directly observed by foreign investors. As said, they have imperfect information on the debtors’ individual portfolio, so that they can not monitor directly the size of the hedging position and the size of the foreign liabilities of each debtor. As seen along the paper, this market imperfection-related assumption is crucial to understand both the market structure and the allocative inefficiency in this model. In particular, it allows us to explain why competitive markets could fail to reallocate efficiently the risk exposure across the home country’s sectors, justifying in some circumstances public intervention through ERRS policies.

Although the results regarding these issues be derived and interpreted in more detail below, it is worth giving here some intuition on how market inefficiency arises in consequence of the information asymmetry cited above. As described in subsection 2.5 below, each home country’s individual maximizes her welfare by choosing the composition of her portfolio, which comprises only her foreign debt and her hedging position. Foreign indebtedness allows her to smooth consumption over time, while trading on the domestic hedge market allows her to change her exchange rate risk exposure. However, as seen in subsections 3.3 and 3.4 below, there is an additional welfare effect behind these portfolio choices: the spread paid by an individual borrower on her foreign debt depends directly on her default probability and this in turn depends on her portfolio. This occurs because: (1) portfolio composition affects the mean and the volatility of the debtor’s wealth distribution and then the range of states of nature in which default is the optimal decision and (2) competition among risk-neutral foreign investors pushes the contractual interest rate $g_0$ to the level at which the expected rate of return - which falls with a higher default probability - equals to the default-risk free interest rate.

Very important, the effect of the portfolio composition on the spread can be seen as a rival and non-excludable ”good” underlying the portfolio positions, so that this model turns out to be a particular case of congestion game. It is rival because the actual default probability of

23 Off-balance accounts as a device to escape from the creditors’ monitoring could justify this assumption as well, mainly in emerging economies lacking a well regulated banking system.

24 Individuals buying hedge have their risk exposure diminished, which brings a welfare gain as they are risk averse. Individuals selling hedge charge a premium in exchange of a higher risk exposure.

25 In congestion games, players use facilities from a common pool and the benefit that a player derives from using the facility depends on the number of users of this facility. In this class of games, decentralized decisions
an individual debtor depends only on her own portfolio, no matter the size of the aggregate positions held by her sector. Therefore, changes perceived by foreign investors in the portfolio of an individual debtor will affect only her spread. In other words, the spread required by creditors from each debtor depends only on the individual portfolio they believe this debtor holds.

It is also non-excludable because the model assumes that foreign investors observe only the aggregate foreign debt and the aggregate hedging position held by each sector and, in addition, they know that all individuals from a same sector are identical. Therefore, if a debtor alone tries to raise the variable $X$ by $\Delta X$, which can be either her foreign debt or her hedging position, foreign investors realize that every debtor in her sector raises $X$ by $\Delta X/N$, where $N$ is the number of individuals in the sector, and then only this amount will be perceived by foreign investors as a rise in her own position. Therefore, although she has actually risen $X$ by $\Delta X$, foreign investors adjust her spread as if she had raised $X$ only by $\Delta X/N$. In addition, since all other debtors in her sector can free ride on her, they also have their spread changed by the same size.

Consequently, in choosing her optimal portfolio, each individual debtor takes into account only the impact of her decisions on her own spread and ignores the additional effect on the spread faced by other debtors. The idea is that once she is not able to take full advantage of the benefits and/or costs of a rise in $X$, she dismisses part of the social effects of her portfolio choices. If all debtors act in the same way, the market allocation is inefficient. More specifically, private markets lead to foreign overborrowing and insufficient risk reallocation across sectors\textsuperscript{26}.

Furthermore, the extent of this market inefficiency increases with $N$: the lower $N$, the closer the social and private effects of individual portfolio choices on spread are. When $N$ is small, $\Delta X/N$ is significant and then the spread each debtor pays will depend to some extent on her individual portfolio choices. In this case, each individual has some market power to set her spread. As $N$ increases, the effect of a rise in $\Delta X$ on the individual spread falls. In the context of this model, we assume that $N$ is large enough to make $\Delta X/N$ close to zero. Therefore, the portfolio choices of each debtor have no effect on her individual spread, so that she takes the spread as given.

\textsuperscript{26}Burnside et al. (1999) examines the foreign debt and hedge decisions of banks in the fixed exchange rate regime. These banks borrow foreign currency-denominated external loans in order to provide domestic currency-denominated loans to domestic firms. They argue that, without government guarantees, it is optimal for banks to hedge completely their exchange rate risk. On the other hand, the existence of governments guarantees eliminates the incentives for hedging and in addition banks tend to increase their exchange rate risk exposure.
2.5 Consumer behavior  Each sector $i$ ($i = T, NT$) consumes both goods in all periods. Then, the sector $i$'s preferences can be represented by the lifetime utility function

$$\ln \left( c^i_0 \right) + \beta E_0 \left[ \ln \left( c^i_1 \right) - (1 - \delta^i) e^i \right], \quad 1 > \beta > 0,$$

$$c^i_t \equiv \left[ c(T)^i_t \right]^{\theta} \left[ c(NT)^i_t \right]^{1-\theta}, \quad 0 < \theta < 1,$$

where $\beta$ is the time-preference factor, $\theta$ is a constant that determines the elasticity of substitution between goods, $c(T)^i_t$ and $c(NT)^i_t$ are the consumption levels of the tradable and the nontradable goods respectively, $c^i_t$ is the composite consumption index and $\delta^i$ is an indicator function, defined as $\delta^i = 0$ if $d^i_0 > 0$ and $\delta^i = 1$ otherwise.

Each sector maximizes (2), subject to the intertemporal budget constraint given by

$$c^i_0 = 1, \quad p^T_0 y^i_0 + d^i_0, \quad (4)$$

$$c^i_1 = \frac{1}{p_1} \left[ p^T_1 y^i_1 - (1 + g^i_0) \delta^i d^i_0 - (y^T_1 - \mu_H) b^0_0 - (y^T_1 - f_0 \mu_H) q^i_0 \right], \quad (5)$$

where $b^T_0 \equiv h_0$ and $b^{NT}_0 \equiv -h_0$, whereas $p^T_1$ and $p^{NT}_1$ are the prices of the tradable and the nontradable goods respectively and $p_t$ is the consumption-based aggregate price index. Assuming that the tradable good is the home country’s numeraire, we have $p^T_1 = 1$. The first term into the brackets in (4)-(5) is the sector $i$’s endowment, measured in tradable goods. The second term is the capital flow with the foreign country. The third and fourth terms in (5) are, respectively, the transfers for the sector $i$ in function of the ERRS policy and of its own individual hedging position acquired in the market.

3 General Equilibrium

This section derives the general equilibrium solution for the model. As discussed in the earlier section, the main result of the model is related to the effect of ERRS policies on the contractual

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27Actually, this elasticity is equal to $\frac{1}{\theta}$.  
28Obviously, $\delta^i = 1$ if $d^i_0 \leq 0$.  
29Formally, $p_t$ is defined as

$$p_t = \varphi \left( p_t^{NT} \right)^{1-\theta} \equiv \min_{c(T)^i_t, c(NT)^i_t} \left\{ \frac{p^T_t c(T)^i_t + p^{NT}_t c(NT)^i_t}{s.a. c^i_t = 1} \right\},$$

where $\varphi \equiv \theta^{-\theta}(1 - \theta)^{\theta-1} > 0$. Note that $p_t$ is a consumption-based index because it is the minimal expenditure required to get $c^i_t = 1$.  
30Obviously, this term in (5) vanishes if the sector $i$ defaults on its foreign debt ($\delta^i = 0$).
interest rate paid by the sector $T$ on its foreign debt. Therefore, we focus only on the cases in which this sector is a foreign debtor at $t = 0$. With this purpose in mind, we assume that $y_{0}^{T} = 0$. Given the logarithmic period utility function in (2), this assumption implies that we will always have $d_{0}^{T} > 0$ in the general equilibrium solution derived below\(^{31}\).

For sake of simplicity, we also assume that $\epsilon^{T} > \epsilon^{NT} = 0$. The sector $NT$ has less incentive to repay its debt than the sector $T$ does. A theoretical justification is that penalties for default could consist mostly in loss or reduction of foreign trade credit, which is the main source of funding to exports. Moreover, as its desutility with default is null, the sector $NT$ has no incentive to repay and hence has no access to the international capital market. Therefore, as seen below, we will have $d_{0}^{NT} = 0$ in equilibrium. It is important to have in mind that such assumption could be dropped without changing the main results of the paper.

