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Working Paper

Non-performing loans, systemic risk and resilience in financial networks

LEM Working Paper Series, No. 2016/08

Provided in Cooperation with:

Laboratory of Economics and Management (LEM), Sant'Anna School of Advanced Studies

Suggested Citation: Bottazzi, Giulio; de Sanctis, Alessandro; Vanni, Fabio (2016) : Non-performing loans, systemic risk and resilience in financial networks, LEM Working Paper Series, No. 2016/08, Scuola Superiore Sant'Anna, Laboratory of Economics and Management (LEM), Pisa

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WORKING PAPER SERIES

Non-performing loans, systemic risk and resilience in financial networks

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2016/08

March 2016

ISSN(ONLINE) 2284-0400

Non-performing loans, systemic risk and resilience in financial networks

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March 1, 2016

Abstract

After the outbreak of the financial crisis in 2007-2008 the level of non-performing loans (NPLs) in the economy has generally increased. However, while in some countries this has been a transitory phenomenon, in others it still represents a major threat for economic recovery and financial stability. The present work investigates the relationship between non-performing loans and systemic risk using a network-based approach. In particular, we analyze how an increase in NPLs at firms level propagates to the financial system through the network of credits and debits. To this end we develop a model with two types of agents, banks and firms, linked one another in a two-layers structure by their reciprocal credits and debits. The model is analyzed via numerical simulations and allows a) to define a synthetic measure of systemic risk and b) to quantify the resilience of the financial system to external shocks, making it particularly useful from a policy point of view. For illustrative purposes, in section 3 we present an application of the model to Italy, Germany, and United Kingdom, using empirically observed data for the three countries.

Keywords: financial crisis, network theory, non-performing loans, resilience, systemic risk

JEL codes: G21, C63, G01, D85

Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 640772 - DOLFINS.

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1 Introduction

Over the last 30 years world's economy has experienced an unparalleled process of globalization, which has lead to a reduction of the world's effective size "from XXL to Small" (Friedman, 2006). In the financial sector this phenomenon has been the cause and the consequence of a strain to diversification which has increased the level of connectivity and complexity of the financial system, with the result that today's financial institutions are directly or indirectly much more connected than ever before.

A non-negligible role in fostering this process has been played by the widespread belief that interconnection of financial markets would have lead to a greater financial stability, as risk would have been reduced via its spreading around the world (Stiglitz, 2010).

However the recent crisis has shown that an higher degree of interconnection is not always desirable to achieve financial stability (Battiston *et al.*, 2012a) and that diversification, rather than curtailing the overall level of risk, has dispersed it, transforming idiosyncratic into systemic risk.

As pointed out by Stiglitz during the IMF conference on interconnectedness in May 2014¹, "the thrust of economic discussion was that diversification, or interconnectedness, was a great thing", however the financial crisis started in 2007-2008 revealed how this belief was wrong since the high number of interconnections between financial intermediaries "facilitated the breakdown" and became "part of the problem". Of the same opinion is Jannet Yellen, current Chair of the Federal Reserve, according to which "interconnections among financial intermediaries are not an unalloyed good. Complex interactions [...] may serve to amplify existing market frictions, information asymmetries, or other externalities" (Yellen, 2013).

The crisis has forced scholars and policy makers to rethink how to promote financial stability paying attention to the interconnections among financial institutions, whose analysis is now considered crucial to gauge systemic risk and to prevent, or at least to dampen, future meltdowns (Schweitzer *et al.*, 2009).

This has led to an intense research activity aimed at better understanding the role of pairwise interactions between financial institutions in propagating and amplifying negative shocks. This field of research goes back to Allen & Gale (2000) and Eisenberg & Noe (2001) and bloomed over the past few years: part of the literature focused primarily on theoretical models of networks, such as Gai & Kapadia (2010), Gai *et al.* (2011), Battiston *et al.* (2012b), Elliott *et al.* (2014), Acemoglu *et al.* (2013), Acemoglu *et al.* (2015), Chinazzi *et al.* (2015)²; while another part devoted its attention to the empirical analysis of interbank networks, such as, Soramäki *et al.* (2007), Iori *et al.* (2008), Bech

¹ "Interconnectedness: Building Bridges between Research and Policy", May 2014, <http://www.imf.org/external/pubs/ft/survey/so/2014/RES052314A.htm> (accessed on 1 July 2015)

² See Chinazzi & Fagiolo (2013) for a survey.

& Atalay (2010), Beltran *et al.* (2015), de Andoain Hidalgo *et al.* (n.d.).

Our paper belongs to the first stream of this growing body of literature and aims at analyzing the level of systemic risk and the resilience of a financial system in a network perspective.

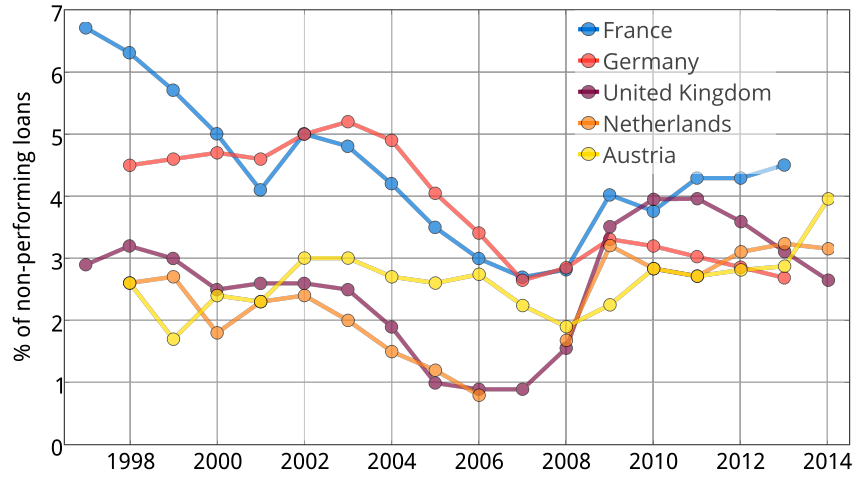
In what follows we develop a network model which simulates how an exogenous shock, represented by an increase in the level of non-performing loans (NPLs)³, can affect the stability of the financial system through the network of credits and debits.

Unlike most of the in this field, we focus on NPLs as source of shock for three main reasons: first, because while in some countries their level is increased with the outbreak of the crisis, to go back subsequently to a level equal or lower the pre-crisis one, in other countries NPLs are still increasing and have become a major concern at policy level; second, because using NPLs allows to measure the intensity of the shock and anchor it to a real observable variable; third, because, notwithstanding the simplifying assumptions of the model, using a real variable to study the response of financial systems allow to draw some practical policy conclusion.

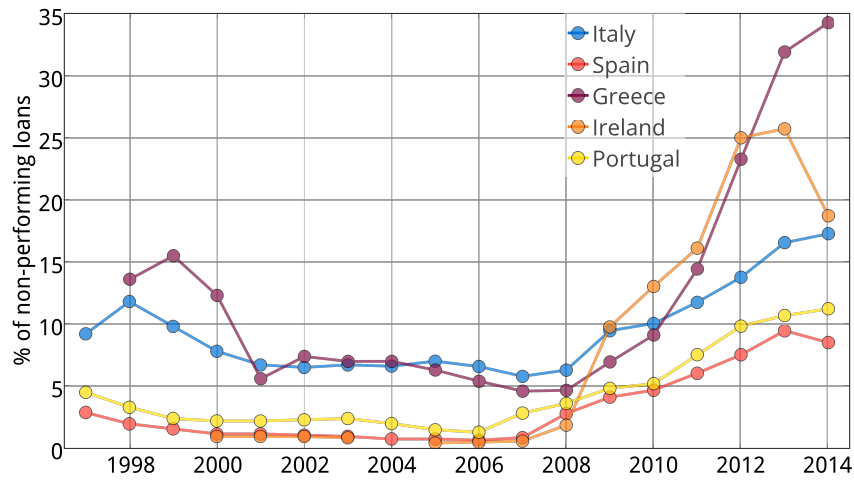
To give a better picture of the phenomenon, Figure 1 shows the percentages of non-performing loans to total gross loans granted by banks in different European countries from 1997 to 2014. By looking at the data it is possible to distinguish two groups of countries: a first group, shown in the top panel, where the financial crisis has had only transitory effects of the level of NPLs, which increased right after the outbreak of the crisis in 2008 and went back thereafter; and a second group, represented in the bottom panel and coinciding mainly with the periphery countries of the Eurozone, where the level of NPLs boomed after the crisis and remained well above levels prior to 2008. In particular, it is worth noting how in countries like Italy, Greece and Portugal NPLs are still today marked by an upward trend, with obvious drawbacks in terms lending provision, economic growth and financial stability.

In what follows we develop a model with two types of agents, banks and firms, linked one another in a two-layer network by their claims and obligations. By means of computer simulations we investigate the relationship between non-performing loans, systemic crisis and resilience, providing a synthetic measure of systemic risk and identifying the shock that a financial system is able to bear. In order to provide a closer matching with reality, we calibrate the model with empirical data for three different countries, Italy, Germany and United Kingdom. Nevertheless, due to the lack of publicly available data on bilateral exposures between banks and between banks and firms, the results of the simulations

³Following the definition of the IMF “a loan is nonperforming when payments of interest and principal are past due by 90 days or more, or at least 90 days of interest payments have been capitalized, refinanced or delayed by agreement, or payments are less than 90 days overdue, but there are other good reasons to doubt that payments will be made in full” (Clarification and Elaboration of Issues Raised by the December 2004 Meeting of the Advisory Expert Group of the Intersecretariat Working Group on National Accounts, International Monetary Fund, June 2005).



