

Reimers, Udo

**Working Paper — Digitized Version**

## A method for solving the decentralized hierarchal multiple objective decision making problem

Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 154

**Provided in Cooperation with:**

Christian-Albrechts-University of Kiel, Institute of Business Administration

*Suggested Citation:* Reimers, Udo (1984) : A method for solving the decentralized hierarchal multiple objective decision making problem, Manuskripte aus den Instituten für Betriebswirtschaftslehre der Universität Kiel, No. 154, Universität Kiel, Institut für Betriebswirtschaftslehre, Kiel

This Version is available at:

<https://hdl.handle.net/10419/175383>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

Manuskripte

aus dem

Institut für Betriebswirtschaftslehre

der Universität Kiel

Nr. 154

A Method for Solving the  
Decentralized Hierarchical  
Multiple Objective Decision Making  
Problem

Udo Reimers

Institut für Betriebswirtschaftslehre  
Olshausenstraße 40 - 60  
2300 Kiel 1  
Federal Republic of Germany

November 1984

# A Method for Solving the Decentralized Hierarchical Multiple Objective Decision Making Problem

Udo Reimers

Institut für Betriebswirtschaftslehre der Universität Kiel  
Olshausenstr. 40-60, 2300 Kiel 1, Federal Republic of Germany

## Abstract

In this paper we are considering an organization with a central unit and several subunits. The central unit has to solve a coordination problem for the whole organization, which can be stated as: How can the decisions concerning future acts needing scarce resources being at the central unit's disposal be coordinated considering multiple objectives of all decision makers? A decomposition principle without employing any information from dual variables is developed first. Next this decomposition principle and a modified STEM version are hybridized to solve the decentralized hierarchical multiple objective decision making problem. The hybrid algorithm is discussed and an illustrative example is given at the end.

Keywords: Allocation: Resources, Decision Theory, Programming: Multiple Criteria

## 1. Introduction

There are many profit and also non-profit organizations with a central unit and several subunits. In these organizations decisions often concern some or all units. Each unit involved in the decision process is represented by one decision maker (DM). Each DM has multiple objectives. Some of them may be the same as those of the central unit, but this is not a necessary condition. The central unit has to solve a coordination problem for the whole organization, which can be stated as: How can the decisions concerning future acts needing scarce resources being at the central unit's disposal be coordinated considering multiple objectives of all DMs?

There are two ways to solve this coordination problem. The first is to formulate an integrated hierarchical multiple objective decision making problem (IHMODM problem) and to determine a compromise solution for all DMs, as

proposed by Reimers /25/ or - with some short-comings - by Hafkamp and Nijkamp /8/. The second is to formulate a multiple objective decision making problem (MODM problem) for every DM. This will be discussed in this paper, which is based on some sections of Reimers /25/. There are several approaches relating to the problem of one DM or multiple DMs with different single objectives or multiple objectives within or without a hierarchical organization (/4/ - /7/, /9/, /10/, /14/, /16/, /20/ - /24/, /26/ - /36/), which all have several deficiencies and are discussed in Reimers /25/.

For convenience let us introduce the following notation: Let  $A$ ,  $C$ ,  $b$ ,  $x$  and  $z$  be  $(M \times N)$ -,  $(L \times N)$ -,  $(M \times 1)$ -,  $(N \times 1)$ - and  $(L \times 1)$ -matrices or vectors, respectively. Let  $l$ ,  $m$ ,  $n$  and  $s$  be the indices according to the objectives, restrictions, activities and units, respectively. Let  $c_l$  be a row vector of  $C$ . Let  $f$  denote a decision variable which measures the weight a subunit's  $s$  proposal  $q_s$  gets in a solution of the central unit's problem. Please note that if we had to use an index combination like  $sq_s$  or  $sl_s$  we will only use the unique notion of  $sq$  or  $sl$ . We will call a solution  $z(x^0)$  of the vector maximum problem  $\max \{ z = Cx \mid Ax = b, x \geq 0 \}$  efficient if there exists no other solution  $z(x')$  with  $z(x') \geq z(x^0)$  and  $z_l(x') > z_l(x^0)$  for at least one  $l \in \{1, \dots, L\}$ .

In the next section a new decomposition algorithm will be introduced. In section 3 a hybrid approach of this decomposition principle and a modified STEM version /1/ will be developed and discussed in section 4. Finally an illustrative example is given.

## 2. A decomposition principle for MODM problems

The traditional price-directive (/2/, /3/), resource-directive (/15/, /17/) or mixed (e.g. /19/) decomposition principles have a major disadvantage for the use in MODM problems: They assume that there is the same single objective for all units. This allows the use of shadow prices derived from the dual variables to determine the unique optimal solution for the whole problem. It is the same solution that would have been computed for the total organizational problem not being decomposed into subproblems.

Instead of a single objective problem we have to consider multiple objective problems without unique solutions. Therefore the shadow prices derived from dual variables by different DMs are incommensurable, even if they had the same objective functions but different preferences. In addition to this no unique shadow prices exist for MODM problems, since within an efficient solution some of the dual variables related to a non-basic variable are positive and others are negative (/11/ - /13/). These are the reasons why we have to outline a new decomposition principle that does not draw on any information from the dual variables. The steps are as following:

- (D1) The central unit determines a starting resource allocation and a minimum resource allocation that does not lead to non-feasible solutions at the subunits' problems and communicates the allocation information to the subunits.
- (D2) The subunits compute the information required by the central unit following the rules (R1), (R2), (R3), (R5) and (R6) described in the next section. Objective function achievements and corresponding resource utilizations are sent to the central unit.
- (D3) The central unit determines the aspiration levels of its MODM problem to be reached in the coordination process. If one of the stopping rules outlined is met, the process will be terminated with (D5), else communication of actualized objective function demands and resource allocations will be continued.
- (D4) Using these new limits, the subunits compute the information required by the central unit considering the rules (R2), (R4) and (R5). After sending this information to the central unit the process is continued with (D3).
- (D5) The coordination process is finished by fixing the terminal objective function demands and resource allocations by the central unit, which communicates them to the subunits. Based on these limits they determine the alternatives to be realized.