### 3.1 Equilibrium conditions for home country’s markets

All home country’s markets clear at $t = 0, 1$, so that

$$
q_{0}^{T} + q_{0}^{NT} = 0 ,
$$

$$
y_{t}^{T} - x_{t} = c(T)^{T}_{t} + c(T)^{NT}_{t} ,
$$

$$
y_{t}^{NT} = c(NT)^{T}_{t} + c(NT)^{NT}_{t} ,
$$

where $x_{t}$ is the home country’s net aggregate exports, which are given by

$$
x_{0} = - \left( d_{0}^{T} + d_{0}^{NT} \right) ,
$$

$$
x_{1} = \left( 1 + g_{0}^{T} \right) \delta^{T} d_{0}^{T} + \left( 1 + g_{0}^{NT} \right) \delta^{NT} d_{0}^{NT} .
$$

It follows from (6) that, in equilibrium, the sectors must have an opposite position of same size in the domestic market for hedging. The market equilibrium conditions for the tradable and the nontradable goods are given by (7) and (8) respectively. As the nontradable good is not exportable by definition, the supply of the tradable good for the home country is equal to the endowment of this good less the home country’s net aggregate exports, whereas the supply of the nontradable good is given only by the endowment of this good. The equilibrium conditions for the home country’s balance of payments are given by (9)-(10): net exports must finance the capital account deficits (and also the interests at $t = 1$). Note that the net amount of wealth

\(^{31}\)As $y_{0}^{T} = 0$, the sector $T$ will have no wealth at $t = 0$ if $d_{0}^{T} = 0$. This is not possible in equilibrium because the marginal utility of consumption goes to infinite when $c_{0}^{T} = 0$. 

transferred to the foreign country at $t = 1$, given by $x_1$, increases with $g_0^i$ and declines with default ($\delta^i = 0, \; i = T, NT$). Note also that a higher $d_0^i$ ($i = T, NT$) causes an increase (decrease) in the tradable good’s supply for the home country at $t = 0$ ($t = 1$).

3.2 Relative Prices By using pure algebra, it follows from (3)-(10) that relative prices in home country are given by

$\begin{align*}
    p_0^{NT} &= \frac{1 - \theta d_0^T + d_0^{NT}}{y_0^{NT}}, \\
    p_1^{NT} &= \frac{1 - \theta y_1^T - (1 + g_0^T) \delta^T d_0^T - (1 + g_0^{NT}) \delta^{NT} d_0^{NT}}{y_1^{NT}}, \\
    p_0 &= \frac{1}{\theta} \left( \frac{d_0^T + d_0^{NT}}{y_0^{NT}} \right)^{1-\theta}, \\
    p_1 &= \frac{1}{\theta} \left[ y_1^T - (1 + g_0^T) \delta^T d_0^T - (1 + g_0^{NT}) \delta^{NT} d_0^{NT} \right]^{1-\theta}.
\end{align*}$

Note in (11)-(14) that the relative price of any good is inversely related to the ratio between the supply of this good and the supply of the other one for the home country. More important is that this result allows us to understand how shocks to $y_1^T$ impact on the wealth of both sectors at $t = 1$. For this, we assume for sake of simplicity that $h_0 = g_0^T = q_0^{NT} = 0$. In this case, we can see in (5) that the wealth of both sectors increases with $y_1^T$. As to the tradable sector, this occurs because the increase in its endowment more than compensate the lower relative price of the tradable good. As to the nontradable sector, its wealth also increase because the relative price of its endowment increases with $y_1^T$, although it does not receive any endowment of the tradable good. Consequently, both sectors have their wealth exposed to shocks to $y_1^T$. This explains why ERRS policies ($h_0 \neq 0$) and trading on the domestic hedge market ($q_i^0 \neq 0$, for $i = T, NT$) give rise to a risk exposure reallocation across sectors.

3.3 Default probability First, we derive the sector $i$’s default probability in country $j$’s belief, denoted by $\pi_j^i$ ($i = T, NT$, $j = H, F$), as a function of all observable variables at $t = 0$, which are given by the vector $z_0 \equiv (d_0^i, g_0^i, q_i^0, f_0)$, and the policy parameter $h_0$. Although both

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32 As explained at the beginning of this section, we will have $d_0^i \geq 0$ ($i = T, NT$) in equilibrium. Then, the prices below are always positive.

33 Note that, by assumption, $p_j^T = 1$ ($t = 0, 1$).

34 As we will see below, it is possible to have $\pi_H^T \neq \pi_F^T$ because in (1) we allow for heterogeneous beliefs about the sector $T$’s expected endowment.

35 As the international capital market is competitive, foreign creditors take $g_0^i$ ($i = T, NT$) as given.
$d_0^i$ and $q_0^i$ refer to portfolio positions of the sector $i$’s representative agent, this does not mean that these positions can be directly observed by foreign investors in the individual portfolio of each sector $i$’s member. As it was assumed above, they can directly observe only the aggregate net foreign debt and the aggregate hedging position of each sector. However, as foreign investors realize correctly that identical individuals have incentive to take the same decisions, they can infer $d_0^i$ and $q_0^i$ indirectly from the aggregate counterparts of these variables.

The sector $i$ repays its debt whenever the utility gain with default, denoted by $\chi^i$, is smaller or equal to the utility loss with penalties for default, given by $\varepsilon^i$. Therefore, the sector $i$ defaults if and only if

$$\chi^i \equiv c_1^i (z_0, \delta^i = 0) - \ln c_1^i (z_0, \delta^i = 1) > \varepsilon^i , \quad (15)$$

where $c_1^i (z_0, \delta^i)$ follows from (5), while $z_0$ and $\delta^i$ were defined above. As to the nontradable sector, since $\varepsilon^{NT} = 0$ by assumption, it follows that $\pi^{NT} = \pi^{NT}_H = 1$ if $d_0^{NT} > 0$. As to the tradable sector, substituting (5) into (15), we have that this sector defaults if and only if $y_T^1 < k$, where

$$k = k (d_0^T, g_0^T, q_0^T, f_0, h_0) \equiv \frac{(1 + g_0^T) d_0^T}{(1 - q_0^T - h_0) [1 - \exp (-\varepsilon^T)]} - \frac{(f_0 g_0^T + h_0) \mu_H}{(1 - q_0^T - h_0)} . \quad (16)$$

Consequently, it follows from (1) that

$$\pi^T_j = \pi^T_j (z_0, h_0) \equiv \text{Pr}_{0,j} [y_T^1 < k] = \frac{k - \mu_j + \eta}{2\eta} , \text{ if } \mu_j - \eta < k < \mu_j + \eta , \quad (17)$$

whereas $\pi^T_j = 0(=1)$ if $k \leq \mu_j - \eta$ ($k \geq \mu_j + \eta$), where $k$ is given by the equation (16).

The figure 1 helps us understand how $\pi^T_j$ is determined by the equations (16) and (17) above. The upper and lower curves are, respectively, the graphs of the period 1-utility, as a function

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36Note that $p_1$ is cancelled out out when we derive (16) from (15). This is possible because each home country’s individual corresponds to a very small fraction of her sector, so that she realizes that her actions, such as default, do not affect the market prices. Moreover, this behavior is anticipated by foreign creditors, so that they also believe that the sector $T$ defaults if and only if $y_T^1 < k$.