(a)



(b)

Figure 1: Percentage of non-performing loans over total gross loans⁴ from 1997 to 2014 in different European countries. Panel a) shows the countries where the level of NPLs remained substantially stable before and after the crisis. Panel b) display the countries where NPLs exploded after 2008 and are still today at levels higher than the pre crisis period. In particular, NPLs in Italy, Portugal and Greece in 2014 still show an upward trend. Source: elaboration of the authors on data from the International Monetary Fund, Global Financial Stability Report.

should be intended only as illustrative of the usefulness of the framework and of the results that it is possible to obtain. Despite this, the model provides a new insight for the emergence of a systemic crisis and represents a first attempt to link together the financial and the real side of the economy, constituting a useful starting point for future research. The model, presented in Section 2, shares the basic structure of Nier *et al.* (2007) and Gai & Kapadia (2010); it further draws from Caccioli *et al.* (2012) as far as regard the introduction of heterogeneity, from Anand *et al.* (2013) for the presence of firms and from Montagna & Lux (2013) in the adoption of a fitness algorithm to generate the network. In Section 3 we discuss the calibration of the model (more details in Appendix 4) and the result of the simulation exercises performed for a baseline setting and three alternative specifications corresponding to Italy, Germany and United Kingdom. Section 4 reviews and concludes the work.

2 The model

Consider an economy composed by N banks and M firms. Assume that the two sets of agents are the nodes of a bipartite network organized in two interconnected layers, where one comprises banks and the other firms. Agents are linked one another through their balance sheets by credits and debits which result from financial transactions: for any node i an incoming link is a credit and an outgoing link is a debit.

Following Nier *et al.* (2007), we represent each bank via a simplified balance sheet structure as the one depicted in Figure 2⁵ and we assume the following relations to hold:

$$A_i^{tot} = A_i^{ib} + A_i^f, \quad (1)$$

$$L_i^{tot} = L_i^{ib} + D_i + K_i, \quad (2)$$

$$A_i^{ib} = \theta \times A_i^{tot}, \quad (3)$$

$$A_i^f = (1 - \theta) \times A_i^{tot}, \quad (4)$$

$$K_i = \eta \times A_i^{tot} \quad (5)$$

where A_i^{ib} represents the interbank assets (i.e. the assets owned towards other banks), A_i^f the external assets (i.e. the assets owned towards firms), A_i^{tot} the total assets, L_i^{ib} the interbank liabilities, D_i the deposits⁶, K_i the net worth and L_i^{tot} the total liabilities. Due to the double-entry bookkeeping system A_i^{tot} is equal to L_i^{tot} for all the values. Finally, θ and η are two constant parameters equal for all banks: the first represents the ratio between the interbank assets and the total assets of a bank, the second the capital ratio. From the imposed relations between the elements of the balance sheet, it follows that a bank i is solvent if the solvency condition

$$K_i = A_i^{ib} + A_i^f - L_i^{ib} - D_i > 0 \quad (6)$$

is satisfied. We assume that banks can lend and borrow from other banks, but can only lend to firms; moreover we assume that firms cannot borrow from each other, but only from banks. These assumptions imply that banks can have both incoming and outgoing links with other banks, but only incoming links form firms. On the other hand, firms can only have outgoing links toward banks and no links with other firms.

At time $t = 0$ each bank $i = 1, \dots, N$ is provided with an amount of interbank assets A_i^{ib} drawn from a power law distribution $\rho(A_i^{ib}) \sim A_i^{ib-\tau_1}$ with bounded support $A_i^{ib} \in [A_{min}^{ib}, A_{max}^{ib}]$. Similarly, to each firm $j = 1, \dots, M$ is assigned a value of total assets F_j^{tot} , distributed according to a power law $\rho(F_j^{tot}) \sim F_j^{tot-\tau_2}$ with $F_j^{tot} \in [F_{min}^{tot}, F_{max}^{tot}]$. We

⁵We do not model the balance sheet structure of firms since our focus is on the consequences of shocks for the financial system.

⁶ D_i is computed as a difference between A_i^{tot} and $L_i^{ib} + K_i$. We label this difference “deposits” in accordance with the current literature, however it must be noted that this is just a convention and that there is no relation between real banks’ deposits and this variable.

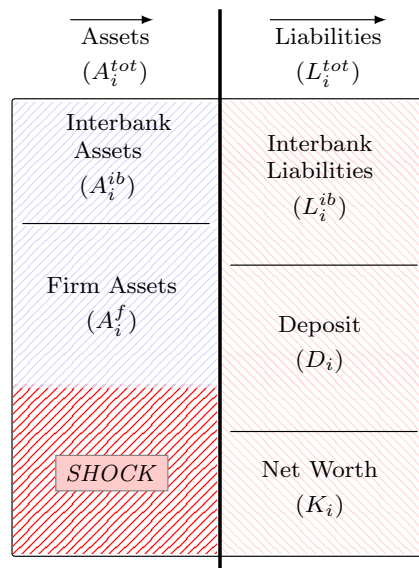


Figure 2: Balance sheet structure. The arrows on the top of the figure indicate the direction of links: incoming links represent an asset, outgoing links represent a liability. The shock affects the portion of the assets held against firms. To cause the default of a node, the initial shock must be higher than the net worth owned by that node. Elaboration of the authors adapted from Haldane & May (2011).

take A_i^{ib} and F_j^{tot} as fitness parameters of the linking functions that we use to generate the network. The algorithm implemented generates the network randomly in a kinetic way⁷, using the probability given by the linking functions to establish whether create an incoming link between two nodes (Caldarelli, 2007). In particular, following Montagna & Lux (2013), we assume the following functional forms:

$$P(A_i^{ib}, A_j^{ib}) = \left(\frac{A_i^{ib}}{A_{max}^{ib}} \right)^\alpha \cdot \left(\frac{A_j^{ib}}{A_{max}^{ib}} \right)^\beta, \quad (7)$$

$$P(A_i^{ib}, F_j^{tot}) = \left(\frac{A_i^{ib}}{A_{max}^{ib}} \right)^\phi \cdot \left(\frac{F_j^{tot}}{F_{max}^{tot}} \right)^\chi. \quad (8)$$

The assumptions about the distribution of banks' and firms' size and about the linking function lead to a scale-free network structure, where the parameters α , β , ϕ and χ define the properties of the network itself. In what follows we assume $\alpha = 1$ and $\beta = 0.25$ for the interbank network and $\phi = 1$ and $\chi = 1$ for the banks-firms network. We do so in order to reproduce the frequently documented feature of disassortative behavior in the interbank network (Iori *et al.* (2006), Soramäki *et al.* (2007)) and to create an assortative behavior in the firms-banks network, which reflect the assumptions that bigger firms have higher possibilities to access credit and that on average tend to have more links (i.e. more credit lines) than small firms.

After having generated the network, we assign a weight to each incoming link: for interbank links, the weight depends on the amount of interbank assets of the creditor (A_i^{ib}) and of the debtor (A_j^{ib}), as well as on the number of incoming links of the creditor. Formally is:

$$w_{i,j}^{\leftarrow} = A_i^{ib} \cdot \frac{A_j^{ib}}{\sum_{k \in \Omega_i} A_k^{ib}}, \quad (9)$$

where Ω_i is the set of nodes linked with i . Similarly, the weights of bank-firm links depend on the external assets of each bank A_i^f , on the level of total assets of firms F_j^{tot} and on the number of incoming links of the bank. Formally is:

$$w_{i,j}^{\leftarrow} = A_i^f \cdot \frac{F_j^{tot}}{\sum_{k \in \Omega_i} F_k^{tot}}. \quad (10)$$

In other words, we distribute the amount of interbank assets and of external assets of each bank proportionally to the size of the debtor. This reflect the assumption that bigger nodes are able to get more credit from banks, which consider them as more trustworthy and less risky. By assigning weights in this way, interbank liabilities of each bank

⁷See Appendix 4 for a more detailed explanation of the algorithm used.

are endogenously determined and we are able to compute all the elements of banks' balance sheet in Figure 2.

Following this procedure we obtain a scale-free network that is bipartite, directed, and weighted. In terms of adjacency matrix it can be represented as follows:

$$A_{(N+M),(N+M)} = \left[\begin{array}{cccc|ccc} 0_{1,1} & a_{1,2} & \dots & a_{1,N} & a_{1,(N+1)} & \dots & a_{1,(N+M)} \\ a_{2,1} & 0_{2,2} & \dots & a_{2,N} & a_{2,(N+1)} & \dots & a_{2,(N+M)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ a_{N,1} & a_{N,2} & \dots & 0_{N,N} & a_{N,(N+1)} & \dots & a_{N,(N+M)} \\ \hline 0_{(N+1),1} & 0_{(N+1),2} & \dots & 0_{(N+1),N} & 0_{(N+1),(N+1)} & \dots & 0_{(N+1),(N+M)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{(N+M),1} & 0_{(N+M),2} & \dots & 0_{(N+M),N} & 0_{(N+M),(N+1)} & \dots & 0_{(N+M),(N+M)} \end{array} \right]$$

In the above matrix each element $a_{i,j}$ represents the debit of node j towards node i . The top-left block represents the interbank network and contains the weights of the links among the N banks: in this block, the row sum gives the interbank liabilities of each bank $L_i^{ib} = \sum_{j=1}^N a_{j,i}$, while the column sum gives the amount of interbank assets owned by each bank $A_i^{ib} = \sum_{j=1}^N a_{i,j}$. The top-right block represents instead the banks-firms network and contains the weights of the links between the N banks and the M firms: in this block, the row sum gives the total debt of each firm, while the column sum gives the amount of external assets owned by each bank $A_i^f = \sum_{j=1}^N x_{i,j}$. The bottom-left and bottom-right blocks are made of zeros because by assumptions we rule out the possibility of firms lending to banks and to other firms. Also the main diagonal of the adjacency matrix has only zeros elements since self-loops, i.e. links starting and ending in the same node, are not allowed (in other words a bank cannot have a credit or debit with itself); instead, consistently with bankruptcy law, we do not net interbank positions, so two banks can be linked with each other in both directions. For illustrative purposes, figure 3 shows an example of the typical network obtained using the previous procedure.

