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Common Cycles in Volatility and Cross Section of Stock Returns

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Common Cycles in Volatility and Cross Section of Stock Returns

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Abstract:

We study the relationship between conditional quantiles of returns and the long-, medium- and short-term volatility in a portfolio of financial assets. We argue that the combination of quantile panel regression and wavelet decomposition of the volatility time series provides us with new insights into the pricing of risk and increases the accuracy of our estimates of re-turn quantiles. Our results contribute to the literature on the risk-return relationship with an emphasis on portfolio management under various investment horizons. Moreover, the analytical framework that we introduce should be applicable to a wide range of problems outside of our research area.

Keywords: Return predictability, Quantiles, Wavelets, Panel data

JEL: C14, C21, C58, G17

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1 Introduction

Estimation of conditional asset return distributions is one of the core tasks in finance. Whether we analyze the overall risk-return profile of an asset or just estimate the Value at Risk (VaR) or the Expected Shortfall (ES), the shape and dynamics of the conditional distributions are crucial. In our work we approach this task using quantile regression (QR), which is one of the methods widely used in VaR estimation. We build on Barunik & Zikes (2015), who (among other contributions) apply QR to the linear relationship between future log-returns and realized volatility (RV). They find that the model performs very well at capturing the dynamics of the respective conditional distributions. Yet, they use the RV time-series as an aggregate, which can hide some interesting patterns in the relationship. Equipped with wavelet transform, we look at the relationship in more detail. As discussed by James B. Ramsey in Gallegati & Semmler (2014), we can often gain “considerable insights into the analysis of economic relationships between variables”, when we apply wavelet transform and look at the dynamics at different time scales. The relationships are especially likely to be scale-dependent in financial market data, where the variables are influenced by investors with both long and short investment horizons. This has been shown many times in the literature, see e.g. Bandi et al. (2016), Dew-Becker & Giglio (2015), Kalyvitis & Panopoulou (2013), Gençay et al. (2005), or from the older papers e.g. Ramsey (1999) or Davidson et al. (1997).

Being motivated by these empirical results, we extend the study of Barunik & Zikes (2015) by applying wavelet transform and decomposing the RV in long-, medium- and short-term components. This enables us to analyze the horizon-specific sources of the return-variance relationship and to test whether the long-term volatility dynamics has the same power as the medium- and short-term volatility in explaining the conditional quantiles of the future log-return distributions.

First, we look at the results for individual assets (see Section 3). Then, we study the implications of our approach for a portfolio of assets. To do so, we apply a panel quantile regression of Koenker (2004) and study the average relationship compared to the results of individual quantile regressions. We are specifically interested in the sources of portfolio risk and whether we can make the portfolio more robust to specific shocks by optimizing its sensitivity to the long-, medium- and short-term RV. We find that there is a common risk structure that defines the sensitivity of conditional quantiles of asset log-returns to the past RV components. This motivates diversification based on the RV decomposition.

To our knowledge, we are the first to come up with such an analysis. Our results enrich the literature on the risk-return relationship and portfolio management under various investment horizons. Yet, the analytical framework that we introduce should be applicable to a wide range of problems outside of our research area.

The paper is organized as follows: In the next section, we describe our methodology, Section 3 introduces our data, in Section 4 we present our results and Section

5 concludes.

2 Methodology

We develop a methodology based on panel quantile regression of asset returns on volatility that is decomposed using wavelet transform. First, we analyze the results for single assets from several industries. Then, we move to panel data, which enables direct testing for the presence of common patterns in the scale-dependence of the return-volatility relationship.

Following Barunik & Zikes (2015), we define a conditional τ -quantile model

$$q_\tau(r_{i,t} | \Omega_{t-1}) = \alpha_i(\tau) + \beta_i(\tau)RV_{i,t-1}. \quad (1)$$

$q_\tau(y_t | \cdot)$ is the conditional τ -quantile of y_t , see e.g. Koenker (2005). Ω_{t-1} denotes the information set available at time $t - 1$, $r_{i,t}$ is the log-return of asset i at time t defined as

$$r_{i,t} = \ln(P_{i,t}) - \ln(P_{i,t-1}) \quad (2)$$

where $P_{i,t}$ is the asset price at time t . $RV_{i,t}$ is the realized volatility of asset i at time t defined as

$$RV_{i,t} = \sqrt{\sum_{m=0}^{M-1} \left(s_{i,t-\frac{m}{M}}^2 \right)} \quad (3)$$

where t is the time index and M stands for the number of intraday log-returns s . See e.g. Andersen et al. (2003).

First, we decompose the realized volatility using wavelet-based multi-resolution analysis (MRA) into short-, medium- and long-term components (RV^S , RV^M , RV^L), so that

$$RV_{i,t} = RV_{i,t}^L + RV_{i,t}^M + RV_{i,t}^S. \quad (4)$$

To do so, we use the Maximal Overlap Discrete Wavelet Transform (MODWT) with *LA8* wavelet and “reflection” boundary conditions (as e.g. Jensen & Whitcher (2014)) and perform a level $J = 3$ MRA. This decomposes the \mathbf{RV} time series into three wavelet details \mathbf{D}_j , $j = 1, 2, 3$ and one smooth \mathbf{S}_3 . Since we work with daily data ($\Delta t = 1$ day), the components of the MRA are associated with changes in averages on physical scales of 1, 2 and 4 days ($\mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3$) and averages over 8 days (\mathbf{S}_3).

$$\mathbf{RV} = \mathbf{S}_3 + \sum_{j=1}^3 \mathbf{D}_j. \quad (5)$$

We then define the short-term volatility component as the first-level details ($\mathbf{RV}^S = \mathbf{D}_1$), the medium-term component as the sum of the second- and third-level details ($\mathbf{RV}^M = \mathbf{D}_2 + \mathbf{D}_3$) and the long-term component as the third-level smooth ($\mathbf{RV}^L = \mathbf{S}_3$). What we get then is Equation 4. Figure 1 shows the RV

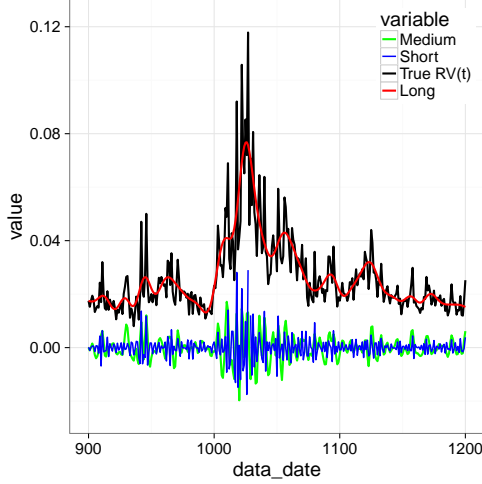


Figure 1: AAPL: Example of RV MRA

decomposition for one of our assets, Apple stock (AAPL). The sum of the individual components gives the true value of RV. For details of the MODWT, MRA and the properties of the resulting time series components see e.g. Percival & Walden (2000), wavelets in the broader context of signal processing are discussed e.g. in Vetterli et al. (2014).