If one of the following stopping rules comes true the coordination process will be finished:

- The central unit has fixed aspiration levels also for its non-bounded objective functions. All aspiration levels are (over-)achieved.
- The central unit communicates to all subunits combinations of objective function demands and resource allocations, which have been transmitted once before during the coordination process.
- None of the subunits sends information to the central unit. This means the subunits have just computed information already known by the central unit.
- During successive iterations the values of non-bounded objective functions remain unchanged or grow less than a critical value determined previously.
- The values of the non-bounded objective functions differ from their individual optimum only by a percentage smaller than the one characterizing a nearly perfect solution.

In addition to these stopping rules the central unit's DM may terminate the iteration process based on his subjective judgement and execute step (D5).

Though it has not yet been mentioned, it is of course possible not only to allocate resources but also to demand them. This is done in the example given at the end of the paper. Demands may grow during the coordination process, even if they were zero in the minimum resource allocation. A resource demand cannot be viewed as a negative resource allocation so far. Nevertheless, when we are concerned with resource allocations in this paper, we are always considering resource demands, too.

### 3. A hybrid approach of the decomposition principle for MODM problems and a modified STEM version

The decentralized hierarchical multiple objective decision making method (DHMODM method) is based on five assumptions:

- (A1) The models of all units can be formulated as linear equalities or inequalities.

(A2) Each unit is participating in the coordination process with one DM.

(A3) The information communicated to the central unit by the subunits are correct in that there is no conscious misinformation.

(A4) The DMs are consistent with regard to their compromise solutions at least during the coordination process.

(A5) Minimum and maximum of an objective function are different.

Let us consider the linear vector maximum problem (LVMP)

$$\begin{aligned}
 \max z_s &= C_s x_s \\
 (1) \quad A_s x_s &= b_s \quad (s = 0, 1, \dots, S) \\
 x_s &\geq 0
 \end{aligned}$$

with  $s = 0$  and  $s = 1, \dots, S$  denoting the central unit and the subunits, respectively. It is not necessary that the  $S+1$  models can be linked as in ordinary decomposition methods.

The DHMODM method requires a more comprehensive communication of information in every cycle than traditional coordination procedures considering only one objective function. It differs from these by being a hybrid of an interactive and a decomposition method. The first is the interactive procedure for determining compromise solutions for the MODM problems, the second is the decomposition procedure for trying to (over-)achieve all aspiration levels of the central unit.

For convenience let us recall the meaning of an efficient solution for the DHMODM problem. We will call  $z^0 = (z_0(x_0^0), z_1(x_1^0), \dots, z_S(x_S^0))^T$  an efficient solution, if there is no other  $z' = (z_0(x_0'), z_1(x_1'), \dots, z_S(x_S'))^T$  with  $z' \geq z^0$  and  $z_{s_l}(x_{s_l}') > z_{s_l}(x_{s_l}^0)$  for at least one  $(s, l) \in \{ (0, 1), \dots, (S, L_S) \}$ . The efficiency concept is applied to the organization as a whole not only to single units of it.

From the past the central unit has made some experiences which allow a first estimation of the correlation between resource allocations and possible objective function achievements. The central unit has to estimate at least one combination, but e.g. different estimations of objective function achievements

for the same resource allocation may be considered. The central problem is

$$(2) \max \quad z_0 = C_0 x_0 + \sum_{s=1}^S \left( \sum_{q=1}^{Q_s} z_{sq} f_{sq} \right)$$

$$(3) \text{ s.t. } \quad A_0 x_0 + \sum_{s=1}^S \left( \sum_{q=1}^{Q_s} b_{sq} f_{sq} \right) = b_0$$

$$(4) \quad \sum_{q=1}^{Q_s} f_{sq} = 1 \quad (s = 1, \dots, S)$$

$$(5) \quad f_{sq} \geq 0 \quad (q = 1, \dots, Q_s; s = 1, \dots, S)$$

$$(6) \quad x_0 \geq 0$$

$$(7) \quad \underline{z}_{0s} - \sum_{q=1}^{Q_s} z_{sq} f_{sq} = 0 \quad (s = 1, \dots, S)$$

$$(8) \quad \underline{b}_{0s} - \sum_{q=1}^{Q_s} b_{sq} f_{sq} = 0 \quad (s = 1, \dots, S)$$

with  $z_{sq}$  and  $b_{sq}$  denoting subunit's  $s$  objective function achievement and corresponding resource utilization in proposal  $q_s$  and  $\underline{z}_{0s}$  and  $\underline{b}_{0s}$  objective function demand and resource allocation by the central unit in any iteration cycle, respectively. In the starting phase of the DHMODM method  $z_{sq}$  and  $b_{sq}$  are estimations of the central unit and  $Q_s$  represents the number of estimations made for subunit  $s$ . To simplify, the objective functions and restrictions in (2) - (8), which are of interest to the central unit only, are not explicitly noted. (7) and (8) do not have to be integrated parts of the central unit's restrictions but may be computed after having determined a compromise solution for (2) - (6). If the first resource allocation leads to non-feasible solutions for some subunits it has to be changed accordingly.

The central unit determines a first, preliminary compromise solution. From this results a resource allocation  $\underline{b}_{0s}$ . In addition to this a resource allocation  $\underline{b}_{0s}$  is fixed.  $\underline{b}_{0s}$  is the minimum resource allocation that ensures the feasibility of subunit's  $s$  problem only. The actual resource allocations during the coordination process will not be less than  $\underline{b}_{0s}$ , unless the central unit gets information allowing this.  $\underline{b}_{0s}$  is determined because the central unit needs additional information about the objective function achievement corresponding to the minimum resource allocation. Communicating  $\underline{b}_{0s}$  and



$\underline{b}_{0s}$  to the subunits terminates the starting phase and opens the iteration phase.

