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Spatial price discrimination with online competition

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Spatial Price Discrimination with Online Competition

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Spatial Price Discrimination with Online Competition

Abstract

This paper introduces an online firm into a spatial duopoly market, where physical firms engage in price discrimination, and consumers are non-uniformly distributed along a linear market. It is shown that in the short-run, the price equilibrium will be uniform, or kinked in the central area, depending on whether the distaste cost of online purchasing is low or high. In the long-run, only the kinked price equilibrium is valid, and physical firms tend to move closer to the densely populated area. Equilibrium price levels and price variations are considerably reduced after the entry of the online firm. Implications on socially optimal locations and zoning policies are also discussed.

Keywords: Spatial price discrimination; location; online competition; price variation; zoning.

JEL Classification Numbers. R32; L13

1 Introduction

Spatial price discrimination is a classical issue since Hoover (1937), who found that discrimination with fixed locations of firms is affected by demand elasticity, transport cost, and the degree of competition. Delivery price discrimination among consumers is common in many industries such as oil and furniture. In a geographical view, the price equilibrium for any spatial location should be the delivery cost for nearby rivals. Hurter and Lederer (1985) and Lederer and Hurter (1986) further include a location stage, and show that one firm will locate at the first quartile, and the other firm locates at the third quartile in equilibrium. Thisse and Vives (1988) discuss the strategic choice of spatial price policy, and show that the price discrimination policy has a robust tendency to be chosen by firms with fixed locations. Recently, Heywood and Ye (2009) show that in a sequential entry game of spatial price discrimination, a public firm can restrict its market in order to induce earlier private entrants to locate near the welfare maximizing points. Barcena-Ruiz and Casado-Izaga (2014) find that the socially optimal locations are more separated as the weight of the total profit of firms increases.

Online shopping is a rising consumption pattern in the past two decades. The motivation of this study is to introduce an online competition into a duopoly structure where firms engage in

price discrimination, and population density is non-uniformly distributed along a unit length market.¹ This study highlights two major differences between online purchasing and traditional physical shopping. One is that online firms usually enact uniform pricing, while physical firms may engage in spatial price discrimination. The other difference is that consumers in an online transaction cannot check products before-hand, and normally wait several days for delivery, which are summarized as a distaste cost for online transactions.² Consequently, the online firm (retailer) is assumed to have location-irrelevant delivering cost *à la* Balasubramanian (1998). If the distaste costs are strong, then the physical firms is less affected, and the price equilibrium is closer to the traditional discrimination results, while if the distaste costs are weak, the uniform pricing will prevail in equilibrium. In other words, the entry of an online retailer may reduce the price differentiation, or even result in geographically uniform pricing! We show that in the short-run, when the locations of physical firms cannot change, the online firm will occupy the two end segments of the linear market, while physical firms serve the center market with uniform pricing (when the waiting cost is low) or kinked pricing near the center and uniform pricing in all other areas. In the long-run, only the latter price pattern is valid. Moreover, the socially optimal locations and zoning policies are also analyzed.

Numerous studies have concerned about the influences of online retailing. Balasubramanian (1998) constructed a circular model *a la* Salop (1979) and analyzed price competition among one direct channel (mail order) and conventional physical stores. The shipping cost is location irrelevant and fixed for the direct channel, while it is increased with distance for conventional physical stores. He found that some consumers may suffer under this environment, and each retailer can compete with the direct channel instead of its nearby retailers. Loginova (2009) analyzed the price and welfare among physical retailers and competitive online retailers with addition of the stipulation that consumers may visit physical stores to collect information and come back home for their online purchasing. She showed that conventional stores may raise their prices unexpectedly in response to the entry of online firms. Empirical studies on competition between physical stores and online firms can be found in Clay *et al.* (2002), Brynjolfsson and Smith (2000), Goolsbee (2001), etc. The current study contributes to previous literatures in combining classical price discrimination and online retailing. In particular, the

¹If population density is uniform along a linear market, an online firm entering the market will make physical firms be indifferent in locating their stores in some range of the linear market and multiple uniform price equilibria appear.

²Another important difference is that online firms provide broader and faster search and comparisons between prices and quality of products, which is not embedded in the current model in order to preserve simplicity.

entry of an online retailer is shown to considerably reduce equilibrium price levels and price variations as well.

The rest of this paper is organized as follows. Section 2 is the benchmark case without the online firm. Section 4 discusses the short-run equilibrium after the entry of the online firm. The long-run equilibrium is analyzed in Section 5. The social optimum is discussed in Section 6. Section 7 provides an analysis on price levels and variations. The zoning policies are analyzed in Section 8. Finally, some concluding remarks are offered in Section 9.