Second, we apply quantile regressions to individual assets, see e.g. Koenker (2005). At this point, the only difference from the model of Barunik & Zikes (2015) is the use of RV components instead of the RV aggregate (\mathbf{RV}).

$$q_\tau(r_{i,t} | \Omega_{t-1}) = \alpha_i(\tau) + \beta_i^S(\tau)RV_{i,t-1}^S + \beta_i^M(\tau)RV_{i,t-1}^M + \beta_i^L(\tau)RV_{i,t-1}^L \quad (6)$$

Figure 2 illustrates how the decomposition changes the AAPL QR results compared to the QR using \mathbf{RV} . The intercept becomes smaller and we see the reversed slope for \mathbf{RV}^S and asymmetric effects of \mathbf{RV}^M . As we discuss later on, these patterns are common to most of the assets in our dataset, as can be seen in Figure 8 in Appendix A. For fitted values using results from the QR for AAPL see Figure 4 in Appendix A.

Finally, we apply the penalized fixed effects panel quantile regression of Koenker (2004):

$$q_\tau(r_{i,t} | \Omega_{t-1}) = \alpha_i(\tau) + \beta^S(\tau)RV_{i,t-1}^S + \beta^M(\tau)RV_{i,t-1}^M + \beta^L(\tau)RV_{i,t-1}^L \quad (7)$$

and estimate it using the algorithm available in the “rqpd” package for R. This follows the method described in Koenker (2004), where the estimates solve the optimization problem:

$$\min_{\alpha, \beta} \sum_{k=1}^{N_q} \sum_{i=1}^m \sum_{t=2}^n \{ \rho_{\tau_k}(r_{i,t} - \alpha_i(\tau_k) - \mathbf{x}_{i,t-1}^T \boldsymbol{\beta}(\tau_k)) \} + \lambda \sum_{i=1}^m |\alpha_i(\tau_k)| \quad (8)$$

$$\mathbf{x}_{i,t}^T \boldsymbol{\beta}(\tau_k) = \alpha_0(\tau_k) + \beta^S(\tau_k)RV_{i,t}^S + \beta^M(\tau_k)RV_{i,t}^M + \beta^L(\tau_k)RV_{i,t}^L \quad (9)$$

$$\rho_\tau(u) = u(\tau - I(u < 0)) \quad (10)$$

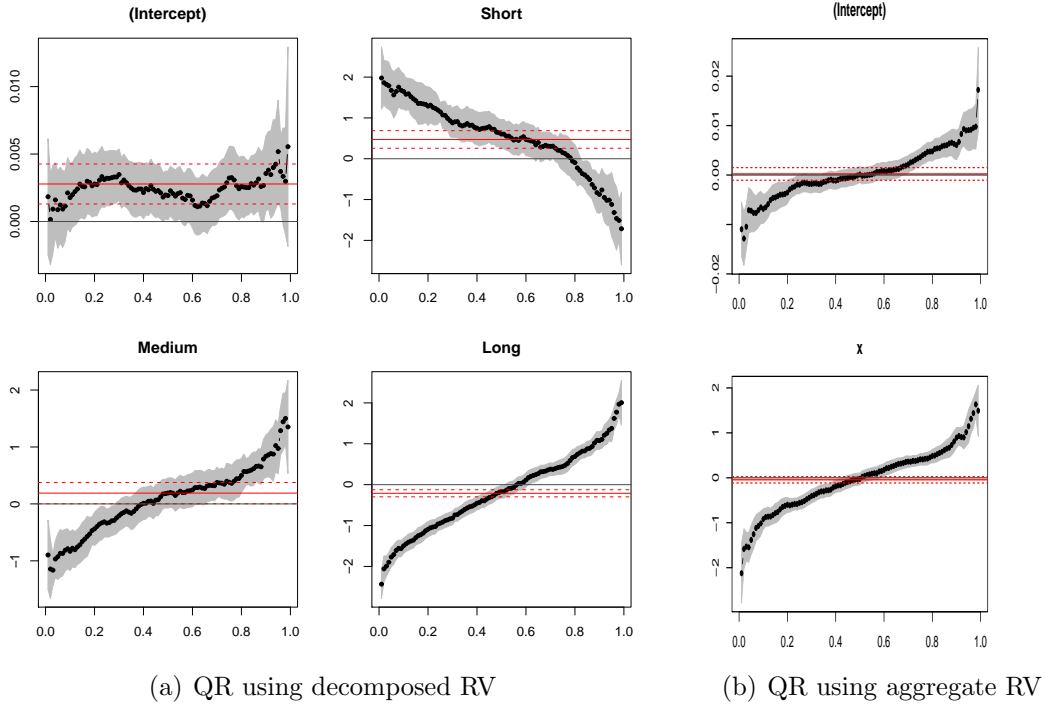


Figure 2: Quantile regression coefficients for AAPL: decomposed RV vs aggregate RV. Grey areas are the 95% confidence intervals. Red lines show the coefficients that we would get using OLS instead of the QR and their 95% confidence intervals.

N_q is the number of quantiles assumed, n is the length of our time series (the same for all assets as we have a balanced panel), m is the number of assets and the constant term for each asset and quantile is composed of the intercept and asset-specific fixed effect: $\alpha_0(\tau) + \alpha_i(\tau)$. As in the case of individual quantile regressions, we estimate the model quantile by quantile ($N_q = 1$) and let the fixed effects $\alpha_i(\tau)$ be quantile-dependent. The size of our dataset permits this approach, see Koenker (2004) or Galvao (2011). ρ_τ is a piecewise linear quantile loss function of Koenker & Bassett Jr (1978) and $\lambda \sum_{i=1}^m |\alpha_i(\tau_k)|$ is the penalty term. The higher the λ , the stronger the pressure on fixed effects to be small. In our application, we set $\lambda = 1$, which is the “rqpd” package default and we obtain fixed effects $\alpha_i(\tau)$ very close to zero. To see if our penalization was not too strong, we compare the results with the case of $\lambda = 0$, where $\alpha_0 = 0$ (i.e. the constant term is equal to α_i) and $\lambda \sum_{i=1}^m |\alpha_i(\tau_k)| = 0$. The resulting asset-specific constant terms $\alpha_0(\tau) + \alpha_i(\tau)$ do not change significantly, which supports our original choice. In general, the search for an optimal λ might be non-trivial, as discussed e.g. in Galvao & Montes-Rojas (2010).

After estimating the model, we construct standard errors, t-statistics, and p-values using bootstrap with 200 replications. We apply the generalized bootstrap of Bose & Chatterjee (2003) with unit exponential weights sampled for each asset, rather than for each observation. The method is available in the “rqpd” package.

For more details about the panel quantile regression see Koenker (2004) or Galvao & Montes-Rojas (2010).

3 Data

We use asset price data for 29 highly liquid U.S. stocks from 8 sectors according to the Global Industry Classification Standard (GICS). The time period is August 2004 to December 2015 with a total of 2825 observations per stock. Table 1 lists all the assets and the respective sectors.

Table 1: Data

Sector	Stocks
Consumer Discretionary	AMZN, CMCSA, DIS, HD, MCD
Consumer Staples	KO, PEP, PG, WMT
Energy	CVX, SLB, XOM
Financials	BAC, C, JPM, WFC
Health Care	JNJ, MRK, PFE
Industrials	GE
Information Technology	AAPL, CSCO, IBM, INTC, MSFT, ORCL, QCOM
Telecommunication Services	T, VZ

The data were checked for possible missing or extreme values. The only drawback is the low number of assets in the individual categories, which hinders cross-industrial analysis.

