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Optimal fiscal policy in the presence of VAT evasion: 
the case of Bulgaria

Aleksandar Vasilev*

February 6, 2018

Abstract

This paper explores the effects of fiscal policy in the presence of a VAT evasion channel, and then compares and contrasts two regimes - the exogenous vs. optimal policy case. To this end, a dynamic general-equilibrium model, calibrated to Bulgarian data (1999-2014), is augmented with a government sector. The main findings from the computational experiments performed in the paper are: (i) The optimal steady-state income tax rate is zero; (ii) The benevolent Ramsey planner provides the optimal amount of the valuable public services, which are now three times lower; (iii) The size of the grey sector is twice lower; (iv) optimal steady-state consumption tax needed to finance the optimal level of government spending is twice lower, as compared to the exogenous policy case.

Keywords: consumption tax, income tax, VAT evasion, general equilibrium, fiscal policy, Bulgaria

JEL Classification Codes: D58, E26, H26

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1 Introduction

Since the early 1990s, many macroeconomic studies have focused on the effects of observed fiscal policy in general equilibrium setups, and in particular comparing and contrasting it to a benchmark-, or ”optimal fiscal policy” regime.\textsuperscript{1} The exercise was used to inform policymakers about the taxation and spending mix in public finances, and how it needs to be adjusted to improve efficiency in the economy. The main focus of the computational experiments performed in those papers, however, has been predominantly on the effects of government purchases (consumption), public investment, and capital and labor taxes. One limitation of that literature is that it overemphasized the distinction between capital and labor income taxation, and abstracted away from consumption, or value-added, taxation (VAT). The other aspect that the literature abstracted from was the tax evasion associated with this category, a phenomenon which is well known to European countries.

Figure 1: Fiscal importance of VAT revenue in Bulgaria (1997-2012)

Furthermore, in Eastern Europe, there was also a move toward a common income tax rate,

\textsuperscript{1}For, example, Chari, Christiano and Kehoe (1994, 1999), and many others.
and reliance on indirect (consumption/VAT and excise) taxation. Mostly due to the absence of qualified tax administration in the early 1990s, Bulgaria, a small Easter European economy, and a EU member-state as of 2007, adopted a public finance model that was built on consumption-based taxation. As seen from Fig. 1 on the previous page, VAT revenue is the major source of tax revenue in Bulgaria, responsible for almost half of the total tax revenue raised.

Compared to consumption-based taxation, which is a tax on demand, income taxation in Bulgaria is of much smaller importance for the budget: for example, over the period 2007-2014, taxation of both individuals and corporations constitutes around 10% of overall tax revenue each. In order to attract foreign investors, and the decrease the incentive to declare income as the one that is levied at a lower rate, as of 2008 both capital and labor income, as well as corporate profits are taxed at the common rate of 10%. Such characteristics lead to a slightly different public finance problem, from the ones typically covered in the public finance literature. In particular, in addition to deciding on the optimal level of public spending, here the fiscal authority is also choosing two tax rates - a common income tax rate, and a tax rate on consumption. Furthermore, the government is running its fiscal policy in the presence of VAT evasion in the economy. The public finance setup, augmented here with VAT evasion channel, is an important variation from the classical approach described in Chari, Christiano and Kehoe (1994, 1999), and thus represents an important contribution to the literature, which could be of interest to policy-makers both in the EU, as well as in Eastern Europe, where the public finance model is based around low income taxes and higher indirect taxes.

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2 The common tax rate on income was introduced in order to discourage individuals from moving income between labor and capital to the category that is taxed at a lower rate.

3 The situation is very similar for other Central and Eastern European economies as well.

4 The other major source of revenue, making around a third of total tax revenues, are social contributions made by both employers and employees.

5 Ignoring income tax evasion does not change our results, as direct taxation is responsible for 20% of total tax revenue, split almost equally between revenue from personal income taxation, and taxing corporate profits. In addition, the Tax Revenue Agency had already implemented measures that curbed income tax evasion, e.g. the flat tax reform itself, introducing a requirement that each labor contract be registered with tax authorities, implementing better risk-adjusted audit strategies, etc.
The paper then proceeds to characterize optimal (Ramsey) fiscal policy in the context of the problem described above and then to evaluate it relative to the exogenous (observed) fiscal policy regime. Similar to earlier literature, e.g. Judd (1985), Chamley (1986), and Zhu (1992), allowing distortionary taxation in a dynamic general-equilibrium framework creates interesting trade-offs: On the one hand, valuable government services directly increase household’s utility. On the other, the proportional income taxes will negatively affect the incentives to supply labor and to accumulate physical capital. In turn, higher taxes reduce not only income, but also consumption, which is actually hit twice due to a second round of taxation, this time at the point of consumption. Both types of taxes lower welfare, both directly, and indirectly, by generating less tax revenue which could be spent on valuable public services. The problem is complicated further due to the presence of a VAT evasion channel, which means that due to some reasons outside the model, the government is not able to collect all its taxes.