2 The Model

Suppose there are two physical firms (1 and 2) locating at $x_1 \in [0, 1]$, and $x_2 \in [0, 1]$, $x_1 \leq x_2$, respectively, and they engage in price discrimination on consumers who are non-uniformly distributed along a linear market with unit length as shown in Figure 1. The price for firm 1 (2) are denoted by p_1 (p_2), respectively. Physical firms have no production cost.³ The transport rate is t , and population density is $f(x) = a - b|2x - 1|$, which is symmetric around the market center, where b represents the spatial gradient of population density with $0 < b \leq 2$, and $a = \frac{b}{2} + 1$ to normalize the total population mass as one unit. Each consumer has inelastic demand for one unit of the homogeneous product. Firms simultaneously choose their locations in the first stage, while they choose their prices simultaneously in the second stage.

To analyze the influence of online competition, consider the entry of an online firm which has no location choice and sets a uniform price for all consumers in the first stage.⁴ Any online buyer suffers a distaste cost z , which includes the inconvenience of checking the product quality before-hand and waiting several days for shipping. Assume that z is not too large, to avoid a redundant online firm throughout this study. In the following, we will discuss the benchmark case with only two physical firms, the short-run equilibrium after the entry of an online firm when the locations of physical firms are fixed, and the long-run equilibrium when the locations of firms are flexible.

³Our results are generally robust when positive production costs are embedded, except additional mathematical complexity.

⁴This setting of game structure can be justified by two reasons. First, in general, the online firm is often a dominant firm in an industry, such as Amazon in the book market. Second, if the online firm sets up its price in the second stage, undercutting is unavoidable and there exists no price equilibrium.

3 The Benchmark Case without Online Competition

Consider the price equilibrium in the second stage. Spatial price discrimination results in an equilibrium price pattern such that at any geographical point, the firm with the lowest delivery cost sets up a discriminatory price equal to the delivery cost of its rival. From Figure 1, the equilibrium prices are: $p_1 = t|x - x_2|$ for $x \in [0, \hat{x}]$ and $p_2 = t|x - x_1|$ for $x \in [\hat{x}, 1]$. The profit

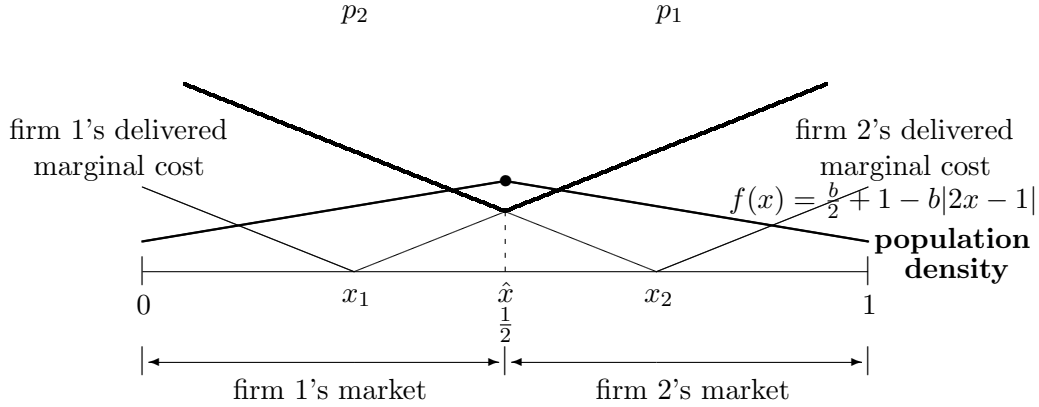


Figure 1: Prices and delivery costs in the benchmark case.

functions for firm 1 and firm 2 are:

$$\pi_1 = \int_0^{\hat{x}} (t|x - x_2| - t|x - x_1|) f(x) dx,$$

$$\pi_2 = \int_{\hat{x}}^1 (t|x - x_1| - t|x - x_2|) f(x) dx.$$

Solving $\partial\pi_1/\partial x_1 = 0$ and $\partial\pi_2/\partial x_2 = 0$ simultaneously in the location stage yields the following proposition.⁵

Proposition 1. *Before the entry of the online firm, the equilibrium locations with spatial price discrimination are $(x_1^*, x_2^*) = (\frac{b-2+\sqrt{b^2+4}}{4b}, 1 - x_1^*)$. Moreover, the locations of firms will be much closer to the market center when population distribution becomes more uneven.*

Proof. Solving $\partial\pi_1/\partial x_1 = 0$ and $\partial\pi_2/\partial x_2 = 0$ simultaneously yields the solution of locations $(x_1^*, x_2^* = 1 - x_1^*)$, which also satisfies the first-order condition $\frac{\partial^2 \pi_i}{\partial x_i^2} \Big|_{x_1=x_1^*, x_2=x_2^*} = \frac{t(b+2-4\sqrt{b^2+4})}{4} <$

⁵Notably, the first-order condition yields another solution $\{x_1 = \frac{b+2+\sqrt{b^2+4}}{4b}, x_2 = 1 - x_1\}$, which in fact violates the second-order condition.