4 Results

Figure 3 summarizes our results from both the individual and panel quantile regression models. The box-plots represent the results of individual regressions, the red line with green 95% confidence interval shows the panel regression results, excluding the fixed effects, which are presented separately. Tables with summary of the panel quantile regression model are in Appendix B.

4.1 Quantile Regressions

First, we look at the individual quantile regressions. All RV components are significant (at 5% significance level) except for quantiles where the $\beta(\tau)$ functions cross zero. Moreover, each RV component contributes in a specific way to the overall fit of the models. We find important common patterns in the coefficient signs and levels. Our results show a significantly decreasing trend of short-term coefficients, increasing and asymmetric medium-term coefficients and increasing long-term coefficients, as shown also in Figure 8. As these patterns are common to all our assets, they are also observable in the panel quantile regression results.

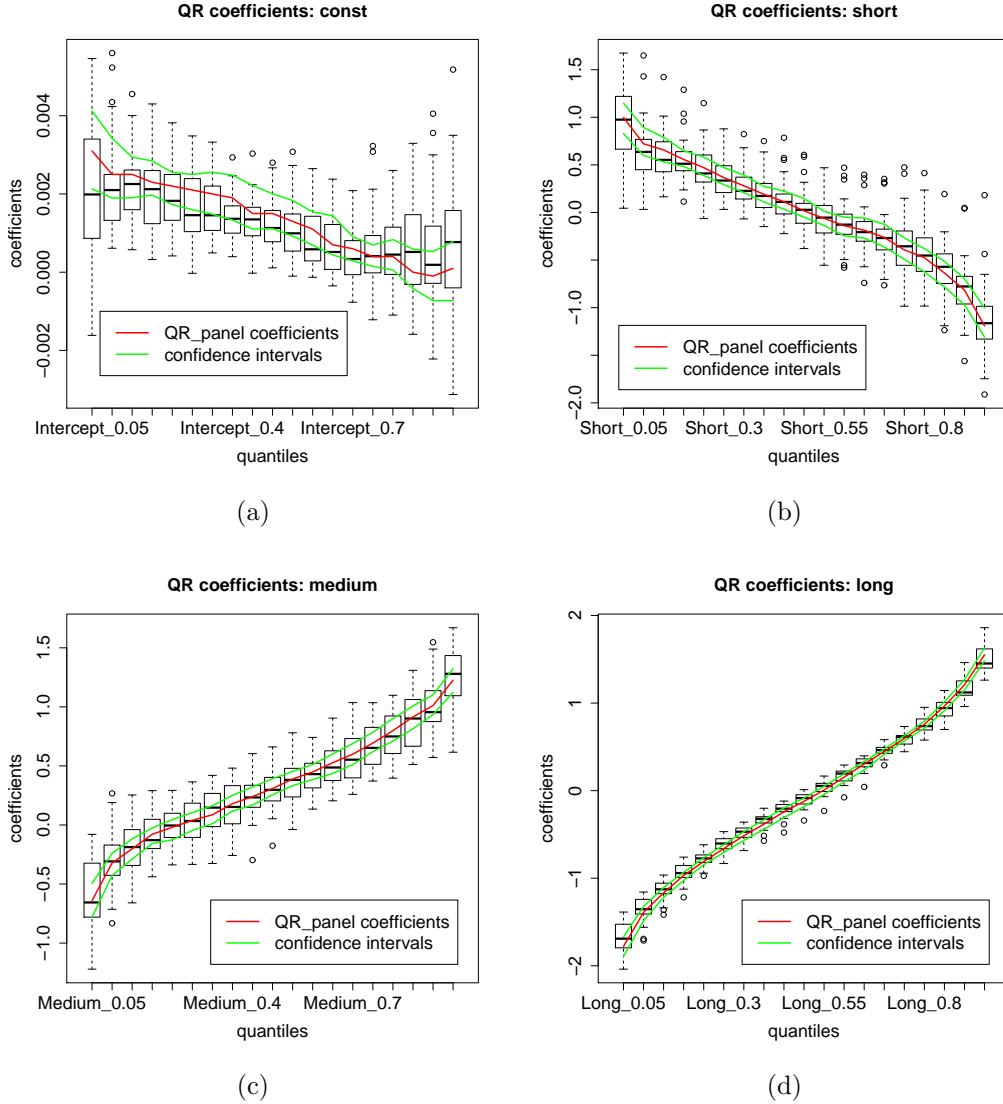


Figure 3: Panel quantile regression coefficients with 95% confidence intervals and boxplots representing coefficients from single-asset quantile regressions

4.2 Panel Quantile Regression

The results of panel quantile regressions are not surprising, given our results for individual assets. Coefficients of all RV components are significant for all quantiles except where the $\beta(\tau)$ crosses zero. This is $\tau = 0.55$ for the RV^L and RV^S and $\tau = 0.3$ for RV^M . The short-term coefficients are decreasing with increasing τ , the medium-term $\beta(\tau)$ is asymmetric and increasing and the long-term $\beta(\tau)$ is increasing. The functions of coefficients $\beta(\tau)$ nicely intersect the box-plots and reveal narrow confidence intervals, especially in the case of RV^L . Fixed effects are very close to zero, for most assets and quantiles, as discussed in Section 2 and summarized in Appendix B and Figure 5(a) in Appendix A. The constant term is therefore very similar for all assets. Moreover, its value is close to zero, espe-

cially for higher quantiles, and thus it does not significantly affect the conditional quantile estimates. This supports our hypothesis of a common structure in the log-return and RV relationship as there is no need for asset-specific compensation of the distribution estimates. The small constant terms show that we can capture most of the features of the empirical log-return distribution using only the decomposed RV.

Next, we look at the values of the RV component coefficients and what they tell us about the $r_{i,t} \sim RV_{i,t-1}$ relationship. See Figure 1 for an illustration of the dynamics of RV components.

- **RV^L**: the coefficients are close to quantiles of the normal distribution $N(-0.1, 1)$, as can be seen in Figure 6(a) in Appendix A. Therefore, the first layer of our estimate of the $r_{i,t}$ conditional distribution is very close to the normal distribution

$$N(-0.1 \times RV_{t-1}^L, 1 \times (RV_{t-1}^L)^2).$$

Parameters of this distribution change relatively slowly over time due to the dynamics of the RV^L .

- **RV^M**: the medium-term component adds the asymmetry for quantiles $\tau < 0.35$, while for higher quantiles the coefficients match the normal distribution with mean and variance around 0.3. Changes in this estimation layer are faster compared to the first layer, as RV^M represents higher frequencies of the RV process. Future quantification of the asymmetry is important. If it proves relevant for forecasting, it could be a sound argument against models assuming normality of the log-return distribution.
- **RV^S**: the short-term component determines the third and last layer of the $r_{i,t}$ conditional distribution estimate. It is the only RV component with decreasing coefficients with values close to $(-1) \times q_{\text{norm}}(\tau)$, where $q_{\text{norm}}(\tau)$ are the quantiles of $N(0, 0.41)$. The negative slope of $\beta^S(\tau)$ is a natural outcome of the dynamics of RV^S . RV^S represents the RV dynamics at highest possible frequencies and is likely to change sign on a daily basis. Hence, today's low short-term volatility predicts tomorrow's high short-term volatility and the coefficient has to be negative. The shape of this layer can change on a daily basis and gives us the finest detail of the estimated conditional distribution of $r_{i,t}$.

