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Regulatory Entry Barriers, Rent-Shifting and the Home Market Effect

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Regulatory Entry Barriers, Rent-Shifting and the Home Market Effect*

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Abstract: I modify a standard model of the home market to introduce entry barriers that create local rents. The existence of rents has relevant implications. First, the home market effect magnifies. Second, when countries are sufficiently unequal in size and rents are sufficiently large, a trade costs reduction reduces the small country's welfare. Third, entry barriers increase the large country's market size and may surprisingly increase its welfare. Fourth, a unilateral increase in import tariffs shifts foreign rents to the home country. This rent shifting effect intensifies the standard production relocation motive for trade policy intervention.

Keywords: Rent Shifting, Home Market Effect, Production Relocation Effect

JEL Classification: F12, F13

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1. Introduction

It is universally accepted that entry barriers exist and that, under their existence, firms create local rents. Furthermore, a large body of research shows these barriers may arise from governmental regulation. Regulation imposes barriers to entry and thus creates profits or rents that accrue to residents (see Section 2 for a detailed literature review). However, the mainstream models of trade have not been able to study the general equilibrium effects of regulatory entry barriers and rents. With the exception of a recent work by Neary (2009), these models have used entry conditions ensuring that entry drives aggregate rents down to zero (note that expected profits equal zero and thus aggregate rents are also null in Melitz’s model, 2003).

Neary (2009) has introduced profits into a general equilibrium model of trade by embedding a Cournot-Nash competition setup in a Ricardian model.1 In the manner of the partial-equilibrium industrial-organization literature, his setup implicitly assumes that entry barriers determine the exogenous number of firms and, therefore, profits are greater than zero. In this paper, I follow Neary’s line of research (2002, 2003a, 2003b and 2009) by introducing rents into Helpman and Krugman’s general equilibrium model of the home market effect (Helpman and Krugman 1985, HK hereafter).2 I consider an exogenous level of entry barriers as Neary (2009) does but fix this level and, following the trade literature, determine endogenously the number of firms. To keep the model tractable, I assume that the level of regulatory entry barriers is the same across countries and provide evidence suggesting this assumption is close to being realistic (i.e. that countries with a similar degree of development but a different market size have a resembling level of regulatory entry barriers). The modified model is used to answer questions that have not been addressed by

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1 See Neary (2002, 2003a, 2003b) for other treatments of profits in general equilibrium.
2 Picard et al. (2004) study the effects of different ownership structures in a model of economic geography. I build on a standard trade model and abstract from differences in ownership structures to focus on the effects of rents on results in the literatures of the home market effect and of trade policy intervention.
Neary. I study the shifting motive for trade protection that is the highlight of the partial equilibrium literature initiated by Brander and Spencer (1981, 1984 and 1985) and investigate standard results in the literature of the home market effect.

When the model is modified to include entry barriers, firms create local rents that accrue to domestic firms, households or governments; all these sorts of rents are consistent with the mechanisms of the model. To the purpose of this paper, the important point is that firms create local rents and thus a country’s income increases with the number of domestic firms. Using this feature of the model, I prove four results. First, the existence of rents magnifies the home market effect. Second, when rents are sufficiently large and countries are sufficiently unequal in size, a trade costs reduction reduces welfare in the small country. Third, surprisingly, an increase in entry barriers raises welfare in the large country under some parameter values. Fourth, the rent shifting effect (Brander and Spencer, 1981, 1984 and 1985) emerges in a general equilibrium model and intensifies the production relocation incentive for trade policy intervention (Venable, 1987 and Ossa, 2011 see Section 2 for a literature review).

In the first part of the paper, I follow HK closely. In their setup, an increase in a country’s world labor share raises its labor earnings and thus its market size. Consequently, entry into the manufacturing becomes more attractive and the country’s share of firms increases proportionally more than its labor share. When firms create rents, the increase in the country’s share of firms raises its rents and, therefore, its income and market size increase by an even greater amount. The increase in market size resulting from higher rents triggers even more entry into the manufacturing sector and, thus, HK’s home market effect magnifies.

The existence of rents also modifies the welfare impacts of a trade costs reduction. In the HK model, a trade costs reduction has a price index increasing effect for the small country (the nation with the smaller labor share). This reduction stimulates entry in the large country, raising the share
of products that is subject to trade costs in the small nation. Despite this price index increasing effect, welfare increases in the small country.

However, the existence of rents triggers two additional welfare effects. First, the trade costs reduction decreases the small country’s rents. Second, the HK price index increasing effect intensifies because the increase in market size resulting from higher rents triggers higher entry in the large country. I show that when rents are large, these two welfare decreasing effects are sufficiently strong that a trade costs reduction reduces the small country’s welfare. That is, the trade costs reduction is no longer Pareto optimal.

I also investigate the effects of an increase in entry barriers. These effects can be decomposed into the welfare impacts of an entry barriers increase under an autarky regime and additional welfare effects due to changes in each country’s share of firms. Entry barriers reduce welfare in autarky as they do in any textbook model. However, in a trade regime, entry barriers increase the large country’s share of firms and, therefore, its price index may decrease and its income may increase. For some parameter values, the increase in the large country’s share of firms is sufficiently large that the entry barriers increase raises its welfare.

In the second part of the paper, I move from HK to Ossa’s approach (Ossa 2011) and interpret trade costs as import tariffs. This strategy allows me to investigate the motives for trade policy intervention in my model. A unilateral increase in import tariffs stimulates entry in the home country and discourages entry in the foreign economy. Since firms create rents, this implies the model features the rent shifting effect that is the highlight of the literature initiated by Brander and Spencer (1981, 1984 and 1985). I show this effect intensifies the relocation motive for trade protection, generating greater incentives for "beggar-thy-neighbor" trade policies than in Ossa’s

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3 This paper abstracts from political and terms-of-trade motives. See Section 2 for a review of the literature dealing with these motives.
setup (2011). In summary, the paper contributes to the early literature initiated by Brander and Spencer and to the literature on the production relocation effect (see Section 2).

The structure of the paper is as follows. Section 2 deals with the link between the paper and relevant literature. In Section 3, I interpret trade costs as non-policy impediments to trade and compare the model with HK’s setup. Section 4 interprets trade costs as tariffs and compares the results with Ossa’s outcomes (2011). Section 5 concludes.

2. Related Literature

The paper is related to the literature on the production relocation motive for trade protection. In the framework of a monopolistic competition model, this motive dates back to Venables (1987) and has been recently studied by Ossa (2011). Ossa shows that governments choose trade policy to reduce entry in the foreign country and thus decrease the domestic price index. He suggests the WTO principles of reciprocity and nondiscrimination help internalize this effect.4

The paper is also related to the literature on the rent-profit shifting effect. This effect dates back to Brander and Spencer’s partial equilibrium models with a fixed number of firms (1981, 1984 and 1985). Brander and Spencer (1985), for instance, show there is a profit shifting motive for export subsidies in a Counot-Nash competition model. Export subsidies improve the strategic position of domestic firms and thus increase its profits at the expense of foreign competitors.5 Recently, Ossa (forthcoming) and Ossa (2012) have introduced the profits shifting in an empirical analysis by building a framework for calibration purposes. In his setups the production relocation effects

4 Baldwin and Nicoud (2000) study production relocation effects in a model with international capital mobility. Melitz and Ottaviano (2008) and Felbermayr et al. (2013) study this effect in a model of heterogeneous firm and the latter extend Demidova and Rodriguez-Clare’s results (2009). On the other hand, Bagwell and Staiger (2009) show that, in the presence of political incentives and production relocation effects, the only rationale for a trade agreement is to remove terms-of-trade effects. My paper abstracts from these effects to highlight the production relocation and profits shifting mechanisms; however its goal is not to neglect the importance of terms-of-trade effects.

5 Governments set export subsidies in the first stage and then firms choose quantity levels.
become profits shifting effects, as he fixes the number of firms (see Mrázová 2011, Bagwell and Staiger 2012 for additional recent treatments of profits shifting).  

Building on Brander and Spencer’s contribution, Neary(2009) embeds a Cournot-Nash setup in a Ricardian (general equilibrium) model with a continuum of sectors. In his setup profits are greater than zero because in each sector the number of firms is fixed.

The model that I present differs from the production relocation literature in two important ways. First, the model features profits shifting and production relocation effects jointly and analyzes their interaction. Second, I study the impact of rents on standard results in the literature of the home market effect. The paper differs from Neary’s work and its subsequent applications in that it does not fix the number of firms. Furthermore, I study the rent shifting effect that is highlight of the literature initiated by Brander and Spencer (1981, 1984 and 1985). The model differs from the rent shifting literature in several of the facts that I have just mentioned but also because it presents a different rent shifting mechanism. An import tariffs increase in this paper impacts rents by increasing the number of profitable domestic firms, rather than by increasing the profits of a given number of domestic producers. In this sense, this paper resembles Haufler and Wooton’s work (1999). Furthermore, I study the impact of rents and rent shifting in a general equilibrium setup.

Finally, the paper relates to the literature showing that regulation acts a barrier to entry and, therefore, creates local rents that accrue to domestic firms, households or governments. Djankov et al. (2002), for instance, show that red tape regulation creates rents accruing to bureaucrats and administrative employees. Ciccone et al. (2007) extend their sample to show that entry is slower

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6 Bagwell and Staiger (2012) show that trade policy triggers local-price externalities associated with profit shifting. See footnote 4 for a discussion on how this paper relates to their work. Mrazova (2011) extends the argument to a model of oligopolistic competition in quantities with $n$ countries and firms.

7 Ossa (2011b) and Ossa (2012) build on general equilibrium, but his framework is designed for calibration purposes and thus does not provide qualitative results.

8 See Bastos and Kreickemeier (2009), Egger and Etzel (2012), Kreickemeier and Meland (2013); Egger and Etzel (2014) for some of the several applications of Neary’s model.

9 In their paper, one instrument is used to attract foreign firms and lump sum taxes extract their profits.
in industries where it is necessary to register land, build facilities, purchase equipment and procure specific licenses. Petts (2009) and Hastings (2010) provide anecdotal evidence that environmental regulation creates rents accruing to regulatory agencies and environmental practitioners.\textsuperscript{10} Fisman and Sarria (2010) use data on the regulations of 77 countries to show that it also yields anticompetitive effects that benefit firms. They show that entry regulation reduces the number of firms and increases their size. Along the same lines, Ryan (2012) demonstrates that the 1990 Amendments to the Clean Air Act reduces competition and benefits incumbent firms in the Portland cement industry. Suzuki (2013) finds that stringent land use regulation raises entry costs, discouraging entry and benefiting incumbent hotels in Texas. Klapper et al. (2006) show that entry regulation increases rents from incumbency in 34 Western and Eastern European countries.