0, $i = 1, 2$. Moreover,

$$\frac{\partial x_1^*}{\partial b} = \frac{\sqrt{b^2 + 4} - 2}{2b^2\sqrt{b^2 + 4}} > 0.$$

□

Note that $x_1^* \in (\frac{1}{4}, \frac{1}{2})$ as per Proposition 1, because the high population density occurs in those areas close to the market center. When b converges to zero, x_1^* will converge to $1/4$; this result shows that the current model is reduced to the classical result in Hurter and Lederer (1985) and Lederer and Hurter (1986) when population density is approaching uniform distribution.

4 Short-Run Equilibrium after the Entry of the Online Firm

In the short-run, there is an online firm (denoted by “0”) entering the market surprisingly, and thus both physical firms cannot relocate their stores. Suppose the online firm has no physical location and sets a uniform price for all consumers. The online firm involves a marginal cost c_0 . Therefore, the short-run equilibrium depends on the size of the distaste cost z . Specifically, there are two cases in the short-run: one is a small z , and the other is a large z .

Let’s start to analyze the case when z is large. The short-run equilibrium can be depicted as Figure 2, where x_{1L} and x_{1R} (x_{2L} and x_{2R}) are the indifferent consumers between the online firm and firm 1 (firm 2), and the online firm occupies the end segments $[0, x_{1L})$ and $(x_{2R}, 1]$, while physical firms serve the center market, where population density is relatively high. The prices under spatial discrimination can be depicted by the red lines in Figure 2, where firm 1 sets prices $p_1(x) = p_0 + z$ for $x \in [x_{1L}, x_{1R}]$ due to the online firm having the second-highest delivery cost and $p_1(x) = t|x - x_2|$ for $x \in [x_{1R}, \hat{x}]$, because firm 2 has the second-highest delivery cost. Firm 2 sets similar prices in the right market. The profit functions become:

$$\begin{aligned}\pi_0 &= \int_0^{x_{1L}} (p_0 - c_0)f(x)dx + \int_{x_{2R}}^1 (p_0 - c_0)f(x)dx, \\ \pi_1 &= \int_{x_{1L}}^{x_{2L}} (p_0 + z - t|x - x_1|)f(x)dx + \int_{x_{2L}}^{\hat{x}} (t|x - x_2| - t|x - x_1|)f(x)dx, \\ \pi_2 &= \int_{x_{1R}}^{x_{2R}} (p_0 + z - t|x - x_2|)f(x)dx + \int_{\hat{x}}^{x_{1R}} (+t|x - x_1| - t|x - x_2|)f(x)dx.\end{aligned}$$

Solving $\partial\pi_0/\partial p_0 = 0$ yields the following results.

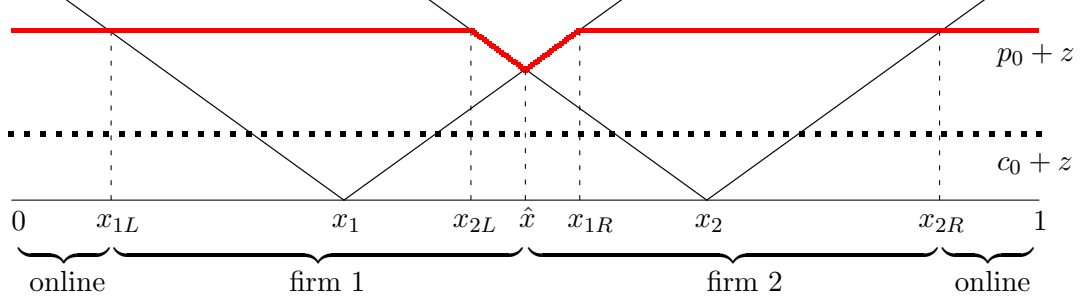


Figure 2: Short-run equilibrium when z is large.

Proposition 2. *When z is large, the online firm occupies the end segments, and the physical firms serve the center market. Moreover, $p_0^* = c - \frac{2}{3}z + \frac{t\sqrt{b^2+4} - \sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2}}{6b}$ and $\partial p_0^*/\partial z < 0$, $\partial(p_0^* + z)/\partial z > 0$, and $\partial p_0^*/\partial b > 0$.*

Proof. Solving $\partial\pi_0/\partial p_0 = 0$ yields the equilibrium price p_0^* by calculations. The equilibrium price should satisfy two additional conditions: The first condition states that the online firm participates in the market, which leads to $z < z_{\max} = \frac{t(b-2+\sqrt{b^2+4})}{4b}$. The second condition ensures that the online firm will not serve the center market, which leads to $z \geq z_{\min} = \frac{t(-5b^2-20-8b+5b\sqrt{b^2+4}+10\sqrt{b^2+4})}{4b(-b+2\sqrt{b^2+4}-2)}$. Henceforth, this type of short-run equilibrium is valid when z is mediated. Moreover, comparative statics shows that

