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Working Paper No. 528

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Poorest Made Poorer?

Decomposing income losses at the bottom of the income distribution during the Great Recession

Michael Savage*

Abstract: On the basis of anonymous (or cross-sectional) analyses, income losses during the Great Recession in a number of European countries were concentrated among the poorest ten per cent of the population. The anonymous approach however, which simply compares the distribution of income at two points of time, can omit important information regarding a change in the distribution of income in a country. Non-anonymous (or longitudinal) analysis, tracking individuals rather than income positions through time, can provide a quite contrasting picture of the distribution of income changes. Focusing on the countries with the largest proportional anonymous losses in income in the bottom decile between 2007 and 2010, a decomposition is proposed that separately identifies the proportion of the anonymous income change that is concentrated on the individuals who remain in the bottom decile during the period of interest (the "stayers" effect), and the component that is the result of changes in the composition of the bottom decile (the "movers" effect). An additional decomposition of the resulting change in social welfare shows that the net welfare outcome depends largely on the treatment of anonymity in the underlying social welfare function, in particular due to the evaluation of the welfare of individuals transitioning between deciles. The net welfare effect, as well as the contribution of stayers and movers, varies widely depending on whether welfare is measured anonymously or non-anonymously and, if the latter approach is used, whether individuals' welfare change is based on their initial income, final income, or some combination of the two.

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1 Introduction

While the depth of recession in Europe since 2007 varied considerably between countries, so too did the distributional impact of recession. Jenkins (2012) conducted a large scale comparison of changes in the distribution of household income in the first two years of recession, and found considerable heterogeneity across countries in the distribution of income losses. Over a slightly longer period, the OECD (2014) compared changes in the distribution of income between 2007 and 2011, and again found considerable variation between countries. For the countries hardest hit by recession however, a predominant pattern emerged from the OECD analysis. For four out of the five countries with the largest decline in average income, the largest proportional income loss across the income distribution was for the poorest ten per cent of the population.

Simply comparing the distribution of income at two points of time, however, can omit important information regarding the profile of income growth in a country. While, for example, the bottom decile had the largest proportional losses across the income distribution in a number of OECD countries between 2007 and 2011, this does not necessarily imply that the individuals in these countries that started out in the poorest ten per cent of the population experienced the largest income losses during the recession. Just as Aaberge et al. (2002) suggested that single year inequality measures can hide important cross-country differences in income mobility, simple cross-sectional comparisons of the distribution of income changes can hide important cross-country differences in the groups of individuals that drive observed cross-sectional income changes.

Large income losses in the bottom decile can be the result of larger than average income losses for individuals who were in the poorest 10 per cent of the population at the beginning of the period of interest. In addition, these income losses can be the result of large falls in income for individuals who started higher up the income distribution, but dropped into the bottom decile during the period under consideration. The first contribution of the paper is to propose a decomposition of income changes in the bottom decile that isolates the impact of these two effects. The method proposed separately identifies: (i) the component of income changes in the bottom decile explained by losses of income for individuals who were in the bottom decile at both the beginning and end of the period (the "stayers effect"), and (ii) the component explained by compositional change: the change in income in the bottom decile caused by individuals dropping into the poorest 10 per cent of the population, replacing the individuals who moved out of the poorest 10 per cent (the "movers effect").

Does it matter to social welfare whose income changes? In other words, does it matter if large income losses at the bottom of the income distribution are driven by individuals that started out in the poorest income percentiles? Often, welfare effects of changes in the distribution of income are measured anonymously, so that the identity, or initial income, of individuals does not affect the overall welfare effect (see Ravallion and Chen (2003), for example). The "anonymous" approach involves drawing welfare implications from income changes at quantiles along the distribution of income between two points in time. However, a growing literature based on removing the anonymity axiom from analysis of the distribution of income suggests that initial position in the distribution of income does matter in evaluating the overall welfare effect.

Bourguignon (2011) developed what he termed non-anonymous Growth Incidence Curves (na-GICs) which plot income growth rates according to individuals' position in the initial income distribution. He motivates the use of na-GICs on the basis that social welfare should logically be defined on both initial and terminal income. Palmisano and Peragine (2015) measured welfare changes due to the redistribution of income using social welfare functions that assign weights to individuals based on their position in the initial income distribution, rather than anonymous

income positions, while Dollar et al. (2015) applied a similar welfare function when examining changes in the distribution of income between countries. Van Kerm (2004), Wagstaff (2005), Jenkins and Van Kerm (2006), Jenkins and Van Kerm (2011), and Duval-Hernandez et al. (2015) all removed the anonymity axiom to examine various aspects of change in the distribution of income. At EU level, policy also takes account of the identity of those with low incomes. One of the "Laeken Indicators", the persistent poverty rate measures the number of currently low income individuals who were also low income (below the poverty line) in at least two of the preceding three years. Implicit in the persistent poverty measure is the belief that those who remain in a low income state over a given time period are more "in need" than those with low income who have recently transitioned from higher up the income distribution.

The second contribution of the paper is therefore to decompose the welfare effect of a change in the distribution of income into a "stayers" effect and a "movers" effect, using an approach based on counterfactual incomes that can be applied to both the anonymous and non-anonymous measurement of welfare. Welfare, measured anonymously, is insensitive to whether income changes are caused by "stayers" or "movers", as identities do not matter. Nonetheless, knowing the group of individuals driving the changes in welfare can be important to policy-makers in order to provide a targeted response. Consider a scenario, for example, where income in the bottom decile fell in real terms between two points in time, resulting in a fall in welfare. If that fall in welfare was driven by income losses for individuals who were in the poorest ten per cent of the population at the beginning of the period, this may require quite a different policy response than if losses are driven by individuals who fell into the bottom decile during the period under consideration.

When welfare is measured non-anonymously, the identities of the individuals driving the income change can have a direct impact on the overall welfare effect of a change in the distribution of income. In this case, as Palmisano and Peragine (2015) argued, it can make a big difference to welfare in society if the poor people in the first period are still the same poor people in the second period. In decomposing the overall welfare effect using a non-anonymous approach, the proposed method can be used to further decompose the "movers effect" into a "movers up effect" (the welfare effect of individuals moving out of the bottom decile) and a "movers down effect" (the welfare effect of individuals moving into the bottom decile). This decomposition allows us to identify whether falls in welfare for the "movers down" are offset by positive welfare effects of the "movers up", and whether welfare changes for "stayers" offset or compound welfare changes of the "movers" groups, for a given specification of the non-anonymous social welfare function.

The decompositions proposed, first of income changes, then of changes in social welfare, aim to provide a link between cross-sectional (anonymous) and longitudinal (non-anonymous) changes in income, particularly at the bottom of the income distribution. Grimm (2007) proposed a similar decomposition of poverty measures to identify the impact of those remaining in poverty, and those moving into and out of poverty on the overall poverty rate. He did not, however, examine the social welfare implications of such a decomposition. By measuring and decomposing the overall welfare effect of a change in the distribution of income, the analysis here shows that the treatment of anonymity in the underlying social welfare function can have a large impact on the resulting welfare effects, and the contribution of different individuals to that welfare effect.

Empirically, the decompositions are applied to a group of countries for whom the largest proportional losses in income between 2007 and 2010 are for the bottom decile. The decompositions provide evidence on whether the poorest individuals in 2007 experienced the largest declines in income by 2010, or if individuals that dropped into the poorest income positions between 2007 and 2010 drove the higher than average losses for the poorest 10 per cent of

the population, for a set of European countries with the most regressive patterns of income loss over that period. Finally, the design of the decomposition is such that the contribution of each group of individuals to the overall change in welfare can be identified, whether welfare is measured with or without the axiom of anonymity.

The paper proceeds as follows: Section 2 presents details of the methods used to decompose income changes in the bottom decile. It then discusses how the welfare effect of an observed change in the distribution of income can be measured, and proposes a decomposition of the overall social welfare changes on both an anonymous and non-anonymous basis. In Section 3 the decomposition methods are applied empirically, and results of income and welfare changes are examined and discussed. Section 4 extends the welfare measurement to allow for final year income and an aversion to income variability over time to be taken into account. In Section 5, the main results of the analysis are summarised and conclusions are drawn.

2 Decomposing the "Stayers Effect" -v- the "Movers Effect"

This section begins by proposing a methodology based on counterfactual incomes that allows us to additively decompose income changes in the bottom income decile¹ into a "stayers effect" and a "movers effect". Section 2.2 discusses how the welfare impact of a change in the distribution of income can be measured. Section 2.3 shows how the overall welfare effect of the income change can be decomposed using a similar counterfactual incomes approach, whether welfare is measured anonymously or non-anonymously. In either case, the decomposition can provide policy relevant information on the main drivers of the overall change in welfare over a given time period.

2.1 Decomposing Income Changes in the Bottom Decile

When comparing income in the bottom decile of the income distribution between two time periods, there are a number of different groups of individuals within that decile that can influence the overall change in income. Table 1 categorises individuals who are in the bottom decile in at least one of year t and year t + n into three separate groups, where n is the number of periods over which the analysis takes place. The "stayers" group is the group of individuals who are in the bottom decile in both year t and year t + n. The "movers up" group are the individuals who are in the bottom decile in year t, but have moved out of the bottom decile by year t + n, while the "movers down" group are the individuals who drop into the bottom decile during the period of analysis².

Using these definitions, year t average income in the bottom decile can be written as:

$$\mu_t^1 = \sigma^s \mu_t^s + (1 - \sigma^s) \mu_t^{mu} \tag{1}$$

¹ For reasons discussed in the Introduction and later in Section 3, the analysis is focussed on income changes in the bottom decile. Of course, this methodology can be easily applied to any other quantile of the income distribution.

² For any n > 1, this methodology ignores the income changes and transitions in intervening years. In the empirical illustration in Section 3 for example, we use n = 4. Table 10 shows that, in this case at least, the stayers group on average are far more likely to spend at least 3 out of 4 years in the bottom decile than either of the movers groups. The stayers are therefore more likely to remain in the bottom decile throughout the period, while the movers are more likely to transition in and out of the bottom decile at a higher frequency. Grimm (2007) uses a similar categorisation of individuals in relation to those staying in, and transitioning out of, poverty in Indonesia and Peru.

