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# Systemic Risk in the European Financial and Energy Sector: Dynamic Factor Copula Approach

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**Abstract:**

In this paper, we perform analysis of systemic risk in the financial and energy sector in Europe. In our investigation, we work with daily time series of CDS spreads. We employ factor copula model with GAS dynamics of Oh and Patton (2016) for estimation purposes of dependency structures between market participants. Based on the estimated models, we perform Monte Carlo simulations in order to obtain future values of CDS spreads, and then we measure probability of systemic events in given time points. We conclude that substantially higher systemic risk is present within the financial sector than in the energy sector. We also find that the most systemic vulnerable financial and energy companies come from Spain.

**Keywords:** Credit Default Swap, Energy Sector, Factor Copula, Financial Sector, Generalized Autoregressive Score Model, Systemic Risk

**JEL:** C53, C55, C58, G17

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## 1. Introduction

The aim of this paper is to investigate and compare systemic risk within the European financial and energy sectors. Because of the growing interconnection of economic markets caused by the globalization process, and recently, because of the global financial crisis, systemic risk is currently highly discussed topic, and these new challenges call for modern approaches to measuring and understanding its nature. Studying systemic risk and connections between market participants is important for various reasons. For example, it may serve as an input for regulatory purposes, portfolio theory, risk management, and many others. Universally acceptable definition of systemic risk does not exist, so there exist various possibilities how to approach this phenomenon. We may distinguish literature focusing on various sources of systemic risk and its causal effects, and literature aiming to provide comprehensive measures of systemic risk. In this paper, we strive to deliver answers to the latter using non-conventional econometric tools. It is well known fact that covariance-based methods assuming gaussianity of data do not adequately characterize nature of the connections present in the real world, such as asymmetry and tail dependence, and thus another tools must be employed in order to capture true dependence relations. In our investigation, we make use of methods that can work with large sets of data, which is essential for the topic, and capture both asymmetry and tail dependencies. We argue that financial and energy sector play an important role in the health of European economy and this paper is the first which investigates and compares their systemic risk using factor copulas and CDS data.

Investigation of systemic risk has been an important topic in economic literature for a long time. Modern econometric approaches and various sources of economic and financial data enables to investigate this research topic from various points of view. In our case, time-varying factor copula functions constitute as the main econometric tool for the investigation of systemic risk in European financial and energy sectors. Copulas have found many applications in finance and economics since their introduction by Sklar (1959). One of the reasons for that is the fact that using cornerstone of copulas theory, Sklar's theorem, copulas allow us to model marginal distributions and the dependence structure separately and thus provide a great flexibility for modeling purposes. Literature that concerns with copula functions in application to the multivariate time series is quite new but rapidly growing. These methods are being used in wide variety of financial and economic problems. For general overview of these models see (Patton, 2012a). As it is expected, if we model time series, we work with conditional expectations, this is formally described in Patton (2006). Very fruitful results are being obtained using copulas particularly in the application to financial time series, which we do in this paper, too. General overview of copula based models for financial time series gives Patton (2009) and Patton (2012b). Estimation procedure of copula models is not straightforward in many cases. For general overview of the estimation strategies of both parametric and semi-parametric copula models see Choroś et al. (2010). Copulas are also widely used in many other fields of finance. For applications in risk management see McNeil

et al. (2015). Wide range of applications in mathematical finance is discussed in Cherubini et al. (2004). For more information about time-varying copula models (including estimations, simulations, etc.) see Manner and Reznikova (2012).

For our investigation purposes, we make use of factor copula model with GAS dynamics suggested by Oh and Patton (2016). This model is a combination of Generalized Autoregressive Score (GAS) model of Creal et al. (2013), and factor copula model of Oh and Patton (2012). Copula model of Oh and Patton (2012) facilitates work with high-dimensional data, and model of Creal et al. (2013) allows us to impose dynamic properties to the model. Obtained model is both flexible and parsimonious at the same time. As mentioned in Oh and Patton (2016), there is a wide range of methods for studying risk and dependence among small number of subjects but substantially less methods for assessing risk and dependence among large number of subjects. Similar econometric approach as the one employed in this paper is suggested by Creal and Tsay (2015). They propose a new class of time-varying copula models that nest many widely used models. They also introduce time-varying factor structure to the model in order to enable to work with large sets of data, and they apply the methods to the CDS time-series of 100 U.S. corporations. Pourkhanali et al. (2016) use vine copulas to investigate systemic risk among 60 international financial institutions classified into 5 rating classes. Work that utilizes sovereign debt CDS data for assessing joint conditional probability of sovereign default in the euro area is Lucas et al. (2014). Authors propose a new methodology, which employs dynamic Generalized Hyperbolic skewed- $t$  conditional density for their analysis, combine it with GAS dynamics, and they find strong time variation in risk dependence and strong impact of important policy announcements on joint and conditional sovereign risk in the Eurozone. Another work that utilizes dynamic copula models for assessing corporate credit is Christoffersen et al. (2014). Work of Kleinow and Moreira (2015) investigates systemic risk in the Eurozone using CDS spreads of European banks. They also strive to explain why some banks are expected to negatively affect systemic risk in the area.

