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**Working Paper**

## Public debt and growth: An assessment of key findings on causality and thresholds

Working Paper, No. 2017-10

**Provided in Cooperation with:**

Department of Economics, University of Massachusetts

*Suggested Citation:* Ash, Michael; Basu, Deepankar; Dube, Arindrajit (2017) : Public debt and growth: An assessment of key findings on causality and thresholds, Working Paper, No. 2017-10, University of Massachusetts, Department of Economics, Amherst, MA

This Version is available at:

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# DEPARTMENT OF ECONOMICS

## Working Paper

**Public Debt and Growth:  
An Assessment of Key Findings  
on Causality and Thresholds**

by

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Deepankar Basu  
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Working Paper 2017-10



**UNIVERSITY OF MASSACHUSETTS  
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Public Debt and Growth:  
An Assessment of Key Findings on Causality and Thresholds

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April 21, 2017

**Abstract**

We provide a comprehensive assessment of the relationship between public debt and GDP growth in the postwar advanced economies. We use the timing of changes in public debt and growth to account for endogeneity, and find little evidence of a negative relationship. Semi-parametric estimates do not indicate any threshold effects. Finally, we reconcile our results with four recent, influential papers that found a substantial negative relationship, especially when public debt exceeds 90 percent of GDP. These earlier results appear to derive mostly from peculiar parametric specifications of nonlinearities, or use of small samples which amplify the influence of outliers.

*JEL classification:* E00, E6, H6, C82.

*Keywords:* public debt, growth.

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# 1 Introduction

During the Great Recession and its aftermath, there has been an intense public debate about the effect of public debt on economic growth. Advocates of austerity have argued that high public debt is a drag on economic growth. Others have argued that the causality flows largely in the other direction: slow economic growth reduces tax revenue and increases the need for public spending, which together increase fiscal deficits. In the latter view, efforts to reduce debt by increasing taxes and decreasing spending during a downturn can actually aggravate macroeconomic problems in economies constrained by insufficient aggregate demand. Critics also argue that the evidence on how public debt affects long term growth is at best weak.

Unfortunately, to date, the academic literature on the relationship between debt and long term growth remains unsettled. There have been a number of important and influential empirical contributions in this literature, including Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015). Based on overlapping samples of developed countries, these four papers make three key arguments. First, high public debt is associated with lower real GDP growth. Second, the relationship is interpreted as a causal effect of debt on growth. Third, there is an important threshold in the debt-to-GDP ratio (typically identified as 90 percent) above which growth drops substantially. In contrast, some recent papers have raised questions about the robustness and the causal content of the debt-growth relationship. Herndon et al. (2014) identifies important problems in the Reinhart and Rogoff (2010) analysis of the bivariate relationship between public debt and growth, while Panizza and Presbitero (2014) uses a specific instrumental variable strategy to argue that the this relationship is not likely causal.<sup>1</sup>

In this paper we examine the relationship between public debt and GDP in developed countries broadly, and provide a full assessment of the evidence on the three key arguments.<sup>2</sup>

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<sup>1</sup>In particular, Panizza and Presbitero (2014) instrument the public debt-to-GDP ratio with the valuation effect brought about by the interaction of foreign currency debt and changes in the nominal exchange rate.

<sup>2</sup>Table 1 describes the papers we review in our study, including the publication outlet and year, whether

Using a comprehensive database of developed countries used across these studies covering the period since 1956, we empirically assess whether there is a robust negative relationship between public debt and growth, whether such a relationship is likely causal, and whether there is any indication of nonlinearity in this relationship. We first provide evidence on the timing of the change in growth and debt to assess causality, and find that higher debt is associated more clearly with past rather than future growth. We then use an ensemble of standard tools to account for reverse causality and unobserved heterogeneity in the relationship between GDP growth and the public debt-to-GDP ratio: using forward (rather than contemporary) growth rates; instrumenting current public-debt-to-GDP ratio with its lag; and controlling for lagged GDP growth rates. We formally establish that these tools do, indeed, mitigate the bias due to reverse causality under a wide range of assumptions about the data generating processes. Moreover, we show that the conditions under which each of these strategies reduce bias from reverse causality are quite different. As a result, if the full ensemble of these specifications suggests a similar estimate, this constitutes a finding that is highly robust to the specific nature of the bi-directional causality.

We find that the contemporaneous bivariate estimate for the 1956-2003 sample is -0.021, suggesting that a 100 percentage point increase in public debt is associated with a 2.1 percentage point reduction in real per capita GDP growth. However, moving from contemporary to future growth as the outcome diminishes the estimate by one third, yielding a bivariate coefficient of -0.014. OLS and IV specifications that control for past growth reduce the original estimate and produce statistically significant estimates of around -0.006. In contrast, fixed effects specifications (OLS or IV) suggest somewhat more negative estimates of around -0.015. When the sample is limited to years since 1970, however, there is no evidence of a negative association between future growth rate and public debt in any specification. This is particularly relevant because this is the period studied by three of the papers we peer-reviewed, and data (country-years).

examine.

We also provide a transparent assessment of nonlinearities and threshold effects by showing the data, and using non-parametric and semi-parametric specifications that make minimal assumptions about functional forms. These plots provide visual confirmation that the relationship between public debt and growth is essentially flat for public debt/GDP exceeding 30 percent when we (1) use forward growth rates, (2) control for past GDP growth, or both. Contemporary public debt is negatively associated with growth in the preceding five years, but there is little evidence of a negative association with the 5-year forward growth rate. We conclude that causality is, thus, more likely to run from GDP growth to public debt than vice versa. There is no sign of systematic thresholds at elevated levels of public debt in the data; 5-year forward growth rates are no lower when public debt crosses 90 percent of GDP.

In addition to providing these preferred estimates based on a superset of the data with a flexible modeling strategy, we examine each of the papers in detail to provide a reconciliation of the results across the papers and with our findings. We find substantial sensitivity of some of the results to choices of model and sample. In particular, we find that use of parametric models of “turning points” (such as a quadratic specification) can sometimes produce highly misleading inference. Some of the studies use a relatively small number of countries, where sometimes a single influential country (e.g., Ireland) is shown to produce a likely spurious suggestion of an inverse-U shaped relationship between public debt and growth. In other cases, we find that a negative relationship requires peculiar choices of controls and specifications that largely disappear when instituting any number of small changes. In contrast, we use a relatively large set of countries, a set of straightforward and standard regression specifications, and transparent visual tools to assess the relationships. This allows us to produce robust inference unlikely driven by peculiar choices sometimes made in the literature.

The rest of the paper is organized as follows. Section 2 lays out the identification problem,

justifies the specifications we employ, and discusses our data and sample. Section 3 presents our main results including from the linear specifications, non- and semi-parametric models, and replication and re-assessment using specific samples used in the key papers in the literature. Section 4 summarizes the key findings and concludes the paper.

## 2 Empirical Strategy

### 2.1 Endogeneity of debt and the identification problem

From a policy perspective, we are interested in estimating the causal effect of higher public debt on growth. The fundamental challenge in identifying this effect is that debt is endogenous, and may be affected by growth. The causal negative relationship from public debt to growth depends on various crowding-out mechanisms. Public borrowing may raise interest rates, which can crowd out private investment or, via exchange rate appreciation, reduce net exports. Some studies also identify inflation associated with government debt as a possible drag on growth, although neither mechanism, from public debt to inflation nor from inflation to growth, is typically elaborated. High public debt also creates potential vulnerability to higher interest rates when public debt is rolled over.

A causal relationship in the other direction, that is, from growth to public debt, is straightforward. Slower economic growth reduces tax collections and increases the need for public spending, e.g., on unemployment insurance. Reduced public revenue and increased public spending constitute an increase in fiscal deficits through the operation of such “automatic stabilizers”.

We formalize the reverse causality problem with the following two equations. The first is growth, which depends on public debt,

$$g_{it} = D_{it}b_1 + u_{it} \tag{1}$$

in which  $g_{it}$  represents the growth rate of  $GDP$  for country  $i$  in year  $t$ ,  $D_{it}$  represents the stock of public debt,  $b_1$  is a parameter, and  $u_{it}$  represents the error term and includes all other controls or contributors to  $GDP$  growth. The second equation describes the evolution of public debt,

$$D_{it} = g_{it}a_1 + v_{it} \tag{2}$$

which introduces the additional parameter  $a_1$  and error term  $v_{it}$ .

In this classic reverse causality situation, the estimate of  $b_1$  is likely biased. The asymptotic bias of the OLS estimator is given by

$$\text{plim } \hat{b}_1 - b_1 = \frac{a_1(1 - a_1b_1)}{a_1^2 + \lambda} \tag{3}$$

where  $\lambda = \frac{\sigma_v^2}{\sigma_u^2}$  is the ratio of the variance of the error terms.

Analyzing the likely size and direction of bias, we note that if  $b_1$  is reasonably small and  $a_1 < 0$ , then the bias is negative, i.e., public debt is estimated to be worse for  $GDP$  growth than it actually is.<sup>3</sup>

Because of concerns about reverse causality, we re-examine the relationship between public debt and growth with the datasets used in the four sources summarized in Table 1.

The most straightforward approach is to consider the sequencing of public debt and growth: which comes first? Of the papers we re-examine, only Reinhart and Rogoff (2010) focuses exclusively on the contemporaneous relationship between public debt and growth, and we contribute a re-analysis with the forward relationship. The other papers, Cecchetti

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<sup>3</sup>There is also a mechanical negative relationship between growth and public debt/GDP in that lower  $GDP$  growth is mechanically associated with a lower denominator in public debt/GDP; so lower growth and higher public debt/GDP can be associated through the common term, without any actual economic impact of public debt on growth. For example, if  $GDP$  is measured with error, then a spurious negative relationship will obtain between growth,  $\log(GDP_t) - \log(GDP_{t-1})$ , and the public debt-to- $GDP$  ratio,  $\text{Public Debt}_t/GDP_t$ . Measurement error in  $GDP_t$  will induce a spuriously negative regression coefficient. A positive error in the measurement of current  $GDP_t$  both increases measured growth and decreases the measured ratio of public debt to  $GDP$ . Thus, measurement error will bias the correlation in a negative direction.



et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015), examine the relationship between contemporary public debt and forward GDP growth. However, a correlation between contemporaneous public debt and future GDP growth may not indicate causality. Hence we introduce additional controls for heterogeneity using lagged growth, and instrumental variables to assess the causal content of the relationship.

We employ three strategies to address the endogeneity between public debt and GDP growth: (1) testing the importance of sequencing by examining leads and lags of GDP growth in relation to public debt; (2) instrumenting contemporary public debt with lagged public debt to break the confounding causal relationship running from growth to public debt; and (3) controlling for lagged GDP growth as an alternative way of weakening the reverse causal relationship, i.e., from growth to public debt. In this section, we describe each of these approaches in more detail. In Appendix A, we formally derive the conditions under which these approaches mitigate the bias in the bivariate estimate of contemporaneous growth on public debt.

### **2.1.1 Forward versus lagged GDP growth**

First, we test temporal sequencing to examine the question of reverse causality. What is the relationship between contemporaneous high public debt and future or past growth, or which comes first, public debt or economic growth? As indicated above, Reinhart and Rogoff (2010) examines exclusively the contemporaneous relationship between current public debt and current one-year GDP growth. However, the other three papers use forward growth as the key dependent variable. Woo and Kumar (2015), Cecchetti et al. (2011), and Checherita-Westphal and Rother (2012) examine forward, or lead, relationships between contemporary public debt and growth in the succeeding five years.

Under what conditions does the use of forward growth addresses the problem of reverse causality? The key issues in correctly identifying  $b_1$ , the strength of the forward and reverse

causality and the autocorrelation of the error terms in each of the equations for GDP growth,  $u$  in Equation 1, and public debt,  $v$  in Equation 2.

Let us suppose reverse causality, i.e.,  $a_1 < 0$ , and no forward causality, i.e.,  $b_1 = 0$ . Autocorrelation in the error term for the growth equation means that a negative shock to growth will both be persistent in itself, i.e., recessions linger, and the shock is passed into the public debt process for a long time, which raises public debt. In a contemporaneous regression, autocorrelation in the growth equation will erroneously lead to the conclusion that public debt is bad for growth, i.e.,  $\hat{b}_1 < 0$ . As we show in Appendix A, averaging growth over several future periods reduces the bias. Autocorrelation in the error term for public debt means that a positive shock to public debt persist for a long time. Under the same supposition about the true value of the parameters,  $b_1$  will be properly estimated.

We systematically examine alternative leading and lagging windows for the outcome variable, real GDP growth, as a function of public debt. We focus on the relationship between public debt and the 5-year past average growth rate, the contemporaneous growth rate, and the 5-year forward average growth rate. A contemporaneous relationship is ambiguous with respect to causality. A leading relationship indicates that causality may run from public debt to growth. A lagging relationship likely indicates reverse causality, from growth to public debt.

### **2.1.2 Controlling for heterogeneity and endogeneity**

In addition to examining the relationship between public debt and forward growth as well as contemporary and lagged growth, we also employ regression specifications that address the endogeneity in the relationship between public debt and economic growth, and control for other sources of heterogeneity.

In the baseline case, for comparability with the existing literature, we focus on a linear relationship between public debt (as a percent of GDP) and GDP growth. Even when the

true relationship is nonlinear, the linear coefficient provides us with a weighted average of the slope of the conditional expectation function. We report the implied change in real average annual growth per 100 percentage points of public debt/GDP.

After we initially examine the bivariate relationship, we include year dummies throughout to account flexibly for secular trends in public debt and growth across the advanced economies.

$$\bar{g}_{i,t+1,t+k} = \beta \cdot d_{i,t} + \gamma_t + \varepsilon_{i,t}, \quad (4)$$

where  $\bar{g}_{i,t+1,t+k}$  is average growth rate of GDP over the succeeding  $k$  years. We also employ specifications which control for lagged growth:

$$\bar{g}_{i,t+1,t+k} = \beta \cdot d_{i,t} + \rho \cdot g_{i,t-1} + \gamma_t + \varepsilon_{i,t} \quad (5)$$

Controlling for lagged growth in a growth equation holds constant recent trends in GDP growth which may be responsible both for the stock of public debt and for continuing economic performance, for example, if there is substantial serial correlation in the data because of inertia in the economy. The control for lagged growth thus accounts directly for past downturns, which may be simultaneously responsible for high debt (because of the accumulation of fiscal deficits) and continued low growth. As an alternative to using lagged growth, we also estimate models with country fixed effects—though this only accounts for time-invariant sources of heterogeneity across countries:

$$\bar{g}_{i,t+1,t+k} = \beta \cdot d_{i,t} + \mu_i + \gamma_t + \varepsilon_{i,t} \quad (6)$$

As an additional approach to account for the endogeneity of public debt, we instrument current public debt with 5 year lagged public debt . The IV specification limits identification to public debt in places where public debt is persistent, i.e., where current public debt is

explained by a history of public indebtedness rather than by macroeconomic shocks.

$$\begin{aligned}
 \text{1st stage: } \quad d_{i,t} &= \alpha \cdot d_{i,t-5} + \mu_{1i} + \gamma_{1t} + \varepsilon_{1i,t} \\
 \text{2nd stage: } \quad \bar{g}_{i,t+1,t+k} &= \beta \cdot d_{i,t} + \mu_{2i} + \gamma_{2t} + \varepsilon_{2i,t}
 \end{aligned} \tag{7}$$

We also estimate analogous IV regressions with lagged growth control instead of country fixed effects.

To summarize our approach to endogeneity in the growth and debt process, we employ three main methods to address endogeneity: using average forward growth as the dependent variable; controlling for lagged growth; and using lagged debt as an excluded exogenous instrument for current debt. Appendix A establishes specific conditions for the time-series properties of the endogenous growth and debt process that illustrate when each of the approaches works best and how to interpret concurrent or divergent results with the three approaches.

The Lagged Growth Control and Average Forward Growth approaches focus identification on different parts of the data-generating process. Controlling for lagged growth focuses identification of the effect of debt on innovations in growth, i.e., persistent growth is controlled for via the inclusion of the lag. Average Forward Growth focuses identification on the persistent portion of growth, i.e., the accumulation of growth over a five-year period. These alternative foci imply different responses of the estimator to alternative values of  $(\rho_u(k), \rho_v(k))$ , where  $\rho_u(k)$  and  $\rho_v(k)$  denote the  $k$ -th order autocorrelation coefficient for  $u$  and  $v$ , respectively. In Appendix A, we illustrate these arguments for the case when the error terms in the growth and debt equations follow stable AR(1) processes, with AR coefficients  $\phi_u$  and  $\phi_v$  respectively.

The Lagged Growth Control and Average Forward Growth estimators have the convenient property that they reduce bias (or are biased towards zero) under opposite assumptions about  $\phi_u$ . Average Forward Growth, focused on the persistent component, performs better when  $\phi_u$  is relatively low and will be biased toward zero only if  $\phi_u$  is relatively high. The Lagged

Growth Control, focused on innovations, performs better when  $\phi_u$  is relatively high and will be biased toward zero only if  $\phi_u$  is relatively low.

This combination means that it cannot be that both of them are biased towards zero (under the data-generating process). If, for example, both reduce the magnitude of the estimate vis-à-vis the baseline OLS estimate, then the reduced magnitude constitutes an improvement on the baseline estimate, and is not a result of specific conditions on  $(\phi_u, \phi_v)$  that bias the estimate toward zero. The similarity of the estimates for the Lagged Growth Control and the Average Forward Growth specification in the empirical analysis imply that the likely values of  $(\phi_u, \phi_v)$  are in the intersection of the zones where each strategy reduces bias.

## 2.2 Non-Linearities and Thresholds

### 2.2.1 Parametric approaches in existing work

All four key papers in the literature that we examine emphasize the importance of nonlinear effects, and an existence of a threshold above which the relationship between public debt and growth becomes more negative. Here we briefly summarize their approaches and what we see as serious shortcomings in each them. A common problem is that none of the papers “show the data” in a transparent fashion that would help us assess the presence and nature of the nonlinearity.

Reinhart and Rogoff (2010), which examines real GDP growth stratified by discrete categories for ranges of the public debt-to-GDP ratio, write, “it is evident that there is no obvious link between debt and growth until public debt reaches a threshold of 90 percent. The observations with debt to GDP over 90 percent have median growth roughly 1 percent lower than the lower debt burden groups and mean levels of growth almost 4 percent lower” (p. 575). As shown in Herndon et al. (2014), the apparent nonlinearity was not a robust

finding and was driven by a number of peculiar choices and errors.

Checherita-Westphal and Rother (2012), Cecchetti et al. (2011), and Woo and Kumar (2015) all use parametric methods to identify nonlinearities in the relationship between public debt and GDP growth. Checherita-Westphal and Rother (2012) uses a quadratic specification and reports the turning point: “The debt-to-GDP turning point of this concave relationship (inverted U-shape) is roughly between 90 and 100% on average for the sample, across all models. This means that, on average for the 12 euro area countries, government debt-to-GDP ratios above this threshold would have a negative effect on economic growth” (p. 1398). Whether the quadratic specification is actually appropriate is not obvious. For example, one implication of their quadratic specification is that the relationship between debt and growth is symmetric around the turning point. If debt really has a negative causal effect above the threshold, a quadratic specification imposes a symmetric positive effect below it.

Woo and Kumar (2015) implements a three-segment linear spline with the segments comprising public debt-to-GDP ratios of 0–30 percent (“low”), 30–60 percent (“medium”), and 60–90 percent (“high”). The breakpoints coincide with those identified by Reinhart and Rogoff (2010) as marking high and low levels of public debt/GDP. Woo and Kumar (2015) finds that the negative relationship between public debt and GDP growth increases at higher levels of public debt.<sup>4</sup>

There is a peculiar aspect of of the Woo and Kumar (2015) specification. While it permits the slopes of the linear segments to vary, the segments are constrained to point to a single intercept on the vertical axis, i.e., at a public debt-to-GDP ratio of zero. The coefficients on the slope of the segments constrained to a single common intercept but without the constraint

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<sup>4</sup>Woo and Kumar (2015) write, “The coefficients of low initial debt (i.e., initial debt\*Dum\_30) are insignificant and even change sign [with some estimation methods]. In the OLS, the coefficient of medium level of debt (initial debt\*Dum\_30–90) is significant at 5 percent, and its estimated coefficient is  $-0.025$ . But they are all insignificant [using the other estimation methods]. By contrast, the coefficients of high debt (initial debt\*Dum\_90) are negative and significant under [all estimation estimation methods except country fixed effects].” We note, however, that these results are based on a pooled sample of advanced and developing economics; they do not separately report results from the spline specification for the advanced economies.

of continuity are not readily interpreted. A schematic of the spline specification issue, which also affects the Cecchetti et al. (2011) threshold analysis, is illustrated in the Appendix in Figure C.1.<sup>5</sup>

Cecchetti et al. (2011) uses a semi-parametric break-point method for identifying the threshold. The study implements a two-segment linear spline with an unknown threshold, with the best-fit threshold identified on the basis of minimizing the sum of squared residuals.<sup>6</sup>

Cecchetti et al. (2011) implement the threshold in their Equation (2):

$$\bar{g}_{i,t+1,t+k} = -\phi y_{i,t} + \beta X_{i,t} + \lambda_- \cdot d_{i,t} \mathbf{I}(d_{i,t} < \tau) + \lambda_+ \cdot d_{i,t} \mathbf{I}(d_{i,t} \geq \tau) + \mu_i + \gamma_t + \varepsilon_{i,t,t+k},$$

where  $\mathbf{I}(\cdot)$  is the indicator function,  $\tau$  is a threshold,  $\bar{g}_{i,t+1,t+k}$  is average growth rate of GDP over the succeeding  $k$  years,  $y$  is level of GDP,  $d$  is public debt/GDP,  $X$  represents a set of controls, and  $\lambda_{-/+}$  is the responsiveness of GDP growth to public debt/GDP below/above the threshold.