$$\begin{aligned} \frac{\partial p_0^*}{\partial z} &= \frac{-4\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2} - 4zb + t\sqrt{b^2+4}}{6\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2}} < 0, \\ \frac{\partial(p_0^* + z)}{\partial z} &= \frac{-4\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2} - 4zb + t\sqrt{b^2+4}}{6\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2}} > 0, \\ \frac{\partial p_0^*}{\partial b} &= -\frac{t(8zb + 3t\sqrt{b^2+4} + 8\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2} - 8t\sqrt{b^2+4})}{12\sqrt{4z^2b^2 - 2t\sqrt{b^2+4}zb + b^2t^2 + 4t^2 - 3bt^2}\sqrt{b^2+4}b^2} > 0, \end{aligned}$$

by the condition $b \leq 2$. □

When z is small, the online firm has a cost advantage, allowing it to grab the center (high density) market segment. The equilibrium prices and market shares can be depicted as Figure 3. The equilibrium prices are equalized for the whole market. The profit function for the online firm becomes

$$\pi_0 = \int_0^{x_{1L}} (p_0 - c_0)f(x)dx + \int_{x_{1R}}^{x_{2L}} (p_0 - c_0)f(x)dx + \int_{x_{2R}}^1 (p_0 - c_0)f(x)dx,$$

and solving $\partial\pi_0/\partial p_0 = 0$ yields the following proposition.

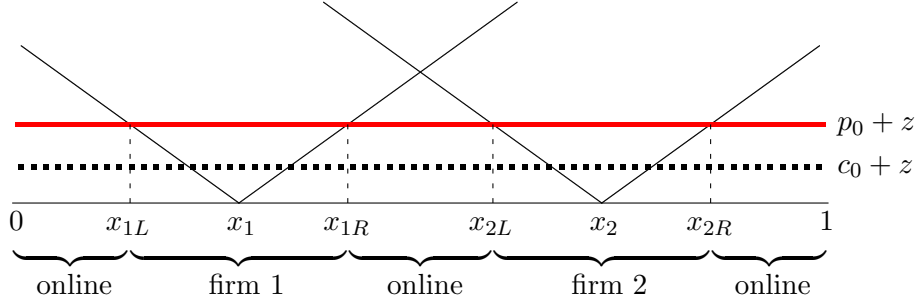


Figure 3: Short-run equilibrium when z is small.

Proposition 3. *When z is small, the short-run equilibrium allows the online firm to occupy both the center segment and the two end segments. Moreover, $p_0^* = c_0 - \frac{z}{2} + \frac{t}{4\sqrt{b^2+4}}$ and $\partial p_0^*/\partial z = -\frac{1}{2} < 0$, $\partial(p_0^* + z)/\partial z = \frac{1}{2} > 0$, $\partial p_0/\partial b < 0$.*

Proof. Solving $\partial\pi_0/\partial p_0 = 0$ yields the equilibrium price p_0^* by calculations. This type of equilibrium requires an additional condition $p_0 + z < t(\hat{x} - x_1)$, meaning that the online firm has a cost advantage in grabbing the central segment. This condition leads to $z < \tilde{z}_{\max} = \frac{t(\sqrt{b^2+4}b + 2\sqrt{b^2+4} - b^2 - 4 - b)}{2\sqrt{b^2+4}b}$. Notably, $\tilde{z}_{\max} > z_{\min}$ when z_{\min} is the critical point in the case of Proposition 2. Therefore, when z is small, this type of equilibrium is valid. Moreover, comparative statics are clear, and we obtain that $\partial p_0^*/\partial z = -1/2$, $\partial(p_0^* + z)/\partial z = 1/2$, and $\partial p_0^*/\partial b < 0$. \square

From Proposition 3, it is easy to have the following corollary.

Corollary 1. *When z is small, the entry of the online firm will induce an equalized price equilibrium.*

The intuition of Corollary 1 is as follows. The online firm is a non-spatial retailer such that it sets a uniform price for all consumers. When z is small, it represents that the online firm has a cost advantage over the physical firms and forces the whole market to have one equilibrium price.

5 The Long-Run Equilibrium

In the long-run, all physical firms are free to their locations. Assume that both physical firms choose their locations and prices, and the online firm sets its price simultaneously.⁶ The

⁶Consider an alternative scenario of sequential decisions. If physical firms simultaneously choose their locations in the first stage, and the online firm sets up its price in the second stage. A similar type of equilibrium

incentive for physical firms to relocate will be examined as the following two lemmas.

Lemma 1. *When z is either small or large, physical firms have incentive to move closer to each other in the short-run equilibrium.*

Proof. When z is small, Proposition 3 illustrates that the online firm occupies both the center segment and the two end segments. Then, the profit function for firm 1 is

$$\pi_1 = \int_{x_{1L}}^{x_{1R}} (p_0 + z - t|x - x_1|)f(x)dx = \frac{(c_0 - p_0 - z)^2(2 - b + 4bx_1)}{2t}.$$

Since $\partial\pi_1/\partial x_1 > 0$, firm 1 has incentive to move closer to the market center. Similar incentive is valid for firm 2.