After having initialized the model, at time $t = 1$ we perturb the system with an exogenous shock consisting by an increase in the level of NPLs. In practice we transform some of the credits provided by banks to firms into be bad loans and we do so by selecting firms at random and assuming that they become unable to meet their obligations until we reach the desired amount of NPLs. More in detail, given an amount x of NPLs: we select a firm at random; again at random we go through its outgoing link one by one; we set the value of the selected link equal to zero and we repeat until an amount of debt equal to x is canceled. If the total debt of the firm is greater than x , the last link considered is simply reduced by the amount necessary to reach x . If instead the the total debt of the firm is lower than x , the procedure continues with another randomly selected firm, until the debts of all the link canceled is equal to x . The idea behind the shock on NPLs is that banks exposed toward defaulted firms incur in a loss which erodes their net worth, potentially forcing

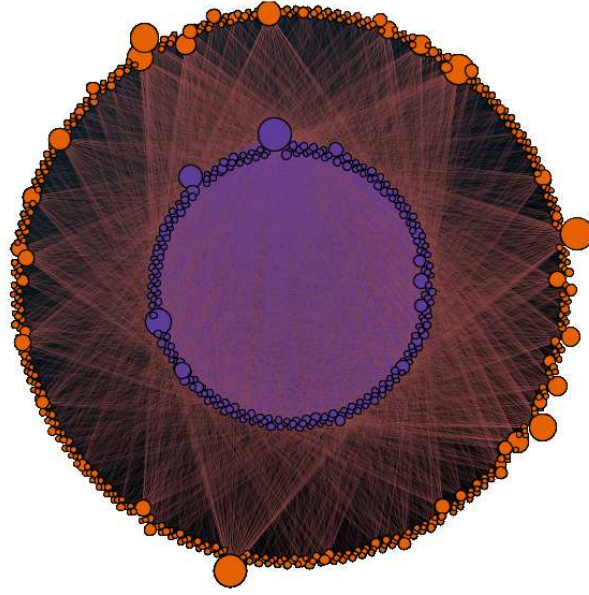


Figure 3: Blue circles represent banks, red circles firms. The size of the circles is proportional to the amount of interbank assets for banks and total assets for firms. Red links represent links from firms to banks, blue links represent links from banks to other banks. Notes that there are no links between firms and that, although it is not possible to (clearly) show in the figure the direction of links, red links can only be directed from firms (out) to banks (in), while blue links among banks can go in both directions (in and out).

them to default. Banks that become insolvent after the shock by being unable to meet condition (6) (that is those banks whose net worth was not enough to absorb the loss), are set into default⁸. Again, defaulted banks are assumed to default on all their liabilities and for all the amount (*no partial recovery*⁹), so the corresponding asset of creditor banks are set equal to zero and their balance sheets are accordingly reduced by the amount lost. In this way the initial firm-level shock transmits at interbank level where failed banks are assumed to default on all of their interbank liabilities, eventually pushing neighbor banks into default. This process continues to iterate until no further bank failures occur. In this way the initial shock can be either absorbed or amplified, eventually triggering a cascade of defaults able to cause a systemic crisis within the financial network.

⁸As pointed out in Gai & Kapadia (2010) it is possible to impose a minimal capital requirement, but this “*would leave our results qualitatively unchanged as it would just lead to a linear rescaling of the balance sheet*”.

⁹As far as concern banks, as pointed out in Gai & Kapadia (2010) “this assumption is likely to be realistic in the middle of a crisis: in the immediate aftermath of a default, the recovery rate and the timing of recovery will be highly uncertain and banks’ funders are likely to assume the worst-case scenario”. Anyway it would be possible to relax this assumption and allow for a partial recovery, so that when a linked bank defaults, the creditors do not lose all their asset, but get some fraction of it, for example a share of the remaining assets proportional to the weight of creditors’ asset over all other liabilities of the defaulted bank.

3 Model calibration and simulation results

Using data from *Bankscope* and *Amadeus* we calibrated the model described in section 2 and we simulated it numerically for a baseline case, where the values of the parameters are the ones usually assumed in the literature, and for the cases of Italy, Germany and United Kingdom, where the values of the parameters are based on empirically observed data. In particular, for each country we derived the capital/total asset ratio η , the interbank assets/total assets ratio θ and the distributions of banks' interbank assets and firms' total assets.

The values of η and θ have been computed as average of the values of the single banks (see Upper (2007)); for the distribution of banks' interbank assets A^{ib} and of firms' total assets F^{tot} , data have been fitted using a power-law distributional model. See Newman (2005), Clauset *et al.* (2009), Bottazzi *et al.* (2015), Bottazzi (2009).

The distributions of A^{ib} and F^{tot} are depicted in Figure 4, while Table 1 reports the model's list of parameters and their values for the four simulations (see Appendix 4 for further details).

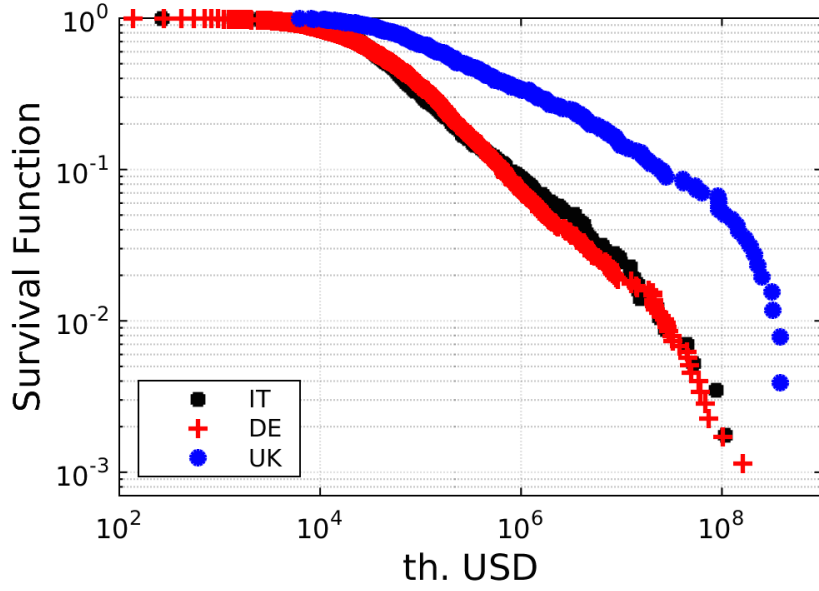
In the simulations we consider a network of 100 banks and 250 firms and different combinations of $AD_{B,B}$ and $AD_{B,F}$. For every pair of values of the average degrees we draw 200 realizations of the network, in each of which we shock the system by increasing the level of NPLs as described by the algorithm in Section 2. We chose the average degree as a key parameter to vary as it gives the average number of counterparts of a node and so is a proxy for the level of interconnectedness of a system (Elliott *et al.*, 2014). Moreover the real value of $AD_{B,B}$ and $AD_{B,F}$ is in general not known¹⁰, so we tested a wide range of reasonable values.

Since we are interested in the risk of a systemic crisis we want to exclude small chain of defaults, for this reason, following Gai & Kapadia (2010), we define a systemic crisis as the occurrence of the default of more than 5% of banks in the network. Given this definition, we compute the frequency of a systemic crisis (F) as the number of times in which more than 5% of banks default over the 200 drawings and the extent of a systemic crisis (D) as the fraction of defaulted banks conditional on contagion over the 5% threshold breaking out, which is therefore a measure of the magnitude of the systemic crisis.

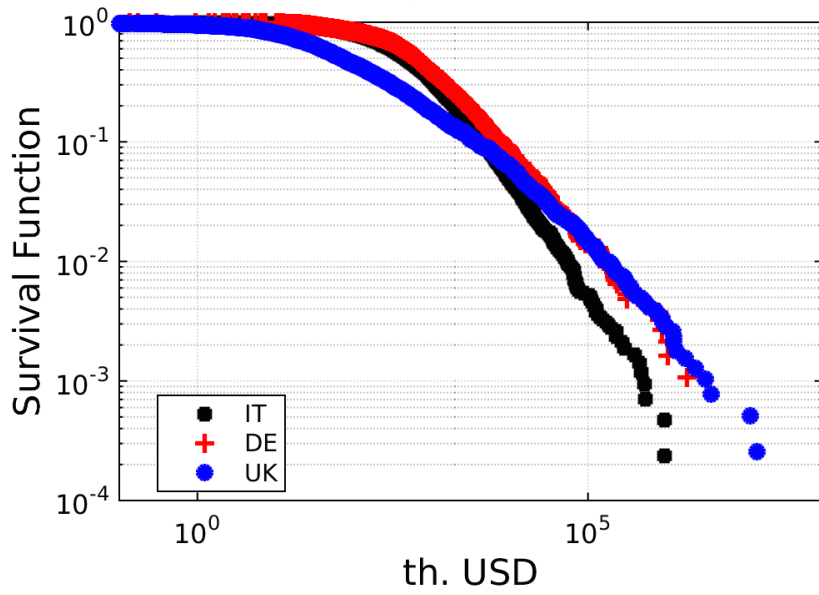
These two quantities allow to define a synthetic statistics for measuring systemic risk (R), which we compute as the product between the frequency of contagion (F) and the extent of contagion (D):

$$R = F \times D \tag{11}$$

¹⁰Few estimates of the interbank average degree are present in the literature, for example Anand *et al.* (2015) finds that the average degree for the German interbank network is 10.5, while Soramäki *et al.* (2007) finds an average degree of 15.2 for the Fedwire interbank payment network.



(a) Distribution of interbank assets (A^{ib})



(b) Distribution of firms' total assets (F^{tot})

Figure 4: Distribution of interbank assets (panel a) and firms' total assets (panel b) for Italy, Germany and United Kingdom in 2013 (log-log scale). Source: computations of the authors based on data from *Bankscope* and *Amadeus*.