Thus, as in Woo and Kumar (2015), Cecchetti et al. (2011) neither knots the spline segments at the threshold nor implements a fully unconstrained spline that permits both an unconstrained discontinuity and a change in slope at the threshold. The resulting specification thus does have a discontinuity at the threshold, but the coefficients on the slope of the segments are constrained because both segments must share a single common  $y$ -intercept.

An important shortcoming of all of the formal methods is that, although they identify turning points from more positive to more negative slopes, they do not distinguish among

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<sup>5</sup>Standard alternatives include a knotted spline that forces continuity (but not differentiability) by knotting the linear segments at the breakpoints or an unconstrained spline that permits both discontinuous jumps at the breakpoints, i.e., threshold effects, and different slopes in the segments. In econometric terms, Woo and Kumar (2015) includes only the interacted terms but not the main effects of the indicators for the public debt/GDP segments.

<sup>6</sup> Cecchetti et al. (2011) reports that “96 percent of GDP is the point estimate of the threshold level. At the 1 percent confidence level, the threshold level lies between 92 percent and 99 percent of GDP — that is, the level at which we estimate that public debt starts to be harmful to growth may be as low as 92 percent of GDP and as high as 99 percent (using 5 percent or 10 percent confidence levels would not change the interval much)” (p. 167).

a wide variety of concave shapes, from cliffs to inverted-v's or u's, all of which exemplify decreasing slope with a turning point or threshold. Even if we both ignore the question of causality and suppose the turning points to be correctly identified, alternative patterns around a threshold would have quite different implications for interpretation and policy. That is, the threshold findings in Cecchetti et al. (2011) and Checherita-Westphal and Rother (2012) are consistent with an inconsequential relationship between public debt and growth in the policy-relevant region, even if the relationship were causal.

### **2.2.2 Our approach using non-parametric and semi-parametric methods**

In contrast to these parametric approaches, in this paper we employ a number of transparent methods to assess and quantify nonlinear effects.

First, we use a bivariate lowess-smoothed regression to summarize the relationship between contemporary public debt and lagged, contemporary, and future GDP growth at alternative levels of public debt and to show the actual pattern of the data without assuming functional forms or thresholds. As in the case of linear regression, an association of higher contemporary public debt with lower past growth implies reverse causality (from growth to public debt), the contemporary association is ambiguous, and an association of higher contemporary public debt with lower future growth would imply causality from public debt to growth. The nonparametric analysis with alternative lagged and forward growth permits a flexible examination of non-linearities.

In addition to showing non-parametric estimates, we also examine the full scatterplot of real GDP growth versus public debt/GDP. Examining the full scatterplot gives a sense of both the relationship between public debt and growth and, importantly, the wide spread of GDP growth experience at every level of public debt.

We use the partial linear model (PLM, (DiNardo and Tobias, 2001)) to implement our preferred regression specification that linearly controls for past GDP growth and time effects,



but permits a fully flexible relationship between public debt and future GDP growth.<sup>7</sup>

$$\bar{g}_{i,t+1,t+k} = f(d_{i,t}) + \rho \cdot g_{i,t-1} + \gamma_t + \varepsilon_{i,t}$$

We also estimate analogous PLM regressions controlling for country fixed effects instead of lagged growth.

A particularly useful feature of the partial linear model is that even with fixed effects it preserves the levels of the variable on the  $x$ -axis. The standard fixed-effects model is equivalent to mean differencing the data, which changes from levels on both the vertical and horizontal axes to deviations from the unit's mean  $x$  and  $y$  values. The partial linear model applies a different approach: ordering the data by  $x$ , the independent variable of interest in the non-parametric relationship; first-differencing the remaining data, both dependent and independent variables, along this ordering; estimating the linear portion of the model, including fixed effects, with the differenced data; and then non-parametrically plotting the  $y$ -residuals from the differenced linear model against the original  $x$ -values. Among the attractions of this method is that it associates (residual) growth with actual level of public debt. So slopes and turning points are preserved at the actual associated level of public debt.<sup>8</sup>

## 2.3 Data on Debt and Growth

In this paper, we use data from International Monetary Fund to construct a dataset of 22 developed countries that largely subsumes the samples used in the key papers in the literature.<sup>9</sup> This dataset includes all of the countries used in the four key papers for which

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<sup>7</sup>We implement the PLM with `plreg` in STATA.

<sup>8</sup>Countries may differ in the historical levels of public debt and growth and in the country-specific relationship between public debt and growth. We do not assess such heterogeneity in the non-linear effect in this paper, instead estimating the average relationship between debt and growth across countries. See Panizza and Presbitero (2013) for a discussion of this issue, which is a potential direction for future research.

<sup>9</sup>We draw these data from Heston et al. (2011) and from the data supplement to Mauro et al. (2013). The Mauro et al. (2013) closely resemble the Abbas et al. (2010), both of which are provided by the IMF. We use the Mauro data based on fewer missing observations for the years and countries in the analysis.

data were available: Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), Woo and Kumar (2015). We refer to these datasets as RR1955, CMZ, CWR and WK, respectively. This set of countries relatively closely resembles the 20 developed countries in Reinhart and Rogoff (2010) with the addition of Iceland and Switzerland, which appear only in Woo and Kumar (2015). We cannot find complete early data for Luxembourg, which appears only in Checherita-Westphal and Rother (2012). The set of 22 countries nonetheless has substantially more countries than the 12 that appear in Checherita-Westphal and Rother (2012) and the 18 that appear in Woo and Kumar (2015) and Cecchetti et al. (2011). We refer to this dataset as the full sample (FS), which covers the 1956–2003 period. In addition, we also replicate our findings using the original subset of countries used in each of the four papers. For the most part, we accept the data definitions and values as presented by the authors. The details of each dataset are given in Table 2, and we discuss them in greater detail in our data appendix. We attempted to re-create each of these datasets based on authors’ description in the paper. In no case were the data archived for replication by the journals or authors.<sup>10</sup>

Our specifications, as we discuss in detail below, depend on leads and lags which limit the availability of data at the endpoints of the series. Regression analysis, lowess plots, and partial linear models use for each data set a sub-sample that excludes the first 5 years, which differs by dataset, and ends in 2003. For RR1955 we limit the sample to 1955 to 2003. We note that our sample for RR1955 thus excludes from the analysis the immediate-postwar observations that were at the center of the Herndon et al. (2014) critique of Reinhart and Rogoff (2010). For CMZ we keep observations from 1985 to 2003. For CWR and WK we keep observations from 1975 to 2003.

As a robustness check, we additionally estimate the models using a larger set of 32

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<sup>10</sup>Requests for replication data to the corresponding authors for Checherita-Westphal and Rother (2012), Woo and Kumar (2015) did not receive a response. Access to the data for Reinhart and Rogoff (2010) is discussed in Herndon et al. (2014).

countries which includes the Eastern European countries that underwent transition to market economies after 1989. The case for inclusion of these countries is mixed, both because of the paucity of years available and because the relationship between gross and net public debt and between public debt and growth may be quite different for these countries. Nonetheless, in the interest of completeness we present the full set of findings for this superset as well.<sup>11</sup>

## 3 Results

### 3.1 Descriptive trends for GDP growth and public debt in advanced economies

Figure 1 shows the count of countries (out of 22) for which the public debt-to-GDP ratio exceeded 60 percent or 90 percent over the period 1955–2010. From the middle 1950s through the late 1960s, the prevalence of high public debt was decreasing, in part because several countries completed a process of reduction of high public debt from World War II levels. By the middle 1970s, no country in our full sample had public debt above 60 percent of GDP. Beginning in the late 1970s we see an increase in the prevalence of higher public debt. By the middle 1990s, thirteen of the 22 countries in the full sample had public debt above 60 percent and the number of countries with public debt above 90 percent of GDP hovered around 5. The prevalence of public debt in excess of either 60 percent or 90 percent of GDP increased sharply after the Great Recession began in 2007–2008.

Figure 2 shows the interquartile range of real per capita GDP growth across the 22 countries in the sample over the period 1955–2010. The end of the golden age around 1970 is

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<sup>11</sup>As we noted above, the papers under examination all concern the postwar growth experience of the advanced economies. But there is variation in the years and countries available by data source. We also examined how differences in sample years and sample countries, as opposed to differences in method, affect the results by constructing common samples between pairs of datasets and computing the main results for the common samples. These results, available from the authors, strengthen our substantive conclusions about the relationship between public debt and growth.

evident in the data. The laggards of the 1960s had growth similar to the fastest growers in the 1980s and 1990s.

Growth slowed sharply in the 1970s when public debt was near its nadir. During and after the 1980s growth remained slow, relative to the postwar boom, and public debt grew. These figures suggest a negative relationship between growth and public debt. The time series plots broadly suggest that high growth accompanies debt decreases in the 1960s and that lower growth preceded the growth of public debt more recently, but it is hard to disentangle the direction of causality.

### **3.2 Regression Results using Linear Models**

Table 3 shows OLS and IV regression results for the linear relationship between public debt and current growth and between public debt and future growth using the full sample. These models specify a linear relationship between public debt/GDP and real per capita GDP growth. The lower panels of Table 3 reports the same results with the sample limited to 1970 and later. We report the expected change in average annual growth rate associated with a 1 percentage-point increase in the public debt-to-GDP ratio.

Reinhart and Rogoff (2010) examines the relationship between public debt and contemporaneous GDP growth, which is most closely captured by the Current Growth panel in Table 3. For example, our coefficient of  $-0.024$  in column 1 indicates that a 100 percentage-point increase in the public debt-to-GDP ratio would be associated with  $-2.4$  percentage point lower real annual GDP growth. The implied effect on growth of a 100 percentage-point increase in the public debt-to-GDP ratio — for example, the change in Japan from its postwar boom and very low debt to the 2000s when its public debt exceeded 100 percent of GDP — implies 2.4 percentage points per year lower real annual GDP growth, a substantial implied reduction in growth associated with higher public debt. When no other controls are used, the results using the full sample and contemporaneous public debt and GDP growth data, i.e.,

the regression specification implied by the Reinhart and Rogoff analysis, find a statistically significant negative relationship between public debt/GDP and current growth.

The introduction of year dummy variables, with results reported in column 2 of row 1, substantially reduces the relationship, halving the coefficient from  $-0.024$  to  $-0.012$ , but preserves statistical significance.

Checherita-Westphal and Rother (2012) reports results for both current growth and 5-year future growth. Checherita-Westphal and Rother (2012) focuses on nonlinear results modeled with a quadratic in public debt/GDP but also reports the results for the linear specification (Models 1(a), 3(a), and 5(a) in Table 2 of CWR). They find that the linear relationship is statistically insignificant — although it is often positive and sometimes large — for both the current and future growth specifications. Cecchetti et al. (2011) and Woo and Kumar (2015) do not report results for the contemporaneous relationship between public debt and growth.

Of greater interest with respect to a causal relationship between public debt and growth are the results for future growth reported in row 2 of Table 3. Table 5 of Cecchetti et al. (2011) reports a statistically significant negative effect on 5-year future growth of  $-0.0164$  ( $p\text{-val} = 0.025$ ) per percentage point of public debt/GDP in the linearly modeled relationship, similar in magnitude to our linear regression results in the pure bivariate specification in column 1 of row 2 of Table 3. However, the Cecchetti et al. (2011) specification additionally includes control variables often found in cross-country growth models, as well as country fixed effects and the lagged level of log GDP.

When we add time dummies to the regression of future growth on public debt, reported in column 2 of row 2, the estimated coefficient falls by half, to  $-0.008$ . When we add one lag of GDP growth as an explanatory variable, the estimated coefficient falls to  $-0.006$  and loses statistical significance.

We then switch from the inclusion of lagged GDP growth as a control to the use of lagged public debt as an instrument for contemporaneous public debt. Again, the logic is to identify

the effect of high debt on growth for chronic and notorious debtors, those countries for which past public debt is a good predictor of current public debt, in contrast to learning from countries whose current public debt reflects only temporary economic straits. The first stage results are reported in Table 6. The coefficient on once-lagged debt, the excluded exogenous instrument, is of the expected positive sign — and approximately 1 throughout, which raises the issue of non-stationarity of public debt — indicating that past debt does partially explain present debt. The  $F$ -statistic for the excluded exogenous instrument is always substantially greater than 10. Overall we conclude that lagged debt is a meaningful predictor for current debt, and we argue that sequencing makes a reasonable case for the exclusion restriction, i.e., that lagged debt is appropriately excluded from a regression of growth on current debt. In the instrumental variables result, the coefficient on public debt rises in magnitude to  $-0.008$  and is at the edge of statistical significance. When both methods of addressing endogeneity are used, the estimated coefficient falls to  $-0.006$  and is not statistically significant. The point estimate implies that 100 percentage points of additional public debt reduce growth by 0.6 percentage point.

Returning to row 1, in which current growth is the dependent variable, and examining the alternative methods of addressing endogeneity, the results are rather similar in magnitude to the results with forward growth as the dependent variable. In columns 3, 4, and 5 of row 1, the estimated coefficient ranges from  $-0.006$  to  $-0.010$  and is statistically significant in all cases. The result gives some confidence in the methods of using lagged GDP growth as a control and lagged public debt as an instrument in that the current growth specification, which is more clearly contaminated by endogeneity, yields essentially the same results as the forward growth specification.

Overall we take row 2, column 5 of Table 3 as our preferred linear specification because it examines forward growth, accounts for inertial business conditions by controlling for lagged growth, and uses the instrument of lagged public debt to identify with “chosen” public debt

rather than endogenous public debt. In this specification the coefficient on public debt/GDP is  $-0.006$  and is significant in the contemporaneous but not forward relationship. Relative to the bivariate contemporaneous estimate of  $-0.024$ , the coefficient is one-fourth as large and not statistically significant.

The final two columns (columns 6 and 7) of Table 3 use country fixed effects, with public debt instrumented by its lag in column 6. We do not include the lag of growth in the fixed-effect models because of the requirement of strict exogeneity in the estimation of fixed-effect panel models (Wooldridge, 2010), which would be violated by correlation between the country fixed effect and the inclusion of a lagged dependent variable as an explanatory variable. The results diverge substantially between datasets and between outcomes of current and future growth. In the Full Sample analysis reported in Table 3, the addition of country fixed effects generally increases the magnitude of the coefficient on public debt, raising it to roughly  $-0.015$  in both forward growth specifications.

In columns 3, 4, 5, 6, and 7 of row 4, which use forward growth as the dependent variable, the coefficient on public debt is effectively zero and never statistically significant, including in the specifications with endogeneity controls and with fixed effects. The implication of row 4 is that the effect of public debt on GDP growth in the post-1970 period is zero.

### 3.3 Results from Alternative Subsamples

Tables 4 and 5 test the sensitivity of the main results to the sample.<sup>12</sup> Because the RR1955 data most closely resemble the Full Sample data, the regression results are in general similar. In the current growth results in Table 4, the RR1955 results range from  $-0.008$  in the specification with both lagged GDP growth as a control and lagged public debt as an instrument to  $-0.024$  in the specification with fixed effects and no other controls. With

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<sup>12</sup>The Full Samples results in Table 4 are not identical to those in rows 1 and 2 of Table 3 because the sample period in Table 4 ends in 2001 to permit the use of identical sampling periods across the datasets. But the results are similar, even if not identical.

forward growth as the dependent variable, reported in row 2 of Table 5, the coefficients remain negative although generally smaller in magnitude than in the current-growth specifications and statistical significance drops away in the three specifications that address endogeneity without fixed effects (columns 3–5 of Table 5). With the RR1955 data, the coefficient on public debt/GDP in the country fixed-effect specification is negative and significant. In the future growth specification with country fixed effects, reported in columns 6 and 7 of Table 5, 100 percentage points of public debt/GDP correspond to a reduction in growth of  $-1.8$  to  $-1.9$  percentage points in the RR1955 data. These are the upper-bound estimates in the future growth regressions, which are reported in Table 5.

The results for the other three datasets are quite different. With one exception, the coefficients in all specifications for the other three datasets, CMZ, CWR, and WK, range from essentially zero to, in several cases, positive and significant. The CMZ data shows a significant positive relationship between public debt and both contemporary and future growth. This is especially surprising because the result for the linear model reported in Cecchetti et al. (2011) included fixed effects and was significantly negative; we return to these results in more detail below. With the CWR data including country fixed effects, the estimated coefficient on public debt is negative with contemporary growth (columns 6 and 7 Table 4) but zero with future growth (columns 6 and 7 of Table 5).

### **3.3.1 Alternative Sample Results**

Here we examine the sensitivity of the results to inclusion of alternative countries and years. In particular, we find that the relationship between public debt and growth weakened after the 1960s. This is important both because these more recent estimates are of greater relevance for policy today, and because most of the literature has focused on this period.

In rows 3 and 4 of Table 3 we limit the sample to post-1970 data in order to test whether the effect of public debt on GDP growth has remained stable over time. The results are quite



striking. In columns 3, 4, and 5 of row 3, which all use current growth as the dependent variable with the three specifications proposed to address endogeneity, the coefficient on public debt ranges from  $-0.001$  to  $-0.003$ . All three are precisely estimated. When country fixed effects are included in columns 6 and 7 of row 3, the estimated coefficient on public debt increases somewhat in magnitude from the estimates without fixed effects. In column 6 with fixed effects but no endogeneity controls, the magnitude is similar in size to the estimate on the post-1955 data. When lagged public debt is introduced as an instrument for contemporaneous public debt, the magnitude of the estimated coefficient falls by about half, to  $-0.007$ . The results for the Reinhart and Rogoff data prove particularly sensitive to the inclusion of the earlier years. We discuss this in more detail below.

In Table 7, we first expand the sample to include the full set of 32 high-income OECD countries, which adds most of the post-socialist European transition economies as well as Chile, Israel, and South Korea. These results are reported in the panels entitled HI56 and HI70. Then we limit this expanded sample by dropping the 6 post-socialist European transition economies, reported in the panels entitled HIXEE55 and HIXEE70.

In our preferred specifications, columns 3–5, there is modestly higher sensitivity of growth to public debt (but not at statistically distinguishable levels) in the post-1955 data when the additional countries are added. In the post-1970 data with the expanded set of countries, the coefficients are somewhat larger than for the post-1970 data from the original set of countries (see Panel FS1970: Future Growth in Table 3), precision decreases sharply, and there is never statistical significance in the estimated relationship between public debt and growth. The fixed-effect results for the post-1970 data for both expanded country samples show a positive but statistically insignificant relationship between public debt and growth.

### 3.4 Alternative Leads and Lags

Figure 3 probes the sensitivity of the regression specifications to alternative lags and leads in the definition of the dependent variable and the inclusion of alternative lags of the dependent variable as control variables. The vertical axis reports coefficient on public debt in linear regression models which vary with alternative leads or lags in the definition of the dependent variable and with controls for alternative lags of dependent variable. Along the horizontal axis, we examine 11 alternative definitions of the window for the outcome variable, from 5 years lagged growth through current growth to 5 years forward growth.

For the cases with contemporaneous or forward growth as the dependent variable, we also examine 3 alternative controls for lagged growth: no control for lagged growth (indicated by green circles); 1-year lagged growth (orange triangles); and 5-year lagged growth (blue diamonds). All models include year dummies. Throughout, we use filled shapes to indicate statistical significance at the 5-percent level.

Figure 3 presents results for the full sample in the upper panel and for RR1955 in the lower panel. The solid green circle in the middle of the upper panel shows the coefficient of  $-0.016$  in the regression of contemporaneous growth on contemporaneous public debt/GDP, which corresponds to row 1, column 2 of Table 4.

The introduction of 1-year lagged growth as a control variable in the contemporaneous regressions reduces the coefficient to a smaller though still-significant value of approximately  $-0.010$  indicated by the filled orange triangle. A control for 5-year lagged growth in the contemporaneous growth equation yields a statistically insignificant coefficient around  $-0.01$  (unfilled square and unfilled diamond) for the relationship between contemporaneous public debt and current growth.

When we examine growing windows of forward growth — without yet controlling for lagged growth — the coefficient on public debt/GDP declines in magnitude from  $-0.016$  to roughly  $-0.012$  as the number of leads increases. The solid green circle at the right edge of

the upper panel shows the coefficient  $-0.012$  in the regression of five-year forward growth on contemporaneous public debt/GDP, which corresponds to row 1, column 2 of Table 5.

With controls for lagged growth, the negative association between public debt and future growth is uniformly somewhat smaller and is statistically significant only in the case of 1-year forward growth with 1-year lagged growth as a control. With eight-year lagged growth as a control, the coefficient on public debt (blue diamonds) decreases from  $-0.008$  in the contemporaneous model to  $-0.003$  for leads two through five and is never statistically significant.

When we examine growing windows of lagged growth as the dependent variable, the coefficients become substantially more negative, around  $-0.02$ , which indicates a strong negative association between past growth and contemporary public debt. That is, public debt is more strongly associated with past low growth than with contemporary or future low growth.

In the lower panel, which uses RR1955 and the same specifications as in the upper panel, the only statistically significant negative relationship between public debt and growth is for the contemporaneous relationship with no controls for lagged growth. Even in this case the statistical significance is weak. The second row of Table 5 shows that public debt is significant only at the 10 percent level. With no control for past growth, the relationship between public debt and future growth at any lead is small and insignificant, falling by about half with a one-year lead and shrinking to essentially zero by five years out.

Any control for lagged growth with every forward lead for growth as the dependent variable yields a statistically insignificant estimated relationship. With both contemporaneous growth and every lead of growth, once a control for the eight-year lag of growth is introduced, the point estimates ranges between  $-0.005$  with contemporaneous growth to  $-0.002$  with any lead and are statistically insignificant throughout. With every lead of growth as the dependent variable and any control for lagged growth, the estimates are always below  $-0.01$

and statistically insignificant for any control for lagged growth and any forwarding more than one year.

