

Lemoy, Rémi; Raux, Charles; Jensen, Pablo

**Conference Paper**

## Where in cities do "rich" and "poor" people live? The urban economics model revisited

56th Congress of the European Regional Science Association: "Cities & Regions: Smart, Sustainable, Inclusive?", 23-26 August 2016, Vienna, Austria

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

*Suggested Citation:* Lemoy, Rémi; Raux, Charles; Jensen, Pablo (2016) : Where in cities do "rich" and "poor" people live? The urban economics model revisited, 56th Congress of the European Regional Science Association: "Cities & Regions: Smart, Sustainable, Inclusive?", 23-26 August 2016, Vienna, Austria, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:

<https://hdl.handle.net/10419/174661>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Where in cities do “rich” and “poor” people live? The urban economics model revisited

Rémi Lemoy<sup>a,b</sup>, Charles Raux<sup>b</sup>, Pablo Jensen<sup>c,d</sup>

<sup>a</sup>University of Luxembourg, Maison des Sciences Humaines, 11 Porte des Sciences, L-4366 Esch-Belval, Luxembourg

<sup>b</sup>LAET (Transport, Urban Planning and Economics Laboratory), CNRS and University of Lyon, Lyon, France

<sup>c</sup>Laboratoire de Physique, Ecole Normale Supérieure de Lyon, CNRS and University of Lyon, Lyon, France

<sup>d</sup>IXXI (Institut Rhônalpin des Systèmes Complexes), Lyon, France

---

## Abstract

In this work, we exploit the power of the Alonso-Mills-Muth (AMM) urban economics model and show that various utility functions and plausible conditions offer alternative explanations of households' location by income within a city. These include the existence of a “rich” center and more complex socio-spatial urban forms, for instance alternating a rich center, poor suburbs and a rich outer ring, which have not yet been derived from the AMM model to our knowledge. In doing so we combine analytical ideas and illustrations by the means of an agent-based model. The hypothesis of a central or non-central amenity is also studied, leading to different insights on the issue.

---

## Introduction

In a widely cited paper, Jan K. Brueckner, Jacques-François Thisse and Yves Zenou (1999) asked “Why is central Paris rich and downtown Detroit poor?” They pointed at the “locational indeterminacy” of the Alonso-Mills-Muth (AMM) monocentric model which identifies two opposing forces: on the one hand the preference of the high income households for housing consumption drives them to the outskirts, on the other hand their high opportunity cost of commuting time drives them toward the job center. Depending on local conditions regarding the evolution of commuting cost and housing consumption with respect to income, rich households may tend to live either in

---

*Email addresses:* remilemoy@gmail.com (Rémi Lemoy), charles.raux@let.ish-lyon.cnrs.fr (Charles Raux), pablo.jensen@ens-lyon.fr (Pablo Jensen)

the center or in the suburbs. In order to overcome this indeterminacy Brueckner, Thisse and Zenou propose an amenity-based theory which links the location of income groups to the spatial pattern of (central) amenities and hence to city idiosyncracies.

The aim of this paper is to enrich the approach of Brueckner, Thisse and Zenou, based on the AMM urban economics model. We show that various utility functions can be used in this framework to obtain a “rich” center, and also more complex socio-spatial urban forms. A special feature of this work is the use of agent-based simulations besides analytical ideas.

Indeed, agent-based models and more generally numerical simulations are interesting tools giving complementary results when related to analytical resolution. We build here on a previous work, Lemoy et al. (2013), which presents an agent-based model reproducing accurately the results of the standard urban economics model (AMM model). We use this agent-based simulation framework with a population of agents separated in two income groups, in order to study the question of the socio-spatial structure of cities. We keep the two “sketch” cities as in Brueckner et al. (1999), that is to say the "European" type city (like Paris for instance), with schematically speaking, a rich center and a poor periphery, and the "North American" city (like Detroit), with a poorer center and richer households in the periphery. This last configuration is the usual result of the AMM model when several income groups are introduced.

Lemoy et al. (2010) present a discussion on the difficulty to represent in a satisfactory way the "European" type city with the standard AMM model. An important reason for this difficulty is the fact that they consider a log-linear (or Cobb-Douglas) utility function, as most studies using concrete examples of utility functions. This function is very convenient for analytical resolution and has some interesting properties for calibration, if one supposes that the budget shares of composite good and housing do not vary with income.

However, this hypothesis can be challenged since the share of income spent on housing is shown empirically to vary with income. In a first stage a Cobb Douglas function is still used, but taking into account various shares of housing expenses for the two income groups. An analytical insight and an illustration with our agent-based model show that, depending on the relative values of the city radius and a critical radius depending on model parameters, a more substantial pattern may emerge, for instance a rich center (“European”-type city) surrounded by a poor suburb *and then* a

rich periphery.