Subunit  $s$  has to solve the problem

$$(9) \quad \max z_s = C_s x_s$$

$$(10) \quad \max z_{0s} = C_{0s} x_s$$

$$(11) \quad \text{s.t.} \quad C_{0s} x_s \geq \underline{z}_{0s}$$

$$(12) \quad A_{0s} x_s \leq \underline{b}_{0s}$$

$$(13) \quad A_s x_s = b_s$$

$$(14) \quad x_s \geq 0$$

$$(15) \quad b_{0s} - A_{0s} x_s = 0$$

Similar to (7) and (8) equation (15) may not be integrated into subunit's  $s$  problem and may be computed after determining a compromise to (9) - (14). The steps of the DHMODM method for a subunit  $s$  are:

(S1) Set  $j := 1$ ;  $i := 0$ ;  $X^{(j)}$  due to (9) - (14) and with respect to the information requirements;  $X^{(i)} := X^{(j)}$ ;  $D^{(i)} := \{1, \dots, L\}$ ;  $E^{(i)} := \emptyset$ ;  $v_l := 1$  for all  $l = 1, \dots, L$ .

(S2) Compute the  $L$  individual optimal efficient solutions  $\hat{z}_l$ . Compute the pay-off-table (PoT) and the minimum objective function values  $\underline{z}_l$ .

(S3) Show the PoT to the DM.

(S4) Compute a compromise solution  $x^{(i)}$  by solving the linear program

$$\min \quad \lambda - \varepsilon \cdot \sum_{l=1}^L c_l x$$

$$\text{s.t.} \quad (\hat{z}_l - \underline{z}_l)^{-1} (z_l(x) - \underline{z}_l) + v_l \lambda \geq 1 \quad \text{for } l \in D^{(i)}$$

$$x \in X^{(i)}$$

$$\lambda \geq 0$$

If  $\lambda > \delta > 0$ , show the DM  $z_l(x^{(i)})$  and continue with (S5), else present the compromise solution and go to (S11).

(S5) Is at least one objective function value  $z_l$  ( $l \in D^{(i)}$ ) satisfactory? If so, go to (S6), else to (S11).

(S6) Are all objective function values  $z_l$  ( $l \in D^{(i)}$ ) satisfactory? If so, go to (S11), else to (S7).

(S7) Is it possible to lower the objective function value for at least one  $l^* \in D^{(i)}$ ? If so, go to (S8), else to (S11).

(S8) The DM has to specify the index  $l^* \in D^{(i)}$  and as a lower bound the aspiration level  $\underline{z}_{l^*} > \underline{z}_{l^*}$ . Set  $D^{(i+1)} := D^{(i)} \setminus \{l^*\}$ ;  $E^{(i+1)} := E^{(i)} \cup \{l^*\}$ . Repeat this step until no index  $l^* \in D^{(i)}$  is specified any more.

(S9) If  $i > 0$ , the DM may specify an index  $l^* \in E^{(i)}$  and the revised lower aspiration level  $\underline{z}_{l^*} > \underline{z}_{l^*}$ , which is the new lower bound for objective function  $l^*$ . Repeat this step, until no index  $l^* \in E^{(i)}$  is specified any more.

(S10) Set  $i := i+1$ ;  $X^{(i)} := \{ x \mid x \in X^{(j)}, z_l(x) \geq \underline{z}_l \text{ with } l \in E^{(i)}, z_l(x) \geq z_l(x^{(i-1)}) \text{ for } l \in D^{(i)} \}$ . Go to (S4).

(S11) Set  $j := j+1$ ;  $v_l = (\hat{z}_l - \underline{z}_l)/(\underline{z}_l - \underline{z}_l)$  with  $l \in E^{(i)}$  and  $\underline{z}_l > \underline{z}_l$ ;  
 $v_l = (\hat{z}_l - \underline{z}_l)/(z_l(x^{(i)}) - \underline{z}_l)$  with  $l \in D^{(i)}$ . Communicate to the central unit the computed information and wait for new limits. Set  $i := 0$ ;  $X^{(j)}$  due to (9) - (14) and with respect to the information requirements;  
 $D^{(i)} := \{1, \dots, L\}$ ;  $E^{(i)} := \emptyset$ . Go to (S2) and start the next iteration cycle or, if the terminal values are on hand, compute the alternatives to be realized and finish the coordination process.

In this process the set of alternatives  $X$  and the matrix  $C$  of objective function coefficients are to be generated due to (9) to (14) with respect to the rules given by the central unit. If objectives of a subunit and the central unit coincide, (9) and (10) may be combined respectively to avoid double inclusion.

Each piece of information to be communicated to the central unit by the subunits consists of vectors of objective function values  $z_{0s}$  and corresponding resource utilizations  $b_{0s}$  computed according to (10) and (15), respectively. In the central unit's problem each of these vectors becomes one column consisting of objective function and restriction coefficients  $z_{sq}$  and  $b_{sq}$ . The subunits have to compute more information and send it to the central unit in the first iteration, just like in the later ones. In the first iteration a subunit has to send to the central unit up to  $2 + 3L_0$  information vectors of this kind, in the latter ones up to  $2 + L_0$ . There are six rules to determine these information vectors. In the first iteration the subunits have to observe all rules except (R4). The rules are:

(R1) Determine the PoT  $l$  due to (10) and (13) - (15). According to this rule the resource utilizations corresponding to the individual maximum values of the central unit's objective functions will be computed only with respect to the subunit's right hand sides. Should it become clear that for a subunit at least one objective function is unbounded the problem can be bounded by giving an upper bound of central resource utilization to the subunit, which always has to be considered. The upper bound might be the total amount of a central resource.

- (R2) Determine the PoT II due to (10) and (12) - (15). Add (11) after the first iteration. The difference to (R1) is that the resource allocation given by the central unit is considered as upper bounds. Please note again that in this case also the real resource utilization is sent to the central unit.
- (R3) Determine the PoT III due to (10) and (12) - (15) where in (12)  $\underline{b}_{0s} = \underline{b}_{0s}$  is set, the minimum resource allocation. From these vectors the central unit gets the information about the direction and extent of changes of the objective function values, if a subunit gets less resources in the following iterations than in the initial allocation.
- (R4) Determine - starting from the second cycle - a compromise solution I due to (9) - (15). According to this rule each subunit determines a compromise solution considering the resource allocation as well as the objective function demands by the central unit. This rule corresponds to (R2) in its formulation for the second and following iterations.
- (R5) Determine a compromise solution II due to (9), (10) and (12) - (15). For the determination of this compromise solution the resource allocation is considered only: the objective function demands are not. This rule corresponds to (R2) in its formulation for the first iteration.
- (R6) Determine a compromise solution III due to (9), (10) and (13) - (15). This compromise solution corresponds to (R1) and should therefore be the one that is to be realized, if the subunit's own restrictions are considered only.