The lower panel of Figure 3, RR1955, shows a pattern nearly identical to that of the upper panel with a negative statistically significant coefficient for contemporaneous growth regressed on contemporaneous public debt/GDP and for forward growth of one or two years if no controls are added. But no other statistically significant coefficients for the relationship of growth and public debt appear when four-year or eight-year lagged growth is included as a control or when forward growth of more than two years is used as the dependent variable.

The remaining three panels of Figure 3 performs the same analysis of alternative forward windows for the Checherita-Westphal and Rother (2012), Cecchetti et al. (2011), and Woo and Kumar (2015) samples. With neither contemporaneous data nor with any forward window is there a statistically significant relationship between GDP growth and public debt/GDP. The estimate for the effect of public debt/GDP on growth is close to zero throughout (and sometimes positive). All of the estimates are close to zero in the bivariate relationship and with controls for one-year, four-year, or eight-year lagged GDP growth.

### **3.5 Summary of Linear Regression Results**

It is useful to summarize the findings from the linear analysis of the hypothesized negative relationship between public debt and growth. In the contemporaneous relationship, the full sample and RR1955 generally yields negative estimated relationships between public debt and growth. The relationship declines when forward growth rather than current growth is used as the dependent variable and when a control for lagged growth is introduced. As the figure showing the relationship between alternatives windows and coefficient estimates demonstrates, the relationship is stronger for past growth and contemporaneous debt than for future growth and contemporaneous debt. The longer forward relationship is weak.

The introduction of lagged public debt as an instrument for contemporaneous public debt

generally reduces the magnitude of the estimated relationship between public debt and GDP growth. For the RR1955 data with current growth as the outcome variable, Table 4 Column 2 shows instrumental variable results some 25 to 50 percent smaller than the simple bivariate results. The coefficient on public debt in the instrumented specification with lagged growth is zero with the CWR data and is actually positive (although not statistically significant) with CMZ data. When both lagged GDP growth is included as a control and contemporaneous public debt is instrumented with past public debt, the magnitude of the coefficient falls still further in the RR1955 data and remains very close to zero in the CMZ and CWR data.

The pattern is similar in Table 5 which reports the IV regression results using the forward lead dependent variable in Columns 4 and 5. The coefficients with CWR and CMZ data is effectively zero (positive but statistically insignificant point estimates). With the Full Sample and RR1955 data, the estimated coefficient ranges between  $-0.008$  and  $-0.012$ , i.e., between one-third and one-half the value of the coefficient for the Full Sample and RR1955 data in Table 4 Column 1.

The introduction of country fixed effects, reported in the last three columns of Tables 4 and 5, yields highly variable results. With the Full Sample and with the RR1955 data, the coefficient on public debt/GDP remains negative, with or without the use of lagged debt as an instrument or the inclusion of time dummies. The implied diminution in forward GDP growth is around  $-0.018$  percentage points of GDP growth per percentage point of public debt/GDP. The coefficient with the CMZ data is large and positive, with 1 percentage point of public debt/GDP associated with a statistically significant positive increase in forward GDP growth of  $+0.022$  to  $+0.024$  percentage points (and  $+0.006$  to  $+0.024$  percentage points in contemporary GDP growth).

Our basic result for the Full Sample data can be found by tracking rows 1 and 2 in Table 3. Table 3, row 1, column 1 describes a contemporaneous, bivariate relationship of  $-0.024$ . As we move to row 2 and examine columns 1–4 the estimated relationship with

the outcome changed to forward growth and with the addition of explanatory variables to control for potential reverse causality, the alternative specifications imply a reduction in the key relationship between public debt and growth by between 50 and 70 percent in the point estimate of the magnitude and from strong statistical significance to insignificance or marginal significance.

The RR1955 data, which closely resemble the full sample, yield similar results as shown in Table 4. For the CMZ, CWR, and WK data, the results in Tables 4 and 5 never include significant negative estimates for the bivariate relationship in either the contemporaneous or forward data. We conclude that the negative relationship between public debt and GDP growth found in Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012) represents artifacts of specification, sample inclusion, and functional form, which we explore in more detail below.

## **3.6 Nonlinearities and Thresholds**

### **3.6.1 Bivariate Relationship between Public debt and Growth**

Although linear regression results are a useful summary of the relationship between public debt and GDP growth, the extant literature has often emphasized non-linearities in the relationship, and in particular identifies important threshold effects in the relationship between public debt-to-GDP ratio and economic growth. For the non-linear analysis, we settled on five-year forward growth as our preferred outcome indicator based on the stability of the linear results reported in Figure 3.

We report results for the Full Sample (FS1956), the Full Sample limited to years after 1970 (FS1970), and the other four papers. Figure 4 shows the bivariate lowess estimates of GDP growth versus public debt/GDP for each dataset. The left panel plots 5-year lagged GDP growth versus current year public debt/GDP, therefore capturing a likely reverse-causal

relationship from growth to public debt. The middle panel of each triptych shows current GDP growth versus current-year public debt-to-GDP ratio. Strikingly, while the contemporaneous relationship is downward sloping across datasets, the slope is generally less pronounced than in the left-hand panels. The fact that contemporary public debt is more strongly associated with past growth rather than current growth is indicative of reverse causality.

The right panel shows future growth, expressed as the annualized average real GDP growth over the succeeding five years versus current-year public debt-to-GDP ratio. Between public debt-to-GDP of 30 percent and 150 percent, the policy relevant range for current debates, the lowest plots indicate a small to zero relationship between public debt and future growth in any of the datasets. In particular, the more recent data in FS1970, CWR, CMZ, and WK, show no relationship.

Although at medium to high levels of public debt there is essentially no relationship between public debt and GDP growth, the relationship between GDP growth and public debt at low levels of public debt varies strongly by dataset. In both the FS and RR1955 analysis, most of the observed 1.5 percentage point decline in GDP growth between 0 and 100 percent of public debt/GDP is, contrary to debt-threshold hypotheses, concentrated in the first 30 percentage points of public debt/GDP. In contrast, in CMZ the relationship between public debt/GDP and GDP growth is sharply positive in the first 30 percentage points of public debt/GDP (and flat thereafter). CWR also shows a modest negative relationship over the lower range (the left tail of the rotated S).

We show the actual data in scatterplots in Figure 8. Using future growth as the outcome variable, the Full Sample data, the RR1955 data, the CMZ data, and the CWR data all show one or more especially fast-growing countries in the medium-high public debt range. In the Full Sample, RR1955, and CWR data, the case is Ireland in the early 1990s, while in the CMZ data, the medium-high-debt, high-growth case is Portugal in the middle 1980s. These potentially anomalous country-years may be particularly important for identification

of turning points in Checherita-Westphal and Rother (2012) and Cecchetti et al. (2011).

### 3.6.2 Semi-Parametric Analysis using a Partial Linear Model

While the bivariate relationships are illustrative, they do not control for the same variables as do the linear regression results. Therefore, we next use a partial linear model to linearly control for these covariates, while allowing a nonparametric relationship between public debt and growth. Results for the Full Sample (FS1956) and the Full Sample after 1970 (FS1970) are shown in Figure 5. These figures show alternative specifications, one with a control for lagged growth and one with country fixed effects. Only FS1956 with country fixed effects shows any indication of a negative relationship between public debt and growth, a roughly linear decline of approximately 1 percentage point of GDP growth over 100 percentage points of public debt-to-GDP ratio. This relationship disappears in the more recent data, FS1970.

Results for all of the datasets are shown in Figures 6 and 7. First, following our linear regressions, we control for recent growth by including 1-lag of GDP growth in the partial linear model shown in Figure 6 and then for country fixed effects in Figure 7.

The lowess curves that control for lagged GDP growth are generally rather flat, indicating little relationship. One exception is CWR where the relationship appears as a shallow rotated-S shape. There is a turning point at roughly 90 percent, but the inverted-U shape at that location is shallow. Accounting for confidence intervals, the estimated function is indistinguishable from a horizontal line. Overall there is little relationship between public debt and future GDP growth once lagged GDP growth is taken into account.

The lowess curves with country fixed effects show substantial variation across datasets. FS1956 and RR1955 show the strongest negative relationship with roughly 1 percentage point of growth lost per 100 percentage points of public debt-to-GDP ratio. There is no indication of a threshold near a public debt-to-GDP ratio of 90 percent. There is no sign of a negative relationship in the FS1970 data. In the case of CMZ there is a sharp increase in



growth as public debt ranges from 16 percent, the lowest value of public debt/GDP in the CMZ data, to 50 percent of GDP followed by a horizontal (non-)relationship. CWR and WK show increasing growth up to a public debt-to-GDP ratio of 100 percent and flat relationship thereafter.

To summarize, the relationship between public debt and forward growth is modest across a variety of specifications. Taken together the semi-parametric plots including country and year fixed effect indicate at most modest reductions in growth which largely disappear in more recent data. Importantly, we find no evidence across various specifications of any clear threshold at elevated levels of public debt, and this is true whether we control for heterogeneity using fixed effects or lagged outcomes. These findings stand in contrast to claims in the four key studies we review here. Therefore, in the next sections we scrutinize the findings of both linear effects and nonlinearities in these paper in greater detail.

### **3.6.3 Replication of Checherita-Westphal and Rother (2012)**

Checherita-Westphal and Rother (2012) report a threshold effect based on a quadratic specification to identify a turning point in the relationship between public debt and GDP growth. The analysis finds the peak of the quadratic to be at a public debt-to-GDP ratio between 90 and 100 percent. It is important to keep in mind what is meant by the “turning point of this concave relationship”. Taken at face value, the implication is that public debt has a growth-enhancing effect up until the turning point. The results reported in the paper imply that a public debt-to-GDP ratio of 60 percent, the maximum public debt level permitted under the Maastricht agreement and subsequent European Stability and Growth pact, reduce growth by roughly one percentage point of annual GDP growth relative to maintaining public debt/GDP at the peak around public debt/GDP of 90 and 100 percent. Another feature of a quadratic is that around the peak the slope is necessarily close to zero, which means that changes in public debt are unlikely to have a large effect on growth.

The lowess plot from the CWR data presents a rotated-S shape, with a trough of annual GDP growth of 2.03 percent at public debt/GDP of 35 percent and a peak of annual GDP growth of 2.73 percent at public debt/GDP of 90 percent. Although the shape is dramatic and the estimate at high public debt/GDP is reasonably precise (in terms of the width of the lowess error bands), the net relationship — both the positive relationship between public debt/GDP of 35 and 90 percent and the negative relationship between public debt/GDP of 90 and 140 — does not amount to much. With relatively close values of annual GDP growth at the trough and peak ( $2.73 - 2.03 = 0.7$  percentage points of GDP growth), there is simply not much room for a strong relationship between public debt and GDP growth. A horizontal line at annual real GDP growth 2.5 would miss some of the nonlinearity, but it would be hard to reject as a characterization of the relationship between public debt and growth in these data.

Visual inspection of the actual scatterplot of CWR data in the CWR panel of Figure 8 indicates a highly visible and potentially influential set of points that trace out a high arc. Thus the actual scatterplot underlying the rotated S-shape of the lowess fit to the CWR data indicates that there are one or two countries with high future growth associated with moderately high public debt. That is, the curve is being pulled up as public debt/GDP approaches 90 percent from below. As we noted above, these outlying cases are Portugal in the middle 1980s and Ireland in the early 1990s.

Focusing on the data points from Ireland, that country's peak growth occurred in the mid-1990s when public debt was between 70 and 80 percent of GDP. We estimate partial linear regressions plots with and without the inclusion of the Ireland in Figure 12. Ireland proves to be fairly influential in generating the rotated-S shape in the relationship, a result that obtains with and without fixed effects. Indeed, without Ireland, the relationship between public debt and growth is essentially flat in the model that controls for lagged growth and modestly upward sloping in the model with fixed country effects.

Ireland has a particularly important effect in the CWR results because the CWR data are limited to twelve countries. The same data for Ireland exert less influence in the Full Sample and the other large datasets simply because they represent a smaller share of the data points. This points to the pitfalls of using a small sample, where inference can be driven by a single influential case.

The results also support the proposition that parametric trend-break tests can be misleading, and so it is important to examine the data directly as well. The nonparametric curve in the CWR sample does turn downward at public debt/GDP above 90 — a fact captured by the maximum of the quadratic in CWR’s regression — but it turns downward from a set of very high growers among countries with moderate-to-high public debt. It is not that growth collapses at public debt/GDP of 90 percent but rather that there are a small number of high growth cases immediately below 90 percent public debt/GDP.

Overall, our re-analysis of the CWR sample suggests that there is little overall relationship between public debt and growth in their sample. There is an indication of a non-monotonic relationship between the variables, which suggests a threshold effect of debt on growth. However, this inference is driven almost entirely due to a combination of a parametric test along with a single influential country (Ireland) in a small sample. Either using more data, or excluding that influential case, suggests there is no such threshold effect.

### **3.7 Sensitivity of years in Reinhart and Rogoff (2010)**

Sensitivity to years was a central issue in the interpretation of the initial Reinhart and Rogoff (2010) finding, with Herndon et al. (2014) noting the particular importance of the treatment of the immediate postwar. In web-published errata to the original paper, Reinhart and Rogoff (2013) bypassed the question of the immediate postwar by selecting 1955 as the starting date for the revised analysis and found a modest negative relationship, without non-linearity, in the contemporaneous relationship between public debt and growth. However, the 1955

starting point substantially predates the starting dates for the other three papers on public debt and growth in the postwar advanced economies (see Tables 1 and 2). In our initial analysis we limited the Reinhart and Rogoff sample to years after 1970 for comparability with the other three papers under analysis, and we found that the negative relationship between public debt and GDP growth largely disappears in the post-1970 data. In this section we assess the sensitivity of the Reinhart and Rogoff results to the specific selection of post-1955 years by comparing the estimated relationship in earlier and later periods.

With the RR1955 data in the linear model, the coefficient on public debt/GDP decline by about one-third to one-half with the change from contemporaneous to future growth and cease to be significant at the 5-percent level (see Table 5). However the point estimate continues to indicate a negative relationship between public debt and future growth in the RR1955 data, and the estimates from the country fixed-effect models using RR1955 data are substantially and significantly negative. Figure 11 explores the source of the negative relationship by splitting the RR1955 data into years through 1965 and years after 1965 with a separate lowess curve for each period.

In Figure 11 with future growth as the outcome variable, the 1955–1965 data show a much more pronounced negative relationship between growth and public debt/GDP than do the post-1965 data. Indeed, post-1965 there is essentially no relationship between public debt and future growth. The RR data stratified by pre- and post-1965 suggest that a handful of countries went from extremely low debt to moderate debt and also had growth slowdowns during 1955-1965. These cases are likely responsible for the negative point estimates in some of the RR results.

To demonstrate that this analysis is not sensitive to the choice of year, Table 8 repeats the linear regression analysis using subsets of the RR1955 data, limited to years after 1960, 1965, and 1970. For every specification, the coefficient declines steadily with the limitation to more recent years. For both the post-1965 and post-1970 subsamples, no specification yields

a statistically significant result and the coefficients are generally small in magnitude.

### 3.8 Replication of Cecchetti et al. (2011)

In the case of Cecchetti et al. (2011), there is a substantial divergence between the large, precise negative estimate of  $-0.0164$  ( $p$ -val= 0.025) in the linear specification of the effect of Government debt on future GDP growth (“Not controlling for banking crises”) reported in their Table 5, row 7, column 2 and our linear estimates in Tables 4 and 5 which were ranged from zero to positive, large, and significant. Indeed in the country fixed-effect specification for forward growth in Table 5, which most closely resemble the Cecchetti et al. (2011) specification reported above, our parameter estimate is  $+0.022$ , i.e., positive, and significant at the 5-percent level (standard error = 0.010). We therefore undertook a more complete replication of Cecchetti et al. (2011)’s more fully specified model. The results are presented in Table 9.

By including country fixed effects and the full set of controls from “standard growth regressions,” we successfully replicated the Cecchetti et al. (2011) result of  $-0.016$  in Table 9, column 2 of the panel titled “Controls Included, Country FE Included.” But note that all estimates in the first three panels — every model without both controls and fixed effects — are zero or positive. That is, the Cecchetti et al. (2011) result indicating a negative effect of public debt on future GDP growth depends precisely on the inclusion of both controls and fixed effects.

Furthermore, Cecchetti et al. (2011) (and we in our best attempt at replication) include among the controls “the log of real per capita GDP at time  $t$  (to capture the ‘catch-up effect’ or conditional convergence of the economy to its steady state)” (p. 159). However, convergence in empirical implementations of Solow models, e.g., Mankiw et al. (1992), means convergence to the income level of the rich countries. In a model with country fixed effects, the inclusion of lagged log level of real per capita GDP effectively models convergence to the

country’s own average income, which is substantially different from Solow-type convergence. Thus the inclusion of once-lagged log level of real per capita GDP is not well motivated by theory. In any case, however, the model should be robust to including more than one lag of the log level of real per capita GDP, that is, robust to a generalization.

Table 9, column 1 of the panel titled “Controls Included, Country FE Included” shows the estimate of the coefficient on public debt/GDP for the CMZ model with country fixed effects and all controls except the lagged log level of real per capita GDP. The estimate of the effect of public debt on GDP growth is +0.018, i.e., positive, and significant at just above the 5-percent level (standard error = 0.009).

As we argued above, a potentially valuable control in these models is the lagged dependent variable, i.e., lagged percent change in real GDP, to capture the reverse-causal effects on public debt of persistent booms or recessions. In column 3 of the same panel we estimate the CMZ model substituting lagged percent change for the lagged log level. Estimation with a control for the lagged change in GDP, i.e.,  $GDP_{t-1} - GDP_{t-2}$ , imposes the restriction of equal, opposite coefficients on lagged and twice-lagged GDP, i.e., on  $GDP_{t-1}$  and  $GDP_{t-2}$ . The point estimate for the effect of public debt on GDP growth is again positive, 0.016, although significant only at the 10-percent level (standard error = 0.010). Finally in column 4, we include  $GDP_{t-1}$  and  $GDP_{t-2}$  separately, which nests both the CMZ model (coefficient on  $GDP_{t-2}$  restricted to zero) and our lagged dependent variable model (coefficient on  $GDP_{t-2}$  restricted to be equal and opposite to the coefficient on  $GDP_{t-1}$ ). In column 4, the nesting model, the effect of public debt on GDP growth is estimated at zero with substantial precision.<sup>13</sup>

A key CMZ result thus appears to be highly fragile with respect to alternative specifications. Only the particular combination of country fixed effects, specific control variables, and once-

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<sup>13</sup>All of these specifications, the lagged log level of real per capita GDP and the lagged percent change in real per capita GDP, potentially violate the strict exogeneity requirement in a fixed-effect model.

lagged log level of real per capita GDP yields a negative estimate for the relationship between public debt and future GDP growth. All reasonable alternative specifications, including the modest generalization of including first and second lags of log level real per capita GDP, generate substantively different results, and none of these show a negative relationship between public debt and future GDP growth.

## 4 Conclusions

Our re-examination of the relationship between public debt and growth in advanced economies finds little evidence to suggest a substantial, causal negative relationship. First, we demonstrate that there is strong indication of a reverse causal relationship from GDP growth to public debt. Indeed, contemporaneous public debt is more strongly correlated with GDP growth in the preceding five years than the growth in the five years in future, suggesting that weak GDP growth probably causes higher public debt. Possible mechanisms include higher deficits, i.e., reduced tax collection and increased public expenditure as well as the mechanical explanation of slow growth in the denominator of public debt/GDP.

We then use a number of standard tools to account for reverse causality in the bivariate regression of growth on public debt: comparing the response of forward and lagged growth rates; instrumenting current public debt/GDP with its lag to learn from chronic debtors; and controlling for lagged GDP growth rates to pick up the lingering effects of booms or recessions. These simple methods of accounting for reverse causality diminish the size of the association by between 50 and 70 percent, with these specifications yielding in most cases coefficients indistinguishable from zero. Temporal sequencing of public debt and GDP growth does not in general support the proposition that higher public debt reduces GDP growth.

Non- and semi-parametric plots provide visual confirmation that the relationship between public debt/GDP and GDP growth is essentially flat for public debt/GDP exceeding 30

percent when we (1) use forward growth rates, (2) control for past GDP growth, or both. There is little evidence of a lower growth rate when public debt exceeds 90 percent of GDP when using the 5-year forward average growth rate.

These results stand in sharp contrast with four key papers in the literature that we review, though consistent with the findings in Panizza and Presbitero (2014). First, despite the remarkably consistent finding of an important growth threshold around 90 percent with alternative methods employed by Reinhart and Rogoff (2010), Cecchetti et al. (2011), Checherita-Westphal and Rother (2012), and Woo and Kumar (2015), our non- and semi-parametric lowess plots (and the full scatterplots) strongly suggest that there is no threshold at elevated levels of public debt.

In the cases of Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012), parametric tests for a threshold identified breaks around a public debt-to-GDP ratio of 90 percent. But these papers insufficiently explored the character of these breaks. In the case of Checherita-Westphal and Rother (2012) the lowess plots show that the threshold slightly above 90 percent public debt/GDP is in fact the peak of a shallow rotated S, although the curve is so shallow as to have no substantive importance. Moreover, the role of a highly influential case (Ireland) in a small sample of 12 countries, along with peculiarities of a quadratic specification, produces a misleading and fragile suggestion of a threshold in the Checherita-Westphal and Rother (2012) sample.