37The subscript $j$ ($j = H, F$) indicates that the probability below is conditioned on the country $j$’s belief about sector $T$’s expected endowment, which is given by $\mu_j$. Note that $\mu_H$ is known by foreign country’s investors because, as we can see at the end of subsection 2.3, this parameter is written on the hedge contract traded in the home market. However, as explained in subsection 2.2, this does not imply that the countries have to agree on the sector $T$’s expected endowment.
of \( y^T \), when the sector \( i \) defaults and when it does not\(^{38}\). Fixed any \( y^T_i \), the utility gain with default, given by \( \chi^i \) in (15), is the vertical difference between these curves. As the marginal utility of consumption is decreasing, we can see in the figure that \( \chi^i \) increases with a lower \( y^T_i \). Intuitively, this means that the utility gain with default increases as debtors get less wealthy. Hence, while the utility loss with default, given by \( \delta^i \), is fixed, the incentives for default increase with a lower \( y^T_i \). As a result, note in the figure that, for \( y^T_i < k \), we have \( \chi^i > \delta^i \), so that it is optimal for the sector \( i \) not to repay. On the contrary, for \( y^T_i > k \), we have \( \chi^i < \delta^i \), so that it is now optimal for the sector \( i \) to repay. At \( y^T_i = k \), we have \( \chi^i = \delta^i \) and in this case we assume that debtors do repay. Therefore, we conclude that \( k \), defined in (16), is the lowest level of \( y^T_i \) at which repayment occurs, so that it can be interpreted as the effective cut-off level of \( y^T_i \) for default. Furthermore, as \( \pi^T_j \) is, by definition, the probability that \( y^T_i < k \), the expression in (17) follows directly from the distribution of \( y^T_i \) in (1).

Note also that a higher \( g^T_0 \) or \( h^T_0 \) shifts the lower curve down, increasing \( \chi^i \) for all \( y^T_i \). Therefore, given that \( \delta^T \) gets unaltered, default will only occur at higher levels of \( y^T_i \), so that \( k \) and hence \( \pi^T_j \) increase. The intuition of this result is very clear: as the utility gain with default increases with the size of the foreign liabilities, the default probability must also increase. This comparative statics helps understand the other results below.

### 3.4 Foreign credit supply

Now, we derive the equilibrium foreign credit supply for the sector \( i \), denoted by \( d^F_i \) \((i = T, NT)\), as a function of the contractual interest rate \( g^T_0 \) and other relevant observable variables\(^{39}\). The variable \( d^F_0 \) is, by definition, the level of \( d_0 \) that meets the following conditions: (C1) all foreign investors currently lending this amount of credit to the sector \( i \) are maximizing profits and (C2) no additional foreign investor has incentive to provide credit to this sector.

As to the sector \( NT \), we saw above that \( \epsilon^NT = 0 \) implies that \( \pi^NT_F = 1 \). Therefore, it is trivial that \( d^F_{NT} > 0 \) is not sustainable in equilibrium: foreign investors never lend to this sector if they expect not to be repaid for sure.

As to the sector \( T \), since foreign investors are risk-neutral, (C1)-(C2) imply that, given \((g^T_0, q^T_0, f_0, h_0)\) with \( g^T_0 \geq r_0 \), we have that \( d^F_T \) solves the equation

\[
k \left( d^F_T, g^T_0, q^T_0, f_0, h_0 \right) = \nu ,
\]

\(^{38}\)As we can see in (15), these functions are given, respectively, by \( \ln c^1_i \left( z_0, \delta^i = 0 \right) \) and \( \ln c^1_i \left( z_0, \delta^i = 1 \right) \).

\(^{39}\)These are the policy parameter \( h_0 \) and other variables in the vector \( z_0 \), as defined above in subsection 3.3.
where the function $k$ is defined in (16) and the constant $\nu$ is defined implicitly by the arbitrage condition

$$\Pr_{0,F} [y_1^T \geq \nu] \left(1 + g_0^T\right) = 1 + r_0.$$  \hspace{1cm} (19)

When $g_0^T \leq r_0$, we have $d_0^{FT} = 0$ because (19) is not met for any positive $d_0^T$. Note in (18)-(19) that $d_0^{FT}$ is such that the expected rate of return on the loans borrowed by the sector $T$ equals the default risk-free interest rate.

The condition (19) sets that, in equilibrium, the lowest level of $y_1^T$ at which repayment occurs must be necessarily equal to $\nu$, which depends solely on $g_0^T$, $r_0$ and the parameters of the distribution of $y_1^T$, as given in (1). Thus, we can properly interpret $\nu$ as the required cut-off of $y_1^T$ for default. As a result, $d_0^{FT}$ is the level of $d_0^T$ that makes $k$, the effective cut-off defined in (16), equal to $\nu$, the required cut-off. Alternatively, $d_0^{FT}$ is the level of $d_0^T$ that makes $\pi_T^F$, the effective default probability in country’s $F$ belief, as defined in (17), equal to $1 - \frac{1 + r_0}{1 + g_0^T}$, which is the required default probability in country’s $F$ belief, as we can infer from (19).

Substituting (16) into (18), we have that $d_0^{FT}$ can be explicitly defined as

$$d_0^{FT} (g_0^T, q_0^T, f_0, h_0) = \left[\nu \left(1 - q_0^T - h_0\right) + \mu_H \left(f_0q_0^T + h_0\right)\right] \frac{1 - \exp \left(-\epsilon_T^T\right)}{1 + g_0^T},$$  \hspace{1cm} (20)

whereas it follows from (17) and (19) that $\nu$ is given by

$$\nu = \mu_F + \eta \left[1 - 2 \frac{1 + r_0}{1 + g_0^T}\right], \text{ if } g_0^T > r_0;$$  \hspace{1cm} (21)

$$\nu = \tau (\mu_F - \eta), \text{ for any } 0 \leq \tau \leq 1, \text{ if } g_0^T = r_0.$$  \hspace{1cm} (22)

An increase in $g_0^T$ has two opposite effects on $d_0^{FT}$. First, a higher $g_0^T$ leads foreign investors to make more profits on the loans they will be actually repaid, so that they have incentive to lend more. Second, as it was mentioned above, sector $T$’s foreign liabilities increase with $g_0^T$, pushing $\pi_T^F$ up and hence leading foreign investors to curb the supply of loans. As a result, for low levels of $g_0^T$, the first effect is dominant, so that the supply curve is increasing in $g_0^T$. However, the second effect gets stronger as $d_0^{FT}$ increases with $g_0^T$, making the supply curve more inelastic. At a certain level of $g_0^T$, the second effect overcomes the first one, so that the supply curve becomes decreasing in $g_0^T$.

Given the purpose of the model, it is important to explain how a change in $h_0$ impacts on $\pi_T^F$ and hence on $d_0^{FT}$. This effect is better illustrated in figure 2, where the upper and lower thin

\footnote{Note below that $g_0^T$, $q_0^T$ and $f_0$ are the only variables in the vector $z_0$, as defined above, on which $d_0^{FT}$ depend.}
curves are, respectively, the graphs of the period 1-utility function with and without default for the case \( h_0 = 0 \), whereas the upper and lower thick curves are, respectively, the graphs of the period 1-utility function with and without default for the case \( h_0 > 0 \). Note in this figure that, when compared to \( h_0 = 0 \), a positive \( h_0 \) makes the sector \( T \)'s wealth increase when \( y_1^T < \mu_H \) and decrease when \( y_1^T > \mu_H \), leading to a rotation in the period 1-utility curve around \( y_1^T = \mu_H \), which gets flatter with \( h_0 > 0 \) than with \( h_0 = 0 \). This occurs either with default, represented by the rotation from the upper thin curve to the upper thick one, or without default, represented by the rotation from the lower thin curve to the lower thick one. Moreover, as the marginal utility of consumption is decreasing, this effect is stronger without default. This is clear in the figure, where, except for \( y_1^T = \mu_H \), the vertical distance between the two lower curves is larger than that between the upper ones. The intuition behind this result is that default makes debtors wealthier and hence they value less changes in their wealth caused by ERRS policies. Therefore, the utility gain with default, given by \( \chi^T \), decreases (increases) for \( y_1^T < \mu_H \) \((y_1^T > \mu_H)\), so that the effect of a higher \( h_0 \) on \( \pi_F^T \) and hence on \( d_0^{FT} \) is ambiguous and depends on whether \( \nu \) - the required cut-off for default in equilibrium, is higher or lower than \( \mu_H \). We have both cases illustrated in figure 2. When \( \nu = \nu_{\text{low}} < \mu_H \) \((\nu = \nu_{\text{high}} > \mu_H)\), the utility gain at \( y_1^T = \nu \) decreases (increases) with a higher \( h_0 \), pushing \( k \) - the effective cut-off for default - and \( \pi_F^T \) down (up). Therefore, since \( g_T^T \) is taken as fixed by foreign investors and \( \pi_F^T \) increases with \( d_0^T \), as seen in the previous subsection, \( d_0^{FT} \) must be higher (lower) in order to bring \( k \) and \( \pi_F^T \) back to their required equilibrium levels, given by \( \nu \) and \( 1 - \frac{1+r_0}{1+g_0} \) respectively.