When z is large, similarly, Proposition 2 shows that the online firm occupies the two end segments and physical firms serve the center segment, thus the profit function for firm 1 becomes

$$\pi_1 = \int_{x_{1L}}^{x_{2L}} (p_0 + z - t|x - x_1|)f(x)dx + \int_{x_{2L}}^{\hat{x}} (t|x_2 - x| - t|x - x_1|)f(x)dx.$$

Then,

$$\left. \frac{\partial\pi_1}{\partial x_1} \right|_{x_1=x_1^*, x_2=x_2^*} = \frac{1}{36bt} \left[6bt^2 - 8z^2b^2 - (4 + b^2)t^2 - 4tzb\sqrt{b^2 + 4} + (t\sqrt{b^2 + 4} - 4zb) \right. \\ \left. \sqrt{4z^2b^2 - 2tzb\sqrt{b^2 + 4} + t^2(b^2 - 3b + 4)} \right],$$

which is zero when $z = z_{\max}$ and is positive when $0 \leq z < z_{\max}$. Therefore, firm 1 profits from moving closer to the center. \square

The price and location equilibrium in the long run can be depicted as in Figure 4, where physical firms set up effective discriminatory prices in the central area: $p_1(x) = t|x - x_2|$, $\forall x \in (x_{2L}, \hat{x}]$ and $p_2(x) = t(x - x_1)$, $\forall x \in [\hat{x}, x_{1R})$. Then the profit functions are:

$$\pi_0 = \int_0^{x_{1L}} (p_0 - c_0)f(x)dx + \int_{x_{2R}}^1 (p_0 - c_0)f(x)dx, \\ \pi_1 = \int_{x_{1L}}^{x_{2L}} (p_0 + z - t|x - x_1|)f(x)dx + \int_{x_{2L}}^{\hat{x}} (t|x - x_2| - t|x - x_1|)f(x)dx, \\ \pi_2 = \int_{x_{1R}}^{x_{2R}} (p_0 + z - t|x - x_2|)f(x)dx + \int_{\hat{x}}^{x_{1R}} (t|x - x_1| - t|x - x_2|)f(x)dx.$$

Solving $\partial\pi_0/\partial p_0 = 0$, $\partial\pi_1/\partial x_1 = 0$, and $\partial\pi_2/\partial x_2 = 0$ simultaneously yield the following results.

will emerge, although physical firms will move closer to the market center to prevent the online firm grabbing the center segment.

Proposition 4 shows that the online firm occupying the two end segments and the physical firms engaging price discrimination between them in the center segment is the long-run equilibrium. This effective result is achieved no matter whether z is large or small. The equilibrium prices cannot be equalized for all areas such as there is a lower price in the center market and uniformly price $p_0 + z$ for other segments.

Corollary 2. *The physical firms locate closer to the market center in the long-run equilibrium.*

Proof. It's clear from Lemma 1. □

This result reveals that the entry of the online firm brings more intense competition and induces physical firms to locate closer to the densely populated area.

The comparative statics is complicated since the equilibrium locations are the solution of a fourth-order polynomial equation (3). In general, the influences of z and b on the locations are consistent in numerical analysis with a broader range of parameter values. The following two figures give our illustrations. Given $c_0 = 0$ and $b = 1$, Figure 5 shows that x_1 is decreasing in z and increasing in t . The physical firms move farther away from the market center as the

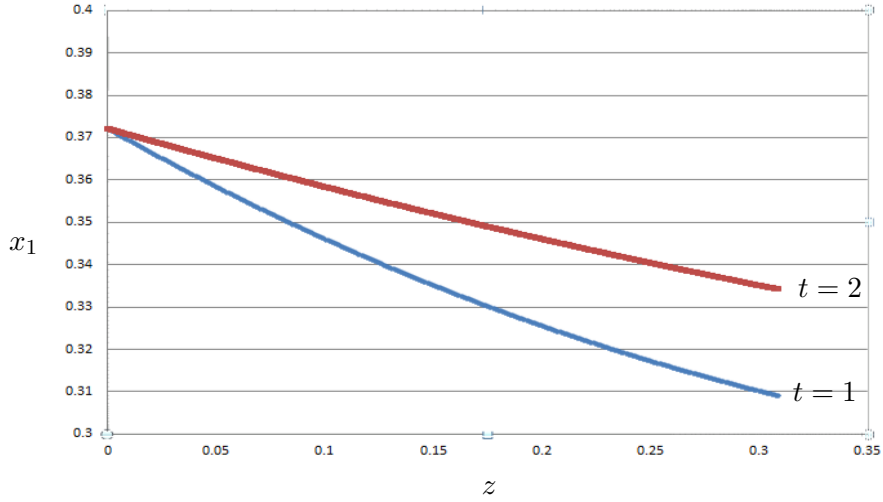


Figure 5: The influence of z on location x_1 .

distaste cost for the online firm increases, because of the increase of the disadvantage of the online firm. In reality, the distaste cost for online firms has been, in general, decreasing in recent years, so the physical firms tend to move closer to the market center to concentrate on densely populated areas. Figure 6 shows the influence of b on the locations of physical firms.