After having computed the information vectors  $z_{sq}$  and  $b_{sq}$  with respect to (R1), (R2), (R3), (R5) and (R6) the subunits communicate them to the central unit. Based on all information given, the central unit starts its first cycle of the iteration phase.

During the iteration phase the central unit gets continuously more knowledge about possible objective function values and corresponding resource utilizations of the subunits. The central unit can try to get higher objective function values by changing the resource distribution among the subunits. Each corresponding combination  $z_{sq}$  and  $b_{sq}$  is added to the problem of the former iteration cycle as a new coefficient column of additional alternatives. The

central unit solves problem (2) - (8) and determines a second compromise solution. In addition to the aspiration levels that have been fixed the DM will be asked to fix aspiration levels according to non-bounded objective functions. The aspiration levels have to range between the values of the actual proposed compromise solution and the perfect solution. This solution to be achieved in the next iterations is at first non-feasible. If an aspiration level according to a non-bounded objective function is just achieved or over-achieved in one of the following iterations then the restrictions are altered so that the respective aspiration level is introduced as right-hand side and the objective function is marked as bounded. The new limits for the subunits yield from (7) and (8). The STEM procedure for the central unit will now be described, but only for those steps that differ from the subunit's ones. Instead of an S the steps of the central unit are marked with a C.

(C1) Set  $j := 0$ ;  $i := 0$ ;  $X^{(j)}$  due to (2) - (8);  $X^{(i)} := X^{(j)}$ ;  
 $D^{(i)} := \{1, \dots, L\}$ ;  $E^{(i)} := \emptyset$ ;  $v_l := 1$  for all  $l = 1, \dots, L$ .

(C4) Determine a compromise proposal  $x^{(i)}$  by solving the linear program

$$\begin{aligned} \min \quad & \lambda - \epsilon \cdot \sum_{l=1}^L c_l x \\ \text{s.t.} \quad & (\hat{z}_l - \underline{z}_l)^{-1} (z_l(x) - \underline{z}_l) + v_l \lambda \geq 1 \quad \text{for } l \in D^{(i)} \\ & x \in X^{(i)} \\ & \lambda \geq 0 \end{aligned}$$

(C4.1) If  $j \geq 2$ , go to (C4.2), else to (C4.7).

(C4.2) If  $z_l(x^{(i)}) \geq \underline{z}_l$  for at least one  $l \in D^{(i)}$ , go to (C4.5), else to (C4.3).

(C4.3) Set  $i := i + 1$ . For all  $l^*$  with  $(z_{l^*}(x^{(i-1)})) \geq \underline{z}_{l^*} \wedge l^* \in D^{(i-1)}$  set

$$\begin{aligned} D^{(i)} &:= D^{(i-1)} \setminus \{l^*\} \text{ and } E^{(i)} := E^{(i-1)} \cup \{l^*\}. \text{ Set } X^{(i)} := \{x \mid \\ x \in X^{(j)}, z_l(x) &\geq \underline{z}_l \text{ with } l \in E^{(i)}, z_l(x) \geq z_l(x^{(i-1)}) \text{ with } l \in D^{(i)}\}. \end{aligned}$$

- (C4.4) If  $D^{(i)} = \emptyset$ , go to (C16), else to (C4) and compute a new compromise solution.
- (C4.5) If for all  $s = 1, \dots, S$   $(\underline{z}_{0s}, \underline{b}_{0s})^{(j)} = (\underline{z}_{0s}, \underline{b}_{0s})^{(j')}$  with  $j' \in \{1, \dots, j-1\}$ , go to (C16), else to (C4.6).
- (C4.6) If all  $z_l(x^{(j)}) - z_l(x^{(j-1)}) \leq \Delta_l$ , go to (C16), else to (C4.7).
- (C4.7) If  $\lambda > \delta > 0$ , go to (C4.8), else to (C16).
- (C4.8) If  $j \geq 2$ , go to (C13), else to (C4.9).
- (C4.9) Show the DM the information  $z_l(x^{(i)})$  and continue with (C5).
- (C11) If  $j = 1$ , go to (C12), else to (C13).
- (C12) Fix for all  $l \in D^{(i)}$  the aspiration levels  $\underline{z}_l$  to be achieved with  $z_l(x^{(i)}) < \underline{z}_l \leq \hat{z}_l$ .
- (C13) Communicate to the subunits the objective function demands and resource allocation computed with respect to (7) and (8).
- (C14) If  $j = 0$ , set  $i := 0$  und  $Q_s := 0$  for all  $s = 1, \dots, S$  and generate with respect to (2) - (8) the first step of the decision problem.
- (C15) Alter the feasible region  $X^{(j)}$  to  $X^{(j+1)}$  by adding the new information got from the subunits and change  $Q_s$  respectively. If there is no information from any subunit, go to (C16), else set  $j := j + 1$  and continue the coordination process with (C4) and the next iteration.

(C16) The coordination process is finished. Communicate to the subunits the terminal resource allocation and objective function demands, which are computed with respect to (7) and (8).

From the second iteration on all cycles until finishing the coordination process have the same characteristics. Based on the new limits the subunits solve their problems with respect to the rules (R2), (R4) and (R5) and inform the central unit about the resulting vectors. If solutions already known to the central unit are computed these will not be communicated a second time. In certain cases it may happen that no information is sent to the central unit by a single subunit. Also, no information is being transmitted if the only feasible solution has the same values as the limits of the central unit. In this case the central unit has forced a 'perfect' solution of the subunit's problem. Communicating these values to the central unit would not lead to any additional information.

The central unit's DM does not have to participate actively in the coordination process after the first cycle, since all information needed from the DM is available. Therefore the coordination process may continue 'automatically' for the central unit until it is finished.