Parameters	Baseline	Italy	Germany	United Kingdom
N	100	100	100	100
M	250	250	250	250
T	200	200	200	200
$AD_{B,B}$	[0:0.5:25]	[0:0.5:25]	[0:0.5:25]	[0:0.5:25]
$AD_{B,F}$	[0:0.5:25]	[0:0.5:25]	[0:0.5:25]	[0:0.5:25]
$A^{ib} \sim$	truncated Pareto	truncated Pareto	truncated Pareto	truncated Pareto
A_{min}^{ib}	5	4.10e+04	1.10e+05	5.15e+04
A_{max}^{ib}	100	3.38e+08	3.87e+08	3.99e+08
τ_1	2	1.59	1.68	1.33
$F^{tot} \sim$	truncated Pareto	truncated Pareto	truncated Pareto	truncated Pareto
F_{min}^{tot}	5	3.77e+03	2.54e+03	6.53e+04
F_{max}^{tot}	100	1.48e+06	1.38e+08	1.69e+08
τ_2	2	1.95	1.73	1.72
η	8%	12%	11%	31%
θ	20%	11%	11%	30%
δ	[0:1.25%:10%]	[0:1.25%:10%]	[0:1.25%:10%]	[0:1.25%:10%]
α	1	1	1	1
β	0.25	0.25	0.25	0.25
ϕ	1	1	1	1
χ	1	1	1	1

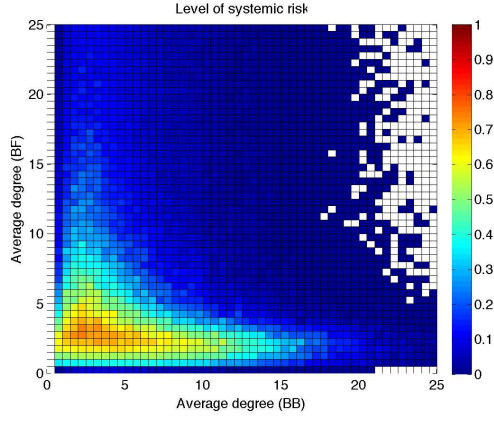
Table 1: The table summarizes the parameters used in the numerical simulations and their values for the four scenarios. The first column list the parameters: N and M are respectively the number of banks and firms in the network; T is the number of realizations for each pair of average degrees, which range and step are shown in the rows $AD_{B,B}$ and $AD_{B,F}$; A^{ib} and F^{tot} are distributed according to a truncated Pareto with exponents respectively given by τ_1 and τ_2 and ranges defined by $[A_{min}^{ib}, A_{max}^{ib}]$ and $[F_{min}^{tot}, F_{max}^{tot}]$; η is the percentage of capital with respect to total asset; θ is the percentage of interbank assets with respect to total assets; δ is the percentage of NPLs over the total amount of loans in the economy (i.e. is the magnitude of the initial shock); α and β are the exponents used in the linking function between banks; ϕ and χ are the exponents used in the linking function for banks and firms.

Figure 5 and 6 summarizes the results of the simulations for the baseline case¹¹. The four panels in Figure 5 show the level of systemic risk in terms of probability associated with different increases in the NPLs over total gross loans. On the x-axis and y-axis we show respectively the interbank average degree $AD_{B,B}$ and the bank-firm average degree $AD_{B,F}$. Low values of $AD_{B,B}$ correspond to a poorly connected interbank network, while higher values correspond to an highly connected network. The same applies to the values of $AD_{B,F}$. White squares in the heat-map corresponds to situations in which the threshold of 5% of default has never been reached in the 200 draws of the simulations. Different colors represent different levels of systemic risk: as shown by the vertical bar on the

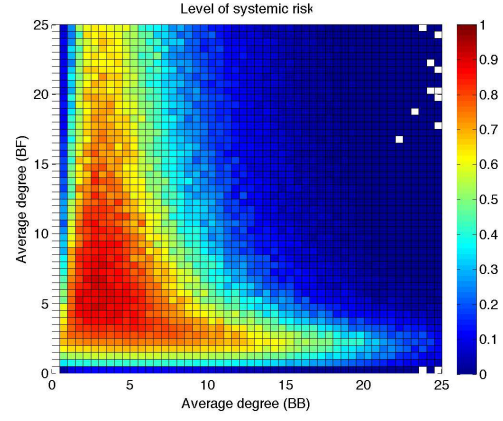
¹¹As reported in Table 1 the values of δ tested go from 0 to 10% with a step of 1.25%. Given the purely illustrative intent of the work in the figures from 5 to 12 we show only some selected charts in order not to overload the reading. The full set of charts is available upon request.

right-hand side of the heat-map, colors towards blue correspond to low levels, while colors toward red to high levels. The figure shows that, on both the axes, the levels of systemic risk first increases and then decreases, showing a non-monotonic behavior and peaking in the bottom-left area of all the panels. Moreover it is possible to see how an higher initial shock does not change the general shape of the plot, but *ceteris paribus* it only increases the risk level, while it preserves the shape and the non-monotonic behavior.

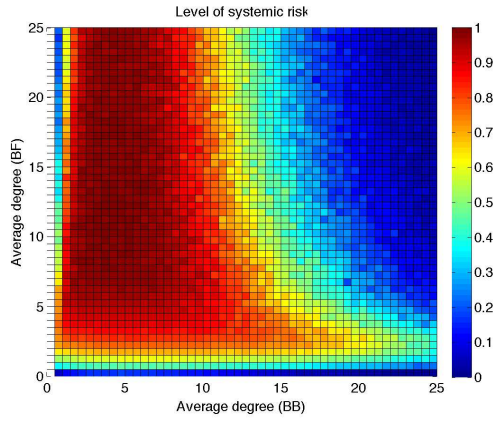
Figure 6 shows the fraction of defaulted banks over the total number of banks present in the network, as a function of the magnitude of the shock. We computed the fraction of defaulted banks as the average over the 200 draws. The results reported in panel in the figure are compiled from performing 200 draws for each value of the shock. In the baseline case it is possible to identify clearly a phase transition for values of the shock between 4% and 5%, so small changes in the shocks in terms of magnitude can have very different consequences in terms of fraction of defaults. Being able to identify such a threshold is particularly important because it tell a lot about the characteristics of the network and is a measure of the resilience of the system to exogenous shock. By comparing the four panels of figure 6 it is also possible to se how changes in the average degrees affect the resilience of the system: *ceteris paribus* an increase in $AD_{B,F}$ moves the phase transition to the right, while an increase of $AD_{B,B}$ increases the high of the curve and so the the fraction of defaulted banks.



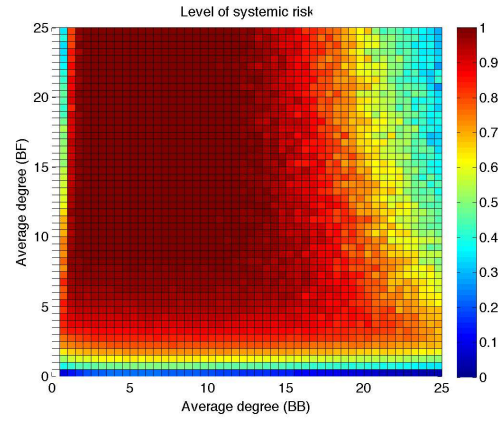
(a) $\delta = 2.5\%$



(b) $\delta = 3.75\%$

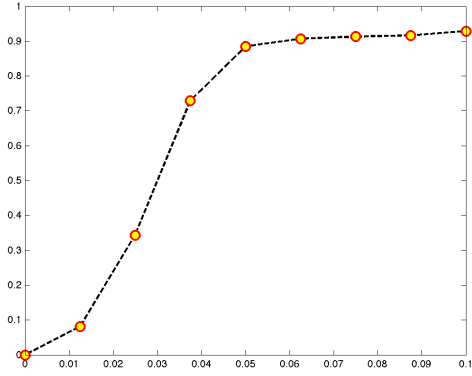


(c) $\delta = 5\%$

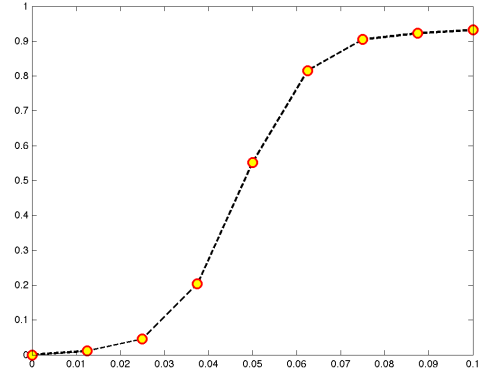


(d) $\delta = 6.25\%$

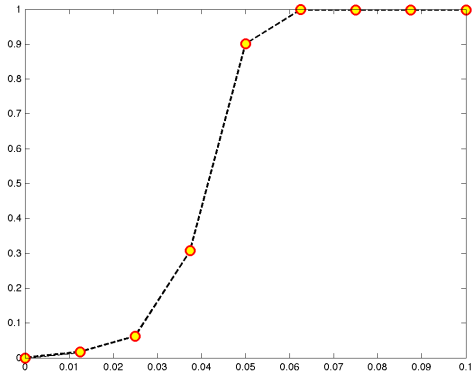
Figure 5: The figure shows the level of systemic risk associated to different level of initial shock in the baseline case.



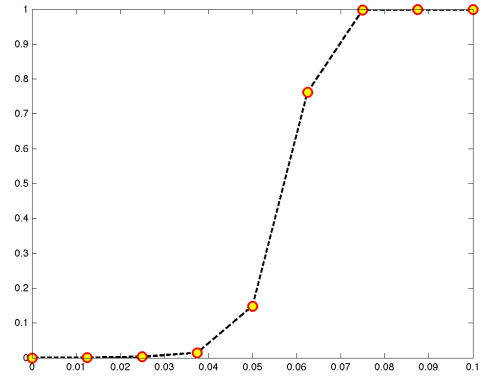
(a) $AD_{BB} = 4, AD_{BF} = 9$



(b) $AD_{BB} = 4, AD_{BF} = 20$



(c) $AD_{BB} = 13, AD_{BF} = 9$



(d) $AD_{BB} = 13, AD_{BF} = 20$

Figure 6: The figure relates to the baseline case and shows the fraction of defaulted banks for different levels of initial shock and different combinations of average degrees.