This last urban form reflects a U-shaped curve of the income as a function of the distance to the city center. This result can be linked to empirical evidence in several big French cities. For instance in Paris metropolitan area the richest districts are observed in the city center, extending up to the western suburbs and outskirts, while other (and especially northern) suburbs gather lower income districts, followed by outskirts gathering again average and upper income districts (François et al. (2011)). Other urban areas like Lyon, Toulouse and Bordeaux show also a concentration of rich households in the city center and similar alternations of lower and upper income districts when going away from the center (Caubel (2005)). In addition, the U-shaped curve can also be seen in older North American cities, like New York, Chicago or Philadelphia (see Glaeser et al. (2008)), which are closer to the "European" pattern than to the usual socio-spatial structure of North American cities.

Since the Cobb-Douglas utility function fixes budget shares, there is no possible substitution between variables. In order to overcome this limitation, we use in a second stage a constant elasticity of substitution utility function (CES, see Chung (1994)), with an exogenous amenity, as in Brueckner et al. (1999). The agent-based model is then able to reproduce a "European" urban social structure. However, here again it is shown that depending on the distance to the center, bid-rent curves of rich and poor households may intersect again, yielding a rich periphery and hence the U-shaped income variation according to the distance from the center. Moreover, thanks to the agent-based model, we illustrate the effect of a displacement of the amenity away from the center and show that more complex socio-spatial structures can be found.

The remainder of the paper is structured as follows. In the first section the agent-based model is briefly presented along with the mechanisms introduced to reach an equilibrium corresponding to the analytical one. Section 2 and section 3 present respectively the analyses with the Cobb-Douglas utility function and the CES utility function. Discussion and conclusion are given in the final section.

## 1. An agent-based model of urban economics

We first present rapidly the numerical simulations used in Lemoy et al. (2013) and in the present work to find the equilibrium of the AMM model in cases where analytical resolution is difficult. The use of these simulations to solve for the equilibrium of the AMM model can be linked to the Monte Carlo method (Binder and Heermann (2010)) or to local search optimization algorithms in computer science (Lenstra (2003)). We use an agent-based framework, where a population of  $N$  agents is given behavior rules which reproduce the competition for land in the AMM model.

Agents live on a 2-dimensional grid representing the urban space, which is polarized by the presence of a central business district (CBD). All agents work in the CBD and commute daily for work, with an associated transport cost which is proportional to the distance traveled. This distance, which we denote by  $x$ , corresponds to the distance between the housing location of an agent and the CBD, and the associated transport cost is then  $tx$ , with  $t$  the transport cost per unit distance.

Agents are divided in two groups of incomes  $Y_r$  and  $Y_p$  for rich and poor, with  $Y_r > Y_p$ . Their income is spent entirely between transport expenses for daily commuting  $tx$ , housing expenses, and the consumption of a composite good  $z$  of unit price, representing all other consumptions which do not depend on  $x$ . Housing expenses are written  $ps$ , with  $p$  the price of a unit surface of housing at the considered location, and  $s$  the surface of the housing lot. Price  $p$  is a variable attached to each cell of the simulation grid. Cells have a fixed surface  $s_{\text{tot}}$ , and the side of a cell is supposed to be of unit length. However cells may welcome several housing lots, that is to say density is endogenous.

Agents wish to maximize their economic welfare, represented by a utility function  $U(z, s)$  depending on consumptions of composite good  $z$  and housing  $s$ . The budget constraint can be written  $Y_i = z_i + t_i x + p s_i$ , with  $i = r, p$  (for rich and poor agents respectively). At given price  $p$  and location  $x$ , agents have optimal composite good and housing consumptions. With a Cobb-Douglas utility function  $U_i = z_i^\alpha s_i^\beta$  with  $\alpha$  and  $\beta$  positive parameters such that  $\alpha + \beta = 1$ , this yields  $z_i = \alpha(Y_i - t_i x)$  and  $p s_i = \beta(Y_i - t_i x)$  for  $i = r, p$  (see Chung (1994)).

The dynamics of this agent-based model consists in a simple move and bidding mechanism. Simulations start with a random configuration: agents are located at random on the simulation space. The prices in all cells are initialized at an agricultural rent  $R_a$ , which is a minimal price in

the system, corresponding to an agricultural use of land. At each time step of the model, an agent and a cell are chosen at random. The agent, having initially utility  $U_0$ , moves into the cell if this one provides him with a higher utility  $U_1$  at its current price  $p_n$ , and enough space. However, the agent needs to propose a bid  $p_{n+1}$  on the price of the cell, which has the following form

$$p_{n+1} = p_n \left( 1 + \varepsilon \frac{U_1 - U_0}{U_0} \right)$$

where  $\varepsilon$  is a positive parameter controlling the magnitude of this bid.