Despite numerous advantages of copula functions, they are not the only econometric tool for estimating and measuring systemic risk. In their influential work of methodological approaches to measuring systemic risk and spillover effects, Adrian and Brunnermeier (2009) propose to use quantile regression for measuring systemic risk. For another modern econometric approaches to the measuring systemic risk, see marginal expected shortfall approach in Brownlees and Engle (2015). More recently, Liu (2016) proposes to use regime switching model for capturing non-linearities in tails of contributions of individual institutions to the systemic risk.

As a data source for the research purposes of assessing and comparing systemic risk in financial and energy sector in Europe, we use daily CDS spreads data between January 2008 and September 2015. Increasing attention paid to the CDS data in general is mainly caused by the fact that CDS spreads can quite well mirror health of the underlying company. Our approach based on these data is to our best knowledge novel and shed light on the differences be-

tween financial and energy sector in this area from a different perspective. We also perform analysis in terms of nationality of the companies and investigate from which European countries come the most systemic vulnerable companies. We estimate two versions of the dynamic factor copula model, one that assumes heterogeneous dependence on the systemic events across companies, and second that assume homogeneous dependence among companies from the same country. Differences of the results obtained from these two models can provide some measure of dependence homogeneity of the companies from the same country.

Computations concerning with dynamic factor copula model were done using Matlab toolbox which accompanied work of Oh and Patton (2016). Other calculations were performed using RStudio with `rugarch`, `tseries`, and `forecast` packages.

## 2. Dynamic Factor Copula for CDS Spreads

### 2.1. Data

In our empirical investigation, we will focus on examining daily credit default swap (CDS) spreads of financial and energy companies in the Europe. CDS is a financial derivative dependent on creditworthiness of one or more reference entities. There are two main participants in the CDS contract: protection buyer and protection seller. Protection buyers protect themselves against potential default of the reference entity or entities in a given time horizon, and protection sellers offer to cover the potential losses arising from the default. Protection buyer pays some premium in order to be protected. This premium is called CDS spread. Usually, CDS spreads are set in the basis points of the protected nominal value. As stated in many works (for example, Christoffersen et al. (2014)), CDS spreads very well reflect market perception of the default probability of the reference entity. If there is a notion that reference entity is more likely to default, related CDS spread is likely to increase, and vice versa. CDS spreads thus contain information about implied default probabilities also called risk neutral probability of default.

It can be simply shown that under some simplifying conditions (constant risk free rate and constant default hazard rate, see Carr and Wu (2010)), CDS spread of reference entity  $i$  at time  $t$  in basis points in terms of real world probability of default is given by

$$S_{it} = 100^2 P_{it}^{\mathbb{P}} M_{it} LGD_{it}, \quad (1)$$

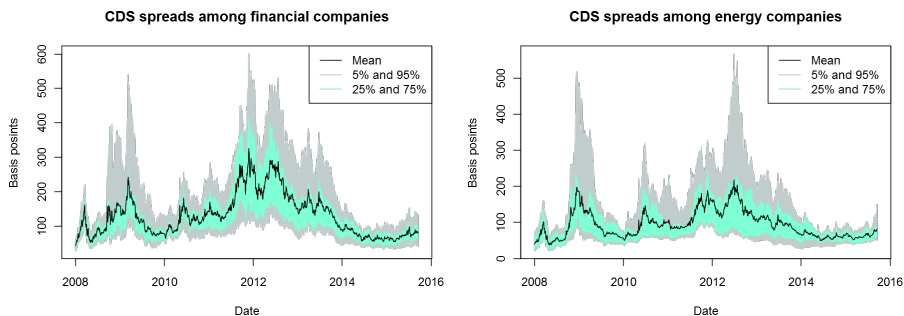
where  $P_{it}^{\mathbb{P}}$  is real world probability of default,  $M_{it}$  is stochastic discount rate also called market price of risk, and  $LGD_{it}$  is loss given default (in many cases  $LGD$  is assumed to be constant and equal to 0.6).