3. Identical Trade Costs and the Home Market Effect

Model Setup

I consider a world with two countries, referred to as Home and Foreign. The two countries are identical except for their labor supply (the share of the world’s labor supply is greater for the large country). Consumers have preferences over a homogeneous non-manufacturing good and a continuum of differentiated manufacturing products. Preferences are given by the following function

\[
U_j = \left[ \sum_{i=0}^{N} z_{ij}^{(\theta-1)/\theta} \right]^{a\theta/(\theta-1)} y_j^{1-\alpha}
\]

where \( y_j \) is the quantity of the homogeneous good consumed in country \( j \), \( \alpha \) is the income share spent on manufacturing products, \( z_{ij} \) is the amount of a product \( i \) consumed in country \( j \) and \( \theta > 1 \) is the elasticity of substitution between the manufacturing products.

\textsuperscript{10} Environmental impact statements create rents if becoming a practitioner is hard (Petts 2009). Rental policies for solar projects may create rents for the Bureau of Land Management (Hastings 2010).
Technologies are also identical across countries and given by the following functions

\[ l_j^Y = q_j^Y, \]  
\[ l_j^M = f + bq_j^M, \]  

where \( l_j^Y \) is the labor requirement for producing \( q_j^Y \) units of the homogeneous good in country \( j \), \( l_j^M \) is the requirement for producing \( q_j^M \) units of a manufacturing good, and \( f \) and \( b \) are the fixed and the marginal labor requirements of manufacturing production, respectively. There is monopolistic competition in the manufacturing product market and perfect competition in the market of the homogeneous good.

Trade costs apply only to manufacturing goods and take the iceberg form. For a unit of a good produced in country \( j' \) to arrive in country \( j \) \( \tau_j > 1 \) units must be shipped and \( \tau_j \) is assumed to be finite for technical convenience. To focus on the impact of rents on standard results in the literature of the home market effect, in this section I follow HK and consider identical trade costs across countries.

The regulatory environment and thus the level of entry barriers are summarized by an exogenous parameter \( \bar{c} \). Being consistent with the evidence presented in Section 2 I assume that entry barriers create local rents. Specifically, I assume that the higher the \( \bar{c} \) level, the higher the amount of rents created by the existence of a firm is. Thus, I write

\[ \pi_i = \bar{c}, \]  

where \( \pi_i \) is the amount of rents created by the existence of a firm and this amount is fixed at a positive value \( \bar{c} > 0 \). When rents take the form of profits, for instance, Equation (4) states that entry barriers are such that entry only drives profits down to \( \bar{c} \). Two remarks deserve to be made. First, the fact that I fix \( \bar{c} \) gives the model the flavor of Neary’s model (2009) and of the Cournot competition setups used in the literature initiated by Brander and Spencer (1981, 1984 and 1985).
These setups fix the number of firms by assuming there is an implicit and exogenous level of entry barriers. Appendix 2 shows that, in a closed economy, an increase in this fixed number of firms generates the same effects as an \( c \) increase in this paper.\(^{11}\) Second, Equation (4) implies there is a unique regulatory environment and thus \( c \) is the same across countries. This assumption is essential to preserve the tractability of the model and to obtain a unique and intuitive equilibrium. Appendix 4 shows the World Bank’s ease of doing business indicator is correlated with a country’s degree of development but not with its market size and, thus, the countries considered in the model have a similar level of regulatory barriers.

Finally, I impose three parameter conditions that ensure incomplete specialization and rule out uninteresting corner solutions.\(^{12}\) I assume that countries’ labor shares lie within a range in the manner of HK (1985), and impose an upper bound on rents per capita (see Appendix 1). These assumptions guarantee that market sizes are sufficiently equal that the manufacturing sector is active in both countries. Finally, an upper bound on \( \alpha \) is imposed to ensure the demand for manufacturing products is sufficiently small that the homogenous good sector is active in both countries (see Appendix 1).

**Trade Equilibrium**

I chose the price of the homogeneous good as the numeraire. This choice, perfect competition in the market of the homogeneous good, identical technologies and incomplete specialization ensure that wages equal one in both countries.

In line with the literature, I consider that, when maximizing profits, manufacturers regard themselves as sufficiently small that they ignore the component of the elasticity of demand that

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\(^{11}\) Appendix 2 proves consistency with the Cournot competition model used in the literature initiated by Brander and Spencer and in Neary (2009) and with the welfare predictions made in Neary (2003b) for a featureless economy.

\(^{12}\) Under the assumptions made, incomplete specialization also rules out terms-of-trade effects.
depends on other firms.\footnote{HK (1985) show this component goes to zero when the number of varieties goes to infinity; however, the literature abstracts from the component when the number of firms is large. Following this approach is not inconsistent with my model since I do not restrict the number of firms.} Profit-maximization then yields the following ex-factory price for a manufacturing good $i$ produced in country $p_{ij} = \theta b / (\theta - 1)$: ex-factory prices are optimally chosen to be identical across countries and firms. I select units such that $\theta / (\theta - 1) = b$ and ex-factory prices equal one in the manner of HK (1985). Thus, Equations (4) can be written as $q_j^M / \theta - f = \bar{c}$ and yields the following output per firm

$$q_j^M = \theta (\bar{c} + f) = q.$$  \hfill (5)

The output per firm determines the labor per firm, which determines, along with market-clearing in the labor and the homogeneous good markets, the number of firms (see Appendix 1 for a complete derivation). This number is written as follows

$$N = [\alpha L_W] / [f\theta + \bar{c}(\theta - \alpha)] < \alpha L_W / \theta f = \bar{N}. \hfill (6)$$

where $L_W$ is the world labor supply and $N$ and $\bar{N}$ are the numbers of firms with entry barriers and under the free entry, respectively. Equations (6)-(7) state that entry barriers increase the output per firm and reduce the number of firms operating in the market. These are the predictions of any textbook model of industrial organization.

Using Equation (5), the market-clearing conditions for manufacturing firms in Home and Foreign can be written as follows

$$q = \alpha I_H P_H^{\theta - 1} + \rho \alpha I_F P_F^{\theta - 1}, \hfill (7)$$
$$q = \rho \alpha I_H P_H^{\theta - 1} + \alpha I_F P_F^{\theta - 1}. \hfill (8)$$

where $\rho = \tau^{1-\theta} < 1$ is a measure of trade costs, $P_j$ and $I_j$ denote country $j$’s price index and its income level in terms of the homogeneous good (income level hereafter). Under the pricing condition displayed above, the price indexes are given by
\[ P_H = [n_H + \rho n_F]^{1/(1-\theta)}, \]  
\[ P_F = [\rho n_H + n_F]^{1/(1-\theta)}, \]  
where \( n_j \) is the numbers of firms in country \( j \). The income levels referred to in Equations (9)-(10) are given by the following expressions

\[ I_H = L_H + n_H \bar{c}, \]  
\[ I_F = L_F + n_F \bar{c}. \]  

Equations (11)-(12) state that income and thus market size in country \( j \) increase with the number of domestic firms. These equations assume that only residents benefit from the rents created by domestic firms. This assumption is by definition fulfilled for rents that accrue to households or governments. Although profits may be in principle accrue to foreign investors, there is large evidence that investor portfolios are disproportionately composed of domestic assets (see French and Poterba 1991; Lutje and Menkhoff 2007 and Strong and Xu (2003) and the literature on Home Equity Bias). Therefore, I hereafter make the simplifying assumption that home bias is full; it is important to note, however, that the mechanisms are robust to introduction of partial home bias as long as country’s \( j \) relative income increases with its number of firms.\(^\dagger\)

Equations (7)-(12) determine equilibrium in the manufacturing product market and hence the number of firms in each country. Following the literature, I present this number by writing country \( j \)’s share of firms \( S_{nj} \) in terms of its labor share \( S_{ij} \) as follows (see Appendix 1)

\[ S_{nj} = n_j/N = \frac{S_{ij}(1 + \rho) - \rho(1 + N\bar{c}/L_W)}/[(1 + \rho) - 2\rho(1 + N\bar{c}/L_W)]. \]  

Note that Equation (13) collapses to the corresponding equation in the HK setup (1985) when \( \bar{c} = 0 \). That is, this paper generalizes their setup by incorporating entry barriers and rents. Simple

\(^\dagger\) I abstract from the effects of different forms of profits distribution (see Picard et al. 2004 for a nice treatment on profits distribution). In particular, in the model each unit of rent is transformed into a unit of expenditure regardless of whether it accrues to domestic firms or to households or the government.
algebra shows the model replicates two of their results. First, $S_{nj}$ increases with $S_{lj}$: The share of firms is greater in large country, the country with the larger labor share (see 3.3.1). Second, the large country’s share of firms increases as trade costs fall (see 3.3.2).

**Comparative Statics**

In this subsection, I perform comparative statics on $S_{lj}$; $\rho$ and $\bar{c}$, holding in each case the other parameters constant. Although I defined $N$ as a finite integer to be consistent with the existence of entry barriers, when performing these exercises, I will take the liberty to treat it as a continuous variable to facilitate the exposition (see Fajgelbaum et al. (2011) for a similar approach). Changes in $S_{lj}$; $\rho$ and $\bar{c}$ can be interpreted as a cross-country comparison, in which the countries belong to identical regions of the world except for the parameter under consideration and the share of the world’s labor supply.

**Comparative Statics on $S_{lj}$.**

Figure 1 shows that rents magnify the home market effect, as formally proved in Appendix 3. This figure displays the relationship between $S_{lj}$ and $S_{nj}$ in the x- and y-axis for the values of $S_{lj}$ under which the manufacturing good sector in both countries (see Appendix 1 for a derivation of $S_I$ and $\bar{S}_I$). The HK and the R segments represent the HK model and my setup, respectively. When $S_{lj} = 1/2$, country $j$’s share of firms equals 1/2 in both models.
In the HK model, an increase in country $j$’s labor share from $S^{1}_{ij}$ to $S^{2}_{ij}$ raises its share of firms. Holding the other parameters constant, the increase in $S_{ij}$ raises labor earnings in country $j$ and its market size. The increase in market size in turn triggers entry into the manufacturing sector and thus $S_{n_j}$ becomes higher. In my setup, the increase in $S_{n_j}$ also raises country $j$’s income and rents. In other words, the initial rise in $S_{ij}$, by increasing the number of firms, increases not only its labor earnings but also raises its rents. The increase in market size resulting from higher rents triggers even more entry into the manufacturing sector; hence, the home market effect magnifies. The following proposition summarizes the results.

