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Working Paper

Fair allocation of disputed properties

Working Paper, No. 2016-10

Provided in Cooperation with:

Department of Economics, University of Massachusetts

Suggested Citation: Ju, Biung-Ghi; Moreno-Ternero, Juan D. (2016) : Fair allocation of disputed properties, Working Paper, No. 2016-10, University of Massachusetts, Department of Economics, Amherst, MA

This Version is available at:

<https://hdl.handle.net/10419/174402>

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DEPARTMENT OF ECONOMICS

Working Paper

Fair Allocation of Disputed Properties

by

Biung-Ghi Ju
Juan D. Moreno-Ternero

Working Paper 2016-10



**UNIVERSITY OF MASSACHUSETTS
AMHERST**

Fair Allocation of Disputed Properties*

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September 26, 2016

Abstract

We model problems of allocating disputed properties as generalized exchange economies. Therein, agents have preferences and claims over multiple goods, and the social endowment of each good may not be sufficient to satisfy all individual claims. We focus on *market-based* allocation rules that impose a two-step procedure: assignment of rights based on claims first, and voluntary exchange based on the assigned rights afterwards. We characterize three focal egalitarian rights-assignment rules that guarantee that the allocation rules are *fair*. We apply our results to problems of greenhouse gas emissions and contested water rights.

JEL numbers: D63, D71.

Keywords: *fairness, claims, no-envy, individual rationality, egalitarianism, efficiency, Walrasian exchange.*

*We are grateful to Masaki Aoyagi, three anonymous referees, Younghwan In, and William Thomson for detailed comments and suggestions. Thanks are also due to Yuki Funaki, Francois Maniquet, Herve Moulin, John Roemer and Roberto Veneziani, as well as the participants at seminars and conferences where earlier versions of this paper has been presented, at Albany, Alicante, Bergen, Bilbao, Brussels, Lisbon, London, Madrid, Odense, Rochester, Salamanca, Seoul, Seville, Tokyo and Valencia. All remaining errors are ours. Biung-Ghi Ju gratefully acknowledges financial supports from the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2013S1A3A2055391) and from the Center for Distributive Justice, Institute of Economic Research at Seoul National University. Financial support from the Spanish Ministry of Economics and Competitiveness (ECO2011-22919, ECO2014-57413-P) is gratefully acknowledged.

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1 Introduction

Fairness and distributive justice are primary concerns in practical procedures for property rights disputes. If a rule is not perceived as fair, its adoption might be jeopardized. Failures to agree on a new international framework dealing with Greenhouse Gas (GHG) emissions after the Kyoto protocol are a point in case. Countries at different stages of economic development have different perceptions of fairness and support different rules. Closing the gap to reach a final resolution is a political economy problem. However, the core issue is an ethical one and investigating it from the perspective of normative economics will facilitate its political resolution.

Resolving property rights disputes usually involves three general principles; namely, *equal division*, *proportional division*, and *equal sacrifice*. Examples can be found in numerous institutional setups (including laws, social, and religious norms) or agreed conventions (such as court settlements for accident damages, or international resolutions on environmental problems). The three principles also underlie the three prominent proposals for the allocation of GHG emission rights.¹

The normative foundation of allocation schemes, such as the above three principles, has been a key subject in the literature of fair allocation. Nevertheless, most studies in this literature either focus on allocating a single good (money) (e.g., O'Neill, 1982; Thomson, 2003, 2015), dismissing the issue of fair initial allocation and its influence on the final allocation of the other goods after the subsequent interactions among claimants, or assume a fixed initial distribution of property rights, without dispute, and investigate end-state fairness (e.g., Pazner and Schmeidler, 1974; Thomson, 2011).² Therefore, they are somewhat limited for the investigation of (end-state and procedural) fairness in some environments with conflicting claims, or property rights dispute.

We provide in this paper a comprehensive framework to investigate fairness in the initial assignment of rights on disputed properties, fairness in the transaction of rights, and fairness of the end-state allocation, as well as the implications of the three categories of fairness and their relations. Our model is an extended model of exchange economies, the extension allowing for

¹The proposal to allocate on an “equal per capita basis” (e.g., Neumayer, 2000) corresponds to equal division, the polluter pays principle (paraphrased as “you broke it, you fix it”; e.g., Singer, 2002) to proportional division, and the principle of equal burden sharing (e.g., Posner and Weisbach, 2010) to equal sacrifice.

²There exist generalizations of the rationing model introduced by O'Neill (1982) to a multidimensional setting, but therein, even though claims refer to multiple types of assets, the endowment to be allocated is still unidimensional (e.g., Ju, Miyagawa, and Sakai, 2007).

conflicts over properties. More precisely, agents have preferences over a finite number of goods, and they also have initial claims on those goods. Furthermore, the available endowment of each good may not be sufficient to satisfy all claims; i.e., the sum of individual claims exceeds the endowment. In other words, we consider rationing problems with multiple goods where agents can have various homogeneous or heterogeneous preferences over those goods.³

Our model is flexible enough to accommodate a variety of real-life problems involving property rights disputes. A running interpretation of the model is as *cap-and-trade* systems to deal with GHG emissions.⁴ For such an interpretation, agents should be considered as GHG-emitting entities, goods as energy and money (with the latter being used to produce the former), claims as their energy consumptions on a base year and their incomes, and the social endowment as the targeted amount for total energy consumption, below the total energy consumption on a base year, and the net total endowment of money after paying the cost of energy production.

Another interpretation of our model is as problems of contested water rights.⁵ In those cases, water supply falls short of satisfying the regular amounts of water usage by the entities who share a water source. There are two goods: water and money. Claims consist of the amount of water used in a base year, and the monetary endowment. Here, dispute exists only over the water rights, not over money.

Various forms of markets have been developed throughout human history. Two fundamental elements encompassing them are private property rights and voluntary exchange.⁶ We precisely define in our context *market-based allocation rules* as those rules determined by these two elements, formalized in our model by the consecutive procedures of *rights-assignment* and *voluntary exchange*. Our main results characterize market-based allocation rules that lead to fair (and efficient) end-state allocations.

End-state fairness is formalized in this context as three variants of *no-envy*, probably the concept with the longest tradition in the theory of fair allocation (e.g., Foley, 1967; Kolm, 1972; Varian, 1974).⁷ *No-envy* is satisfied if no agent prefers anyone else's consumption to her

³This is the main difference with respect to bankruptcy problems dealing with the allocation of a single good (e.g., Thomson, 2003, 2015) where all agents are presumed to have the unique monotonic preferences over the single good.

⁴The term is broadly used to describe a policy that sets caps on the emissions of (GHG-emitting) entities and allows trading in the resulting emissions allowances (e.g., Stavins, 2008).

⁵A different analysis of these problems is Ansink and Weikard (2009).

⁶For instance, as Hodgson (2008, p.326) puts it, 'Together these specifications (put forward by the Austrian school economist Ludwig van Mises) amount to a definition of the market that embraces all forms of trade or exchange that involve private property, defined loosely as assets under private control'. This view is also shared by Adam Smith, Karl Marx, Friedrich Hayek, etc. (see Hodgson, 2008).

⁷It was also used in moral philosophy as a basic test for resource egalitarian allocations (e.g., Dworkin, 1981).

own. A parallel comparative notion of fairness, defined through interpersonal comparisons of (absolute) sacrifices gives rise to the notion of *sacrifice-no-envy*. Likewise, we define *relative-no-envy* through interpersonal comparison of relative sacrifices (or rewards).

The *rights-assignment* procedure assigns property rights by means of adjudicating claims under resource constraints.⁸ *Voluntary exchange* yields *individually rational* end-state allocations, leaving no one worse off than at her endowment (determined by her property rights). The rights-assignment is not based on preferences, which makes the procedure informationally simple. The voluntary exchange procedure relies on individual preferences and endowments but is not restricted by claims or by any constraint (other than *individual rationality*). Hence it can take the form of any system of decision making providing each agent with the freedom of exercising the property rights over her endowment. Therefore, focusing on market-based allocation rules, we achieve the advantage of simple information processing and freedom of choice. Our market-based allocation rules actually guarantee agents the welfare lower bounds determined by their individual property rights. Furthermore, we show that the idea of guaranteeing agents certain welfare lower bounds requires the use of market-based allocation rules.

Empirical support for market-based allocation rules has been reported in numerous practical contexts. For instance, Field (2008) investigates the effect of the urban land property rights assignment on the labor supply in Peru. Doremus (2013) investigates the effects of market-based strategies in the context of fisheries. Aghakouchak et al. (2014) deal with the economic role of water rights management and water markets during the Australian drought from 1997 until 2009.

Our main results show that market-based allocation rules so constructed, lead to *fair* (and efficient) end-state allocations only when the rights-assignment procedure is carried out by one of the three focal rationing rules, known as *constrained equal awards*, *constrained equal losses*, and *proportional*.

The rest of the paper is organized as follows. We set up the model in Section 2. We provide our main results in Section 3. We analyze an important application of our model in Section 4 and conclude in Section 5. We defer the proofs to the appendix.

⁸We impose *solidarity* in this procedure, modeled by the combination of two axioms, known as *resource monotonicity* and *consistency*, as suggested by Moreno-Ternero and Roemer (2006, 2012), among others.

2 The model and definitions

Let $\mathbb{N} \equiv \{1, 2, \dots\}$ be the domain of potential agents and $\mathcal{N} \equiv \{N \subset \mathbb{N} : 2 \leq |N| < \infty\}$ the family of finite non-empty subsets of $\mathbb{N}$ with at least two agents.⁹ There are ℓ privately appropriable and infinitely divisible goods. Each agent $i \in \mathbb{N}$ has a preference relation R_i defined on $\mathbb{R}_+^\ell$, which satisfies the classical conditions of rationality (completeness and transitivity), continuity (lower and upper contour sets are closed), strong monotonicity, and convexity (upper contour sets are convex).¹⁰ Let $\mathcal{R}$ denote the domain of such admissible preferences.¹¹

Consider population $N \in \mathcal{N}$ with n agents. A collective resource, or social *endowment*, $\Omega \in \mathbb{R}_{++}^\ell$ is an ℓ -vector indicating the available amount of each good. Each agent $i \in N$ has a *claim* (vector) $c_i \in \mathbb{R}_+^\ell$ on the endowment. We consider problems in which the aggregate claim of the agents exceeds (or equals) the social endowment of each good. Formally, the set of agents $N \in \mathcal{N}$, the social endowment $\Omega \in \mathbb{R}_{++}^\ell$, the profile of claims $c \equiv (c_i)_{i \in N} \in \mathbb{R}_{++}^{n\ell}$, such that $\Omega \leq \sum_{i \in N} c_i$, and the profile of preference relations $R \equiv (R_i)_{i \in N} \in \mathcal{R}^N$ constitute an *economy* $e \equiv (N, \Omega, c, R)$.¹² If $\Omega = \sum_{i \in N} c_i$, then e is a standard *exchange economy* and c would be interpreted as a profile of *individual endowments*. If $\ell = 1$ and $\Omega < \sum_{i \in N} c_i$, then e is a standard *bankruptcy problem* (e.g., O'Neill, 1982).

Let $\mathcal{E}(N, \Omega)$ denote the domain of economies with population N and social endowment Ω , and $\mathcal{E}$ the domain of all economies. Let $\bar{\mathcal{E}} \subset \mathcal{E}$ be the domain of exchange economies. We often denote the claims profile for an exchange economy (the endowment profile then) by $\omega \equiv (\omega_i)_{i \in N}$ instead of $c \equiv (c_i)_{i \in N}$, and omit Ω from the description of the economy.¹³

As a motivating example for this model, suppose there are n agents sharing a river for water supply. There are two goods: water w and some other “representative” good x . The amount of water agent i utilizes, when there is no water shortage, is denoted by $\bar{w}_i$. This is i ’s claim on water. Suppose that, due to a drought, the aggregate water to be shared among the countries is reduced to $E \leq \sum_{i \in N} \bar{w}_i$. Let ω_i^x be agent i ’s endowment of the representative good x and $c_i \equiv (\omega_i^x, \bar{w}_i)$ her claim. Let $\Omega = (\sum_{i \in N} \omega_i^x, E)$ be the social endowment.¹⁴ Finally, agents have

⁹Denote by $|N|$ the cardinality of set N .