From the previous equations for credit spreads, it is obvious that increase of the CDS spread is caused either by increase of the objective probability of default, increase of the loss given default, or by increase of market price of default risk. It is expected that raw CDS spreads show some heavy autoregressive persistence with near unit root. In order to eliminate this feature, we work

with the log-differences of CDS spreads. We may also assume that  $LGD_{it}$  and  $M_{it}$  are constant through time, implying that changes in CDS spreads directly reflect changes in objective probabilities of default. This approach is then used in the interpretation of the results in the paper, but as an increase of any of the three terms represents negative impact on the given firm, this separation of the objective probability of default is not necessary for interpretation of the results.

Data are taken between January 2008 and September 2015 yielding totally 2011 observations. Number of companies from the financial sector is equal to 27 and number of companies from the energy sector is equal to 18. List of all companies along with basic descriptive statistics of their CDS spreads can be found in Appendix A in Tables A.13 and A.16. All the companies are constituents of iTraxx Europe CDS index with 5 year maturity. Data were obtained from the Bloomberg Terminal. We will separately estimate dynamic factor copula models for both sectors and discuss the systemic risk and the differences between those two sectors. Both sectors will be compared with respect to the connectedness and its evolution through time, and with respect to the related risk and its evolution.

Figure 1: Time series of CDS spreads of financial and energy companies.

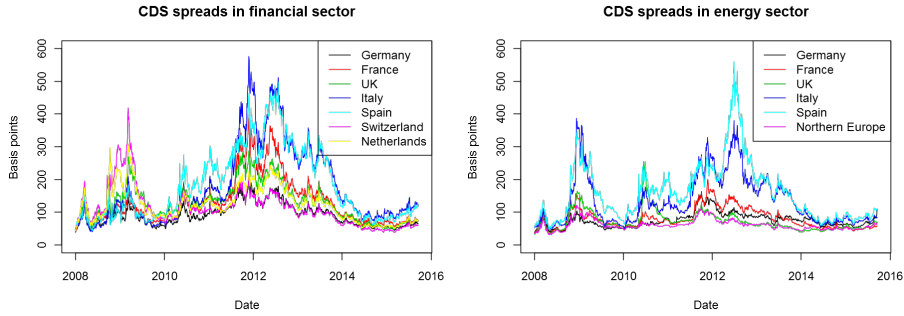


From Figure 1, we observe that spreads in the financial sector move slightly more jointly together than spreads within the energy sector, especially during the sovereign debt crisis in the year 2012 with confidence bands and mean value of the spreads reacting more dramatically within the periods of crisis in this sector. This is natural as connectedness among the financial subjects is expected to be higher than connectedness within energy sector participants. Spreads in the Figure 2 are averaged with respect to nationality of the companies. We see that since the 2010 the highest average CDS spreads are observed within companies from Italy and Spain in both financial and energy sector. From both Figures we notice two peaks of CDS spreads, at the turn of years 2008 and 2009 at the peak of the financial crisis, and in year 2012 when sovereign debt crisis was at the highest level.

Figure 3 depicts average log-differences among financial and energy sector. We observe volatility clusters, so in order to properly model the time series we will have to employ some conditional volatility model, such as GARCH model,

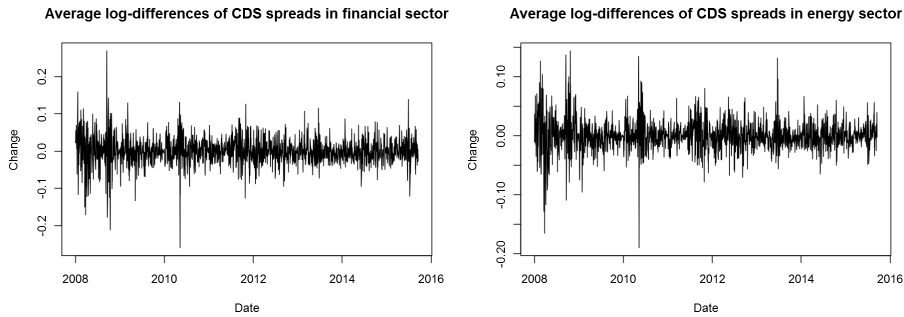


Figure 2: Time series of average CDS spreads of financial and energy companies according to their nationality.



in order to obtain valid models.