#### 4. Discussion

In hierarchical organizations there is a need for coordination if the subunits share scarce resources. Since the traditional coordination procedures have only been developed with respect to single objective problems, they can only be employed to multiple objective problems in a modified version, if these are solved via a compromise function to be minimized or maximized. To reach an 'optimal' solution it is necessary that all DMs have identical objectives and preferences. To overcome this restrictive assumption it is necessary to develop and employ a new decomposition principle especially designed for MODM problems as done in the DHMODM method. Other assumptions like complete knowledge of the subunits' models (/31/, /32/) or announcement of all subunits' objective functions resigning own central unit's objective functions (/22/) are not necessary with the DHMODM method. On the other hand, it

cannot guarantee the determination of 'the optimal solution', because it does not exist with the exception of the perfect solution, which is of no interest here.

We have made five assumptions to employ the DHMODM method. (A1) may be considered as a little restrictive but on the other hand it has some advantages. (A1) allows the DHMODM method to be implemented with little effort in all organizations using ordinary linear programming. Only taking linear problems into account leads to the simple and easy to understand algorithm for the complex and difficult to analyze MODM coordination problem within hierarchical decentralized organizations. (A2) avoids that we are concerned with group decisions inside the DHMODM method. If we want to cancel (A2) we have to hybrid the DHMODM and a modification of the IHMODM method (/25/).

(A3) seems to be very restrictive but it will not be too restrictive in practical applications. The DMs cannot get advantages for several planning periods (e.g. years) by cheating the central unit, since the DMs cannot be interested in communicating objective function achievements corresponding to resource utilizations which are not realizable. This would lead to non-feasible solutions and would be recognized at once or at most after one period by the central unit. Cheating may only be considered as reasonable by a subunit when determining the minimum resource allocation  $\underline{b}_{0s}$  to ensure the feasibility of subunit's  $s$  problem. To exclude this possibility the central unit has to establish adequate incentives by control and sanctions. (A4) and (A5) are assumptions for technical reasons only, having no substantial meaning to the DHMODM method.

Since the DHMODM method does not employ shadow prices, the subunits get the possibility to alter their decision models at any time during the coordination process. Applying this, the subunits have to repeat all computations already done and communicate to the central unit the fact of having altered the model and all revised information vectors. The central unit replaces the old by the new information and continues computing. If the reported changes seem to have a substantial influence upon the central unit's objective function values than the central unit may be forced to determine a new PoT and new aspiration levels to be achieved with respect to all available information. The possibility of model alteration for a single subunit during the coordination process is not given in traditional decomposition methods.



In the DHMODM method the resources are allocated not only with respect to a single objective but with respect to multiple objectives of the central unit. If the central unit wants to coordinate the subunits with respect to its objectives, it is of course necessary to announce the objectives to the subunits. This means that the central unit has to inform the subunits about its objectives, but not vice versa. To ensure comparability of subunits' objective function achievements the central unit has to give hints and rules to the subunits as to how the coefficients of the objective functions have to be determined.

The central unit does not only allocate resources to the subunits but also demands minimum objective function achievements to realize its own objectives. It cannot drop these demands, since any convergence of the DHMODM method in direction towards the solution point to be achieved could not be ensured otherwise. Else the subunits might use the central resources to fulfill their own objectives and give only little respect to the central unit's objectives. Thereby the central unit would not (or just hardly) be able to allocate the resources with respect to its objectives. In this case the subunits might argue they had kept all limits of the central unit, but central unit's objective function achievements had not been required and therefore the resources had been used efficiently with respect to own and additional central unit's objectives.

We have just mentioned the term 'convergence of the DHMODM method'. We will discuss this idea next. In this context the notion of convergence has to be restricted from the beginning, since the DHMODM method cannot attain a fixed optimal solution in a finite number of cycles. The reason is that on the one hand there is no 'optimal' solution and on the other hand the interactive part of the DHMODM method, strictly speaking, does not converge. The interactive part may be characterized only as 'practical' converging, since each DM will terminate it after some time. The weights  $v_j$  fixed in step (S11) shall serve this practical convergence. From the second iteration on the weights represent the structure of aspiration levels fixed in the preceding cycle and allow the DM to determine the new compromise solution faster than without these weights. It cannot be ensured that the central unit's aspiration levels are actually achieved, but a solution close to them is aimed.

Every compromise solution of a subunit is an efficient one, even if a single subunit is considered. This statement cannot be given with respect to the

central unit, if it is considered by itself. During the coordination process just some objective function values shall be increased without decreasing other ones. However, considering the central unit and subunits together every solution is efficient in the sense of section 3, since a rise without reduction of central unit's objective functions is possible only if at least one objective function value of at least one subunit has to be lowered to enable raising one objective function value of any one subunit.

As it is usual in linear programming, we assume that the set of feasible solutions is convex. From this premise the central unit in the DHMODM method may conclude, that, if it knows two or more feasible solutions of a subunit, every convex combination of these solutions is feasible, too, and can be realized by the subunit. The solutions are represented for the central unit by the vectors of objective function achievement and corresponding resource utilization. This means that (16) is valid for every combination of resource allocations for any subunit by the central unit:

If  $x_q \in X = \{ x \mid Ax = b, x \geq 0 \}$ , then with  $A_0 x_q = b_{0q}$  every

$$\underline{b}_0 = \sum_{q=1}^Q f_q b_{0q} = \sum_{q=1}^Q f_q A_0 x_q$$

$$(16) \quad \text{s.t.} \quad \sum_{q=1}^Q f_q = 1$$

$$f_q \geq 0 \quad \text{for all } q = 1, \dots, Q$$

is a feasible resource allocation with respect to (12).

A corresponding relation can be established for the objective function values whereby it has to be considered that the vector  $z'$  of a convex combination of already computed objective function vectors  $z_q$  is at best equal to or may even be less than an objective function vector  $z^*$ , which is computed maximizing (10) - (14) with respect to the same convex combination of objective function demands and resource allocations.

Every  $x_q$  feasible to (10) - (14) is dependent upon the objective function demands  $\underline{z}_0$  and the resource allocations  $\underline{b}_0$ . For

$$(17) \quad z_q = z(x_q(\underline{z}_0, \underline{b}_0)) \text{ it is valid that}$$

$$z' = \sum_{q=1}^Q f_q z_q = \sum_{q=1}^Q f_q z(x_q(\underline{z}_0, \underline{b}_0)) \leq z(x(\sum_{q=1}^Q f_q \underline{z}_0, \sum_{q=1}^Q f_q \underline{b}_0)) = z^* .$$

We see from (16) and (17) why every convex combination of objective function demands and resource allocations for the subunits by the central unit is feasible and can be realized. Even step (C12) can be discussed better employing (17). In this step the DM of the central unit is asked to determine aspiration levels for yet non-bounded objective functions. The aspiration levels have to range between the actual compromise solution and the perfect solution. This is done to be able to utilize the effect of (17), since it is an important task of the DHMODM method to improve the central unit's objective function achievements. If all aspiration levels are (over-)achieved, the DHMODM method will be finished.