This bidding mechanism describes how price increases in attractive cells. It competes with the decreasing price of cells which are fully or partially vacant. Indeed, vacancy indicates that these cells are not attractive at their current price  $p_n$ , which the landowner decreases to  $p_{n+1}$  following

$$p_{n+1} = p_n - (p_n - R_a \times 0.9) / T_p$$

where  $T_p$  is a parameter controlling the speed of this decrease of prices. This formula yields an exponential decrease of the price, which goes to the minimal price corresponding to the agricultural rent  $R_a$  after a finite number of steps. If the price reaches  $R_a$ , this decrease stops.

As Lemoy et al. (2013) show, this agent-based system reaches a discrete version of the analytical equilibrium of the AMM model. Parameters  $\varepsilon$  and  $T_p$  can have a dependency on the occupation rate of cells in order to accelerate the convergence to the equilibrium. Throughout this paper, we will use the values  $\varepsilon = 0.5$  and  $T_p = 100$ . These behavior rules provide a robust mechanism pushing the agent-based system towards the equilibrium of the AMM model. Indeed, this equilibrium can also be reached in cases where analytical treatment is difficult, as in the case of our present study of the socio-spatial structure of cities.

## 2. A critical radius for income related location

As was shown in the previous section, the Cobb-Douglas utility function has the property to fix the shares of income (net of transport cost) spent by the agent on different items. However, the empirical literature shows an evolution with income of the share of income spent by households on housing. On average, when income increases, the share of income spent on housing decreases: see

for instance Accardo and Bugeja (2009) or Polacchini (1999) for evidence respectively in France and Paris region; Cervero et al. (2006) for evidence in the USA.

Moreover, the value of time increases with income with an elasticity smaller than one (see for instance Small (1992); Wardman (2001a,b)). In this section, we study the influence of these factors together – evolution with income of the value of time and of the share of income spent on housing – on the location choices of households within the AMM model. We still use a Cobb-Douglas utility function. However, we suppose here that the share of income spent on housing and composite good depends on income. With two income groups, this is taken into account by introducing different parameters  $\alpha_r, \beta_r$  and  $\alpha_p, \beta_p$  in the utility functions of rich and poor agents. We still have  $\alpha_r + \beta_r = \alpha_p + \beta_p = 1$ , but rich agents spend a smaller part of their income on housing:  $\beta_r < \beta_p$ .

Following empirical literature the value of time is assumed to be higher for the rich income group. As a consequence, we postulate that the (unit distance) transport cost  $t_r$  of rich agents is higher than the transport cost  $t_p$  of poor agents,  $t_r > t_p$ , due to the difference of time costs, supposing that the monetary part of the transport cost is the same for both income groups.

### 2.1. Critical distance and city boundary

The condition which determines the social structure of the city is detailed in Fujita (1989) (or Goffette-Nagot et al. (2000)): in the case of a continuum of income groups, the income group located near the center is the high income group if the income elasticity of marginal transport cost (pulling force) is higher than the income elasticity of the demand for housing (pushing force). If the income elasticity of the demand for housing is higher, then the low income group is located near the center.

In our case where only two income groups are considered, the ratio of the marginal transport cost to the demand for housing should be compared between the two income groups, as detailed in Appendix A. The group with the higher ratio in absolute value will be located closer to the center (see Appendix A). In our case then, the high income group is located near the center ("European-type" city) if

$$\frac{t_r}{\beta_r(Y_r - t_r x)} > \frac{t_p}{\beta_p(Y_p - t_p x)} \quad (1)$$

Condition (1) can also be written

$$\frac{t_r}{t_p} > \frac{\beta_r(Y_r - t_r x)}{\beta_p(Y_p - t_p x)}$$

This inequality can be interpreted as follows: to have rich agents located near the center, the ratio of transport costs of rich and poor agents must be higher than the ratio of housing surfaces. An immediate and important consequence of this inequality comes from the fact that the right-hand side depends on the distance  $x$  to the center, whereas the left-hand side is constant, as we use a linear transport cost, proportional to the distance to the CBD. This condition yields richer urban forms in the model than just rich households in the center and poorer ones in the periphery, or vice versa.