¹⁰As usual, we denote by P_i the strict preference associated with R_i , and by I_i the corresponding indifference relation.

¹¹As preferences are continuous, we can represent them by continuous real-valued functions, and it is sometimes convenient to do so. For each $i \in N$, let $U_i : \mathbb{R}_+^\ell \rightarrow \mathbb{R}$ be such a representation of agent i ’s preferences, and let $U \equiv (U_i)_{i \in N}$. These representations will not have cardinal significance.

¹²Our mathematical notation $x \leq y$ to relate vectors $x, y \in \mathbb{R}_{++}^\ell$ means that $x_i \leq y_i$ for each $i = 1, \dots, \ell$. Thus, we are implicitly saying that no commodity exceeds the corresponding aggregate claim.

¹³For each $N \in \mathcal{N}$, $M \subset N$, and $z \in \mathbb{R}_{++}^{n\ell}$, we denote $z_M \equiv (z_i)_{i \in M}$. Furthermore, for ease of notation, if $M = N \setminus \{i\}$, for some $i \in N$, we let $z_{-i} \equiv z_M$.

¹⁴Note that there is no conflict with regard to the ownership of the x -good.

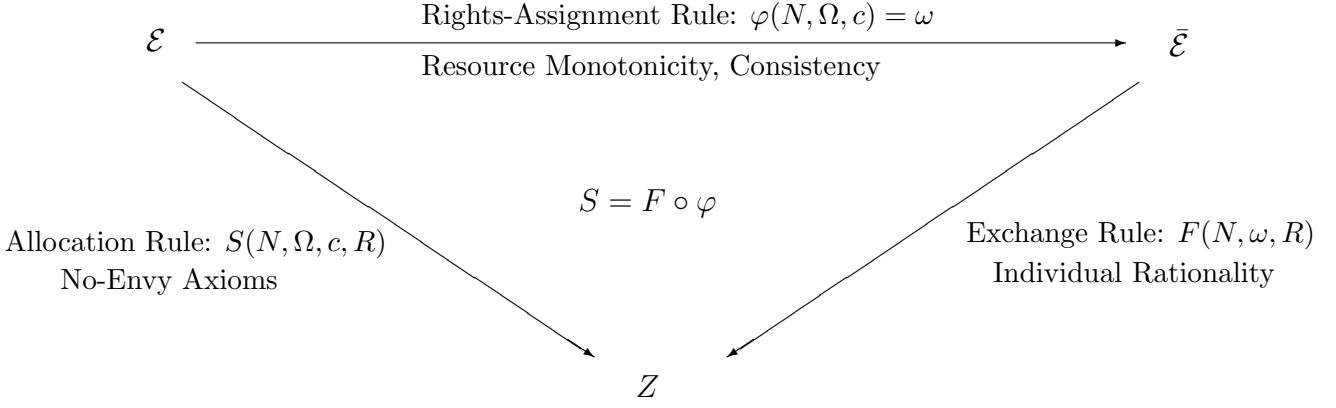


Figure 1. Market-based allocation rules. A market-based allocation rule S is the result of applying a rights-assignment rule φ (possibly, satisfying *resource monotonicity* and a *consistency* axiom), and an exchange rule F satisfying *individual rationality*, such that, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$, $S(e) = F(N, \varphi(N, \Omega, c), R)$. We endorse that they satisfy a *no-envy* axiom.

preferences over the two goods and the profile of their preferences is denoted by R . Thus, an economic environment in this case is also formalized by $e \equiv (N, \Omega, c, R) \in \mathcal{E}$, as in the model just introduced.

A (feasible) *allocation* for $e \in \mathcal{E}(N, \Omega)$ is a profile of individual consumption bundles $z \equiv (z_i)_{i \in N} \in \mathbb{R}_+^{n\ell}$ such that $\sum_{i \in N} z_i = \Omega$. Let $Z(N, \Omega) \equiv \{z \in \mathbb{R}_+^{n\ell} : \sum_{i \in N} z_i = \Omega\}$ be the set of all allocations for economies in $\mathcal{E}(N, \Omega)$. Let $Z \equiv \bigcup_{N \in \mathcal{N}} \bigcup_{\Omega \in \mathbb{R}_{++}^\ell} Z(N, \Omega)$. Given $e \equiv (N, \Omega, c, R)$, an allocation $z \in Z(N, \Omega)$ is (Pareto) *efficient* if there is no other allocation z' that makes an agent better off and everyone else at least as well off, that is, for some $i \in N$, $z'_i P_i z_i$ and for each $j \in N \setminus \{i\}$, $z'_j R_j z_j$. An *allocation rule* $S: \mathcal{E} \rightarrow Z$ associates with each economy $e \equiv (N, \Omega, c, R)$ a non-empty set of allocations, i.e., a non-empty subset of $Z(N, \Omega)$.

We are mostly interested in allocation rules that are defined by the following two consecutive procedures: First, a *rights-assignment procedure* mapping the non-preference information (N, Ω, c) of each economy $e \equiv (N, \Omega, c, R)$ into a profile of individual endowments $\omega \equiv (\omega_i)_{i \in N}$ and, second, an *exchange procedure* determining final allocations for the resulting exchange economy (N, ω, R) . This allows us to scrutinize the relationship between principles of fairness imposed in each of these two procedures and principles of end-state fairness. Figure 1 presents a diagram illustrating this approach.

2.1 Claims adjudication

A problem of adjudicating claims, briefly a *claims problem*, consists of a set of agents, a social endowment, and a profile of conflicting claims. Formally, it is a triple (N, Ω, c) such that $\Omega \leq \sum_{i \in N} c_i$. Let $\mathcal{C}$ denote the domain of all claims problems. A *rights-assignment rule* $\varphi: \mathcal{C} \rightarrow$

$\bigcup_{N \in \mathcal{N}} \mathbb{R}_+^{n_\ell}$ associates with each claims problem $(N, \Omega, c) \in \mathcal{C}$, “individual property rights” over the social endowment, that is, an allocation $\varphi(N, \Omega, c) \in Z(N, \Omega)$, with $\varphi_i(N, \Omega, c) \leq c_i$ for each $i \in N$. This inequality condition is called *claims-boundedness*, and it requires that individual property rights not exceed individual claims.¹⁵ We often refer to $\varphi(N, \Omega, c)$ as the (individual) endowment profile, set by rule φ , for the claims problem (N, Ω, c) . A rights-assignment rule φ then converts each economy $e \equiv (N, \Omega, c, R)$ into an exchange economy $e' \equiv (N, \varphi(N, \Omega, c), R)$.

We now define three central rights-assignment rules in the literature. The *constrained equal awards rule* φ^{CEA} splits the social endowment of each good as equally as possible, subject to the condition that no agent is awarded more than her claim. The *constrained equal losses rule* φ^{CEL} splits the social endowment of each good so as to make the losses of all agents as equal as possible, subject to the condition that no agent gets a negative amount (of any good). The *proportional rule* φ^{PRO} equalizes, for each good, the ratios between awards and claims among agents. Formally, for each problem $(N, \Omega, c) \in \mathcal{C}$, each $i \in N$, and each $l \in \{1, \dots, \ell\}$,

$$\varphi_{il}^{CEA}(N, \Omega, c) = \min\{c_{il}, \mu_l\},$$

where $\mu_l > 0$ is chosen so that $\varphi^{CEA}(N, \Omega, c) \in Z(N, \Omega)$;

$$\varphi_{il}^{CEL}(N, \Omega, c) = \max\{0, c_{il} - \lambda_l\},$$

where $\lambda_l > 0$ is chosen so that $\varphi^{CEL}(N, \Omega, c) \in Z(N, \Omega)$; and

$$\varphi_{il}^{PRO}(N, \Omega, c) = \frac{\Omega_l}{\sum_{j \in N} c_{jl}} c_{il}.$$

We now present axioms for rights-assignment rules. First, when there is more to be divided, nobody should lose.¹⁶ Formally,

Resource Monotonicity. A rights-assignment rule φ satisfies *resource monotonicity* if, for each $(N, \Omega, c) \in \mathcal{C}$ and each $\Omega \leq \Omega'$,

$$\varphi(N, \Omega, c) \leq \varphi(N, \Omega', c).$$

¹⁵This does not exclude the possibility of an end-state allocation of a good exceeding the claimed amount, which may result after the subsequent exchange procedure. Note that there is also an implicit (lower) bound condition in the range of the rights-assignment rule, which precludes agents from obtaining negative amounts.

¹⁶This axiom reflects a solidarity principle whose formalization in axiomatic work can be traced back to Roemer (1986).

An implication of *resource monotonicity* is that when the social endowment of a certain good does not change, the allocation of that good should remain unaffected by increases in the social endowment of the other goods.

Another widely applied principle in the axiomatic literature, known as *consistency*, relates the allocation made by a rule at a given problem to the allocation at the “reduced” problems, faced by each subgroup of agents, in which the endowment is the sum of the amounts that have been awarded to them. It requires that the application of the rule to each reduced problem yield the allocation that the subgroup obtained in the original problem.¹⁷ Here we consider two weaker principles pertaining to the exclusion of either agents with zero awards or agents with full awards.¹⁸

The agents who receive nothing in a given dimension of the original problem have nothing to contribute in the ensuing reduced problem. Thus, they may not be of interest to the other agents in the process of reassessment of the original resolution. The next axiom requires that excluding such agents should not alter how much the others get.

Zero-Award-Out-Consistency. A rights-assignment rule φ satisfies *zero-award-out-consistency* if, for each $(N, \Omega, c) \in \mathcal{C}$, each $M \subset N$, and each $l \in \{1, \dots, \ell\}$, if, for each $i \in M$, $\varphi_{il}(N, \Omega, c) = 0$, then, for each $j \in N \setminus M$,

$$\varphi_{jl}(N \setminus M, \sum_{k \in N \setminus M} \varphi_k(N, \Omega, c), c_{N \setminus M}) = \varphi_{jl}(N, \Omega, c).$$

Similarly, the agents whose claims were fully honored will not be interested in any further reassessment. The next axiom requires that dismissing them should not affect how much the others get.

Full-Award-Out-Consistency. A rights-assignment rule φ satisfies *full-award-out-consistency* if, for each $(N, \Omega, c) \in \mathcal{C}$, each $M \subset N$, and each $l \in \{1, \dots, \ell\}$, if, for each $i \in M$, $\varphi_{il}(N, \Omega, c) = c_{il}$, then, for each $j \in N \setminus M$,

$$\varphi_{jl}(N \setminus M, \sum_{k \in N \setminus M} \varphi_k(N, \Omega, c), c_{N \setminus M}) = \varphi_{jl}(N, \Omega, c).$$

¹⁷The reader is referred to Thomson (2012) for a thoughtful discussion of the normative underpinnings underlying the notion of consistency (which are also connected to the principle of solidarity), as well as for a list of references dealing with this notion.

¹⁸The two principles were mentioned in passing by Thomson (2014) for the specific case of (unidimensional) bankruptcy problems. More recently, Harless (2016) has used them in the axiomatic characterization of a broad family of rules for such a context. We consider them here for our more general (multidimensional) framework.

It should be noted that the three rights-assignment rules described above satisfy these two axioms (as well as resource monotonicity).

2.2 Exchange

An *exchange rule*, $F: \bar{\mathcal{E}} \rightarrow Z$, associates with each exchange economy $e \equiv (N, \omega, R) \in \bar{\mathcal{E}}$ a non-empty set of feasible allocations, $F(e) \subseteq Z(N, \Omega)$. Exchange rules are studied extensively in the literature. The best known is the *Walrasian (exchange) rule*, F^W , which associates with each exchange economy the set of allocations in which each agent maximizes her preference satisfaction over her budget set. Formally, for each $e = (N, \omega, R) \in \bar{\mathcal{E}}$, $z^* \equiv (z_i^*)_{i \in N} \in F^W(e)$, if there exists a price vector $p \in \mathbb{R}_+^\ell$, such that, for each $i \in N$, $z_i^* \in B(\omega_i, p) = \{z_i \in \mathbb{R}_+^\ell : p \cdot z_i \leq p \cdot \omega_i\}$ and for each $z_i \in B(\omega_i, p)$, $z_i^* R_i z_i$. These allocations are called *Walrasian allocations*.