4.3 Aggregate vs. Decomposed RV

A natural benchmark for our model is the penalized fixed effects panel quantile regression of conditional quantiles of $r_{i,t}$ on $RV_{i,t-1}$:

$$q_\tau(r_{i,t} | \Omega_{t-1}) = \alpha_i(\tau) + \beta^{RV}(\tau)RV_{i,t-1}. \quad (11)$$

We estimate the model as described in Equation 8. For fixed effect estimates see Figure 5(b), for constant terms and RV coefficients $\beta^{RV}(\tau)$ see Figure 6. As was the

case of $\beta^L(\tau)$, $\beta^{RV}(\tau)$ is very close to the quantile function of a normal distribution, only this time the variance is lower. The forecast of the conditional distribution of the r_t therefore has two layers. The first is a fixed quantile function given by the constant terms. The second layer are the quantiles of the normal distribution $N(-0.04RV_{t-1}, 0.69(RV_{t-1})^2)$, where RV_{t-1} is the most recent observation of RV.

As the benchmark is a special case of our model where all three $\beta(\tau)$ are equal, our model should be always superior in terms of fit. Moreover, it provides a detailed analysis of the log-return \sim RV relationship. The benchmark model lacks a term capturing asymmetry and the effect of daily RV oscillation.

4.4 Sector-specific patterns

As we have shown, for all assets there are common patterns in the log-return \sim RV relationship. Yet, there are still slight differences among individual assets. When we look at the differences closer, we find that the similarity is stronger among assets from the same industry. Although our dataset is not sufficient for analyzing the differences between individual sectors, our results are a good motivation for a proper analysis in the future. For now, we present Figure 7 in Appendix A which shows the boxplots for two sectors, namely “finance” and “Consumer Discretionary”, and our panel quantile regression results estimated using the whole portfolio. The deviations from our panel results are not excessive, yet there is a visible difference between the coefficients from the two distinct sectors.

4.5 Common Cycles in Risk

Similar structure in the log-return \sim RV relationship provides a diversification opportunity. The more are the MRA components of two assets correlated, the more risky it is to add these two in a portfolio without any compensation. In case of equities, this is consistent with standard means of diversification across industries, as companies within one industry have very often a similar risk structure and any realized risk might affect companies within the industry similarly. This means that in fact, our RV components are proxies for all the factors affecting asset returns. We only take three variables summarizing all the risks at three time horizons, the short-, medium- and long-term. This brings our work close to the factor models of Fama & French (1993) and others. The question if our approximation is a sufficient basis for an effective diversification in real portfolio management is to be asked in the follow-up research.

5 Conclusion

In this paper we propose to model conditional distribution of asset log-returns as a linear combination of past long-, medium- and short-term RV components. First, we estimate the quantile regression for each asset separately, then we estimate a fixed effect panel quantile regression for a portfolio of assets. We find that there is a common structure in the log-return \sim RV relationship. For all assets, the long-term

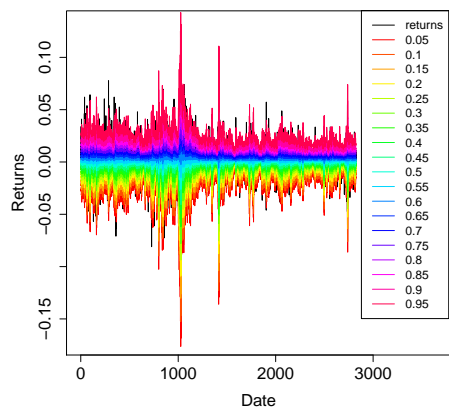
component, which is a smoothed RV time series, is the most important building block. It gives us for each observation the parameters of the normal distribution that is our first layer of the estimated future log-return distribution. The medium-term component that covers changes in RV on physical scales of 2-4 days then adds asymmetry to the distribution. The third layer is the short-term component that reflects high-frequency cycles of RV time series, especially the pattern that an increase in short-term RV is usually followed by its decrease on the next day. Overall the estimate performs better than that provided by quantile regression on aggregated RV. The common structure is very important from an analytical point of view, as it explains where the asymmetry in the log-return~RV comes from, as well as it shows why the RV aggregate cannot work that well in the model. Moreover, it opens a new approach to portfolio diversification that can be based upon the MRA of available assets and on the slight differences of the panel quantile regression parameters across industries. Future research in this area should bring interesting results.

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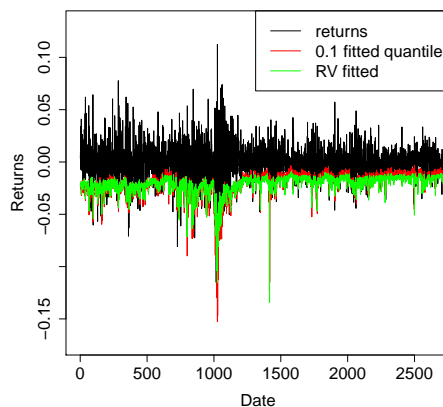
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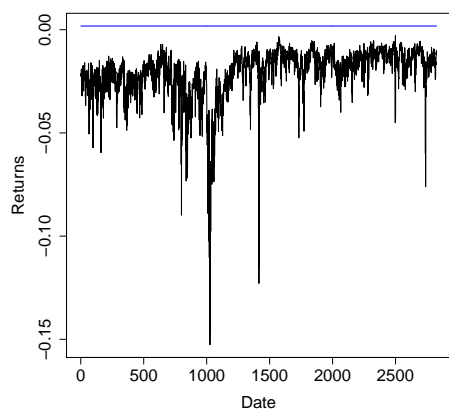
A Appendix A



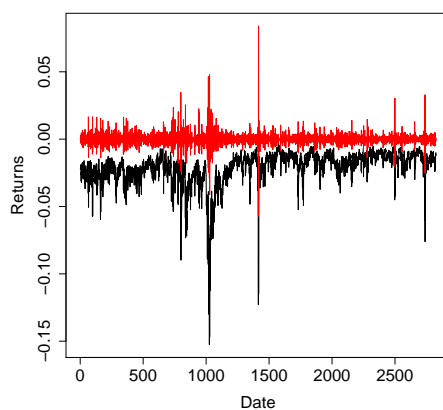
(a) Fitted values across the whole distribution



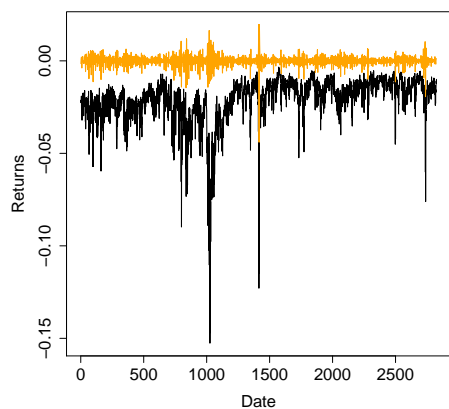
(b) True r_t (black), estimate using decomposed RV (red), estimate using aggregated RV (green)



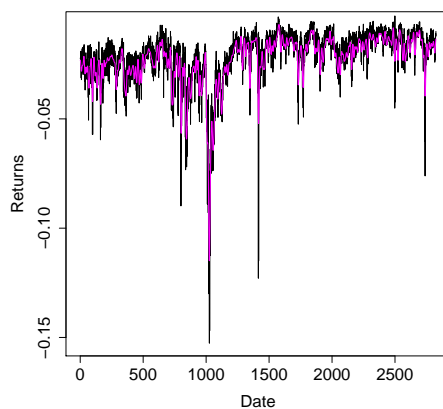
(c) Effect of the *Constant* on the fitted value (black)



(d) Effect of the RV^{Short} on the fitted value (black)



(e) Effect of the RV^{Medium}



(f) Effect of the RV^{Long}

Figure 4: Fitted values: multiple quantiles, in more detail $\tau = 0.10$. The first panel shows fitted values for all assumed quantiles. The second panel shows only the $\tau = 0.10$ quantile estimate, where we compare the results from QRs using RV and decomposed RV . The other four panels show the decomposition of the latter estimate into the effects of the individual regressors: constant, RV^S , RV^M and RV^L .