Table 1: Definitions of "Transition Groups"

	Voon t	Voor t + m
	Year t	Year $t+n$
Stayers	In Bottom Decile	In Bottom Decile
Movers Up	In Bottom Decile	In Decile 2 - 10
Movers Down	In Decile 2 - 10	In Bottom Decile

where μ_t^1 is average income in decile 1 at year t, σ^s is the share of the bottom decile occupied by stayers, μ_t^s is the average income of the stayers in year t, and μ_t^{mu} is the average income of movers up in year t.

Similarly, we can write average income in the bottom decile in year t + n as:

$$\mu_{t+n}^{1} = \sigma^{s} \mu_{t+n}^{s} + (1 - \sigma^{s}) \mu_{t+n}^{md} \tag{2}$$

where μ_{t+n}^{md} is the average income of the movers down group in year t+n.

Using Equations (1) and (2), the proportional change in average income in the bottom decile between year t and year t + n can be written as:

$$\delta_{pc}^{1} = \frac{\mu_{t+n}^{1}}{\mu_{t}^{1}} - 1 \tag{3}$$

while the absolute change in income can be written as:

$$\delta_{abs}^{1} = \mu_{t+n}^{1} - \mu_{t}^{1} \tag{4}$$

By defining two counterfactual income scenarios, we can isolate the impact of the changes in income of the various groups identified in Table 1. CF_1 is the distribution of income if the income of the "stayers" group in year t + n is held constant at its year t value, and all other incomes are allowed to change. Average income in the bottom decile in this first counterfactual income scenario can be calculated as:

$$\mu_{cf1}^{1} = \sigma^{s} \mu_{t}^{s} + (1 - \sigma^{s}) \mu_{t+n}^{md}$$
(5)

The "movers effect", or the change in average income in the bottom decile if only the income of those transitioning into and out of the bottom decile changed, can therefore be calculated as:

$$\delta_{cf1_{pc}}^{1} = \frac{\mu_{cf1}^{1}}{\mu_{t}^{1}} - 1 \tag{6}$$

in proportional terms, or as:

$$\delta_{cf1_{abs}}^1 = \mu_{cf1}^1 - \mu_t^1 \tag{7}$$

in absolute terms.

Conversely, the second counterfactual income distribution, CF_2 , is the distribution of income when only the incomes of the movers up and movers down is held constant at their year t value. All other incomes are allowed to vary to their year t + n values. In this case, average income in the bottom decile can be calculated as:

$$\mu_{cf2}^{1} = \sigma^{s} \mu_{t+n}^{s} + (1 - \sigma^{s}) \mu_{t}^{mu}$$
(8)

The "stayers effect" can therefore be calculated as:

$$\delta_{cf2_{pc}}^1 = \frac{\mu_{cf2}^1}{\mu_t^1} - 1 \tag{9}$$

in proportional terms, or as:

$$\delta_{cf2_{ch_2}}^1 = \mu_{cf2}^1 - \mu_t^1 \tag{10}$$

in absolute terms.

The proportion of the overall change in income that can be attributed to the "stayers effect" and the "movers effect" is straightforward to calculate, based on the fact that:

$$\delta^1 = \delta_{cf1}^1 + \delta_{cf2}^1 \tag{11}$$

for either the proportional or absolute change in income³.

2.2 Socially Evaluating a Change in the Distribution of Income

Growth Incidence Curves (GICs) are commonly used to assess the welfare implications of a given distributional pattern of income growth (or decline). GICs simply plot the change in mean income (in absolute or proportional terms) of each quantile of the income distribution. Between year t and year t+1 for example, the GIC shows the change in quantile i's mean income in year t to quantile i's mean income in year t+1. Ravallion and Chen (2003) and Son (2004) showed that where the GIC of a growth pattern is everywhere above the GIC of another growth pattern, the first growth pattern first order dominates the other. For any non-negative social weight function, the first growth pattern is preferred to the second⁴(Jenkins and Van Kerm 2011).

A growing literature on the distributional impact of income changes suggests that not only does the overall pattern of growth on a cross-sectional basis matter, but so too do the identities of individuals within the distribution of income. Non-anonymous GICs (na-GICs, Bourguignon 2011), also known as mobility profiles (Jenkins and Van Kerm 2011), are used to compare the distribution of growth experienced by individuals based on their initial income, rather than the income growth at certain quantiles of the income distribution. The na-GIC graphs the relationship between income growth and initial rank in the income distribution. As in the anonymous case, first order dominance exists in the non-anonymous case when an na-GIC lies somewhere above and nowhere below another. A distribution of (non-anonymous) income changes is socially preferred to another, therefore, when one na-GIC first-order dominates the other, for any non-negative social welfare weights.

Duval-Hernandez et al. (2015) examined the relationship between the anonymous and non-anonymous income growth profiles. They argued that while inequality in cross-sectional studies can increase or decrease over any given time period, panel income changes are almost always progressive in nature, with the highest income gains (or smallest income losses) experienced by those who are initially poorest. They showed that income growth can be both pro-poor (from a non-anonymous basis) and, at the same time, inequality can rise due to the effect of reranking.

A number of recent contributions (Bourguignon (2011), Jenkins and Van Kerm (2011), Palmisano and Peragine (2015), among others) propose the use of social evaluation functions to compare distributions of non-anonymous growth that capture a range of social preferences. The following five properties of the social evaluation function, proposed by Jenkins and Van Kerm

³ See Appendix A for proof.

⁴ Second order dominance can be examined by comparing cumulate GICs. See Section 3.

(2011), allow growth to be evaluated taking the identity of individuals into account capturing certain social preferences. The first and second properties, that the social evaluation is the sum of individual level income growth evaluations and that it satisfies replication invariance, imply that the overall social evaluation is a per capita average of individual social evaluations. The third property is that the evaluation of growth is sensitive to how individual growth is distributed along the *initial* ranking of individuals, so that a social weight depends on the individual's income in the base year⁵. Fourth, the welfare function is directional, so that $\delta(x,y) = -\delta(y,x)$. Finally, to capture a social preference for progressive growth, Jenkins and Van Kerm suggest the social weights to be positive and (weakly) declining in initial income. Palmisano and Peragine (2015) discuss a similar set of properties of non-anonymous welfare functions, in particular allowing for the case where the welfare function should account for horizontal inequality concerns.

A number of social evaluation functions have been proposed based on initial income rank rather than initial income level (including Jenkins and Van Kerm (2011), Palmisano and Peragine (2015), Palmisano (2015)). The distinction between these social evaluation functions and Atkinson-Bourguignon social welfare functions (based on income levels rather than rank) is important when making welfare comparisons across countries or across time when the marginal distributions are not equal in the initial period. Jenkins and Van Kerm, and Bourguignon (2011), show that this distinction is unimportant when comparing growth over identical base-period incomes. While the identical base-period income condition is unlikely to hold in practice across countries or across different time periods, by design it will hold when comparing counterfactual income changes based on an initial distribution of income.

The dominance conditions provide unambiguous orderings of income changes only in certain circumstances. When the GICs or na-GICs cross, only partial orderings of growth processes are possible. Further assumptions regarding the social welfare function are required to derive complete orderings. These assumptions can be imposed through restrictions on the profile of social weights and use of a scalar measure of welfare. Jenkins and Van Kerm label these scalar measures "indices of progressivity-adjusted growth".

Palmisano and Peragine (2015) and Dollar et al. (2015) show that such a scalar measure of the welfare impact of a given growth process can be evaluated by:

$$W = \frac{\sum_{i=1}^{I} q^i v^i \delta^i}{\sum_{i=1}^{I} q^i v^i}$$

$$\tag{12}$$

where q^i is the proportion of the population in quantile i, I is the number of quantiles in the population, and v^i is the profile of social welfare weights⁶. Equation (12) evaluates growth accounting for both the size and the vertical redistribution impact of growth. As we restrict $v^i \geq 0$, an increase in income for decile i always results in $W \geq 0$, all else equal. Equation (12) can be evaluated either anonymously (see Duclos and Araar (2003), for example) or non-anonymously (see Palmisano and Peragine (2015) or Dollar et al. (2015), for example). In the anonymous case, δ^i measures the change between mean real income in quantile i in year t and mean real income in quantile i in year t + n. When dropping the axiom of anonymity,

⁵ In Section 4 we examine the welfare consequences of dropping this property.

⁶ In the case where I is equal to the number of individuals in the population (P), each individual is assigned their own "welfare weight". When i < P, each individual within quantile i is assigned an equivalent weight. Horizontal equity concerns can be addressed by allowing v^i to vary within quantiles - see Palmisano and Peregine (2015).

 δ^i measures the change in mean real income between year t and year t+n for the group of individuals in quantile i in year t.

The social welfare weights, v^i , capture the social preferences for aggregating individual income changes. Different restrictions on v^i capture different social preferences for redistribution. Palmisano and Peregine discuss a number of axioms that can capture a range of desired properties in the evaluation of income growth. The restriction of $v^i \geq 0$, for example, satisfies the axiom of "pro-growth", so that an increase in income for any individual will, at worst, leave overall welfare unchanged. A preference for pro-poor growth can be captured by restricting $v^i \geq v^{i+1}$, while a further restriction for a preference for diminishing pro-poor growth can be captured by setting $v^i - v^{i+1} \geq v^{i+1} - v^{i+2}$. When examining the relationship between growth, inequality and social welfare, Dollar et al. (2015) showed that a specification of v^i motivated by the World Bank's "shared prosperity" goal⁷ would be to set $v^i = 1$ for the bottom four deciles, and $v^i = 0$ otherwise. Of course, in the anonymous setting v^i weights the income change for income position i (for example, the bottom decile), whereas in the non-anonymous setting v^i weights the income change of an individual or group of individuals, usually (but not always) based on rank in the initial distribution of income. In the empirical application in Section 3, a number of specifications of v^i are examined in both the anonymous and non-anonymous setting.

2.3 Decomposing the Social Evaluation of Growth

The income decomposition proposed in Section 2.1 is a decomposition of the anonymous change in income in the bottom decile over a given time period. It can, however, provide useful policy-relevant information when measuring the welfare effect both anonymously and non-anonymously.