We shall also consider other rules that are not Walrasian, yet satisfy the following basic condition for *voluntary* exchange:

Individual Rationality. An exchange rule F satisfies *individual rationality* if, for each $(N, \omega, R) \in \bar{\mathcal{E}}$, each $z \in F(N, \omega, R)$, and each $i \in N$,

$$z_i R_i \omega_i.$$

A trivial instance of an *individually rational* rule is the *No-trade rule*, which always selects the endowment profile, i.e., $F^{NT}(N, \omega, R) = \{\omega\}$, for each $(N, \omega, R) \in \bar{\mathcal{E}}$. Another example is the *Core rule*, F^C , selecting the allocations upon which no coalition of agents can improve.¹⁹ It is well known that the Walrasian rule is a selection of the Core rule, i.e., for each $(N, \omega, R) \in \bar{\mathcal{E}}$, $F^W(N, \omega, R) \subseteq F^C(N, \omega, R)$.

The composition of a rights-assignment rule φ and an exchange rule F gives rise to an allocation rule $S(\cdot) \equiv F \circ \varphi$, that is, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$,

$$S(e) = F(N, \varphi(N, \Omega, c), R).$$

When the exchange rule satisfies *individual rationality*, we refer to the rules so constructed as *market-based* allocation rules. The exchange rule relies on individual preferences and endowments but is not restricted by claims or by any constraint other than *individual rationality*.

¹⁹A coalition of agents $S \subseteq N$ can improve upon an allocation z , when there is a feasible allocation $(z'_i)_{i \in S}$ for coalition S , which makes at least one agent in S better off than at z and all other agents in S at least as well off as at z .

Hence it can take the form of any system of decision making where each agent makes an optimal decision enjoying the freedom of exercising the property rights over her endowment.

2.3 End-state fairness

One of the fundamental notions in the theory of fair allocation is envy-freeness, which can be traced back to Tinbergen (1953) and Foley (1967). The concept has come to play a central role in the theory of fair allocation.²⁰ An allocation satisfies *no-envy*, or is said to be *envy-free*, if no agent prefers someone else's consumption to her own. The next axiom requires that allocation rules should yield only envy-free allocations. Formally,

No-Envy. An allocation rule S satisfies *no-envy* if, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$ and each $z \in S(e)$, there is no pair $i, j \in N$ such that $z_j P_i z_i$.

The above notion disregards claims. The following ones will base envy comparisons on them.

Given an allocation $z \in Z$ and an agent $i \in N$, we call $c_i - z_i$ the *sacrifice* agent i makes at z . An allocation satisfies *sacrifice-no-envy*, or is said to be *sacrifice-envy-free*, if no agent prefers someone else's sacrifice to her own. The next axiom requires that allocation rules should yield only *sacrifice-envy-free* allocations. Formally,

Sacrifice-No-Envy. An allocation rule S satisfies *sacrifice-no-envy* if, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$ and each $z \in S(e)$, there is no pair $i, j \in N$, such that $c_i - (c_j - z_j) \in \mathbb{R}_+^\ell$ and $[c_i - (c_j - z_j)] P_i z_i$.

This notion extends *fair net trades*, introduced by Schmeidler and Vind (1972) for exchange economies, to our problems with disputed properties.

Instead of measuring sacrifices with differences (i.e., in absolute terms), we could measure them with ratios (i.e., in relative terms). Given an allocation $z \in Z$ and an agent $i \in N$, let $c_i/z_i \equiv (c_{i1}/z_{i1}, \dots, c_{i\ell}/z_{i\ell})$ be the *relative sacrifice* agent i makes at z , whenever it is well-defined.²¹ A dual concept is *relative compensation* $z_i/c_i \equiv (z_{i1}/c_{i1}, \dots, z_{i\ell}/c_{i\ell})$. Interpersonal comparisons of relative sacrifices, or relative compensations, provide the same notion of no-envy. Here we consider relative compensations.²² An allocation satisfies *relative-no-envy*, or is said

²⁰See, for instance, Kolm (1972), Pazner and Schmeidler (1974), Feldman and Kirman (1974), and recent surveys, such as Fleurbaey and Maniquet (2011) and Thomson (2011).

²¹That includes the case in which $c_{i\ell} > 0$ and $c_{i\ell}/z_{i\ell} = c_{i\ell}/0 \equiv \infty$.

²²If $z_{i\ell} = 0 = c_{i\ell}$, relative compensation is not well-defined. In this case, we adopt the convention that i is considered to be getting zero relative compensation with regard to good ℓ . If we do not follow this convention, the results we provide in Section 3.3 would require an additional mild consistency axiom to hold.

to be *relative-envy-free*, if no agent prefers someone else's relative compensation to her own. The next axiom requires that allocation rules should yield only *relative-envy-free* allocations. Formally,

Relative-No-Envy. An allocation rule S satisfies *relative-no-envy* if, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$, and each $z \in S(e)$, there is no pair $i, j \in N$, such that $c_i \times (z_j/c_j) P_i z_i$, where the product $\times$ is to be defined as the coordinate-wise multiplication.²³

3 The results

We start this section with a result stating that the family of market-based allocation rules is characterized by an axiom formalizing lower bounds on individual welfare levels, together with an invariance axiom. Lower bound axioms have a long tradition in normative economics and, in particular, are frequently invoked in the literature on fair allocation (e.g., Thomson, 2011). They model the idea that (end-state) outcomes should guarantee agents at least a predetermined welfare level. Precisely, we propose that, for each claims problem, there exists a feasible allocation ω , bounded above by the claims profile, such that the allocation rule guarantees all agents the welfare level they would achieve at ω . Formally,

Lower Bound Property. An allocation rule S satisfies the *lower bound property* if for each claims problem $(N, \Omega, c) \in \mathcal{C}$, there exists a *welfare lower bound*, i.e., an allocation $\omega \in Z(N, \Omega)$ with $\omega \leq c$ such that for each $R \in \mathcal{R}^N$, each $z \in S(N, \Omega, c, R)$, and each $i \in N$,

$$z_i R_i \omega_i.$$

The welfare lower bound for a claims problem, if it exists, is unique, which is shown at the proof of Proposition 1.

Given this property, claims can put a restriction on the choice of allocations through affecting the welfare lower bound. They may further restrict the choice only to compromise freedom of choice. Hence we require them not to impose any further restriction, as stated in the next axiom. It requires that the same allocations should be chosen for any pair of economies with the same preferences and welfare lower bound, despite their differences in claims.

Invariance. An allocation rule S satisfies *invariance* if, for each pair of claims problems

²³For each pair $x = (x_1, \dots, x_\ell), y = (y_1, \dots, y_\ell) \in \mathbb{R}_+^\ell$, let $x \times y \equiv (x_1 y_1, \dots, x_\ell y_\ell)$.

(N, Ω, c) and $(N, \Omega, c') \in \mathcal{C}$ sharing a common welfare lower bound,

$$S(N, \Omega, c, R) = S(N, \Omega, c', R).$$

for each $R \in \mathcal{R}^N$.

The *lower bound property* and *invariance* together characterize the family of market-based allocation rules, as stated in the following result.

Proposition 1 *An allocation rule satisfies the lower bound property and invariance if and only if it is a market-based allocation rule.*²⁴

The proofs of all the results of this section, including Proposition 1, are relegated to the appendix. We devote the rest of the section to axiomatic characterizations of market-based allocation rules involving the three focal rights-assignment rules (constrained equal awards, constrained equal losses, and proportional). The main axioms are the three notions of end-state fairness introduced above.

3.1 No-envy and the constrained equal awards rule

A trivial way of guaranteeing *no-envy* is to divide equally. However, equal division disregards both preferences and claims. Thus, it is typically inefficient and often violates *claims-boundedness*. Consequently, it does not always qualify as an end-state allocation obtained by a market-based allocation rule.

It is well-known that, in exchange economies with individual endowments, *no-envy* and *individual rationality* may be incompatible. If endowments are extremely unequal, any *individually rational* exchange makes an agent with a small endowment envy another agent with a large endowment. The same incompatibility prevails in our economies, when the distribution of claims is very unequal. Therefore, we require *no-envy* only for those economies in which claims are not too unevenly distributed. Formally, consider the domain of economies, denoted by $\mathcal{E}^0$, where the equal division satisfies *claims-boundedness*, i.e., $\mathcal{E}^0 \equiv \{(N, \Omega, c, R) \in \mathcal{E} : \text{for all } i \in N, \Omega/n \leq c_i\}$. Let $\mathcal{C}^0$ be the corresponding domain of claims problems, i.e., $\mathcal{C}^0 \equiv \{(N, \Omega, c) \in \mathcal{C} : \text{for all } i \in N, \Omega/n \leq c_i\}$.

²⁴We thank a referee who pointed out a mistake in the earlier version of the proposition without *invariance*, which is essential for the equivalence. Counterexamples are available upon request.

The first result of this section is that, to guarantee *no-envy*, we should focus on the domain of economies $\mathcal{E}^0$. A domain is said to be a *maximal domain* on which a market-based allocation rule satisfies *no-envy*, if there is a market-based allocation rule satisfying *no-envy* on the domain, and there is no such rule on any larger domain.

Proposition 2 *The domain $\mathcal{E}^0$ is the unique maximal domain on which a market-based allocation rule satisfies no-envy. Moreover, there is an efficient market-based allocation rule satisfying no-envy on this domain.*

The intuition behind Proposition 2 is the following. Suppose that the rights-assignment rule chooses unequal endowments for a pair of agents, i and j . This is necessarily the case in any economy outside the maximal domain, due to *claims-boundedness*. As the rights-assignment rule ignores preferences, this unequal rights-assignment applies for all admissible preference profiles within our domain. Then, there exists a profile of preferences for which j prefers i 's endowment to her own. Now, the end-state allocation, obtained from these endowments through any *individually rational* trade, can be so close to the endowments that j still prefers i 's bundle after trading. Therefore, the unequal rights-assignment for such an economy makes envy inevitable. This explains the first statement of the proposition, as well as why the rights-assignment rule necessarily chooses equal division (which coincides with the constrained equal awards outcome on the maximal domain) whenever possible, as stated in Theorem 1.

As for the second statement, one can just resort to the composition of the constrained equal awards rule and the Walrasian exchange rule. On the maximal domain, such a market-based allocation rule yields *envy-free* and *efficient* end-state allocations. *No-envy* holds because individual budget sets are identical across agents and each agent selects her most preferred bundle in her budget set. *Efficiency* follows from the First Fundamental Theorem of Welfare Economics.

The composition of the constrained equal awards rule and the Walrasian exchange rule is akin to the *Walrasian rule from equal division*, which has emerged as a focal allocation rule in the literature on fair allocation and distributive justice (e.g., Dworkin, 1981; Yaari and Bar-Hillel, 1984; Roemer, 1996; Fleurbaey and Maniquet, 2011). When the equal division is dominated by the claims bound of all agents, they coincide. For classical exchange economies, the Walrasian rule from equal division has been axiomatized in several ways. In particular, Thomson (1988) shows that the Walrasian rule from equal division or its “subcorrespondences” are the only exchange rules that are *envy-free*, *efficient* and independent of “non-essential”

entries of new members.

Now, there are infinitely many *envy-free* and *efficient* allocations at typical economies within the maximal domain $\mathcal{E}^0$. Thus, there are infinitely many allocation rules satisfying *efficiency* and *no-envy* on such a domain. Nevertheless, when one focuses on market-based allocation rules, generated from rights-assignment rules satisfying *resource monotonicity* and *full-award-out-consistency*, only those adopting equal division for the rights-assignment remain, as stated in the next result.

Theorem 1 *Given a rights-assignment rule φ satisfying resource monotonicity and full-award-out-consistency, the following statements are equivalent:*

1. *For some individually rational exchange rule F , the market-based allocation rule $S \equiv F \circ \varphi$ satisfies no-envy on $\mathcal{E}^0$.*
2. *φ is the constrained equal awards rule.*

Hence, the constrained equal award rule is the only rights-assignment rule, among those satisfying *resource monotonicity* and *full-award-out-consistency*, which generates, together with an *individually rational* exchange rule, a market-based allocation rule satisfying *no-envy* on $\mathcal{E}^0$. We emphasize that the proof of Theorem 1 relies on two additional axioms for rights-assignment rules; namely, *resource monotonicity* and *full-award-out-consistency*. Nevertheless, dropping them does not change the result on the maximal domain $\mathcal{E}^0$ (see Corollary 1 in the appendix).