For illustrative purposes in figures from 7 to 12 we show the results of the simulations for the cases of Italy, Germany and United Kingdom. In general the results obtained for the three countries are more “noisy” than the ones obtained for the baseline case and, for example, it is not always easy to identify a clear phase transition as it is instead possible to do in figure 6 (this is true especially for the United Kingdom where the trend appears linear). Nevertheless, the same considerations made for the baseline apply also to the other three cases, both in terms on non-monotonic behavior of the systemic risk (with levels and position varying in the $AD_{B,B}$, $AD_{B,F}$ space according to country characteristics) and for what regards the effects of an increase in the average degrees on the fraction of defaulted banks over the total number of banks.

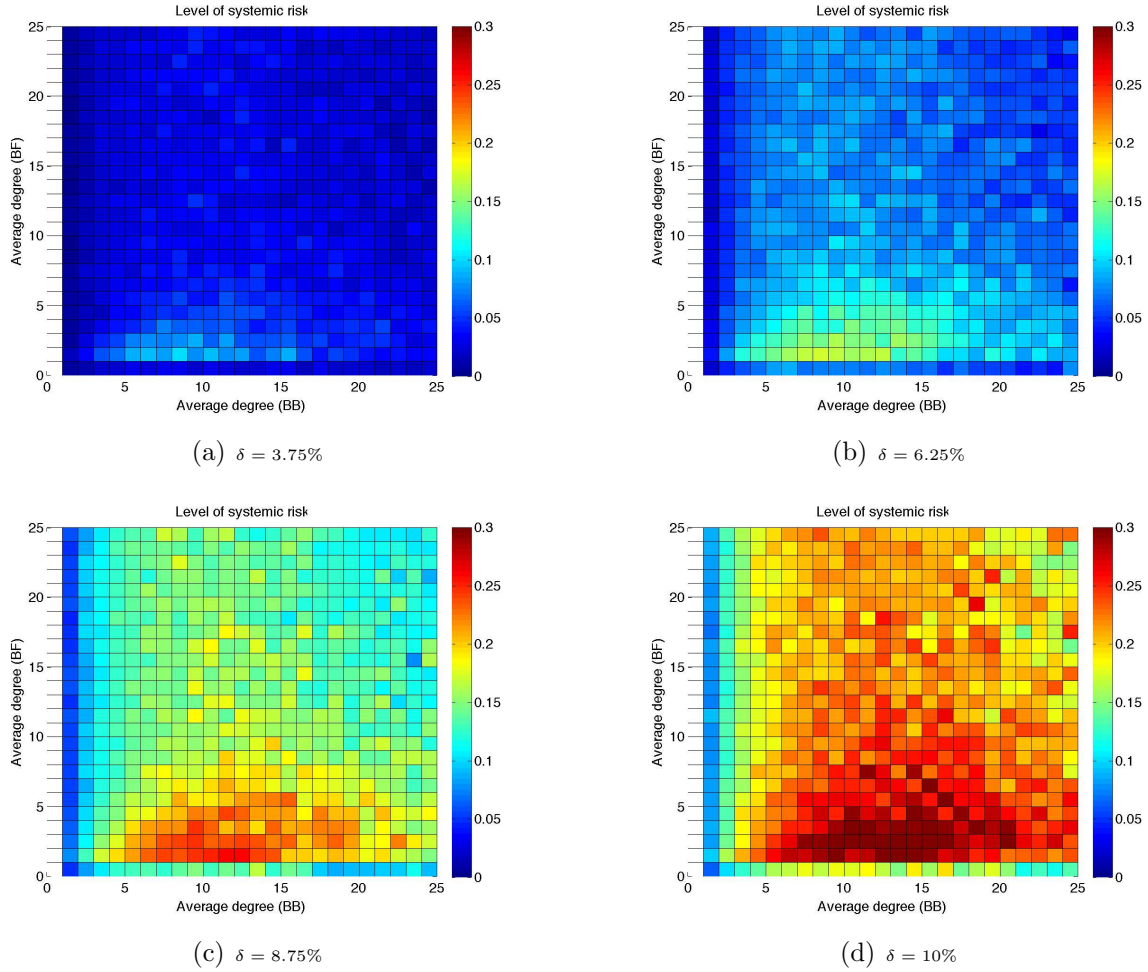
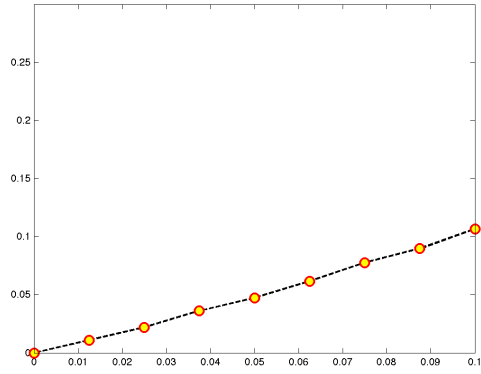
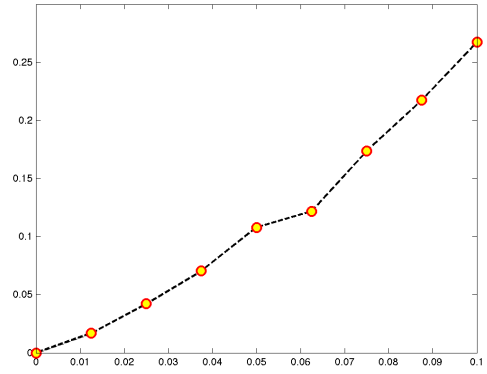


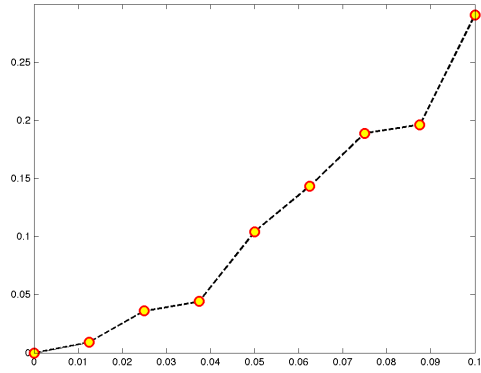
Figure 7: The figure shows the level of systemic risk associated to different level of initial shock in the case of Italy.



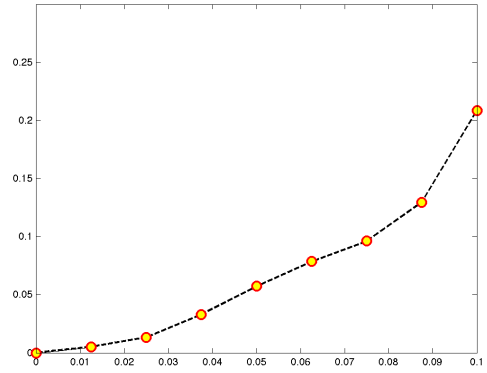
(a) $AD_{BB} = 1, AD_{BF} = 3$



(b) $AD_{BB} = 4, AD_{BF} = 9$



(c) $AD_{BB} = 4, AD_{BF} = 14$



(d) $AD_{BB} = 13, AD_{BF} = 20$

Figure 8: The figure relates to the case of Italy and shows the fraction of defaulted banks for different levels of initial shock and different combinations of average degrees.

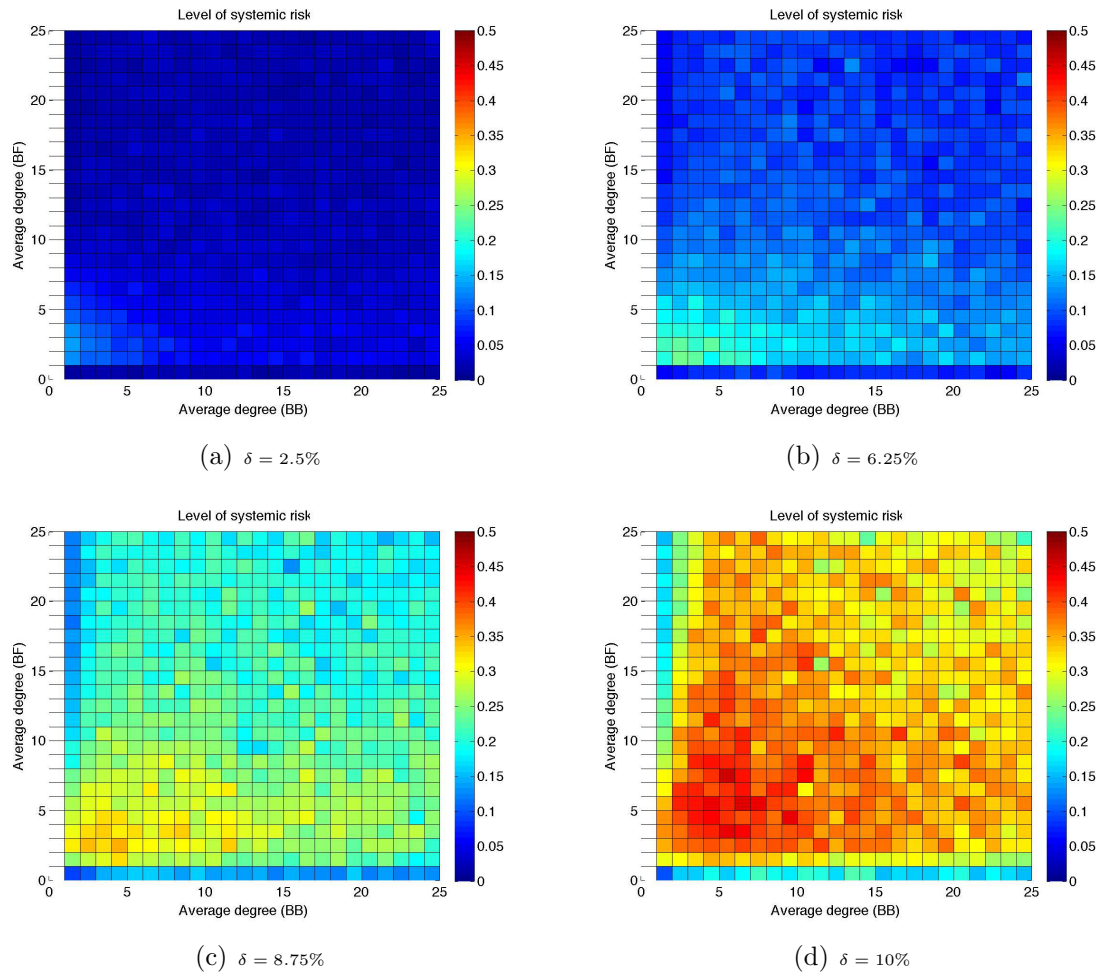
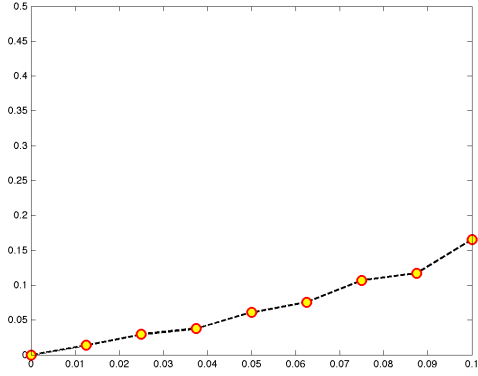
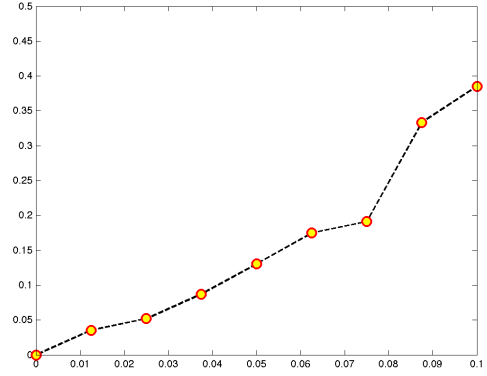