In the anonymous setting, the income decomposition results can be directly mapped onto the GICs, so that the welfare implications, for a given underlying social welfare function, can be easily inferred. Because welfare evaluation of growth requires information on the full distribution of income, not just income in the bottom decile, incomes for individuals not in the bottom decile in either year t or t+n are allowed to vary to their year t+n value in both CF_1 and CF_2 . Then, by comparing the GICs of CF_1 and CF_2 , we can examine whether the overall welfare effects are driven by the income changes of those moving into and out of the bottom decile (the movers effect) or the income changes of those remaining in the bottom decile (the stayers effect). In practice, only the portion of the GIC below the 10th percentile (the bottom decile) will differ between the GICs of CF_1 and CF_2 , so the two counterfactual GICs will not cross. Contrary to the GICs, the na-GICs of the two counterfactual scenarios outlined may cross, so that an unambiguous ranking of welfare effects may not be possible.

The scalar measures of welfare may therefore be required in the non-anonymous setting to provide an unambiguous ranking of welfare changes. When evaluating the overall welfare effect using the non-anonymous scalar measures, the movers effect can be further decomposed into a "movers up" effect and a "movers down" effect by defining four counterfactual income scenarios, CF_1^{na} to CF_4^{na} . CF_2^{na} allows only stayers income to change, holding all other income constant. In CF_1^{na} and CF_3^{na} , we allow "movers up" and "movers down" incomes to change respectively. Finally, in CF_4^{na} , we allow all other incomes to change. We label the impact of CF_4^{na} the "other" effect, as it captures the income changes of all individuals who are not in the bottom decile in either year t or year t+1. Similarly, in the anonymous setting using scalar measures of welfare change, we can isolate the impact of "other" income changes by defining CF_3^a as the

⁷ "Shared prosperity" is defined as growth in average incomes of those in the bottom 40 percent of the income distribution in each country in the developing world. See World Bank (2013).

Person	Transition Group	CF_1^a	CF_2^a	CF_3^a	CF_1^{na}	CF_2^{na}	CF_3^{na}	CF_4^{na}
1	Stayer	y_t	y_{t+1}	y_t	y_t	y_{t+1}	y_t	y_t
2	Mover Up	y_{t+1}	y_t	y_t	y_{t+1}	y_t	y_t	y_t
3	Mover Down	y_{t+1}	y_t	y_t	y_t	y_t	y_{t+1}	y_t
4	Other	u_t	y_t	y_{t+1}	y_t	y_t	y_t	y_{t+1}

Table 2: Counterfactual Incomes for Decomposition of Scalar Measures of Social Welfare

 CF^a is the distribution of counterfactual income used to decompose the W^a CF^{na} is the distribution of counterfactual income used to decompose the W^{na}

distribution of income holding both stayers and movers income constant, and allowing all other incomes to change. CF_1^a is the distribution of income when only movers' income are allowed to vary to their year t+n value, while CF_2^a is the distribution of income when only stayers' incomes are allowed to vary to their year t+n value.

Table 2 summarises how the counterfactual incomes used in the decomposition of the scalar measure of welfare are distributed in practice. Person 1 is a stayer, so only her income is allowed to vary in CF_2^a and CF_2^{na} . Person 2 is a "mover up", so only his income is allowed to vary in CF_1^{na} . Person 3 is in the movers down group, so only his income is allowed to vary in CF_3^{na} . When evaluating W anonymously, movers up and movers down effects cannot be separately identified, so person 2 and person 3's income vary together in CF_1^a to capture the combined "movers" effect. Person 4 is neither a stayer nor a mover, so her income change is captured in CF_3^a and CF_4^{na} . In the anonymous setting, this decomposition provides evidence on whether the overall change in welfare, for a given specification of v^i , is driven by welfare changes for individuals who began the period in the poorest 10 per cent of the population, or welfare changes caused by a change in the composition of the bottom decile (reranking). In the non-anonymous setting, the question of interest becomes, for a given specification of v^i , does the positive welfare impact of the movers up offset the negative welfare impact of the movers down, and do the stayers have a positive or negative impact upon overall welfare in society?

To isolate the welfare impact of each of the transitions groups' income, we evaluate different counterfactual welfare change scenarios, based on the counterfactual incomes described above.

$$W_{cf\alpha} = \frac{\sum_{i=1}^{n} q^i v^i \delta_{cf\alpha}^i}{\sum_{i=1}^{n} q^i v^i}$$

$$\tag{13}$$

where $\alpha = 1, 2, 3$ or 4 in the non-anonymous case, and $\alpha = 1, 2$ or 3 in the anonymous case. As before, this decomposition is additive, so that $W = \sum_{\alpha} W_{cf\alpha}$. The contribution of each transition group is therefore straightforward to identify.

3 Decomposing Income and Welfare Changes in 5 European Countries

Between 2007 and 2010, the bottom decile in a number of European countries experienced larger than average losses in income, on a cross-sectional basis. In this section, the methods described in the previous sections are applied to a set of these countries to examine whether the larger than average losses in income in the bottom decile were driven by losses in income for the individuals that began the period in the poorest ten per cent of the population, or by a change in the composition of the poorest ten per cent, in each country. The welfare implications

of the observed patterns of income change are then examined, first by comparing the GICs and na-GICs in each country, and then by examining the scalar measures of social welfare, under each counterfactual scenario.

3.1 Data

The decompositions are applied using EU-SILC data⁸. Given that the decomposition requires longitudinal data, the analysis is based on the 2011 longitudinal file, although the 2008 and 2011 cross-sectional files are also used for descriptive and cross-checking purposes. As EU-SILC records data from the previous calendar year (with the exception of Ireland and the UK), the 2011 longitudinal file provides income information from the period 2007 to 2010.

In applying the decompositions, the 4 year balanced panel is used, so that full information on incomes and decile transitions in each year between 2008 (2007 incomes, year t) and 2011 (2010 incomes, year t + n) is available. To reduce the impact of measurement error, we trim the top and bottom percentile of the income distribution in each year of data⁹. The measure of income used is equivalised household disposable income. The equivalence scale used is 1 for the first adult, 0.66 for subsequent adults, and 0.33 for children aged 14 or less¹⁰. All incomes are expressed in real terms using the price index reported by the OECD¹¹.

3.2 The Distribution of Income Growth

OECD (2014) compared the mean proportional loss in income between 2007 and 2011 with losses for the bottom and top ten per cent of the income distribution¹² on a cross-sectional (anonymous) basis in 33 OECD countries. Here we focus on the specific group of European countries where average income declined over the period, and proportional income losses were larger than average at the bottom of the income distribution. We examine changes in the distribution of income in Spain, Italy, Greece, Hungary and Estonia who all share this pattern of income losses¹³. Figure 1a confirms that, on the basis of the EU-SILC data, the largest proportional losses in income between 2007 and 2010 were for the poorest 10 per cent of the population in each of these countries (in Greece, the 4th decile experienced similar proportional losses in income as the poorest decile). Comparing absolute changes in income (Figure 1b), the pattern of losses is somewhat different, with the largest absolute losses in income in the bottom decile ranging from being the largest across the income distribution (in Spain) to being smallest across the income distribution (in Greece).

 $^{^{8}\,}$ See Appendix B for a more complete discussion of the data.

⁹ If no observation was in the top or bottom percentile for more than one year, this step would remove 8 per cent of the sample for each country. If there was no reranking between percentiles in any country, 2 per cent of the sample would be dropped. In practice, the proportion of observations dropped ranges between 3.1 per cent (in Italy) and 4.5 per cent (in Spain). See Appendix B for further sensitivity tests regarding measurement error in the data.

 $^{^{10}}$ A number of alternative equivalence scales exist, such the "OECD Scale" (1, 0.7, 0.5), the "OECD-modified scale" (1, 0.5, 0.3), and the "square root" scale (square root of household size), each of which would lead to slightly different measures of equivalised household income. The scale used in the analysis is the Irish National Scale used by the Central Statistics Office (CSO) of Ireland.

¹¹ OECD.stat consumer price index

 $^{^{12}}$ see Appendix Figure 10.

¹³ According to the OECD (2014) and Callan et al. (2014), the distribution of income losses in Ireland over this period also fits this pattern. However, Ireland is not present in the 2011 EU-SILC longitudinal data, so is not included in the analysis. See Savage et al. (2015) for an analysis of changes in the distribution of income in Ireland during the Great Recession.

(a) Proportional Change

(b) Absolute Change

Fig. 1: Change in Real Equivalised Disposable Household Income by Decile - 2007 to 2010

Source: Author analysis of EU-SILC 2011 longitudinal file.

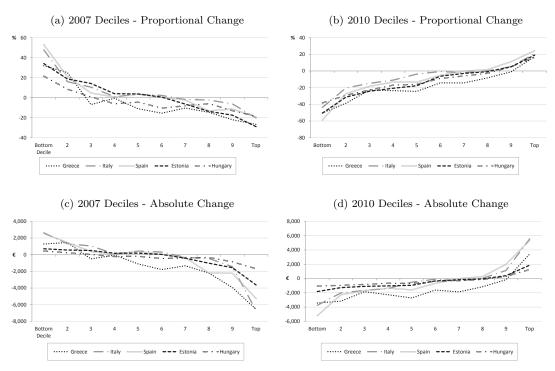
The question remains: what lies behind the larger than average proportional losses for the poorest 10 per cent in these countries? Did the individuals that began this period in the bottom decile experience larger than average losses in income? Or did individuals dropping into the bottom decile drive the larger than average losses in income? The non-anonymous GICs in Figures 2a and 2c show that on average the income of the individuals in the bottom decile at the beginning of the period grew substantially over the following four years, both in proportional and absolute terms. Despite the bottom decile suffering a decline in income between 2007 and 2010 in Spain and Italy, individuals that were in the bottom decile in these countries at the beginning of the period saw their income increase by about 50 per cent, on average. Similarly, in Hungary, Greece and Estonia, those that started in the bottom decile saw their incomes grow by between 20 and 40 per cent.

Rather than ranking individuals by initial year decile, Figures 2b and 2d rank individuals by final year decile. The resulting curves show that in each country, individuals that end up in the bottom decile suffered significant losses in income, significantly larger than the cross-sectional losses shown in Figure 1.

The divergence between the GICs and na-GICs presented in Figures 1 and 2 suggest the presence of significant mobility across the income distribution between 2007 and 2010. A number of income mobility indices have been developed to measure the degree of changes in incomes, or movement of individuals in the income distribution, over a time period. Jäntti and Jenkins (2013) and Fields and Ok (1996) provide comprehensive reviews of a range of measures of mobility. Jantti and Jenkins showed that while several measures of mobility exist that range from summary measures of pure income changes (ignoring reranking) to measures that summarise the degree of reranking (ignoring actual income changes), the income mobility literature has not reached a consensus in the way cross-sectional inequality literature has. When examining mobility, they recommend use of straightforward descriptive measures of mobility, in particular the use of transition matrices.