The optimal fiscal policy problem discussed in this paper is to choose consumption and a common income tax rate to finance both utility-enhancing and redistributive government expenditure, while at the same time minimizing both the allocative distortions created in the economy, as a result of the presence of proportional taxation, and the amount of VAT evasion. The main findings from the computational experiments performed are: (i) The optimal steady-state income tax rate is zero; (ii) The benevolent Ramsey planner provides the optimal amount of the utility-enhancing public services, which are now three times lower; (iii) The optimal steady-state consumption tax needed to finance the optimal level of government spending is twice lower, as compared to the exogenous policy case.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized equilibrium system, Section 4 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the optimal taxation (Ramsey) policy problem, and evaluates the long-run effects on the economy. Section 6 concludes the paper.
2 Model Description

For the most part, the model follows the setup in Vasilev (2016a): The novelty here is in the computation of the optimal fiscal policy in the presence of VAT evasion. There is a unit mass of households who derive utility out of consumption, leisure and public services. The time available to households can be spent enjoying leisure, or on either productive or opportunistic activities leading to VAT evasion. The benefit from rent-seeking behavior is measured in terms of the share of extracted VAT payments, which is absorbed by each household. Thus, the government is assumed to be inefficient, and not being able to collect all the tax revenue, and will thus spend less on valuable public purchases and government transfers. On the production side, there is a representative firm, which produces a homogenous final good, which could be used for either consumption, investment, or government purchases.

2.1 Households

There is a unit mass of one-member households in the economy, indexed by $i$. Each household $i$ maximizes the following utility function:

$$\sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{it} + \gamma \ln (1 - h_{it}) + \phi \ln g_{it} \right\},$$

where $c_{it}$ denotes household’s $i$ private consumption in period $t$, $h_{it}$ are non-leisure hours in period $t$, $g_{it}$ is per-household consumption of public services, $0 < \beta < 1$ is the discount factor, $\gamma > 0$ is the relative weight that each household attaches to leisure, and $\phi$ is the relative weight that each household attaches to public services.

Each household $i$ starts with an initial stock of physical capital $k_{i0}$, and chooses how much to add to it in the form of new investment. Every period physical capital depreciates at a rate $\delta$, $0 < \delta < 1$. The law of motion for physical capital is described by the following equation:

$$k_{i,t+1} = i_{it} + (1 - \delta)k_{it}. \quad (2.2)$$

The real interest rate is $r_t$, hence the before-tax capital income of household $i$ in period $t$ equals $r_t k_{it}$. 

5
In addition to capital income, each household can generate labor income. However, not all hours are spent in productive activities: only $\eta_{it}$ share, $0 < \eta_{it} < 1$, is dedicated to working in the representative firm, where the hourly wage rate is $w_t$, so labor income equals $w_t \eta_{it} h_{it}$. The remaining hours, $(1 - \eta_{it})h_{it}$, are used to engage in activities, whose aim is to evade paying consumption taxes.\(^6\) The reward from engaging in VAT evasion is a certain share $\theta$, $0 < \theta < 1$, of the lost aggregate VAT tax revenue from the government, which adds to the household’s income.\(^7\) The ”prize,” or the rent, obtained as a result of the opportunistic behavior, $R_{it}$, is represented by the following technology, which is akin to the one used in Angelopoulos et al. (2009, 2011) and Vasilev (2016b):

$$R_{it} = \tau^e C_t \frac{(1 - \eta_{it}) h_{it}}{\sum_i (1 - \eta_{it}) h_{it}},$$

where $\tau^e$ is the VAT/consumption tax rate, $C_t$ denotes aggregate consumption, and $\tau^e C_t$ represents total VAT revenue in period $t$. Since the individual household is assumed to be small relative to the aggregate, $C_t$ is taken as given. The fraction $\frac{(1 - \eta_{it}) h_{it}}{\sum_i (1 - \eta_{it}) h_{it}}$ is the endogenous probability of winning the ”prize” (or getting a larger per-household ”slice” of the rent pie).\(^8\) This probability is positively related to the own time spent evading taxes, and negatively related to the time other households’ spend in tax evasion.\(^9\)

Next, household $i$’s problem can be now recast as follows:

$$\max \{c_{it},\eta_{it},k_{i,t+1}\}_{t=0}^{\infty} \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_{it} + \gamma \ln[1 - (1 - \eta_{it}) h_{it} - \eta_{it} h_{it}] + \ln g_t^e \right\}$$

s.t.

$$(1 + \tau^e) c_{it} + k_{i,t+1} - (1 - \delta) k_{it} = (1 - \tau^y) [w_t \eta_{it} h_{it} + r_t k_{it} + \pi_{it}] + g_t^e + \frac{\theta \tau^e C_t (1 - \eta_{it}) h_{it}}{\sum_i (1 - \eta_{it}) h_{it}},$$

\(^6\)In data, $1 - \eta$ is taken as a proxy to the ”hidden employment” share.