In the case of both Cecchetti et al. (2011) and Woo and Kumar (2015) the estimated spline specification appears not to correspond with the authors' intent. In Cecchetti et al. (2011) we find no robustness of the result in the linear specification. The negative relationship appears only in a very particular specification, which does not have particular prior claims. Modest variants yield altogether different results. Lowess plots using the data for Cecchetti et al. (2011) indicate that GDP growth is flat or even increasing in public debt/GDP. In the case of Woo and Kumar (2015), the lowess specification does not indicate any important



break around 90 percent.

We conclude with three main points. First, employing a range of methods, including temporal sequencing, linear models, and semi-parametric models, gives some robustness in the effort to measure the relationship between public debt and growth. The overall finding is that the effect of public debt on GDP growth is small. For the data beginning in 1955, a 100-point increase in the public debt-to-GDP ratio causes no more than a  $-0.8$  percentage point decrease in annual real GDP growth, without statistical significance in most specifications. The effect of public debt on growth is essentially zero after 1970.

Second, there is an important inconsistency between the conclusions of the papers in the literature which appear to uniformly suggest a negative causal relationship. If we simply consider a bivariate relationship between forward growth and public debt, there is essentially no relationship between these two since 1970, the period considered in much of the recent relationship. This means that the simple bivariate findings of a negative relationship in Reinhart and Rogoff (2010) simply do not show up in the time period or samples of Cecchetti et al. (2011), Woo and Kumar (2015), and Checherita-Westphal and Rother (2012). The conclusions in those paper of a negative relationship require either use of very particular set of controls, or questionable use of tests of trend breaks which are driven by small samples, outliers and parametric choices.

Third, many policy decisions to confront public debt via austerity have hinged on the presumption of a threshold. There is no evidence of a public debt threshold above which growth is substantially reduced in any of the data, using any method. Our findings underscore the importance of looking at the data themselves rather than relying on opaque parametric tests to derive conclusions about how public debt affects growth.

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**Table 1:** *Papers on public debt and economic growth*

Dataset, Publication, and Data Source	Peer Reviewed	Countries	Years
<b>RR1955</b> Reinhart and Rogoff (2010) <i>American Economic Review Papers &amp; Proceedings</i> and Reinhart and Rogoff (2013) Errata: Growth in a Time of Debt . Data: authors.	No	20	1955–2009
<b>CMZ</b> Cecchetti et al. (2011) Economic Symposium Conference Proceedings, Federal Reserve Bank of Kansas City; and BIS Working Paper No. 352. Data: OECD.	No	18	1980–2009
<b>CWR</b> Checherita-Westphal and Rother (2012) <i>European Economic Review</i> . Data: AMECO.	Yes	12	1970–2008
<b>WK</b> Kumar and Woo (2010) and Woo and Kumar (2015) IMF Working Paper and <i>Economica</i> . Data: PWT 7.0, IMF WEO, IMF Public Finances in Modern History Fiscal Prudence and Profligacy Database.	Yes	21	1970–2008

*Notes.* The table reports publication information, listed data sources, and the count of countries and years for each of the papers. Where applicable, both working paper and published versions are cited.

**Table 2:** *Countries included, by data source*

	Full Sample (1956–2003)	RR1955 (1955–2003)	CMZ (1985–2003)	CWR (1975–2003)	WK (1975–2003)
Australia	✓	✓	✓		✓
Austria	✓	✓	✓	✓	✓
Belgium	✓	✓	✓	✓	✓
Canada	✓	✓	✓		✓
Denmark	✓	1976–79	✓		✓
Finland	✓	✓	✓	✓	
France	✓	1973–77	✓	1970–76	✓
Germany	✓	✓	✓	✓	✓
Greece	✓	1955–69	✓	✓	✓
Iceland	✓				✓
Ireland	✓	✓		✓	
Italy	✓	✓	✓	✓	✓
Japan	✓	✓	✓		✓
Luxembourg				✓	
Netherlands	✓	1955–56	✓	1970–74	✓
New Zealand	✓	✓			
Norway	✓	✓	✓		
Portugal	✓	✓	✓	1970–72	✓
Spain	✓	1959–79	✓	✓	✓
Sweden	✓	✓	✓		✓
Switzerland	✓				✓
UK	✓	✓	✓		✓
USA	✓	✓	✓		✓

*Notes.* The table reports the countries and years used in our analysis. ✓ indicates availability of data for the full set of years listed at the top of the column; otherwise a list of years reports missing data for the country.

**Table 3:** *Public Debt and Growth: Regression Results for Full Sample*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>FS1956: Current Growth</b>							
Public Debt/GDP	-0.021*** (0.004)	-0.014*** (0.004)	-0.007** (0.003)	-0.010 (0.007)	-0.007 (0.005)	-0.025*** (0.006)	-0.019*** (0.007)
Observations	1056	1056	1034	946	946	1056	946
<b>FS1956: Future Growth</b>							
Public Debt/GDP	-0.014*** (0.005)	-0.009* (0.004)	-0.006 (0.004)	-0.008 (0.007)	-0.007 (0.007)	-0.016*** (0.006)	-0.016* (0.009)
Observations	1056	1056	1034	946	946	1056	946
<b>FS1970: Current Growth</b>							
Public Debt/GDP	-0.009** (0.003)	-0.005* (0.003)	-0.001 (0.002)	0.003 (0.006)	0.001 (0.004)	-0.019** (0.007)	0.002 (0.013)
Observations	748	748	726	638	638	748	638
<b>FS1970: Future Growth</b>							
Public Debt/GDP	0.002 (0.004)	0.001 (0.003)	0.002 (0.004)	0.003 (0.008)	0.003 (0.007)	0.000 (0.007)	0.008 (0.016)
Observations	748	748	726	638	638	748	638
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

*Notes.* Current growth and future 5-year average growth in real per capita GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . FS1956 refers to the Full Sample of countries for 1956–2003; FS1970 refers to the Full Sample of countries for 1970–2003.

**Table 4: Public Debt and Current Growth: Regression Results by Dataset**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>FS1956</b>							
Public Debt/GDP	-0.021*** (0.004)	-0.014*** (0.004)	-0.007** (0.003)	-0.010 (0.007)	-0.007 (0.005)	-0.025*** (0.006)	-0.019*** (0.007)
Observations	1056	1056	1034	946	946	1056	946
<b>RR1955</b>							
Public Debt/GDP	-0.021*** (0.006)	-0.016*** (0.005)	-0.009** (0.004)	-0.010 (0.009)	-0.007 (0.006)	-0.025*** (0.007)	-0.021*** (0.007)
Observations	931	931	908	824	822	931	824
<b>CWR</b>							
Public Debt/GDP	-0.004 (0.005)	-0.006 (0.005)	-0.002 (0.004)	0.002 (0.010)	0.002 (0.005)	-0.012 (0.018)	0.039 (0.026)
Observations	335	335	323	275	275	335	275
<b>CMZ</b>							
Public Debt/GDP	-0.004 (0.004)	-0.003 (0.004)	0.001 (0.002)	0.003 (0.006)	0.002 (0.003)	0.007 (0.014)	0.173 (0.119)
Observations	342	342	324	252	252	342	252
<b>WK</b>							
Public Debt/GDP	0.000 (0.003)	-0.001 (0.003)	0.000 (0.003)	0.001 (0.004)	0.000 (0.003)	-0.002 (0.012)	0.022 (0.022)
Observations	522	522	504	432	432	522	432
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

*Notes.* Current growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \* p<0.1, \*\* p<0.05, \*\*\* p<0.01. For references to the datasets see Section 2.3.

**Table 5: Public Debt and Future Growth: Regression Results by Dataset**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>FS1956</b>							
Public Debt/GDP	-0.014*** (0.005)	-0.009* (0.004)	-0.006 (0.004)	-0.008 (0.007)	-0.007 (0.007)	-0.016*** (0.006)	-0.016* (0.009)
Observations	1056	1056	1034	946	946	1056	946
<b>RR1955</b>							
Public Debt/GDP	-0.013* (0.007)	-0.011* (0.006)	-0.007 (0.006)	-0.004 (0.010)	-0.003 (0.009)	-0.019*** (0.007)	-0.012 (0.009)
Observations	929	929	905	825	822	929	825
<b>CWR</b>							
Public Debt/GDP	0.004 (0.008)	-0.002 (0.007)	0.001 (0.007)	0.004 (0.012)	0.004 (0.009)	0.018 (0.015)	0.056*** (0.015)
Observations	335	335	323	275	275	335	275
<b>CMZ</b>							
Public Debt/GDP	0.003 (0.005)	-0.000 (0.004)	-0.000 (0.004)	-0.002 (0.005)	-0.002 (0.005)	0.019* (0.010)	0.074* (0.040)
Observations	342	342	324	252	252	342	252
<b>WK</b>							
Public Debt/GDP	0.002 (0.004)	-0.001 (0.003)	-0.001 (0.003)	-0.001 (0.004)	-0.001 (0.004)	0.006 (0.011)	0.016 (0.021)
Observations	522	522	504	432	432	522	432
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

*Notes.* Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \* p<0.1, \*\* p<0.05, \*\*\* p<0.01. For references to the datasets see Section 2.3.



**Table 6: First-Stage Results for IV Specifications**

	Current Growth			5 Year Future Growth		
	(1)	(2)	(3)	(4)	(5)	(6)
<b>FS1956</b>						
L5.Public Debt/GDP	0.879*** (0.047)	0.873*** (0.048)	0.837*** (0.064)	0.879*** (0.047)	0.873*** (0.048)	0.837*** (0.064)
Observations	946	946	946	946	946	946
F-Stat	347	329	172	347	329	172
<b>RR1955</b>						
L5.Public Debt/GDP	0.885*** (0.038)	0.877*** (0.039)	0.865*** (0.048)	0.886*** (0.038)	0.877*** (0.039)	0.865*** (0.048)
Observations	824	822	824	825	822	825
F-Stat	542	501	326	543	501	326
<b>CWR</b>						
L5.Public Debt/GDP	0.983*** (0.046)	0.984*** (0.022)	0.735*** (0.185)	0.983*** (0.046)	0.984*** (0.022)	0.735*** (0.185)
Observations	275	275	275	275	275	275
F-Stat	456	1969	16	456	1969	16
<b>CMZ</b>						
L5.Public Debt/GDP	0.893*** (0.070)	0.899*** (0.059)	0.260* (0.155)	0.893*** (0.070)	0.899*** (0.059)	0.260* (0.155)
Observations	252	252	252	252	252	252
F-Stat	165	233	3	165	233	3
<b>WK</b>						
L5.Public Debt/GDP	1.000*** (0.057)	1.002*** (0.058)	0.574*** (0.140)	1.000*** (0.057)	1.002*** (0.058)	0.574*** (0.140)
Observations	432	432	432	432	432	432
F-Stat	303	299	17	303	299	17
Year FE	Y	Y	Y	Y	Y	Y
Country FE			Y			Y

*Notes.* Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . For references to the datasets see Section 2.3.

**Table 7:** *Public Debt and Future Growth: Regression Results for Expanded Sample of High Income OECD Countries*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>HI1955</b>							
Public Debt/GDP	-0.016*** (0.005)	-0.014** (0.006)	-0.011** (0.005)	-0.015* (0.008)	-0.013** (0.007)	-0.010 (0.007)	-0.008 (0.010)
Observations	1238	1238	1206	1079	1078	1238	1079
<b>HIXEE1955</b>							
Public Debt/GDP	-0.015*** (0.006)	-0.011* (0.006)	-0.008* (0.005)	-0.012 (0.008)	-0.011 (0.007)	-0.010 (0.007)	-0.007 (0.010)
Observations	1178	1178	1152	1049	1048	1178	1049
<b>HI1970</b>							
Public Debt/GDP	-0.007 (0.006)	-0.011* (0.006)	-0.008 (0.005)	-0.010 (0.008)	-0.009 (0.007)	0.007 (0.007)	0.019 (0.015)
Observations	904	904	872	745	744	904	745
<b>HIXEE1970</b>							
Public Debt/GDP	-0.004 (0.007)	-0.007 (0.007)	-0.004 (0.005)	-0.006 (0.009)	-0.005 (0.008)	0.007 (0.007)	0.020 (0.016)
Observations	844	844	818	715	714	844	715
Year FE		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y		Y		
IV: 5yr Lag Debt				Y	Y		Y
Country FE						Y	Y

*Notes.* Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . For references to the datasets see Section 2.3.

**Table 8:** *Public Debt and Future Growth: Regression Results for Different Subsamples of RR1955*

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>Year &gt;= 1955</b>							
Public Debt/GDP	-0.013*	-0.011*	-0.007	-0.003	-0.004	-0.012	-0.019***
	(0.007)	(0.006)	(0.006)	(0.009)	(0.010)	(0.009)	(0.007)
Observations	929	929	905	822	825	825	929
<b>Year &gt;= 1960</b>							
Public Debt/GDP	-0.011	-0.007	-0.005	-0.001	-0.002	-0.010	-0.016**
	(0.007)	(0.006)	(0.006)	(0.010)	(0.010)	(0.009)	(0.007)
Observations	841	841	818	734	737	737	841
<b>Year &gt;= 1965</b>							
Public Debt/GDP	-0.008	-0.005	-0.003	0.004	0.003	-0.002	-0.014*
	(0.006)	(0.006)	(0.006)	(0.010)	(0.011)	(0.010)	(0.008)
Observations	751	751	728	644	647	647	751
<b>Year &gt;= 1970</b>							
Public Debt/GDP	-0.002	-0.003	-0.002	0.004	0.003	0.003	-0.009
	(0.006)	(0.006)	(0.006)	(0.010)	(0.011)	(0.012)	(0.009)
Observations	661	661	638	556	559	559	661
Year Fixed Effects		Y	Y	Y	Y	Y	Y
Control: 1yr Lag Gwth			Y	Y			
IV: 5yr Lag Debt				Y	Y	Y	
Country Fixed Effects						Y	Y

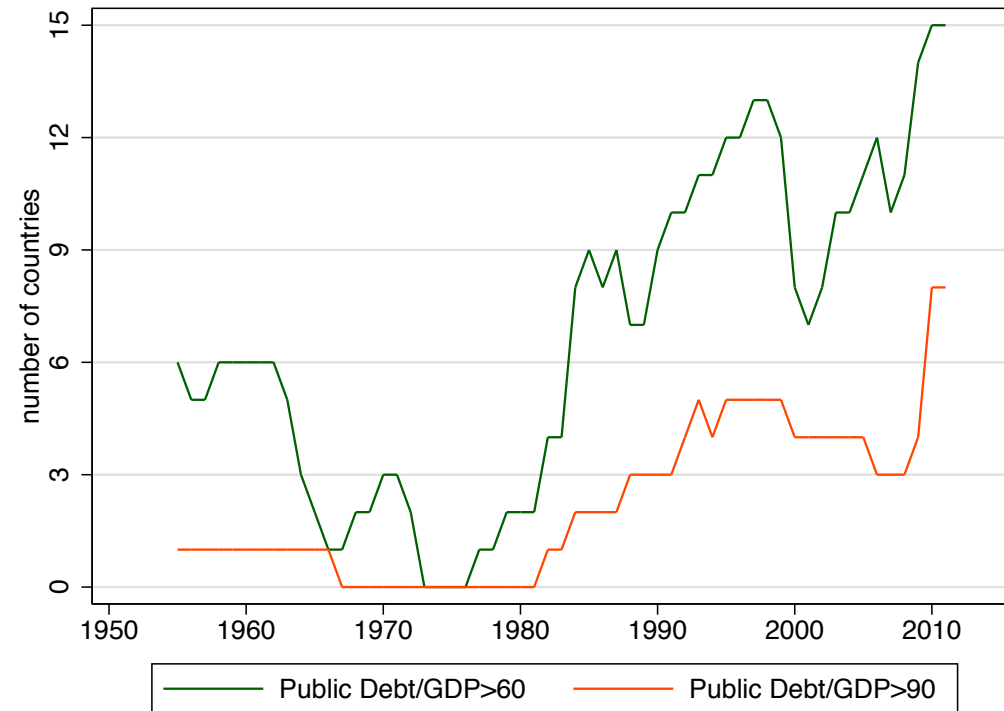
*Notes.* Future 5-year average growth in real GDP explained by the public debt-to-GDP ratio. For references to the datasets see Section 2.3. Robust standard errors, clustered by country, in parentheses below parameter estimates. Significance levels are as follows: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

**Table 9:** *Replication of Regression Results in Cecchetti et al. (2011)*

	(1)	(2)	(3)	(4)
<hr/>				
<u>Controls Excluded, Country FE Excluded</u>				
Public Debt/GDP	-0.000 (0.004)	-0.001 (0.004)	-0.000 (0.004)	-0.000 (0.004)
Observations	342	342	324	306
<hr/>				
<u>Controls Excluded, Country FE Included</u>				
Public Debt/GDP	0.019* (0.010)	-0.007 (0.012)	0.020* (0.010)	-0.000 (0.004)
Observations	342	342	324	306
<hr/>				
<u>Controls Included, Country FE Excluded</u>				
Public Debt/GDP	0.000 (0.006)	-0.002 (0.005)	0.001 (0.006)	-0.000 (0.004)
Observations	342	342	324	306
<hr/>				
<u>Controls Included, Country FE Included</u>				
Public Debt/GDP	0.009 (0.008)	-0.016*** (0.006)	0.008 (0.007)	-0.000 (0.004)
Observations	342	342	324	306
Year FE	Y	Y	Y	Y
1-Lag GDP		Y		Y
2 Lags of GDP				Y
1-Lag Growth of GDP			Y	

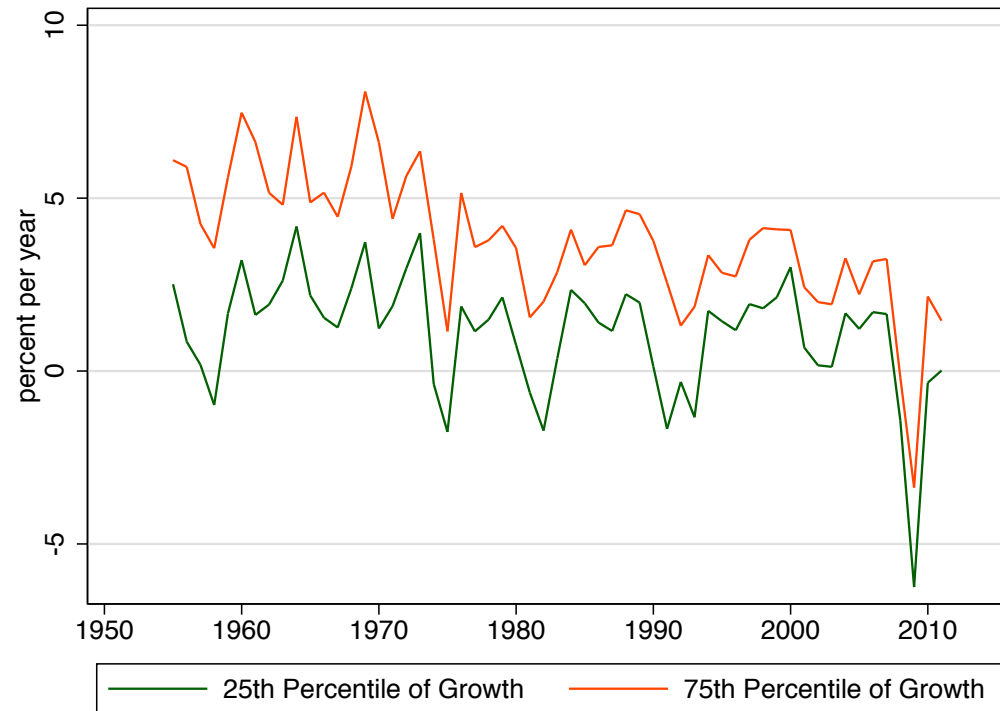
*Notes.* Future 5-year average growth in real per capita GDP explained by the public debt-to-GDP ratio. Additional controls include the variables used by CMZ: savings, growth of population, years of schooling, trade openness, the rate of inflation, dependency ratio, and liquid liabilities. Robust standard errors, clustered by country, appear in parentheses below parameter estimates. Significance levels are as follows: \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ . Standard errors in parentheses.

**Figure 1:** *Prevalence of high public debt, Full Sample*



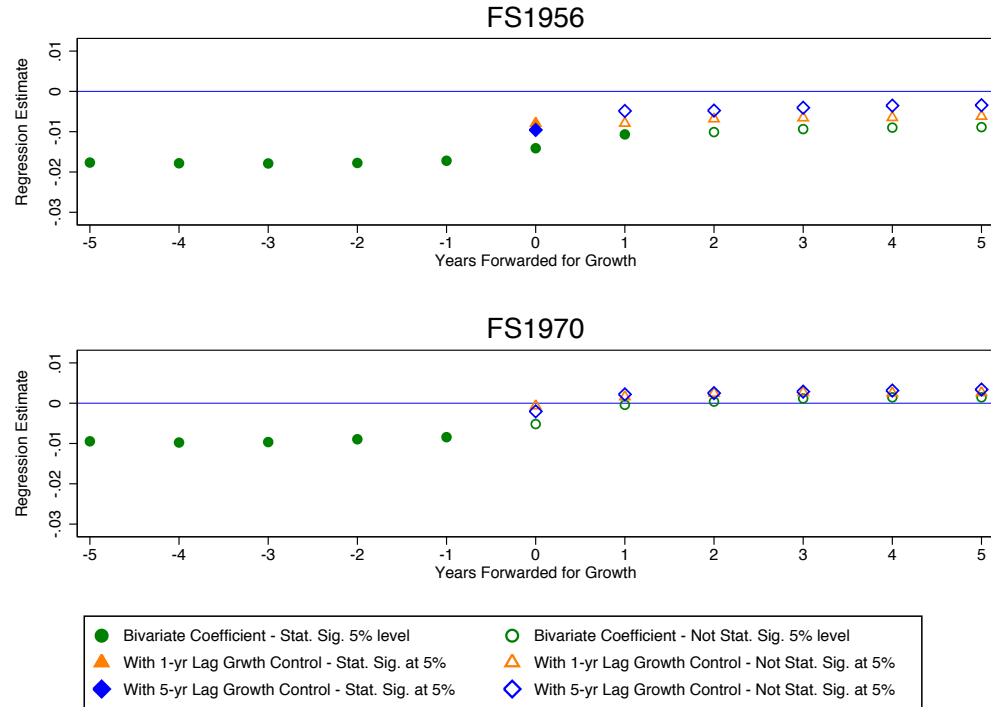
*Notes.* The figure shows the count of countries out of the 22 countries in the Full Sample with public debt-to-GDP ratio above 60 percent or above 90 percent between 1955 and 2010.