Tables 3 and 4 summarise such transition matrices for each country, focusing on transitions into and out of the bottom decile. According to Table 3, in 4 out of the 5 countries, between 40 and 45 per cent of individuals that were in the bottom decile in 2007 remained in the bottom decile in 2010. The exception is Spain, where just under 1 in 3 individuals that were in the bottom decile in 2007 were also in the bottom decile in 2010. Canto (2000), Canto and Ruiz (2014) and Ayala and Sastre (2008) previously found that income mobility in Spain is relatively

Fig. 2: Change in Real Equivalised Disposable Household Income 2007 to 2010, by 2007 Deciles (a and b) and 2010 Deciles (c and d)



Source: Author calculations based on EU-SILC 2011 longitudinal file.

Table 3: Transitions out of bottom decile - "stayers" and "movers up" - 2007 to 2010

Decile	Greece	Italy	Spain	Estonia	Hungary
1	41.9	44.5	30.5	44.7	42.9
2-3	38.9	36.2	41.2	35.4	36.4
4+	19.2	19.4	28.2	19.9	20.7

Source: Author analysis of EU-SILC 2011 longitudinal file.

high compared with other developed countries¹⁴. In all countries, the majority of individuals who started in the bottom decile in 2007 remained in the bottom 30 per cent of the income distribution in 2011.

In Table 4, the frequency at which individuals dropped into the bottom decile is examined. Again, Spain stands out as a country with a particularly high level of mobility between deciles. There is some evidence that "movers down" came from slightly higher deciles than the deciles that "movers up" transitioned into. Just less than 1 in 4 "movers down" in Italy and Hungary transitioned from above the 3rd decile, while about 1 in 3 "movers down" in Spain and Estonia came from above the 3rd decile. Again however, the majority of those in the bottom decile in

¹⁴ As shown in Appendix B, only 7 per cent of longitudinal sample in Spain have income below the bottom decile cut-off derived from the cross-sectional data. Therefore, a degree of uncertainty exists regarding the reliability of the Spanish transition results. See Appendix Table 9.

Table 4: Transitions into bottom decile - "stayers" and "movers down" - 2007 to 2010

Decile	Greece	Italy	Spain	Estonia	Hungary
1	41.6	44.6	30.4	45.1	42.8
2-3	31.1	31.8	34.5	21.0	34.2
4+	27.3	23.7	35.1	33.9	22.9

Source: Author analysis of EU-SILC 2011 longitudinal file.

Table 5: P-ratios 2007 and 2010

	2007	2010	2007	2010	2007	2010
	2007	2010	2007	2010	2007	2010
	P90,	/P10	P50,	/P10	P9	0/P50
Greece	4.2	4.1	2.1	2.1	2.0	1.9
Italy	3.7	3.9	2.0	2.1	1.8	1.9
Spain	4.1	4.7	2.1	2.3	1.9	2.0
Estonia	3.8	4.1	1.9	2.0	2.0	2.1
Hungary	2.8	3.2	1.7	1.8	1.7	1.8

Source: Author analysis of EU-SILC 2011 longitudinal file.

2010 in each country were in the bottom 30 per cent of the income distribution in 2007^{15} . Of course, in countries with a relatively unequal distribution of income, an individual may require a larger change in income to transition between deciles than in countries with a more equal distribution of income. Despite having relatively high frequency of decile transitions however, Table 5 shows that Greece and Spain were also among the countries with the highest p-ratios in both 2007 and 2010, particularly at the p90/p10 ratio.

To begin to understand what is driving the larger than average cross-sectional income losses in the bottom decile over this period, Table 6 compares the income of the various transition groups in 2007 and 2010. Unsurprisingly, the movers up and movers down groups had the largest percentage change in income in each country. On average, individuals moving out of the bottom decile saw their income increase by between 49 per cent (Hungary) and 85 per cent (Italy, Spain). Individuals dropping into the bottom decile saw their incomes fall by between 47 per cent (Hungary) and 66 per cent (Spain). The percentage change in stayers income varies between the countries. In two countries, Spain and Hungary, stayers income fell by at least as much as the overall fall in income in the bottom decile. In the other four countries, the decline in stayers income was considerably less than the overall fall in income in the bottom decile. In the next section, we apply the decomposition approach described in Section 2.1 to identify how much of the overall fall in income in the bottom decile between 2007 and 2010 can be attributed to the income falls for stayers and how much can attributed to the income falls of the movers.

¹⁵ Slight differences emerge in the proportion of "stayers" in Tables 3 and 4 (top row of each table) due to small changes in the number of individuals in the bottom decile in each year of the survey.

Table 6: Mean Real Income of Individuals in Bottom Decile - Grouped by Transition Group

	2007	2010	% Change	2007	2010	% Change
		Spain	,		Gree	ce
Stayer	4,687	3,602	- 23	3,930	3,479	- 11
Move Up	4,880	9,025	85	4,103	6,642	62
Move Down	10,641	3,658	- 66	8,881	3,314	- 63
		Italy			Hunge	ary
Stayer	$5,\!133$	4,930	- 4	1,961	1,614	- 18
Move Up	5,803	10,702	84	2,081	3,105	49
Move Down	11,371	4,770	- 58	3,396	1,792	- 47
		Estoni	ia			
Stayer	1,969	1,792	- 9			
Move Up	2,215	$3,\!655$	65			
Move Down	4,951	1,773	- 64		1.01	

Source: Author analysis of EU-SILC 2011 longitudinal file.

3.3 Income Decomposition Results

Figure 3 shows that for four out of the five countries analysed here, the majority of the decline in income in the bottom decile was a result of falls in income for those dropping into the bottom decile. Only in Hungary was the balance between the contribution of the stayers and the movers relatively equal. In 4 out of 5 countries, falls in income for individuals who remain in the bottom decile in both 2007 and 2010 contributed less than 30 per cent of the overall fall in income in the bottom decile. In Italy, the contribution of the stayers group was particulary small, with almost 11 percentage points of the overall 12 per cent decline (or 6575 out of almost 660) in income resulting from falls in income for individuals dropping into the bottom decile from higher up the income distribution. Only in Hungary was the contribution made by stayers to the overall income fall close to 50 per cent.

The comparison between Spain and Hungary is of particular interest. In both of these countries, "stayers" income fell by at least as much as the overall fall in income in the bottom decile. Despite this, the results of the decomposition for the two countries show that in Spain only a quarter of the overall fall income in the bottom decile was explained by falls in "stayers" income, whereas in Hungary the same group explained about half of the overall fall income. The reason, of course, for the difference between the two countries the rate of transitions in and out of the bottom decile in the two countries. Just above 30 per cent of the bottom decile were "stayers" in Spain, compared to close to 45 per cent in Hungary. The value of the decomposition applied here is shown by the contrasting results for these two countries.

(a) Proportional Income Change

(b) Absolute Income Change

Fig. 3: Decomposition of Income Changes in Bottom Decile - 2007 to 2010

Source: Author calculations based on EU-SILC 2011 longitudinal file.

3.4 Decomposing the Welfare Effect

For four out of five countries examined, 70 per cent or more of the larger than average decline in income for the bottom decile between 2007 and 2010 was the result of income losses for individuals dropping into the bottom decile. So if losses at the bottom of the income distribution were predominantly caused by income losses for those falling from higher up the income distribution, and if those at the bottom of the income distribution either experienced relatively small declines in income, or transitioned higher up the income distribution during the recession, what was the overall impact on welfare in society?

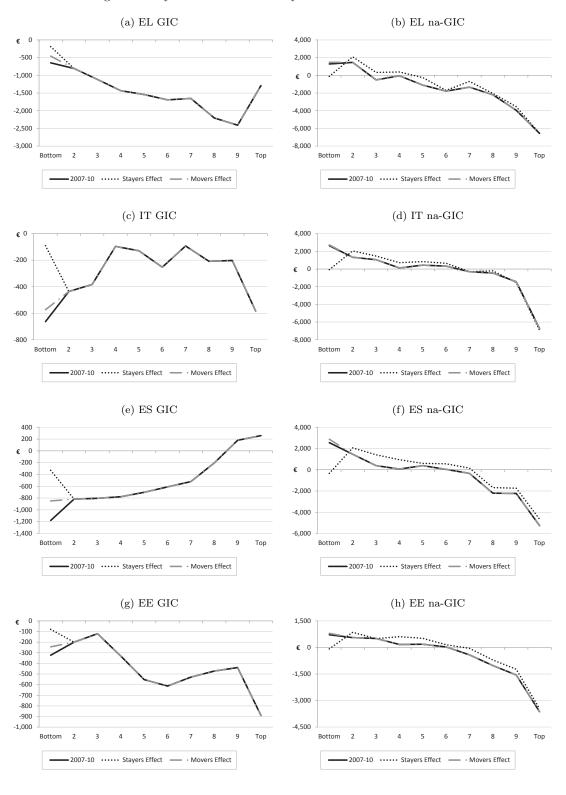
We can start by examining the GICs and na-GICs for each of the counterfactual scenarios outlined previously¹⁶. The income decomposition results can be directly mapped onto the GICs, as can be seen in the first column of Figure 4. For each country, the GIC associated with the stayers effect first-order dominates the GIC associated with the movers effect¹⁷. For Hungary, however, the difference between the GICs is marginal due to the stayers effect and the movers effect each contributing approximately 50 per cent of the overall fall in income in the bottom decile. Therefore, for any anonymous social welfare function with non-negative welfare weights, in four of the five countries analysed falling incomes of those dropping into the bottom decile resulted in a greater fall in welfare than income falls for individuals that started in the poorest 10 per cent of the population.