\(^7\)Parameter $\theta$ could be also interpreted as the efficiency of the rent-seeking technology.

\(^8\)More precisely, the VAT evasion in this context is evasion of the sales tax at the retail level, when the merchant does not record the sale and performs the transaction in cash, thus avoiding the tax. Examples of such activities are buying groceries from street vendors, purchasing smuggled cigarettes, and generally any transaction where neither a fiscal receipt, nor an invoice are issued.

\(^9\)This game-theoretic representation captures the bargaining between the merchant and the customer over the distribution of the evaded tax amount.
where $\tau^y$ is the proportional income tax rate ($0 < \tau^y < 1$), levied on both labor and capital income, $g_{it}$ is household $i$'s government transfer, and $\pi_{it}^f$ is household $i$'s claim on the firm’s profit. The problem generates the following optimality conditions:

$$c_{it} : \frac{1}{c_{it}} = \lambda_t (1 + \tau^c) \tag{2.6}$$

$$k_{i,t+1} : \lambda_t = \beta \lambda_{t+1} [1 + (1 - \tau^y) r_{t+1} - \delta] \tag{2.7}$$

$$(1 - \eta_{it})h_{it} : \frac{\gamma}{1 - h_{it}} = \lambda_t \theta \tau^c C_t \frac{1}{\sum_i (1 - \eta_{it})h_{it}} \tag{2.8}$$

$$TVC : \lim_{t \to \infty} \beta^t \lambda_t k_{i,t+1} = 0, \tag{2.9}$$

where $\lambda_t$ is the Lagrangean multiplier attached to household $i$’s budget constraint in period $t$.

The interpretation of the first-order conditions above is standard: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second equation is the "Euler condition," which describes how each household chooses to allocate physical capital over time. Next, at the margin, each hour spent working for the firm should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. Similarly, at the margin, an hour spent rent-seeking should equate the benefit to the utility cost. The last condition is called the "transversality condition" (TVC): it is a boundary condition, which needs to be imposed to eliminate explosive solutions.

### 2.2 Firm

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is assumed to be Cobb-Douglas that uses both physical capital, $k^f$, and labor hours, $h^f$. The firm maximizes static profit

$$\Pi_t = A (k^f_t)^\alpha (h^f_t)^{1-\alpha} - r_t k^f_t - w_t h^f_t, \tag{2.10}$$

where $A$ denotes the level of technology, which in this application will be held fixed. Since the firm rents the capital from households, the problem of the firm collapses to a sequence

---

10In the optimality conditions below, total hours will be fixed, $h_{it} = \bar{h}$ (in order to determine the distribution of time between activities); therefore, determining the productive time, $\eta_{it}$, or setting evasion hours, $(1 - \eta_{it})h_{it}$, are equivalent choices to be made by each household.
of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, i.e.:

\[ k_t^f : \alpha \frac{y_t}{k_t^f} = r_t, \]  
\[ h_t^f : (1 - \alpha) \frac{y_t}{h_t^f} = w_t. \]  

(2.11)  
(2.12)

### 2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption in order to finance spending on utility-enhancing government purchases. However, due to the presence of VAT evasion (which could be due to inefficiencies in the way tax officials operate), the government is able to collect only \( 1 - \theta \) share of the consumption tax revenue. The government budget constraint is as follows:

\[ g_{c,t} + \sum_i g_{it}^f = \left(1 - \theta\right) \sum_i R_{it} + \tau^y \left( w_t \sum_i \eta_{it} h_{it} + r_t \sum_k k_{it} \right), \]  

(2.13)

where \( (1 - \theta)R_{it} \) is the proportion of the consumption tax revenue collected from each household by the government.\(^{11}\) Government transfers would be determined residually in each period so that the government budget is always balanced.\(^{12}\)

### 2.4 Market Clearing

In addition to the optimality conditions from the household’s and firm’s problem, as presented in the previous subsections, and the government budget constraint above, we need to impose consistency among the different decisions. More specifically, this would require that in equilibrium (i) aggregate quantities equal the sum of individual allocations, and (ii)

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\(^{11}\)Given that the measure of households is unity, the proportion of hours also equals to one.