3.5 Foreign credit demand

Given \( (g_T^T, g_0^{NT}, h_0) \), the vector \( (d_0^i, q_0^i, f_0)_{i=T,NT} \) on which the home country rests in equilibrium, denoted by \( (d_H^i, q_H^i, f_H^0)_{i=T,NT} \), meets the following conditions: (C3) both sectors maximize the lifetime utility function in (2)-(3) subject to the intertemporal budget constraint in (4)-(5), (C4) all home country’s markets clear in both periods, namely, the equilibrium conditions in (6)-(10) are satisfied and (C5) period 0-expectations about the relative prices \( p_1^{NT} \) and \( p_1 \) are formed rationally\(^{41}\).

In order to meet (C3), the equilibrium solution must satisfy the marginal conditions of opti-

\(^{41}\)This implies that, in equilibrium, period 0 - expectations about relative prices are conditioned on \( (d_0^i, q_0^i, f_0)_{i=T,NT} = (d_H^i, q_H^i, f_H^0)_{i=T,NT} \).
mization with respect to \( d_0^i \) and \( q_0^i \), which are given, respectively, by\(^{42}\)

\[
\frac{1}{p_0 c_0} - (1 + g_0^i) \beta E_{0,H} \left[ \frac{1}{p_1 c_1^i} \right] = 0, \quad i = T, NT; \tag{23}
\]

\[
E_{0,H} \left[ \frac{y_T^i - f_0^i \mu_H}{p_1 c_1^i} \right] = 0, \quad i = T, NT. \tag{24}
\]

Note that such conditions were derived with \( g_0^i \) having been taken as given by sector \( i \)'s individuals. As explained in subsection 2.4, this price-taking behavior in turn follows directly from the assumption that foreign investors can observe, for each sector, only the aggregate net foreign debt and the aggregate hedging position. Individual portfolio choices can not be observed directly. Therefore, as there is a large number of participants in each sector deciding on their actions in a decentralized way, they correctly realize that the impact of their individual choices on the aggregate portfolio of her sector and hence on the contractual interest rate is irrelevant.

In order to meet (C4)-(C5), we must substitute (4)-(5) and (11)-(14) into (23)-(24). As a result, we get an equation system that, together with (6), solves for \( \lambda, q_0^H, q_0^T, f_0 \) \(^{43}\). For sake of simplicity, we assume a vector of parameters \( \Phi_H \equiv (\lambda, \theta, \epsilon_T, \mu_H, \eta) \) \(^{44}\) with \( \epsilon_T \) - the desutility with default - so large that, for any \( g^T_0 \), the sector \( T \) does never have incentive to default in equilibrium, even when \( y_T^i \) reaches its lower bound. As we can note from (15)-(17), this means that the home country reaches an equilibrium solution at a vector \( (d_0^H, q_0^H, f_0^H) \) \(^{i=NT} \) such that, in its own belief, the utility gain with default is always smaller than the desutility with penalties for default, namely, \( \pi_H^T = 0 \) in equilibrium. This assumption can be dropped without changing the main results of the model, which are presented in the next section\(^ {45}\). The existence of a vector \( \Phi_H \) which assures an equilibrium solution with \( \pi_H^T = 0 \) is proved in the appendix, where we still show that in this case \( d_0^{H,T} \) is given by

\[
d_0^{H,T}(g_0^T, h_0) = \left[ \frac{1 - \exp\left(-\epsilon_T\right)}{1 + g_0^T} \right], \tag{25}
\]

such that \( \lambda = \lambda(h_0), q_0^{H,T} = q_0^{H,T}(h_0) \) and \( f_0^H = f_0^H(h_0) \) are defined as the solution of the equation system (A2) through (A4) in the appendix. We can see in this system that \( \lambda, q_0^{H,T} \)

\(^{42}\)The subscript \( H \) indicates that the expectation below is conditioned on the home country’s belief about sector \( T \)’s endowment, which is given by \( \mu_H \).

\(^{43}\)More precisely, the home economy is in equilibrium at a vector \( (d_0^i, q_0^i, f_0) \) if and only if this vector is a solution for this system. The sufficiency follows from the strict concavity of the lifetime utility in (2).

\(^{44}\)These are the only relevant parameters for the sector \( T \)’s problem of portfolio choice. Note that \( \mu_F \notin \Phi_H \) as it refers to country \( F \)’s beliefs.

\(^{45}\)As it will be clear in the next section, we just need the possibility for heterogeneous beliefs, as given in (1), so that \( \mu_F \) can be lower than \( \mu_H \).
\( f_0^H \) are written just in function of \( h_0 \) because they do not depend on \( q_0^T \) and \( g_0^{NT} \). This in turn implies that \( d_0^{H,T} \) does not depend on \( q_0^{NT} \) in (25). Moreover, it follows from these results that \( d_0^{H,T}, q_0^{H,T} \) and \( f_0^H \) are derived independently of the equilibrium solution for \( q_0^{NT} \) and \( d_0^{NT} \). Therefore, in order to getting \( q_0^{H,NT} \) and \( d_0^{H,NT} \), it is enough to substitute \( d_0^{H,T}, q_0^{H,T} \) and \( f_0^H \) into (6) and into the equation (23) for \( i = NT \) respectively.

It is trivial in (25) that \( d_0^{H,T} \) decreases with a higher \( g_0^T \), the contractual interest rate. More interesting is that \( d_0^{H,T} \) also depends on the ERRS policy parameter \( h_0 \) and on the hedging position \( q_0^{H,T} \). This occurs because these variables impact on the period 1-wealth volatility and hence on the incentives that individuals have to smooth consumption over time: they are less encouraged to transfer wealth to present when they feel less confident about period 1- wealth. Therefore, we can conclude that a change in \( h_0 \) shifts both the foreign credit’s supply and demand curves (as functions of \( g_0^T \)), given in (20) and (25) respectively. This in turn implies that the effect of a higher or lower \( h_0 \) on the equilibrium level of \( g_0^T \) is ambiguous, as it depends on the parameters of the model, which determine ultimately the relative strength of a change in \( h_0 \) on those curves.

### 3.6 General equilibrium solution

The general equilibrium solution for the vector of endogenous variables \( z_0 \equiv (d_i^0, g_i^0, q_i^0, f_0)_{i = T, NT} \), denoted by \( \bar{z}_0 \equiv (\bar{d}_0^i, \bar{g}_0^i, \bar{q}_0^i, \bar{f}_0)_{i = T, NT} \), in function of the policy parameter \( h_0 \) and the vector of structural parameters \( \Phi \equiv (\beta, \theta, \epsilon^T, \mu_H, \mu_F, \eta) \), is defined as

\[
\bar{f}_0 = f_0(h_0, \Phi) = f_0^H(h_0), \quad (26)
\]
\[
\bar{g}_0^i = \bar{g}_0^i(h_0, \Phi) = q_0^{H,i}(h_0), \quad i = T, NT, \quad (27)
\]
\[
\bar{d}_0^T = \bar{d}_0^T(h_0, \Phi) = d_0^{H,T}(\bar{g}_0^T, h_0) = d_0^{E,T}(\bar{g}_0^T, \bar{q}_0^T, \bar{f}_0, h_0), \quad (28)
\]
\[
\bar{d}_0^{NT} = \bar{d}_0^{NT}(h_0, \Phi) = d_0^{H,NT}(\bar{g}_0^T, \bar{q}_0^{NT}, h_0) = 0, \quad (29)
\]

where \( \bar{g}_0^i = \bar{g}_0^i(h_0, \Phi) \) for \( i = T, NT \). Next, we sketch the derivation of \( \bar{z}_0 \). First, it follows from (20)-(22), (25) and (26)-(28) that\(^{46}\)

\[
\bar{g}_0^T = \frac{2(1 + r_0) \eta}{\mu_F - [\lambda(\mu_H - \eta) - \eta]} - 1, \quad \text{if} \quad \lambda(\mu_H - \eta) - \eta < \mu_F < \lambda(\mu_H - \eta) + \eta, \quad (30)
\]
\[
\bar{g}_0^T = r_0, \quad \text{if} \quad \lambda(\mu_H - \eta) + \eta \leq \mu_F; \quad (31)
\]