Indeed, a critical distance  $x_c$  from the center can be defined, at which arc elasticities  $\varepsilon(t, Y)$  and  $\varepsilon(s, Y)$  are equal. Beyond this distance of the center, the direction of the inequality (1) changes. As a consequence, condition (1) can also be written

$$x < x_c = \frac{\beta_p Y_p t_r - \beta_r Y_r t_p}{t_r t_p (\beta_p - \beta_r)}$$

Then different social patterns can be observed in this model, depending on the value of  $x_c$  and on the radius of the city  $x_b$ , the latter being fixed by the boundary conditions of the model, see Fujita (1989) for instance.

If  $x_c \leq 0$ , the model gives the standard result of the AMM model with two income groups: poor agents are located in the center of the city and rich agents in the periphery. This is usually seen as a representation of a "North American" city. On the contrary, if  $x_c \geq x_b$ , the social pattern is reversed. Rich households are located in the center, and poorer ones in the periphery. This can be seen as a "European"-type city.

A different result can be found if  $0 < x_c < x_b$ <sup>1</sup>. Indeed in this case, rich households tend to locate closer to the center than poor households when their commuting distance  $x$  is such that  $0 \leq x < x_c$ , and further from the center than poor households when  $x_c < x < x_b$ . We leave to further study the

---

<sup>1</sup>Note that in the case where the marginal transport costs of both income groups are taken as equal,  $t_r = t_p$ , the income elasticity of the share  $\beta$  of income spent on housing has to be inferior to  $-1$  to have a positive critical radius  $x_c > 0$ .



complete analytical treatment of this case, which would for instance compare the bid rent curves of rich and poor agents within this model and show that they can intersect two times. In the following section, we use agent-based simulations to illustrate the results of this model in this special case where the critical radius is within the city boundary  $0 < x_c < x_b$ .

## 2.2. Emergence of a rich periphery in a "European"-type city

The values of parameters used in the following simulations with the agent-based model are presented in table 1. The corresponding value of the distance  $x_c$  defined in the previous section

| Parameters          | Description  | Value      |
|---------------------|--|------------|
| $\alpha_p, \beta_p$ | Preferences for composite good and housing of poor agents  | 0.7; 0.3   |
| $\alpha_r, \beta_r$ | Preferences for composite good and housing of rich agents  | 0.78; 0.22 |
| $Y_r, Y_p$          | Incomes of rich and poor agents                            | 450, 300   |
| $t_r, t_p$          | Transport cost (per unit distance) of rich and poor agents | 12, 10     |
| $N_r, N_p$          | Number of rich and poor agents                             | 1000       |
| $R_a$               | Agricultural rent  | 5          |
| $s_{tot}$           | Surface of a cell  | 45         |

Table 1: Parameters of the model.

is  $x_c \simeq 9.4$ . Let us note that in the simulations the side of a cell corresponds to a unit distance. We present the results of simulations of our model using different population sizes. In this way we model the emergence of a rich periphery in a "European"-type city. The evolution of the urban social structure in this case is shown on figure 1. From an initial configuration with a small popula-

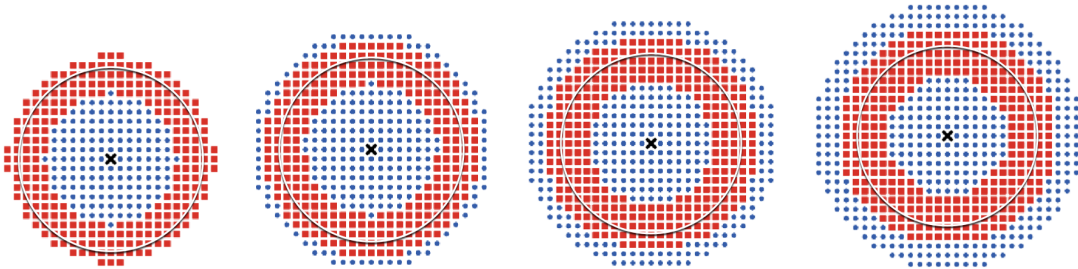


Figure 1: Evolution of the equilibrium urban social structure with a growing population size:  $N_r = N_p = 1000, 1600, 2200, 2800$  from left to right. The other parameter values are given in table 1. Radius  $x_c \simeq 9.4$  is indicated by a white circle. Rich agents are represented by discs, and poor ones by squares.

tion size, where rich agents are in the center of the city and poor agents in the periphery, the growth of the population size results in an increase of the city size and in the formation of a rich periurban area. Meanwhile, the surface of the rich central area decreases slightly.