\(^{12}\)Government consumption would be also residually determined from the resource constraint.
output, capital and labor markets all clear, or for all $t$:

$$\sum_i \left[ c_{it} + k_{i,t+1} - (1 - \delta)k_{it} \right] + g_{it}^c = y_t$$  \hspace{1cm} (2.14)

$$\sum_i c_{it} = C_t$$  \hspace{1cm} (2.15)

$$\sum_i g_{it}^f = g_t^f$$  \hspace{1cm} (2.16)

$$\sum_i k_{it} = k_t^f = K_t$$  \hspace{1cm} (2.17)

$$\sum_i \eta_{it} h_{it} = h_t^f.$$  \hspace{1cm} (2.18)

### 2.5 Dynamic Competitive Equilibrium (DCE)

For a given level of technology $A$, average tax rates $\{\tau^c, \tau^y\}$, initial individual capital endowments stock $k_{i0}, \forall i$, and aggregate allocations $\{C_t, K_t\}_{t=0}^{\infty}$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_{it}, i_{it}, k_{it}, \eta_{it}, h_{it}\}_{t=0}^{\infty}$ for each household $i$, input levels $\{k_t^f, h_t^f\}$ chosen by the firm in each time period $t$, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) each household $i$ maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

#### 2.5.1 Symmetric DCE

In the general, non-symmetric, case it is very difficult to solve the system defined in the subsection above. More specifically, the model in its general formulation can generate a multitude of distributions of capital stock holdings across households, and in this sense, the equilibrium is indeterminate. Therefore, we will concentrate on a particular equilibrium, one in which all households are identical, or the symmetric solution. This requires setting $k_{i0} = k_0$, and imposing symmetry in the DCE system for all $i$, which in turn greatly simplifies the optimality conditions derived above. Since the model features a unit mass of households, this produces $c_{it} = C_t, k_{it} = K_t, h_{it} = h_t, \eta_{it} = \eta_t$, etc. In addition, in the symmetric equilibrium every household will receive an equal share of the pie, or the rent from VAT evasion will be spread uniformly (note that total evasion equals $\theta R_t = \theta \tau^c C_t$).
To calibrate the model to Bulgarian data, we will focus on the period after the introduction of the currency board (1999-2014). Data on output, consumption and investment was collected from National Statistical Institute (2015), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2015). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, the discount factor, $\beta = 0.956$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 3.491$. The labor share parameter, $\alpha = 0.429$, was obtained from Vasilev (2015a) as the average value of labor income in aggregate output over the period 1999-2014.

The relative weight attached to the utility out of leisure in the household’s utility function, $\gamma = 1.652$, was calibrated to match the fact that in steady-state consumers would supply one-third of their time endowment to working. The weight attached to public goods is set to $\phi = 0.25$ to reflect the fact that households value public consumption four times less than private consumption.\textsuperscript{13} The depreciation rate of physical capital in Bulgaria, $\delta = 0.05$, was taken from Vasilev (2015b). It was estimated as the average depreciation rate over the period 1999-2014. The share of working time used in the VAT evasion technology, $1 - \eta = 1/3$, was set as the average hidden employment share as estimated by Center for the Study of Democracy (2015).\textsuperscript{14} Finally, the average income tax rate was set to $\tau^y = 0.22$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008, plus the average rate of employee’s social security contributions, which are treated as effective taxes on labor. Finally, the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$.\textsuperscript{15} Table 1 on the next page summarizes the values of all model parameters used in the paper.

\textsuperscript{13}The value of $\phi$ is also in line with Vasilev (2016a), who found the same proportion in spending efficiency of these two categories of consumption.

\textsuperscript{14}Angelopoulos et al. (2011) find a similar value for Mexico, $\eta = 2/3$.

\textsuperscript{15}Here we abstract away from excise taxes and import duties.
Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.956</td>
<td>Discount factor</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.429</td>
<td>Capital Share</td>
<td>Data average</td>
</tr>
<tr>
<td>$1 - \alpha$</td>
<td>0.571</td>
<td>Labor Share</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.652</td>
<td>Relative weight attached to leisure</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.250</td>
<td>Relative weight attached to public goods</td>
<td>Set/Estimated</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.050</td>
<td>Depreciation rate on physical capital</td>
<td>Data average</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.670</td>
<td>Share of working hours used productively</td>
<td>Set/Estimated</td>
</tr>
<tr>
<td>$\tau^y$</td>
<td>0.220</td>
<td>Average tax rate on income</td>
<td>Data average</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.200</td>
<td>VAT/consumption tax rate</td>
<td>Data average</td>
</tr>
</tbody>
</table>

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 on the next page. The steady-state level of technology, $A$, was normalized to unity. Next, the model matches consumption-to-output ratio by construction; the investment and government purchases ratios are also closely approximated. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function.