**Proposition 1.** Under the assumptions stated in 3.1, the derivative of country $j$’s share of firms with respect to its labor share is greater in my model than in Helpman and Krugman's setup: $dS_{n_j}/dS_{ij} > S^{HK}_{n_j}/dS^{HK}_{ij} = (1 + \rho)/(1 - \rho)$.

**Proof.** See Appendix 3.

The impact of a trade costs reduction on welfare, or analogously of an increase in $\rho$, can be decomposed into two types of effects (see Appendix C). The first type is analogous to the effects
of a trade costs reduction in the HK setup. The second type refers to additional welfare effects due to the existence of rents. The following Lemma states the result by studying proportional changes in utility.

Lemma 1. \[ \frac{dV_j}{d\rho} / V_j = \frac{dV_j^{HK}}{d\rho} / V_j^{HK} + \delta_j \], where \( V_j \) denotes the indirect utility function of country \( j \), \( \frac{dV_j^{HK}}{d\rho} / V_j^{HK} \) refers to the welfare effects of an increase in \( \rho \) in the HK model and \( \delta_j \) refers to the additional effects due to the existence of rents.

Proof. See Appendix 3.

I proceed by studying the sign \( \frac{dV_j^{HK}}{d\rho} / V_j^{HK} \). In the HK model a trade costs reduction exerts conflicting effects on the price index of the small country. On the one hand, the price of foreign goods in the domestic market falls. On the other hand, there is a price index increasing effect: The large country’s share of firms rises and, therefore, the share of goods that is subject to trade costs in the small country increases. This price index increasing effect is sufficiently mild that a trade costs reduction reduces the small country’s price index and increases its welfare. The following remark states the result and the fact that the two types of effects are price index-decreasing for the large country.

Remark 1. In Helpman and Krugman’s model, a trade costs reduction decreases the price index and, therefore, increases welfare in both countries under incomplete specialization. Thus, \( \frac{dV_j^{HK}}{d\rho} / V_j^{HK} \) in Lemma 1 is positive for both countries.

Proof. See Appendix 3.

In this model, a trade costs reduction impacts rents so that, unlike in the HK setup, its effect on welfare is not limited to \( \frac{dV_j^{HK}}{d\rho} / V_j^{HK} \). The trade costs reduction reduces entry in the small country as it does in HK. This implies that the number of firms and rents fall in this country; by
the same token, the number of firms and the amount of rents increase in the large country.\textsuperscript{15} The increase in market size resulting from higher rents triggers higher entry in the large country: That is, the share of manufacturing goods that is subject to trade costs in the small country increases by a greater amount than in HK and, therefore, their price index increasing effect magnifies. Furthermore, the fall in rents decreases the small country’s income. Proposition 2 states that, when the magnification of HK’s price index increasing effect and the fall in the small country’s income are sufficiently large, a trade costs reduction reduces welfare in the small country. This proposition also states that the large country’s welfare always increases and is written as follows

Proposition 2. Holding the other parameters constant, a trade costs reduction increases welfare in the large country under incomplete specialization. Under the assumptions stated in 3.1, there exists a set of parameter values for which the terms on Lemma 1 fulfill the following conditions: $\delta_j < 0$ and $| \delta_j | > | [dV_{ij}^{HK}/d\rho]/V_{ij}^{HK} |$ for the small country. Thus, in contrast with the HK model, the trade costs reduction decreases welfare in the small country (see Appendix 3 for the precise set). Formally, if $S_{ij} > 1/2$, then $dV_j/d\rho > 0$; for the set of parameter values, it is also true that if $S_{ij} < 1/2$, then $dV_j/d\rho < 0$.

\textit{Proof.} See Appendix 3.

Example 1. If rents are sufficiently large so that $\bar{\omega} > \omega^p$ and $\theta > \theta^p$ also holds, then $dV_j/d\rho < 0$ for a sufficiently small country. Importantly, $\theta > \theta^p$ is a mild restriction because $\theta^p < 2$ and $\theta$ is, by definition, greater than 1.

\textit{Proof.} See Appendix 3.

The lower bound on $\bar{\omega}$ ensures that the fall in rents in the small country is sufficiently large (so that the model is substantially different from HK’s setup). The lower bound on $\theta$ guarantees that

\textsuperscript{15} As its share of firms increases so does its number of firms since $\rho$ changes have no effects on $N$. 

14
the price index increasing effect is strong: When consumers in the small country substitute a great amount of domestic products for foreign goods, the increase in the share of products that is subject to trade costs is large.

*Comparative Statics on $\bar{c}$*

An increase in $\bar{c}$ generates two types of welfare effects (see Appendix 3). The first type is analogous to the effects of an increase in $\bar{c}$ under an autarkic regime. The second type refers to additional welfare effects due to changes in each country’s share of firms. The following Lemma states the result by studying proportional changes in utility

Lemma 2. $[\partial V_j / \partial \bar{c}]_j / V_j = [\partial V_j^A / \partial \bar{c}]_j / V_j^A + \theta_j$, where $[\partial V_j^A / \partial \bar{c}]_j / V_j^A$ refers to the change in welfare induced by an increase in $\bar{c}$ in autarky and $\theta_j$ refers to the welfare effects associated with changes in country $j$’s share of firms.

*Proof. See Appendix Sections 2 and 3.*

In an autarky economy, entry barriers hamper entry into the manufacturing sector and, therefore, reduce the number of firms or varieties; the reduction in product variety increases the price index. On the other hand, entry barriers lessen the intensity of competition and the manufacturing sector is consequently more profitable; therefore, rents and income increase. This income increasing effect is sufficiently mild that entry barriers reduce welfare in autarky, as stated in the following Lemma

Lemma 3. Entry barriers reduce welfare in autarky. Thus, the term $[\partial V_j^A / \partial \bar{c}]_j / V_j^A$ in Lemma 2 is negative for both countries.

*Proof. See Appendix 2.*
The welfare effects of an increase in $\bar{c}$ are not limited to $[dV_j^A/d\bar{c}]/V_j^A$. There are additional effects associated with changes in each country’s share of firms and captured by the $\theta_j$ term from Lemma 2. The following summarize the sign of these effects

Lemma 4. Entry barriers increase the large country’s share of firms and reduces the share of the small country. Formally, if and only if $S_{lj} > 1/2$, then $S_{nj}/d\bar{c} > 0$ and thus $\theta_j > 0$; if and only if $S_{lj} < 1/2$, then $S_{nj}/d\bar{c} < 0$ and thus $\theta_j < 0$.

Proof. See Appendix 3.

The intuition for Lemma 4 goes as follows. An increase in $\bar{c}$ increases the relative market size of the country with the largest number of firms, the large country. Thus, market entry then becomes more attractive in relative terms and, therefore, an increase in $\bar{c}$ raises the large country’s share of firms: $\theta_j > 0$ for this country but $\theta_j < 0$ for the small country.

Since $\theta_j$ is negative for the small country, an increase in $\bar{c}$ unambiguously reduces its welfare. On the other hand, the large country’s share of firms increases and thus its income may increase and its price index may fall. Specifically, as $\theta_j$ is sufficiently large to offset $[dV_j^A/d\bar{c}]/V_j^A$, entry barriers increase the large country’s welfare as stated in the following Proposition 3.

Proposition 3. Entry barriers reduce welfare in the small country under incomplete specialization. There exist a set of parameter values for which $|\theta_j| > |[dV_j^A/d\bar{c}]/V_j^A|$ so that the increase in entry barriers raises welfare in the large country (see Appendix 3 for the precise set). Formally, if $S_{lj} < 1/2$, then $dV_j/d\bar{c} < 0$; for the set of parameter values, it is also true that if $S_{lj} > 1/2$, then $dV_j/d\bar{c} > 0$.

Proof. See Appendix 3.

Example 2. If $\theta = 4.6$ and $\alpha = 0.188$ (the estimates found in Dekle et al. 2007 and used in Ossa 2012) and $\bar{c} = 45; f = 100$ and $\rho = 0.7$, then $dV_j/d\bar{c} > 0$ for a sufficiently large country $j$. 

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However, if this economy is not sufficiently open to trade so that $\rho = 0.6$ rather than $\rho = 0.7$, then $dV_j/d\bar{c} < 0$, regardless of country size.\textsuperscript{16}

Proof. See Appendix 3.

In Example 2 only when the economy is sufficiently open to trade, $\theta_j$ more than offsets $[dV_j^A/d\bar{c}]/V_j^A$, the welfare effects of entry barriers under an autarky regime.

4. Trade Costs as Import Tariffs

Model Setup

The utility functions; the production technologies; the market structures and the level of entry barriers are the same as in Section 3. Moving from the HK setup (1985) to Ossa’s model (2011), in this section I interpret the iceberg costs as import tariffs. In particular, I define $\rho_j = \tau_j^{1-\theta}$ and $\rho_{j'} = \tau_{j'}^{1-\theta}$ as the tariffs of countries $j$ and $j'$, respectively, and assume they are finite for technical convenience.\textsuperscript{17}

Finally, I make assumptions to ensure incomplete specialization. The assumptions are analogous to those made in Section 3: Countries’ labor shares are assumed to lie within a range and rents per capita as well $\alpha$ are assumed to be lower than an upper bound (see Appendix 5 for the precise restrictions).

Trade Equilibrium

I chose the price of the homogeneous good as the numeraire and select units in the manner of Helpman and Krugman (1985). These choices ensure, together with incomplete specialization;

\textsuperscript{16} Due to the amount of non-linearities involved in the proof, providing parameter restrictions to show the set mentioned in Proposition 3 is non-empty yields restrictions that are not necessarily intuitive. Hence, I prefer to provide numbers that lie within this set. This simple strategy does the job.