Figure 3: Average log-differences of CDS spreads of financial and energy companies.



From the Figure 2, we observe that average spreads of financial companies from different European countries move more jointly together than the average spreads of energy companies. This also suggests that the financial sector is more connected than the energy sector.

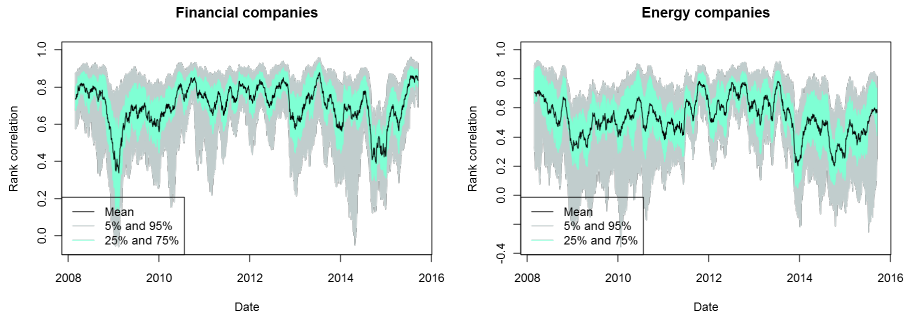
Naive analysis of connectedness within the financial and energy sector gives Table 1, which contains average rank correlation among all pairs of time series of log-differences of CDS spreads within the sectors (702 pairs in the financial sector and 306 pairs in energy sector). We can see that average rank correlation during the whole observed time period among financial sector is equal to 0.703 against 0.542 within the energy sector. This supports our hypothesis that financial sector is more connected with higher potential systemic risk arising from this sector. Figure 4 depicts rolling window of length 40 days of average rank correlation between pairs of subjects within the financial and energy sector along with their empirical quantiles. We do not observe any steep peaks of the correlations, only 2 major slumps at the turn of years 2008 and 2009, and at the turn of years 2014 and 2015 within the financial sector. In the energy sector we

observe decrease of correlations since the end of year 2013. But in both sectors, we also observe a rapid increase of correlations in year 2015.

Table 1: Rank correlations among financial and energy sector.

	Mean	5%	25%	Median	75%	95%
Financial sector	0.703	0.523	0.670	0.707	0.759	0.814
Energy sector	0.542	0.299	0.484	0.577	0.626	0.690

Figure 4: Moving rank correlation over last 40 days among financial and energy companies.



Now, we have to choose proper modeling approach and estimate its parameters. This will be crucial for our analysis of systemic risk as we will employ simulation approach to the estimation of the systemic risk. When we are estimating models for time series data, we assume that type of the underlying process is the same for all the subjects within given sector, but the parameters for each subject are estimated separately.

## 2.2. Financial Sector

When analyzing financial sector, we take into consideration 27 time series of major financial institutions from the Europe. As discussed earlier, time series of CDS spreads are non-stationary although they do not exactly follow random walk as they are bounded by 0 from below and by 1 (100%) from above. Table A.11 describes cross distribution of the series in levels. The whole list of financial companies that are included in our analysis is featured in Appendix A in Table A.13. From Table A.11 in Appendix A, we see that average value of CDS spreads across the companies during the period was 130.65 bps. Non-stationarity of the series is supported by the estimated first order autocorrelation coefficients, the smallest value of the coefficient across the spreads is 0.985. The mean of 1<sup>st</sup> order autocorrelation coefficient is equal to 0.995. This also supports Augmented Dickey-Fuller test which rejects unit-root in only 6 cases and KPSS test rejects stationarity in every case. In order to properly model the series we have to stationarize them. In order to do so, we take log-differences of the series.

Cross-sectional distribution of the log-differences is characterized by the Table A.12 in Appendix A. One of the interesting observations is very high excess kurtosis across financial companies, which is equal to 11.31. We also see that average skewness across the financial sector is slightly positive, but estimates across the companies lie in a wide range between -2.12 and 3.71.