The after-tax return, net of depreciation, $\tilde{r} = (1 - \tau^y) r - \delta$, is also very closely captured by the model. The models also correctly predicts the magnitude of VAT tax evasion relative to output, which as computed by the Center for the Study of Democracy (2015) is close to 9% of GDP. Lastly, the model predicts that the government is not able to collect 63% of its revenue, which is almost two-thirds of total revenue This number, although too high when compared to other EU member states, is in line with the number for Greece in Angelopoulos et al. (2009).
Table 2: Data Averages and Long-run Solution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Steady-state output</td>
<td>N/A</td>
<td>0.568</td>
</tr>
<tr>
<td>$c/y$</td>
<td>Consumption-to-output ratio</td>
<td>0.674</td>
<td>0.674</td>
</tr>
<tr>
<td>$i/y$</td>
<td>Investment-to-output ratio</td>
<td>0.201</td>
<td>0.175</td>
</tr>
<tr>
<td>$g^c/y$</td>
<td>Government cons-to-output ratio</td>
<td>0.159</td>
<td>0.151</td>
</tr>
<tr>
<td>$w\eta h/y$</td>
<td>Labor income-to-output ratio</td>
<td>0.571</td>
<td>0.571</td>
</tr>
<tr>
<td>$r k/y$</td>
<td>Capital income-to-output ratio</td>
<td>0.429</td>
<td>0.429</td>
</tr>
<tr>
<td>$h$</td>
<td>Share of time spent working</td>
<td>0.333</td>
<td>0.333</td>
</tr>
<tr>
<td>$\tilde{r}$</td>
<td>After-tax net return on capital</td>
<td>0.056</td>
<td>0.057</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Scale parameter, rent-seeking technology</td>
<td>N/A</td>
<td>0.628</td>
</tr>
<tr>
<td>$\theta \tau^c c/y$</td>
<td>VAT evasion-to-output ratio</td>
<td>0.090</td>
<td>0.085</td>
</tr>
</tbody>
</table>

5 The Ramsey problem (Optimal fiscal policy under full commitment)

In this section, we solve for the optimal fiscal policy scenario under full commitment. More specifically, the government will be modelled as a benevolent planner, who has the same preferences as the people in the economy, i.e. it will choose to maximize the household’s utility function, while at the same time taking into account the optimality conditions by both the household and the firm, or the equations describing the DCE.\textsuperscript{16} The fiscal instruments at government’s disposal are consumption and income tax rate, and the level of public consumption spending.\textsuperscript{17} In this section we allow only for distortionary, or proportional, taxes, thus the optimal allocations are only "second-best."\textsuperscript{18} In addition, it will be assumed that the government can also fully and credibly commit to the future sequence of taxes and spending.

\textsuperscript{16}Note that when the household and the firm are making optimal choices, they are taking all fiscal policy variables as given. Also note that the benevolent government treats everyone the same, i.e., we have already imposed the symmetry in the constraints.

\textsuperscript{17}Note that the government transfers will be held fixed at the level computed from the equilibrium under the exogenous policy case.

\textsuperscript{18}In case the government is allowed to use lump-sum taxation, it can achieve the first-best (Pareto) allocation.
In order to solve the previous problem we set up the corresponding Lagrangean,

Under the Ramsey framework, the choice variables for the government are \( \{c_t, g_t^c, k_{t+1}, w_t, r_t\}_{t=0}^{\infty} \) plus the two tax rates \( \{\tau_t^c, \tau_t^y\}_{t=0}^{\infty} \). The initial conditions for the state variable \( \{k_0\} \) as well as the sequence of government transfers \( \{g_t^f\}_{t=0}^{\infty} \) and the fixed level of total factor productivity \( A \) are taken as given. The optimal policy problem is then recast as a setup where the government chooses after-tax input prices \( \tilde{r}_t \) and \( \tilde{w}_t \) directly, where

\[
\tilde{r}_t \equiv (1 - \tau_t^y)r_t \tag{5.1}
\]
\[
\tilde{w}_t \equiv (1 - \tau_t^y)w_t \tag{5.2}
\]

Thus, government budget constraint is now represented by

\[
\tau_t^c c_t + Ak_t^\alpha h_t^{1-\alpha} - \tilde{r}_t k_t - \tilde{w}_t h_t = g_t^c + g_t^f \tag{5.3}
\]

The Ramsey problem then becomes

\[
\max_{\{c_t, n_t, k_{t+1}, g_t^c, \tilde{w}_t, \tilde{r}_t, \tau_t^c\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - (1 - \eta_t)h_t - \eta_t h_t) + \phi \ln g_t^c \right\} \tag{5.4}
\]

s.t.