It should still be discussed whether the values of parameters we use could be representative of real agents, even in a rough way, as this model is not calibrated on real data. As  $Y_r = Y_p \times 1.5$ , for a variation of income of 50%, the variation in (unit distance) transport cost is taken as high as 20%, with  $t_r = t_p \times 1.2$ . We suppose that this variation is mainly due to the variation in transport time cost. And the corresponding variation of the share of income spent on housing is a decrease of roughly 27%, with  $\beta_r \simeq \beta_p \times 0.73$ .

Supposing that the variation of the global transport cost corresponds only to a variation of the transport time cost, and that monetary and time costs are of the same magnitude for poor agents, the variation of the transport time cost is then an increase of 40% for an increase in income of 50%. This corresponds to an (arc) income elasticity of the transport time cost that is smaller than one, as reported above in the empirical literature. We conclude that these parameter values are consistent with empirical data.

### **3. More complex socio-spatial urban forms: central city revisited**

We explore in this section another way to make a rich center emerge within the AMM model. Let us first clarify the challenge. Housing is usually considered to be a normal good, whose consumption increases with income. With only one transport mode (associated with a given monetary transport cost) and no time cost, the result of the AMM model corresponds to the "North American" city, because condition (1) is not verified. The main issue is then to find a realistic model where the reverse configuration emerges. In this goal, Brueckner et al. (1999) introduce a central amenity in the AMM model, which can be linked in reality to the features which make the centers of European cities attractive, like Paris. Let us note however that Brussels for instance has a different socio-spatial structure (see Goffette-Nagot et al. (2000)). Brueckner et al. (1999) cite three types of amenities: natural (parks or rivers for instance), historical (e.g. monuments) and modern ones (theaters, swimming pools, etc.). But a central amenity is not enough to account for a "European"-type city in the AMM model: in a model using a Cobb-Douglas utility function, a central amenity

does not reverse the social pattern (see Lemoy et al. (2010)).

### 3.1. Analytical discussion

To obtain this reversal, two conditions are given by Brueckner et al. (1999): the marginal valuation of amenities, after optimal choice of the housing consumption, must rise faster with income than housing consumption, and the gradient of the amenity function must be negative and large in absolute value. An example of utility function satisfying the first condition is given by the constant elasticity of substitution (CES) utility function, which can be written:

$$U_{\text{CES}}(z, s, a(y)) = \left( \alpha z^{-\rho} + \beta s^{-\rho} + (1 - \alpha - \beta) a(y)^{-\rho} \right)^{-1/\rho}$$

where the same notations are used as previously for  $z$  and  $s$ ,  $\alpha$  and  $\beta$  are now such that  $\alpha + \beta \leq 1$ , and  $\rho$  is a real parameter.  $a(y)$  is a function describing the amenity, depending on the distance  $y$  to the amenity center. The budget constraint is the same as in the previous section, with  $t_r = t_p = t$ :  $Y_i = z_i + tx + ps_i$  with  $i = r, p$ .  $\sigma = 1/(1 + \rho)$  is a parameter which is linked to the elasticity of substitution between variables (see Chung (1994)). Brueckner et al. (1999) show that the first condition (the marginal valuation of amenities rising faster with income than housing consumption) is valid if  $\sigma < 1$ . We use this CES utility function here, with  $\rho = 0.3$  ( $\sigma \simeq 0.77$ ), so that this condition is verified.

### 3.2. Variations on (non-)central amenity, rich center and rich periphery

The amenity function  $a(y)$ , depending on the distance  $y$  to the amenity center, must also be chosen. We take the same function as Lemoy et al. (2010): a decreasing exponential  $a(y) = 1 + a_0 \exp(-y/b)$ , with  $a_0$  determining the magnitude of the amenity at its origin and  $b$  the characteristic distance of the amenity decrease. The optimal consumption of land (or housing) conditional on price  $p$  and location  $x$  is

$$s_i = \frac{Y_i - tx}{(\alpha p / \beta)^\sigma + p}$$

for  $i = r, p$  (see Chung (1994)). This expression is used in the agent-based model presented in section 1, replacing the corresponding expression for the Cobb-Douglas function.