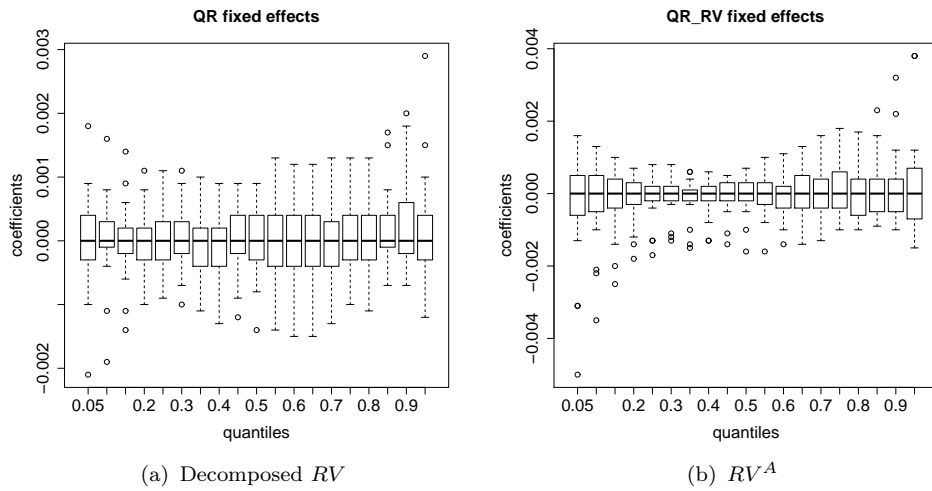
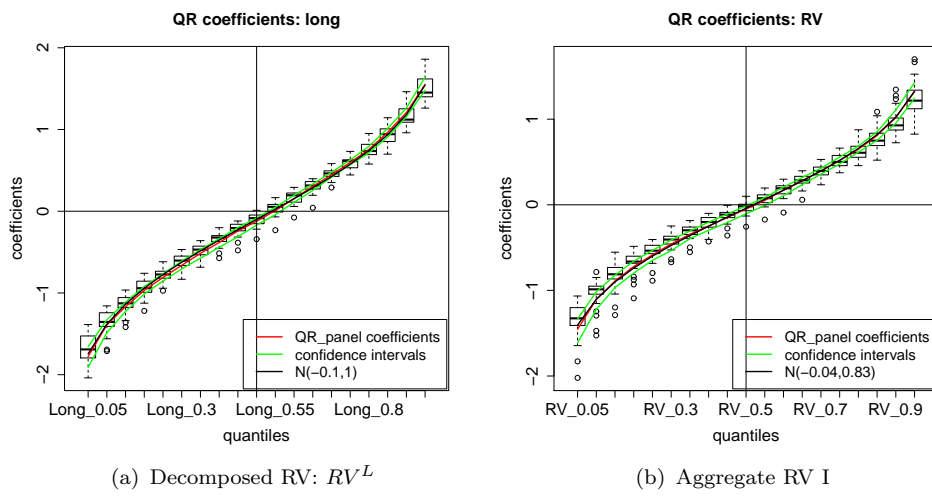
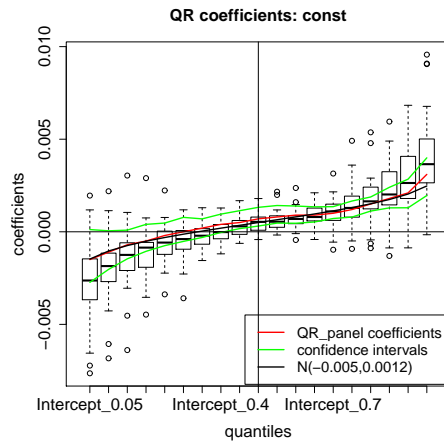


Figure 5: Panel quantile regression: Fixed effects



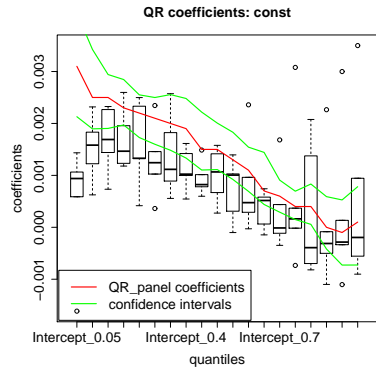
(a) Decomposed RV: RV^L

(b) Aggregate RV I

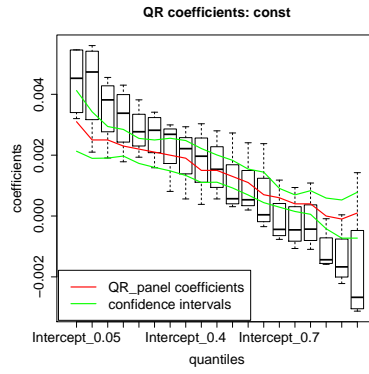


(c) Aggregate RV II

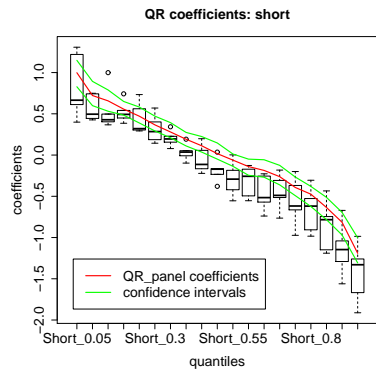
Figure 6: Panel quantile regression coefficients with 95% confidence intervals vs normal distribution $N(\text{mean}, s.d.)$: decomposed and aggregate RV