\(^{46}\)There is no equilibrium if \( \mu_F \leq \lambda(\mu_H - \eta) - \eta \). In this case, the credit demand curve relies on the right of the supply curve and there is no intercept between them.
where \( \lambda = \lambda(h_0) \) was defined in subsection 3.5. Note that \( \bar{g}_0^T > r_0 \) in (30)\(^{47} \). Second, substituting \( \tilde{f}_0, \bar{q}_0^T \) and \( \bar{g}_0^T \), as defined above, into (28), we get \( \bar{d}_0^T \). Third, substituting \( \bar{q}_0^T \) into (29), we get \( \bar{g}_0^{NT} \). Fourth, it follows from (6) that \( \bar{g}_0^T > r_0 \) in (30).\(^{49} \) Second, substituting \( \bar{f}_0, \bar{q}_0^T \) and \( \bar{g}_0^T \), as defined above, into (28), we get \( \bar{d}_0^T \). Third, substituting \( \bar{g}_0^T \) into (29), we get \( \bar{g}_0^{NT} \). Finally, the other endogenous variables - exports, prices and consumption - can be derived directly from \( \bar{z}_0 \) through the equations (3)-(5), (7)-(14) and the solution of the optimization problem in footnote 29. Note that all conditions (C1)-(C5) in subsections 3.4 and 3.5 are met when \( z_0 = \bar{z}_0 \): both the home and foreign countries are in equilibrium. Moreover, conditions (26)-(29) set that the foreign credit market is in equilibrium when \( (g_0^T, g_0^{NT}) = (\bar{g}_0^T, \bar{g}_0^{NT}) \).

As shown along the proof of the proposition in the appendix, since \( \pi_T^H = 0 \) in equilibrium, it follows from (16)-(17) that \( \lambda \leq 1 \). Moreover, as \( \lambda \) does not depend on \( \mu_F \), it follows from (30)-(31) that the term \( \lambda (\mu_H - \eta) + \eta \) is the cut-off level of \( \mu_F \) for a spread to be paid in equilibrium. Note also that \( \lambda \leq 1 \) implies that a spread is paid if and only if \( \mu_F < \lambda (\mu_H - \eta) + \eta \leq \mu_H \), i.e., a necessary and sufficient condition for a positive spread is that foreign investors are sufficiently more pessimistic than home debtors about the sector \( T \)'s performance and ability to repay. To better understand the case in (30), note that, although the sector \( T \) has never incentive to default on a debt amounted to \( (1 + r_0^T) d_0^{HT} (r_0^T, h_0) \), foreign investors do not share this view when \( \mu_F \) is sufficiently lower than \( \mu_H \), since in this case they realize that the sector \( T \) is not able to repay all this debt in the lowest levels of \( y_T^T \). This in turn implies that \( \pi_T^F > 0 \) and hence the arbitrage condition in (19) is not observed for \( g_0^T = r_0 \). Therefore, foreign investors will provide less credit than the amount demanded by the sector \( T \), pushing \( g_0^T \) up. Faced with a higher \( g_0^T \), the sector \( T \) will demand less credit and foreign investors will be willing to supply more credit. The market equilibrium will only occur at \( g_0^T = \bar{g}_0^T \), when condition (28) is met.

---

\(^{47}\)Since \( \mu_F < \lambda (\mu_H - \eta) + \eta \) in (30), we have that \( 2\eta > \mu_F - [\lambda (\mu_H - \eta) - \eta] \). Thus, \( \frac{1+g_0^T}{1+r_0} > 1 \).

\(^{48}\)To understand the equilibrium condition in (29), remember that, as seen in subsections 3.3 and 3.4, the assumption of that \( \epsilon^{NT} = 0 \) implies that \( d_0^{F,NT} > 0 \) is not possible in equilibrium. Therefore, given \( (\bar{g}_0^T, h_0) \), we should set \( \bar{g}_0^{NT} \) such that \( d_0^{H,NT} = 0 \) in (29). In this case, we can also set \( d_0^{F,NT} = 0 \), since it is optimal for foreign investors to lend nothing when they expected not to be repaid for sure.

\(^{49}\)To verify this result, note that \( \mu_H > \eta \) in (1).

\(^{50}\)According to (25), this is the amount of foreign credit that the sector \( T \) wishes to borrow when \( g_0^T = r_0 \).
4 Welfare effect of ERRS policies

This section derives and interprets the welfare effects of ERRS policies. As explained in subsection 2.3, such policies in this model amount to set \( h_0 = 0 \). More precisely, we show that ERRS policies may be or not Pareto-improving interventions and that this depends, among other factors, on how much pessimist foreign investors are with respect to the sector \( T \)’s ability to repay, i.e., the extent \( \mu_F \) is below \( \mu_H \). We assume that the world economy rests initially on a general equilibrium solution as the one defined in the previous section. Analytical tractability restricts us to examine interventions that consist in small changes of \( h_0 \) around 0.

First, we define \( V^i \) as the sector \( i \)’s lifetime utility as a function of \( h_0 \) and \( \Phi \), so that

\[
V^i = V^i(h_0; \Phi) \equiv U^i [\bar{z}_0, h_0], \quad i = T, NT
\]

where \( \bar{z}_0 = \bar{z}_0(h_0) \), as defined in subsection 3.6 above, is the general equilibrium solution for the vector of endogenous variables \( z_0 \equiv (d_0, g_0, q_0, f_0)_{i=T,NT} \), while

\[
U^i(z_0,h_0) \equiv \ln \left( c^i_0 \right) + \beta E_0 \left[ \ln \left( c^i_1 \right) \right]
\]

where \( c^i_t (i = T, NT, t = 0, 1) \), written as a function of \( z_0 \) and \( h_0 \), is determined by (4)-(5) and (11)-(14). More intuitively, the function \( V^i \) gives the sector \( i \)’s lifetime utility when the world economy rests on a general equilibrium solution for a given \( h_0 \) and \( \Phi \).

Next, we examine the optimality of a departure of \( h_0 \) from 0. Such an intervention leads to a Pareto-improvement if and only if \( \Delta V^i \equiv V^i(h_0; \Phi) - V^i(0; \Phi) \geq 0 \) for \( i = T, NT \), with strict inequality for at least one sector. We just analyze small enough changes in \( h_0 \) to be well approximated by a first-order Taylor expansion, so that

\[
\Delta V^i \cong \frac{\partial V^i(0; \Phi)}{\partial h_0} h_0, \quad i = T, NT
\]

whereas

\[
\frac{\partial V^T(0; \Phi)}{\partial h_0} = K(\Phi) + L(\Phi) ; \quad \frac{\partial V^NT(0; \Phi)}{\partial h_0} = -\left( \frac{\theta}{1-\theta} \right) K(\Phi) + L(\Phi) ,
\]

where

\[
K(\Phi) \equiv \frac{\partial U^T}{\partial h_0} = -\left( \frac{1-\theta}{\theta} \right) \frac{\partial U^{NT}}{\partial h_0} = -E_{0,H} \left[ \frac{\gamma_1 - \mu_H}{\gamma_1 - (1 + \bar{g}_0) d_0^T} \right] > 0 ; \quad \frac{\partial U^T}{\partial g_0^T} = \frac{\partial U^{NT}}{\partial g_0^T} \frac{\partial g_0^T}{\partial h_0}
\]

(37) (38)
and

\[ \frac{\partial U^T}{\partial \tilde{g}_T} = \frac{\partial U^{NT}}{\partial \tilde{g}_0} = -\theta \beta E_{0,H} \left[ y^T_1 - \frac{\tilde{d}_T}{y^T_1 - (1 + g^T_0) \tilde{d}_0} \right] = -\frac{\theta}{1 + g^T_0} < 0, \tag{39} \]

where all the derivatives in (37)-(39) are evaluated at \( \tilde{z}_0 = \tilde{z}_0(0) \) and \( h_0 = 0 \).

The first derivative in (37) is the direct effect of a higher \( h_0 \) on the sector \( T \)'s welfare, holding \( \tilde{g}_T \) constant: its sign is positive as this sector has its wealth volatility decreased\(^{51} \). Note the opposite sign of this effect on the sector \( NT \)'s welfare. This shows that a change in \( h_0 \) leads to a risk exposure reallocation across home country’s sectors. As one can see in subsection 2.3, the reason for this is that the wealth of both sectors increases with a higher \( y_1^T \) and decreases with a lower \( y_1^T \), so that hedging a sector against shocks to \( y_1^T \) rises necessarily the risk exposure of the other one.