3.2 Sacrifice-no-envy and the constrained equal losses rule

A trivial way of guaranteeing *sacrifice-no-envy* is to allocate equally the total loss (sacrifice) society has to bear. Nevertheless, the equal sacrifice allocation is typically inefficient, as it disregards preferences. Moreover, it may yield negative consumptions for some agents, in which case it is not feasible.

Claims are the maximum sacrifices agents can afford and so agents with relatively small claims tend to experience small sacrifices whereas agents with relatively large claims tend to experience large sacrifices. For such economies, any market-based allocation rule may yield allocations violating *sacrifice-no-envy*. Therefore, we require *sacrifice-no-envy* only for those economies where claims are not too unequally distributed. Formally, for each $(N, \Omega, c) \in \mathcal{C}$, the *equal sacrifice allocation* divides the total sacrifice $(\sum_{j \in N} c_j - \Omega)$ equally among agents, assigning to each agent i the bundle $c_i - (\sum_{j \in N} c_j - \Omega)/n$. Consider the domain of economies,

denoted by $\mathcal{E}^*$, where the equal sacrifice allocation is feasible, i.e., $\mathcal{E}^* \equiv \{e = (N, \Omega, c, R) \in \mathcal{E} : \text{for each } i \in N, 0 \leq c_i - (\sum_{j \in N} c_j - \Omega)/n\}$. Let $\mathcal{C}^*$ be the corresponding domain of claims problems.

The first result of this section is that, if we are interested in guaranteeing *sacrifice-no-envy*, we should focus on $\mathcal{E}^*$. Such a domain is maximal in that respect. In other words, a market-based allocation rule will generate allocations with envy in sacrifices, for any economy outside the domain $\mathcal{E}^*$. No other domain has that property.

Proposition 3 *The domain $\mathcal{E}^*$ is the unique maximal domain on which a market-based allocation rule satisfies sacrifice-no-envy. Moreover, there is an efficient market-based allocation rule satisfying sacrifice-no-envy on this domain.*

The intuition behind the first statement of Proposition 3 can be provided as done earlier for Proposition 2. As for the second statement, note that, on the maximal domain, the composition of the equal sacrifice allocation and the Walrasian exchange yields *sacrifice-envy-free* and *efficient* end-state allocations. *Sacrifice-no-envy* holds in the resulting end-state allocations because the equal sacrifice allocation equalizes the initial sacrifices at the endowments, and the Walrasian exchange provides equal trading opportunities across agents. Therefore, as all agents face equal opportunities for end-state sacrifices (sums of the initial equal sacrifice at endowments and a Walrasian trade), and choose optimally, no one would prefer someone else's sacrifice to her own. *Efficiency* follows from the First Fundamental Theorem of Welfare Economics.

The composition of the constrained equal losses rule and the Walrasian exchange rule does not have a counterpart in the literature on fair allocation, as claims, which are collectively not feasible, are the standard for making fairness comparisons and for determining the endowment profile from which Walrasian exchange takes place. Nevertheless, this rule and the end-state fairness concept (*sacrifice-no-envy*) used for the characterization, resemble the Walrasian rule and the axiom of fair net trade, or *trade-no-envy*, introduced by Schmeidler and Vind (1972) in exchange economies. The Walrasian rule is shown to be the unique rule satisfying “coalitional” *trade-no-envy* if the number of agents is sufficiently large (Gabszewicz, 1975).

Without *resource monotonicity*, the maximal domain result in Proposition 3 does not hold. There are economies where *sacrifice-no-envy* can be satisfied by making agent's sacrifices incomparable. For example, consider an allocation z where for each pair $i, j \in N$, there is a good l such that $c_{il} = 0$ and $c_{jl} - z_{jl} > 0$. Then z is *sacrifice-envy-free* simply because agent

i 's bundle obtained after making j 's sacrifice has a negative component (good l) and so it is incomparable to z_i . There are economies where the incomparability of sacrifices occurs at any feasible allocation,²⁵ and adding these economies to $\mathcal{E}^*$, we may find a larger domain for a *sacrifice-envy-free* market-based allocation rule. *Resource monotonicity* prevents allocation rules from abusing the “trick” of making sacrifices incomparable, which allows us to get the maximal domain result stated in the proposition.

Now, infinitely many allocation rules satisfy *efficiency* and *sacrifice-no-envy* within the maximal domain $\mathcal{E}^*$. Nevertheless, when one focuses on market-based allocation rules generated from rights-assignment rules satisfying *resource monotonicity* and *zero-award-out-consistency*, only those adopting equal sacrifice for the rights-assignment remain, as stated in the next result.

Theorem 2 *Given a rights-assignment rule φ satisfying resource monotonicity and zero-award-out-consistency, the following statements are equivalent:*

1. *For some individually rational exchange rule F , the market-based allocation rule $S \equiv F \circ \varphi$ satisfies sacrifice no-envy on $\mathcal{E}^*$.*
2. *φ is the constrained equal losses rule.*

Hence the constrained equal losses rule is the only rights-assignment rule, among those satisfying *resource monotonicity* and *zero-award-out-consistency*, which generates, together with an *individually rational* exchange rule, a market-based allocation rule satisfying *sacrifice-no-envy* on $\mathcal{E}^*$. We emphasize that the proof of Theorem 2 relies on two additional axioms for rights-assignment rules; namely, *resource monotonicity* and *zero-award-out-consistency*. Nevertheless, dropping them does not change the result for two-agent economies without excessive claims (that is, when everyone's claim on each good is below the social endowment), as we explain later in Remark 1.

3.3 Relative-no-envy and the proportional rule

We explore in this section the implications of *relative-no-envy*. In contrast with the previous two sections, one can find a rights-assignment rule that, when composed with an *individually rational* exchange rule, generates an allocation rule satisfying *relative-no-envy* on the “universal” domain $\mathcal{E}$. A simple example is the composition of the proportional rights-assignment and the no-trade

²⁵Examples are all two-agents economies where the equal sacrifice allocation gives each agent a negative amount of a good.

exchange rule. Thus, the maximal domain on which a market-based allocation rule satisfies *relative-no-envy* is the universal domain. This is in contrast with the counterpart results for *no-envy* (Proposition 2) and *sacrifice-no-envy* (Proposition 3). Nevertheless, as stated in the next proposition, *relative-no-envy* and *efficiency* are incompatible, which is also in contrast with the mentioned results.²⁶

Proposition 4 *There is a market-based allocation rule satisfying relative-no-envy on the universal domain. However, relative-no-envy and efficiency are incompatible.*

In contrast with the results of the previous sections involving other notions of *no-envy*, the Walrasian rule cannot be used to prove the compatibility of *relative-no-envy* and *efficiency*, as Walrasian allocations from the proportional rights-assignment may violate *relative-no-envy*, as we show. In the case of *no-envy*, the (Walrasian) budget sets given by the equal division provide equal opportunities for consumptions, which guarantees *no-envy* at the end-state Walrasian allocations. Likewise, the budget sets provide equal opportunities for trades. Thus, when the endowment profile is chosen at the equal sacrifice allocation, the budget sets provide equal opportunities for “absolute sacrifices”, which guarantees *sacrifice-no-envy* at the end-state Walrasian allocations. However, the budget sets given by the proportional rights-assignment do not provide equal opportunities for “relative sacrifices”. This is why Walrasian exchange, though composed with the proportional rights-assignment, does not guarantee *relative-no-envy*.

Now, we are ready to state the third characterization result.

Theorem 3 *Given a rights-assignment rule φ , the following statements are equivalent:*

1. *For some individually rational exchange rule F , the market-based allocation rule $S \equiv F \circ \varphi$ satisfies relative-no-envy.*
2. *φ is the proportional rule.*

Hence the proportional rule is the only rights-assignment rule that generates, together with an *individually rational* exchange rule, a market-based allocation rule satisfying *relative-no-envy*.

Partly motivated by the somewhat different nature of the results for *relative-no-envy*, we conclude by considering an alternate method of measuring “rates” of sacrifices, or rewards, and

²⁶As it can be checked in its proof, the incompatibility remains valid in the standard model of exchange economies.

the corresponding axiom of no-envy. As we shall see, and in contrast with *relative-no-envy*, this new notion is compatible with *efficiency*.

Formally, let z be a feasible allocation for an economy $e \equiv (N, \Omega, c, R)$. For each $i \in N$, denote by p^i a *supporting normal vector of i 's indifference set at z_i* , that is, for each x with $x R_i z_i$, $p^i \cdot x \geq p^i \cdot z_i$. Then, p^i gives agent i 's marginal rate of substitution between any two goods. It is a price vector representing i 's subjective valuation of goods at the margin, up to normalization. Note that $\frac{z_i}{p^i \cdot c_i}$ is the vector consisting of the amounts of each good agent i gets for each unit value of her claim, namely, *per value consumption*. Likewise, from i 's point of view, $\frac{z_j}{p^i \cdot c_j}$ is the vector consisting of the amounts of each good agent j gets for each unit value (measured by i 's price vector) of j 's claim. The comparative axiom of fairness with regard to these vectors (per value consumptions) can be defined as follows.

We say that z satisfies *ratio-no-envy* at $e \equiv (N, \Omega, c, R)$ if there is no pair $i, j \in N$ such that for each supporting normal vector p^i of i 's indifference curve at z_i ,

$$p^i \cdot c_i \times \frac{z_j}{p^i \cdot c_j} P_i z_i.$$

It turns out that there exist allocations satisfying *ratio-no-envy* and *efficiency*. They can be obtained using the concept of *proportional Walrasian allocation* (e.g., Thomson, 1992) defined next for our setting. An allocation z is a *proportional Walrasian allocation* for $e \equiv (N, \Omega, c, R) \in \mathcal{E}$ if there is a price vector p such that z is obtained as a Walrasian allocation with individual income profile $(w_i)_{i \in N}$ determined by dividing the value of the social endowment $p \cdot \Omega$ in proportion to the values of claims, $(p \cdot c_i)_{i \in N}$, i.e., for each $i \in N$,

$$w_i \equiv \frac{p \cdot c_i}{p \cdot \sum_{j \in N} c_j} p \cdot \Omega.$$

Note that equilibrium price vector p is a “common” supporting normal vector of the indifference set of each agent at z . Using p , it can be shown that the proportional division allows all agents to have equal opportunities of per value consumption, which makes the proportional Walrasian allocations *ratio-envy-free*.

Proposition 5 *There is an allocation rule satisfying ratio-no-envy and efficiency on the universal domain.*

Proposition 4 showed that *relative-no-envy* (the counterpart of the notions of *no-envy* and *sacrifice-no-envy*) is incompatible with *efficiency*. Proposition 5 shows that the alternative of

ratio-no-envy can be reconciled with *efficiency*. Nevertheless, this result for *ratio-no-envy* calls for a different approach to our problems, as the proportional Walrasian rule assigns individual incomes using an equilibrium price vector that depends on preferences. This is discarded by our two-step approach where any information on preferences is not needed for rights-assignment. A similar route could be taken to define alternatives to *no-envy* and *sacrifice-no-envy*, in which prices would also be involved in the determination of initial incomes. This seems unnecessary given that the two original concepts can indeed be combined with *efficiency*, as shown by Propositions 2 and 3.

4 An application: Greenhouse gas emission

As mentioned in the introduction, an interpretation of our model is one of *cap-and-trade* systems for greenhouse gas (GHG) emission. We develop such an interpretation in this section.

Consider a simple model of energy production. There are two goods: the energy good e and money m . The energy good is produced by using money as the input good. The production technology exhibits constant returns to scale with marginal cost $\kappa \geq 0$. Each agent $i \in N$ is endowed with $\bar{m}_i$ units of money and i 's preferences are represented by a quasi-linear utility function $u_i(m_i, e_i) = m_i + v_i(e_i)$, where $v_i(\cdot)$ satisfies $v'_i(\cdot) > 0$, $v''_i(\cdot) < 0$, and $\lim_{e_i \rightarrow \infty} v'_i(e_i) = 0$. Given energy price $p > 0$, agent i attains her maximal preference satisfaction at $(\bar{m}_i - pe_i^*, e_i^*)$, where e_i^* is the unique solution to $p = v'_i(e_i)$. Let $d_i(p) \equiv (v'_i)^{-1}(p)$ be i 's energy demand function and $D(p) \equiv \sum_{i \in N} d_i(p)$ the market demand function. Under these conditions, it is not difficult to show that there is a unique competitive equilibrium with energy price $p = \kappa$ and allocation $(m_i^{CE}, e_i^{CE})_{i \in N}$ such that, for each $i \in N$, $e_i^{CE} \equiv d_i(\kappa)$ and $m_i^{CE} = \bar{m}_i - \kappa e_i^{CE}$.