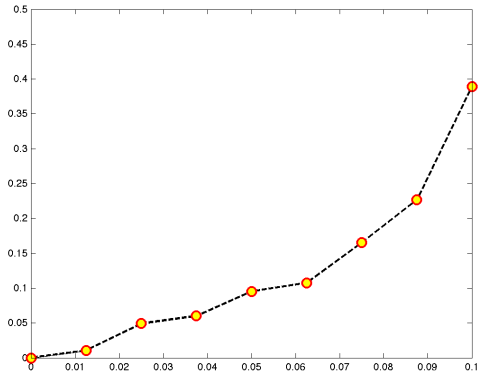
Figure 9: The figure shows the level of systemic risk associated to different level of initial shock in the case of Germany.



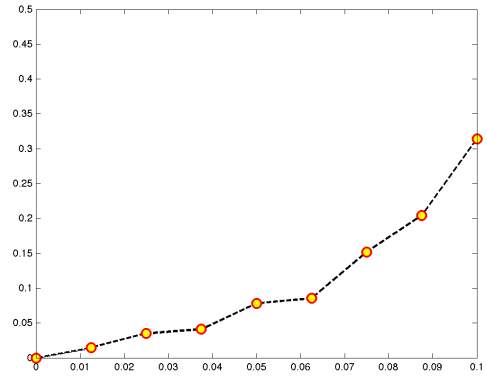
(a) $AD_{BB} = 1, AD_{BF} = 3$



(b) $AD_{BB} = 4, AD_{BF} = 9$



(c) $AD_{BB} = 8, AD_{BF} = 14$



(d) $AD_{BB} = 13, AD_{BF} = 20$

Figure 10: The figure relates to the case of Germany and shows the fraction of defaulted banks for different levels of initial shock and different combinations of average degrees.

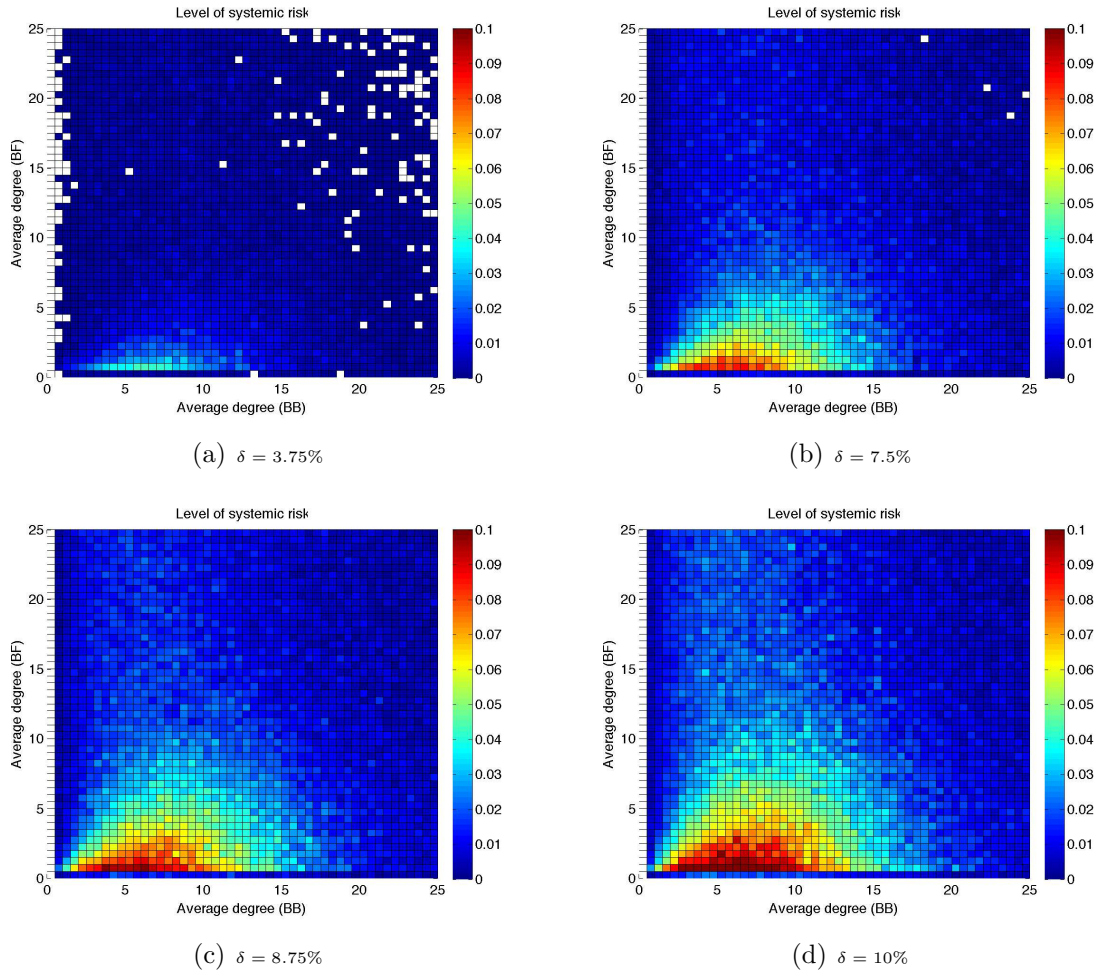
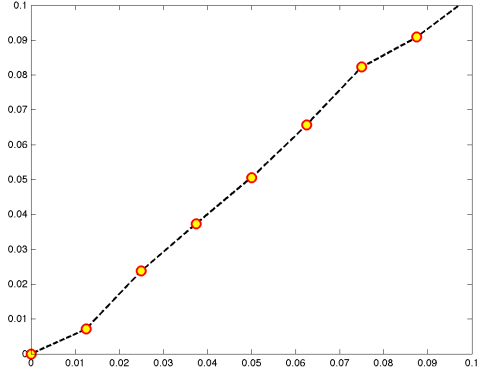
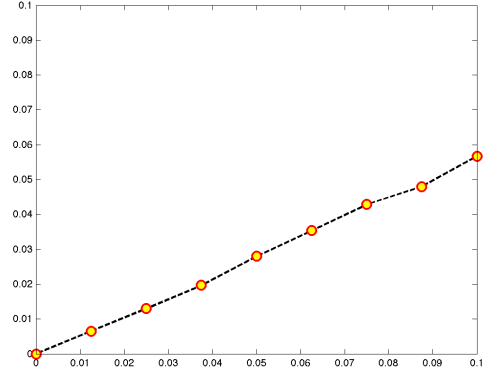


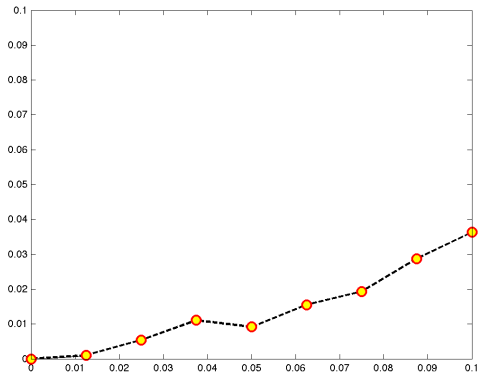
Figure 11: The figure shows the level of systemic risk associated to different level of initial shock in the case of the United Kingdom.



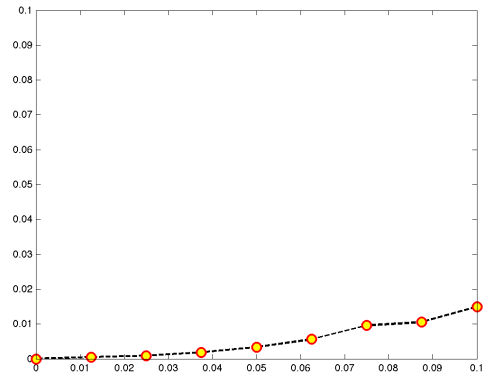
(a) $AD_{BB} = 1, AD_{BF} = 3$



(b) $AD_{BB} = 4, AD_{BF} = 3$



(c) $AD_{BB} = 8, AD_{BF} = 14$



(d) $AD_{BB} = 13, AD_{BF} = 20$

Figure 12: The figure relates to the case of United Kingdom and shows the fraction of defaulted banks for different levels of initial shock and different combinations of average degrees.