No unambiguous ranking of welfare effects is possible when comparing the na-GICs associated with the stayers and movers effects. In each country the na-GICs cross after the 10th percentile of the income distribution, displayed on the horizontal axis of each figure. This is because holding constant the income of movers up reduces income growth among the individuals that were initially in the bottom decile, so that the na-GIC associated with the movers effect is higher than the na-GIC associated with the stayers effect for the section of the curves up to the 10th percentile. Conversely, holding the income of the movers down constant increases income growth in deciles 2 to 10, so that the na-GIC associated with the movers effect is lower than

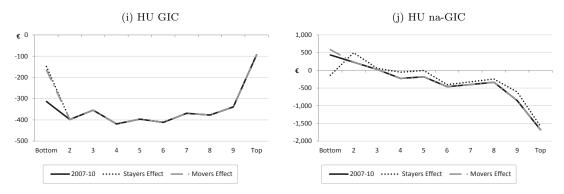
¹⁶ Following Palmisano and Peragine (2015) and Bourguignon (2011), we evaluate changes in social welfare using absolute income changes. Palmisano (2015) shows that proportional income changes can also be used when drawing welfare implications based on certain assumptions regarding the underlying social welfare evaluation function. Results of Section 3 are therefore replicated using proportional incomes changes in the Appendix. Qualitative conclusions remain robust to those drawn from the absolute income change analysis.

¹⁷ From Section 2.3, one (na-)GIC dominates another when it lies nowhere below and somewhere above the other.

Fig. 4: Anonymous and Non-Anonymous Growth Incidence Curves







the na-GIC associated with the stayers effect for the section of the curves beyond the bottom decile¹⁸.

3.5 Scalar Measures of Social Welfare

In cases where GICs or na-GICs cross, further restrictions on the social welfare function are required to provide unambiguous rankings of welfare changes. These restrictions are imposed through restrictions on v^i , so that they satisfy some or all of the axioms discussed by Ravallion and Chen (2003) in the anonymous setting, and Jenkins and Van Kerm (2011) and Palmisano and Peragine (2015) in the non-anonymous setting, among others. In each case, we normalise the welfare weights so that $\sum_i v^i = 1$.

The first set of welfare weights, v^i , used in this analysis are derived from an Atkinson-type utility function. Salas and Rodriguez (2013) class the utilitarian approach "probably the most widely used class of welfare functions in the income distribution literature". Using this approach, individual welfare can be measured as:

$$U^{i}(y^{i}) = \frac{k(y^{i})^{1-e}}{1-e} \quad if \ e \ge 0 \ and \ e \ne 1$$
 (14)

$$U^{i}(y^{i}) = klog(y^{i}) \qquad if \quad e = 1 \tag{15}$$

where k is a normalisation parameter. e is an inequality aversion parameter, with higher values of e increasing the concavity of the utility function in income. At its most straightforward interpretation, the welfare weights simply reflect each individuals private marginal utility of income. Higher values of e therefore place more weight on the welfare of the poorest individuals. Sen (1973) suggests that as well as simply being interpreted as private individual utility, U^i can be the "component of social welfare corresponding to person i, being itself a strictly concave function of income" ¹⁹. v^i can therefore be further interpreted as being the product of two terms,

¹⁸ Social welfare implications, based on more restrictive social preferences, can be drawn if second-order dominance exists (one cumulative na-GIC is nowhere below and somewhere above another). On inspection, no second-order dominance exists between the stayers and movers effects in any country analysed here, further necessitating use of scalar measures of welfare.

¹⁹ In practice, y^i refers to the mean income in decile i (μ^i), so that each individual in decile i is assigned the same welfare weight.

the first representing the private marginal utility of income, and the second representing $\frac{\partial W}{\partial U^i}$, the elasticity of social welfare with respect to utility of individual i^{20} .

Specifying the welfare weights in such a manner ensures that v^i satisfies the axiom of propoor growth (Palmisano and Peragine (2015)) or a social preference for progressive income growth (Jenkins and Van Kerm (2011)) as v^i decreases (weakly) in rank in the initial income distribution. The implication is that the transfer of a small amount of income to an individual in decile i from an individual in decile i + 1 would not result in a decrease in overall welfare, all else equal.

Four different sets of weights are specified by allowing e to take on values of 0, 1, 2 and 5. A value of e=0 represents the extreme Utilitarian case whereby all income changes are weighted equally, so $W=\bar{\delta}=\sum_{i=1}^{I}\delta^{i}/I$, and the overall welfare impact will be equal whether evaluated anonymously or non-anonymously (though the stayers and movers effects will not necessarily be equal between the two approaches). A value of e=5 is closer to the Rawlsian case where only the welfare of the poorest agents matter in the evaluation of social welfare²¹.

Figure 5 shows the overall change in welfare between 2007 and 2010 in each country, and the contribution of each of the transition groups to this change, as e ranges from 0 to 5. Panels a and b show the decomposition of the scalar measure of welfare change using the Utilitarian weights with e=0, when welfare is measured anonymously and non-anonymously respectively. In each case, the resulting welfare change is simply $\bar{\delta}$. When the welfare effect is measured anonymously with e=0, the negative welfare effect in each country was driven largely by income losses for deciles 2 to 10. The picture is somewhat different when welfare is measured non-anonymously, where the large changes in income for movers up and movers down drive the overall welfare effect. In each case with e=0, welfare losses for movers down offset any welfare gains due to movers up in all five countries.

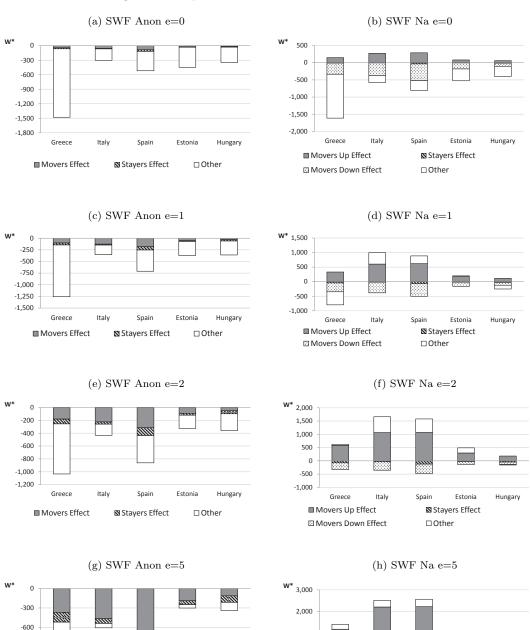
As the value of e increases (panels c to h), welfare changes for those in the bottom decile make a larger contribution to the overall change in welfare, as more weight is placed on the income changes in the bottom decile. In the anonymous case, larger values of e result in a more negative overall welfare effect, driven by welfare losses for stayers and movers. The ratio between the stayers effect and the movers effect remains constant for any value of e, but the two effects combined make up a larger share of the overall welfare effect. With e=5, welfare changes for those outside the bottom decile have very little impact on the overall welfare effect. With the normalisation $\sum_i v^i = 1$, as e approaches infinity, the welfare weight on the bottom decile approaches 1, and the anonymous welfare decomposition converges on the standard income decomposition in Figure 3. As e increases in the non-anonymous case, more social weight is placed on the welfare of the initially poorest so that the welfare gains of the movers up begin to offset the welfare losses of the movers down. For values of e>0, the overall welfare effect of the change in the distribution of income over the period becomes positive in each country.

When measured anonymously, income changes between 2007 and 2010 resulted in a decrease in welfare in each of the countries. The decomposition of these welfare changes showed that both stayers and movers contributed to the overall negative welfare effect. For four of the five countries, income losses for those dropping into the bottom decile caused a significantly larger share of the overall welfare loss than income losses for individuals who began the period in

 $^{^{20}\,}$ See Ahmad and Stern (1984) for discussion in a tax reform setting.

²¹ Dollar et al. (2015) suggested that a specification of $v^i = 1$ for i = 1, ..., 4, and $v^i = 0$ otherwise captures the World Bank's Shared Prosperity goal. This set of weights satisfies the pro-growth axiom discussed by Palmisano and Peragine (2015) and Jenkins and Van Kerm (2011), so that the welfare ordering based on GIC and na-GIC dominance are fully applicable. Results based on this specification of v^i were qualitatively similar a value of e = 2 in the utilitarian setting, and are available on request.

Fig. 5: Decomposition of Scalar Social Welfare Function



1,000

-1,000

Greece

■ Movers Up Effect

☑ Movers Down Effect

Italy

Estonia

Stayers Effect

□Other

-900 -1,200

■ Movers Effect

■ Stayers Effect

□ Other

the poorest 10 per cent of the population. In Hungary, each group contributed approximately equal proportions to the overall welfare loss for each specification of the social welfare function examined.

A quite different pattern emerges when welfare is measured non-anonymously. With the exception of the Utilitarian weights with e=0, the non-anonymous approach suggests that the majority of countries experienced an increase in welfare between 2007 and 2010. The results of the decomposition show that gains in income for individuals moving out of the bottom decile are the primary reason for this increase in welfare, particularly at the higher levels of inequality aversion.

4 Accounting for Final Year Income when Socially Evaluating Income Growth

When evaluated non-anonymously, the social welfare evaluation functions examined thus far have been evaluated on the basis of individuals' rank in the initial distribution of income. The choice of using the initial distribution of income to rank individuals, while intuitively appealing, is essentially arbitrary. It is questionable, as Palmisano (2015) argued, to give priority to the income growth of the initially poor individuals over the income growth of the finally poor²². Palmisano showed that the choice of reference period can have a significant impact upon the welfare implications drawn from a given change in the distribution of income.

Within the current framework, we can account for both initial year ranking and final year ranking in the social evaluation of growth in two ways. First is through the aggregation of individuals into deciles. The na-GICs used thus far have shown the relationship between initial income and income growth, where individuals have been grouped into decile based on their ranking in the initial distribution of income. By grouping individuals into deciles based on their ranking in the final year income distribution, the na-GIC_f shows the relationship between final year rank and income growth. Based on this approach, Palmisano (2015) proposed that growth path A first order dominates growth path B, taking account of both initial year and final year rankings, when na-GIC(A) first order dominates na-GIC(B) and na-GIC_f(A) first order dominates na-GIC_f(B). As shown in Section 3.4, the na-GICs associated with the stayers and movers effects cross for all countries, so that the first order dominance conditions required by Palmisano (2015) do not hold for any country analysed here.

Again therefore, we must turn to scalar measures to draw unambiguous first-order welfare implications when taking account of initial and final year income. We introduce new restrictions on the social weights, v^i , to account for initial and final year income. Social weights based on 'permanent' (longitudinally-averaged) income, rather than initial year income, capture the scenario where the social evaluator takes account of both initial and final year incomes in evaluating income growth. Rather than assuming that the welfare of initially poorer individuals takes precedence over finally poor individuals, 'permanent' income based welfare weights place most weight on the welfare of individuals with the lowest permanent income²³.