**Figure 2:** *Interquartile range of real per capita GDP growth, Full Sample*



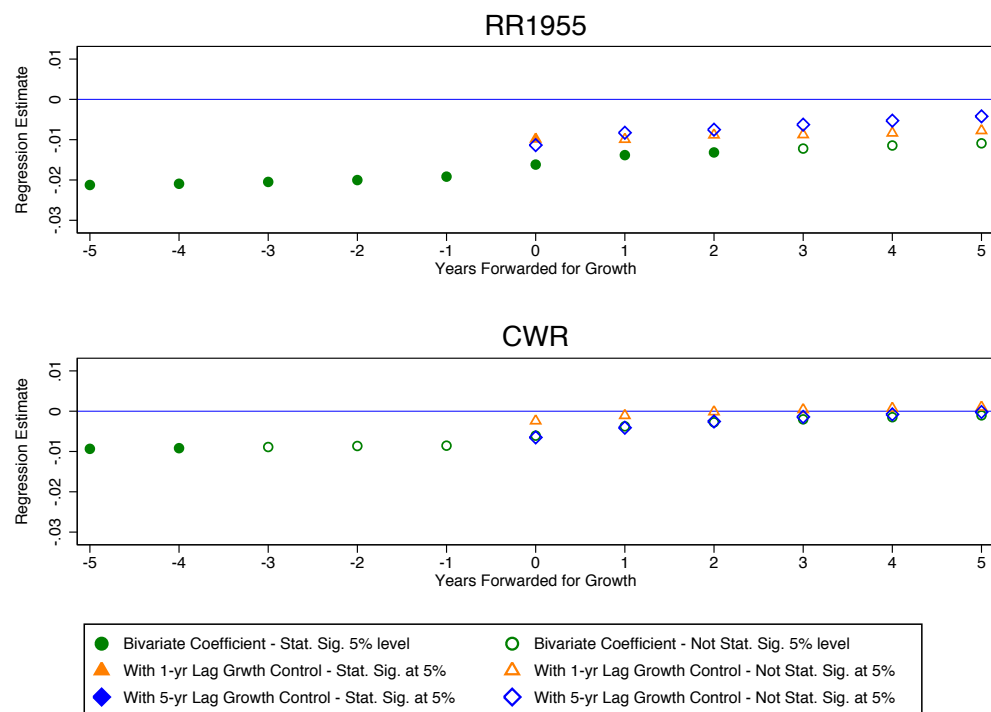
*Notes.* The figure shows the interquartile range of real per capita GDP growth among the 22 countries in the Full Sample between 1955 and 2010.

**Figure 3:** *Coefficient on Public Debt-to-GDP Ratio with Alternative Windows*



*Notes.* The plot shows coefficients from regressions of real GDP growth on current public debt when the time window for averaging GDP growth varies from  $-5$  to  $+5$  years. When the dependent variable is contemporaneous or forward growth, alternative controls for lagged growth include no control and controls for 1- and 5-year lagged growth. All specifications include year dummies. Filled markers indicate statistical significance at the 5 percent level with country-clustered standard errors. For references to the datasets see Section 2.3.

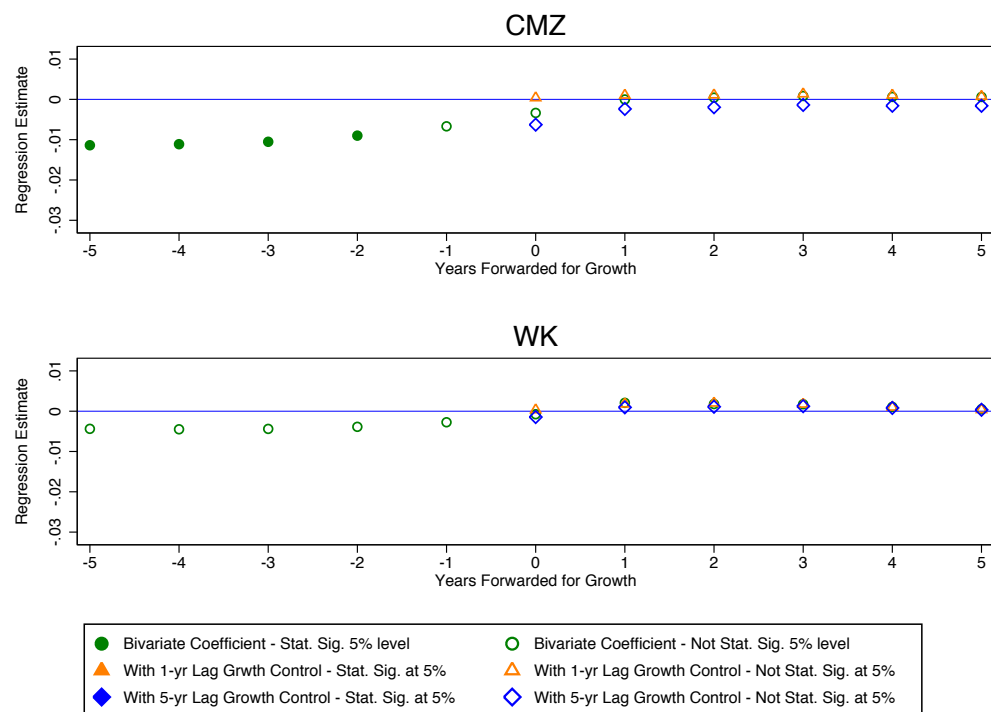
**Figure 3:** *Coefficient on Public Debt-to-GDP Ratio with Alternative Windows (continued)*



*Notes.* The plot shows coefficients from regressions of real GDP growth on current public debt when the time window for averaging GDP growth varies from  $-5$  to  $+5$  years. When the dependent variable is contemporaneous or forward growth, alternative controls for lagged growth include no control and controls for 1- and 5-year lagged growth. All specifications include year dummies. Filled markers indicate statistical significance at the 5 percent level with country-clustered standard errors. For references to the datasets see Section 2.3.

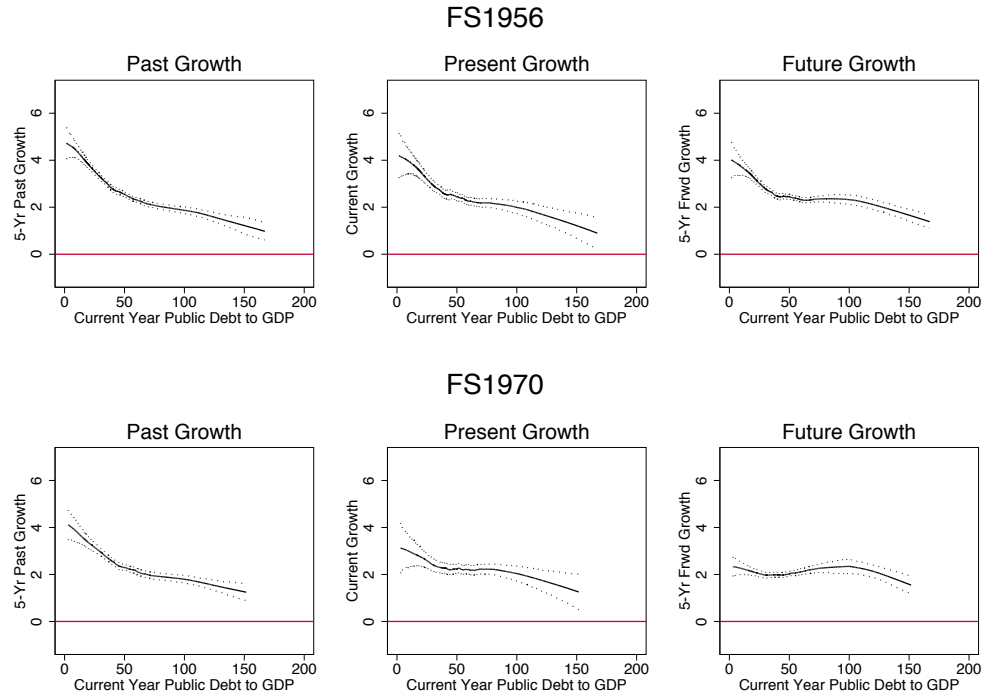


**Figure 3:** *Coefficient on Public Debt-to-GDP Ratio with Alternative Windows (continued)*



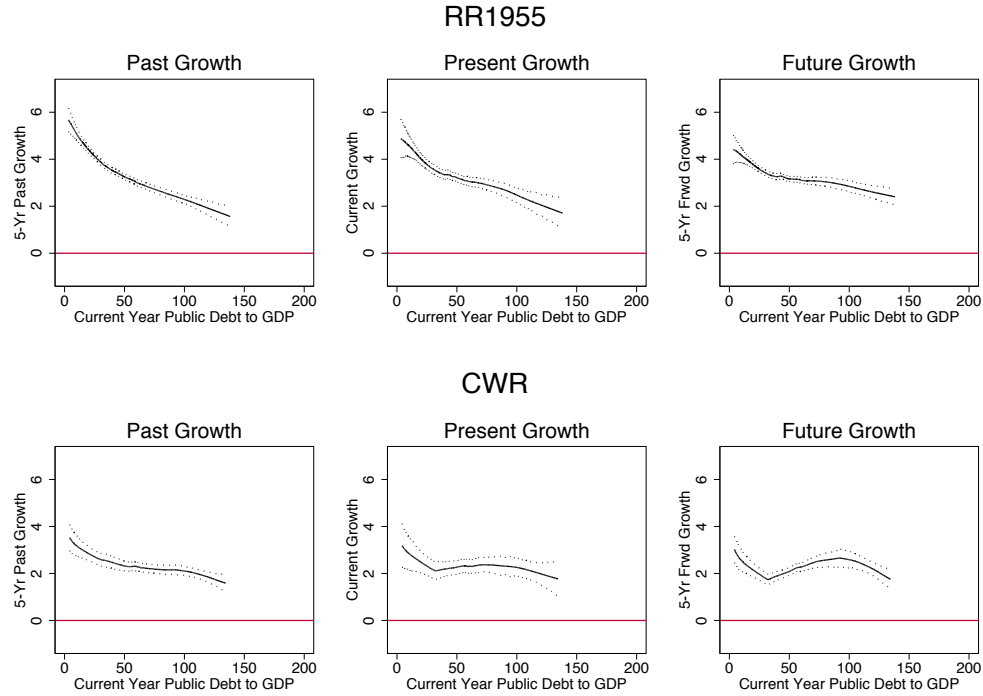
*Notes.* The plot shows coefficients from regressions of real GDP growth on current public debt when the time window for averaging GDP growth varies from  $-5$  to  $+5$  years. When the dependent variable is contemporaneous or forward growth, alternative controls for lagged growth include no control and controls for 1- and 5-year lagged growth. All specifications include year dummies. Filled markers indicate statistical significance at the 5 percent level with country-clustered standard errors. For references to the datasets see Section 2.3.

**Figure 4:** *Public Debt-Growth Lowess Plots*



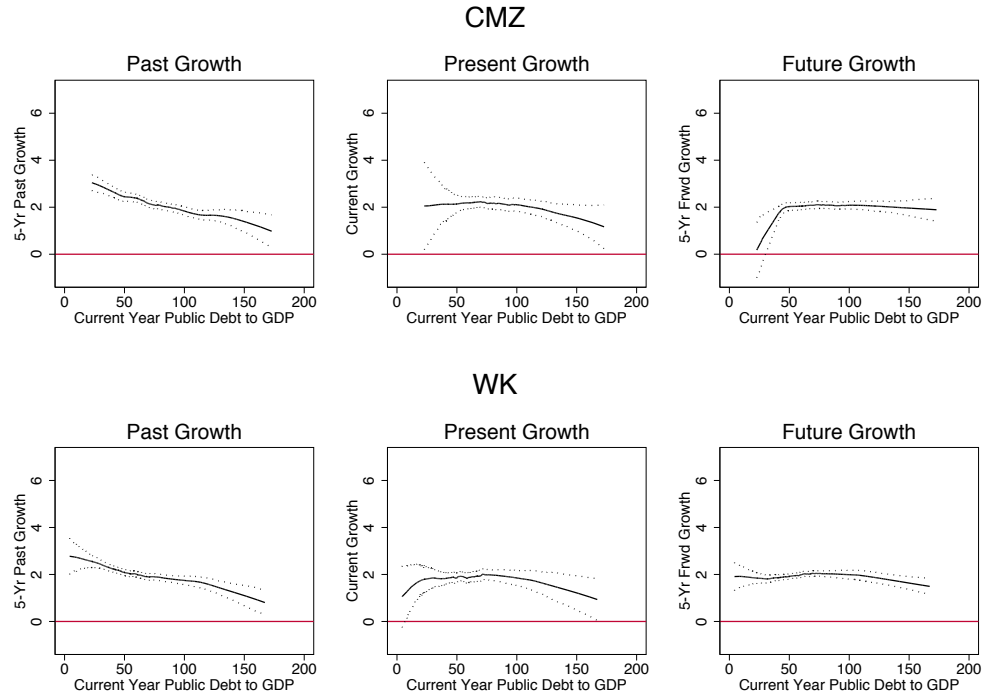
*Notes.* Lowess plots for the bivariate relationship between public debt and growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 4:** *Public Debt-Growth Lowess Plots (continued)*



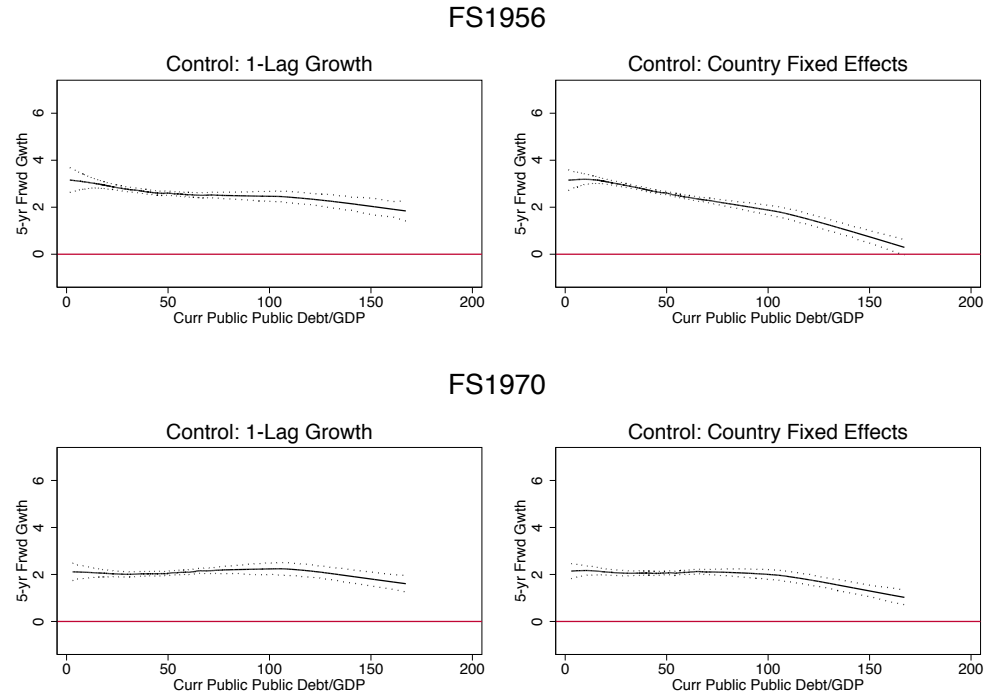
*Notes.* Lowess plots for the bivariate relationship between public debt and growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 4:** *Public Debt-Growth Lowess Plots (continued)*



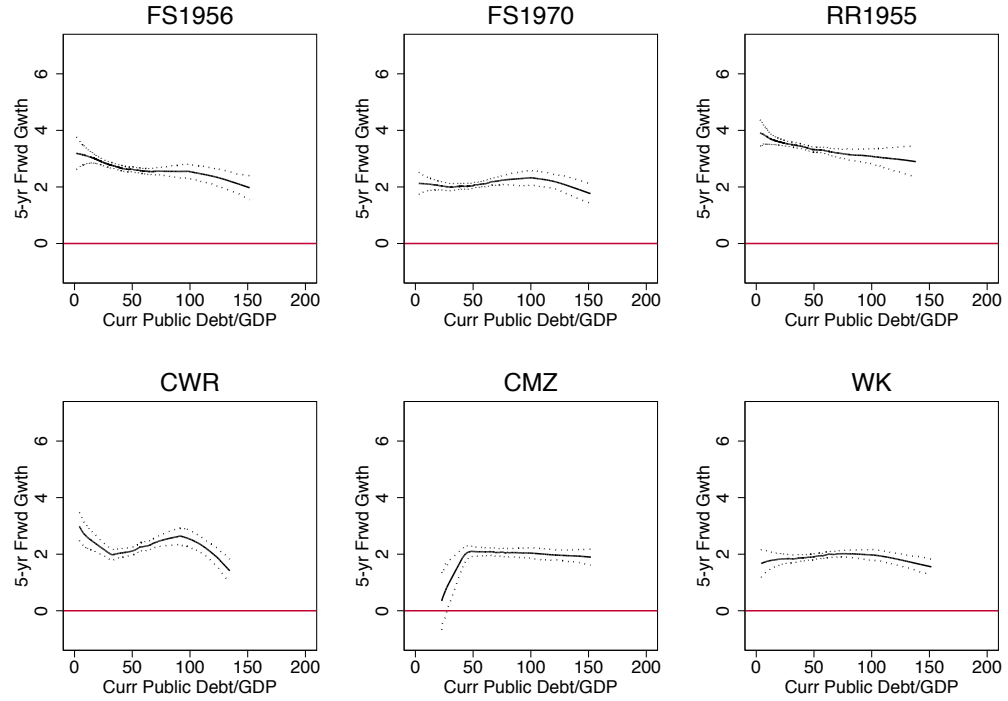
*Notes.* Lowess plots for the bivariate relationship between public debt and growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 5:** *Public Debt-Future Growth Partial Linear Regression*



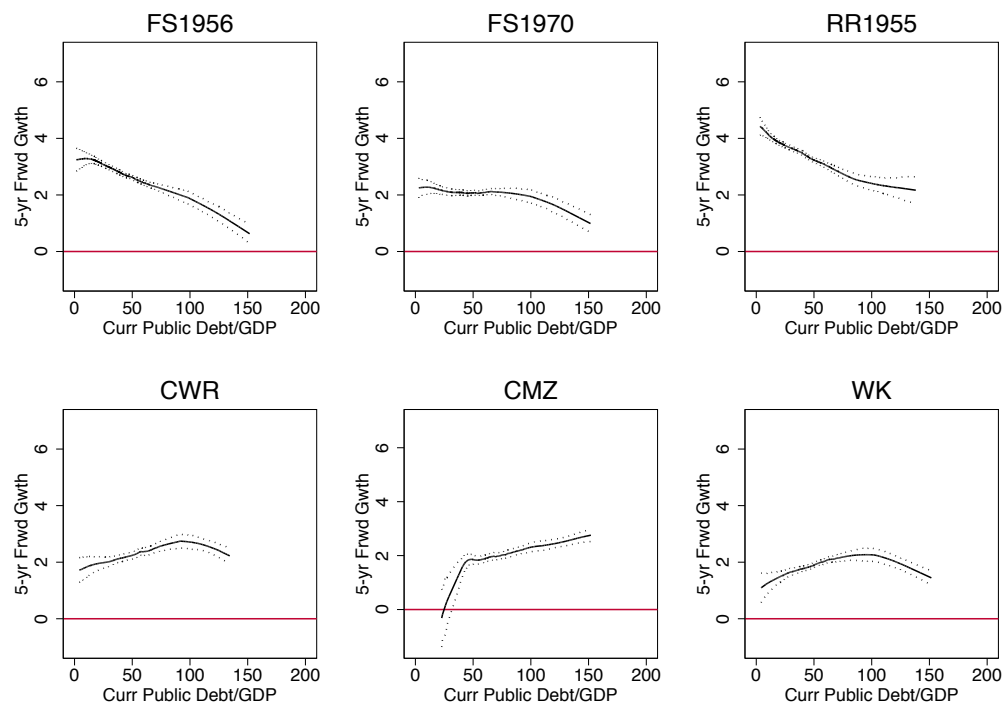
*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth. Left panel shows the model with 1-lagged growth and year fixed effects; right panel shows the model with country and year fixed effects. FS1956 refers to the Full Sample 1956--2003; FS1970 refers to the Full Sample 1970--2003. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 6:** *Public Debt-Future Growth Partial Linear Regression Controlling for Lagged Growth*