Given $c_0 = 0$ and $z = 0.3$, the equilibrium location x_1^* is increasing in b and t . This is because the central area has more demand as b increases, and it induces physical firms to move closer to the central area.

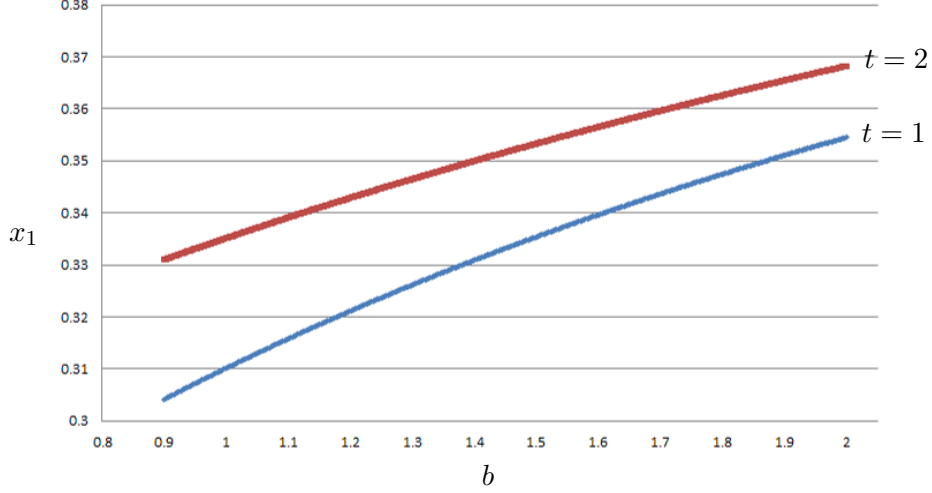


Figure 6: The influence of b on location x_1 .

6 The Social Optimum

For the benchmark case, the socially desirable locations can be derived via minimizing the total transportation costs (TTC):

$$\text{TTC} = \int_0^{\hat{x}} t|x - x_1|f(x)dx + \int_{\hat{x}}^1 t|x - x_2|f(x)dx.$$

we then have the following corollary:

Corollary 3. *Before the entry of the online firm, the equilibrium locations are socially desirable.*

Proof. Solving $\partial \text{TTC} / \partial x_1 = 0$ and $\partial \text{TTC} / \partial x_2 = 0$ simultaneously yields the solution $x_1 = x_1^*$ and $x_2 = x_2^*$, which minimize TTC. \square

With online competition, the socially optimal location is solved by minimizing the total transportation cost and total distaste costs:

$$\text{TTC} = \int_0^{x_{1L}} z f(x)dx + \int_{x_{1L}}^{\hat{x}} t|x - x_1|f(x)dx + \int_{\hat{x}}^{x_{2R}} t|x - x_2|f(x)dx + \int_{x_{2R}}^1 z f(x)dx.$$

From the socially optimal perspective, the online firm only serves the end segments instead of occupying a part of the central area, because physical firms provide lower transportation costs than the distaste cost for their nearby consumers, and therefore, physical firms should serve the densely populated area. The following proposition describes the socially desirable locations.

Proposition 5. *The socially desirable locations are $(x_1^o, x_2^o) = (\frac{tb-2t-4zb+\sqrt{t^2(2+b)^2+32z^2b^2}}{4tb}, 1-x_1^o)$, which are much closer to the market center than in the benchmark case.*

Proof. Solving $\partial\text{TTC}/\partial x_1 = 0$ and $\partial\text{TTC}/\partial x_2 = 0$ simultaneously yields the solution x_1^o and x_2^o . Moreover, comparing x_1^o with x_1^* yields

$$x_1^o - x_1^* = \frac{\sqrt{t^2(2+b)^2 + 32z^2b^2} - (42b + t\sqrt{b^2 + 4})}{4tb},$$

which is positive from the assumption of $b \leq 2$. \square

The intuition of Proposition 5 is that the physical transportation costs for the end segments are relatively high and should be served by the online firm. Moreover, physical firms have locational advantages for their nearby consumers, and this advantage is enlarged in the central area, which is highly populated. The following result compares the locations of firms in the long-run equilibrium and those under the socially optimum.