²² Consider the extreme case, for example, of a two-person society where individuals simply swap incomes between two time periods. In this case, for any strictly declining profile of welfare weights (or any positive level of inequality aversion in the welfarist setting), welfare will increase when measured non-anonymously based on initial income rank.

²³ According to Atkinson and Bourguignon (2000), any social welfare function defined over y_t can also be defined over 'permanent' income. In practice, 'permanent' income weights can be calculated by substituting $\sum_{i=1}^{T} y_t^i/T$ for y^i in Equations (14) and (15), and measuring the welfare weights accordingly. It is straightforward

to allow for discounting incomes from different time periods, though for clarity incomes are not discounted here. The same approach applies to the use of 'stability-equivalent' income. The weights in these cases can be directly

Longitudinally-averaged income assumes perfect substitutability between income in different time periods. In the presence of imperfect capital markets however, this may not be a reasonable assumption to make. Individuals may have a preference for a smooth income profile, for example, to enable a stable level of consumption between periods. The assumption of risk averse individuals, who would sacrifice expected income for income certainty, can also be used to motivate an aversion to income variability. Cruces (2005) and Creedy et al. (2013) suggest that a welfare metric for each individual that takes account of aversion to variability in income over time can be measured by assuming an additive time-separable evaluation function for each individual:

$$\omega(y) = \frac{(y)^{1-\rho}}{1-\rho} \qquad if \quad \rho \neq 1 \tag{16}$$

$$\omega(y) = \log(y) \qquad if \quad \rho = 1 \tag{17}$$

where ρ is a sensitivity parameter capturing the degree of aversion to variability in income, assumed to be constant across individuals. These functions lead to a "stability equivalent" income, \tilde{y} , which is a money metric welfare measure showing the income level, if received in every period, that would lead to same utility as the observed income stream.

$$\tilde{y} = \left[\frac{1}{T} \sum_{t=1}^{T} y_t^{1-\rho}\right]^{\frac{1}{1-\rho}} \quad if \quad \rho \neq 1$$
 (18)

$$\tilde{y} = \prod_{t=1}^{T} y_t^{\frac{1}{T}} \qquad if \quad \rho = 1 \tag{19}$$

A value of $\rho=0$ is the special case where there is no aversion in income variability over time, capturing the 'permanent' income scenario described above. At the other extreme, when $\rho \to \infty$, $\tilde{y} = min(y_1, ..., y_T)$. The relative values of ρ and e, the inequality aversion parameter in the Atkinson utility functions, determine whether inequality aversion of the social planner is high enough to overcome individuals' aversion to income variability over time (Creedy, 2012). Table 7 shows the results of the decomposition when v^i is calculated on the basis of \tilde{y} rather than initial year income, for a range of values of e and ρ . In each case, compared to the initial year income results with an equivalent value of e (shown in the first column (e=1) and fifth column (e=2) of Table 7), the inclusion of ρ leads to the movers up representing a smaller share of the net welfare change, with the movers down representing a larger share of the net welfare effect. This is due to the initial year v^i specification placing a relatively higher weight on movers up due to their initial low ranking in the income distribution. The overall impact of using \tilde{y} instead of initial y is therefore to reduce net social gain of reranking of bottom decile individuals. Results are relatively insensitive to the value of the ρ parameter.

Table 7 aggregated individuals based on initial year rank. The choice remains whether to group individuals into deciles based on the initial or final year income. Table 8 performs the same decomposition of the welfare effect when welfare is evaluated based on final year ranks. Again the first and fifth columns show the results when welfare is measured based on a single year income distribution, in this case final year incomes. The net welfare effect becomes negative in each country when individuals are ranked on their final year incomes for all values of e and ρ analysed here. The decomposition of the results shows that the negative net welfare effect is

interpreted as the social evaluator's judgement on the weight given to the welfare of the individual, rather than the marginal utility of income. 'Permanent income'-based weights can also be derived by the use of CES-like utility functions in which base- and final- year income are substitutes (Jenkins and Van Kerm (2011)).

Table 7: Decomposition of Scalar Social Welfare Function, W(.), with Aversion to Income Variability - Initial Year deciles

				Estonia				
			e=1				e=2	
	v(y)	$v(ilde{y}, ho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
Stayers	-17	-15	-16	-16	-30	-25	-26	-26
Move U	173	154	156	157	297	252	257	261
Move D	-145	-151	-151	-151	-112	-124	-123	-123
Other	27	-22	-26	-29	192	155	150	146
Overall	38	54	-37	-38	347	258	258	259
				Greece				
			e=1	Greece			e=2	
	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, ho_1)$	$v(\tilde{y}, \rho_2)$	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
Stayers	-43	-37	-37	-38	-75	-61	-62	-63
Move U	333	290	292	294	584	475	482	489
Move D	-305	-313	-313	-312	-252	-275	-275	-274
Other	-451	-576	-575	-574	40	-119	-113	-108
Overall	-466	-637	-633	-629	296	20	32	44
				Spain				
			e=1				e=2	
	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(ilde{y}, ho_2)$
Stayers	-71	-59	-61	-63	-123	-97	-101	-105
Move U	621	518	535	549	1068	847	883	914
Move D	-432	-452	-449	-446	-356	-430	-416	-404
Other	264	210	214	217	510	522	513	506
Overall	381	216	238	257	1099	841	878	911
				Hungary				
			e=1				e=2	
	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
Stavers	-28	-26	-26	-26	-46	-42	-43	-44
Move U	112	101	102	103	183	166	169	172
Move D	-95	-96	-96	-95	-92	-102	-101	-101
Other	-298	-147	-146	-146	-18	-44	-43	-42
Overall	-359	-167	-166	-164	26	-23	-19	-15
				Italy				
	()		e=1	(~ \	I ()		e=2	(~ \
Ct -	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	v(y)	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
Stayers	-20	-17	-18	-18	-35	-28	-29	-30
Move U	605	517	534	549	1064	847	882	913
Move D	-364	-374	-371	-370	-319	-359	-347	-338
Other	391	346	345	344	598	619	602	589
Overall	612	472	490	505	1308	1079	1108	1133

Note 1: Columns titled v(y) show W when v^i are based on initial year income, y. Note 2: Columns titled $v(\tilde{y}, \rho_c)$ show W when v^i are based on \tilde{y} , with $\rho = c$.

driven by movers down being significantly larger than the movers up effect. Analogous to the results presented in Table 7, accounting for longer term income reduces the negative welfare impact of the movers down, and increases the positive impact of movers up. As ρ increases however, this effect is partially reversed due to a decrease in the value of \tilde{y} (converging on $min(y_1,...,y_T)$ as ρ grows) for those in the bottom decile in the final year - the stayers and movers down - relative to the rest of the income distribution.

The choice between making welfare weights a function of initial year incomes or permanenttype incomes (or, indeed, final year incomes) is not unlike the index problem in generating

Table 8: Decomposition of Scalar Social Welfare Function, W(.), with Aversion to Income Variability - Final Year deciles

$\begin{array}{c c c c c c c c c c c c c c c c c c c $					Estonia				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			ϵ				ϵ		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	Stayers		-14						-27
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move U				84			81	78
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move D	-407	-315	-335	-353	-729	-494	-549	-597
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Other	-562		-549	-547	-566	-631	-614	-597
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Overall	-906	-794	-815	-833	-1258	-1065	-1107	-1144
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					Greece				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			ϵ	=1			ϵ	=2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Stayers	-43	-34	-36	-38		-54	-59	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move U	144	150	148	147	121	143	138	134
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move D	-746	-595	-629	-660	-1315	-935	-1030	-1115
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Other	-1579	-1580	-1570	-1560	-1450	-1609		-1534
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Overall	-2224	-2060	-2087	-2112	-2719	-2456	-2524	-2580
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					Spain				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			ϵ	=1	1. 1.		ϵ	=2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Stayers						-90		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move U	235	260	255	250	169	244	221	203
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move D	-1232	-847	-934	-1012	-2265	-1331	-1530	-1709
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Other	-840	-829	-813	-800	-883	-1150	-1048	-968
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Overall	-1920	-1473	-1556	-1630	-3133	-2328	-2460	-2590
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					Hungary				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			ϵ	=1	G J		ϵ	=2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Stayers				-27				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move U	51	53	53	53	41	49	49	49
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Move D	-185	-157	-161	-164	-308	-247	-264	-278
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Other	-433	-438	-435	-433	-457	-513	-517	-519
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Overall	-597	-567	-569	-571	-773	-752	-775	-793
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					Italy				
Stayers -22 -16 -16 -16 -40 -25 -26 -28 Move U 248 264 263 262 197 253 256 258 Move D -882 -632 -648 -661 -1608 -994 -1060 -1117			ϵ	=1			ϵ	=2	
Stayers -22 -16 -16 -16 -40 -25 -26 -28 Move U 248 264 263 262 197 253 256 258 Move D -882 -632 -648 -661 -1608 -994 -1060 -1117		$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$	$v(y_f)$	$v(\tilde{y}, \rho_0)$	$v(\tilde{y}, \rho_1)$	$v(\tilde{y}, \rho_2)$
Move U 248 264 263 262 197 253 256 258 Move D -882 -632 -648 -661 -1608 -994 -1060 -1117	Stayers								
Move D -882 -632 -648 -661 -1608 -994 -1060 -1117		248	264	263	262	197	253	256	258
	Move D	-882	-632	-648	-661	-1608	-994	-1060	-1117
Otner -714 -655 -656 -657 -805 -904 -921 -934	Other	-714	-655	-656	-657	-805	-904	-921	-934
Overall -1370 -1038 -1057 -1073 -2255 -1669 -1752 -1821	Overall	-1370	-1038	-1057	-1073	-2255	-1669	-1752	-1821

Note 1: Columns titled $v(y_f)$ show W when v^i are based on final year income, y_f .

Note 2: Columns titled $v(\tilde{y}, \rho_c)$ show W when v^i are based on \tilde{y} , with $\rho = c$.

cost-of-living indices. Basing the welfare weights on initial year incomes (or ranking) can be viewed as providing an upper bound on the welfare gain following a change in the distribution of income between two periods, much like the Laspeyres Cost-of-Living Index which provides a cost of living index based on the initial expenditure patterns. Basing welfare weights on final year incomes (or ranking) can be related to the Paasche Cost-of-Living Index, which is based on final year expenditure patterns. It can therefore be seen as a lower bound on the welfare gain (or lower bound on the welfare loss) following a change in the distribution of income between

two periods. The welfare weight based on permanent or variability-adjusted income lies between these two extremes.