The first city shape shown on figure 2 illustrates the fact that the urban social structure can indeed be reversed under these conditions. The other subfigures present the urban shape in this

model when the work center and the amenity are not at the same location. Figure 2 shows that this

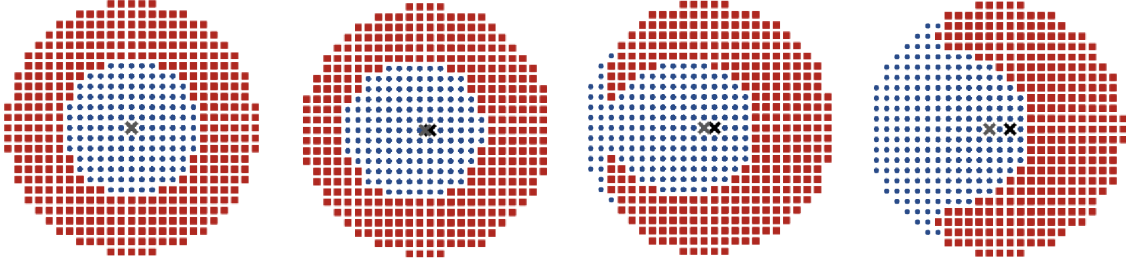


Figure 2: Illustration of the result of Brueckner et al. (1999), and evolution of the equilibrium urban social structure when the amenity is pushed to the west of the center. Distance  $d$  between the CBD (black "x") and the amenity (gray "x") is  $d = 0; 0.5; 1; 2$  from left to right. Other parameter values are given in table 2. Rich agents are represented by discs, and poor ones by squares.

model is sensitive to a small displacement of the amenity, which can be seen as a weakness of the amenity center added here. Indeed in real cities, amenity and employment centers are probably not found in the exact same locations. As the results show, even a small distance between both centers has an important impact on the urban socio-spatial structure in this model. The parameters used are given in table 2.

| Parameters            | Description  | Default value |
|-----------------------|--|---------------|
| $\alpha, \beta, \rho$ | Parameters of the CES utility function             | 0.3           |
| $Y_r, Y_p$            | Incomes of rich and poor agents                    | 450, 300      |
| $t$                   | Transport cost (unit distance)                     | 10            |
| $N_r, N_p$            | Number of rich and poor agents                     | 2000          |
| $R_a$                 | Agricultural rent                                  | 5             |
| $s_{tot}$             | Surface of a cell                                  | 100           |
| $a_0$                 | Amenity function at the origin                     | 5             |
| $b$                   | Characteristic distance of decrease of the amenity | 4             |

Table 2: Default parameters of the model reproducing the result of Brueckner et al. (1999)

Then we illustrate a fact which is not studied by Brueckner et al. (1999), but appears very easily in our agent-based simulations. This result is in direct correspondence with the one presented in the previous section: the condition which guarantees that the high income group is located near the center depends on the distance to the center. So that it may be verified close to the center and not

further away. More precisely, we suppose with Brueckner et al. (1999) that there is a first radius  $x_1$  at which the bid rent curves of rich and poor agents intersect. The condition mentioned before, that the gradient of the amenity function is negative and large in absolute value, is verified. So that rich agents are located at  $x \leq x_1$  and poorer ones at  $x \geq x_1$ .

But we suppose now that there is a second radius  $x_2 > x_1$  at which these bid rent curves intersect. With the decreasing exponential form of the amenity function we have chosen, the amenity gradient decreases in absolute value when  $x$  increases. This is why both curves can intersect again. So that at this second intersection  $x_2$ , the gradient of the amenity function can be small in absolute value, and rich agents are now driven to the periphery at  $x \geq x_2$ , while poor agents are located closer to the center, at  $x \leq x_1$ .

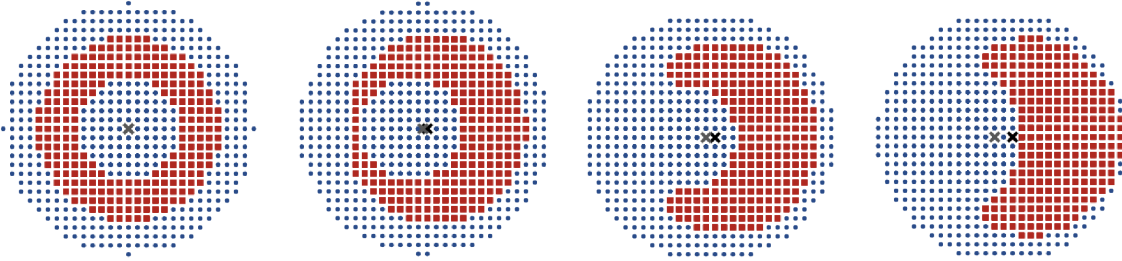


Figure 3: Evolution of the equilibrium urban social structure when the amenity is pushed to the west of the center. The distance  $d$  between the CBD and the amenity is  $d = 0; 0.5; 1; 2$  from left to right. The other parameter values are given by table 2, except  $a_0$  which has here a smaller value:  $a_0 = 3$ . Same symbols as figure 2.