An upper bound of  $z^*$  in (17) and the aspiration levels fixed in step (C12) is the perfect solution  $\hat{z}_0$ , since the individual maximums of the objective functions are also known for that case, if allocation of scarce central resources is not restrictive for a subunit. The objective function values are computed with respect to rule (R1) and cannot be exceeded during the coordination process, except in case the models are altered.

The convergence of the DHMODM method in its decomposition part towards the direction of a certain point is given by one of the two following facts. Either with respect to (17) new vectors of objective function values and consequently new edges expand the objective function polyhedron during the iteration cycles or no better solution can be computed. The expanding edges are outside the polyhedron of the preceding iteration and approximate the aimed solution point. Therefore the central unit either is able to complete the knowledge of its own objective function polyhedron or terminates the process at latest, when the polyhedron is not expanded any more.

The convergence of the DHMODM method at least towards the direction of the aimed solution point is implied by using a column generation scheme as in the Dantzig and Wolfe algorithm (/2/, /3/). New columns are added to the

central unit's problem from cycle to cycle in both methods. The solution is computed by using all columns. Dantzig and Wolfe show that the optimal solution is attained in a finite number of iteration cycles (/3/, p. 772). This nice mathematical property cannot be shown for the DHMODM method, since the number of efficient solutions, which may be combined, is infinite. The similarity between the decomposition algorithms of Dantzig and Wolfe and the DHMODM method consists merely of determining a solution by column combination and column generation from cycle to cycle.

Contrary to the traditional coordination principle, the decomposition algorithm of the DHMODM method does not use shadow prices at any time, but rather uses quantities. Like in resource-directive methods the central unit allocates resources to and - in this case additionally - demands objective function achievements from the subunits. These do not answer with the communication of shadow prices but send like in price-directive methods the possible objective function achievements with respect to the given resource allocation and - in this case additionally - the corresponding resource utilizations.

Under the condition that the initial resource allocations ensure the feasibility of subunits' problems the limits set during the coordination process save the feasibility. Though it cannot be shown that the decomposition algorithm of the DHMODM method terminates in a finite number of iterations, it will converge in only few iterations on the aimed solution point. This is to be expected, since the aspiration levels of the central unit will be closely approximated in the beginning iterations with respect to (17) and the rules (R2) and (R3) by demanding objective function values.

This hypothesis is based on the voluminous communication of information between central unit and subunits. In the Dantzig and Wolfe method the central unit gets  $S$  information at most, in the Maier and Vander Weide method  $2S$  at most, but in the DHMODM method in the first iteration up to  $S(2+3L_0)$  and in the following ones up to  $S(2+L_0)$ . Since the differences between the methods of Maier and Vander Weide on the one hand and Dantzig and Wolfe on the other hand with respect to the total number of required iterations and the closer approximation to the optimum (/18/, pp. 186 - 212) are only based on the differences in communicating information, it is to be expected that a more comprehensive information basis causes just more obvious results with respect to the hypotheses. By this, the DHMODM method enables the

central unit's DM to terminate the coordination process after a few iterations only and additionally to exchange information based on quantities only, both properties being highly application-oriented.

Next we will discuss some possible modifications of the DHMODM method. Similar to traditional decomposition methods, the central unit does not take much care of the subunits' objectives and objective function achievements in the DHMODM method dealt with so far. If the central unit wants to enable the subunits to fulfill their objectives with a certain degree of autonomy, this can be done by some simple modifications of the DHMODM method.

The subunits gain more autonomy, if the central unit demands less information only concerning its objectives. If the central unit resigns completely the information due to the rule (R1) and (R2), its knowledge and assessment of possible objective function achievements are based only on the subunits' compromise solutions. The information of the central unit consists of alternatives accepted by the subunits, only. If the central unit resigns additionally on (R4) and by that on objective function demands, its task is reduced to determining a feasible resource allocation.

Indeed the subunits are coordinated with respect to the central unit's objectives in both kinds of modification, but it is now impossible to compute the individual maximum objective function values of the central unit, since this cannot be determined from compromise solutions. The number of iterations will increase on the one hand, but the subunits will rather accept the decision of the central unit on the other hand.

It is obvious that the DHMODM method is able to represent various kinds of decision processes. On the one hand, the central unit may achieve its objectives without considering the subunits'. On the other hand, the subunits' objective function achievements may determine the central unit's objective function achievement.

In future developments it might be of interest to consider the decomposition principle outlined in this paper to be applied to coordination methods with single objective integer or mixed-integer programming problems as well as including integer or mixed integer problems in the DHMODM method.

### 5. An illustrative example

We will consider an organization consisting of three units, the central unit and two subunits. The central unit has two objectives and controls two resources which are allocated to or demanded from the subunits. Each subunit has three objectives, two of which are the same as of the central unit.

The problem of subunit 1 is due to (9) - (14)

$$\begin{aligned}
 \max \quad z_1 &= x_1 + 2x_2 \\
 \max \quad z_2 = z_{01} &= 2x_1 + 12x_2 \\
 \max \quad z_3 = z_{02} &= 18x_1 + 3x_2 \\
 \text{s.t.} \quad & -2x_1 + x_2 + x_3 = 10 \\
 & x_1 + 4x_2 + x_4 = 76 \\
 & 4x_1 + 6x_2 + x_5 = 150 \\
 & 2x_1 + 2x_2 + x_6 = 61 \\
 & 4x_1 - 2x_2 + x_7 = 59 \\
 & -x_1 + 2x_2 \leq \underline{b}_{01} \\
 & 5x_1 + x_2 \leq \underline{b}_{02} \\
 & x_1, \dots, x_7 \geq 0
 \end{aligned}$$