Proposition 6. *The long-run equilibrium locations are more separated than those in the socially optimum when p_0 is not high.*

Proof. Substituting the socially optimal locations (x_1^o, x_2^o) into the first-order condition of firm 1 in the long-run equilibrium yields

$$\left. \frac{\partial \pi_1}{\partial x_1} \right|_{(x_1^o, x_2^o)} = -(p_0 - c_0) \left(\sqrt{t^2(2+b)^2 + 32z^2b^2} + 2bc_0 - 8zb - 2bp_0 \right),$$

which is negative when p_0 is not large and $p_0 > c_0$. Therefore, firm 1 has incentive to move to the left from x_1^o , and then the equilibrium locations are more separated than the socially desirable locations. \square

This result is clear in that physical firms can take more market share from the online firm by moving toward the left. Although our analysis requires the condition that p_0 is not large, the result of location comparison is generally valid for a broader range of parameters in numerical calculations.

7 Price Levels and Variations with Online Competition

In this section, a novel perspective is explored to compare the population-weighted average price (EP) and the standard division of price (SD):

$$\begin{aligned} \text{EP} &= \int_0^1 p(x) \cdot f(x) dx, \\ \text{SD}^2 &= \int_0^1 (p - \text{EP})^2 f(x) dx, \end{aligned}$$

where $p(x)$ is the equilibrium prices, described by the red line in Figure 1, 2, and 4. The case with a small z , which leads to completely equal prices, will be ignored in this study. In the benchmark case, the EP and SD^2 are:

$$\begin{aligned} \text{EP} &= \int_0^{\hat{x}} (t|x_2 - x|) f(x) dx + \int_{\hat{x}}^1 (t|x - x_1|) f(x) dx \\ &= -\frac{-12t + b^2t - 24c_0b - 12tb + 6t\sqrt{b^2 + 4}}{24b}, \\ \text{SD}^2 &= \int_0^{\hat{x}} (t|x_2 - x| - \text{EP})^2 f(x) dx + \int_{\hat{x}}^1 (t|x - x_1| - \text{EP})^2 f(x) dx \\ &= -\frac{t^2(b^2 - 12)}{576}. \end{aligned}$$

Intuitively, the average price level in the short-run equilibrium is lower than that in the benchmark case, because the market competition is enhanced after the entry of the online firm. Similarly, the price variation is also reduced, as the competition is more intense due to the online participation. This result provides an empirical implication to test the influence of online competition on the average price level and price variation.

Since EP and SD are not analytical in the long-run equilibrium, numerical analysis is employed and illustrated as Table 1, where $c_0 = 0$ and $t = 1$, and SD is multiplied by 10. Note that from Table 1, the EP and SD in the short-run equilibrium and in the long-run equilibrium are significantly lower than those in the benchmark case. Moreover, EP and SD in the long-run equilibrium are either greater or smaller than those in the short-run equilibrium.

8 Zoning Policies

This section provides the implication of zoning policies following Bárcena-Ruiz and Casado-Izaga (2014), in which the online retailer is not considered and the population is uniformly distributed. Consider now a regulator enacts an optimal zoning at stage 0, and online retailer

Table 1: The numerical examples for EP and VP.

	Benchmark case		Short-run equilibrium		Long-run equilibrium	
	EP	SD ($\times 10$)	EP	SD ($\times 10$)	EP	SD ($\times 10$)
$b = 1, z = 0.1$	0.399	1.382	0.196	0.003	0.206	0.127
$b = 1, z = 0.2$	0.399	1.382	0.246	0.138	0.249	0.220
$b = 1, z = 0.3$	0.399	1.382	0.286	0.327	0.286	0.333
$b = 1.5, z = 0.1$	0.354	1.301	0.194	0.050	0.198	0.174
$b = 1.5, z = 0.2$	0.354	1.301	0.244	0.256	0.243	0.322
$b = 1.5, z = 0.3$	0.354	1.301	0.280	0.486	0.286	0.011
$b = 2, z = 0.1$	0.313	1.179	0.182	0.080	0.183	0.185
$b = 2, z = 0.2$	0.313	1.179	0.231	0.322	0.229	0.365
$b = 2, z = 0.3$	0.313	1.179	0.266	0.586	0.265	0.590

sets its uniform price on stage 1. We first solve for the socially optimal symmetric locations $x_1^o(\alpha)$ and $x_2^o(\alpha) = 1 - x_1^o$, where α is a weighted parameter for the regulator who maximizes the social welfare function

$$W = \alpha(\pi_1 + \pi_2 + \pi_0) + (1 - \alpha)CS, \quad \alpha \in [0, 1], \quad (4)$$

where CS is the consumer surplus.

When $x_1 \leq 1/4$, the equilibrium pattern is either described similar to Figure 3, where $x_{1L} \geq 0$, or the physical retailers occupy two end market segments, where $x_{1L} < 0$. When $1/4 < x_1 \leq 1/2$, the equilibrium pattern is either described similar to Figure 2, where $x_{1R} > 1/2$, or described as Figure 3, where $x_{1R} \leq 1/2$. Note that the choices of p_0 could corresponds to the above four possible equilibrium patterns, which results in considerable calculation complexity. We depict our numerical simulation results with parameters values $v = 1$, $b = 0.02$, $z = 0.15$, $t = 1$, $c_0 = 0$, where a small value of b is used to approximate and compare with the uniform case in Bárcena-Ruiz and Casado-Izaga (2014). Figure 7 illustrates the socially optimal locations of physical firms, which are quite different from those in Bárcena-Ruiz and Casado-Izaga (2014, p.664). When α is small, x_1^o and x_2^o are separated and far away from the center point, shown in Bárcena-Ruiz and Casado-Izaga (2014).