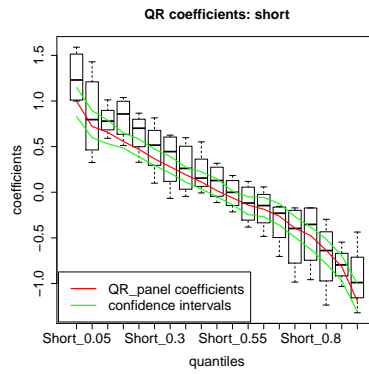
(a) *Constant*—Consumer Discretionary



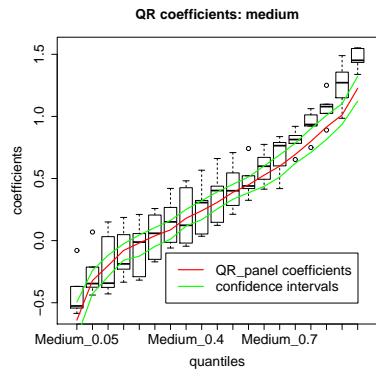
(b) *Constant*—Finance



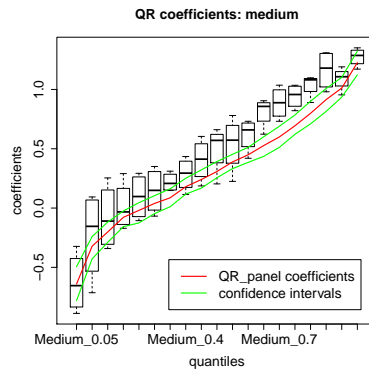
(c) RV^{Short} —Consumer Discretionary



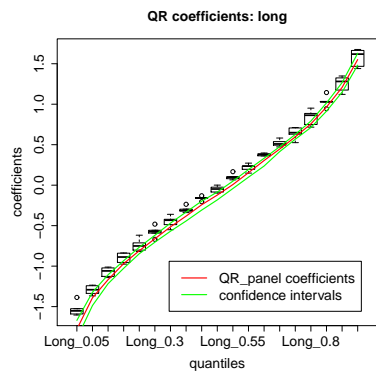
(d) RV^{Short} —Finance



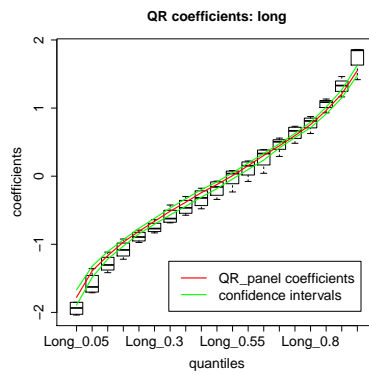
(e) RV^{Medium} —Consumer Discretionary



(f) RV^{Medium} —Finance



(g) RV^{Long} —Consumer Discretionary



(h) RV^{Long} —Finance

Figure 7: Individual QR boxplots for sectors: Consumer Discretionary and Finance compared to Panel QR results (with 95% confidence intervals)

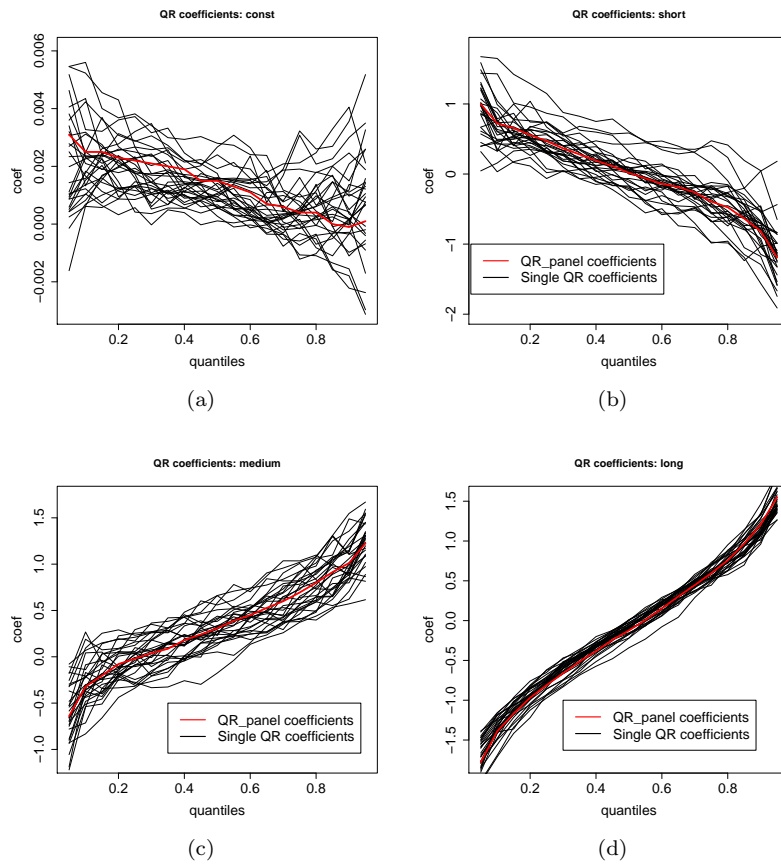


Figure 8: Quantile regression coefficients: non-summarized regression coefficients for all assets.

B Appendix A

Table 2: Quantile regression coefficients

quantile	0.05	0.1	0.15	0.2	0.25	0.3	0.35
const	0.0031 (5e-04)	0.0025 (4e-04)	0.0025 (3e-04)	0.0023 (2e-04)	0.0022 (2e-04)	0.0021 (2e-04)	0.002 (3e-04)
short	0.9983 (0.0929)	0.7218 (0.0714)	0.6571 (0.0676)	0.5569 (0.0511)	0.4714 (0.0417)	0.3661 (0.044)	0.2799 (0.052)
medium	-0.6405 (0.0809)	-0.3215 (0.0439)	-0.2018 (0.0452)	-0.0786 (0.0345)	-0.0195 (0.035)	0.039 (0.0368)	0.0864 (0.0347)
long	-1.7786 (0.0564)	-1.3909 (0.0404)	-1.1675 (0.0302)	-0.9685 (0.0244)	-0.8099 (0.0197)	-0.6617 (0.0221)	-0.5148 (0.0259)
quantile	0.4	0.45	0.5	0.55	0.6	0.65	
const	0.0019 (3e-04)	0.0015 (3e-04)	0.0015 (2e-04)	0.0013 (2e-04)	0.0011 (2e-04)	7e-04 (2e-04)	
short	0.1917 (0.0413)	0.1127 (0.05)	0.0173 (0.0413)	-0.0613 (0.0415)	-0.1395 (0.0529)	-0.1862 (0.0456)	
medium	0.1801 (0.0356)	0.2404 (0.0388)	0.3082 (0.0337)	0.389 (0.0329)	0.449 (0.0366)	0.5271 (0.0393)	
long	-0.3825 (0.0283)	-0.2419 (0.0274)	-0.1225 (0.0234)	0.0076 (0.0257)	0.1537 (0.0276)	0.3044 (0.0202)	
quantile	0.7	0.75	0.8	0.85	0.9	0.95	
const	6e-04 (2e-04)	4e-04 (2e-04)	4e-04 (2e-04)	0 (3e-04)	-1e-04 (3e-04)	1e-04 (4e-04)	
short	-0.2619 (0.0559)	-0.3928 (0.0506)	-0.4745 (0.062)	-0.6358 (0.0673)	-0.8153 (0.0675)	-1.1913 (0.0775)	
medium	0.5995 (0.0448)	0.6937 (0.0419)	0.7987 (0.0461)	0.9146 (0.0509)	1.0117 (0.0463)	1.2261 (0.0508)	
long	0.4461 (0.0174)	0.5964 (0.0173)	0.757 (0.0192)	0.9653 (0.0241)	1.207 (0.0317)	1.5513 (0.0411)	