\textsuperscript{17} Modeled this way, tariffs do not generate any revenue. This assumption is essential for the model’s tractability as it is in Ossa (2011)
perfect competition in the homogeneous good market and identical production technologies, that
wages and ex-factory prices equal one in both countries.

The market-clearing condition in the homogeneous good market is the same as in Section 3 and,
therefore, \( q \) and \( N \) are given by Equations (5) and (6). Tariffs modify the market-clearing
conditions for manufacturing firms in Home and Foreign, which are written as follows

\[
q = \frac{\alpha I_H}{g_H} + \rho_F \frac{\alpha I_F}{g_F}, \quad (14)
\]

\[
q = \rho_H \frac{\alpha I_H}{g_H} + \frac{\alpha I_F}{g_F}, \quad (15)
\]

where \( g_H \) and \( g_F \) are the following decreasing monotonic transformations of the price indexes

\[
g_H = P_H^{1-\theta} = n_H + \rho_H n_F, \quad (16)
\]

\[
g_F = P_F^{1-\theta} = \rho_F n_H + n_F, \quad (17)
\]

Clearing in the manufacturing product market determines the number of firms in each country
(see Appendix 5 for the derivation). Country’s \( j \) share is written as follows

\[
S_{nj} = n_j/N = \frac{[S_{ij}(1 - \rho_j \rho_j \rho_j) - \rho_j (1 - \rho_j)(1 + N\bar{v}/L_W)]}{[(1 - \rho_j \rho_j) - (\rho_j + \rho_j - 2\rho_j \rho_j)(1 + N\bar{v}/L_W)]}. \quad (18)
\]

**Comparative Statics**

Comparative statics on \( \rho_j \) and \( \rho_{ji} \) show the effects of a unilateral increase in import tariffs (see
Appendix 6 for the analytical results). In particular, these exercises show that introducing entry
barriers into Ossa’s model (2011) generates rent shifting effects.

I begin by studying the effects of a unilateral tariffs increase in Ossa’s setup. In his model, an
increase in tariffs exerts two competing effects on the price index, the sole endogenous variable
affecting welfare. On the one hand, it increases the price of foreign manufacturing goods in the
domestic market; this effect to which Ossa calls the import price effect raises the price index. On
the other hand, since the price of foreign manufacturing goods increases, domestic consumers shift
expenditure toward domestic goods. This implies a higher demand for domestic goods and a lower demand for foreign manufacturing products and, therefore, triggers entry in the domestic country and reduces the number of foreign firms. In other words, the tariffs increase reduces the share of goods that is subject to tariffs in the domestic country. This effect to which Ossa calls the production relocation effect reduces the price index and more than offsets the price import effect as stated in the following remark.

Remark 2. In Ossa’s model (2011), a unilateral increase in import tariffs reduces the price index of the domestic country, and thus increases its welfare: In terms of proportional changes in the $g$ function, $[dg_j^{os}/d\rho_j]/g_j^{os} < 0$ and, therefore, $[dV_j^{os}/d\rho_j]/V_j^{os} < 0$, where the superscript $os$ denotes Ossas’s model.

Proof. See Ossa (2011).

The model presented in this paper also features the import price and the production relocation effects. Appendix 6 shows that country $j$’s price index is decreasing in its own tariffs. Hence, this paper shows that Ossa’s results (2011) subsist in the presence of entry barriers and rents.

Furthermore, a tariffs increase also affects welfare through its impact on rents in this model. Since the tariffs increase stimulates entry in the domestic country and reduces entry in the foreign economy, it increases rents in the former and reduces rents in the latter. Therefore, introducing entry barriers into a Krugman (1980) type of environment generates the rent shifting that is the highlight of the literature initiated by Brander and Spencer (1981,1984, 1985). In contrast with this literature, rent shifting emerges in a general equilibrium model in this paper. The following proposition states the result in terms of proportional changes in rents.

Proposition 4. Holding the other parameters constant, a unilateral increase in tariffs raises rents in the domestic country and decreases rents in the foreign economy: In terms of proportional changes, $[dn_j\bar{c}/d\rho_j]/[n_j\bar{c}] < 0$ and $[dn_j\bar{c}/d\rho_j]/[n_j\bar{c}] > 0$. 
Proof. See Appendix 6.

The impact of rent shifting on welfare is not limited to the impact of rent shifting on income. The reduction in market size resulting from lower rents generates a greater decrease in entry and the number of foreign firms: That is, the reduction in the share of products subject to tariffs in the domestic country is greater than in Ossa’s setup. To put it differently, the existence of rents magnifies Ossa’s production relocation effect, generating a greater reduction in the price index. The following Proposition summarizes the result

Proposition 5. \( \frac{dg_j/d\rho_j}{g_j} = \frac{dg_j^{os}/d\rho_j}{g_j^{os}} + \omega_j \), where \( \omega_j < 0 \) refers to the magnification of the price index decreasing effect.

Proof. See Appendix 6.

The following corollary summarizes how rent shifting and the magnification of the price index decreasing effect modify the motives for import tariffs

Corollary 1. A unilateral increase in import tariffs raises utility proportionately more in this model than in Ossa’s setup: \( | \frac{dV_j/dV_j}{V_j} | > | \frac{dV_j^{os}/dV_j}{V_j^{os}} | \).

Proof. Propositions 4 and 5.

Since rent shifting and the magnification of the price index decreasing effects are absent in Ossa’s setup, the motive for import tariffs is stronger in this model (measured by the proportional changes in utility). This result suggests that, in the absence of a trade agreement, governments have greater incentives to raise tariffs unilaterally. Thus, it reinforces Ossa’s idea (2011) that the WTO principles of reciprocity and nondiscrimination help overcome inefficient equilibria.

5. Concluding Remarks

The theory of the home market effect has inspired a large body of theoretical and empirical research. In this paper, I augmented the definition of market size to account for rents that accrue to
residents of the home country. The main conclusion is that rents make market size more relevant than predicted by the exiting literature. The existence of rents magnifies the home market effect, indicating that market size has a greater role in explaining industrial agglomeration.

Market size can also be critical to classifying winners and losers from trade. A trade costs reduction may reduce the small country’s welfare and, thus, this reduction may no longer be Pareto optimal. Along the same lines, an increase in entry barriers can raise welfare in the large country by increasing its market size. The existence of rents also modifies the motives for trade protection. A rents shifting motive for import tariffs emerges, thereby generating greater incentives for "beggar-thy-neighbor" trade policies.
6. REFERENCES


7. Appendix Section

Appendix 1

Equilibrium in the Homogenous Good Market

The world supply of the homogeneous good $S_W^Y$ is given by the amount of labor not employed for producing varieties. Using $q$ displayed in (6) and the corresponding amount of labor employed per manufacturer, I write $S_W^Y = L_W - N[q(\theta - 1)/\theta + f].$ The world demand arises from utility maximization and equals $D_W^Y = (1 - \alpha)l_W,$ where $l_W = L_W + N\bar{c}$ is the world income. Substituting for $q$ in $S_W^Y$ and equating it to the demand yields

$$N = \frac{[\alpha L_W]}{[\theta(\bar{c} + f) - \alpha\bar{c}]}.$$  \hspace{1cm} (1.1)

This is the same expression as in Equation (6). Note that $\theta(\bar{c} + f) - \alpha\bar{c} > 0$ so the denominator in (A1) is positive and $N$ is well-defined.

Equilibrium in the Manufacturing Products Markets

The system of Equations (7) and (8) can be written as $l_j P_j^{\theta-1} = l_j P_j^{\theta-1}.$ Substituting for $l_j P_j$ and for $P_j,$ I solve for $n_j$ to obtain $n_j = [N(L_j - (L_j + N\bar{c})\rho)]/[L_W(1 - \rho) - 2N\bar{c}\rho].$ Dividing the numerator and the denominator by $L_W$ and the numerator by $N$ yields

$$S_{nj} = n_j/N =$$

$$[S_{lj}(1 + \rho) - \rho(1 + N\bar{c}/L_W)]/[((1 + \rho) - 2\rho(1 + N\bar{c}/L_W)].$$

This is the expression displayed in Equation (13).

Parameter Restrictions

Let me write (A2) in terms of the exogenous parameter as follows

$$S_{nj} = n_j/N =$$

$$[S_{lj}(f\theta(\bar{c} + f) - \alpha\bar{c})(1 + \rho) - \rho\theta(\bar{c} + f)]/[f\theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))].$$
The manufacturing products sector is active in both countries when $S_{nj}$ lies between zero and one. Mathematically, there exist two ways of ensuring this condition. Consider first the option in which the numerator in (1.2') is positive and smaller than the denominator. This happens when the following assumptions hold

\[ S_{ij} < \bar{S}_{ij} = \frac{[f\theta + \bar{c}(\theta - \alpha(1 + \rho))]}/[(\theta(\bar{c} + f) - \alpha\bar{c})(1 + \rho)]; \quad (1.3) \]

\[ S_{ij} > S_{ij} = \frac{(\bar{c} + f)\theta\rho}/[(\theta(\bar{c} + f) - \alpha\bar{c})(1 + \rho)]. \]

To ensure that $\bar{S}_{ij} > S_{ij}$ I make the following assumption

\[ f\theta(1 - \rho) + \bar{c}[\theta(1 - \rho) - \alpha(1 + \rho)] > 0 \quad \text{or, equivalently,} \quad (1.4) \]

\[ N\bar{c}/L_w < 1/2\rho - 1/2, \quad (1.4') \]

which is the upper bound on rents per capita referred to in Subsection 3.1.

Note that $\bar{S}_{ij} < 1$ and $S_{ij} > 0$ always hold because $\theta(\bar{c} + f) - \alpha\bar{c} > 0$. Furthermore, $S_{ij} < 1$ and $\bar{S}_{ij} > 0$ when $f\theta + \bar{c}(\theta - \alpha(1 + \rho)) > 0$, which is implied by (1.4).

Consider now the second mathematical way of ensuring that $S_{nj}$ lies between zero and one; when the numerator in (1.2') is negative and smaller than the denominator in absolute terms. This occurs when $S_{ij} > S_{ij} > \bar{S}_{ij}$, and requires the opposite sign in (1.4) so $S_{ij} > \bar{S}_{ij}$ (it requires $f\theta(1 - \rho) + \bar{c}[\theta(1 - \rho) - \alpha(1 + \rho)] < 0$). However, when the sign in (1.4) is reversed, $\bar{S}_{ij} < 0$ and $S_{ij} > 1$. Hence, (1.3)-(1.4) are the only set of restrictions with bounds on country $j$'s labor share that lie between zero and one. Using $S_{ij} = 1 - S_{ij}$, the same conditions can be written in terms of the labor share of country $j'$.