Cap-and-trade The total energy consumption should be reduced to $E \leq \sum_{i \in N} e_i^{CE}$, via a cap-and-trade scheme. Such a scheme requires *permits*, which can be traded in a competitive market, to consume units of energy. A *cap-and-trade equilibrium* consists of a permit assignment $(\omega_i^e)_{i \in N}$, an energy price $p^* = \kappa$, a permit price r^* and an allocation $(m_i^*, e_i^*)_{i \in N}$ such that $\sum_{i \in N} \omega_i^e = E = \sum_{i \in N} e_i^*$, and, for each $i \in N$, (m_i^*, e_i^*) maximizes agent i 's utility, within the budget set given by her money endowment and the value of her permit at those prices.

Augmenting the concept with redistributive transfers, an *extended cap-and-trade equilibrium* consists of a permit assignment $(\omega_i^e)_{i \in N}$, an income transfer $(\tau_i)_{i \in N}$, an energy price $p^* = \kappa$, a permit price r^* , and an allocation $(m_i^*, e_i^*)_{i \in N}$ such that $\sum_{i \in N} \omega_i^e = E = \sum_{i \in N} e_i^*$, $\sum_{i \in N} \tau_i = 0$

and, for each $i \in N$, (m_i^*, e_i^*) maximizes agent i 's utility, within the budget set given by her money endowment, her income transfer and the value of her permit at those prices.

The extended cap-and-trade schemes are thus composed of the assignment of permits and transfers, and *individually rational* market exchange of the two goods and permits. They resemble our market-based allocation rules, and the first procedure of assigning emission rights and transfers plays a critical role in making the final outcome fair. We will apply our results here to obtain unique fair assignments in this setting.

The current model of energy production differs from our model dealing with exchange economies (without production and without permits). Hence, for the application of our results, we need to establish a correspondence between the two models. We next show that, after an appropriate transformation, the current model of energy production can be treated as an example of our model, and the cap-and-trade equilibrium as a Walrasian equilibrium.

Claims adjudication and exchange For each $i \in N$, let $\bar{e}_i$ denote the benchmark energy consumption for agent i .²⁷ Then, i 's claim is given by $c_i \equiv (\bar{m}_i, \bar{e}_i)$.²⁸ Let $\Omega \equiv (\sum_{i \in N} \bar{m}_i - \kappa E, E)$ be the social endowment, and R a profile of quasilinear preferences, as described above. Thus, an economic environment in this case is also formalized by $e \equiv (N, \Omega, c, R)$, as in Section 2. We then have the following correspondence between extended cap-and-trade equilibria and Walrasian equilibria, whose proof is provided also in the appendix.

Proposition 6 *The allocation obtained as an extended cap-and-trade equilibrium with permit assignment $(\omega_i^e)_{i \in N}$ and transfer $(\tau_i)_{i \in N}$ coincides with the allocation obtained as a Walrasian equilibrium from the (rights-assignment) endowment profile $(\bar{m}_i + \tau_i - \kappa \omega_i^e, \omega_i^e)_{i \in N}$.*

For ease of exposition, we focus on economies where each agent claims sufficiently large amounts of both goods and, thus, concentrate on the domain $\mathcal{E}^0 \cap \mathcal{E}^*$.

It then follows from Theorem 1 and Proposition 6 that, under the presence of *resource monotonicity* and *full-award-out-consistency* for the rights-assignment, *the extended cap-and-trade scheme satisfies no-envy only if all agents are assigned equal amounts of permits and*

²⁷This reflects current energy needs and satisfies $\sum_{i \in N} \bar{e}_i = \sum_{i \in N} e_i^{CE}$. In particular, one may consider the equilibrium allocation as the benchmark energy needs, i.e., $\bar{e}_i = e_i^{CE}$, for each $i \in N$.

²⁸Note that, in the context of reducing GHG emissions, the Kyoto Protocol imposes no emission reduction on developing nations, which, in our model, means assigning a greater amount of emission permits than the claimed amount (emission in a base year) by each developing nation. This is the key criticism to the Kyoto Protocol by Posner and Weisbach (2010, p.3). Our model imposes *claims boundedness* at the initial stage of rights assignment. But, as mentioned above, it does not preclude the possibility of agents obtaining amounts above their claims after the second stage of exchange from initial rights takes place.

*transfers equalizing their (post-transfer) money endowments.*²⁹

Similarly, it follows from Theorem 2 and Proposition 6 that, under the presence of *resource monotonicity* and *zero-award-out-consistency* for the rights-assignment, *the extended cap-and-trade scheme satisfies sacrifice-no-envy only if all agents are assigned the permits and transfers equalizing their (energy and money) sacrifices.*³⁰

Finally, the proportional rights-assignment is obtained as the initial endowment of the cap-and-trade scheme when each agent receives the proportional permit assignment and the proportional transfer.³¹ Unlike the two rights-assignments considered earlier, Theorem 3 and the proportional rights-assignment cannot be applied to advocate the corresponding cap-and-trade scheme.

5 Concluding remarks

We have presented a general model of exchange economies to study the allocation of disputed properties, while accommodating the three levels in which fairness can be scrutinized in this context; namely, fairness in the initial assignment of rights on disputed properties, fairness in the transaction of the assigned rights, and fairness of the end-state allocation. We have focused, in such a context, on allocation rules that are represented by a composition of a rights-assignment rule (to assign each profile of conflicting claims an endowment profile) and Walrasian or other *individually rational* exchange rules. Our main results are characterizations of three egalitarian rights-assignment rules. In particular, we have characterized two rights-assignment rules (constrained equal awards rule and constrained equal losses rule) that, when composed with Walrasian exchange, give rise to *efficiency*, as well as two related forms of *no-envy*. The third rights-assignment rule (the proportional rule) gives rise to a third form of *no-envy* which is, however, incompatible with *efficiency*.

Prior investigation on using *no-envy* as both procedural and end-state principles of fairness has produced negative results (e.g., Kolm; 1972; Feldman and Kirman, 1974). For instance, the combination of *envy-free* initial allocation (equal division) and a sequence of *envy-free* trades may lead to a core allocation with envy. Our three main characterization results impose three versions of *no-envy* as the principle of end-state fairness and obtain *no-envy* (with some

²⁹That is, for each $i \in N$, $\omega_i^e = E/n$ and $\tau_i = \sum_{j \in N} \bar{m}_j/n - \bar{m}_i$.

³⁰That is, for each $i \in N$, $\omega_i^e = \bar{e}_i - \{\sum_{j \in N} \bar{e}_j - E\}/n$ and $\tau_i = \kappa (\bar{e}_i - \{\sum_{j \in N} \bar{e}_j\}/n)$.

³¹That is, for each $i \in N$, $\omega_i^e = E\bar{e}_i/\sum_{j \in N} \bar{e}_j$ and $\tau_i = \kappa E (\bar{e}_i/\sum_{j \in N} \bar{e}_j - \bar{m}_i/\sum_{j \in N} \bar{m}_j)$.

constraints) of the initial allocation as an implication. We do not insist on *no-envy* as a requirement for rights-assignment. Instead, we impose other standard axioms (such as *claims-boundedness*, *resource monotonicity*, and *consistency* for rights-assignment rules, and *individual rationality* and *efficiency* for exchange rules) in the two procedures.³²

One might argue that some of our requirements for the two procedures are not entirely appealing in our setting. For instance, we acknowledge that *resource monotonicity* uses the objective standard of comparison of the effects of resource changes on individuals, instead of using the subjective standard of well-being. Now, this makes the first step of our process informationally simple. We believe that this is, actually, a merit instead of a shortcoming of our approach. This parsimony may be problematic if it would lead to incompatibility with other important preference-related requirements. But our results are mostly constructive. Furthermore, without *resource monotonicity*, similar results for two-agent economies hold.

In standard exchange economies (with homothetic preferences), Moulin and Thomson (1988) consider *resource monotonicity* as an end-state axiom. They show that it is not compatible with *efficiency* and *no-envy*. Therefore, requiring the end-state *resource monotonicity* is too strong in our framework. We impose it only in the rights-assignment procedure. That is, when the resource increases, every agent, whatever her preferences are, should be at least as well-off as earlier at the rights-assignment stage. Nevertheless, agents may be worse off in the end-state; so the market-based allocation rule violates end-state resource monotonicity. But this worsened end-state is due to their own voluntary decision in the exchange procedure, and so it is not a moral concern in our approach. This way, *resource monotonicity* becomes less demanding and allows us to generate constructive conclusions.

Alternatively, we could have considered a weakening of *resource monotonicity* stating that when the social endowment of a certain good does not change, the allocation of that good should remain unaffected by increases in the social endowment of the other goods. This axiom would be enough to prove our results. Due to the fact that the original *resource monotonicity* axiom has a long tradition in axiomatic work (especially in the literature dealing with claims problems or fair allocations), we have decided to use that axiom in the paper. Other related axioms, with the same informational requirements and equally parsimonious, would not be valid to obtain our results. An instance would be an axiom stating that, as the endowment increases, nobody receives a smaller amount of every good (which allows for the opportunity

³²Our model and the procedural approach follow the lesson on procedural fairness delineated by Thomson (2011, pp.419-422).

that, upon the increase of the endowment of one good, an agent receives a smaller award of another good, provided she received more of the good which the endowment increased, which seems natural in some situations).

In standard exchange economies, Thomson (1983) is also concerned with the three levels of justice: fair initial position (endowment), fair trade, and fair end-state. In his approach, the principle of fair trade plays a central role and the principle of fair initial position is formulated through the exchange of the initial positions (as in the definition of no-envy) and their objections based on the principle of fair trade from any reshuffled position. His main result says that if one accepts Walrasian trade to be a fair trade rule and the possibility of exchanging initial positions, then the only fair initial position is equal division. Our Theorem 1 reinforces this conclusion in the extended framework and using a different procedural approach. Unlike Thomson (1983), we use *no-envy* as the end-state fairness axiom and characterize egalitarian rights-assignment rules for the first procedure.

Our model is also related to a model introduced by Thomson (1992, 2007).³³ The aim in that model is to formulate notions of consistency for exchange economies augmented by a social deficit. Mathematically, the model of “deficit-sharing exchange economies” (on p.184, Thomson, 2007) is identical to ours, if the deficit T in his model is defined to be the difference between the sum of the claims and the aggregate endowment in our model. However, we do not impose consistency on our allocation rule, but only on our rights-assignment rules. Thus, due to the *claims-boundedness* property of rights-assignment rules, defining consistency is not an issue in our context.

The validity of our three end-state fairness axioms should be judged context by context. In some applications, claims may be perceived as indicating “credits”, and in other applications, as indicating “discredits”. In bankruptcy problems, claims are amounts invested; the greater they are, the more credits are attributed to the claimants. On the other hand, in allocating pollution permits, claims may represent current or past emissions and greater claims mean greater discredits, or greater responsibility for environmental damage. When claims are credits, they deserve special attention in judging fairness and so *no-envy*, which disregards claims, does not qualify as a valid criterion. In such a scenario, our results pin down the proportional and the constrained equal losses rules as the only fair rights-assignment rules. When claims are discredits, using them in judging fairness, as in *sacrifice-no-envy* and *relative-no-envy*, does not agree with our moral intuition. Then, our results point towards constrained equal awards

³³See also Peleg (1996) and Korthues (2000).

as the unique fair rights-assignment rule. This is our justification for proposing equal per capita allocation of GHG emission rights. The axiom of *no-envy* does not directly involve the allocation of rights; it emphasises the welfare consequences of this allocation upon individual agents. Section 4 gives a formal presentation of this case as an example of our model.

6 Appendix

We provide in this appendix the proofs of all the results stated throughout the paper. We start with the proof of Proposition 1.