Table 3: Panel quantile regression: Fixed effects I

quantile	0.05	0.1	0.15	0.2	0.25	0.3	0.35
AAPL	-0.0021 (4e-04)	-0.0019 (3e-04)	-0.0014 (2e-04)	-8e-04 (1e-04)	-5e-04 (1e-04)	-6e-04 (1e-04)	-6e-04 (1e-04)
AMZN	6e-04 (3e-04)	4e-04 (1e-04)	2e-04 (1e-04)	1e-04 (1e-04)	5e-04 (1e-04)	3e-04 (1e-04)	3e-04 (1e-04)
BAC	0 (0)	0 (0)	1e-04 (0)	0 (0)	3e-04 (1e-04)	4e-04 (1e-04)	1e-04 (1e-04)
C	4e-04 (2e-04)	0 (1e-04)	2e-04 (1e-04)	2e-04 (1e-04)	2e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)
CMCSA	8e-04 (2e-04)	4e-04 (1e-04)	5e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)	1e-04 (0)	0 (0)
CSCO	7e-04 (2e-04)	8e-04 (1e-04)	6e-04 (1e-04)	7e-04 (1e-04)	8e-04 (1e-04)	8e-04 (1e-04)	7e-04 (1e-04)
CVX	4e-04 (2e-04)	4e-04 (2e-04)	3e-04 (1e-04)	1e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)	3e-04 (1e-04)
DIS	0.0018 (4e-04)	0.0016 (3e-04)	0.0014 (2e-04)	0.0011 (2e-04)	0.0011 (2e-04)	0.0011 (2e-04)	0.001 (2e-04)
GE	0 (1e-04)	2e-04 (1e-04)	1e-04 (1e-04)	2e-04 (1e-04)	5e-04 (1e-04)	3e-04 (1e-04)	2e-04 (0)
GOOG	-2e-04 (1e-04)	-1e-04 (0)	3e-04 (1e-04)	3e-04 (1e-04)	2e-04 (0)	3e-04 (1e-04)	1e-04 (0)
HD	1e-04 (1e-04)	-1e-04 (1e-04)	-3e-04 (1e-04)	-1e-04 (1e-04)	-1e-04 (0)	0 (0)	1e-04 (0)
IBM	-5e-04 (1e-04)	-4e-04 (1e-04)	-5e-04 (1e-04)	-4e-04 (1e-04)	-4e-04 (1e-04)	-2e-04 (1e-04)	-2e-04 (1e-04)
INTC	-2e-04 (1e-04)	-1e-04 (1e-04)	-2e-04 (1e-04)	-4e-04 (0)	-3e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)
JNJ	-3e-04 (1e-04)	-1e-04 (1e-04)	-4e-04 (1e-04)	-3e-04 (1e-04)	-3e-04 (1e-04)	-3e-04 (1e-04)	-3e-04 (1e-04)
JPM	-1e-04 (1e-04)	1e-04 (1e-04)	0 (0)	-1e-04 (0)	-1e-04 (1e-04)	-1e-04 (1e-04)	-3e-04 (1e-04)
KO	0 (1e-04)	3e-04 (1e-04)	0 (0)	1e-04 (0)	1e-04 (1e-04)	2e-04 (1e-04)	2e-04 (1e-04)
MCD	4e-04 (1e-04)	4e-04 (1e-04)	2e-04 (1e-04)	1e-04 (0)	-2e-04 (0)	-2e-04 (1e-04)	-3e-04 (1e-04)
MRK	2e-04 (1e-04)	1e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)	-3e-04 (0)	-3e-04 (1e-04)	-4e-04 (1e-04)
MSFT	-0.001 (2e-04)	-0.0011 (1e-04)	-0.0011 (2e-04)	-0.001 (1e-04)	-9e-04 (1e-04)	-0.001 (1e-04)	-0.001 (1e-04)
ORCL	-5e-04 (2e-04)	-4e-04 (2e-04)	-6e-04 (2e-04)	-3e-04 (1e-04)	-5e-04 (1e-04)	-7e-04 (1e-04)	-0.0011 (1e-04)
PEP	-5e-04 (2e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)	-1e-04 (1e-04)	0 (0)	1e-04 (0)	1e-04 (0)
PFE	-7e-04 (2e-04)	0 (0)	0 (0)	1e-04 (1e-04)	-1e-04 (0)	-2e-04 (1e-04)	-4e-04 (0)
PG	-1e-04 (1e-04)	-1e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)	-2e-04 (0)	0 (0)	0 (0)
QCOM	4e-04 (1e-04)	3e-04 (1e-04)	1e-04 (0)	0 (0)	0 (0)	-2e-04 (0)	-4e-04 (1e-04)
SLB	9e-04 (4e-04)	5e-04 (3e-04)	9e-04 (2e-04)	8e-04 (2e-04)	9e-04 (1e-04)	9e-04 (2e-04)	8e-04 (2e-04)
T	-7e-04 (1e-04)	0 (0)	-1e-04 (1e-04)	0 (0)	3e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)
VZ	0 (1e-04)	-1e-04 (1e-04)	-1e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)	-2e-04 (1e-04)	-4e-04 (1e-04)
WFC	-3e-04 (1e-04)	-1e-04 (1e-04)	0 (0)	2e-04 (1e-04)	1e-04 (0)	0 (0)	0 (0)
WMT	6e-04 (2e-04)	1e-04 (1e-04)	0 (1e-04)	-1e-04 (0)	-3e-04 (1e-04)	-4e-04 (0)	-2e-04 (1e-04)

Table 4: Panel quantile regression: Fixed effects II

quantile	0.4	0.45	0.5	0.55	0.6	0.65
AAPL	-4e-04 (1e-04)	-1e-04 (1e-04)	0 (1e-04)	-1e-04 (1e-04)	1e-04 (1e-04)	3e-04 (1e-04)
AMZN	3e-04 (1e-04)	4e-04 (1e-04)	2e-04 (1e-04)	1e-04 (1e-04)	1e-04 (1e-04)	1e-04 (1e-04)
BAC	3e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	5e-04 (1e-04)
C	1e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)	3e-04 (1e-04)	2e-04 (1e-04)	0 (0)
CMCSA	0 (0)	0 (0)	-2e-04 (0)	0 (0)	-1e-04 (1e-04)	0 (0)
CSCO	7e-04 (1e-04)	7e-04 (1e-04)	7e-04 (1e-04)	8e-04 (1e-04)	5e-04 (1e-04)	4e-04 (1e-04)
CVX	4e-04 (1e-04)	6e-04 (1e-04)	5e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	5e-04 (1e-04)
DIS	9e-04 (2e-04)	8e-04 (2e-04)	9e-04 (2e-04)	0.0013 (2e-04)	0.0012 (2e-04)	0.0012 (2e-04)
GE	4e-04 (1e-04)	5e-04 (1e-04)	4e-04 (1e-04)	3e-04 (1e-04)	4e-04 (1e-04)	5e-04 (1e-04)
GOOG	2e-04 (0)	4e-04 (1e-04)	2e-04 (1e-04)	4e-04 (0)	4e-04 (1e-04)	4e-04 (1e-04)
HD	1e-04 (0)	1e-04 (0)	1e-04 (1e-04)	2e-04 (0)	2e-04 (1e-04)	0 (0)
IBM	-3e-04 (1e-04)	-2e-04 (1e-04)	-1e-04 (1e-04)	-1e-04 (0)	-1e-04 (0)	-1e-04 (0)
INTC	-5e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)	-6e-04 (1e-04)
JNJ	-3e-04 (1e-04)	-1e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)	-4e-04 (1e-04)	-3e-04 (1e-04)
JPM	-2e-04 (1e-04)	0 (1e-04)	-1e-04 (1e-04)	0 (1e-04)	0 (0)	0 (0)
KO	2e-04 (1e-04)	2e-04 (1e-04)	0 (1e-04)	0 (0)	0 (0)	-1e-04 (1e-04)
MCD	-2e-04 (1e-04)	0 (0)	-2e-04 (0)	-1e-04 (0)	-1e-04 (0)	-1e-04 (0)
MRK	-4e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (0)	-4e-04 (1e-04)	-4e-04 (0)	-5e-04 (1e-04)
MSFT	-0.0012 (1e-04)	-9e-04 (1e-04)	-8e-04 (1e-04)	-8e-04 (1e-04)	-9e-04 (1e-04)	-9e-04 (1e-04)
ORCL	-0.0013 (1e-04)	-0.0012 (1e-04)	-0.0014 (1e-04)	-0.0014 (1e-04)	-0.0015 (1e-04)	-0.0015 (1e-04)
PEP	-1e-04 (0)	2e-04 (1e-04)	0 (0)	1e-04 (0)	2e-04 (1e-04)	1e-04 (0)
PFE	-4e-04 (1e-04)	-5e-04 (1e-04)	-6e-04 (0)	-4e-04 (1e-04)	-5e-04 (1e-04)	-4e-04 (1e-04)
PG	1e-04 (0)	4e-04 (1e-04)	4e-04 (1e-04)	5e-04 (1e-04)	5e-04 (1e-04)	3e-04 (0)
QCOM	-5e-04 (1e-04)	-4e-04 (1e-04)	-5e-04 (1e-04)	-6e-04 (1e-04)	-8e-04 (1e-04)	-6e-04 (1e-04)
SLB	8e-04 (2e-04)	9e-04 (1e-04)	7e-04 (1e-04)	0.001 (2e-04)	7e-04 (2e-04)	6e-04 (1e-04)
T	2e-04 (1e-04)	3e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)
VZ	-4e-04 (1e-04)	-2e-04 (1e-04)	-4e-04 (1e-04)	-4e-04 (1e-04)	-4e-04 (1e-04)	-5e-04 (1e-04)
WFC	0 (0)	-1e-04 (0)	-3e-04 (1e-04)	-4e-04 (1e-04)	-4e-04 (1e-04)	-2e-04 (0)
WMT	-4e-04 (0)	-3e-04 (0)	-4e-04 (0)	-5e-04 (1e-04)	-6e-04 (1e-04)	-7e-04 (1e-04)