The negative sign of the derivatives in (39) indicates that the welfare of both sectors increases with a fall in \( \tilde{g}_T \). The intuition behind this result is that the reduction of the sector \( T \)'s foreign liabilities, caused by a lower level of \( \tilde{g}_T \), not only increases the wealth of this sector, but also allows the home country as a whole to export less in order to finance its foreign liabilities, increasing thereby the domestic supply of the tradable good. Therefore, the sector \( NT \) also takes advantage of a lower \( \tilde{g}_T \) through the increase in the relative price of its endowment.

We still have to examine the expression for \( \frac{\partial \tilde{g}_T}{\partial h_0} \) in (38), whose size will determine whether or not a change in \( h_0 \) is a Pareto-improving intervention. For this, we examine in subsections 4.2 and 4.3 below both cases in which the sector \( T \) pays and does not pay a spread on its foreign debt in equilibrium. Before this, however, it is very helpful to examine in subsection 4.1 what would happen if we dropped the assumption of information asymmetry about debtors’ individual portfolios. This result works as a benchmark which help us explain why ERRS may be efficient when this assumption is introduced\(^{52} \).

\(^{51}\)This can be proved by using the Jensen’s inequality.

\(^{52}\)In focusing only on Pareto-improving interventions along this section, we ignore the whole issue of "distributional weights". In practice, however, the implementation of ERRS policies should depend, among other things, on the policymaker’s preferences. This issue can be formally addressed by assuming that the home country’s government maximizes a social welfare function given by \( W(V^T,V^{NT}) \), which is increasing with respect to the individual welfare of each sector. It follows from (32) that we can write it as a function of the policy parameter \( h_0 \), so that \( W(h_0;\Phi) = W[V^T(h_0;\Phi),V^{NT}(h_0;\Phi)] \). In this case, marginal ERRS policies, which consist in a very small increase in \( h_0 \), will be implemented if and only if the derivative of the function above is different from zero. In addition, note that the sign of this derivative will determine which sector must have its risk exposure decreased.
4.1 Impossibility for Pareto-improvement with perfect information

Suppose that foreign investors have perfect information about the individual hedging position of all sector $T$’s debtors. In this case, the contractual interest rate they require from each debtor will depend only on her individual hedging position, which can now be directly monitored. In this case, as the default probability of each debtor depends on her own risk exposure, debtors with different hedging positions will pay different rates. Therefore, when each debtor chooses the size and the sign of her hedging position, she has incentive to take into account the effect of this decision on the cost of her foreign debt. Given the limited structure of the model, this means that no benefit or cost of this decision is ignored by market participants. As a consequence, the risk exposure is efficiently reallocated by private markets, so that ERRS policies will never bring a Pareto improving for the home country. This point is well illustrated when we derive for this case the marginal condition of optimization with respect to $q^T_0$, which is given by

$$E_{0,H} \left[ \frac{\sigma^T_1 - f_0 \mu_H}{p_1} \frac{1}{c^T_1} \right] + E_{0,H} \left[ \frac{\delta^T_0 \frac{\partial q^T_0}{\partial q^T_0}}{p_1} \frac{1}{c^T_1} \right] = 0 \quad (40)$$

This condition must be met in equilibrium with full information. Note that the second term on the left-hand side of the equation (40) is the marginal welfare change due to the effect of the risk reallocation on the sector $T$’s contractual interest rate. As it was explained in subsection 3.5, this term does not exist in condition (24) because sector $T$’s debtors take $q^T_0$ as given when information about their individual portfolios is asymmetric. Based on the condition (40), we can see why ERRS policies are not Pareto-improving interventions under perfect information about individual hedging positions. For this, suppose on the contrary that a small change in $h_0$ brings a welfare gain for both sectors when the economy is in equilibrium with $h_0 = 0$. In this case, the equilibrium condition (40) could not have been met. The reason is that private markets are expected to provide incentives for trading, without a need for public intervention, by intervention. As a particular case, suppose that $W(V^T, V^{NT}) = \lambda V^T + (1 - \lambda)V^{NT}$, where $\lambda \in (0, 1)$ is the weight of the sector T’s welfare in government preferences. So, by using (35)-(36), the derivative of $W(h_0; \Phi)$ with respect to $h_0$, when evaluated at $h_0 = 0$, is given by

$$\frac{\partial W \left(0; \Phi \right)}{\partial h_0} = \left( \frac{\lambda - \theta}{1 - \theta} \right) K \left( \Phi \right) + L \left( \Phi \right) = \frac{\lambda}{1 - \theta} K \left( \Phi \right) + \frac{\partial V^{NT} \left(0; \Phi \right)}{\partial h_0}.$$ 

Suppose that there is $\lambda = \lambda^*$ such that the derivative above is null. Thus, since $K > 0$ and $K$ and $L$ do not depend on $\lambda$, government should set $h_0 > 0 \ (h_0 < 0)$ when $\lambda > \lambda^*$ ($\lambda < \lambda^*$). As we can see in the second equality, if $\lambda > \lambda^*$, government provides hedge to the sector $T$, even if $V^{NT}$ falls with a higher $h_0$. This example shows that government preferences can be such that intervention takes place even if it does not bring a Pareto-improvement, i.e., even when it has opposite effects on the sectors’ welfare.
if market participants are able to take full advantage of the benefits and costs of an additional risk reallocation.

4.2 Equilibrium with $\bar{g}_0^T = r_0$. This is the case described in (31), in which foreign investors are not so pessimistic about $\mu_F$ to require a positive spread. In this case, a marginal change in $h_0$ has no effect on $\bar{g}_0^T$\footnote{We just consider changes in $h_0$ so small that the inequality $\lambda (\mu_H - \eta) + \eta \leq \mu_F$ still holds after them.}: only $\bar{d}_0^T$ is affected by the shift induced by ERRS policies on the foreign credit’s supply and demand curves. Therefore, it follows from (38) that $L (\Phi) = 0$ and hence $\Delta V^T$ and $\Delta V^{NT}$, given by (34)-(36) respectively, have opposite signs. This means that there is no scope for Pareto-improvement when $\bar{g}_0^T = r_0$ because $\bar{g}_0^T$ is already at its lowest possible level. A higher (lower) $h_0$ will cause a net welfare loss for the nontradable (tradable) sector as it had its risk exposure increased without having been compensated by a fall in $\bar{g}_0^T$. This result allows us to conclude that, given the limited structure of the model, a positive spread in equilibrium, which can be shrunk by ERRS policies, is a necessary condition for these policies to be Pareto-improving interventions.

4.3 Equilibrium with $\bar{g}_0^T > r_0$. This is the case described in (30), in which foreign investors are so pessimistic about $\mu_F$ that they require a spread. In this case, we show below that there is a range for $\mu_F$ such that a marginal change in $h_0$ brings a Pareto-improvement for the home country. First, it follows from (30) that

$$\frac{\partial \bar{g}_0^T (\Phi)}{\partial h_0} = \frac{2 (1 + r_0) \lambda' (0) (\mu_H - \eta) \eta}{(\mu_F - \gamma)^2}, \tag{41}$$

where $\gamma \equiv \lambda (0) (\mu_H - \eta) - \eta$. Next, substituting (30), (39) and (41) into (38) and noting that $\lambda' (0)$ does not depend on $\mu_F$, we have that\footnote{Just the limit to right in (42) below is considered because, as seen in (30)-(31), a general equilibrium solution exists only for $\mu_F > \gamma$.}

$$\lim_{\mu_F \to \gamma^+} |L (\Phi)| = \lim_{\mu_F \to \gamma^+} \left| \frac{\theta \lambda' (0) (\mu_H - \eta)}{\mu_F - \gamma} \right| = \infty_+. \tag{42}$$

Since $K (\Phi)$ in (37) is finite and does not depend on $\mu_F$, it follows from (35)-(36) and (42) that there are low enough levels of $\mu_F$ for both $\Delta V^T$ and $\Delta V^{NT}$ in (34) to have the same sign. In these cases, we can also infer from (34)-(36) and (38)-(39) that $\Delta V^T$ and $\Delta V^{NT}$ are positive if the change in $h_0$ has the same sign of $L (\Phi)$ and then the opposite sign of the derivative in (41) - i.e., if the sign of the change in $h_0$ is such that it causes a reduction in $\bar{g}_0^T$. Therefore,
we prove that ERRS policies may be Pareto-improving interventions. Note, however, that this occurs only under the circumstances that foreign investors are enough overpessimistic, i.e., $\mu_F$ must be sufficiently lower than $\mu_H$. In order to understand this result, note first in (30) that $\bar{g}_0^T$ increases with a lower $\mu_F$: as foreign investors are more pessimistic about sector $T$’s performance and ability to repay, the supply curve in (20) shifts left, pushing $\bar{g}_0^T$ up. Moreover, as we saw in subsection 3.4, this curve becomes more inelastic as $g_0^T$ increases. Consequently, as $\mu_F$ decreases, a change in $h_0$, shifting the supply curve, has a stronger impact on $g_0^T$.