Country $j$ produces the homogeneous good if its labor supply is sufficiently large to fit the maximum feasible number of varieties $n_j^{max}$. I call this labor supply $L_j^{max}$ and write

\[ N_l^M = L^W \alpha[(c + f)\theta - \bar{c}]/[\theta(\bar{c} + f) - \alpha\bar{c}], \]

where $l^M$ is the amount of labor per manufacturer.
A sufficient condition for country \( j \) to produce the homogeneous good in an incomplete specialization equilibrium is then \( L_j^{\text{max}} / L_W < S_{lj} \). To ensure this condition holds I impose the following upper bound on \( \alpha \)

\[
\alpha < \left[ \rho \theta (\bar{c} + f) \right] / \left[ \theta (\bar{c} + f) - \bar{c}(1 + \rho) \right]
\]  

Using \( S_{lj'} = 1 - S_{lj} \), the same conditions can be written for country \( j' \).

Appendix 2

*Consistency with Cournot Competition Model used in Brander and Spencer and in Neary (2009), with Neary’s (2003b) and Lemma 3*

I show that introducing entry barriers through Equation (3) makes my model consistent with the Cournot competition model used in the literature initiated by Brander and Spencer and in Neary (2009) and also in Neary (2003b) (see footnote 9). I build upon an autarky (closed) economy to isolate the increase in \( \bar{c} \) from home market effects.

Following the same logic as in Section 3, I choose the price of homogeneous good as the numeraire and make the same selection of units as HK (1985). Thus, the wage and the price of a manufacturing product in the autarky economy equal 1. The equation analogous to (3) yields \( q = \theta (\bar{c} + f) \) and \( t^M = q(\theta - 1)/\theta + f \).

The number of firms is determined by market-clearing in the homogenous good market. The supply of this good is given by \( S^Y = L - N[q(\theta - 1)/\theta + f] \). The demand results from utility maximization and equals \( D^Y = (1 - \alpha)I \), where \( I = L + N\bar{c} \) denotes income in autarky. Substituting for \( q \) in the supply and equating it the demand yields

\[
N = [\alpha L] / [\theta (\bar{c} + f) - \alpha \bar{c}] = P^{1-\theta},
\]  

where \( N \) is the number of firms and \( P \) is the ideal price index. Simple algebra on (2.1) shows that \( dN/d\bar{c} = L\alpha(\alpha - \theta) / [\theta (\bar{c} + f) - \bar{c}\alpha]^2 < 0 \); An increase in entry barriers decreases \( N \) and
increases the price index. Similarly, a reduction in the number of firms in the Cournot competition model increases the price charged by the firms. Given (2.1), the income level in the autarky economy equals

\[ I = \frac{[\theta (\bar{c} + f) L]}{[\theta (\bar{c} + f) - \bar{c} \alpha]}. \tag{2.2} \]

Simple algebra on (B2) shows that \( \frac{dI}{d\bar{c}} = \frac{dN\bar{c}}{d\bar{c}} = fL\bar{\alpha}\theta /[(\bar{c} + f)\theta - \bar{c} \alpha]^2 < 0 \): An increase in \( \bar{c} \) raises rents and income. Similarly, a reduction in the number of firms in the Cournot competition model increases profits at the firm and at industry levels. Given (2.1) and (2.2), the indirect utility function is written as follows

\[ V = \alpha^\alpha (1 - \alpha)^{1-\alpha} P^{-\alpha} I = \frac{[(L\alpha(\theta(\bar{c} + f) - \bar{c} \alpha))^{\alpha/(\theta - 1)}(\bar{c} + f)L\theta]/[\theta(\bar{c} + f) - \bar{c} \alpha].} \tag{2.3} \]

I take the derivative of \( V \) with respect to \( \bar{c} \). To facilitate the comparison of these results and the outcomes presented in Appendix 3, I divide this derivative by \( V \) and write

\[ \frac{dV/d\bar{c}}{V} = -\alpha[(\theta - \alpha)\bar{c} + f(1 - \alpha)]/[\theta(\bar{c} + f) - \bar{c} \alpha](\theta - 1)(\bar{c} + f)] < 0 \tag{2.4} \]

Since \( dV/d\bar{c} \) is smaller than 0, an increase in \( \bar{c} \) reduces welfare. A reduction in the number of firms reduces welfare in the Cournot competition model. Furthermore, this the same welfare effect a reduction in the number of firms has in Neary (2003b) in the so-called featureless economy (when not all sectors have the same technology parameter). Equation (2.4) proves Lemma 3 and, together with (3.16), shows Lemma 2.

**Herfindahl Hirschman Index**

Using Equation (2.1) I write the the Herfindahl Hirschman index (HHI) as follows

\[ HHI = \sum_{i=0}^{N} s_i^2 = 1/N, \tag{2.5} \]

where \( s_i \) is firm \( i \)'s market share. Since \( dN/d\bar{c} < 0 \), an increase \( \bar{c} \) raises concentration.

**Appendix 3**

*Proposition 1 and Lemma 4*
Employing (1.2), I write the derivative of $S_{nj}$ with respect to $S_{lj}$ as follows

$$
\frac{dS_{nj}}{dS_{lj}} = \frac{\theta(\bar{\epsilon} + f) - \bar{c} \alpha(1 + \rho)}{[(1 - \rho)(f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))].}
$$

$dS_{nj}/dS_{lj} > 0$ because the term $f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))$ in the denominator is positive under (1.4) and the numerator is always positive. The same derivative in the $HK$ model is given by $dS_{nj}^{HK}/dS_{lj}^{HK} = (1 + \rho)/(1 - \rho)$. Subtracting $dS_{nj}^{HK}/dS_{lj}^{HK}$ from (3.1) yields $2\bar{c}\alpha \rho(1 + \rho)/[(1 - \rho)(f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho)))]$, where the inequality arises from condition (1.4) and shows that the Home market effect magnifies. This proves Proposition 1.

Employing (1.2), I write the derivative of $S_{nj}$ with respect to $\bar{\epsilon}$

$$
\frac{dS_{nj}}{d\bar{\epsilon}} = \frac{f(-1 + 2S_{lj})\alpha \rho(1 + \rho)}{[(1 - \rho)(f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))].}
$$

$dS_{nj}/d\bar{\epsilon} > 0$ if and only if $S_{lj} > 1/2$; $dS_{nj}/d\bar{\epsilon} < 0$ if and only if $S_{lj} < 1/2$. This is Lemma 4.

**Comparative Statics on \( \rho \): Proposition 2; Example 1 and Lemma 1**

Model with Rents Greater than 0

Country $j$’s indirect utility function is written as follows

$$
V_j = \alpha^\alpha (1 - \alpha)^{1-\alpha} p_j^{-\alpha} I_j; \quad (3.3)
$$

$$
I_j = \frac{\theta(\bar{\epsilon} + f)L_j(S_{lj}(\theta(\bar{\epsilon} + f) - \alpha \bar{\epsilon}(1 - \rho) - \bar{c} \alpha \rho))}{[(\theta(\bar{\epsilon} + f) - \alpha \bar{\epsilon})(f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))].}; \quad (3.4)
$$

$$
p_j^{1-\theta} = \frac{\alpha(1 + \rho)L_j(S_{lj}(\theta(\bar{\epsilon} + f) - \alpha \bar{\epsilon}(1 - \rho) - \bar{c} \alpha \rho))}{[(\theta(\bar{\epsilon} + f) - \alpha \bar{\epsilon})(f \theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))].}; \quad (3.5)
$$
The numerator in (3.4) is positive: \( \theta(\bar{e} + f)L_j \) is positive and \( S_{lj}(\theta(\bar{e} + f) - \alpha \bar{e})(1 - \rho) - \bar{e} \alpha \rho \) is positive. The latter term is positive because \( S_{lj} > S_{lj}^{WD} = \bar{e} \alpha \rho /[(\theta(\bar{e} + f) - \alpha \bar{e})(1 - \rho)] \) and, therefore, \( S_{lj} \) is always greater than \( S_{lj}^{WD} \) in an incomplete specialization equilibrium. The denominator in (3.4) is also positive because \( \theta(\bar{e} + f) - \alpha \bar{e} \) and \( f \theta(1 - \rho) + \bar{e}(\theta(1 - \rho) - \alpha(1 + \rho)) \) are positive under (1.4). Since the numerator and the denominator in (3.4) are positive, \( L_j \) is well-defined. Following the same logic, the numerator and the denominator in (3.5) are greater than zero for the same reasons and the price index is well-defined. The derivative of \( V_j \) with respect to \( \rho \) is written as follows

\[
\frac{[dV_j/d\rho]}{V_j} = A + \delta_j;
\]

\[
A = \alpha /[(1 + \rho)(\theta - 1)];
\]

\[
\delta_j = \frac{[(2S_{lj} - 1)\alpha \bar{e}(\alpha + \theta - 1)(\theta(\bar{e} + f) - \alpha \bar{e})]/}{[(S_{lj}(\theta(\bar{e} + f) - \alpha \bar{e})(1 - \rho) - \bar{e} \alpha \rho)(\theta - 1)(f \theta(1 - \rho) + \bar{e}(\theta(1 - \rho) - \alpha(1 + \rho))].}
\]

Note that \( A \) is positive and equal to \( [dV_j^{HK}/d\rho]/V_j^{HK} \) (see (3.15)). This proves Lemma 1 and, together with (3.15), demonstrates Remark 1.

The denominator in \( \delta_j \) is positive because \( (\theta - 1) \) is positive; \( f \theta(1 - \rho) + \bar{e}(\theta(1 - \rho) - \alpha(1 + \rho)) \) is greater than zero under (1.4) and \( S_{lj}(\theta(\bar{e} + f) - \alpha \bar{e})(1 - \rho) - \bar{e} \alpha \rho \) is positive whenever \( S_{lj} > S_{lj}^{WD} \). As for the numerator in \( \delta_j \), \( \alpha \bar{e}(\alpha + \theta - 1)(\theta(\bar{e} + f) - \alpha \bar{e}) \) on the right hand side is positive so that the sign of the numerator depends on the sign of \( 2S_{lj} - 1 \). Hence, based on the sign of \( 2S_{lj} - 1 \), we are left with two cases

If country \( j \) is the large country so that \( S_{lj} > 1/2 \), \( \delta_j \) and \( dV_j/d\rho \) are greater than 0. This proves the first part of Proposition 2.
If country $j$ is the small country so that $S_{lj} < 1/2$, $\delta_j$ is negative and $dV_j/\rho$ can be smaller than zero. I show that $dV_j/\rho$ is negative if country $j$'s labor share is lower than a threshold $S_{ij}^\rho$.