The problem of subunit 2 is due to (9) - (14)

$$\begin{aligned}
 \max \quad z_1 &= 4x_1 + 10x_2 \\
 \max \quad z_2 = z_{01} &= 3x_1 + 15x_2 \\
 \max \quad z_3 = z_{02} &= 12x_1 + 8x_2 \\
 \text{s.t.} \quad & -x_1 + x_2 + x_3 = 15 \\
 & -x_1 + 2x_2 + x_4 = 33 \\
 & x_1 + 4x_2 + x_5 = 87 \\
 & x_1 + x_2 + x_6 = 30 \\
 & 2x_1 + x_2 + x_7 = 45 \\
 & x_1 - x_2 + x_8 = 9
 \end{aligned}$$

$$\begin{aligned}
 -6x_1 + 2x_2 &\leq \underline{b}_{01} \\
 x_1 + 4x_2 &\leq \underline{b}_{02} \\
 x_1, \dots, x_8 &\geq 0
 \end{aligned}$$

The central unit has at its disposal 33 units of the first and 161 units of the second resource. Subunit 1 gets 13 and 81 units, subunit 2 gets 20 and 80 units as initial allocation and zero units in each case as minimum allocation. With respect to these limits the subunits compute the information required by the central unit. The results are:

#### Subunit 1:

Information due to							
	(R1)		(R2)		(R3)	(R5)	(R6)
q	1	2	3	4	5	6	7
$z_{01}$	224	166	216	52.71	0	206	214
$z_{02}$	126	391.5	264	286.93	0	240	298.5
$b_{01}$	32	1	20	12.07	0	17	17
$b_{02}$	38	110.5	76	80	0	80	85.5

#### Subunit 2:

Information due to						
	(R1)	(R2)	(R3)	(R5)	(R6)	
q	1	2	3	4	5	6
$z_{01}$	321	189	250	0	294	268
$z_{02}$	164	252	136	0	224	208
$b_{01}$	-2	-90	8	0	-44	-28
$b_{02}$	87	54	81	0	81	87

The central unit has to consider the problem yielding from these results and with respect to (2) - (6):

$$\max z_1 = 224f_{11} + 166f_{12} + 216f_{13} + 52.71f_{14} + 0f_{15} + 206f_{16} + 214f_{17} + 321f_{21} + 189f_{22} + 250f_{23} + 0f_{24} + 294f_{25} + 268f_{26}$$

$$\max z_2 = 126f_{11} + 391.5f_{12} + 264f_{13} + 286.93f_{14} + 0f_{15} + 240f_{16} + 298.5f_{17} + 164f_{21} + 252f_{22} + 136f_{23} + 0f_{24} + 224f_{25} + 208f_{26}$$

$$\text{s.t.} \quad 32f_{11} + f_{12} + 20f_{13} + 12.07f_{14} + 0f_{15} + 17f_{16} + 17f_{17} - 2f_{21} - 90f_{22} + 8f_{23} + 0f_{24} - 44f_{25} - 28f_{26} + x_1 = 33$$

$$38f_{11} + 110.5f_{12} + 76f_{13} + 80f_{14} + 0f_{15} + 80f_{16} + 85.5f_{17} + 87f_{21} + 54f_{22} + 81f_{23} + 0f_{24} + 81f_{25} + 87f_{26} + x_2 = 161$$

$$f_{11} + f_{12} + f_{13} + f_{14} + f_{15} + f_{16} + f_{17} = 1$$

$$f_{21} + f_{22} + f_{23} + f_{24} + f_{25} + f_{26} = 1$$

$$f_{11}, \dots, f_{17}, f_{21}, \dots, f_{26}, x_1, x_2 \geq 0$$

The pay off table is

	$z_1$	$z_2$
$\hat{x}_1$	545	344.71
$\hat{x}_2$	290	631.35

The first compromise proposal is  $z = (444.4, 515.85)^T$ .

This is not accepted as satisfactory by the DM. He sets as aspiration levels 450 and 540 for the first and second objective function, respectively. He finishes the first iteration by communicating the information to the subunits. The vectors of subunit 1 are  $\underline{z}_{01} = (183.4, 311.85)^T$  and  $\underline{b}_{01} = (10.3, 88.75)^T$  and of subunit 2  $\underline{z}_{02} = (261, 204)^T$  and  $\underline{b}_{02} = (-42, 72)^T$ . Subunit 2 has to place at the central units disposal 42 units of the first resource instead of getting any.



Both subunits reformulate their problems with respect to the new limits and the rules. Subunit 1 computes with respect to all rules a perfect solution ( $x_1 = (15.2, 12.75)^T$ ), which is enforced by the limits of the central unit. Therefore subunit 1 sends no information to the central unit. Subunit 2 computes also a perfect solution ( $x_2 = (12, 15)^T$ ), if it observes the objective function demands. Due to (R5) it computes a compromise solution without considering the demands with increasing the value of the second objective function for account of the first one and sends only this information back to the central unit.

The DM of the central unit notices that an improvement of both objective functions as compared to the first compromise proposal is impossible. The DM terminates the coordination process after the first iteration by communicating the limits of the first cycle as final.

From this example it is obvious that by the voluminous information exchange one has to expect a fast convergence towards the direction of the (unknown) solution point. In this example, the DM of the central unit forced his compromise solution with no influence on the part of the DMs of the subunits after all.

#### References:

- /1/ Benayoun, R., Montgolfier, J. de, Tergny, J., and Larichev, O., "Linear Programming with Multiple Objective Functions: Step Method (STEM)", Mathematical Programming 1 (1971) 366 - 375
- /2/ Dantzig, G.B., and Wolfe, P., "Decomposition Principle for Linear Programs", Operations Research 8 (1960) 101 - 111
- /3/ Dantzig, G.B., and Wolfe, P., "The Decomposition Algorithm for Linear Programs", Econometrica 29 (1961) 767 - 778
- /4/ Dudnikov, E.E., and Molostvov, V.S., "Multicriteria Problems in Pooling Resources", in: C. Carlsson, and Y. Kochetkov (eds.), Multiple Criteria Decision Making, North Holland, Amsterdam, 1983, 157 - 170
- /5/ Freeland, J.R., "A Note on Goal Decomposition in a Decentralized Organization", Management Science 23 (1976) 100 - 102
- /6/ Freeland, J.R., and Baker, N.R., "Goal Partitioning in a Hierarchical Organization", Omega 3 (1975) 673 - 688
- /7/ Geoffrion, A.M., and Hogan, W.W., "Coordination of Two-Level Organizations with Multiple Objectives", in: A.V. Balakrishnan (ed.), Techniques of Optimization, Academic Press, New York, 1972, 455 - 466