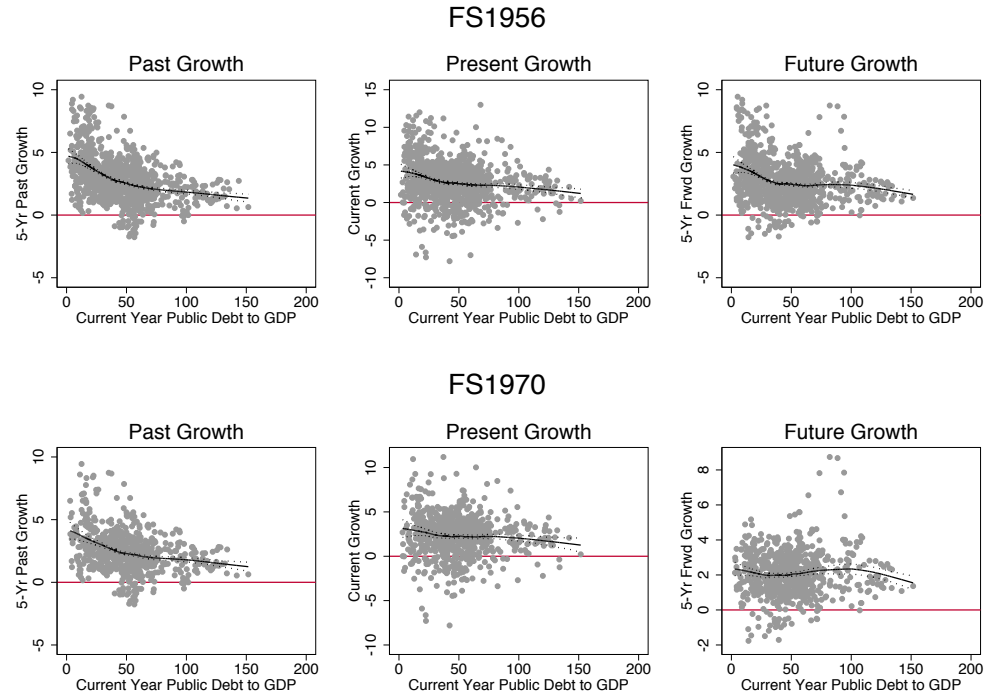
*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 7:** *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects*



*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

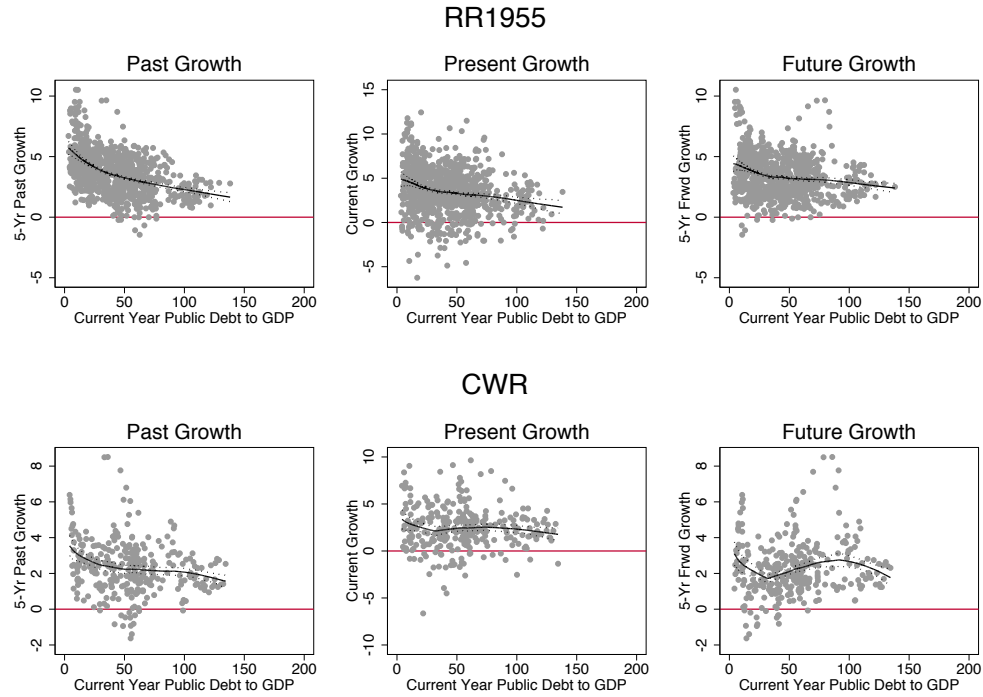
**Figure 8:** *Public Debt-Growth Lowess with Scatterplot*



*Notes.* Lowess plots for the bivariate relationship between public debt and real GDP growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

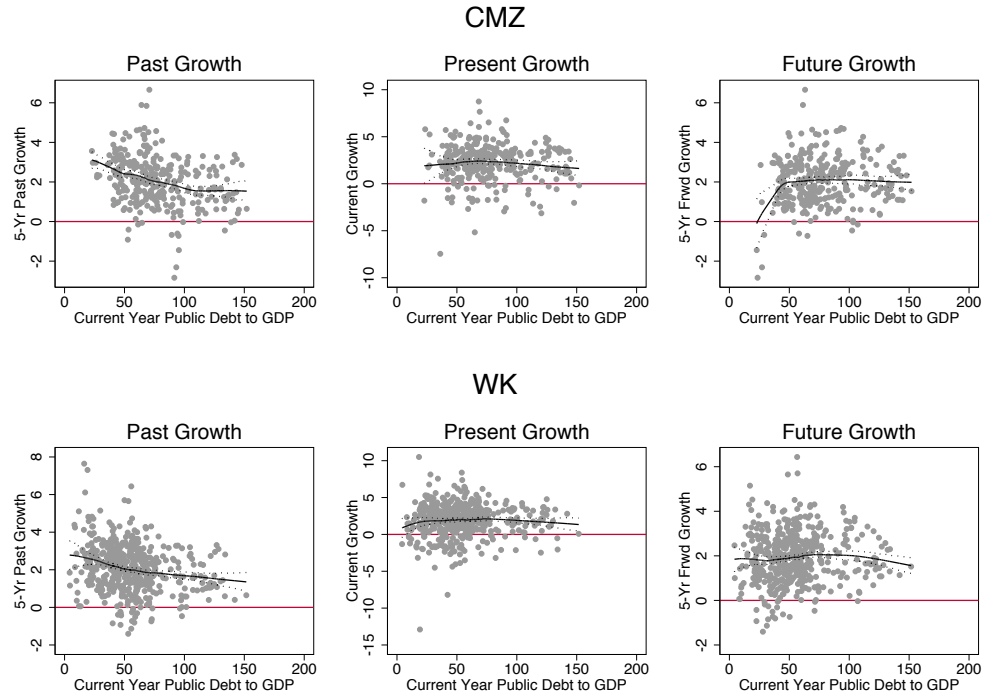


**Figure 8:** *Public Debt-Growth Lowess with Scatterplot (continued)*



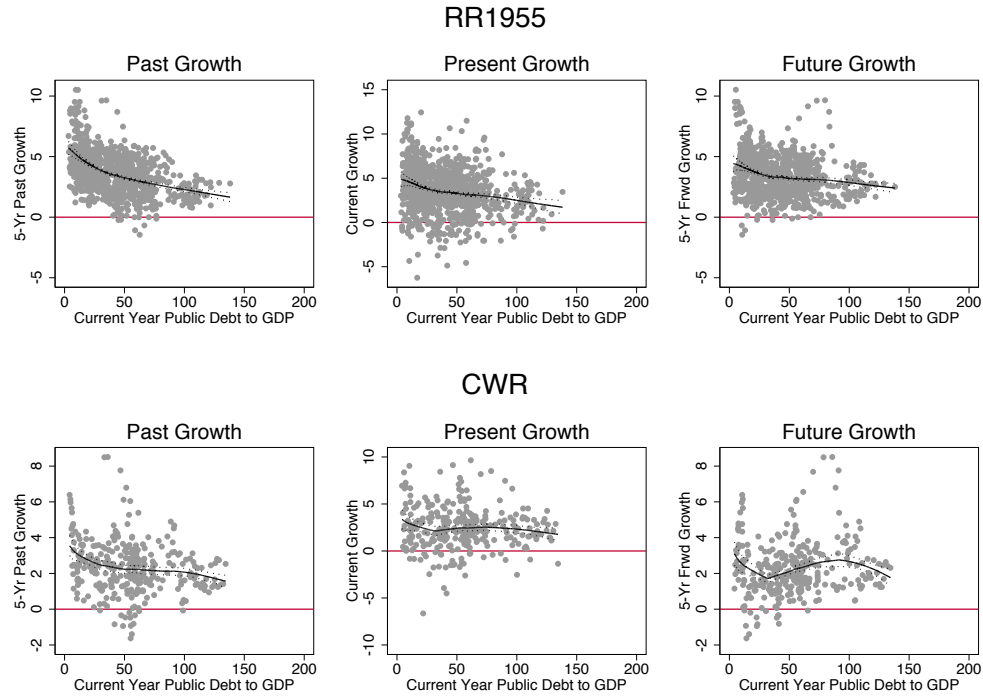
*Notes.* Lowess plots for the bivariate relationship between public debt and real GDP growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 8:** *Public Debt-Growth Lowess with Scatterplot (continued)*



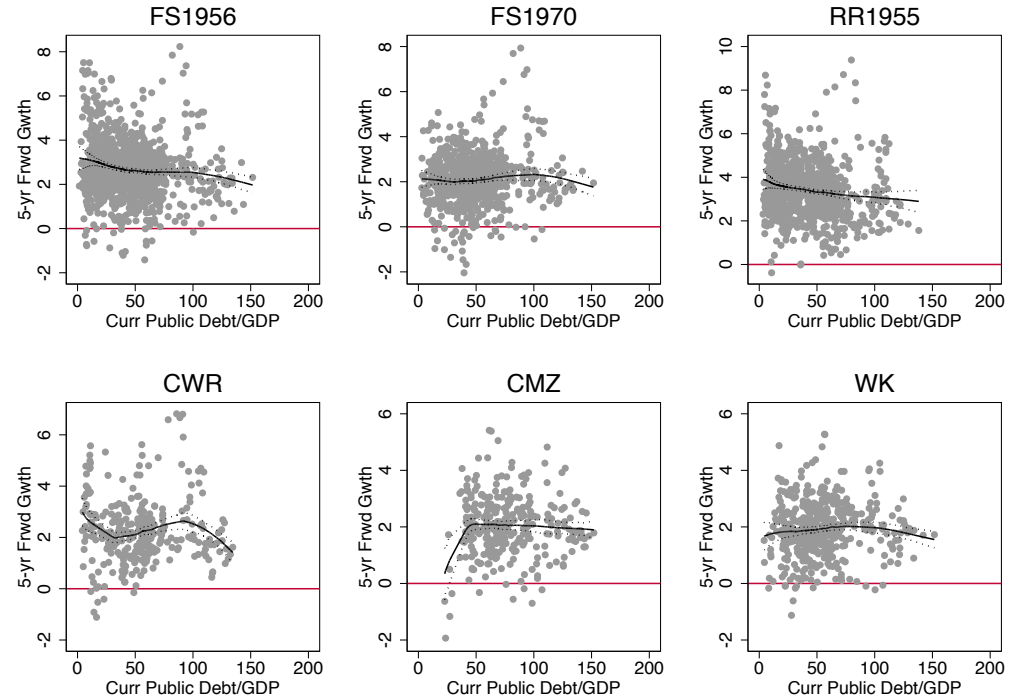
*Notes.* Lowess plots for the bivariate relationship between public debt and real GDP growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 8:** *Public Debt-Growth Lowess with Scatterplot (continued)*



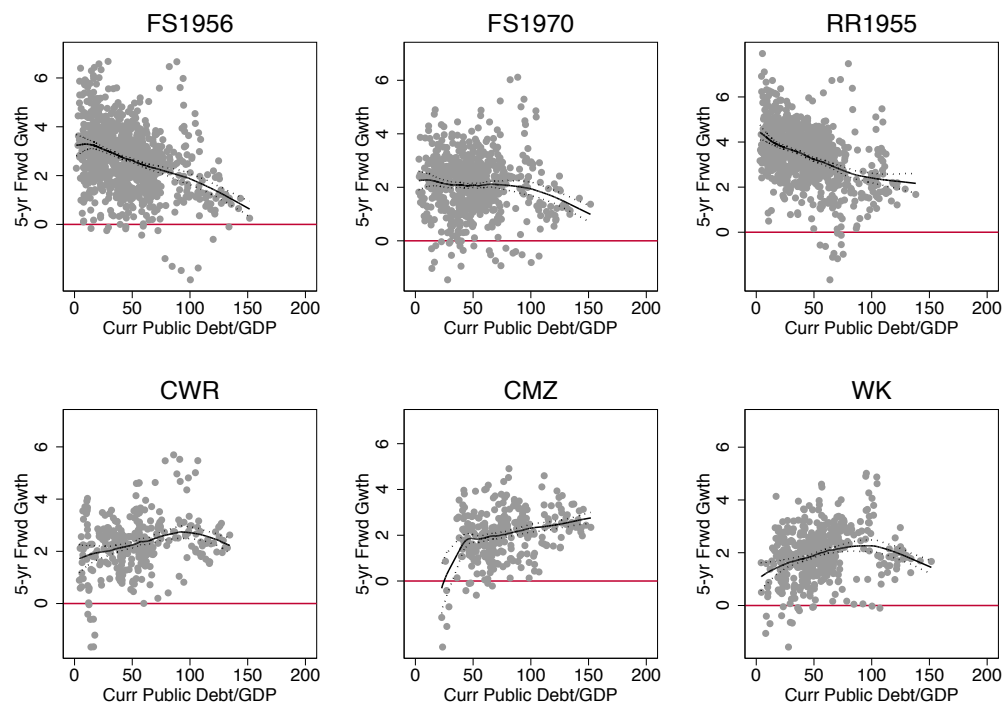
*Notes.* Lowess plots for the bivariate relationship between public debt and real GDP growth. For references to the datasets see Section 2.3. In each row, the relationship between contemporary public debt and average real GDP growth for the preceding five years is show in the left panel, between contemporary public debt and contemporary real GDP growth in the center panel, between average real GDP growth for the following five years in the right panel. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 9:** *Public Debt-Future Growth Partial Linear Regression Controlling for Lagged Growth with Scatterplot*



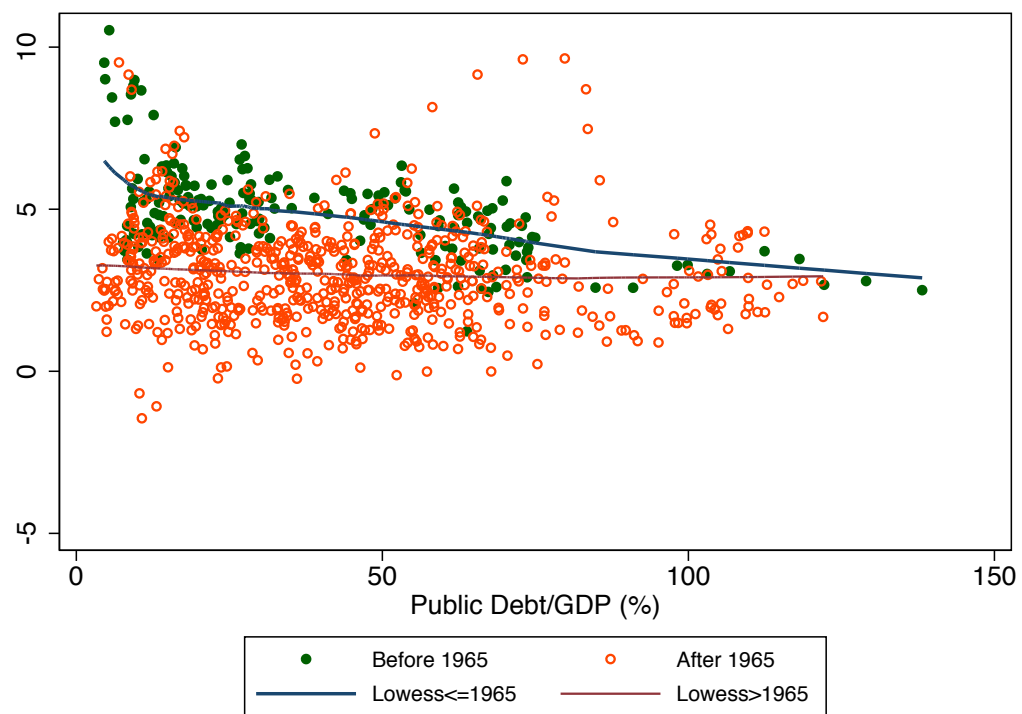
*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 10:** *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects with Scatterplot*



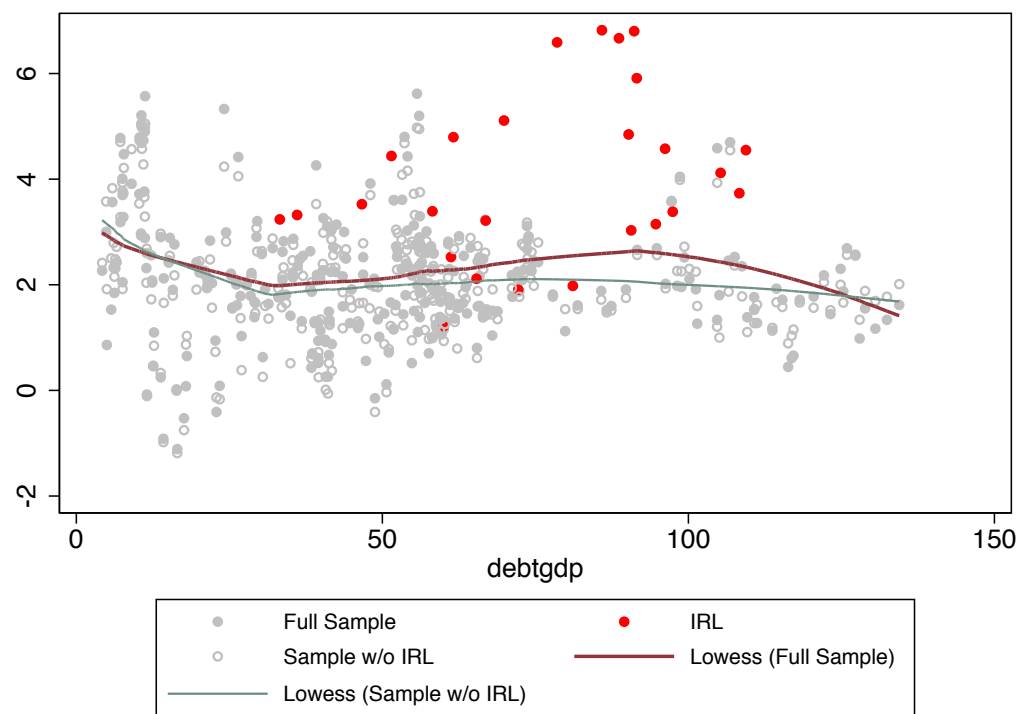
*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.

**Figure 11:** *Public Debt-Future Growth Partial Linear Regression for RR1955, by pre/post-1965*



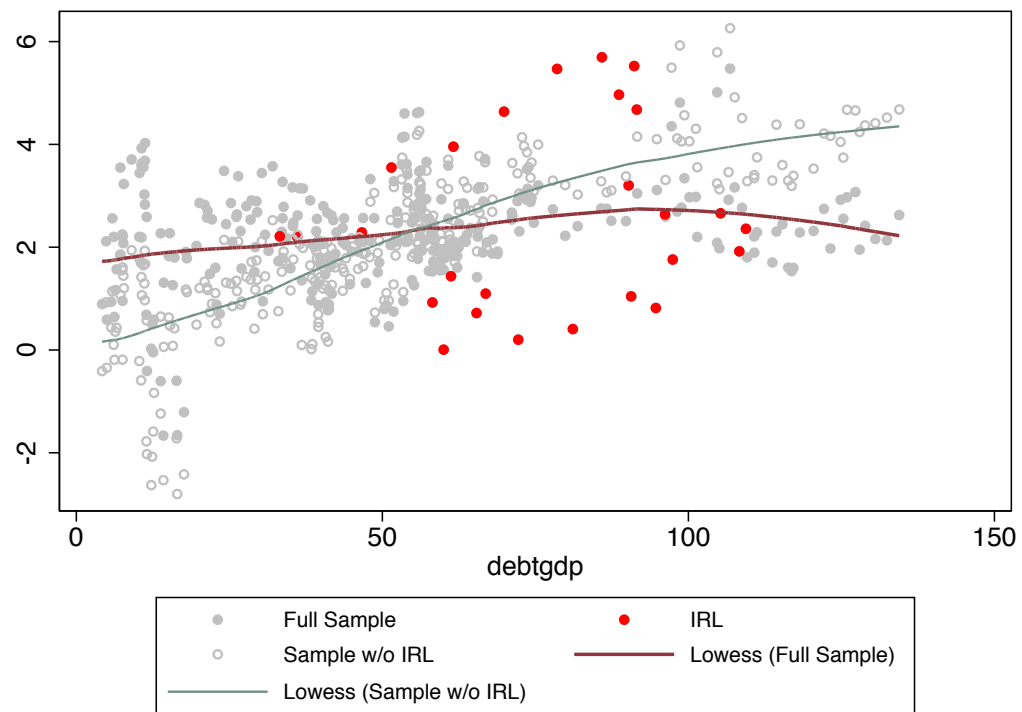
*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. Future For references to the datasets see Section 2.3.

**Figure 12:** *Public Debt-Future Growth Partial Linear Regression for CWR with \mathcal{E} without Ireland*



*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth controlling for one-year lagged real GDP growth and including year dummies. Future For references to the datasets see Section 2.3.

**Figure 13:** *Public Debt-Future Growth Partial Linear Regression with Country Fixed Effects for CWR with  $\setminus \mathcal{E}$  without Ireland*



*Notes.* Partial Linear Regression Residual Lowess plots for the relationship between public debt and five-year future average real GDP growth including country and year dummies. For references to the datasets see Section 2.3. Bootstrapped confidence intervals based on 250 repetitions.