Proof of Proposition 1. We first show that the welfare lower bound for a claims problem, when it exists, is unique. Suppose, by contradiction, that there is an allocation rule S , and a claims problem (N, Ω, c) , for which there exist two welfare lower bounds ω and ω' . Then, there exist $i \in N$ and $p \in \mathbb{R}_{++}^\ell$ such that $p \cdot \omega_i < p \cdot \omega'_i$. Let $R \in \mathcal{R}^N$ be such that

- (i) R_i is the linear preference relation represented by $u(x) \equiv p \cdot x$,
- (ii) For each $h \neq i$, R_h is strictly convex and strongly monotonic, and
- (iii) ω is Pareto efficient at R .

Now, suppose that there is $z \in S(N, \Omega, c, R) \setminus \{\omega\}$. As ω is a welfare lower bound, it follows that, for each $j \in N$, $z_j R_j \omega_j$. As $z \neq \omega$, it follows by (iii) that $z_j I_j \omega_j$, for each $j \in N$. Let $\lambda \in (0, 1)$ and $z^\lambda = \lambda z + (1 - \lambda)\omega$. By (i), $z_i^\lambda I_i \omega_i$. By (ii), $z_j^\lambda P_j \omega_j$, for each $j \in N$. Altogether, we have a contradiction with (iii). Hence, $S(N, \Omega, c, R) \equiv \{\omega\}$. Now, by (i), $\omega'_i P_i \omega_i$, which contradicts the assumption that ω' was a welfare lower bound for (N, Ω, c) .

Next, to prove the “only-if” part, let S be an allocation rule satisfying the *lower bound property* and *invariance*. Let $\varphi: \mathcal{C} \rightarrow \bigcup_{N \in \mathcal{N}} \mathbb{R}_+^{n\ell}$ be such that, for each $(N, \Omega, c) \in \mathcal{C}$, $\varphi(N, \Omega, c) = \omega$, where ω is the welfare lower bound (for (N, Ω, c)) whose existence is guaranteed by the *lower bound property*. As shown earlier, the welfare lower bound is unique and so, φ is a well-defined rights-assignment rule. Let Φ denote the image of φ and $\bar{\mathcal{E}}^\Phi$ the domain of exchange economies generated by endowment profiles in Φ . Then, let $F: \bar{\mathcal{E}} \rightarrow Z$ be such that, for each $(N, \omega, R) \in \bar{\mathcal{E}}$,

$$F(N, \omega, R) = \begin{cases} S(N, \Omega, c, R), & \text{if } (N, \omega, R) \in \bar{\mathcal{E}}^\Phi \\ \{\omega\}, & \text{otherwise,} \end{cases}$$

where $c \in \mathbb{R}_{++}^{n\ell}$ is such that $\varphi(N, \Omega, c) = \omega$. By *invariance*, the value of F is independent of choosing another claims profile c' with $\varphi(N, \Omega, c') = \omega$. Hence, F is well-defined. By the *lower*

bound property, F is *individually rational*. Therefore, as $S(N, \Omega, c, R) = F(N, \varphi(N, \Omega, c), R)$, for each $(N, \Omega, c, R) \in \mathcal{E}$, it follows that S is a market-based allocation rule.

Conversely, let S be a market-based allocation rule. Then, there exist a rights-assignment rule φ and an exchange rule F satisfying *individual rationality*, such that, for each $e \equiv (N, \Omega, c, R) \in \mathcal{E}$, $S(e) = F(N, \varphi(N, \Omega, c), R)$. For each $(N, \Omega, c) \in \mathcal{C}$, let $\omega \equiv \varphi(N, \Omega, c)$. Then, by *claims-boundedness*, $\omega \leq c$. Furthermore, by *individual rationality*, it follows that, for each $R \in \mathcal{R}^N$, each $z \in S(e)$, and each $i \in N$, $z_i R_i \omega_i$. Hence, ω is the welfare lower bound for (N, Ω, c) , which shows that S satisfies the *lower bound property*. Furthermore, as S is the composition of φ and F , it follows that, for each $c' \in \mathbb{R}_{++}^\ell$ with $\varphi(N, \Omega, c') = \omega$, $S(N, \Omega, c, R) = S(N, \Omega, c', R)$, for each $R \in \mathcal{R}^N$. Thus, S satisfies *invariance*. ■

We now present a preliminary lemma that is used for proving Proposition 2 and Theorem 1.

Lemma 1 *Let φ be a rights-assignment rule and let F be an individually rational exchange rule. If the market-based allocation rule $S \equiv F \circ \varphi$ satisfies no-envy, then, for each $(N, \Omega, c) \in \mathcal{C}$, $\varphi(N, \Omega, c) = (\Omega/n, \dots, \Omega/n)$.*

Proof. Let F and φ be as in the lemma. Suppose, by contradiction, that there exists $(N, \Omega, c) \in \mathcal{C}$ such that $\omega \equiv \varphi(N, \Omega, c) \neq \Omega/n$. Then, as $\sum_{k \in N} \omega_k = \Omega$, there exist $i, j \in N$ and $p \in \mathbb{R}_{++}^\ell$ such that $p \cdot \omega_i < p \cdot \frac{\Omega}{n} < p \cdot \omega_j$. Let $e = (N, \Omega, \omega, R) \in \mathcal{E}$ be such that R_i is represented by $U_i: x \rightarrow p \cdot x$, R_h is strictly convex for each $h \in N \setminus \{i\}$, and ω is Pareto efficient at R . Then, ω is the only feasible allocation that satisfies individual rationality.³⁴ Hence, $\omega = F(N, \omega, R)$. As $p \cdot \omega_j > p \cdot \omega_i$, agent i prefers j 's bundle to his own, contradicting no-envy. ■

The first statement of Proposition 2 follows immediately from Lemma 1. As for its second statement, it suffices to consider the constrained equal awards rule composed with the Walrasian exchange rule.

Proof of Theorem 1. The composition of the constrained equal awards rule and the Walrasian exchange rule proves that the second statement implies the first statement. In order to prove the converse, let φ be a rights-assignment rule satisfying *resource monotonicity* and *full-award-out-consistency* that, when composed with an *individually rational* exchange rule F , leads to a market-based allocation rule $S \equiv F \circ \varphi$ satisfying *no-envy* on $\mathcal{E}^0$.

³⁴This is because if there is any other *individually rational* allocation z , a convex combination of z and ω will be a Pareto improvement of ω , contradicting Pareto efficiency of ω .

For each $l \in \{1, \dots, \ell - 1\}$, let $\mathcal{C}(l) \equiv \{(N, \Omega, c) \in \mathcal{C} : \forall k \geq l + 1, \forall i \in N, \Omega_k/|N| \leq c_{ik}\}$. Let $\mathcal{C}(\ell) \equiv \mathcal{C}$ and $\mathcal{C}(0) \equiv \mathcal{C}^0$. We show that for each $k = 0, 1, \dots, \ell$, φ coincides with φ^{CEA} on $\mathcal{C}(k)$, by induction on the number of commodities k . By Lemma 1, φ coincides with φ^{CEA} on $\mathcal{C}(0)$. Let $l \in \{1, \dots, \ell\}$. Suppose, by induction, that φ coincides with φ^{CEA} on $\mathcal{C}(k)$ for each $k \leq l - 1$. We prove that φ coincides with φ^{CEA} on $\mathcal{C}(l)$.

Let $N \in \mathcal{N}$ and $c \equiv (c_i)_{i \in N} \in \mathbb{R}_{++}^{n\ell}$. Let $\mathcal{C}(N, c)(l)$ be the domain of claims problems in $\mathcal{C}(l)$ with population N and claims profile c . In what follows, we prove that φ coincides with φ^{CEA} on $\mathcal{C}(N, c)(l)$. For ease of exposition, assume, without loss of generality, that $N = \{1, \dots, n\}$ and that claims on the l -th commodity are increasingly ordered, i.e., $c_{1l} \leq \dots \leq c_{nl}$. For each $m \in \{1, 2, \dots, n - 1\}$, let³⁵

$$\mathcal{C}(N, c)(l)^m = \{(N, \Omega, c) \in \mathcal{C}(N, c)(l) : \sum_{i \leq m-1} c_{il} + (n - m + 1)c_{ml} \leq \Omega_l \leq \sum_{i \leq m} c_{il} + (n - m)c_{m+1,l}\}.$$

Furthermore, let

$$\mathcal{C}(N, c)(l)^0 = \{(N, \Omega, c) \in \mathcal{C}(N, c)(l) : \Omega_l \leq nc_{1l}\}.$$

Then, it is straightforward to see that $\mathcal{C}(N, c)(l) = \bigcup_{m=0}^{n-1} \mathcal{C}(N, c)(l)^m$. For each $m = 0, 1, \dots, n - 1$ and each $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^m$,

$$\varphi_l^{CEA}(N, \Omega, c) = (c_{1l}, \dots, c_{ml}, \lambda_m, \dots, \lambda_m), \quad (1)$$

where $\lambda_m = [\Omega_l - \sum_{i=1}^m c_{il}]/(n - m)$.

We now show, by induction on the number of agents m , that for each $m \in \{0, 1, \dots, n - 1\}$ and each $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^m$, $\varphi(N, \Omega, c) = \varphi^{CEA}(N, \Omega, c)$. Note that, as $\mathcal{C}(N, c)(l)^0 \subset \mathcal{C}(l - 1)$, φ coincides with φ^{CEA} on $\mathcal{C}(N, c)(l)^0$. Let $m \in \{1, 2, \dots, n - 1\}$. Suppose that if $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^{m-1}$, $\varphi(N, \Omega, c) = \varphi^{CEA}(N, \Omega, c)$. Let $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^m$. We aim to show that $\varphi(N, \Omega, c) = \varphi^{CEA}(N, \Omega, c)$.

Let $\Omega'_l \equiv \sum_{i \leq m-1} c_{il} + (n - m + 1)c_{ml}$ and $\Omega' \equiv (\Omega'_l, \Omega_{-l})$. Then, $(N, \Omega', c) \in \mathcal{C}(N, c)(l)^{m-1} \cap \mathcal{C}(N, c)(l)^m$. As $(N, \Omega', c) \in \mathcal{C}(N, c)(l)^{m-1}$,

$$\varphi(N, \Omega', c) = \varphi^{CEA}(N, \Omega', c). \quad (2)$$

As, for each $l' \neq l$, $\Omega'_{l'} = \Omega_{l'}$, then by *resource monotonicity*, $\varphi_{l'}(N, \Omega, c) = \varphi_{l'}(N, \Omega', c)$ and

³⁵We use the notational convention that $\sum_{i \leq 0} c_{il} = 0$.

$\varphi_{l'}^{CEA}(N, \Omega, c) = \varphi_{l'}^{CEA}(N, \Omega', c)$. Hence, using (2), we obtain: for each $l' \neq l$,

$$\varphi_{l'}(N, \Omega, c) = \varphi_{l'}^{CEA}(N, \Omega, c). \quad (3)$$

We only have to show that (3) holds also for $l' = l$. As $(N, \Omega', c) \in \mathcal{C}(N, c)(l)^m$, then by (1) and (2), for each $i \leq m$, $\varphi_{il}(N, \Omega', c) = c_{il}$. As $\Omega'_l \equiv \sum_{i \leq m-1} c_{il} + (n - m + 1)c_{ml} \leq \Omega_l$ (the inequality holds because $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^m$), then by *resource monotonicity* and *claims boundedness*, for each $i \leq m$, $\varphi_{il}(N, \Omega, c) = c_{il}$. Hence, by (1), for each $i \leq m$, $\varphi_{il}(N, \Omega, c) = \varphi_{il}^{CEA}(N, \Omega, c)$.

If $m = n - 1$, then it follows from the above result and the resource constraint that $\varphi_{nl}(N, \Omega, c) = \varphi_{nl}^{CEA}(N, \Omega, c)$, completing the proof.