4 Conclusions

This paper studies the relationship between non-performing loans and financial stability in a network perspective. In particular, we analyze how an increase in NPLs at firms level affects the financial system through the network of credits and debits. To this end we develop a model with two types of agents, banks and firms, linked one another by their credits and debits.

In order to illustrate the model we calibrated it for a baseline case and for the cases of Italy, Germany, and United Kingdom starting from empirically observed data on the banking sector and on firms for the three countries. The analysis of the data showed that the distributions of banks' interbank assets and firms' total assets follow a power law distribution and that the ratios "equity over total asset" and "interbank assets over total assets" are respectively 12% and 11% in Italy, 11% and 11% in Germany and 31% and 30% in the United Kingdom (figures for 2013).

By means of simulations we studied the impact of an exogenous shock represented by an increase in NPLs in terms of systemic risk and resilience of the financial system. We find that the level of systemic risk varies with the level of interconnectedness of the network in a non-monotonic way and that it peaks for low and intermediate degrees of connectivity, rather than for high degrees, with differences in terms of levels and exact position of the peak in the AD_{BB}, AD_{BF} space depending on the specific case.

In terms of resilience, the simulations show the existence of a phase transition for the baseline case, Italy and Germany (not clear for the United Kingdom), so small variations in the magnitude of the initial shocks can have very different consequences in terms of fraction of defaults.

Although the model presented is a simplified representation of the dynamics that lead to the emergence of systemic risk, we argue that in presence of data on bilateral exposures between banks and firms, the framework developed can help to assess the level of risk to which the system is exposed and to evaluate its resilience to exogenous shocks, thus providing useful guidance to policy makers when facing with decisions about financial stability, such as whether the level of NPLs is critical and exposes the whole system to a high risk, whether to intervene to reduce the amount of NPLs in the financial system, whether to create incentives/disincentives to modify the position of the network in the "average degrees space" or whether to implement regulations about the values of η and θ .

Appendix A

The dataset used builds on the data provided by *Bankscope* and *Amadeus*, two database produced by *Bureau van Dijk* which collect financial information respectively on banks and on public and private companies.

From *Bankscope* we extracted information on interbank assets (A^{ib}) and banks' total assets (A^{tot}), while from *Amadeus* we got the data about firms' total assets (F^{tot}). As far as concerns banks, our dataset covers the period from 2000 to 2014, while for firms the dataset goes from 2005 to 2014. The cut off date for the observations is 31 March 2015.

Bankscope and *Amadeus* report financial informations about banks and firms according to their consolidation level. In order to avoid double counting issues and keep banking groups as much aggregated as possible, we selected data associated to a consolidation code *U1*, *C2* or *C1*¹². However, as pointed out in Duprey (2012) in relation to *Bankscope* - the same holds for *Amadeus* -, even after having considered these consolidation aspects, it still remains possible to face another double counting issue, which is impossible to solve without information about the ownership for all the years corresponding to the date at which the observations have been recorded. Ownership data are provided by *Bankscope*, but they require an extra license; moreover ownership data are in the cross-section for the current years, so to get the evolution over time of ownership structure in order to include the evolution of parent/subsidiaries relation over time, it is necessary to use the updated version of the database at that time. The data can therefore suffer of a bias, which is however small and negligible for the purposes of the present work.

From the dataset described above we estimated the values reported in table 1 which we used in the simulations. In tables from 2 to 7 we reported the estimates of those values for all the years available in the dataset. In the simulations we used the estimated values for 2013, as they are the most recent with a high coverage of the sample. Indeed not for all the banks and the firms present in *Bankscope* and *Amadeus* the values of the variables of our interest are reported, so to avoid using in the simulations estimates based of few observations, we computed a measure of coverage of the sample (see last 1-2 columns in the tables below), which is nothing else that the fraction of banks/firms for which the values of the variables are reported, over the total number of banks/firms surveyed. From the tables below it is possible to see that in 2013 the coverage of interbank assets, banks' total assets and firms' total assets are respectively 0.87, 0.88 and 0.88 for Italy, 0.94, 0.95 and 0.50 for Germany, 0.48, 0.85 and 0.93 for the United Kingdom.

In the following tables: a) τ_1 , $\tau_{A^{tot}}$ and τ_2 indicate the exponent of the power law distribution respectively for the interbank assets, the total assets of banks and the total assets of firms; b) A_{min}^{ib} and A_{max}^{ib} are the estimated minimum and maximum values of the in-

¹²For more informations about this classification reference should be made directly to the guide provided by *Bureau van Dijk*.

terbank assets, A_{min}^{tot} and A_{max}^{tot} for banks' total assets and F_{min}^{tot} and F_{max}^{tot} for firms' total assets; c) $sample_{A^{ib}}$, $sample_{A^{tot}}$, $sample_{F^{tot}}$ represent the total number of observations for banks and firms in the database; d) θ is the average ratio between interbank assets and total assets; e) η is the average ratio between capital and total asset.

Table 2: Italy

Year	τ_1	A_{min}^{ib}	A_{max}^{ib}	$sample_{A^{ib}}$	$\tau_{A^{tot}}$	A_{min}^{tot}	A_{max}^{tot}	$sample_{A^{tot}}$	θ	η	$coverage_{A^{ib}}$	$coverage_{A^{tot}}$
2000	1.33	1.21e+03	2.41e+07	15	1.46	4.42e+04	1.68e+08	15	0.18	0.43	0.02	0.02
2001	1.38	3.35e+03	8.57e+06	20	1.35	3.53e+03	1.58e+08	20	0.27	0.40	0.03	0.03
2002	1.47	1.09e+04	7.27e+06	21	1.48	4.77e+04	1.55e+08	21	0.34	0.36	0.03	0.03
2003	1.54	1.47e+04	1.04e+07	16	1.46	2.15e+04	1.84e+08	16	0.42	0.34	0.02	0.02
2004	1.30	6.27e+03	1.99e+07	22	1.85	8.79e+06	2.17e+08	22	0.36	0.27	0.03	0.03
2005	1.55	9.08e+03	7.92e+07	530	1.59	2.09e+05	9.29e+08	531	0.11	0.12	0.80	0.80
2006	1.53	1.25e+04	9.74e+07	543	1.58	2.63e+05	1.08e+09	544	0.11	0.13	0.82	0.82
2007	1.53	1.37e+04	1.15e+08	551	1.58	3.49e+05	1.50e+09	555	0.11	0.14	0.83	0.84
2008	1.51	5.00e+04	1.02e+08	564	1.58	3.88e+05	1.46e+09	566	0.12	0.13	0.85	0.86
2009	1.54	1.60e+04	1.18e+08	562	1.58	3.88e+05	1.34e+09	564	0.11	0.12	0.85	0.85
2010	1.55	1.44e+04	1.72e+08	583	1.61	4.53e+05	1.24e+09	586	0.11	0.13	0.88	0.89
2011	1.55	1.54e+04	2.75e+08	595	1.61	4.62e+05	1.20e+09	600	0.10	0.12	0.90	0.91
2012	1.57	6.24e+04	3.61e+08	591	1.60	5.35e+05	1.22e+09	596	0.11	0.12	0.89	0.90
2013	1.59	4.10e+04	3.38e+08	573	1.61	5.19e+05	1.17e+09	579	0.11	0.12	0.87	0.88
2014	1.48	5.84e+04	2.42e+08	222	1.56	1.74e+06	1.02e+09	226	0.12	0.11	0.34	0.34

Table 3: Germany

Year	τ_1	A_{min}^{ib}	A_{max}^{ib}	$sample_{A^{ib}}$	$\tau_{A^{tot}}$	A_{min}^{tot}	A_{max}^{tot}	$sample_{A^{tot}}$	θ	η	$coverage_{A^{ib}}$	$coverage_{A^{tot}}$
2000	1.85	6.24e+04	8.48e+07	1290	2.07	9.14e+05	6.46e+08	1294	0.12	0.06	0.69	0.70
2001	1.85	4.55e+04	7.86e+07	1292	2.08	1.15e+06	6.31e+08	1298	0.12	0.06	0.69	0.70
2002	1.90	1.19e+05	6.01e+07	1285	2.04	1.06e+06	5.52e+08	1290	0.13	0.06	0.69	0.69
2003	1.81	7.94e+04	6.65e+07	1264	1.97	1.19e+06	5.98e+08	1271	0.12	0.07	0.68	0.68
2004	1.77	7.79e+04	1.18e+08	1286	1.95	1.44e+06	6.37e+08	1296	0.13	0.07	0.69	0.70
2005	1.75	6.48e+04	1.02e+08	1564	1.91	1.14e+06	5.82e+08	1578	0.13	0.07	0.84	0.85
2006	1.68	6.81e+04	2.51e+08	1618	1.80	1.04e+06	2.07e+09	1635	0.14	0.08	0.87	0.88
2007	1.70	1.14e+05	3.02e+08	1641	1.81	1.39e+06	2.83e+09	1660	0.16	0.08	0.88	0.89
2008	1.73	1.35e+05	3.10e+08	1649	1.79	1.22e+06	3.07e+09	1664	0.17	0.08	0.89	0.89
2009	1.68	9.72e+04	3.47e+08	1701	1.80	1.28e+06	2.16e+09	1712	0.14	0.08	0.91	0.92
2010	1.67	8.10e+04	3.72e+08	1749	1.80	1.27e+06	2.55e+09	1764	0.14	0.09	0.94	0.95
2011	1.69	1.00e+05	4.71e+08	1793	1.81	1.31e+06	2.80e+09	1809	0.15	0.10	0.96	0.97
2012	1.69	1.13e+05	3.82e+08	1790	1.82	1.39e+06	2.67e+09	1806	0.12	0.10	0.96	0.97
2013	1.68	1.10e+05	3.87e+08	1758	1.82	1.43e+06	2.22e+09	1774	0.11	0.11	0.94	0.95
2014	1.30	1.12e+04	3.40e+08	289	2.12	8.47e+07	2.07e+09	293	0.15	0.12	0.16	0.16