# Online Appendix (Not for Publication)

This Online Appendix presents:

- A. A formal derivation of conditions under which the bias in the estimate of the impact of debt on growth is reduced by (1) using forwarded growth, (2) controlling for lagged dependent variable, and (3) instrumenting public debt with lagged public debt.
- B. Description of our dataset
- C. A schematic of the spline specification used in Cecchetti et al. (2011) and Woo and Kumar (2015) in Figure C.1

## A Bias in Estimation of the Debt-Growth Relationship

### A.1 Basic Setup

Let  $g_{it}$  and  $D_{it}$  denote de-measured growth (annual growth rate of real gross domestic product) and public debt (ratio of public debt to nominal gross domestic product) respectively. The structural model of bi-directional causal relationships between growth and debt, i.e., the “true” data generating process (DGP), is given by the following simultaneous equation model

$$D_{it} = a_1 g_{it} + v_{it} \tag{8}$$

and

$$g_{it} = b_1 D_{it} + u_{it} \quad (9)$$

where the structural errors are distributed as  $v_{it} \sim (0, \sigma_v^2)$  with  $k^{\text{th}}$ -order autocorrelation coefficient  $\rho_v(k)$ ,  $u_{it} \sim (0, \sigma_u^2)$  with  $k^{\text{th}}$ -order autocorrelation coefficient  $\rho_u(k)$ , and for all integers  $k = 0, 1, 2, \dots$ ,  $\mathbb{E}(v_{i,t+k}u_{i,t+k}) = 0$ , where  $\mathbb{E}(y)$  refers to the expected value of the random variable,  $y$ .

The reduced form model corresponding to the structural model in (8) and (9) is given by

$$D_{it} = \left\{ \frac{a_1}{1 - a_1 b_1} \right\} u_{it} + \left\{ \frac{1}{1 - a_1 b_1} \right\} v_{it} \quad (10)$$

and

$$g_{it} = \left\{ \frac{1}{1 - a_1 b_1} \right\} u_{it} + \left\{ \frac{b_1}{1 - a_1 b_1} \right\} v_{it}. \quad (11)$$

We use the following notation:  $\text{plim } z_n$  refers to the probability limit of the random variable  $z_n$  when the appropriate index,  $n$ , goes to infinity;  $|x|$  refers to the absolute value of a real number  $x$ . Our maintained assumption throughout this paper is stated as

**Assumption 1.** *For the model in (8) and (9),  $a_1 < 0$ ,  $b_1 < 0$ , and  $a_1 b_1 < 1$ .*

Let  $\hat{b}_1$  denote the OLS estimator of  $b_1$  in (9) and let  $\lambda = (\sigma_v^2/\sigma_u^2)$ ; then, assuming that the relevant exogeneity conditions of the structural errors in (8) and (9) hold, we have

$$\text{plim } \hat{b}_1 = \frac{\mathbb{E}(D_{it}g_{it})}{\mathbb{E}(D_{it}^2)} = \frac{a_1 + b_1\lambda}{a_1^2 + \lambda}. \quad (12)$$

The estimator  $\hat{b}_1$  in the above equation is indexed by the sample size and the probability limit is computed with the sample size approaching infinity. But here, and below, we suppress explicit indexing with the sample size for notational simplicity.

Using the above expression for the OLS estimator of  $b_1$ , we can see that the bias of the

OLS estimator (due to reverse causality) is given by

$$\text{plim } \hat{b}_1 - b_1 = \frac{a_1(1 - a_1 b_1)}{a_1^2 + \lambda} < 0, \quad (13)$$

where the inequality holds because of assumption 1. Hence, bias in the OLS estimator is negative, i.e., the “true” impact of debt on growth is smaller in magnitude than what the OLS estimator shows.

## A.2 Conditions for Bias Reduction

Let  $\tilde{b}_1$  be an estimator of  $b_1$  arrived at using some specification other than (9) or some method other than OLS. We are interested in understanding the conditions under which this alternative specification (or method) takes us closer to the “true” parameter  $b_1$  *from below*, i.e., while the direction of asymptotic bias in  $\tilde{b}_1$  remains the same as for the OLS estimator  $\hat{b}_1$ , i.e., negative, its magnitude is reduced. This would be ensured when  $\text{plim } \tilde{b}_1$  is bounded from below by the probability limit of the OLS estimator and from above by the “true” value of the parameter, i.e.,

$$\text{plim } \hat{b}_1 < \text{plim } \tilde{b}_1 < b_1. \quad (14)$$

We limit ourselves to downward biased estimators because our analysis suggests that the effect of debt on growth is less negative than claimed in the extant literature. The opposite case, where estimators could be upward biased, is of limited interest to us.

While we derive conditions for bias reduction in terms of the correlation coefficients of the two error terms - the error term in the growth and in the debt equation, we illustrate these conditions for a specific case: when the error terms for the growth and debt equations follow stable AR(1) processes with AR coefficients,  $\phi_u$  and  $\phi_v$  respectively. For each method, we illustrate the condition for bias reduction by depicting the locus of  $(\phi_u, \phi_v)$  on the unit square for which bias is reduced, i.e., the approach improves on OLS if  $(\phi_u, \phi_v)$  falls in the

shaded range. We generate Figure 14 with plausible values for  $a$ ,  $b$ , and  $\lambda$ :  $a = -20$ , which implies that a severe recession with growth of -4 percent per year generates debt of 80 percent;  $b = -0.012$ , which implies that a public debt-to-GDP ratio of 100 percent generates a severe recession with growth of -1.2 percent per year; and  $\lambda = 16$ , under the supposition that a standard deviation of public debt is 8 percentage points of GDP and the standard deviation of GDP growth is 2 percentage points.

## B Bias Reduction in Alternative Strategies

We will use the general conditions listed above in (14) to derive *sufficient* conditions for reduction in the magnitude of the negative bias in alternative estimation strategies.

### B.1 k-period Forwarding of Growth

Let  $k$  be any positive integer. Many papers in the emerging literature on the growth-debt relationship use forwarding of the dependent variable (growth) to reduce the bias due to reverse causality. While it might be intuitively clear that forwarding reduces the bias, we would like to investigate the question more rigorously here and ask: under what conditions does  $k$ -period forwarding of the dependent variable in (9) reduce the bias in the OLS estimator?

To proceed, note that with  $k$ -period forwarding of the dependent variable, the following equation is estimated

$$g_{i,t+k} = b_{11}D_{it} + \varepsilon_{it}. \quad (15)$$

instead of (9). Let  $\hat{b}_{11}$  be the OLS estimator of  $b_{11}$  in (15); then we have

$$\text{plim } \hat{b}_{11} = \frac{\mathbb{E}(D_{it}g_{t+k})}{\mathbb{E}(D_{it}^2)} = \frac{a_1\rho_u(k) + b_1\lambda\rho_v(k)}{a_1^2 + \lambda} \quad (16)$$

where the second equality comes from using the expression for  $D_{it}$  and  $g_{i,t+k}$  corresponding

to the “true” DGP in (8) and (9), and

$$\rho_v(k) = \frac{\mathbb{E}(v_{it}v_{i,t+k})}{\sigma_v^2}$$

is the  $k$ -th order autocorrelation coefficient in  $v_{it}$ , and

$$\rho_u(k) = \frac{\mathbb{E}(u_{it}u_{i,t+k})}{\sigma_u^2}$$

is the  $k$ -th order autocorrelation coefficient in  $u_{it}$ . We assume that the structural errors,  $u_{it}$  and  $v_{it}$ , have non-negative autocorrelation coefficients and state this as

**Assumption 2.** For all  $j = 0, 1, 2, \dots$ ,  $\rho_v(j) \geq 0$  and  $\rho_u(j) \geq 0$ .

**Proposition 1.** If assumption 1 and 2 holds, and

$$a_1 \{\rho_u(k) - a_1 b_1\} < b_1 \lambda \{1 - \rho_v(k)\} \tag{17}$$

then  $k$ -period forwarding reduces bias from below, i.e.,  $\text{plim } \hat{b}_1 < \text{plim } \hat{b}_{11} < b_1$ .

*Proof.* Since  $\lambda > 0$ ,  $0 \geq \rho_u(k) \geq 1$ ,  $0 \geq \rho_v(k) \geq 1$ ,  $a_1 < 0$  and  $b_1 < 0$ , we have  $a_1 \rho_u(k) > a_1$  and  $b_1 \lambda \rho_v(k) > b_1 \lambda$ , so that

$$a_1 + b_1 \lambda < a_1 \rho_u(k) + b_1 \lambda \rho_v(k)$$

and hence that

$$\text{plim } \hat{b}_1 = \frac{a_1 + b_1 \lambda}{a_1^2 + \lambda} < \frac{a_1 \rho_u(k) + b_1 \lambda \rho_v(k)}{a_1^2 + \lambda} = \text{plim } \hat{b}_{11}$$

using the expressions in (12) and (16). Thus, the probability limit of the alternative estimator

is bounded from below by the OLS estimator. To see the upper bound, note that if

$$a_1 \{\rho_u(k) - a_1 b_1\} < b_1 \lambda \{1 - \rho_v(k)\}$$

then

$$a_1 \rho_u(k) + b_1 \lambda \rho_v(k) < b_1 a_1^2 + b_1 \lambda$$

so that

$$\text{plim } \hat{b}_{11} = \frac{a_1 \rho_u(k) + b_1 \lambda \rho_v(k)}{a_1^2 + \lambda} < b_1.$$

This completes the proof. □

To get some intuitive understanding of the result in proposition 1, let us compare the expressions for the OLS estimator in (12) and the k-period forwarded estimator in (16). Note two extreme cases regarding the magnitudes of the autocorrelations in the structural errors: (a) when the autocorrelations tend towards unity from below, i.e.,  $(\rho_u(k), \rho_v(k)) \uparrow (1, 1)$ , the k-period forwarded estimator collapses to the OLS estimator; (b) when the autocorrelations tend towards zero from above, i.e.,  $(\rho_u(k), \rho_v(k)) \downarrow (0, 0)$ , the k-period forwarded estimator tends towards zero (which leads to positive bias, because the true parameter is negative, i.e.,  $b_1 < 0$ ). Hence the k-period forwarded estimator reduces the magnitude of the negative bias when the autocorrelations in the two structural errors are bounded away sufficiently strongly from the extreme values of 0 and 1, i.e., the errors need to be correlated with its past values but not too strongly. The condition in (17) gives the precise way in which this bounding away is needed to achieve reduction in bias from below. Since,  $0 < \rho_u(k), \rho_v(k) < 1$ , combinations of these two autocorrelations, i.e.,  $(\rho_u(k), \rho_v(k))$ , fall in the unit square on the positive part of the 2D plane. The condition in (17) defines the subset of this unit square which would be consistent with reduction in the magnitude of the negative bias.

## B.2 k-period Average Forwarding

For any variable  $x_{it}$ , let  $k$ -period average forwarding be denoted by  $\tilde{x}_{it}$ , i.e.,

$$\tilde{x}_{it} = \frac{1}{k} \sum_{j=0}^{k-1} x_{i,t+j}.$$

With  $k$ -period average forwarding of the dependent variable, instead of (9), the following equation is estimated,

$$\tilde{g}_{i,t+k} = b_{12}D_{it} + \varepsilon_{it}. \quad (18)$$

Let  $\hat{b}_{12}$  be the OLS estimator of  $b_{12}$  in (18), and let

$$\tilde{\rho}_v(k) = (1/k) \left\{ \sum_{j=0}^{k-1} \rho_v(j) \right\} = (1/k) \left\{ 1 + \sum_{j=1}^{k-1} \rho_v(j) \right\},$$

and

$$\tilde{\rho}_u(k) = (1/k) \left\{ \sum_{j=0}^{k-1} \rho_u(j) \right\} = (1/k) \left\{ 1 + \sum_{j=1}^{k-1} \rho_u(j) \right\};$$

then we have

$$\text{plim } \hat{b}_{12} = \frac{\mathbb{E}(D_{it}\tilde{g}_{i,t+k})}{\mathbb{E}(D_{it}^2)} = \frac{a_1\tilde{\rho}_v(k) + b_1\lambda\tilde{\rho}_u(k)}{a_1^2 + \lambda} \quad (19)$$

**Proposition 2.** *If assumption 1 holds, and*

$$a_1 \{ \tilde{\rho}_u(k) - a_1 b_1 \} < b_1 \lambda \{ 1 - \tilde{\rho}_v(k) \} \quad (20)$$

*then  $k$ -period average forwarding reduces bias from below, i.e.,  $\text{plim } \hat{b}_1 < \text{plim } \hat{b}_{12} < b_1$ .*

*Proof.* The proof follows the same logic as the proof of the previous proposition when  $\rho_v(k)$  and  $\rho_u(k)$  are replaced with  $\tilde{\rho}_v(k)$  and  $\tilde{\rho}_u(k)$ , respectively.  $\square$

Equation 19 gives insight into the effectiveness of the approach of average forwarding for recovering the true value of  $b_1$ . If  $\rho_u(k)$  and  $\rho_v(k)$  are both close to one, the estimate of  $b_1$

based on  $k$ -period average forwarding approaches the OLS estimate,  $\text{plim } \hat{b}_{12} = \text{plim } \hat{b}_1$ , and average forwarding is, like OLS, negatively biased. If  $\rho_u(k)$  and  $\rho_v(k)$  are both close to zero, the estimate of  $b_1$  based on  $k$ -period average forwarding approaches zero, i.e.,  $\text{plim } \hat{b}_{12} = 0$ , and assuming  $b_1 < 0$ , average forwarding is positively biased. This result is not surprising because the average forwarding estimator focuses on the persistent component of growth in GDP. If there is little persistent component, i.e.,  $\rho_u(k) = 0$ , then the approach does not yield useful results. For intermediate values of  $\rho_u(k)$  and  $\rho_v(k)$ , average forwarding will recover the true value of  $b_1$ .

We can illustrate these conditions for bias reduction with a specific example. Suppose  $u$  and  $v$  both follow AR(1) processes so that

$u_{it} = \phi_u u_{i,t-1} + \epsilon_{it}^1$  and  $v_{it} = \phi_v v_{i,t-1} + \epsilon_{it}^2$ , where  $|\phi_u| < 1$ , and  $|\phi_v| < 1$  so that the AR(1) processes are stable. Then, if we use 5-year average forwarding, the condition for bias reduction, 20, becomes  $a_1 \left\{ \frac{1-\phi_u^5}{5(1-\phi_u)} - a_1 b_1 \right\} - b_1 \lambda \left\{ 1 - \frac{1-\phi_v^5}{5(1-\phi_v)} \right\} < 0$ .

In the upper left-hand panel of Figure 14, we plot the function  $F(\phi_u, \phi_v) = a_1 \left\{ \frac{1-\phi_u^5}{5(1-\phi_u)} - a_1 b_1 \right\} - b_1 \lambda \left\{ 1 - \frac{1-\phi_v^5}{5(1-\phi_v)} \right\}$  on the unit square, using the following parameter values  $a_1 = -20$ ,  $b_1 = -0.012$ ,  $\lambda = 16$ , and indicate the region on the unit square where the value of the function is negative. This area represents the combination of  $(\phi_u, \phi_v)$  for which the estimator based on the use of Average Forward Growth as the dependent variable improves on the OLS estimate (with contemporaneous growth as the dependent variable).



### B.3 Lagged Debt as an IV for Current Debt

In this case, we estimate (9) using  $D_{i,t-5}$  as an instrument for  $D_{it}$ .<sup>14</sup> Let  $\hat{b}_{13}$  be the IV estimator for  $b_1$  in (9). We have,

$$\text{plim } \hat{b}_{13} = \frac{\mathbb{E}(g_{it}D_{i,t-5})}{\mathbb{E}(D_{it}D_{i,t-5})} = \frac{a_1\rho_u(5) + b_1\lambda\rho_v(5)}{a_1^2\rho_u(5) + \lambda\rho_v(5)} = \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} \quad (21)$$

where  $\omega(5) = (\rho_v(5)/\rho_u(5))$  is the ratio of the autocorrelation coefficients of order 5.

**Proposition 3.** *If assumption 1 and 2 holds, and  $\omega(5) > 1$ , then using  $D_{i,t-5}$  as an instrument for  $D_{it}$  in (9) reduces the bias from below, i.e.,  $\text{plim } \hat{b}_1 < \text{plim } \hat{b}_{13} < b_1$ .*

*Proof.* Note that the upper bound is always satisfied, because  $a_1 < 0$  and  $a_1b_1 < 1$  implies that

$$\text{plim } \hat{b}_{13} = \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} < b_1.$$

To see that the lower bound is satisfied note that  $\omega(5) > 1$  implies

$$a_1\lambda\{1 - \omega(5)\}(1 - a_1b_1) > 0$$

so that

$$a_1\lambda\{1 - \omega(5)\} > \lambda a_1^2 b_1\{1 - \omega(5)\}.$$

Adding  $a_1^3 + b_1\lambda^2\omega(5)$  to both sides of the above we get

$$\{a_1 + b_1\lambda\omega(5)\}\{a_1^2 + \lambda\} > \{a_1 + b_1\lambda\}\{a_1^2 + \lambda\omega(5)\}$$

which shows that

$$\text{plim } \hat{b}_1 = \frac{a_1 + b_1\lambda}{a_1^2 + \lambda} < \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} = \text{plim } \hat{b}_{13} \quad (22)$$

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<sup>14</sup>We have used the 5-th lag because that has been used in the literature. There is no reason why one could not use any other lag. The results derived here are valid for any lag,  $k = 1, 2, 3, \dots$

This completes the proof. □

An intuitive understanding of the result in proposition 3 can be obtained if we re-write the expression for the IV estimator in (21) as

$$\text{plim } \hat{b}_{13} = \frac{\mathbb{E}(g_{it}D_{i,t-5})}{\mathbb{E}(D_{it}D_{i,t-5})} = \frac{a_1 + b_1\lambda\omega(5)}{a_1^2 + \lambda\omega(5)} = \frac{[a_1/\omega(5)] + b_1\lambda}{[a_1^2/\omega(5)] + \lambda}.$$

Consider two extreme cases. First, when  $\omega(5) = 1$ , i.e., when the 5th order autocorrelation coefficient in error term in the growth equation and the debt equation are the same, the IV estimator collapses to the OLS estimator, i.e.,

$$\text{plim } \hat{b}_{13} = \text{plim } \hat{b}_1.$$

Second, when  $\omega(5)$  becomes large, the IV estimator converges to the “true” value of the underlying parameter, i.e., when  $\omega(5) \rightarrow \infty$

$$\text{plim } \hat{b}_{13} = b_1.$$

Thus, we can see that the crucial condition that reduces the magnitude of the negative bias in the IV estimator is the relative magnitude of the autocorrelation coefficients of the error term in the growth equation,  $u_{it}$ , and the error term in the debt equation,  $v_{it}$ .

What is the intuition here? Since the IV estimator is the ratio of the reduced form and the first stage partial effects, the numerator in the expression above captures the reduced form effect of lagged debt on contemporary growth, and the denominator captures the first stage effect of lagged debt on contemporary debt. From the numerator we see that the reduced form effect is a weighted average of  $a_1$  and  $b_1$ , with relative magnitudes of autocorrelation coefficients and variances functioning as weights.<sup>15</sup> When the magnitude of the autocorrelation

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<sup>15</sup>To be more precise, the reduced form effect is the numerator divided by the variance of  $D_{i,t-5}$ . But since

coefficient for  $u_{it}$  (error in the growth equation) increases relative to  $v_{it}$  (error term in the debt equation), then the contribution of  $a_1$  (the reverse causal effect of growth on debt) to the reduced form effect falls, i.e., the confounding effect of the reverse casual relationship is neutralized better by the IV.

In one extreme case, when autocorrelation coefficient of  $u_{it}$  (error in the growth equation) is the same as the the autocorrelation coefficient of  $v_{it}$  (error term in the debt equation), the IV is useless because the confounding effect of the reverse causal effect is in full force. In the other extreme case, when the autocorrelation coefficient of  $u_{it}$  (error in the growth equation) is infinitely larger than the the autocorrelation coefficient of  $v_{it}$  (error term in the debt equation), the confounding effect of the reverse causal channel is perfectly tamed, and the IV estimator takes us to the “true” value.

To summarize the relationship between the autocorrelation parameters of the data-generating process and the effectiveness of IV as an identification strategy, we observe that when the ratio  $\rho_u(5)/\rho_v(5)$  is approximately one, the IV estimate approaches the OLS estimates, i.e.,  $\text{plim } \hat{b}_{13} = \text{plim } \hat{b}_1$ . When the ratio  $\rho_u(5)/\rho_v(5)$  is large, then the IV estimate approaches the true value of  $b_1$ , i.e.,  $\text{plim } \hat{b}_{13} = b_1$ . Intuition for this result follows from the standard conditions for identification with instrumental variables. There must be some correlation between the excluded exogenous instrument and the endogenous regressor, i.e.,  $\rho_v(5)$  cannot be too small, but the correlation cannot be so strong that we fail to break the endogenous relationship, i.e., the excluded exogenous instrument must not be as bad as the endogenous regressor.

Continuing with our example of AR(1) errors, the condition for bias reduction becomes:  
 $\phi_u^5 - \phi_v^5 < 0$ .

In the lower left-hand panel of Figure 14, we plot the function,  $F(\phi_u, \phi_v) = \phi_u^5 - \phi_v^5$ , on the unit square (with the same parameter values as in the upper left figure) and indicate the 

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this variance is a constant, one can consider it as a normalization factor.

area on the unit square for which the IV estimator improves on the OLS estimate (and the value of the function is negative).