An urban structure similar to the last configuration presented in the previous section can be observed in this case: a part of the high income group lives in the center, encompassed by a ring where the low income group is located. And further away lives the rest of the high income group, which benefits less of the amenity, but can afford bigger housing surfaces thanks to the lower land prices. This configuration is presented on figure 3, which uses the same parameter values as figure 2 (given by table 2), except that the intensity of the amenity function at its origin is smaller:  $a_0 = 3$ . It can be noted that more complex amenity functions than the decreasing exponential form could lead to social patterns which are even more complex than the patterns presented on figure 3.

Here also, we explore the results of the model when the amenity is not at the same location as the CBD, and we find that the city's social structure is very sensitive to this small displacement

of the amenity. Paris is an empirical illustration of this case where various urban amenities like monuments, museums and parks are located on the western half-side along with richer population (François et al. (2011)).

## **Discussion and conclusion**

Two different frameworks are used with two income groups within the AMM model and yield a hybrid configuration corresponding to a "European" city structure with a rich suburb, which has different origins in each case.

In the first one, with a Cobb-Douglas utility function where the evolution with income of the budget share of housing is taken into account, analytical calculations on income elasticities lead us to define a critical distance at which the relative locational behaviors of income groups changes. Below this distance, the higher value of time of rich agents, combined with their lower budget share for housing, leads them to choose small housing lots in the city center. Beyond this critical distance, the other part of the high income group is driven to the periphery by the desire to have bigger housing lots, allowed by smaller price in the periphery.

In the second framework, a study of the introduction of a central amenity in the AMM model, combined with a CES utility function, allows us to confirm the results of Brueckner et al. (1999), and to go further. A compromise between smaller housing lots and the benefit of a central amenity leads a part of rich households to live in the city center, giving the same outcome as previously. However, beyond a specific distance the influence of the amenity is weak and the preference for housing takes over to drive another part of the rich households to the outskirts.

Indeed, like the first model, this second model can have as an outcome a "European"-type urban social structure, with rich agents in the center, poorer ones in the suburbs, and in addition an outer ring of rich households. The first framework has the advantage, when compared with the second one, to avoid the exogenous introduction of an amenity, which is a bit frustrating from a modeling point of view. In addition, because it does not have an exogenous amenity (which could still be added in further work), its results are not sensitive to the location of the amenity, contrary to Brueckner et al. (1999)'s framework in two dimensions. On the other hand, this second framework can model richer, non-isotropic urban structures.

Several perspectives of work can be drawn. The first and most important one concerns the issue of the calibration of urban models, to test more precisely their link with empirical data. In particular, the ideas studied here regarding European and North American city models should be interesting to test on empirical data from both continents, if these can be gathered. This issue of the calibration of urban models is a difficult one, which is probably not enough treated in the literature.

Another perspective is linked to the historical evolution of cities through population size and transport cost (see for instance LeRoy and Sonstelie (1983)). Moreover, introducing two transport modes with various costs and speed in the model presented in section 2 is also an interesting perspective.

### Acknowledgements

The authors thank gratefully Florence Goffette-Nagot (GATE, CNRS and University of Lyon) for her valuable comments at various stages of this research.

### Appendix A. Socio-spatial structure and bid rent

In the case of a monocentric AMM model with two income groups<sup>2</sup>, let us derive here condition (1), which determines which income group is located closer to the center (see also Fujita (1989) or Goffette-Nagot et al. (2000)). The welfare of households is described by a utility function  $U(z, s)$  depending on their consumption of a composite good  $z$  and of housing surface  $s$ . These households also have a budget constraint, which expresses the fact that their income  $Y$  is spent entirely between their consumption of composite good  $z$ , their transport expenses  $T(x)$  and their housing expenses  $ps$ :  $Y = z + T(x) + ps$ , where  $p$  is the rent per unit surface.

Due to their competition for land, agents are led to pay a rent corresponding to their bid rent  $\Psi(x, u)$ , which is the maximum rent (per unit surface) which they can pay at a distance  $x$  from the center, when their utility is at a level  $u$ . From the budget constraint, this bid rent can be expressed as

$$\Psi(x, u) = \max_{z, s} \left( \frac{Y - T(x) - z}{s}, \text{ s. t. } U(z, s) = u \right).$$

---

<sup>2</sup>This approach can be extended to more than two income groups.

Let us now suppose that the expression of the utility function  $U(z, s)$  can be inverted in order to express the consumption of composite good  $Z(s, u)$  as a function of the consumption of housing  $s$  and of the utility level  $u$ . Then the bid rent can be rewritten as

$$\Psi(x, u) = \max_s \left( \frac{Y - T(x) - Z(s, u)}{s} \right).$$

Using the envelope theorem, the gradient of the bid rent  $\partial\Psi/\partial x$  can be computed:

$$\frac{\partial\Psi}{\partial x} = -\frac{T'(x)}{S(x, u)},$$

where  $T'$  is the derivative of  $T$  and  $S(x, u)$  is the consumption of land which maximizes the bid rent at distance  $x$  and utility level  $u$ . As the transport cost  $T(x)$  is increasing with distance  $x$ , this gradient is negative.