If $m < n - 1$, let $M \equiv \{1, \dots, m\}$ and $\bar{\Omega} \equiv \Omega - \sum_{i \in M} \varphi_i(N, \Omega, c)$. Consider $(N \setminus M, \bar{\Omega}, c_{N \setminus M})$. Note that, for each $l' \geq l + 1$ and each $i = 1, \dots, m$, $\varphi_{il'}(N, \Omega, c) = \Omega_{l'}/n$ and, for each $i \in N$, $\Omega_{l'}/n \leq c_{il'}$. Then, for each $i \in N$, $\bar{\Omega}_{l'}/(n - m) = \Omega_{l'}/n \leq c_{il'}$. Moreover, from $\bar{\Omega}_l/(n - m) = (\Omega_l - \sum_{i=1}^m c_{il})/(n - m) \leq c_{m+1,l} \leq \dots \leq c_{nl}$ (the first inequality holds because $(N, \Omega, c) \in \mathcal{C}(N, c)(l)^m$), it follows that $(N \setminus M, \bar{\Omega}, c_{N \setminus M}) \in \mathcal{C}(l - 1)$. Thus, $\varphi(N \setminus M, \bar{\Omega}, c_{N \setminus M}) = \varphi^{CEA}(N \setminus M, \bar{\Omega}, c_{N \setminus M})$.

By *full-award-out-consistency*, for each $i \in N \setminus M$, $\varphi_{il}(N, \Omega, c) = \varphi_{il}(N \setminus M, \bar{\Omega}, c_{N \setminus M})$ and $\varphi_{il}^{CEA}(N, \Omega, c) = \varphi_{il}^{CEA}(N \setminus M, \bar{\Omega}, c_{N \setminus M})$. Therefore, for each $i = m + 1, \dots, n$, $\varphi_{il}(N, \Omega, c) = \varphi_{il}^{CEA}(N, \Omega, c)$, completing the proof that (3) holds also for $l' = l$. ■

The proof of Lemma 1 and, thus, that of Proposition 2, rely on neither *resource monotonicity* nor *full-award-out-consistency*. It is clear then that the same result as in Theorem 1 holds on $\mathcal{E}^0$, without imposing the two solidarity axioms for rights-assignment rules. Formally,

Corollary 1 *Given a rights-assignment rule φ defined on domain $\mathcal{C}^0$, the following statements are equivalent:*

1. *For some individually rational exchange rule F , the market-based allocation rule $S \equiv F \circ \varphi$, defined on domain $\mathcal{E}^0$, satisfies no-envy.*
2. *φ is the constrained equal awards rule.*

We now move to the results in Section 3.2. In order to prove Proposition 3 and Theorem 2, we need a preliminary lemma.

Lemma 2 *Let φ be a rights-assignment rule satisfying resource monotonicity, and let F be an exchange rule satisfying individual rationality. If the market-based allocation rule $S \equiv F \circ \varphi$ satisfies sacrifice-no-envy, then, for each $(N, \Omega, c) \in \mathcal{C}$, $\varphi(N, \Omega, c) = (c_i - (\sum_{i \in N} c_i - \Omega)/n)_{i \in N}$.*

Proof. Let F and φ be as in the lemma. Let $\omega^{es} \equiv (c_i - (\sum_{i \in N} c_i - \Omega)/n)_{i \in N}$ be the equal sacrifice allocation. Suppose, by contradiction, that $\omega = \varphi(N, \Omega, c) \neq \omega^{es}$. Then, there are $i, j \in N$ such that $\omega_i - c_i \neq \omega_j - c_j$. We distinguish two cases:

Case 1. $c_i - [c_j - \omega_j] \in \mathbb{R}_+^\ell$.

Subcase 1.1. Neither $c_i - [c_j - \omega_j] \succeq \omega_i$ nor $c_i - [c_j - \omega_j] \preceq \omega_i$.

Then there is $p \in \mathbb{R}_{++}^\ell$ such that

$$p \cdot \omega_i = p \cdot (c_i - [c_i - \omega_i]) < p \cdot (c_i - [c_j - \omega_j]).$$

Let $R \in \mathcal{R}^N$ be such that (i) R_i is the linear preference represented by $u(x) \equiv p \cdot x$, (ii) for each $h \neq i$, R_h is strictly convex and strongly monotonic, and (iii) ω is Pareto efficient at R . Then, by the strict convexity of some preferences, ω is the only individually rational allocation at (N, ω, R) . Therefore, $\omega = F(N, \omega, R)$. Then, the above inequality says that agent i prefers the sacrifice made by agent j , contradicting *sacrifice-no-envy*.

Subcase 1.2. $c_i - [c_j - \omega_j] \succeq \omega_i$ or $c_i - [c_j - \omega_j] \preceq \omega_i$.

When the former (latter) inequality holds, using the same argument as in Subcase 1.1, we can show that there is an exchange economy with endowment profile ω , where ω is the only individually rational allocation and so it is chosen by F . Then, agent i (agent j) prefers the sacrifice of agent j (agent i), again contradicting *sacrifice-no-envy*.

Case 2. $c_i - [c_j - \omega_j] \notin \mathbb{R}_+^\ell$.

In this case, for some $k = 1, \dots, \ell$, $c_{ik} - [c_{jk} - \omega_{jk}] < 0$. Then, as $c_{ik} - [c_{ik} - \omega_{ik}] = \omega_{ik} \geq 0 > c_{ik} - [c_{jk} - \omega_{jk}]$, it follows that $\omega_{ik} - c_{ik} > \omega_{jk} - c_{jk}$. Let Ω' be such that, for each $l \neq k$, $\Omega'_l \equiv \sum_{i \in N} c_{il}$ and $\Omega'_k \equiv \Omega_k$. Let $\omega' \equiv \varphi(N, \Omega', c)$. Then, by *claims-boundedness*, for each $h \in N$ and all $l \neq k$, $\omega'_{hl} = c_{hl}$. As $\Omega'_k = \Omega_k$, it follows by *resource monotonicity* that, for each $h \in N$, $\omega'_{hk} = \omega_{hk}$. Then, $\omega'_{ik} - c_{ik} = \omega_{ik} - c_{ik} > \omega_{jk} - c_{jk} = \omega'_{jk} - c_{jk}$ and, for each $l \neq k$, $\omega'_{il} - c_{il} = 0 = \omega'_{jl} - c_{jl}$. Therefore $\omega'_i - c_i \succeq \omega'_j - c_j$ and so $c_j - [c_i - \omega'_i] \in \mathbb{R}_+^\ell$. Then, again, we can adapt the argument of Case 1, switching the roles of i and j and replacing (N, Ω, c) and ω with (N, Ω', c) and ω' , respectively. ■

The first statement of Proposition 3 follows immediately from Lemma 2. As for its second

statement, it suffices to consider the constrained equal losses rule composed with the Walrasian exchange rule.

Proof of Theorem 2. The composition of the constrained equal losses rule and the Walrasian exchange rule proves that the second statement implies the first statement. In order to prove the converse, let φ be a rights-assignment rule satisfying *resource monotonicity* and *zero-award-out-consistency* that, when composed with an *individually rational* exchange rule F , leads to a market-based allocation rule $S \equiv F \circ \varphi$ satisfying *sacrifice-no-envy* on $\mathcal{E}^*$.

For each $l \in \{1, \dots, \ell - 1\}$, let $\mathcal{C}^*(l) \equiv \{(N, \Omega, c) \in \mathcal{C} : \forall k \geq l + 1, \forall i \in N, (\sum_{j \in N} c_{jk} - \Omega_k)/|N| \leq c_{ik}\}$. Let $\mathcal{C}^*(\ell) \equiv \mathcal{C}$ and $\mathcal{C}^*(0) \equiv \mathcal{C}^*$. We show, by induction on the number of goods l , that for each $l = 0, 1, \dots, \ell$, φ coincides with φ^{CEL} on $\mathcal{C}^*(l)$. By Lemma 2, φ coincides with φ^{CEL} on $\mathcal{C}^*(0)$. Let $l \in \{1, \dots, \ell\}$. Suppose, by induction, that φ coincides with φ^{CEL} on $\mathcal{C}^*(k)$ for each $k \leq l - 1$. We prove that φ coincides with φ^{CEL} on $\mathcal{C}^*(l)$.

Let $N \in \mathcal{N}$ and $c \equiv (c_i)_{i \in N} \in \mathbb{R}_{++}^{n\ell}$. Let $\mathcal{C}^*(N, c)(l)$ be the domain of claims problems in $\mathcal{C}^*(l)$ with population N and claims profile c . In what follows, we prove that φ coincides with φ^{CEL} on $\mathcal{C}^*(N, c)(l)$. For ease of exposition, assume, without loss of generality, that $N = \{1, \dots, n\}$ and that claims on the l -th commodity are increasingly ordered, i.e., $c_{1l} \leq \dots \leq c_{nl}$. For each $m \in \{1, 2, \dots, n - 1\}$, let

$$\mathcal{C}^*(N, c)(l)^m = \{(N, \Omega, c) \in \mathcal{C}^*(N, c)(l) : \sum_{i=m+1}^n c_{il} - (n-m)c_{m+1,l} \leq \Omega_l \leq \sum_{i=m}^n c_{il} - (n-m+1)c_{ml}\}.$$

Furthermore, let

$$\mathcal{C}^*(N, c)(l)^0 = \{(N, \Omega, c) \in \mathcal{C}^*(N, c)(l) : \sum_{i=1}^n c_{il} - nc_{1l} \leq \Omega_l\}.$$

Then, it is straightforward to see that $\mathcal{C}^*(N, c)(l) = \bigcup_{m=0}^{n-1} \mathcal{C}^*(N, c)(l)^m$. For each $m = 0, 1, \dots, n - 1$ and each $(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^m$,

$$\varphi_l^{CEL}(N, \Omega, c) = (0, \dots, 0, c_{m+1,l} - \mu_m, \dots, c_{nl} - \mu_m), \quad (4)$$

where $\mu_m \equiv (\sum_{i=m+1}^n c_{il} - \Omega_l)/(n - m)$.

We now show, by induction on the number of agents m , that for each $m \in \{0, 1, \dots, n - 1\}$ and each $(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^m$, $\varphi(N, \Omega, c) = \varphi^{CEL}(N, \Omega, c)$. Note that, as $\mathcal{C}^*(N, c)(l)^0 \subset \mathcal{C}^*(l - 1)$, φ coincides with φ^{CEL} on $\mathcal{C}^*(N, c)(l)^0$. Let $m \in \{1, 2, \dots, n - 1\}$. Suppose that if

$(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^{m-1}$, $\varphi(N, \Omega, c) = \varphi^{CEL}(N, \Omega, c)$. Let $(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^m$. We aim to show that $\varphi(N, \Omega, c) = \varphi^{CEL}(N, \Omega, c)$.

Let $\Omega'_l \equiv \sum_{i=m}^n c_{il} - (n - m + 1)c_{ml}$ and $\Omega' \equiv (\Omega'_l, \Omega_{-l})$. Then $(N, \Omega', c) \in \mathcal{C}^*(N, c)(l)^{m-1} \cap \mathcal{C}^*(N, c)(l)^m$. As $(N, \Omega', c) \in \mathcal{C}^*(N, c)(l)^{m-1}$,

$$\varphi(N, \Omega', c) = \varphi^{CEL}(N, \Omega', c). \quad (5)$$

As, for each $l' \neq l$, $\Omega'_{l'} = \Omega_{l'}$, then by *resource monotonicity*, for each $i \in N$, $\varphi_{il'}(N, \Omega, c) = \varphi_{il'}(N, \Omega', c)$ and $\varphi_{il'}^{CEL}(N, \Omega', c) = \varphi_{il'}^{CEL}(N, \Omega, c)$. Hence, using (5), we obtain: for each $l' \neq l$,

$$\varphi_{l'}(N, \Omega, c) = \varphi_{l'}^{CEL}(N, \Omega, c). \quad (6)$$

We only have to show that (6) holds also for $l' = l$. As $(N, \Omega', c) \in \mathcal{C}^*(N, c)(l)^m$, then by (4) and (5), for each $i \leq m$, $\varphi_{il}(N, \Omega', c) = 0$. As $\Omega'_l \equiv \sum_{i=m}^n c_{il} - (n - m + 1)c_{ml} \geq \Omega_l$ (the inequality holds because $(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^m$), then by *resource monotonicity* and non-negativity, for each $i \leq m$, $\varphi_{il}(N, \Omega, c) = 0$. Hence, by (4), for each $i \leq m$, $\varphi_{il}(N, \Omega, c) = \varphi_{il}^{CEL}(N, \Omega, c)$.

If $m = n - 1$, then it follows from the above result and the resource constraint that $\varphi_{nl}(N, \Omega, c) = \varphi_{nl}^{CEL}(N, \Omega, c)$, completing the proof.