## 6.4 Lagged Growth as an Additional Control

In this case we estimate the following model

$$g_{it} = b_{14}D_{it} + b_2g_{i,t-1} + \varepsilon_{it} \quad (23)$$

instead of (9). Let  $\hat{b}_{14}$  be the OLS estimator of  $b_{14}$  in (23); then

$$\text{plim } \hat{b}_{14} = \frac{\mathbb{E}(g_{i,t-1}^2) \mathbb{E}(D_{it}g_{it}) - \mathbb{E}(D_{it}g_{i,t-1}) \mathbb{E}(g_{it}g_{i,t-1})}{\mathbb{E}(g_{i,t-1}^2) \mathbb{E}(D_{it}^2) - \mathbb{E}(D_{it}g_{i,t-1}) \mathbb{E}(D_{it}g_{i,t-1})}. \quad (24)$$

The expression for the probability limit of  $\hat{b}_{14}$  can be simplified to the following:

$$\text{plim } \hat{b}_{14} = \frac{(a_1 + b_1\lambda) - [a_1\rho_u(1) + b_1\rho_v(1)\lambda]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda} [a_1\rho_u(1) + b_1\rho_v(1)\lambda]^2}. \quad (25)$$

**Proposition 4.** *If Assumptions 1 and 2 hold, and if*

$$a_1 [1 - \rho_u(1)] + b_1 [1 - \rho_v(1)] < a_1^2 b_1 + b_1 \lambda - \frac{b_1}{1 + b_1^2 \lambda} [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2 \quad (26)$$

then using  $g_{i,t-1}$  as an additional control reduces the bias from below, i.e.,  $\text{plim } \hat{b}_1 < \text{plim } \hat{b}_{14} < b_1$ .

*Proof.* Since  $[a_1\rho_u(1) + b_1\rho_v(1)\lambda] < 0$ , and  $(1 + b_1^2\lambda) > 0$ ,

$$\text{plim } \hat{b}_1 = \frac{(a_1 + b_1\lambda)}{(a_1^2 + \lambda)} < \frac{(a_1 + b_1\lambda) - [a_1\rho_u(1) + b_1\rho_v(1)\lambda]}{(a_1^2 + \lambda) - \frac{1}{1+b_1^2\lambda} [a_1\rho_u(1) + b_1\rho_v(1)\lambda]^2} = \text{plim } \hat{b}_{14}$$

so that the lower bound is always satisfied. On the other hand if

$$a_1 [1 - \rho_u(1)] + b_1 [1 - \rho_v(1)] < a_1^2 b_1 + b_1 \lambda - \frac{b_1}{1 + b_1^2 \lambda} [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2$$

then

$$a_1 [1 - \rho_u(1)] + b_1 [1 - \rho_v(1)] < \left\{ a_1^2 + \lambda - \frac{1}{1 + b_1^2 \lambda} [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2 \right\} b_1$$

The expression in the braces on the RHS is positive because

$$(a_1^2 + \lambda) (1 + b_1^2 \lambda) > [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2.$$

Hence, we can divide through by the expression in the braces to get

$$\text{plim } \hat{b}_{14} = \frac{(a_1 + b_1 \lambda) - [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]}{(a_1^2 + \lambda) - \frac{1}{1 + b_1^2 \lambda} [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2} < b_1$$

This completes the proof. □

To understand the intuition behind the result in proposition 4 let us re-write the expression for the estimator in (25) as

$$\text{plim } \hat{b}_{14} = \frac{a_1 [1 - \rho_u(1)] + b_1 \lambda [1 - \rho_v(1)]}{(a_1^2 + \lambda) - \frac{1}{1 + b_1^2 \lambda} [a_1 \rho_u(1) + b_1 \rho_v(1) \lambda]^2} \quad (27)$$

Consider two extreme cases. First, when  $\rho_u(1) = \rho_v(1) = 1$ , the estimator converges to 0, which gives a positively biased estimate (because the “true” value of the parameter is negative, i.e.,  $b_1 < 0$ ). Second, when  $\rho_u(1) = \rho_v(1) = 0$ , the estimator coincides with the OLS estimator,  $\hat{b}_1$ . Thus, for the estimator  $\hat{b}_{14}$  to reduce the magnitude of the negative bias, the autocorrelation in the error terms must be bounded away from the extreme values of 0 and 1,

which is analogous to the case of  $k$ -period forwarding that we discussed in proposition 1. The expression in (26) gives the precise domain of values of the combination of autocorrelations,  $(\rho_u(1), \rho_v(1))$ , on the unit square on the positive part of the 2D plane that is needed to ensure the reduction in the magnitude of the negative bias.

One way to get a better intuitive grasp of the issues here is to use a “partialling out” interpretation of the estimator  $\hat{b}_{14}$ . Using the Frisch-Waugh-Lovell theorem,  $\hat{b}_{14}$  is the OLS estimator from a regression of contemporary growth,  $g_{it}$ , on the residual obtained from an auxiliary regression of contemporary debt,  $D_{it}$ , on lagged growth,  $g_{i,t-1}$ . Hence, the numerator of the expression in (25) is the covariance of contemporary growth with the residual, and the denominator is the variance of the residual. Comparing the expression in (25) with the analogous expression in (12), we see that that the numerator increases (becomes less negative) and the denominator increases. Intuitively, including lagged growth as an additional control has two effects (if the conditions stated in proposition 4 are satisfied): first, it soaks up some of the variation in the error term in (9) leading to a lower variance in the resulting error term; and second, it makes the covariance between contemporary growth and the residual of the auxiliary regression less negative than the covariance of contemporary growth and contemporary debt. The net result is that inclusion of a lagged growth term, under these conditions, reduces the magnitude of the negative bias.

Equation 27 gives insight into the conditions of effectiveness of the approach of controlling for lagged growth for recovering the true value of  $b_1$ . If  $\rho_u(1)$  and  $\rho_v(1)$  are both close to 1, the estimate of  $b_1$  based on controlling for lagged growth approaches zero, i.e.,  $\text{plim } \hat{b}_{14} = 0$ , and assuming  $b_1 < 0$ , average forwarding is positively biased. If  $\rho_u(1)$  and  $\rho_v(1)$  are both close to zero, the estimate of  $b_1$  based on controlling for lagged growth approaches the OLS estimate,  $\text{plim } \hat{b}_{14} = \text{plim } \hat{b}_1$ , and controlling for lagged growth is, like OLS, negatively biased.

This result is not surprising because the estimator controlling for lagged growth focuses identification on growth innovations. If there is little innovation, i.e.,  $\rho_u(1) = 1$ , then the

approach does not yield useful results. For lower values of  $\rho_u(1)$  and  $\rho_v(1)$ , controlling for lagged growth will recover the true value of  $b_1$ .

Continuing with our example of AR(1) errors, the condition for bias reduction, 26, becomes:

$$a_1 [1 - \phi_u] + b_1 [1 - \phi_v] - a_1^2 b_1 - b_1 \lambda + \frac{b_1}{1+b_1^2 \lambda} [a_1 \phi_u + b_1 \lambda \phi_v]^2 < 0$$

In the upper right-hand panel of Figure 14, we plot, using the same parameter values as used in the other figures, the function  $F(\phi_u, \phi_v) = a_1 [1 - \phi_u] + b_1 [1 - \phi_v] - a_1^2 b_1 - b_1 \lambda + \frac{b_1}{1+b_1^2 \lambda} [a_1 \phi_u + b_1 \lambda \phi_v]^2$  on the unit square, and indicate the combination of  $(\phi_u, \phi_v)$  for which the value of the function is negative, and hence the estimator based on the inclusion of a Lagged Growth Control improves on the OLS estimate without the Lagged Growth Control.

## C Taking Stock of the Bias Reduction Strategies

The lower-right panel of Figure 14 shows the alternative loci for which the three identification approaches improve on the OLS estimates. With our plausible values of  $a = -20$ ,  $b = -0.012$ , and  $\lambda = 16$ , the intersection of the loci — the area in which all three of the methods, Lagged Growth Control, Average Forward Growth, and Instrumental Variables, yield improvements over the OLS estimate (which is represented by the dark shaded trapezoid-like region) — occupies a substantial share of the unit square.

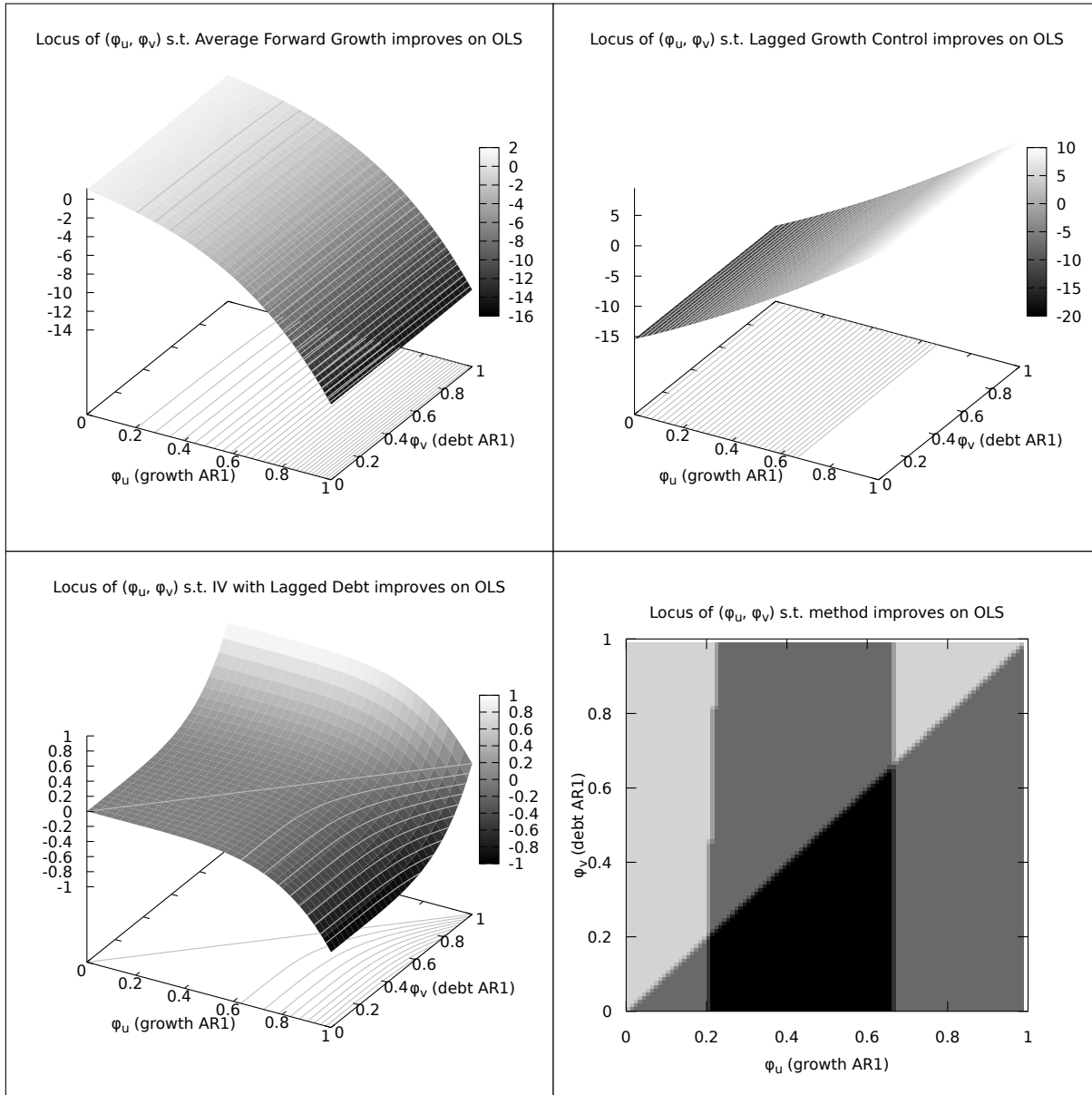
But the implication of the bias-reduction analysis is stronger. As we note above, the Lagged Growth Control and the Average Forward Growth approaches focus identification on different parts of the data-generating process. Controlling for lagged growth focuses identification of the effect of debt on innovations in growth, i.e., persistent growth is controlled for via the inclusion of the lag. Average Forward Growth focuses identification on the persistent portion of growth, i.e., the accumulation of growth over a five-year period. These alternative focuses imply different responses of the estimator to alternative values of  $(\phi_u, \phi_v)$ .

The Lagged Growth Control and Average Forward Growth estimators have the convenient property that they reduce bias (or are biased towards zero) under opposite assumptions about  $\phi_u$ . As the upper-left panel of Figure 14 illustrates, Average Forward Growth, focused on the persistent component, performs better when  $\phi_u$  is relatively high and will be biased toward zero only if  $\phi_u$  is relatively low. As the upper-right panel of Figure 14 shows, the Lagged Growth Control, focused on innovations, performs better when  $\phi_u$  is relatively low and will be biased toward zero only if  $\phi_u$  is relatively high.

The combination means that it cannot be that both of them are biased towards zero (under the data-generating process). If both reduce the magnitude of the estimate vis-à-vis the baseline OLS estimate—which is the case empirically—then the reduced magnitude constitutes an improvement on the baseline OLS, and cannot a result of specific conditions on  $(\phi_u, \phi_v)$  that bias the estimate toward zero. That is, the similarity of the estimates for the Lagged Growth Control and the Average Forward Growth specification in the empirical analysis imply that the likely values of  $(\phi_u, \phi_v)$  are in the intersection of the zones where each strategy reduces bias.



**Figure 14: Conditions for Bias Reduction**



*Notes.* The panels illustrate the effect of each of three bias reduction strategies and, in the lower-right panel, the intersection of the three approaches. For Average Forward Growth (the upper-left panel), the shaded locus of improvement  $(\phi_u, \phi_v)$  is derived from Equation 20. For Lagged Growth Control (the upper-right panel), the shaded locus of improvement  $(\phi_u, \phi_v)$  is derived from Equation 26. For Instrumental Variables (the lower-left panel), the shaded locus of improvement  $(\phi_u, \phi_v)$  is derived from Equation 22. These figures were produced using the following values:  $a = -20, b = -0.012, \lambda = 16$ , and under the assumption that the error terms in the growth and debt equations follow AR(1) processes with coefficients  $\phi_u$  and  $\phi_v$  respectively.

## Appendix B Data Appendix

In this appendix, we describe our various data sources in greater detail. A word about the two datasets derived from Reinhart and Rogoff (2010) is in order. Reinhart and Rogoff published related public-use data on their website, but the actual data used in Reinhart and Rogoff (2010) were made available by Reinhart and Rogoff to the authors of Herndon et al. (2014) in spreadsheet format. Herndon et al. released a data and code package based on these data from their website, and this release forms the basis of our analysis. This dataset covers the time period 1946–2009 and includes the 20 countries indicated in column 2 of Table 2. Following the authors’ recommendation in Reinhart and Rogoff (2013), we use a subset of these data that limits the data to 1955–2003 and refer to this dataset as RR1955. In the data from Reinhart and Rogoff, public debt is measured as gross central government debt as a percentage of GDP, and growth is measured as the annual growth rate of real GDP (not per capita).

The dataset for Cecchetti et al. (2011) was downloaded from the website of the Bank of International Settlements.<sup>16</sup> This data set, which we call CMZ, covers the 18 countries indicated in column 2 of Table 2 for the period 1980–2009. As explained in Cecchetti et al. (2011), the data come either from the OECD website or from national sources. In this dataset, public debt is measured as gross liabilities of general government valued at market prices on a non-consolidated basis (as a percentage of GDP), and growth is measured as the annual growth rate of per capita real GDP. Other variables, e.g., school enrollment and the level of real GDP, were included on separate spreadsheets.

We reconstructed the dataset for Checherita-Westphal and Rother (2012) by downloading data from the Annual macro-economic database (AMECO) website of the European Commission’s Directorate General for Economic and Financial Affairs (DG ECFIN) for the following

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<sup>16</sup>See <http://www.bis.org/publ/work352.htm>.

variables: (1) per capita real GDP (gross domestic product at 2010 reference levels per head of population), (2) gross public debt as a percentage of GDP (i.e., general government consolidated gross debt as a percentage of GDP at current prices).<sup>17</sup> This data set, which we call CWR, runs over the period 1970–2008 and includes the 12 countries indicated in column 3 of Table 2. In CWR, public debt is measured as general government consolidated gross debt, and growth is measured as the annual growth rate of per capita real GDP.

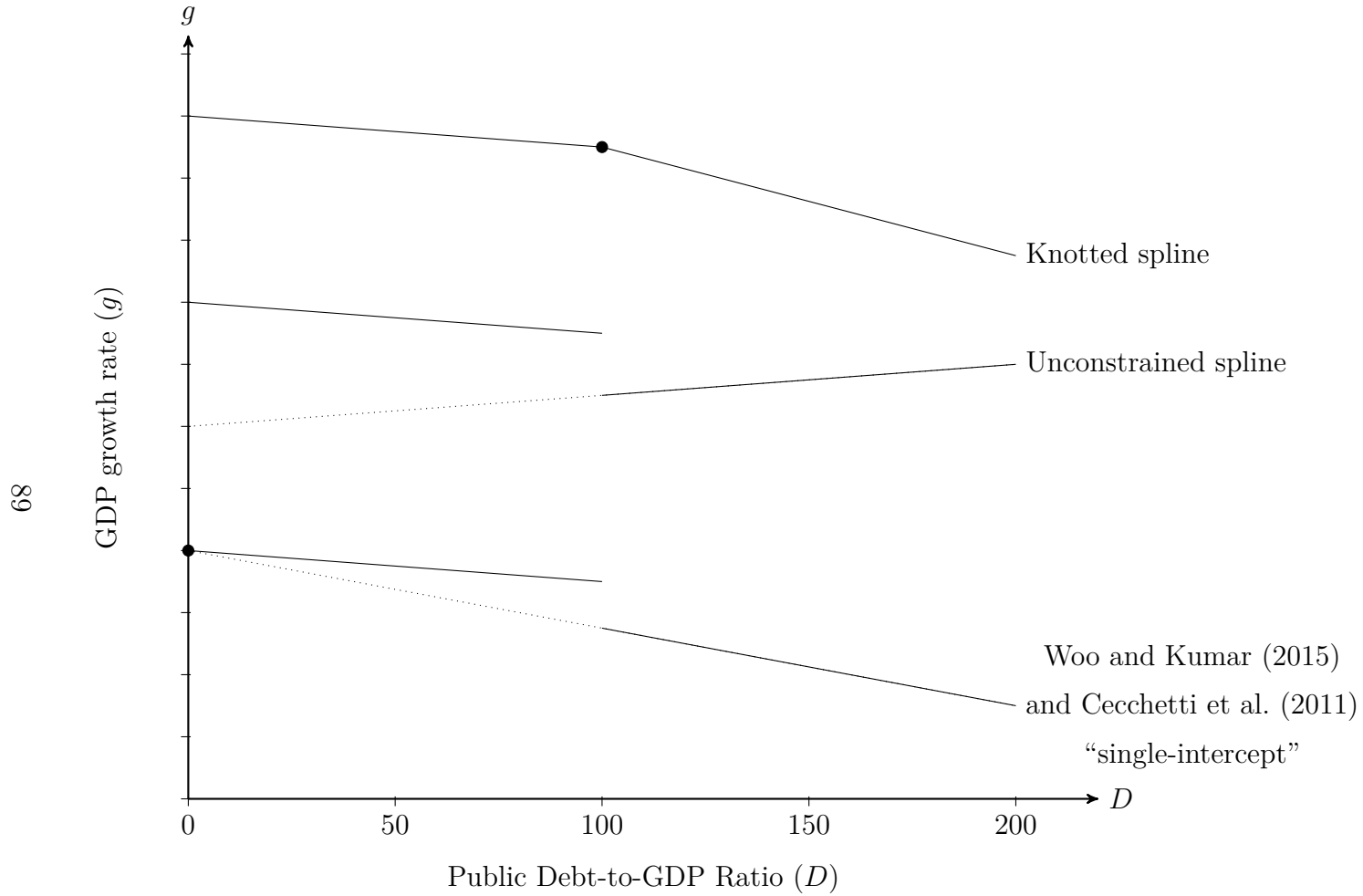
We also attempted to reconstruct the advanced economies dataset used by Woo and Kumar (2015). Although Woo and Kumar (2015) cites the IMF World Economic Outlook (WEO) data, the version that we found on the IMF website includes the key macroeconomic and fiscal variables for all countries but all data are limited to 1980–present, while Woo and Kumar (2015) presents results for 1970–present. We reconstructed the data, which we refer to as WK, by merging the WEO data (WEOOct2015all, the set available from the IMF website in October 2015) with data from the Penn World Tables version 7.0 (Heston et al., 2011) and data on fiscal variables for all countries from 1800–present from the IMF Public Finances in Modern History Fiscal Prudence and Profligacy Database (Mauro et al., 2013).<sup>18</sup>

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<sup>17</sup> Data were downloaded on 4/14/2016 from [http://ec.europa.eu/economy\\_finance/ameco/user/serie/SelectSerie.cfm](http://ec.europa.eu/economy_finance/ameco/user/serie/SelectSerie.cfm).

<sup>18</sup>In no case were the data archived for replication by the journals or authors. Requests for replication data to the corresponding authors for Checherita-Westphal and Rother (2012), Woo and Kumar (2015) did not receive a response. Access to the data for Reinhart and Rogoff (2010) is discussed in Herndon et al. (2014).

Figure C.1: Schematic of Spline Specifications



*Notes.* The diagram illustrates two commonly used spline specifications and the partially constrained spline specification used in Woo and Kumar (2015) and Cecchetti et al. (2011). In the top specification, the spline is knotted at the break point to permit a continuous function with a change in slope. In the middle specification, both a discontinuity and a change in slope are permitted. In the third specification, the two segments are constrained a single common intercept with the vertical axis.