Table 4: United Kingdom

Year	τ_1	A_{min}^{ib}	A_{max}^{ib}	$sample_{A^{ib}}$	$\tau_{A^{tot}}$	A_{min}^{tot}	A_{max}^{tot}	$sample_{A^{tot}}$	θ	η	$coverage_{A^{ib}}$	$coverage_{A^{tot}}$
2000	1.48	3.85e+04	2.93e+07	99	1.63	6.64e+05	2.26e+08	138	0.39	0.26	0.18	0.26
2001	1.96	3.17e+05	9.89e+06	103	1.69	9.54e+05	1.46e+08	146	0.40	0.27	0.19	0.27
2002	3.09	1.30e+06	1.06e+07	106	1.69	1.50e+06	1.59e+08	159	0.39	0.28	0.20	0.30
2003	2.30	6.69e+05	1.30e+07	110	1.75	7.88e+05	2.32e+08	175	0.38	0.28	0.20	0.33
2004	1.49	3.55e+05	3.23e+08	153	1.49	8.31e+05	1.38e+09	245	0.36	0.25	0.28	0.46
2005	1.53	5.55e+05	3.33e+08	203	1.51	2.36e+06	1.59e+09	326	0.34	0.24	0.38	0.61
2006	1.50	8.16e+05	4.07e+08	203	1.48	2.32e+06	1.96e+09	342	0.35	0.23	0.38	0.64
2007	1.53	1.31e+06	4.51e+08	207	1.48	2.53e+06	3.81e+09	363	0.35	0.24	0.39	0.68
2008	1.45	4.57e+05	2.62e+08	218	1.42	6.98e+05	3.50e+09	385	0.32	0.25	0.41	0.72
2009	1.43	4.99e+05	3.11e+08	228	1.44	8.48e+05	2.75e+09	399	0.32	0.28	0.42	0.74
2010	1.34	6.65e+04	3.83e+08	243	1.45	1.07e+06	2.45e+09	436	0.32	0.30	0.45	0.81
2011	1.35	6.85e+04	3.14e+08	254	1.45	1.07e+06	2.56e+09	462	0.32	0.30	0.47	0.86
2012	1.36	7.87e+04	3.54e+08	257	1.47	1.55e+06	2.69e+09	468	0.31	0.30	0.48	0.87
2013	1.33	5.15e+04	3.99e+08	256	1.48	2.36e+06	2.67e+09	458	0.30	0.31	0.48	0.85
2014	1.27	2.51e+04	2.81e+08	131	1.34	5.70e+05	2.63e+09	177	0.19	0.21	0.24	0.33

Table 5: Italy

Year	τ_2	F_{min}^{tot}	F_{max}^{tot}	$sample_{F^{tot}}$	$coverage_{F^{tot}}$
2005	1.98	3.75e+03	8.32e+05	1681	0.35
2006	1.86	1.62e+03	1.12e+06	1875	0.39
2007	1.88	1.56e+03	8.72e+06	2832	0.59
2008	1.91	2.81e+03	9.97e+06	3120	0.65
2009	1.93	3.17e+03	8.82e+06	3398	0.70
2010	1.93	3.36e+03	8.97e+06	3738	0.77
2011	1.91	5.50e+03	9.56e+06	4066	0.84
2012	1.96	3.81e+03	1.52e+06	4219	0.87
2013	1.95	3.77e+03	1.48e+06	4259	0.88
2014	1.85	2.84e+03	8.70e+05	213	0.04

Table 6: Germany

Year	τ_2	F_{min}^{tot}	F_{max}^{tot}	$sample_{F^{tot}}$	$coverage_{F^{tot}}$
2005	1.63	5.36e+02	1.27e+08	1574	0.43
2006	1.71	1.44e+03	1.28e+08	2468	0.67
2007	1.71	1.09e+03	1.37e+08	2623	0.71
2008	1.72	1.09e+03	1.57e+08	2891	0.78
2009	1.72	1.87e+03	1.53e+08	3098	0.84
2010	1.75	2.80e+03	1.53e+08	3250	0.88
2011	1.76	2.63e+03	1.53e+08	3468	0.94
2012	1.73	3.35e+03	1.40e+08	2410	0.65
2013	1.73	2.54e+03	1.38e+08	1859	0.50
2014	1.48	3.36e+02	1.55e+08	62	0.02

Table 7: United Kingdom

Year	τ_2	F_{min}^{tot}	F_{max}^{tot}	$sample_{F^{tot}}$	$coverage_{F^{tot}}$
2005	1.52	5.94e+02	2.12e+08	1695	0.40
2006	1.59	4.28e+03	2.32e+08	1878	0.45
2007	1.49	5.07e+02	2.42e+08	2094	0.50
2008	1.51	5.32e+02	2.35e+08	2317	0.55
2009	1.75	1.43e+05	1.68e+08	2538	0.60
2010	1.65	2.21e+04	2.00e+08	2798	0.67
2011	1.56	4.28e+03	2.37e+08	3122	0.74
2012	1.57	3.91e+03	1.95e+08	3508	0.84
2013	1.72	6.53e+04	1.69e+08	3884	0.93
2014	1.45	4.19e+01	1.41e+07	1649	0.39

Appendix B

In what follows we provide some definitions for the network concepts mentioned in the paper, as well as a description of the model used to generate the network and of the algorithm implemented in the simulations. For a more complete treatment of network concepts see Newman (2010) and Jackson (2010), while a more formal explanation of the network model used see Caldarelli (2007) and for an analytical solution of the kinetic formation of fitness based networks see Bottazzi & Vanni (2016).

Definitions

In the following definitions we refer to a directed network X with N nodes and L links. The adjacency matrix and the weighted adjacency matrix associated to the network are respectively $A_{N,N}$ and $B_{N,N}$, with $a_{i,j} \in A_{N,N}$ and $b_{i,j} \in B_{N,N}$.

Degrees of a node In directed networks nodes have both an in-degree and an out-degree, which represent respectively the number of incoming links (i.e. links pointing in toward the node) and the number of outgoing links (i.e. links starting from the node). Formally the in-degree k^{in} and the out-degree k^{out} of a node i are:

$$k_i^{in} = \sum_{j=1}^N a_{ij} \quad k_i^{out} = \sum_{j=1}^N a_{ji} \quad (12)$$

In our model an incoming link represents an asset for the node, while an outgoing link represents a liability, so for each node i the total assets A_i and the total liabilities L_i can be written as:

$$A_i = \sum_{j=1}^N b_{ij} \quad L_i = \sum_{j=1}^N b_{ji} \quad (13)$$

Average degree The average degree of a node in a network is the number of links that a generic node has on average. Formally is:

$$\bar{k} = \frac{1}{N} \sum_{j=1}^N k_j^{in} = \frac{1}{N} \sum_{j=1}^N k_j^{out} = \frac{L}{N} \quad (14)$$

The average degree is therefore a measure of the average number of counterparts of a node and so a proxy for the interconnectedness of the network. In this sense, to compute the average degree of a scale-free network (namely a network structure whose degree distribution follows a power law) is not problematic. A concept closely related to the one of average degree and often used in its place is the one of density (or completeness). The

density d is computed as the fraction of active links over the total number of possible links. It is therefore possible to establish a direct relationship between the density and the average degree of a network:

$$d = \frac{L}{N(N-1)} = \frac{\bar{k}}{N-1} \quad (15)$$

Assortativity and disassortativity Assortativity (or assortative mixing) is the tendency of nodes to attach to others nodes that are similar to them in some respect. Usually assortativity is computed in terms of node’s degree, but the measure of similarity may vary. A network is then said to be assortative if this property holds and disassortative in the opposite case.

Fitness based networks

The model described in section 2 belongs to the class of network models where the attachment rules are governed by intrinsic node fitness. This fitness is a measure of attractiveness of a node and so of the probability of forming a link. Our model is based on the one described in Caldarelli *et al.* (2002) and Caldarelli (2007) and adapted to interbank networks in Montagna & Kok (2013). However, while in these works the model described is *static*, in the sense that the network is formed in “one shot” by assigning a probability to each of the possible links and then randomly “activating” them, in this paper we implemented a *kinetic* version of the model, where the network is constructed starting with N isolated nodes and connect two nodes at time. In this way we can control the average degree \bar{k} adding L links to the network one by one as shown in Bottazzi & Vanni (2016). The next section describes the algorithm implemented in the simulations.

Algorithm of the shock procedure

```

for each couple of average degrees  $AD_{BB}$  and  $AD_{BF}$ 
  for each level of the shock  $\delta$ 
    for each of the  $T$  realizations
      ▷ draw a value of  $A^{ib}$  for every bank from a truncated Pareto distribution
      ▷ draw a value of  $F^{tot}$  for every firm from a truncated Pareto distribution
      ▷ generate a two layer network with the desired  $AD_{BB}$  and  $AD_{BF}$ 
      ▷ shock the network with the target level of NPLsa
      ▷ initialize counter equal to 0
      while counter < target level of NPLs
        ▷ randomly select a firm
        for every link of the firmb
          ▷ randomly select a link
          if value of the link < difference between target NPLs and counter
            ▷ set the value of the selected link equal to 0
            ▷ update counter
          else subtract to the link the difference between target NPLs and counter
            ▷ update counter and exit while
          end
        end
      end
      ▷ initialize flag banks' default equal to 1c
      while flag banks' default = 1
        ▷ set flag banks' default equal to 0
        for every bank
          ▷ update balance sheet
          if equity <= 0
            ▷ bank defaults and its outgoing links are set to 0d
            ▷ set flag banks' default equal to 1
          end
        end
      end
    end
  end
  ▷ compute the fraction of defaulted banks
  ▷ compute the frequency of contagion
  ▷ compute the extent of contagion
  ▷ compute the level of systemic risk
end

```

^aTarget level NPLs = $\delta \sum_i^N A_i^f$

^bRecall that firms have only outgoing links pointing toward banks, each of which represent a liability for the firm and an asset for the bank.

^cNote that setting the flag for banks' default equal to 0 does not mean assuming *ex ante* the default of any bank, it is only a way to enter the while loop.

^dWhich are interbank assets of their creditors.

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