Table 5: Panel quantile regression: Fixed effects III

quantile	0.7	0.75	0.8	0.85	0.9	0.95
AAPL	6e-04 (2e-04)	0.0011 (1e-04)	0.0013 (1e-04)	0.0017 (2e-04)	0.002 (1e-04)	0.0029 (3e-04)
AMZN	1e-04 (0)	0 (0)	-1e-04 (1e-04)	0 (1e-04)	-5e-04 (1e-04)	-0.001 (2e-04)
BAC	6e-04 (1e-04)	6e-04 (1e-04)	5e-04 (1e-04)	6e-04 (1e-04)	6e-04 (1e-04)	0 (1e-04)
C	0 (0)	1e-04 (1e-04)	2e-04 (1e-04)	3e-04 (1e-04)	-1e-04 (1e-04)	-3e-04 (1e-04)
CMCSA	0 (0)	0 (0)	0 (0)	0 (0)	2e-04 (1e-04)	7e-04 (2e-04)
CSCO	3e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	3e-04 (1e-04)	1e-04 (1e-04)	0 (1e-04)
CVX	3e-04 (1e-04)	4e-04 (1e-04)	3e-04 (1e-04)	0 (0)	-1e-04 (1e-04)	2e-04 (1e-04)
DIS	0.0013 (1e-04)	0.0013 (1e-04)	0.0013 (2e-04)	0.0015 (2e-04)	0.0018 (2e-04)	0.0015 (3e-04)
GE	4e-04 (1e-04)	5e-04 (1e-04)	4e-04 (1e-04)	6e-04 (1e-04)	6e-04 (1e-04)	8e-04 (2e-04)
GOOG	7e-04 (1e-04)	8e-04 (1e-04)	7e-04 (1e-04)	8e-04 (1e-04)	0.001 (2e-04)	4e-04 (1e-04)
HD	1e-04 (0)	-1e-04 (0)	2e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)	3e-04 (1e-04)
IBM	0 (0)	0 (0)	0 (0)	1e-04 (1e-04)	0 (1e-04)	2e-04 (1e-04)
INTC	-6e-04 (1e-04)	-5e-04 (1e-04)	-7e-04 (1e-04)	-4e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)
JNJ	-3e-04 (1e-04)	-3e-04 (1e-04)	-3e-04 (1e-04)	0 (1e-04)	-2e-04 (1e-04)	1e-04 (1e-04)
JPM	0 (0)	1e-04 (0)	0 (0)	2e-04 (1e-04)	2e-04 (1e-04)	-3e-04 (1e-04)
KO	-1e-04 (0)	-2e-04 (1e-04)	-3e-04 (1e-04)	-1e-04 (1e-04)	-1e-04 (1e-04)	-3e-04 (1e-04)
MCD	-2e-04 (0)	-2e-04 (0)	-4e-04 (1e-04)	-3e-04 (1e-04)	-7e-04 (1e-04)	-0.001 (1e-04)
MRK	-4e-04 (1e-04)	-5e-04 (1e-04)	-3e-04 (1e-04)	-2e-04 (1e-04)	-5e-04 (1e-04)	-8e-04 (1e-04)
MSFT	-6e-04 (1e-04)	-4e-04 (1e-04)	-4e-04 (1e-04)	0 (1e-04)	0 (1e-04)	2e-04 (1e-04)
ORCL	-0.0013 (2e-04)	-0.001 (1e-04)	-0.0011 (1e-04)	-7e-04 (1e-04)	-4e-04 (1e-04)	-2e-04 (1e-04)
PEP	2e-04 (0)	1e-04 (0)	1e-04 (1e-04)	2e-04 (0)	8e-04 (2e-04)	0.001 (2e-04)
PFE	-4e-04 (1e-04)	-2e-04 (0)	-3e-04 (1e-04)	-1e-04 (1e-04)	1e-04 (1e-04)	-1e-04 (1e-04)
PG	2e-04 (0)	4e-04 (1e-04)	4e-04 (1e-04)	8e-04 (1e-04)	4e-04 (1e-04)	4e-04 (1e-04)
QCOM	-5e-04 (1e-04)	-5e-04 (1e-04)	-5e-04 (1e-04)	-1e-04 (1e-04)	0 (1e-04)	-2e-04 (1e-04)
SLB	4e-04 (1e-04)	2e-04 (1e-04)	1e-04 (1e-04)	0 (1e-04)	6e-04 (2e-04)	3e-04 (2e-04)
T	4e-04 (1e-04)	6e-04 (1e-04)	6e-04 (1e-04)	7e-04 (1e-04)	6e-04 (1e-04)	5e-04 (1e-04)
VZ	-5e-04 (1e-04)	-5e-04 (1e-04)	-6e-04 (1e-04)	-2e-04 (1e-04)	-2e-04 (1e-04)	-3e-04 (1e-04)
WFC	-1e-04 (1e-04)	-1e-04 (0)	-1e-04 (0)	1e-04 (0)	0 (0)	-1e-04 (1e-04)
WMT	-6e-04 (1e-04)	-7e-04 (0)	-7e-04 (1e-04)	-6e-04 (1e-04)	-6e-04 (1e-04)	-0.0012 (2e-04)

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