Intuitively, when the regulator very concerns the consumer surplus (α is small), the socially optimal regulation is to creates competition among three firms by proper dispersed locations. Specifically, when two physical retailers agglomerate at the center point as the optimal lo-

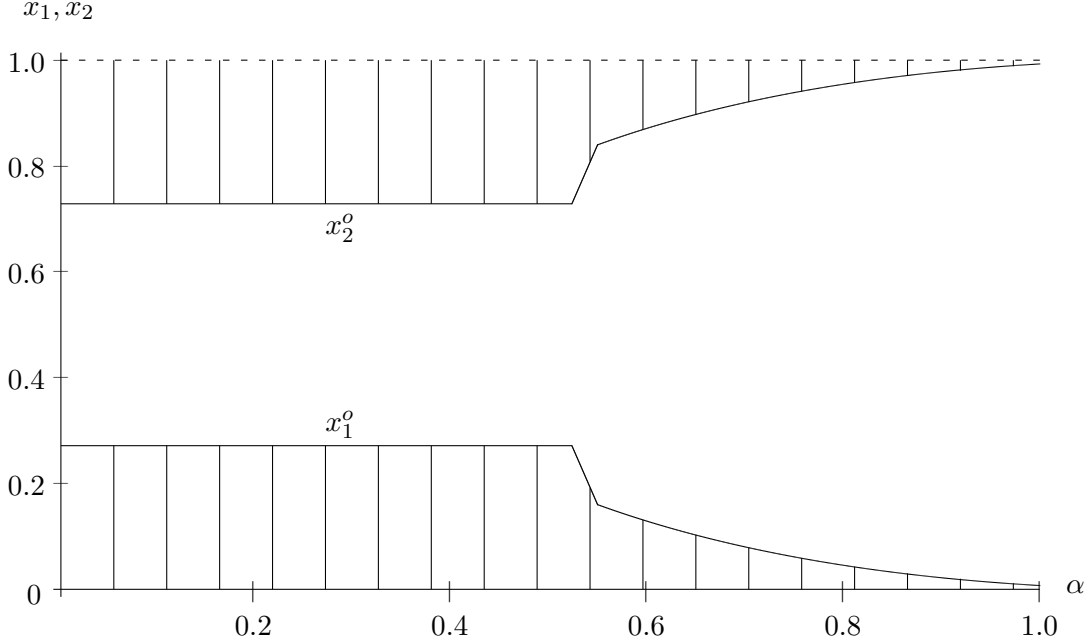


Figure 7: Firms' optimal locations and allowed locations.

cation in Bárcena-Ruiz and Casado-Izaga (2014). Although physical firms engage maximal competition, yet the online retailer enjoys the benefit of avoiding competition with physical retailers and sets a high price (p_0) to consumers. Thus, agglomeration at the center point is not beneficial to consumers. However, when the regulator very concerns the profits of retailers (α is large), the socially optimal locations should be close to the end points, where the market shares of physical retailers are minimized and the online retailer enjoy a near monopoly market (include the densely areas) which result in a maximal total profits. Intuitively, the monopoly profit is obvious larger than the sum profits of tripolists, and the online retailer becomes almostly a monopoly in this case. Although our optimal locations are the same as Bárcena-Ruiz and Casado-Izaga (2014) when α approaches to one, yet their result is due to avoiding competition between duopolists.

After some calculations, it is shown that physical retailers have incentive to move close to the market center where population is more dense than other areas. Therefore, the zoning policy should restrict physical retailers from locating near the market center as shown in the shaded areas (Figure 7). Notably, comparing with Bárcena-Ruiz and Casado-Izaga (2014), we do not have the optimal allowed locations near the market center, because the socially optimal locations are sufficiently separated as per online competition in our framework.

9 Conclusions

This paper introduces an online firm entering a spatial duopoly market where non-uniform population density is assumed and price discrimination is engaged in by the physical duopoly firms, and the price and location equilibrium in the short-run (immobile locations) and in the long-run are analyzed. Social welfare is also discussed. It is shown that in the short-run, the price equilibrium will be either all uniform or kinked (price discrimination) at the center area, depending on the relative size of the distaste cost for online purchasing. In the long-run, only the kinked pricing pattern is valid, and physical firms will move closer to the center to keep the densely populated area's market share. Moreover, the long-run equilibrium locations are more separated than the socially optimum when the online price is not high. The average price and price variation will fall after the entry of an online firm. Finally, the socially optimal zoning is keeping sufficiently separated locations between physical retailers.

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