5 Summary and Discussion

While cross-sectional analyses of changes in the distribution of income provide evidence on how changes in income affect different quantiles of the income distribution, it remains unclear from such analyses how the incomes of specific individuals are affected. When reranking of individuals across the income distribution occurs, a longitudinal perspective is required to determine the extent to which observed patterns of anonymous income changes affected individuals based on their initial or final ranking in the income distribution. Focusing on income changes at the bottom of the income distribution, a decomposition of cross-sectional income changes in the bottom decile of the income distribution into a stayers effect and a movers effect was proposed. The stayers effect is the contribution to the observed change in mean income in the bottom decile made by individuals who were initially in the bottom decile, and remained there at the end of the period. The movers effect is the component of the overall change in mean income in the bottom decile that was the result of a change in the composition of the bottom decile. The decomposition complements a growing literature, including Wagstaff's (2005) and Jenkins and Van Kerm's (2006) decompositions of inequality measures and Grimm's (2007) decomposition of poverty measures, that use longitudinal data to provide evidence on the drivers of crosssectional results.

Whether the identity of the individuals driving the observed change in income matters to the overall change in welfare in society depends on whether the welfare effect is measured anonymously or non-anonymously. In either case, the decompositions can be applied to the change in welfare to identify the group of individuals driving the overall effect. When welfare is measured anonymously, the decomposition provides evidence on whether overall welfare change, for a given social welfare function, was driven by a change in welfare for the stayers, or whether a change in the composition of the bottom decile drove the overall change. When welfare is measured without the axiom of anonymity, the decomposition shows whether welfare gains for individuals moving out of the bottom decile were offset by welfare losses for individuals moving into the bottom decile, and whether stayers made a positive or negative contribution to overall welfare change.

The decompositions, first of income changes in the bottom decile, then of overall welfare effects, were applied to a group of European countries for whom the largest proportional income losses between 2007 and 2010, on a cross-sectional basis, were concentrated among the poorest 10 per cent of the population. For four out of five of the countries analysed, the relatively large income losses in the bottom decile were driven by a change in the composition of the bottom decile, rather than income losses for individuals who began the period in the bottom decile. When measured anonymously, the welfare change for four of the five countries was therefore also driven by individuals dropping into the bottom decile during the period, particulary at higher levels of inequality aversion. Hungary was the outlier, where the income loss in the bottom decile was due, in almost equal proportions, to income losses for individuals that were initially in the bottom decile and individuals that dropped into the bottom decile during the period.

When measured non-anonymously, the overall welfare effect and the results of the decomposition were shown to depend on how each individual's welfare was evaluated, and whether deciles were assigned based on initial or final year rank. Somewhat paradoxically given the depth of recession throughout Europe during the period considered, when the welfare effect of the change in the distribution of income was measured non-anonymously based on initial year

incomes and income change, welfare increased in all countries for the majority of specifications of the underlying social welfare function. However, as argued by Palmisano (2015), there seems no a priori reason to base welfare evaluations on initial year incomes rather than viable alternatives. The positive welfare effect was reduced or entirely offset by allowing social weights to depend on 'permanent income' (also allowing for an aversion to income variability between periods), or by grouping individuals into deciles based on their final year ranking rather than their initial year ranking. The aim of the analysis was not to resolve the question of which approach is most "correct" to use, the answer to which will differ depend on moral or philosophical judgement. Rather the aim was to highlight that results can differ significantly depending upon the assumption of anonymity or non-anonymity in the underlying social welfare function, and in the case where the latter is assumed, upon how non-anonymity is interpreted (in terms of initial/final period weights or some combination of the two).

The paper examined the extent to which the individuals who were initially among the poorest ten per cent of the population in a range of European countries suffered the largest income losses during the early years of the Great Recession. To do so, a decomposition was proposed which allowed us to differentiate between the contribution individuals that remain in the bottom decile make to overall income and welfare changes, and the contribution of individuals that transition between deciles make to overall income and welfare changes, during the period of interest. On average, in all countries examined the real income of the stayers group fell between 2007 and 2010. However, the results showed that in all but one country, the majority of the large falls in income for the poorest ten per cent of the population on a cross-sectional basis were not driven by falls in income for the initially poorest individuals, but by those falling into the bottom decile. The net welfare implications, therefore, depend largely on the treatment of anonymity in the underlying social welfare function.

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Appendices

A Proof of Additivity of Income Decomposition

The absolute change in income in the bottom decile can be written as (which is Equation (4) in the text):

$$\delta_{abs}^1 = \mu_{t+n}^1 - \mu_t^1 \tag{20}$$

Adding μ_t^1 to each side and rearranging we get:

$$\delta_{abs}^1 = \mu_t^1 + \mu_{t+n}^1 - \mu_t^1 - \mu_t^1 \tag{21}$$

Using Equations (1) and (2) to replace the first two terms on the right hand side of Equation (21) we get:

$$\delta_{abs}^{1} = \sigma^{s} \mu_{t}^{s} + (1 - \sigma^{s}) \mu_{t}^{mu} + \sigma^{s} \mu_{t+n}^{s} + (1 - \sigma^{s}) \mu_{t+n}^{md} - \mu_{t}^{1} - \mu_{t}^{1}$$
 (22)

Rearranging gives:

$$\delta_{abs}^{1} = \sigma^{s} \mu_{t}^{s} + (1 - \sigma^{s}) \mu_{t+n}^{md} - \mu_{t}^{1} + \sigma^{s} \mu_{t+n}^{s} + (1 - \sigma^{s}) \mu_{t}^{mu} - \mu_{t}^{1}$$
(23)

which can be rewritten as:

$$\delta_{abs}^{1} = \mu_{cf1}^{1} - \mu_{t}^{1} + \mu_{cf2}^{1} - \mu_{t}^{1} \tag{24}$$

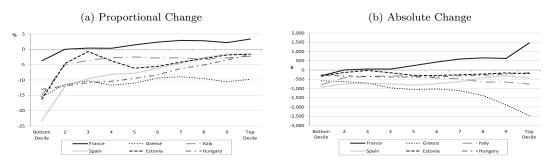
Therefore, we have:

$$\delta^{1}_{abs} = \delta^{1}_{cf1_{abs}} + \delta^{1}_{cf2_{abs}} \tag{25}$$

which is Equation (11) in the text, in absolute income change form. We can simply divide both sides by μ_t^1 to prove additivity in the proportional income change case, so that:

$$\delta_{pc}^{1} = \delta_{cf1_{pc}}^{1} + \delta_{cf2_{pc}}^{1} \tag{26}$$

Fig. 6: Change in Real Equivalised Disposable Household Income by Decile - 2007 to 2010 - Cross Sections 08 to 11



Source: Author calculations based on EU-SILC 2008 and 2011 cross-section files.

B Data Appendix

In this appendix, we present some summary statistics and sensitivity tests related to the data used in this analysis. For more details on the EU-SILC data, see Lohmann (2011).

When analysing poverty rates using the EU-SILC data, Jenkins and Van Kerm (2014) derive the poverty lines from the cross-sectional data, and use this value of the poverty line when analysing persistent poverty with the longitudinal data. This is motivated by the fact that the larger sample size in the cross sectional data is likely to produce more reliable estimates of median income. As a sensitivity test, we mirrored this approach by deriving decile cut-offs from the cross-sectional data and applying them to the longitudinal data. One implication of this approach is that often there is not exactly 10 per cent of the population in each decile, so that σ^s can vary slightly between years. We therefore assumed that $\sigma^s = \sigma^s_t = \sigma^s_{t+n}$ at year t values. Results were robust to this test²⁴. Table 9 shows the proportion of the longitudinal sample in each of the 2007 deciles using the decile cut-offs from the cross-sectional data. In most cases, there is between 9 and 11 per cent of the population in each decile. The lowest deciles in Spain seem to be under-represented in the longitudinal data.

Figure 6 replicates Figure 1 in the text, but using the cross-sectional data rather than the longitudinal data. The decompositions are performed on the longitudinal data, so are decompositions of the patterns show in Figure 1.

Table 9: Proportion of LT Observations in Each Decile - 2008 LT sample with CS decile cut-offs

Decile	Greece	Italy	Spain	Estonia	Hungary
Bottom Decile	8.2	9.6	7.3	9.3	10.2
2	8.9	10.5	7.9	9.5	11.8
3	9.9	10.6	10.2	9.3	8.7
4	8.7	10.9	10.6	8.8	10.4
5	11.4	9.1	10.4	9.1	9.6
6	9.5	11.3	11.0	9.6	9.1
7	11.6	9.4	9.2	11.3	10.1
8	9.9	9.6	11.4	10.6	8.2
9	12.7	9.7	11.0	12.2	10.1
Top Decile	9.2	9.2	11.0	10.3	11.7

Source: Author analysis of EU-SILC 2011 longitudinal file.

Some concerns about the reliability of the EU-SILC longitudinal data should be recognised. Krell et al. (2015) and Jenkins and Van Kerm (2015) examined the quality of the income information and poverty information

²⁴ We also tested the sensitivity of the results to setting σ^s equal to the year t + n value. Results are also robust to this sensitivity check.

Table 10: Years in bottom decile by transition group

Years in Bottom Decile	Stayers (%)	Movers Up (%)	Movers Down (%)	Other (%)
		Gr	reece	
0	-	-	-	93
1	-	46	65	6
2	2	32	19	1
3+	98	22	16	0
	100	100	100	100
		It	aly	
0	-	-	-	95
1	-	50	45	4
2	14	27	37	1
3+	86	24	18	0
	100	100	100	100
		S_{I}	oain	
0	-	-	-	93
1	-	56	54	7
2	29	32	34	1
3+	71	12	12	0
	100	100	100	100
		Est	tonia	
0	-	-	-	92
1	-	43	49	7
2	14	40	23	1
3+	86	17	28	0
	100	100	100	100
		Hu	ngary	
0	-	-	-	93
1	-	49	32	5
2	13	31	35	2
3+	87	20	33	0
	100	100	100	100

respectively in the longitudinal EU-SILC. Using 2005 to 2009 EU-SILC data, Krell et al. found that, while EU-SILC "is an unparalleled data source of major importance for research", that there were significant variances in income measures in the cross-sectional and longitudinal data. They found that this was particulary true in countries that derive their income information from registers 25. Jenkins and Van Kerm (2015) found that attrition from the survey can result in wide bounds for poverty estimates coming the longitudinal sample. Both Krell et al. and Jenkins and Van Kerm made recommendations to improve the quality and reliability of the EU-SILC longitudinal data. While Table 11 shows that the summary statistics of a number of key variables exhibit quite similar distributions when compared between the cross-sectional and longitudinal files used in this analysis, the results and recommendations of the two papers discussed here should be borne in mind when interpreting results.