\[
\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}}[1 - \delta + \tilde{r}_{t+1}] \tag{5.5}
\]
\[
\frac{\gamma}{1 - h_t} = \frac{\tilde{w}_t}{(1 + \tau_t^c) c_t} \tag{5.6}
\]
\[
\frac{\gamma}{1 - h_t} = \theta (1 + \tau_t^c) (1 - \eta_t) h_t \tag{5.7}
\]
\[
A_t k_t^\alpha (\eta_t h_t)^{1-\alpha} = c_t + k_{t+1} - (1 - \delta) k_t + g_t^c \tag{5.8}
\]
\[
\tau_t^c c_t + A_t k_t^\alpha (\eta_t h_t)^{1-\alpha} - \tilde{r}_t k_t - \tilde{w}_t \eta_t h_t = g_t^c + g_t^f \tag{5.9}
\]

In order to solve the previous problem we set up the corresponding Lagrangean,

\[
\mathcal{L} = \max_{\{c_t, n_t, k_{t+1}, g_t^c, \tilde{w}_t, \tilde{r}_t, \tau_t^c\}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1 - (1 - \eta_t)h_t - \eta_t h_t) + \phi \ln g_t^c \right\}
\]

\[
+ \lambda_1^1[-c_{t+1} + c_t (1 - \delta + \tilde{r}_{t+1})]
\]
\[
+ \lambda_1^3[\gamma (1 + \tau_t^c) (1 - \eta_t) h_t - \theta \tau_t^c (1 - h_t)]
\]
\[
+ \lambda_1^4[A_t k_t^\alpha (\eta_t h_t)^{1-\alpha} - c_t - k_{t+1} + (1 - \delta) k_t - g_t^c]
\]
\[
+ \lambda_1^5[\tau_t^c c_t + A_t k_t^\alpha (\eta_t h_t)^{1-\alpha} - \tilde{r}_t k_t - \tilde{w}_t \eta_t h_t - g_t^c - g_t^f] \tag{5.10}
\]
The optimality conditions are as follows:

\[
c_t : -\frac{\lambda_1}{\beta} + \frac{1}{c_t} + \lambda_1^1(1 - \delta + \bar{r}_{t+1}) - \lambda_4^t + \lambda_5^t \bar{c}_t = 0 \tag{5.11}
\]

\[
\eta_t : \frac{\gamma}{1 - h_t} - \lambda_3^3 \gamma (1 + \tau_t) h_t + \lambda_4^t (1 - \alpha) y_t + \lambda_5^t \left[ \frac{(1 - \alpha) y_t}{\eta_t} - \bar{w}_t h_t \right] = 0 \tag{5.12}
\]

\[
k_{t+1} : -\frac{\lambda_1}{\beta} + \lambda_4^t [r_t + 1 - \delta] + \lambda_5^t [r_t - \bar{r}_t] = 0 \tag{5.13}
\]

\[
g_t^c : \frac{\phi}{g_t^c} = \lambda_4^t + \lambda_5^t \tag{5.14}
\]

\[
\tau_t^c : \lambda_5^t \gamma (1 - \eta_t) h_t - \lambda_3^3 \theta (1 - h_t) + \lambda_5^t c_t = 0 \tag{5.15}
\]

\[
\bar{r}_t : \frac{\lambda_1}{\beta} c_{t-1} = \lambda_5^t k_t \tag{5.16}
\]

We can also add the equations for the auxiliary variables, namely

\[
y_t = A_t k_t^\alpha (\eta_t h_t)^{1-\alpha} \tag{5.17}
\]

\[
y_t = c_t + k_{t+1} - (1 - \delta) k_t + g_t^c \tag{5.18}
\]

\[
i_t = k_{t+1} - (1 - \delta) k_t \tag{5.19}
\]

\[
r_t = \frac{y_t}{k_t} \tag{5.20}
\]

\[
w_t = (1 - \alpha) \frac{y_t}{\eta_t h_t} \tag{5.21}
\]

### 5.1 Steady-state Ramsey with evasion

In this section we focus on the steady-state Ramsey allocations in the presence of VAT evasion channel. Evaluating optimality conditions and constraints in steady-state produces
the following:

\[ -\frac{\lambda^1}{\beta} + \frac{1}{c} + \lambda^1(1 - \delta + \tilde{r}) - \lambda^4 + \lambda^5 \tau^c = 0 \quad (5.22) \]
\[ \frac{\gamma}{1 - \eta} - \lambda^3 \gamma (1 + \tau^c) h + \lambda^4 (1 - \alpha) \frac{y}{\eta} + \lambda^5 \left[ \frac{(1 - \alpha) y}{\eta} - \tilde{w} h \right] = 0 \quad (5.23) \]
\[ -\frac{\lambda^4}{\beta} + \lambda^4 [r + 1 - \delta] + \lambda^5 [r_t - \tilde{r}] = 0 \quad (5.24) \]
\[ \frac{\phi}{g^c} = \lambda^4 + \lambda^5 \quad (5.25) \]
\[ \lambda^3 \gamma (1 - \eta) h - \lambda^3 \theta (1 - h) + \lambda^5 c = 0 \quad (5.26) \]
\[ \frac{\lambda^1 c}{\beta} = \lambda^5 k \quad (5.27) \]
\[ y = Ak^\alpha (\eta h)^{1-\alpha} \quad (5.28) \]
\[ y = c + \delta k + g^c \quad (5.29) \]
\[ i = \delta k \quad (5.30) \]
\[ r = \alpha \frac{y}{k} \quad (5.31) \]
\[ w = (1 - \alpha) \frac{y}{\eta h} \quad (5.32) \]