To find $S_{ij}^\rho$, I assume this threshold is consistent with incomplete specialization; that is I assume that $S_{ij}^\rho > S_{ij}$ and will then check under which conditions this assumption holds. $S_{ij}^\rho$ can then be found by solving for the value of $S_{lj}$ that equates $dV_j/\rho$ to 0 equals

$$
S_{ij}^\rho = \frac{\bar{c}[(\alpha(2 + \rho) + (\theta - 1)(1 + \rho))B - \alpha((\bar{c} + f)\theta + \bar{c}\alpha)\rho^2]}{[(B)(C)];}
$$

$$
B = (\theta(\bar{c} + f) - \bar{c}\alpha);
$$

$$
C = f\theta(1 - \rho)^2 + \bar{c}(\theta(3 + \rho^2) + \alpha(1 + \rho)^2 - 2(1 + \rho)).
$$

I must now show that $dV_j/\rho < 0$ for any $S_{lj}$ such that $S_{lj} < S_{lj} < S_{ij}^\rho$. For this purpose, I write

$$
dV_j/[(\rho dS_{lj})] = \frac{[(\theta(\bar{c} + f) - \bar{c}\alpha)\bar{c}\alpha(\alpha + \theta - 1)]}{[S_{lj}(1 - \rho)((\bar{c} + f)\theta - \bar{c}\alpha) - \bar{c}\alpha\rho^2].}
$$

Because $S_{lj} = S_{lj}^{WD}$, the numerator and the denominator in (3.8) and thus $dV_j/[(\rho dS_{lj})]$ are greater than 0. Because $dV_j/[(\rho dS_{lj})] > 0$, we have that $dV_j/\rho < 0$ for any $S_{lj}$ such that

$$
S_{lj} < S_{lj} < S_{ij}^\rho. \text{ Equation (3.8), by showing that } dV_j/[(\rho dS_{lj})] \text{ is monotonic for all values } S_{lj} > S_{lj}^{WD}, \text{ it demonstrates that } S_{ij}^\rho \text{ is the only value of } S_{lj} \text{ such that } S_{lj} > S_{lj}^{WD} \text{ that equates } dV_j/\rho \text{ to 0.}
$$

I must now show that $S_{ij}^\rho$ is consistent with incomplete specialization; that is, I must show that there exist a set of parameter values for which $S_{ij}^\rho > S_{lj}$, and thus I write

$$
(3.9) \quad S_{ij}^\rho - S_{lj} =
$$
\[ [(D)(E)]/ 
\] 
\[ [(B)(C)(F)]; \]

\[ B \text{ and } C: \text{defined above;} \]
\[ D = f\theta(1 - \rho) + \overline{c}(\theta(1 - \rho) - \alpha(1 + \rho)); \]
\[ E = \overline{c}((\theta + \alpha)(1 + \rho)^2 - 1 - \rho) - f\theta(1 - \rho); \]
\[ F = 1 + \rho. \]

\(B\) and \(F\) are unambiguously positive and \(D\) is positive under condition (1.4). Furthermore, \(C\) is also positive because \(\theta\) is always greater than \(\overline{\theta}_E < 1\), where \(\overline{\theta}_E = [(1 + \rho)(2 - (1 + \rho))]/[3 + \rho^2]\) is the value of \(\theta\) above which \(C > 0\). Since \(B; F; D\) and \(C\) are positive, the sign of \(S_{ij}^P - S_{ij}\) depends exclusively on the sign of \(E\).

To fix ideas, any set of parameter values for which \(E > 0\) generates consistency with incomplete specialization. I demonstrate that this is set of parameters is non-empty by imposing a mild restriction on \(\theta\) and a lower bound on \(\overline{\theta}\). Whereas the former restriction guarantees that the term \((\theta + \alpha)(1 + \rho)^2 - 1 - \rho\) is positive, the latter restriction ensures that this term multiplied by \(\overline{\theta}\) is greater than \(f\theta(1 - \rho)\). The restrictions are

\[(3.10) \quad \theta > \theta^\rho = [(1 + \rho)(1 - \alpha(1 + \rho))]/[1 + \rho^2] < 2\]
\[ \overline{\theta} > \overline{\theta}^\rho = [f\theta(1 - \rho)/(\theta(1 + \rho^2) + \alpha(1 + \rho)^2 - (1 + \rho)]. \]

Finally, I must show that \(\overline{\theta}^\rho\) and (3.10) are consistent with the upper bound that I have imposed in (1.4). For this purpose, I write assumption (1.4) as follows

(1.4'') \(i\)f \(\alpha \leq \theta(1 - \rho)/(1 + \rho)\): I do not assume anything
\(i\)f \(\alpha > \theta(1 - \rho)/(1 + \rho)\): I assume \(\overline{\theta} < \overline{\theta}_{A4} = [f\theta(1 - \rho)]/[\alpha(1 + \rho) - \theta(1 - \rho)]. \)

When \(\alpha \leq \theta(1 - \rho)/(1 + \rho)\); Equation (A4') imposes no restriction on \(\overline{\theta}\) and, therefore, \(\overline{\theta}^\rho\) is consistent with this Equation. When \(\alpha > \theta(1 - \rho)/(1 + \rho)\), on the other hand, (1.4') requires \(\overline{\theta}\) to be lower than \(\overline{\theta}_{A4}\). In this case, \(\overline{\theta}^\rho\) must be lower than \(\overline{\theta}_{A4}\). To prove this condition holds, I write the following expression

32
(3.11) \[ \bar{c}_{44} - \bar{c}^0 = \]
\[ \frac{\left[ (\alpha + \theta - 1)f\theta(1 - \rho) \right]}{\left[ (\alpha + \theta - \theta(1 - \rho))((\theta + \alpha)(1 + \rho)^2 - 1 - \rho) \right]} \].

The numerator in (3.11) is always positive. As for the denominator, \(\alpha(1 + \rho) - \theta(1 - \rho)\) is positive whenever \(\alpha > \theta(1 - \rho)/(1 + \rho)\) and (1.4’) restricts the value of \(\bar{c}\). Therefore, the denominator is positive and (3.10) is consistent with (1.4’) when the other term on the right, \((\theta + \alpha)(1 + \rho)^2 - 1 - \rho\), is greater than zero. Note that this term is positive is positive whenever \(\theta > \theta^0\) and, therefore, (3.10) is always consistent with (1.4’). This proves Proposition 2 by showing the set of parameters that makes \(E > 0\) is non-empty. It also shows Example 1.

HK Model: Remark 1 and 2nd Part of Lemma 1

Plugging \(\bar{c} = 0\) in (3.2) I obtain the following indirect utility function in the HK model

(3.12) \[ V_j^{HK} = \alpha^\alpha(1 - \alpha)^{1 - \alpha}p_j^{1-\alpha}I_j; \]
(3.13) \[ I_j^{HK} = L_jS_{ij} \]
(3.14) \[ p_j^{HK1-\theta} = \left[ L_jS_{ij}(1 + \rho)\alpha \right]/[f\theta] \]

Thus, \([dV_j^{HK}/d\rho]/V_j^{HK}\) is written as follows

(3.15) \[ [dV_j^{HK}/d\rho]/V_j^{HK} = \frac{A}{(1 + \rho)(\theta - 1)} \]

The fact that \(A = \alpha/[((1 + \rho)(\theta - 1))\) is the expression in (3.6) proves Lemma 1. The fact that this expression is positive, regardless of \(S_{ij}\), proves Remark 1.

Comparative Statics on \(\bar{c}\): Lemma 2, Proposition 3; Examples 2 and 3

Employing Equations (3.3)-(3.5) I write \(dV_j/d\bar{c}\) as follows

(3.16) \[ \frac{dV_j/d\bar{c}}{V_j} = G + \theta_j \]
\[ G = -\alpha[(\theta - \alpha)\bar{c} + f(1 - \alpha)]/[(\theta(\bar{c} + f) - \bar{c}\alpha)(\theta - 1)(\bar{c} + f)]; \]
\[ \theta_j = [(2S_{ij} - 1)\alpha f(\alpha + \theta - 1)(1 - \rho)\rho]/ \]
\[ [(S_{ij}(\theta(\bar{c} + f) - \alpha\bar{c})(1 - \rho) - \bar{c}\alpha\rho)(\theta - 1)(f\theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho))]. \]
Note that $G$ is the same as the expression in (2.4) and, therefore, equals $[dV_j^A/d\bar{c}]/V_j^A$: this proves Lemma 2. As shown above $[dV_j^A/d\bar{c}]/V_j^A < 0$.

The denominator in $\theta_j$ is positive: $(\theta - 1)(f\theta(1 - \rho) + \bar{c}(\theta(1 - \rho) - \alpha(1 + \rho)))$ is positive under (A4) and $S_{ij}(\theta(\bar{c} + f) - \alpha\bar{c})(1 - \rho) - \bar{c}\alpha\rho$ is positive because $S_{ij} > S_{ij}^{WD}$ (see (C6) in this Appendix for a detailed explanation). As for the numerator in $\theta_j$, the expression $a\theta f(\alpha + \theta - 1)(1 - \rho)\rho$ is positive. Based on the sign of the remaining term in the numerator, $2S_{ij} - 1$, we are then left with two cases

If country $j$ is the small country so that $S_{ij} < 1/2$, $\theta_j$ and $dV_j/d\bar{c}$ are negative. This proves the first part of Proposition 3.

If country $j$ is the large country so that $S_{ij} > 1/2$, $\theta_j$ is greater than zero and $dV_j/d\rho$ can positive. I will show that $dV_j^H/d\bar{c}$ is positive if country $j$’s labor share is greater than a threshold $S_{ij}^{\rho}$.