In order to model the series, we employ ARMA-GARCH models. As a conditional mean model we use the following adjusted AR(1) model given by the following specification

$$X_{i,t} = \phi_{0,i} + \phi_{1,i}X_{i,t-1} + \phi_{m,i}X_{m,t-1} + e_{i,t} \quad (2)$$

where  $X_{i,t}$  is log return of company  $i$  on day  $t$  and  $X_{m,t-1}$  is average return among financial companies from the previous day, this should capture the spill-over effect within the sector. For the conditional variance model we use modified GJR-GARCH model

$$\sigma_{i,t}^2 = \omega_i + \beta_i\sigma_{i,t-1}^2 + \alpha_i e_{i,t-1}^2 + \delta_i e_{i,t-1}^2 \mathbf{I}\{e_{i,t-1} > 0\} \quad (3)$$

and

$$e_{i,t} = \sigma_{i,t}\epsilon_{i,t}, \quad \epsilon_{i,t} \sim iid \text{skew } t(\nu, \psi). \quad (4)$$

The modification of the GJR-GARCH model lies in changing the inequality sign in the indicator variable as an increase of the CDS spreads (positive return) represents a bad news and higher variance is expected after this positive return. Standardized residuals are modeled as random variables from skewed student's  $t$  distribution of Hansen (1994). Skewed student's  $t$  distribution is more flexible distribution than, for example, normal distribution because it enables to mirror some of the prominent features of the financial data, such as skewness and fat tails. Even if the standardized residuals are not significantly skewed or do not poses fat tails, student's  $t$  distribution mirror that by estimating appropriate values of asymmetry and kurtosis parameters.

Summary of the estimated models can be found in Table 2 and complete description of the estimated models for every company is included in Appendix A in Table B.17. From the Table 2 we see that the coefficient for lagged market return is positive and its mean value is significantly different from 0. This supports our hypothesis of the spill-over effect within the sector. Another noticeable feature of the estimated models is slightly negative value of average of skewness parameters of the skewed student's  $t$  distribution.

In order to test goodness-of-fit of the models, we firstly verify if there is no significant autocorrelation among the residuals. We use standardized residuals and perform Ljung-Box (LB) test to verify that. We do not reject the null hypothesis of no autocorrelation among the residuals up to the 10th lag in any case. The adequacy of the variance model is tested using LB test for squared standardized residuals up to the 10th lag and we also do not reject the null hypothesis in any case.

Table 2: Results of the estimated ARMA-GARCH models among financial companies.

	Mean	Min	25%	Median	75%	Max
$\phi_0 \times 10^4$	2.047	-2.119	0.736	1.947	3.728	5.726
$\phi_1$	-0.004	-0.251	-0.055	-0.001	0.039	0.156
$\phi_m$	0.248	0.094	0.218	0.259	0.298	0.350
$\omega \times 10^4$	0.686	0.318	0.487	0.592	0.871	1.218
$\alpha$	0.113	0.064	0.093	0.112	0.124	0.222
$\beta$	0.828	0.733	0.808	0.838	0.847	0.882
$\delta$	0.043	-0.005	0.029	0.035	0.058	0.088
$\nu$	5.035	3.696	4.660	5.023	5.238	6.485
$\psi$	-0.026	-0.055	-0.038	-0.023	-0.016	0.017

### 2.3. Energy Sector

We consider 18 energy companies from the Europe when assessing systemic risk in the energy sector. Same as in the case of financial companies, list of all energy companies taken into consideration in our analysis along with basic descriptive statistics of their CDS spreads is included in Appendix A in Table A.16. First, we test the series for stationarity. Augmented Dickey-Fuller rejects the null hypothesis that series has unit root in only 3 cases. KPSS test rejects the null hypothesis that the series is stationary in every case. Descriptive statistics concerning with levels of CDS spreads can be found in Table A.14 in Appendix A. We observe smaller value of average CDS spreads across energy companies than across financial companies with value 94.36 bps, which is approximately 40 bps less than in the case of financial companies.

Descriptive statistics on log-differences summarizes Table A.15 in Appendix A. We also observe a large value of average kurtosis of log-differences, even higher than within financial sector, which equals to 17.08. Average skewness of log-differences in the energy sector is substantially higher than in the financial sector with average value being equal to 0.82.

In case of energy companies, we follow the same procedure as in the case of financial companies. Given the fact of non-stationary character of time series of CDS spreads, we take logarithm of the series and difference them. Then, we model the obtained stationary series as ARMA-GARCH processes. In case of energy firms, we use modified AR(3) model of the following form

$$X_{i,t} = \phi_{0,i} + \sum_{j=1}^3 \phi_{j,i} X_{i,t-j} + \phi_{m,i} X_{m,t-1} + e_{i,t}, \quad (5)$$

in which we also include market return  $X_{m,t-1}$  from the previous period. Higher order of the AR process is used for modeling log-differences of CDS spreads within the energy sector than in the case of financial companies. This need of higher order of AR process for modeling purposes can be caused, for example, by the lower liquidity of CDS spreads of the energy companies. Conditional