Let us consider now two income groups 1 and 2, with incomes  $Y_1$  and  $Y_2$ , with two different bid rent curves  $\Psi_1(x, u)$  and  $\Psi_2(x, u)$ . Then because of the competition for housing, these curves need to intersect somewhere in the city if both groups are to be located inside the city. Let us call  $x$  the distance at which the curves intersect. As the gradients of bid rents are negative, the income group which will be located closer to the center (at distances smaller than  $x$ ) is the one whose bid rent has the largest gradient (in absolute value): income group 1 is located closer to the center if

$$\frac{T'_1(x)}{S_1(x, u)} > \frac{T'_2(x)}{S_2(x, u)},$$

which leads to condition (1) with a Cobb-Douglas utility function (see Chung (1994)) and a linear transport cost.

## References

- Accardo, J., Bugeja, F., 2009. Le poids des dépenses de logement depuis vingt ans, in: Cinquante ans de consommation en France. Insee, Paris, pp. 33–47.
- Binder, K., Heermann, D., 2010. Monte Carlo simulation in statistical physics: an introduction. Springer Science & Business Media.
- Brueckner, J., Thisse, J., Zenou, Y., 1999. Why is central Paris rich and downtown Detroit poor? An amenity-based theory. *European Economic Review* 43, 91–107.



- Caubel, D., 2005. Disparités territoriales infra-communales (IRIS-2000) selon les niveaux de vie et les positions sociales sur les aires urbaines de Lyon, Bordeaux, Paris, Toulouse, Dijon, Pau, Agen et Villefranche-sur-Saône. XLIème Colloque de l'ASRDLF "Villes et territoires face aux défis de la mondialisation". <http://halshs.archives-ouvertes.fr/halshs-00095751>.
- Cervero, R., Chapple, K., Landis, J., Wachs, M., Duncan, M., Scholl, P.L., 2006. MAKING DO: How Working Families in Seven U.S. Metropolitan Areas Trade Off Housing Costs and Commuting Times. Technical Report. UC Berkeley: Institute of Transportation Studies (UCB).
- Chung, J.W., 1994. Utility and production functions: theory and applications. Blackwell Publishers.
- François, J.C., Ribardièrre, A., Fleury, A., Mathian, H., Pavard, A., Saint-Julien, T., 2011. Les disparités de revenus des ménages franciliens - Analyse de l'évolution entre 1999 et 2007. Rapport pour la DREIA Ile-de-France. <http://halshs.archives-ouvertes.fr/halshs-00737156>.
- Fujita, M., 1989. Urban Economic Theory. Cambridge University Press.
- Glaeser, E.L., Kahn, M.E., Rappaport, J., 2008. Why do the poor live in cities? The role of public transportation. *Journal of Urban Economics* 63, 1 – 24.
- Goffette-Nagot, F., Thomas, I., Zenou, Y., 2000. Structure urbaine et revenu des ménages. *Economica*, Paris. chapter 10. *Economie Géographique : approches théoriques et empiriques*.
- Lemoy, R., Raux, C., Jensen, P., 2010. An agent-based model of residential patterns and social structure in urban areas. *Cybergeog: European Journal of Geography, Systems, Modelling, Geostatistics*, article 512. <http://cybergeog.revues.org/index23381.html>.
- Lemoy, R., Raux, C., Jensen, P., 2013. Exploring the polycentric city with multi-worker households: an agent-based microeconomic model. <https://hal.archives-ouvertes.fr/hal-00602087>.
- Lenstra, J.K., 2003. Local search in combinatorial optimization. Princeton University Press.
- LeRoy, S.F., Sonstelie, J., 1983. Paradise lost and regained: Transportation innovation, income, and residential location. *Journal of Urban Economics* 13, 67–89.

Polacchini, A., 1999. Les dépenses des ménages franciliens pour le logement et les transports. Recherche Transports Sécurité 63.

Small, K.A., 1992. Urban transportation economics, in: Regional and Urban Economics II. Harwood Academic Publishers.

Wardman, M., 2001a. A review of British evidence on time and service quality valuations. Transportation Research Part E: Logistics and Transportation Review 37, 107 – 128.

Wardman, M., 2001b. Inter-temporal variations in the value of time. ITS Leeds Working Paper 566.