To find $S_{ij}^{\rho}$, I assume this threshold is consistent with incomplete specialization; that is I assume that $S_{ij}^{\rho} > S_{ij}$ and will then check under which conditions this assumption holds. $S_{ij}^{\rho}$ can then be found by solving for the value of $S_{ij}$ that equates $dV_j/d\rho$ to 0 and equals

$$S_{ij}^{\rho} = \frac{\rho[H + J + K + M]}{[O + Q + R]};$$

$$H = -f^3\theta^2(1 - \alpha - \theta)(1 - \rho);$$

$$J = \bar{c}\theta^2(2(1 - \theta) - \alpha)(1 - \rho);$$

$$K = \bar{c}^3\alpha(\alpha - \theta)(\alpha(1 + \rho) - \theta(1 - \rho));$$

$$M = \bar{c}^2f(-\alpha^2(1 + \theta(1 - \rho) + \rho) + \alpha\theta^2(1 - \rho) - (\theta - 1)\theta^2(1 - \rho) + \alpha^3(1 + \rho));$$

$$O = (\theta(\bar{c} + f) - \alpha\bar{c})^2;$$
\[ Q = 1 - \rho; \]
\[ R = f((1 - \alpha)(1 + \rho) - 2\theta\rho) - \bar{c}(\alpha(1 + \rho) - \theta(1 - \rho)); \]

I must now prove that \( dV_j/d\bar{c} > 0 \) for any \( S_{ij} \) such that \( S_{ij}^c < S_{ij} < S_{ij}^{\bar{c}} \). For this purpose, I write

\[ dV_H/d[\bar{c}dS_{ij}] \]
as follows

\[\begin{align*}
    dV_j/d[\bar{c}dS_{ij}] &= \frac{f\alpha\theta(\alpha + \theta - 1)(1 - \rho)\rho}{[S_{ij}(1 - \rho)(\theta(\bar{c} + f) - \bar{c}\alpha) - \bar{c}\alpha\rho]^2}.
\end{align*}\]  

(3.18)

Because \( S_{ij} \neq S_{ij}^{WD} \) the numerator and denominator in (3.18) and thus \( dV_j/d[\bar{c}dS_{ij}] \) positive. Since \( dV_j/d[\bar{c}dS_{ij}] > 0 \), \( dV_j/d\bar{c} < 0 \) for any \( S_{ij} \) such that \( S_{ij}^c < S_{ij} < S_{ij}^{\bar{c}} \). Equation (3.18), by showing that \( dV_j/d[\bar{c}dS_{ij}] \) is monotonic for all values \( S_{ij} > S_{ij}^{WD} \), shows that \( S_{ij}^c \) is the only value of \( S_{ij} \) such that \( S_{ij} > S_{ij}^{WD} \) that equates \( dV_j/d\bar{c} \) to 0.

I must now show that \( S_{ij}^c \) is consistent with incomplete specialization; that is, I must show there exist a set of parameter values for which \( S_{ij}^c < S_{ij}^{\bar{c}} \) and thus write

\[\begin{align*}
    S_{ij}^{\bar{c}} - S_{ij}^c &= \frac{[(D)(S + T + U)]}{[(O)(R)(V)]}.
\end{align*}\]  

(3.19)

\[ D; O and R: defined \text{ above} \]
\[ S = -c^2(\alpha - \theta)(\bar{c}(\theta(1 - \rho) - \alpha(1 + \rho)) \]
\[ T = f^2\theta(1 - \rho)(\theta\rho - (1 - \alpha)(1 + \rho)) \]
\[ U = -cf(\alpha - \theta(1 - \rho))(\theta(\rho - 1) - (1 - \alpha)(1 + \rho)) \]
\[ V = 1 - \rho^2 \]

\( O; V \text{ and } D \) are positive under (1.4). Thus, the sign of \( S_{ij}^{\bar{c}} - S_{ij}^c \) then depends on the sign of the ratio \((S + T + U)/R\); any set of parameter values that makes this ratio greater than 0 generates consistency with incomplete specialization. To prove that this set is non-empty I could follow the same strategy as in the comparative statics on \( \rho \) and provide an analytical proof. The drawback of
this strategy is that the non-linearities present in $S; T; U$ and $R$ make the relationships between these terms and the parameters non-monotonic and, therefore, force me to impose restrictions on all the model’s parameters. Rather than following this strategy then, I will pick a vector of parameter values and show that this vector makes $(S + T + U)/R > 0$. This simpler strategy does the job.

I use parameter values given in example 2: I take $\theta = 4.6$ and $\alpha = 0.188$ from Dekle et al. (2007) and Ossa (2012), and set $\bar{c} = 45; f = 100$ and $\rho = 0.7$ for the remaining parameters. Note that this vector of parameter values fulfills (1.3)-(1.5). These values yield $S_{i,j}^{c} = 0.58$ and $(S + T + U)/R = 0.002 > 0$. On the other hand, when I use the same values but change $\rho$ goes from 0.7 to 0.6 (so that (A3)-(A5) still hold), I obtain $S_{i,j}^{c} = 0.646$ and $(S + T + U)/R = -0.026 < 0$.

Appendix 4

The ease of doing business ranking from the World Bank is used as a proxy for regulatory entry barriers. This ranking is based on 11 Doing Business (DB) indicators that measure the complexity of regulation; the time and cost of achieving a regulatory goal or complying with regulation; the extent of legal protections of property and different dimensions of employment regulation. I choose the year 2012 to take advantage of the improvement in the methodology that occurred that year. G.D.P. and G.D.P. per capita in P.P.P. terms are used as proxies for market size and labor productivity, respectively. These data are retrieved from the World Economic Outlook of the IMF for 2011 because the DB indicators constructed in 2012 are for this year. I constraint my sample to the 182 countries for which information on all the variables is available; then I rebuild the ranking of countries implied by the DB indicators so that it takes values 1-182.

Table 1 displays the results. The ranking is more correlated with G.D.P. per capita than with G.D.P.; whereas the former variable is significant at the 1% level (column 2), the latter variable is
significant at the 1% level (column 1). Interestingly, when the two variables are included in the regression, G.D.P. per capita remains significant, but GDP is not significant at any level.

Table 1
CROSS-COUNTRY DIFFERENCES IN ENTRY BARRIERS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.D.P.</td>
<td>-0.007**</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td>b/se</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>G.D.P. per cápita</td>
<td>-0.002***</td>
<td>-0.002***</td>
<td></td>
</tr>
<tr>
<td>b/se</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>94.375***</td>
<td>123.448***</td>
<td>123.699***</td>
</tr>
<tr>
<td>b/se</td>
<td>-4</td>
<td>-4.09</td>
<td>-4.1</td>
</tr>
</tbody>
</table>

* p<0.05, **p<0.01, ***p<0.001
b/se : Point estimate divided by standard errors.

Notes: Correlations between the ease of doing business ranking with G.D.P. (Model 1); G.D.P. per capita (Model 2) and both G.D.P. and G.D.P. per capita (Model 3).

Appendix 5

Equilibrium in the Manufacturing Products Markets

The system of Equations (14)-(15) can be written as \((1 - \rho_j)l_j/g_j = (1 - \rho_{j'})l_{j'}/g_{j'}\). Substituting for the income levels and for the price indexes I obtain \(n_j = [N(L_j(1 - \rho_j) - (L_{j'} + N\bar{c})\rho_j(1 - \rho_{j'}))]/[L_W(1 - \rho_j)(1 - \rho_{j'}) - N\bar{c}((\rho_j + \rho_{j'} - 2\rho_j\rho_{j'}))]\). Dividing the numerator and the denominator by \(L_W\) and the numerator by \(N\) yields

\[
S_{nj} = n_j/N = (5.1)
\]

\[
[S_{ij}(1 - \rho_{j'}\rho_j) - \rho_j(1 - \rho_{j'})(1 + N\bar{c}/L_W)]/

[(1 - \rho_{j'}\rho_j) - (\rho_j + \rho_{j'} - 2\rho_j\rho_{j'})(1 + N\bar{c}/L_W)].
\]

This is the expression displayed in Equation (18).

Parameter Restrictions

Let me write (5.1) in terms of the exogenous parameter as follows

\[
S_{nj} = n_j/N = (5.1')
\]

\[
[S_{ij}(1 - \rho_{j'}\rho_j)((\bar{c} + f)\theta - \bar{c}\alpha) - \theta\rho_H(1 - \rho_{j'})(\bar{c} + f)]/

(\bar{c} + f)\theta - \bar{c}\alpha) - \theta\rho_H(1 - \rho_{j'})(\bar{c} + f)]/
\]

37
\[ [\theta(f + c)(1 - \rho_j)(1 - \rho_j') - \tilde{c}\alpha(1 - \rho_j\rho_j)]. \]

The manufacturing products sector is active in both countries when \( S_{nj} \) lies between zero and one. Mathematically, there exist two ways of ensuring this condition. Consider first the option in which the numerator in (5.1') is positive and smaller than the denominator. This happens when the following assumptions hold

\[ S_{lj} < S_{lij} = \frac{\theta(\tilde{c} + f)(1 - \rho_j')(1 - \rho_j\rho_j) - \tilde{c}\alpha(1 - \rho_j\rho_j')}{[\theta(\tilde{c} + f) - \alpha\tilde{c}] (1 - \rho_j\rho_j)}; (5.2) \]

\[ S_{lj} > S_{lij} = \frac{\theta(\tilde{c} + f)\rho_j(1 - \rho_j')}{[\theta(\tilde{c} + f) - \alpha\tilde{c}] (1 - \rho_j\rho_j)}. \]

To ensure that \( S_{lij} > S_{lj} \) I make the following assumption

\[ \theta(\tilde{c} + f)(1 - \rho_j)(1 - \rho_j') - \tilde{c}\alpha(1 - \rho_j\rho_j') > 0. \]

Note that \( S_{lij} < 1 \) and \( S_{lj} > 0 \) always hold because \( \theta(\tilde{c} + f) - \alpha\tilde{c} > 0 \). Furthermore, \( S_{ij} < 1 \) and \( S_{lij} > 0 \) when \( \theta(\tilde{c} + f)(1 - \rho_j') - \tilde{c}\alpha(1 - \rho_j\rho_j') > 0 \), which is implied by (5.3).