Here Eqs. (5.22)-(5.32) directly follow from equations (5.11)-(5.21). Note that since in steady-state \( \frac{\lambda^i}{\beta} = \lambda^i [r + 1 - \delta] \), it follows that \( r = \tilde{r} \), which means that \( \tau^y = 0 \). But then it follows that \( w = \tilde{w} \), since both factors of production are taxed at the same rate. Note also that with a fixed degree of evasion parameter, consumption tax rate is again residually determined from the government budget constraint. Table 3 on the next page reports the results and compares the observed vs. the optimal fiscal policy regime.

Compared to the exogenous policy case, under optimal fiscal policy the benevolent government sets the income tax rate to zero, as in Judd (1985), Chamley (1986), and Zhu (1992), which leads to a higher capital in steady-state. Since we hold total hours (which could be distributed between working or evasion activities) fixed in this scenario, and the share of productive hours increases under the optimal policy case, steady-state output under the second-best equilibrium is also higher, the same upward change is observed in investment, private and public consumption. Note that the share of productive hours increases due to the increase in the marginal product of labor, or the wage, which is a direct consequence of the increased capital stock. In other words, productive hours are reallocated to the official
Table 3: Exogenous vs. Ramsey policy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Data</th>
<th>Exo. policy</th>
<th>Ramsey</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>Steady-state output</td>
<td>N/A</td>
<td>0.568</td>
<td>0.981</td>
</tr>
<tr>
<td>$c/y$</td>
<td>Consumption-to-output ratio</td>
<td>0.674</td>
<td>0.674</td>
<td>0.724</td>
</tr>
<tr>
<td>$i/y$</td>
<td>Investment-to-output ratio</td>
<td>0.201</td>
<td>0.175</td>
<td>0.224</td>
</tr>
<tr>
<td>$k/y$</td>
<td>Capital-to-output ratio</td>
<td>3.491</td>
<td>3.491</td>
<td>4.475</td>
</tr>
<tr>
<td>$g^c/y$</td>
<td>Government cons-to-output ratio</td>
<td>0.159</td>
<td>0.151</td>
<td>0.052</td>
</tr>
<tr>
<td>$w\eta h/y$</td>
<td>Labor income-to-output ratio</td>
<td>0.571</td>
<td>0.571</td>
<td>0.571</td>
</tr>
<tr>
<td>$rk/y$</td>
<td>Capital income-to-output ratio</td>
<td>0.429</td>
<td>0.429</td>
<td>0.429</td>
</tr>
<tr>
<td>$h$</td>
<td>Share of time spent working</td>
<td>0.333</td>
<td>0.333</td>
<td>0.333</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Share of productive time</td>
<td>0.670</td>
<td>0.670</td>
<td>0.948</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>After-tax net return on capital</td>
<td>0.056</td>
<td>0.057</td>
<td>0.046</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Scale parameter, rent-seeking technology</td>
<td>N/A</td>
<td>0.627</td>
<td>0.627</td>
</tr>
<tr>
<td>$\tau^y$</td>
<td>Income tax rate</td>
<td>0.220</td>
<td>0.220</td>
<td>0.000</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>Consumption tax rate</td>
<td>0.200</td>
<td>0.200</td>
<td>0.098</td>
</tr>
<tr>
<td>$\theta \tau^c c/y$</td>
<td>VAT evasion-to-output ratio</td>
<td>0.265</td>
<td>0.085</td>
<td>0.045</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Welfare gain</td>
<td>-</td>
<td>0.000</td>
<td>0.625</td>
</tr>
</tbody>
</table>

sector, instead of being used for rent-seeking.\textsuperscript{19} The real interest rate is also lower which is a function of the higher capital stock, which overcompensates for the absence of income taxation.

Note that the only source of revenue is consumption taxation. Since it is a non-distortionary tax, in the Ramsey framework its rate will be determined residually to achieve government budget balance. Since public consumption is now lower, and the level of government transfers is held equal to its level from the exogenous policy, the consumption tax rate can drop by half to less than 10%, which also decreases by half the size of the grey economy. This feeds back into the share of productive hours $\eta$ and leads to the reallocation of productive hours to the official sector, as described above.