As seen above, Pareto-improvement does not always require a positive change in $h_0$. As explained in subsection 3.5, this occurs because a change in $h_0$ has an ambiguous effect on $g_0^T$, which depends on the relative strength of its impact on the foreign credit’s supply and demand curves (as functions of $g_0^T$). Therefore, although the optimality of ERRS policies does require a fall in the spread faced by the tradable sector, it is not necessarily this sector that must have its risk exposure reduced in order to push $g_0^T$ down. This is the case only when the derivative in (41) is negative. Otherwise, it is the risk exposure of the nontradable sector that must be reduced through a lower $h_0$.

Once there is a domestic market for hedging in the home country, a very important question is still to be answered: given that home country’s residents can trade privately their risk exposures, why do they fail to internalize the welfare effect of a lower debt cost into their allocative decisions? In other words, why isn’t the effect in (38) incorporated into the marginal conditions of optimization (23)-(24)? As explained in subsection 2.4, the model assumes that foreign investors can observe only the aggregate foreign debt and the aggregate hedging position of each home country’s sector. They can not monitor the individual risk exposure and the individual foreign liabilities of each home country’s debtor separately. Therefore, if an individual debtor buys more hedge in the domestic market, she can not prevent her sector as a whole from free riding on her by sharing the shrinking effect on the spread of this change in her portfolio. Moreover, as each sector is composed by a large number of identical individuals, this implies that the impact of a rise in her hedging position on the spread she pays and hence on her welfare is negligible. Consequently, when she chooses the size of this position, she has no incentive to take into account the effect of this decision on the level of $g_0^T$. The model then turns out to be a particular case of congestion game, so that the amount of risk exposure privately reallocated across sectors is below the socially optimal level. It is necessary to be clear that imperfect information is a necessary assumption for ERRS policies to be efficient. Without it, as explained in subsection 4.1, intervention is pointless because home country’s residents will have incentive to incorporate
all benefits and costs of the hedging position acquired in the domestic hedge market into their allocative decisions, so that the risk exposure will be efficiently reallocated across sectors by competitive markets.

It is important to stress that a change in $h_0$ brings a Pareto-improvement only for a certain range of $\mu_F$. For not sufficiently low levels of $\mu_F$, the fall in $g_T^T$ is not large enough to bring a welfare gain that fully compensates the welfare loss of the sector having its risk exposure increased. In this case, ERRS policies do not bring a Pareto-improvement, even if the sector $T$ pays a spread that is affected by ERRS policies and in addition foreign investors have imperfect information about the individual portfolio of home debtors. We can then conclude that spread and imperfect information are necessary, but not sufficient, conditions for Pareto-improvement.

As seen above, it is still necessary that ERRS policies cause a large enough fall in the spread. But which determines the extent of this effect? In this model, spread is paid only because foreign investors are overpessimistic and in addition the more pessimist they are, the larger the impact of those policies on the spread. As a result, intervention must not be necessarily efficient whenever there is heterogeneous beliefs about repayment. In addition, foreign investors must be sufficiently more pessimistic that home debtors about the ability of the latter to repay. In other words, it is not sufficient to have $\mu_F < \mu_H$. It is also necessary that $\mu_F$ be sufficiently lower than $\mu_H$. The conclusion is that, although there are circumstances under which ERRS policies can be efficient, there are much more cases in which this does not occur.

It is important to have in mind that the results above do not allow us to conclude that heterogeneous beliefs are, under any circumstances, a necessary condition for ERRS policies to bring a Pareto-improvement. This assumption was introduced into this model because it is the way through which the spread is caused by overpessimism and it is just this fact that allows us to insert the discussion around the optimality of ERRS policies into the literature on imperfect information-related market failures in the world capital markets. As seen in subsection 4.1, although homogeneous beliefs ($\mu_F = \mu_H$) in this model implies that there is no scope for Pareto-improvement, this occurs only because we assume in subsection 3.5, for the sake of simplicity, that the penalties for default are so large that debtors have no incentive to default, so that $g_T^T$ is already in its lowest possible level $r_0$. Suppose now that beliefs are homogeneous, but both foreign and home countries are equally so pessimistic about repayment that a spread is paid in equilibrium. Although the optimality of ERRS policies in this case is not addressed by this paper, we think that they could still bring a Pareto-improvement under some circumstances. This is because the market still fails to reach an efficient exchange rate risk sharing if we maintain
the assumption, described in subsection 2.4, that foreign creditors are imperfectly informed on the individual hedging position of each home country’s debtor. Looking more deeply into this question could be a topic for further research.

Finally, ERRS policies could have costs that must be taken into account by governments gauging their optimality. Besides the bureaucratic costs and other ones associated to errors in policy evaluation and implementation, distortionary taxation can be borrowed from literature on public debt management as another important drawback of this kind of intervention\textsuperscript{55}. Other different type of cost has to do with the process through with expectations are formed, since intervention could keep foreign investors from learning over time with their own expectational error\textsuperscript{56}.

\textsuperscript{55}Bohn (1990) suggests the reasons why this could take place. Note first that a large part of the government receipts comes from taxes on the labor income, which encourage taxpayers to spend wasteful resources trying to evade or shelter income. This excessive burden of the taxation leads to a social welfare loss that can be measured in this model in terms of wasted endowment. Note next that this excessive burden could increase if ERRS policies make public expenditures vulnerable to shocks to $y_T$. To see how this could occur, suppose that the excessive burden is an increasing and convex function of the tax rate on the labor income. In this case, if a complet set of Arrow-Debreu contingent securities existed, the optimal tax rule would be hold the tax rate constant. However, in a context of incomplete markets, government would be forced to change the tax rate to keep its budget balanced whenever they face a shock to the public expenditures. Under these circumstances, the excessive burden and then the social welfare loss increase with the volatility of the public expenditures, which in this model is determined by the shocks to $y_T$. If this volatility is enough high, ERRS policies could become unattractive.

\textsuperscript{56}We know that overpessimism in this model occurs when foreign creditors underestimate the expected future sector $T$’s performance, so that $\mu_F$ is pushed down from $\mu_H$. Moreover, we have implicitly assumed that home country’s residents form their expectations correctly. Therefore, as default probability in foreign creditors’ belief rises with a lower $\mu_F$, overpessimism implies that foreign creditors expect default at a frequency higher that the one supported by home country’s fundamentals. In this case, suppose in addition that foreign creditors update their expectations as new information on default arrives and that reliable and timely information on the realization of $y_T$ is hard to be collected or provided. Given this context, we compare the cases in that ERRS policies are implemented and are not. If there is no intervention when overpessimism arises, foreign creditors will learn over time that default does not occur so often as they expected and then they will revise their expectations on home country’s fundamentals upwards. Therefore, even with short-term welfare losses, the alternative of no intervention has the long-term benefit of making foreign creditors expectations become less sensitive to false rumors hitting the market. On the other hand, suppose that intervention does occur whenever foreign creditors are overpessimistic and that in addition it is effective to squeeze the spread. Now, the learning process above is impaired as foreign creditors will wrongly conclude that the frequency of default is low because of the government intervention, when in true this occurs because the home country’s fundamentals are not so bad as they expected. In this case, ERRS policies would keep the market from learning on its own expectational errors. Therefore, if
Costs associated to ERRS policies leads us to figure out alternative policies to cope with the imperfect information-related market failures in the model. As an example, a public effort could be done to provide timely and credible information. This strategy could, at least to some extent, attenuate the lack of information. First, foreign investors would be better informed about home country’s fundamentals. In this case, it is less likely that herd behavior will lead foreign investors to run away from home country’s securities when this decision is not supported by an actual deterioration in fundamentals. Second, foreign investors could have access to more disaggregated information and hence be also able to monitor the individual portfolio of each debtor. In this case, market would be efficient to reallocate exchange rate risk, so that intervention would be unnecessary.