variance is modeled by the same GJR-GARCH model as in the case of the financial companies. This model is of the following form

$$\sigma_{i,t}^2 = \omega_i + \beta_i \sigma_{i,t-1}^2 + \alpha_i e_{i,t-1}^2 + \delta_i e_{i,t-1}^2 \mathbf{I}\{e_{i,t-1} > 0\}. \quad (6)$$

and

$$e_{i,t} = \sigma_{i,t} \epsilon_{i,t}, \quad \epsilon_{i,t} \sim iid \text{skew } t(\nu, \psi). \quad (7)$$

Summary of the estimation results is captured in Table 3 and the whole set of estimated parameters can be found in Appendix B in Table B.18. Estimated parameter of the lagged market return is significantly different from 0 and positive supporting our hypothesis of presence of spill-over effect in the energy sector. Average value of GJR-GARCH parameter  $\delta$  is estimated to be significant and higher than in the financial sector with average values being equal to 0.070 and 0.043 in the financial and energy sector, respectively. This suggests that volatility reacts more strongly on the previous day residual in case of the energy companies.

Table 3: Results of the estimated ARMA-GARCH models among energy companies.

	Mean	Min	25%	Median	75%	Max
$\phi_0 \times 10^4$	2.591	-1.333	1.550	2.417	3.745	6.385
$\phi_1$	-0.065	-0.466	-0.158	-0.075	0.032	0.230
$\phi_2$	-0.031	-0.232	-0.045	-0.027	-0.001	0.055
$\phi_3$	-0.050	-0.121	-0.075	-0.044	-0.030	0.022
$\phi_m$	0.283	-0.030	0.212	0.325	0.373	0.497
$\omega \times 10^4$	0.444	0.010	0.255	0.352	0.558	1.341
$\alpha$	0.127	0.047	0.106	0.123	0.138	0.214
$\beta$	0.810	0.729	0.794	0.807	0.824	0.937
$\delta$	0.070	-0.010	0.044	0.068	0.093	0.203
$\nu$	3.972	2.312	3.774	4.060	4.348	4.889
$\psi$	-0.044	-0.086	-0.048	-0.043	-0.036	0.010

We follow the same procedure as in the case of financial companies and perform goodness-of-fit tests of the estimated models. We reject the null hypothesis of no autocorrelation among standardized residuals up to 10 lag in 2 cases, which gives us a satisfactory result. Adequacy of the variance model tested by the LB test for the squared standardized residuals is not rejected in any case.

Now, when we have estimated both conditional mean and variance models for both sectors, we can compare the sectors with respect to their estimated volatility. Average estimated volatility during the observed period was 0.039 and 0.030 within the financial and energy sector, respectively. GARCH processes are mean-reverting so we can also calculate estimated unconditional values of the variance implied by the volatility model for each series according to the formula

$$\sigma_i^2 = \frac{\omega_i}{1 - \alpha_i - \beta_i - \frac{\delta_i}{2}}. \quad (8)$$

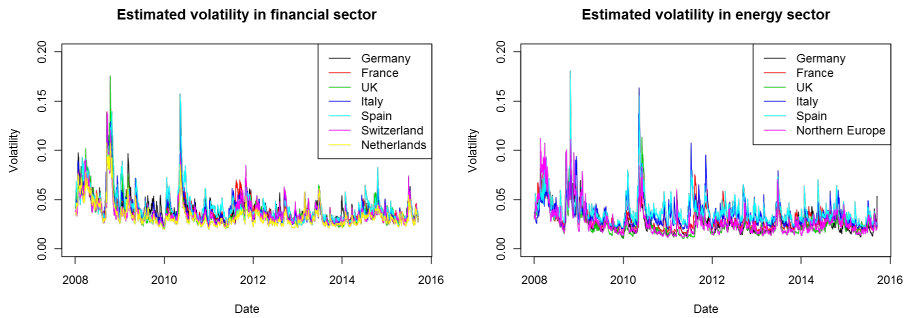
Median value of the unconditional volatility (square root of variance) is then equal to 0.043 within the financial sector and 0.040 within the energy sector. Both these results suggest slightly higher degree of uncertainty within the financial sector than in the energy sector. Table 4 summarizes average volatility with respect to nationality of the companies. We observe that volatility in the financial sector is higher for every country than in the energy sector. The highest average volatility in both sectors is observed within Spanish companies.