\textsuperscript{19}In a way, total hours and productive hours react in the same way. That is why total hours are held fixed; otherwise the model produces indeterminacy.
Lastly, there is a substantial improvement of welfare that can be realized by moving from the equilibrium under the exogenous policy regime to the equilibrium with optimal fiscal policy. Welfare gain, measured in terms of additional consumption ($\xi$), is almost 0.625, which means that in order to make people as well off as they are under the Ramsey regime, the benevolent government needs to increase the steady-state consumption under the exogenous policy case by two-thirds to make them indifferent to the allocation under Ramsey regime.\footnote{The expression behind $\xi$ is derived in the Appendix.}

Overall, our results are new and could be of interest to policy makers, as previous research had ignored the important dimension of VAT evasion and its relevance for fiscal policy.

The limitations of the study should be acknowledged. For example, in the optimal policy case, the government has no way of directly affecting the degree of VAT evasion. It seems reasonable to assume that the government can spend more on enforcement of the tax laws by hiring more tax inspectors. Another reason for the VAT evasion might be the high consumption tax rate itself. Unfortunately, endogeneizing $\theta$ and making it respond to either the level of the consumption tax rate itself, and/or to spending on law and order, and especially solving for the optimal policy turns to be a complicated problem. That is why here we decided to compute the optimal policy for the case when the degree of evasion parameter is being held fixed. Possible extensions along the lines above are left for future research.

### 6 Conclusions

This paper characterized optimal fiscal policy in the presence of a VAT evasion channel, and evaluated it relative to the exogenous (observed) one. The results were evaluated in light of consumption vs. income taxation debate, the issue of optimal provision of valuable public services, and the effect of fiscal policy on the size of VAT evasion. To this end, a dynamic general-equilibrium model, calibrated to Bulgarian data (1999-2014), was set up with a richer public finance side. Bulgarian economy was chosen as a case study due to its dependence on consumption taxation as a source of tax revenue, and the prevalence of VAT evasion. The main findings from the computational experiments performed in the paper are:
(i) The optimal steady-state income tax rate is zero; (ii) The benevolent Ramsey planner provides the optimal amount of the utility-enhancing public services, which are now three times lower; (iii) The optimal steady-state consumption tax needed to finance the optimal level of government spending is twice lower, as compared to the exogenous policy case.

References


**Appendix: Derivation of the welfare gain measure**

In this section we derive the formula behind the steady-state consumption gain measure, $\xi$, used in the computational experiment. In other words, by what percent we need to increase household’s steady-state consumption under the exogenous (observed) fiscal policy, in order to make it indifferent to the allocations from the optimal fiscal policy regime? That requires

$$
\sum_{t=0}^{\infty} \beta^t \left\{ \ln(1 + \xi)c^e + \gamma \ln(1 - h^e) + \phi \ln(g^e)c^e \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \ln c^o + \gamma \ln(1 - h^o) + \phi \ln(g^o)c^o \right\},
$$

19
where "e" denotes an allocation from the exogenous (or "observed") policy case, while "o" is an allocation obtained under the optimal policy case. Since we focus on the long-run consumption gain, it follows that

$$\sum_{t=0}^{\infty} \beta^t \ln(1 + \xi) + \gamma \ln(1 - h^e) + \phi \ln(g^c)^e = \sum_{t=0}^{\infty} \beta^t \ln(1 + \xi) + \gamma \ln(1 - h^o) + \phi \ln(g^o)^o,$$

or

$$\frac{1}{1 - \beta} \{\ln(1 + \xi) + \gamma \ln(1 - h^e) + \phi \ln(g^c)^e\} = \frac{1}{1 - \beta} \{\ln(1 + \xi) + \gamma \ln(1 - h^o) + \phi \ln(g^o)^o\}.$$

Cancel the common multiplier to obtain

$$\ln(1 + \xi) + \ln(c^o - c^e) + \phi \ln(g^o)^o = \ln(c^o - c^e) + \phi \ln(g^o)^o.$$ 

Rearrange terms, and after some algebra one can obtain

$$\ln(1 + \xi) = \ln\left[\frac{c^o}{c^e}\right] + \gamma \ln\left[\frac{1 - h^o}{1 - h^e}\right] + \phi \ln\left[\frac{(g^o)^o}{(g^c)^e}\right],$$

$$\ln(1 + \xi) = \ln\left[\left(c^o/c^e\right)\left(1 - h^o/1 - h^e\right)^\gamma\left[g^o/g^c\right]^\phi\right],$$

$$1 + \xi = \left[c^o/c^e\right]\left[1 - h^o/1 - h^e\right]^{\gamma}\left[g^o/g^c\right]^\phi,$$

$$\xi = \left[1 - h^o/1 - h^e\right]^{\gamma}\left[g^o/g^c\right]^\phi - 1,$$