- /8/ Hafkamp, W., and Nijkamp, P., "Multiobjective modelling for economic-environmental policies", Environment and Planning A 13 (1981) 7 - 18
- /9/ Haimes, Y.Y., and Tarvainen, K., "Hierarchical-Multiobjective Framework for Large Scale Systems", in: P. Nijkamp, and J. Spronk (eds.), Multiple Criteria Analysis - Operational Methods, Gower, Aldershot, 1981 201 - 232
- /10/ Hall, W.A., and Haimes, Y.Y., "The Surrogate Worth Trade-Off Method with Multiple Decision-Makers", in: M. Zeleny (ed.), Multiple Criteria Decision Making, Springer, Berlin, Heidelberg, New York, 1976, 207 - 233
- /11/ Isermann, H., "The Relevance of Duality in Multiple Objective Linear Programming", in: M.K. Starr, and M. Zeleny (eds.), Multiple Criteria Decision Making, North Holland, Amsterdam, 1977, 241 - 262
- /12/ Isermann, H., "Duality in Multiple Objective Linear Programming", in: S. Zionts (ed.), Multiple Criteria Problem Solving, Springer, Berlin, Heidelberg, New York, 1978, 274 - 285
- /13/ Isermann, H., "On Some Relations between a Dual Pair of Multiple Objective Linear Programs", Zeitschrift für Operations Research 22 (1978) 33 - 41
- /14/ Isermann, H., "Investment and Financial Planning in a General Partnership", in: M. Grauer, and A.P. Wierzbicki (eds.), Interactive Decision Analysis, Springer, Berlin, 1984, 175 - 185
- /15/ Kate, A. ten, "Decomposition of Linear Programs by Direct Distribution", Econometrica 40 (1972) 883 - 898
- /16/ Korhonen, P., Wallenius, J., and Zionts, S., Some Thoughts on Solving the Multiple Decision Maker / Multiple Criteria Decision Problem and an Approach, Working Paper No. 414, School of Management, State University of New York at Buffalo, Buffalo, N.Y., USA, revised July 1980
- /17/ Kornai, J., and Lipták, T., "Two Level Planning", Econometrica 33 (1965) 141 - 169
- /18/ Leichtfuß, R., Kapitalbudgetierung in divisionalisierten Unternehmen, Gabler, Wiesbaden, 1984
- /19/ Maier, S.F., and Vander Weide, J.H., "Capital Budgeting in the Decentralized Firm", Management Science 23 (1976) 433 - 443
- /20/ Narula, S.C., and Nwosu, A.D., A Dynamic Programming Solution for the Hierarchical Linear Programming Problem, Research Report No. 37-82-P10, Rensselaer Polytechnic Institute, Troy, N.Y., USA, 1982
- /21/ Narula, S.C., and Nwosu, A.D., "Two-Level Hierarchical Programming Problem", in: P. Hansen (ed.), Essays and Surveys on Multiple Criteria Decision Making, Springer, Berlin, Heidelberg, New York, 1983, 290 - 299
- /22/ Nijkamp, P., and Rietveld, P., "Multi-Objective Multi-Level Policy Models - An Application to Regional and Environmental Planning", European Economic Review 15 (1981) 63 - 89

- /23/ Rabenstein, H.U., Dezentrale Planung: Ausgewählte Modelle und Methoden mit Ziel- und Ressourcenvorgaben, Hain, Königstein/Ts., 1979
- /24/ Rasmusen, H.J., "Multilevel planning with conflicting objectives", Swedish Journal of Economics 76 (1974) 155 - 170
- /25/ Reimers, U., Koordination von Entscheidungen in hierarchischen Organisationen bei mehrfachen Zielsetzungen, Lang, Frankfurt am Main, 1985
- /26/ Rietveld, P., Multiple Objective Decision Methods and Regional Planning, North Holland, Amsterdam, 1980
- /27/ Ruefli, T.W., "A Generalized Goal Decomposition Model", Management Science 17 (1971) B505 - B518
- /28/ Ruefli, T.W., "Behavioral Externalities in Decentralized Organizations", Management Science 17 (1971) B649 - B657
- /29/ Ruefli, T.W., "Linked Multi-Criteria Decision Models", in: J.L. Cochrane, and M. Zeleny (eds.), Multiple Criteria Decision Making, University of South Carolina Press, Columbia, S.C., USA, 1973, 406 - 415
- /30/ Shima, T., Tarvainen, K., and Haimes, Y.Y., Multiobjective Hierarchical Overlapping Coordination, Paper presented at the Fifth International Conference on Multiple Criteria Decision Making, Mons, Belgium, 1982
- /31/ Tarvainen, K., and Haimes, Y.Y., "Hierarchical-Multiobjective Framework for Energy Storage Systems", in: J.N. Morse (ed.), Organizations: Multiple Agents with Multiple Criteria, Springer, Berlin, Heidelberg, New York, 1981, 424 - 446
- /32/ Tarvainen, K., and Haimes, Y.Y., "Coordination of Hierarchical Multiobjective Systems: Theory and Methodology", IEEE Transactions on Systems, Man, and Cybernetics, SMC-12 (1982) 751 - 764
- /33/ Tarvainen, K., Haimes, Y.Y., and Lefkowitz, I., "Decomposition methods in multiobjective discrete-time dynamic problems", Automatica 19 (1983) 15 - 28
- /34/ Whitford, D.T., and Davis, W.J., "A Generalized Hierarchical Model of Resource Allocation", Omega 11 (1983) 279 - 291
- /35/ Winkofsky, E.P., Baker, N.R., and Sweeney, D.J., "A decision Process Model of R&D Resource Allocation in Hierarchical Organizations", Management Science 27 (1981) 268 - 283
- /36/ Yu, P.L., "A Class of Solutions for Group Decision Problems", Management Science 19 (1973) 936 - 946