Table 4: Average estimated volatility within financial and energy sector averaged with respect to nationality.

Financial		Energy	
Nationality	Volatility	Nationality	Volatility
Germany	0.041	Germany	0.029
France	0.040	France	0.031
UK	0.038	UK	0.025
Italy	0.040	Italy	0.036
Spain	0.042	Spain	0.039
Switzerland	0.037	Northern Europe	0.027
Netherlands	0.034		

Figure 5 depicts average estimated volatility among European financial and energy sectors with respect to the nationality of the companies. In both sectors we observe 2 main peaks of volatility. One at the end of 2008 and one in the middle of 2010.

Figure 5: Average estimated volatility among countries within financial and energy sector.



#### 2.4. Dependence Structure

For modeling purposes of dependence structure, we use copula models. From mean-variance equations, we obtain estimated residuals and these are then transformed into uniformly distributed random variables using probability integral transform as

$$U_{i,t} \equiv F_{i,t} \left( \frac{X_{i,t} - \mu_{i,t}(\phi_{i,0})}{\sigma_{i,t}(\phi_{i,0})}; \phi_{i,0} \right), \quad i = 1, 2, \dots, n \quad (9)$$

where  $\mu_{i,t}$  is estimated conditional mean of  $X_{i,t}$ ,  $\sigma_{i,t}$  is its estimated conditional standard deviation, and  $F_{i,t}$  is marginal distribution function of given time series. As stated earlier, in our case the residuals are assumed to have skewed Student's  $t$  distribution.

Then, the conditional distribution of  $\mathbf{U}_t | \mathcal{F}_{t-1}$  is the conditional copula of  $\mathbf{X}_t | \mathcal{F}_{t-1}$ , i.e.

$$\mathbf{U}_t | \mathcal{F}_{t-1} \sim \mathbf{C}_t(\gamma) \quad (10)$$

where  $\gamma$  is a vector of copula parameters. Because of the high dimensionality of the data, we have to employ some sort of dimension-reduction tool. In this paper, we follow model of Oh and Patton (2016). Their methods impose factor structure for dependence modeling purposes in order to obtain feasible models. This modeling procedure allows us to use copula models in the cases where large collection of data are analyzed. Besides that, it serves very well for our purpose of investigation systemic risk as we can interpret dependence on the common factor as a vulnerability to the systemic events.

We use two different dynamic factor copula models. The first one is characterized by the heterogeneous dependence on the common factor for time series of latent variables  $\mathbf{Y}_t \equiv (Y_{1,t}, \dots, Y_{n,t})^T$ , and posses the following form

$$\begin{aligned} Y_{i,t} &= \lambda_{i,t}(\gamma_\lambda) Z_t + \epsilon_{i,t}, \quad i = 1, 2, \dots, n \\ Z_t &\sim F_Z(\gamma_z), \quad \epsilon_t \sim iid \quad F_\epsilon(\gamma_\epsilon), \quad Z_t \perp \epsilon_{i,t}, \end{aligned} \quad (11)$$

and the second one where we group the companies according to their nationality and assume homogeneous dependence on the common factor among them, so the block equidependence model is

$$\begin{aligned} Y_{i,t} &= \lambda_{g(i),t} Z_t + \epsilon_{i,t}, \quad i = 1, \dots, N \\ Z_t &\sim F_Z(\gamma_z), \quad \epsilon_t \sim iid \quad F_\epsilon(\gamma_\epsilon), \quad Z_t \perp \epsilon_{i,t}, \quad g = 1, 2, \dots, G, \end{aligned} \quad (12)$$

where  $g(i)$  is the group in which variable  $i$  belongs to and  $G$  is a number of the groups. Rest of the variables are defined as follows:  $F_Z(\gamma_z)$  is the univariate distribution for the common factor and  $F_\epsilon(\gamma_\epsilon)$  are univariate distributions for the idiosyncratic variables. Both these distributions are parametric. Weights on the common factor  $\lambda_{i,t}(\gamma_\lambda)$  are time-varying and by increasing factor loading, *ceteris paribus*, we obtain increase of dependence rate among variables. Set of the copula parameters is denoted  $\gamma = (\gamma_z^T, \gamma_\epsilon^T, \gamma_\lambda^T)^T$ . In our case, distribution for the common factor is skewed student's  $t$  distribution with parameter  $\nu$  governing the shape of the distribution and parameter  $\psi$  governing the skewness of the distribution, and distribution for the idiosyncratic factors is student's  $t$  distribution, thus we call this model skew  $t-t$  copula. This allows to model both asymmetrical and heavy-tailed dependencies across the variables.