Consider now the second mathematical way of ensuring that \( S_{nj} \) lies between 0 and 1: when the numerator in (5.1') is negative and smaller than the denominator in absolute terms. This occurs when \( S_{lij} > S_{lij} > S_{lj} \), and requires the opposite sign in (5.3) so that \( S_{lij} > S_{lij} \). However, when the sign in (5.3) is reversed, \( S_{lij} < 0 \) and \( S_{lij} > 1 \). Hence, (5.2)-(5.3) are the only set of restrictions with bounds on country \( j \)'s labor share that lie between 0 and 1. Using \( S_{lij} = 1 - S_{lij} \), the same conditions can be written for country \( j' \).

If a country has enough labor to fit the maximum feasible amount of varieties \( n_{H}^{max} \) and \( n_{F}^{max} \), it will produce the homogeneous good; by symmetry, \( n_{H}^{max} = n_{F}^{max} = n^{max} \). A sufficient condition for a country to produce the homogeneous good then is that the amount of labor required for producing all varieties \( NL_M/L_W \) (relative to \( L_W \)) is lower than its minimum possible labor share under incomplete specialization. For country \( j \), the minimum possible labor share is given by \( S_{lij} \).
as defined in (5.2) and, for country \(j'\), this labor share is given by the following expression \(S_{lj'} = 1 - \tilde{S}_{lj'}\). I ensure that \(S_{lj} > NlM/LW\) and \(S_{lj'} > NlM/LW\) hold by imposing an upper bound on \(\alpha\).

I obtain two values value of \(\alpha\): I equate \(S_{lj}\) to \(NlM/LW\) to obtain one value and \(S_{lj'}\) to \(NlM/LW\) to obtain the other, where \(NlM = L\alpha[(c + f)\theta - \bar{c}]/[\theta(\bar{c} + f) - \alpha\bar{c}]\), where \(NlM\) is the amount of labor required for producing all varieties for both Home and Foreign. This exercise leads me to make the following assumption

if \(\rho_j < \rho_{j'}\): I assume \(\alpha < \rho_{j'}(1 - \rho_{j})\theta(\bar{c} + f)/(\theta(\bar{c} + f) - \bar{c})(1 - \rho_{j}\rho_{j'})\),

(5.4)

if \(\rho_j > \rho_{j'}\): I assume \(\alpha < \rho_j(1 - \rho_{j'})(\bar{c} + f)/(\theta(\bar{c} + f) - \bar{c})(1 - \rho_{j}\rho_{j'})\).

Appendix 6

Rents: Proposition 4

\(N\) is independent of \(\rho_j\) and \(\rho_{j'}\). I will take advantage of this feature of the model to prove Propositions 4 and 5 taking the total number of firms as given. This strategy will simplify notation.

Assumptions (5.2)-(5.3) are written as follows in terms of \(N\)

\[S_{lj} < \tilde{S}_{lj} = [1 - \rho_j\rho_{j'} - \rho_{j'}(1 - \rho_{j})(1 + N\bar{c}/LW)]/[1 - \rho_{j}\rho_{j'}];\]  
\[S_{lj'} > \tilde{S}_{lj} = [(1 + N\bar{c}/LW)\rho_j(1 - \rho_{j'})]/[1 - \rho_{j}\rho_{j'}].\]  
\[(1 - \rho_{j'}\rho_j) - (\rho_{j} + \rho_{j'} - 2\rho_j\rho_{j}) (1 + N\bar{c}/LW)) > 0.\ or\ equivalently\ (5.3')\]

\[N\bar{c}/LW < [(1 - \rho_j)(1 - \rho_{j'})]/[\rho_j + \rho_{j'} - 2\rho_j\rho_{j}].\]  
(5.3’’)

which is the upper bound on rents per capita referred to in Subsection 4.1.

The rents of country \(j\) equal \(n\bar{c} = NS_{lj}\tilde{c}\), which is written as follows

\[NS_{lj}\tilde{c} = \]

\[N\bar{c}(S_{lj}(1 - \rho_{j}\rho_{j'}) - \rho_{j'}(1 - \rho_{j})(1 + N\bar{c}/LW))/\]

\[(1 - \rho_{j'}\rho_j) - (\rho_{j} + \rho_{j'} - 2\rho_j\rho_{j})(1 + N\bar{c}/LW)].\]

The expression in (5.5) is well-defined because \(0 < S_{nj} < 1\) under (E2)-(E4). I take the derivative of (5.5) with respect to \(\rho_j\), holding the other parameters constant and write
\[ \frac{[d(NS_{ij}\tilde{c})/d\rho_j]/[NS_{ij}\tilde{c}]}{[(1 + N\tilde{c}/L_W)(1 - \rho_j)(1 - S_{ij})(1 - \rho_{ij}) - \rho_{ij}N\tilde{c}/L_W)]} = \] \hspace{1cm} (5.6) 

\[ -\frac{1}{((1 - \rho_j'\rho_j) - (\rho_j + \rho_j' - 2\rho_j,\rho_j)(1 + N\tilde{c}/L_W))(S_{ij}(1 - \rho_j,\rho_j) - \rho_j(1 - \rho_j)(1 + N\tilde{c}/L_W))}. \]

The numerator in (5.6) is negative: \( N\tilde{c}(1 + N\tilde{c}/L_W)(1 - \rho_j) \) is positive and \((1 - S_{ij})(1 - \rho_j) - \rho_jN\tilde{c}/L_W \) is positive when \( S_{ij} < S_{ij}^{\rho_j} = \left[ 1 - \rho_j'(1 + N\tilde{c}/L_W) \right][1 - \rho_{ij}] \), which always holds with incomplete specialization because \( S_{ij}^{n\tilde{c}} > S_{ij} \). Since these terms are positive and there is a minus in front, the denominator in (5.6) is negative. The denominator is positive since \( S_{ij}(1 - \rho_j,\rho_j) - \rho_j(1 - \rho_j')(1 + N\tilde{c}/L_W) \) is positive under (5.2) and \((1 - \rho_j'\rho_j) - (\rho_j + \rho_j' - 2\rho_j,\rho_j)(1 + N\tilde{c}/L_W) \) is positive under (5.3). Hence, \( \left[ d(NS_{ij}\tilde{c})/d\rho_j]/[NS_{ij}\tilde{c}] = [d\tilde{c}/d\rho_j]/[n\tilde{c}] \right) < 0 \), proving the 1st part of Proposition 4.

The derivative of (5.5) with respect to \( \rho_j \), holding the other parameters constant equals

\[ \frac{[d(NS_{ij}\tilde{c})/d\rho_j]/[NS_{ij}\tilde{c}]}{[(1 + N\tilde{c}/L_W)(1 - \rho_j)(1 - S_{ij}(1 - \rho_j) - \rho_jN\tilde{c}/L_W)]} = \] \hspace{1cm} (5.7) 

\[ -\frac{1}{((1 - \rho_j'\rho_j) - (\rho_j + \rho_j' - 2\rho_j,\rho_j)(1 + N\tilde{c}/L_W))(S_{ij}(1 - \rho_j,\rho_j) - \rho_j(1 - \rho_j)(1 + N\tilde{c}/L_W))}. \]

The denominator in (5.7) is positive for the same reasons as the denominator in (5.6) is positive. The numerator is also positive because \((1 + N\tilde{c}/L_W)(1 - \rho_j) \) is positive and \( S_{ij}(1 - \rho_j) - \rho_jN\tilde{c}/L_W \) is positive when \( S_{ij} > S_{ij}^{\rho_j} = \left[ \rho_jN\tilde{c}/L_W \right][1 - \rho_j] \), which always holds in an incomplete specialization equilibrium because \( S_{ij}^{n\tilde{c}} < S_{ij} \) under (E3). Hence, \( \left[ d(\tilde{c})/d\rho_j]/[n\tilde{c}] \) \( [NS_{ij}\tilde{c}] = [d\tilde{c}/d\rho_j]/[n\tilde{c}] > 0 \), proving the 2nd part of Proposition 4.

**,Price Indexes: Proposition 5**

The \( g_j \) function is written as

\[ g_j = \] \hspace{1cm} (5.8) 

\[ N[(S_{ij}(1 - \rho_j) - \rho_jN\tilde{c}/L_W)(1 - \rho_j,\rho_j)]/. \]
\[(1 - \rho_j' \rho_j) - (\rho_j + \rho_j' - 2\rho_j \rho_j)(1 + N\bar{c}/L_W)\]

The numerator in (5.8) is positive because \(S_{ij} > S_{ij}'\) in an incomplete specialization equilibrium. The denominator is positive under (5.3'). Hence, (5.8) is positive and \(g_j\) is well-defined. The derivative of this expression with respect to \(\rho_j\) is

\[
\frac{dg_j}{d\rho_j} = W + \omega_j
\]

\[
W = -\rho_j/(1 - \rho_j \rho_j)
\]

\[
\omega_j =
-\left[N\bar{c}/L_W((1 - S_{ij})(1 - \rho_j') - \rho_j N\bar{c}/L_W)/((S_{ij}(1 - \rho_j)' - \rho_j N\bar{c}/L_W))((1 - \rho_j' \rho_j) - (\rho_j + \rho_j' - 2\rho_j \rho_j)(1 + N\bar{c}/L_W))\right]
\]

\(W\) is smaller than 0 and equals \([dg_j^{os}/d\rho_j]/g_j^{os}\) (see (5.11)). This proves, together with (5.11) the first part of Proposition 5.

The numerator in \(\omega_j\) is negative: \((1 - S_{ij})(1 - \rho_j') - \rho_j N\bar{c}/L_W\) is positive because \(S_{ij} < S_{ij}'\) and there is a minus in front. The denominator is positive: the term on the left \(S_{ij}(1 - \rho_j)' - \rho_j N\bar{c}/L_W\) is positive because \(S_{ij} > S_{ij}'\) and the term on the right is positive under (E3). Hence, \(\omega_j < 0\), which proves part of Proposition 5.

To finish the proof, I plug \(\bar{c} = 0\) in (5.8) and write the \(g\) function in Ossa’s model as follows \([dg_j^{os}/d\rho_j]/g_j^{os}\)

\[
g_j^{os} = \left[S_{ij}(1 - \rho_j' \rho_j)\right]/[1 - \rho_j]
\]

(5.10)

The derivative of (E10) with respect to \(\rho_j\) is written as follows

\[
\frac{dg_j^{os}}{d\rho_j} \cdot g_j^{os} = -\rho_j/(1 - \rho_j \rho_j) = W
\]

(5.11)

This is the expression that appears in (5.9). This proves Proposition 5.