If $m < n - 1$, let $M \equiv \{1, \dots, m\}$ and $\bar{\Omega} \equiv \Omega - \sum_{i \in M} \varphi_i(N, \Omega, c)$. Consider $(N \setminus M, \bar{\Omega}, c_{N \setminus M})$.

Note that, for each $l' \geq l + 1$ and each $i \in N$, $\varphi_{il'}(N, \Omega, c) = c_{il'} - (\sum_{j \in N} c_{jl'} - \Omega_{l'})/n$ and $(\sum_{j \in N} c_{jl'} - \Omega_{l'})/n \leq c_{il'}$. Then, for each $i \in N$, $(\sum_{j \in N \setminus M} c_{jl'} - \bar{\Omega}_{l'})/(n - m) = (\sum_{j \in N} c_{jl'} - \Omega_{l'})/n \leq c_{il'}$. Moreover, from $(\sum_{j \in N \setminus M} c_{jl} - \bar{\Omega}_l)/(n - m) = (\sum_{j \in N \setminus M} c_{jl} - \Omega_l)/(n - m) \leq c_{m+1, l} \leq \dots \leq c_{nl}$ (the first inequality holds because $(N, \Omega, c) \in \mathcal{C}^*(N, c)(l)^m$), it follows that $(N \setminus M, \bar{\Omega}, c_{N \setminus M}) \in \mathcal{C}^*(l - 1)$. Thus $\varphi(N \setminus M, \bar{\Omega}, c_{N \setminus M}) = \varphi^{CEL}(N \setminus M, \bar{\Omega}, c_{N \setminus M})$.

By *zero-award-out-consistency*, for each $i \in N \setminus M$, $\varphi_{il}(N \setminus M, \bar{\Omega}, c_{N \setminus M}) = \varphi_{il}(N, \Omega, c)$ and $\varphi_{il}^{CEL}(N \setminus M, \bar{\Omega}, c_{N \setminus M}) = \varphi_{il}^{CEL}(N, \Omega, c)$. Therefore, for each $i \geq m + 1$, $\varphi_{il}(N, \Omega, c) = \varphi_{il}^{CEL}(N, \Omega, c)$, completing the proof that (6) holds also for $l' = l$. ■

Remark 1 Let $\bar{\mathcal{E}}^{2\text{-agent}}$ be the domain of economies with two agents having their claims below the social endowment, i.e., $\bar{\mathcal{E}}^{2\text{-agent}} = \{(N, \Omega, c, R) \in \mathcal{E} : |N| = 2 \text{ and, for each } i \in N, c_i \leq \Omega\}$, and $\bar{\mathcal{C}}^{2\text{-agent}}$ the corresponding claims domain. Note that $\bar{\mathcal{E}}^{2\text{-agent}} \subset \mathcal{E}^*$ and $\bar{\mathcal{C}}^{2\text{-agent}} \subset \mathcal{C}^*$. Let $(N, \Omega, c) \in \bar{\mathcal{C}}^{2\text{-agent}}$, and $\omega \equiv \varphi(N, \Omega, c)$. Then, if $N \equiv \{i, j\}$, $0 \leq c_i - \omega_i = c_i - [\Omega - \omega_j] \leq c_i - [c_j - \omega_j]$. Thus, Case 2 in the proof of Lemma 2 never occurs. As that is the only case where *resource monotonicity* is used, Lemma 2 and Proposition 3 prevail on $\bar{\mathcal{E}}^{2\text{-agent}}$ without resource

monotonicity. Consequently, for this domain, Theorem 2 will hold even without imposing the two solidarity axioms for the rights-assignment procedure.

We now move to the results in Section 3.3.

Proof of Proposition 4. A simple example of a market-based allocation rule satisfying *relative-no-envy* on the whole domain is the rule obtained by composing the proportional rights-assignment rule and the no-trade exchange rule. To prove the second statement, consider an exchange economy with two agents and two goods. Let $c_1 \equiv (200/3, 100/3)$ and $c_2 \equiv (100/3, 200/3)$. Let $\Omega \equiv (100, 100)$. Preferences are represented by $u_1 : (x_1, x_2) \rightarrow \alpha x_1 + x_2$ and $u_2 : (x_1, x_2) \rightarrow \beta x_1 + x_2$, with $1/2 < \alpha < \beta < 2$. Then, the set of *efficient* allocations is $\{((0, x_2), (100, 100 - x_2)) : x_2 \in [0, 100]\} \cup \{((x_1, 100), (100 - x_1, 0)) : x_1 \in [0, 100]\}$. We show that no *efficient* allocation in this economy satisfies *relative-no-envy*. First, consider the *efficient* allocations $((0, x_2), (100, 100 - x_2))$ with $x_2 \in [0, 100]$. All these allocations fail *relative-no-envy* due to agent 1's envy. If agent 1 gets the relative compensation of agent 2, her consumption becomes $(2 \times 100, (100 - x_2)/2)$. As $\alpha > 1/2$, then $(400\alpha + 100)/3 > 100$. This implies $x_2 < 200\alpha + 50 - x_2/2$, which means $u_1(0, x_2) < u_1(2 \times 100, (100 - x_2)/2)$, i.e., agent 1 prefers the relative compensation of agent 2 to her own. In the case of efficient allocations $((x_1, 100), (100 - x_1, 0))$ with $x_1 \in [0, 100]$, using $\beta < 2$ and the same argument as above for agent 2, we can show that they fail *relative-no-envy* due to agent 2's envy. ■

Proof of Theorem 3. The composition of the proportional rule and the no-trade exchange rule proves that the second statement implies the first statement. In order to prove the converse, let φ be a rights-assignment that, when composed with an *individually rational* exchange rule F , leads to a market-based allocation rule $S \equiv F \circ \varphi$ satisfying *relative-no-envy*. Suppose, by contradiction, that there exists $(N, \Omega, c) \in \mathcal{C}$ such that $\omega \equiv \varphi(N, \Omega, c) \neq \varphi^{pro}(N, \Omega, c)$. Thus, there exist $i, j \in N$, and $\hat{l} \in \{1, \dots, \ell\}$, such that $\omega_{i\hat{l}} > \Omega_{\hat{l}} \cdot \frac{c_{i\hat{l}}}{\sum_{h \in N} c_{h\hat{l}}}$ and $\omega_{j\hat{l}} < \Omega_{\hat{l}} \cdot \frac{c_{j\hat{l}}}{\sum_{h \in N} c_{h\hat{l}}}$. Then $c_{i\hat{l}} > 0$, as otherwise the first inequality contradicts *claims boundedness*. Likewise, $c_{j\hat{l}} > 0$, as otherwise the second inequality contradicts non-negativity. Hence $\frac{\omega_{i\hat{l}}}{c_{i\hat{l}}} > \frac{\omega_{j\hat{l}}}{c_{j\hat{l}}}$. Then, there exists a vector of prices $p \in \mathbb{R}_{++}^\ell$ such that $p \cdot (c_j \times \frac{\omega_i}{c_i}) > p \cdot \omega_j$. Let $e = (N, \Omega, c, R) \in \mathcal{E}$, where R is such that R_j is represented by $U_j : x \rightarrow p \cdot x$, R_h is strictly convex for each $h \in N \setminus \{j\}$, and ω is Pareto efficient at R . Then, ω is the only feasible allocation that satisfies individual rationality. Hence, $\omega = F(N, \omega, R)$. Now, as $p \cdot c_j \times \frac{\omega_i}{c_i} > p \cdot \omega_j$, agent j envies i 's relative sacrifice, a contradiction. ■

Proof of Proposition 5. Let $e \equiv (N, \Omega, c, R) \in \mathcal{E}$. For each price p , denote by $B(w_i(p, c, \Omega), p)$ the Walrasian budget set given by the income $w_i(p, c, \Omega) \equiv \frac{p \cdot c_i}{p \cdot \sum_{j \in N} c_j} p \cdot \Omega$, i.e.,

$$B(w_i(p, c, \Omega), p) = \{x \in \mathbb{R}_+^\ell : p \cdot x \leq w_i(p, c, \Omega)\}.$$

Let z be a *proportional Walrasian equilibrium* allocation, i.e., there exists a price p such that, for each $i \in N$, $z_i \in B(w_i(p, c, \Omega), p)$ and, for each $x \in B(w_i(p, c, \Omega), p)$, $z_i R_i x$.³⁶ Thus, p supports the indifference set of each $i \in N$ at z_i . Note that $\sum_i z_i = \Omega$ and

$$\frac{w_i(p, c, \Omega)}{w_j(p, c, \Omega)} = \frac{p \cdot c_i}{p \cdot c_j}.$$

This means that agent i can afford the bundle $\frac{p \cdot c_i}{p \cdot c_j} \times z_j$. As z_i maximizes i 's preference satisfaction over her budget set,

$$z_i R_i p \cdot c_i \times \frac{z_j}{p \cdot c_j}.$$

Thus, ratio-no-envy is satisfied. Note that e was an arbitrary economy and, thus, no domain restrictions were imposed for the *proportional Walrasian* allocation rule just constructed. Therefore, we have shown the existence of an allocation rule, defined on the universal domain of economies, which generates efficient and ratio-envy-free allocations. ■

We conclude with the proof of Proposition 6 in Section 4.1.

Proof of Proposition 6. For each energy price q , denote by $B^W((\bar{m}_i, \bar{\omega}_i), q)$ the Walrasian budget set given by the endowment of money and energy $(\bar{m}_i, \bar{\omega}_i)$, i.e.,

$$B^W((\bar{m}_i, \bar{\omega}_i), q) = \{(m_i, e_i) \in \mathbb{R}_+^2 : m_i + q \cdot e_i \leq \bar{m}_i + q \cdot \bar{\omega}_i\}.$$

Similarly, for each pair of energy and permits prices (p, r) , denote by $B^{CT}((\bar{m}_i, \bar{\omega}_i), (p, r))$ the cap-and-trade budget set given by the endowment of money and permits $(\bar{m}_i, \bar{\omega}_i)$, i.e.,

$$B^{CT}((\bar{m}_i, \bar{\omega}_i), (p, r)) = \{(m_i, e_i) \in \mathbb{R}_+^2 : m_i + (p + r) \cdot e_i \leq \bar{m}_i + r \cdot \bar{\omega}_i\}.$$

Let $(m_i^*, e_i^*)_{i \in N}$ be an extended cap-and-trade equilibrium allocation with permits assignment $(\omega_i^e)_{i \in N}$ and transfers $(\tau_i)_{i \in N}$. Let κ and r^* be the equilibrium prices of energy and permit.

³⁶The existence of proportional Walrasian equilibria is shown by Peleg (1996) and Korthues (2000). They consider the framework where the total claims of each good may be more or less than the total endowment; thus, all economies in our model are examples of theirs.

Let $q^* \equiv \kappa + r^*$. Then,

$$B^W((\bar{m}_i + \tau_i - \kappa\omega_i^e, \omega_i^e), q^*) = B^{CT}((\bar{m}_i + \tau_i, \omega_i^e), (\kappa, r^*)). \quad (7)$$

Thus, (m_i^*, e_i^*) maximizes i 's preferences over $B^W((\bar{m}_i + \tau_i - \kappa\omega_i^e, \omega_i^e), q^*)$. The permits market-clearing condition $\sum_{i \in N} e_i^* = E$ implies the energy-market clearing and we conclude that $(m_i^*, e_i^*)_{i \in N}$ is a Walrasian equilibrium with endowment profile $(\bar{m}_i + \tau_i - \kappa\omega_i^e, \omega_i^e)_{i \in N}$ and energy price q^* .

Conversely, let $(m_i^*, e_i^*)_{i \in N}$ be a Walrasian equilibrium with money-energy endowment profile $(\omega_i^m, \omega_i^e)_{i \in N}$ and energy price q^* . For each $i \in N$, let $\tau_i \equiv \omega_i^m - \bar{m}_i + \kappa\omega_i^e$. Let $r^* \equiv q^* - \kappa$. Then, by (7), (m_i^*, e_i^*) maximizes i 's preferences over $B^{CT}((\bar{m}_i + \tau_i, \omega_i^e), (\kappa, r^*))$. Finally, it follows from the market clearing condition for the Walrasian equilibrium that the permits-market clears, i.e., $\sum_{i \in N} e_i^* = E$. ■

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