5 Conclusion

We know that ERRS policies are not Pareto-improving interventions under full information and perfect competitive markets. Therefore, the model derives under which circumstances these policies may bring a Pareto improvement for a indebted small open economy. There is a need for market imperfections and several other pre-conditions. In the model, the tradable sector pays a spread on its foreign debt because foreign investors are relatively overpessimistic about repayment and in addition they observe only the aggregate exchange rate risk exposure of the tradable and the nontradable sectors. As foreign investors are not able to monitor the risk exposure of a particular debtor, the shrinking effect on the spread of a higher hedging position against exchange rate shocks can be regarded as a rival and non-excludable "good", so that our model is a particular case of congestion game. Consequently, competitive markets lead to a suboptimal reallocation of the exchange rate risk exposure across the home country. Based on Calvo and Mendoza (2000a,b), the imperfect information-related market imperfections on which this model relies could be supported by financial globalization in a context of institutional constraints, which keep foreign investors from taking full advantage of costly information, while diversification continues to be an optimal strategy.

However, Pareto-improvement also requires that the welfare loss of the sector having its ERRS policies are always triggered to avoid short-term distortions caused by overpessimism, this alternative can no more be abandoned, unless the government accepts short-term welfare losses while the learning process cited above is not fully achieved. As a result, since intervention has social costs associated to its implementation, it could be better for the government, in a long-term perspective, to leave the market works alone.
wealth volatility increased is lower than the welfare gain provided by a smaller spread. This in turn only takes place when foreign investors are very pessimistic about the home economy’s ability to repay so that the credit supply curve is very little responsive to the contractual interest rate. Otherwise, ERRS policies do not bring a Pareto-improvement, even if spread and market failures do exist.

References


Appendix

**Proposition 1** Consider a vector $\Phi_H \equiv (\beta, \theta, \epsilon^T, \mu_H, \eta)$ with a sufficiently large $\epsilon^T$ and a parameter $h_0$ close enough to zero. Then, the home country relies in equilibrium on a vector $(d_0^H, q_0^H, f_0^H)_{i=T,NT}$ such that the sector $T$ has never incentive to default, namely, $\pi_T^H = 0$ in equilibrium.

**Proof.** For a given $(d_0^i, q_0^i, f_0^i)_{i=T,NT}$, we define $x$ as the solution of the equation $(\mu_H - \eta) x = k$, where $k$, the effective cut-off level of $y_1^T$ for default, is given by (16). Substituting (16) into the equation above, we have that

$$(1 + q_0^T) d_0^T = [x(\mu_H - \eta)(1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0)] [1 - \exp (-\epsilon^T)]. \quad (A1)$$

Moreover, we know from (1) that, in home country’s belief, the lowest possible level for $y_1^T$ is $\mu_H - \eta$. According to (17), this implies that $\pi_T^H = 0$ if and only if $x \leq 1$.

As we saw in subsection 3.5, the home country reaches an equilibrium at a vector $(d_0^i, q_0^i, f_0^i)_{i=T,NT}$ if and only if this vector meets the marginal conditions (23)-(24), where consumption and relative prices are given, respectively, by (4)-(5) and (11)-(14). Consequently, it follows from the results above that there is an equilibrium solution for the home country with $\pi_T^H = 0$ if and only if there is a vector $(x, q_0^T, f_0) = \left[\lambda(h_0), q_0^{H,T}(h_0), f_0^H(h_0)\right]$, with $\lambda(h_0) \leq 1$, that solves the system

$$E_{0,H} \left[ \frac{y_1^T - f_0^H \mu_H}{y_1^T (1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0) - [x(\mu_H - \eta)(1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0)] [1 - \exp (-\epsilon^T)]} \right] = 0 ; \quad (A2)$$

$$E_{0,H} \left[ \frac{y_1^T - f_0^T \mu_H}{y_1^T (1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0) - \left[\frac{1}{\theta} x(\mu_H - \eta)(1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0)\right] [1 - \exp (-\epsilon^T)]} \right] = 0 ; \quad (A3)$$

$$\beta E_{0,H} \left[ \frac{1}{[x(\mu_H - \eta)(1 - q_0^T - h_0) + \mu_H (f_0 q_0^T + h_0)] [1 - \exp (-\epsilon^T)]} \right] = 0 , \quad (A4)$$

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where (A2)-(A4) are derived by substituting (A1) into the equation (23) for \( i = T \) and into the equation (24) for \( i = T, NT \). In particular, when \( h_0 = 0 \), it follows from (A2)-(A4) that there is an equilibrium solution with \( \pi_H^T = 0 \) if and only if there is a vector \((x, q_0^T, f_0) = \left[ \lambda(0), q_0^{H,T}(0), f_0^H(0) \right] \), with \( \lambda(0) \leq 1 \) and \( q_0^{H,T}(0) = 0 \), such that \( \lambda(0) \) and \( f_0^H(0) \) solve the system

\[
E_{0,H} \left[ \frac{y_1^T - f_0 \mu_H}{x (\mu_H - \eta) [1 - \exp (-\epsilon^T)]} \right] = 0; \quad \text{(A5)}
\]

\[
\frac{1}{x (\mu_H - \eta) [1 - \exp (-\epsilon^T)]} - \beta E_{0,H} \left[ \frac{1}{y_1^T - x (\mu_H - \eta) [1 - \exp (-\epsilon^T)]} \right] = 0. \quad \text{(A6)}
\]

According to section 4, we just examine the welfare properties of ERRS policies that consist in very small changes of \( h_0 \) around 0. Therefore, in order to prove the proposition, it is sufficient to show that, for a large enough \( \epsilon^T \), the equation (A6) is solved for \( x = \lambda(0) \leq 1 \). For this, we define the function \( A(x; \Phi_H) \) as the left-hand side of (A6), so that

\[
A(x; \Phi_H) = \frac{1}{x (\mu_H - \eta) [1 - \exp (-\epsilon^T)]} - \frac{\beta}{2\eta} \ln \left( \frac{\mu_H + \eta}{\mu_H - \eta} \right) - x (\mu_H - \eta) \left[ 1 - \exp (-\epsilon^T) \right]. \quad \text{(A7)}
\]

It is trivial to see in (A7) that

\[
\lim_{x \to 0^+} A(x; \Phi_H) = \infty_+; \quad (A8)
\]

\[
\lim_{x \to \xi^-} A(x; \Phi_H) = \infty_-, \quad (A9)
\]

where \( \xi \equiv 1/ \left[ 1 - \exp (-\epsilon^T) \right] > 1 \) as \( \epsilon^T > 0 \). Moreover, by using the Leibnitz’s rule, we have that

\[
\frac{\partial A(x; \Phi_H)}{\partial x} < 0. \quad (A10)
\]

It follows from (A8)-(A10) that the graph of the function \( A(x; \Phi_H) \) intercepts the horizontal axis at an unique point \( x = \lambda(0) \) between 0 and \( \xi \), which hence solves the equation (A6)- i.e., there is an unique \( \lambda(0) \) such that \( A(\lambda(0); \Phi_H) = 0 \). We still have to show that, given \((\beta, \theta, \mu_H, \eta)\), there is a \( \epsilon^T \) so large that \( \lambda(0) \leq 1 \). For this, it is sufficient to see in (A7) that

\[
\lim_{\epsilon^T \to \infty} A(1; \Phi_H) = \infty_-. \quad (A11)
\]

---

Note that (A6) does not depend on \( f_0 \). After we get \( \lambda(0) \) from (A6), we substitute it into (A5) in order to get \( f_0^H(0) \).
Textos para discussão

A lista completa dos textos para discussão do Departamento de Economia da PUC-Rio, bem como os arquivos eletrônicos dos artigos mais recentes (publicados a partir de 1997) pode ser obtida no nosso site no endereço HTTP://www.econ.puc-rio.br