 $^{^{25}\,}$ None of the countries in the present analysis use income register data.

Table 11: Summary Statistics 2011 - Longitudinal Sample (4-year balanced) and Cross Section Sample

	Gre	Greece	Ita	Italy	Spain	ain	Estonia	nia	Hungary	gary
	CS	TT	CS	TT	CS	TT	CS	TT	CS	TT
		7	Age Distribution	ribution						
<16	15.5	14.0	15.4	12.4	16.1	14.3	16.4	13.3	16.0	14.8
<30	16.0	17.4	14.6	14.9	16.1	15.6	20.3	21.1	17.9	18.6
<45	23.2	23.9	22.7	21.6	25.7	23.5	20.3	19.4	20.5	22.1
<65	25.9	25.6	26.9	28.8	25.1	26.6	25.9	28.0	29.6	27.7
65+	19.4	19.1	20.5	22.3	17.0	20.0	17.1	18.2	16.1	16.8
		Deg	Degree of Urbanisation	rbanisat	ion					
Densely populated area	43.3	35.1	43.4	43.5	50.9	49.3	48.6	49.5	30.8	30.8
Intermediate area	13.8	8.7	40.8	39.9	22.0	23.1	$_{\rm n/a}$	$^{\mathrm{n/a}}$	21.5	21.8
Thinly populated area	42.9	56.2	15.8	16.6	27.1	27.6	51.5	50.5	47.7	47.5
			Housing	Tenure						
Outright owner	60.2	64.4	57.4	62.1	47.7	52.0	67.4	72.1	2.99	69.4
Owner paying mortgage	15.9	17.0	15.7	12.7	32.0	32.8	16.4	14.4	23.3	22.5
Renting, Free Accom.	23.9	18.6	26.9	25.1	20.3	15.2	16.2	13.5	10.0	8.1
			Sex	×						
Female	50.8	50.7	51.4	51.5	50.6	29.0	54.2	54.2	52.7	53.3
	Sel	Self-Defined	d Current	t Econo	Economic status	ıs				
Employee	27.0	27.5	35.0	30.5	39.3	39.9	49.1	51.2	39.4	41.5
Self-Employed	12.9	14.1	9.6	10.0	5.9	7.5	3.9	3.6	5.5	4.9
Unemployed	13.0	12.4	6.4	7.9	13.5	11.3	7.1	5.9	7.9	9.7
Retired	27.0	27.9	27.0	27.9	27.0	27.9	27.0	27.9	27.0	27.9
Other	20.0	18.1	21.9	23.7	14.3	13.4	12.9	11.5	20.1	18.2
			Marital	Status						
Never married	25.7	27.9	28.9	28.4	30.8	29.7	36.7	35.1	29.3	31.6
Married	63.2	62.2	56.3	57.0	56.4	59.7	40.1	41.9	49.6	47.2
Separated, Widowed, Divorced	11.1	10.0	14.9	14.6	12.8	10.6	23.3	23.0	21.2	21.2
			Income	me						
Mean	10,349	10,360	15,325	15,303	11,963	12,681	5,512	5,425	4,273	4,271
Median	9,193	9,148	13,918	13,889	10,502	11,178	4,740	4,686	3,870	3,813
p10	4,302	4,244	6,631	6,710	4,559	5,177	2,398	2,325	2,132	2,063
06d	17,632	17,203	25,855	25,649	21,521	22,587	9,832	9,350	6,845	7,029
$_{ m p10/p90}$	0.24	0.25	0.26	0.26	0.21	0.23	0.24	0.25	0.31	0.29
Source: Author calculations based on EU-SILC 2011 longitudinal file and cross-sectional files.	d on EU-S	SILC 2011	longitud	inal file a	nd cross-s	ectional fi	les.			

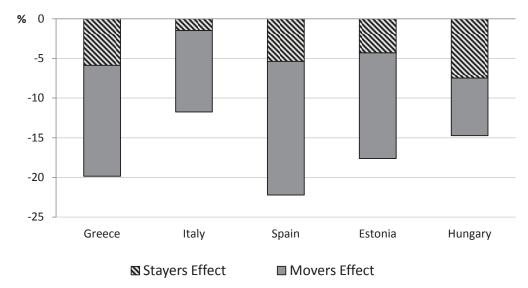


Fig. 7: Decomposition of Income Changes - No Self-Employed - Percentage Change in Income

Source: Author Analysis of EU-SILC 20111.

Given that individuals are aggregated into deciles in this analysis, measurement error is less likely to significantly affect results than if percentiles or a lower level of aggregation was used. Nonetheless, we carried out two sensitivity checks to ensure that measurement error was not driving the results found in the paper. The first test, proposed by Fields et al. (2003) and applied in Palmisano (2015) and Grimm (2007), identifies the proportion of measurement error in the income data that would be required to "undo" to downward slope in the na-GICs. According to Grimm, values outside a range of 10 per cent to 30 per cent suggest that results are robust to measurement error, while Palmisano used a threshold of 10 per cent. Even based on the more stringent threshold used by Grimm, only in Hungary can it not be excluded with certainty that measurement error is responsible for the observed progressive pattern of non-anonymous income changes. If we assume particularly high levels of correlation between initial year income and measurement error, then the downward sloping na-GIC for Spain may also be due to measurement error. In the majority of cases however, the results suggested that the variance of measurement error would need to be between 40 and 150 per cent to undo the downward slope.

The second sensitivity test is motivated by Jenkins and Van Kerm (2011) who suggested that incomes of the self-employed were more likely to be measured with error than the income of employees. We therefore drop from the analysis any households that receive any self-employment income in any year from 2007 to 2010. Again, results remain robust to this check (see Figure 7).

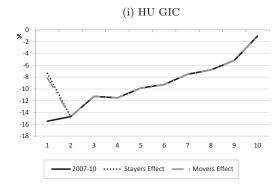
C Decomposition of the Social Evaluation of Proportional Income Changes

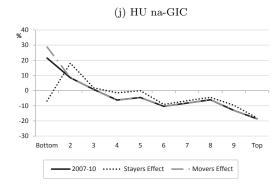
Figures 8 and 9 replicate Figures 4 and 5 in the main text, using proportional income changes rather than absolute income changes. In each case, qualitative results remain largely unchanged.

(a) EL GIC (b) EL na-GIC 50 %40 % -2 30 20 -8 -10 -10 -20 -12 -14 -16 -30 -18 -40 ■ 2007-10 ······ Stayers Effect · Movers Effect Movers Effect 2007-10 ····· Stayers Effect (c) IT GIC (d) IT na-GIC _% 0 %⁶⁰ 50 40 30 -6 20 10 -8 0 -10 -10 -12 -20 -14 -30 Bottom ■ 2007-10 ······ Stayers Effect · Movers Effect ■ 2007-10 ······ Stayers Effect · Movers Effect (e) ES GIC (f) ES na-GIC 75 **%** 60 % ₀ 45 -5 -10 30 15 -15 -20 0 -25 -15 -30 -30 Тор Bottom 2007-10 ····· Stayers Effect — · Movers Effect ■ 2007-10 ······ Stayers Effect — · Movers Effect (g) EE GIC (h) EE na-GIC % ⁰
-2
-4
-6 %⁵⁰ 40 30 20 10 -8 0 -10 -10 -12 -20 -14 -30 -16 -40 2007-10 ······ Stayers Effect · Movers Effect ■ 2007-10 ······ Stayers Effect — · Movers Effect

Fig. 8: Anonymous and Non-Anonymous Growth Incidence Curves - Proportional

Fig. 8: Cont'd.





Greece

■ Movers Effect

Spain

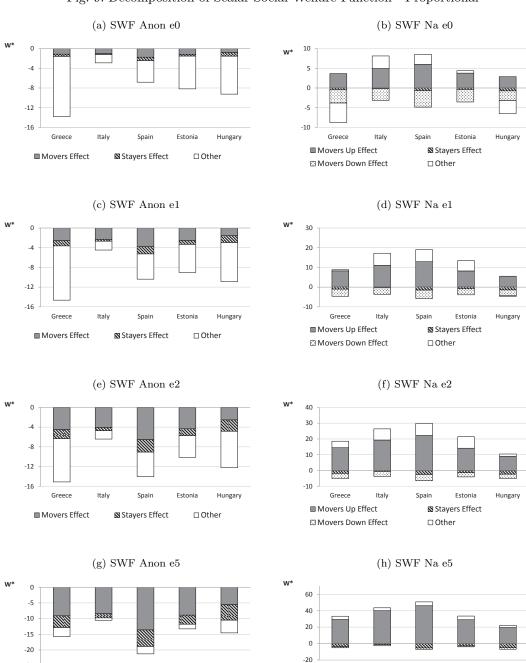
■ Stayers Effect

Estonia

□ Other

Hungary

Fig. 9: Decomposition of Scalar Social Welfare Function - Proportional



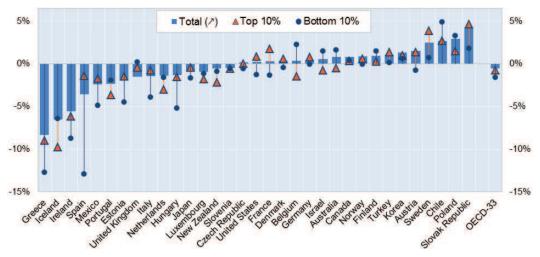
Estonia

■ Stayers Effect

Greece

■ Movers Up Effect

Fig. 10: Annual percentage changes in household disposable income between 2007 and 2011, by income group



Source: OECD (2014).

		Title/Author(s)
Year	Number	ESRI Authors/Co-authors <i>Italicised</i>
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	517	How Do External Costs Affect Pay-As-Bid Renewable Energy Connection Auctions? Niall Farrell and Mel T. Devine