The time-varying structure of the models employed in this paper follows specification of the GAS types models proposed by Creal et al. (2013). These

models are general type of models with time-varying parameters and encompass, for example, GARCH type models or autoregressive conditional duration models. Various other dynamic structures are proposed in the literature. In case of GARCH-type volatility models, time-varying parameter evolves as a function of lagged values of residuals. In stochastic volatility models parameter evolves as a latent time series process. The GARCH-type models are more favorable from estimation complexity point of view. In case of our factor model we do not have any obvious choice how to specify the evolution of the time varying parameter and which function of lagged observables to use. The GAS model enables to overcome this obstacle. GAS model for some copula model as proposed in Oh and Patton (2016) may be defined by the following specification

$$\begin{aligned}
\mathbf{U}_t | \mathcal{F}_{t-1} &\sim \mathbf{C}(\delta_t(\gamma)) \\
\delta_t &= \omega + B\delta_{t-1} + A s_{t-1} \\
s_{t-1} &= S_{t-1} \nabla_{t-1} \\
\nabla_{t-1} &= \frac{\partial \log \mathbf{c}(u_{t-1}; \delta_{t-1})}{\partial \delta_{t-1}},
\end{aligned} \tag{13}$$

where  $\delta_t$  is a time-varying parameter dependent on fixed parameter  $\gamma$  and  $S_t$  is a scaling matrix, e.g. the inverse Hessian or its square root. As it is the case that in most of the factor copula applications, the likelihood function is not known, we must calculate the vector  $\nabla_t$  via numerical integration.

We employ model of Oh and Patton (2016), which is a combination of factor copula model proposed in Oh and Patton (2012) and GAS model of Creal et al. (2013). Model employs GAS dynamics in the factor copula model to allow for time-varying factor loading parameters. On the other hand, in order to make the model parsimonious we assume constant values of parameters of the common and idiosyncratic variables distributions, i.e.  $\gamma_z$  and  $\gamma_\epsilon$  does not vary through the time. For modeling purposes of idiosyncratic variable we use standardized student's  $t$  distribution.

We apply general framework from equation 13 to the factor loading parameter  $\lambda_{i,t}$ . But by simply applying that, we would have to estimate  $N + 2N^2$  parameters. In order to make the model feasible, we adopt principles from DCC model of Engle (2002) and set matrices of  $\beta_{i,j}$  and  $\alpha_{i,j}$ ,  $i, j = 1, 2, \dots, N$  to be diagonal matrices with constant scalars,  $\alpha$  and  $\beta$ , respectively, on the diagonal. Scale matrix is then set to be identity matrix,  $S_t = I_t$ . This finally yields the following equation for development of parameter  $\lambda_{i,t}$  for the heterogeneous dependence model in log terms

$$\begin{aligned}
\log \lambda_{i,t} &= \omega_i + \beta \log \lambda_{i,t-1} + \alpha s_{i,t-1}, \quad i = 1, 2, \dots, N \\
s_{i,t} &\equiv \frac{\partial \log \mathbf{c}(\mathbf{u}_t, \boldsymbol{\lambda}_t, \nu_z, \phi_z, \nu_\epsilon)}{\partial \log \lambda_{i,t}} \\
\boldsymbol{\lambda}_t &\equiv [\lambda_{1,t}, \dots, \lambda_{N,t}].
\end{aligned} \tag{14}$$



and for the block equidependence model we obtain following specification

$$\begin{aligned} \log \lambda_{g(i),t} &= \omega_g(i) + \beta \log \lambda_{g(i),t-1} + \alpha s_{g(i),t-1}, \quad g(i) = 1, 2, \dots, G \\ s_{g(i),t} &\equiv \frac{\partial \log \mathbf{c}(\mathbf{u}_t, \boldsymbol{\lambda}_t, \nu_z, \phi_z, \nu_\epsilon)}{\partial \log \lambda_{g(i),t}} \\ \boldsymbol{\lambda}_t &\equiv [\lambda_{1,t}, \dots, \lambda_{G,t}]. \end{aligned} \quad (15)$$

Dynamics for the heterogeneous dependence model implied by the equations 14 demands estimation of  $N+2$  parameters for the GAS dynamics and 3 parameters in total for the shape of the common and idiosyncratic variables, yielding  $N+5$  parameters to be estimated. In case of block equidependence model, number of parameters to be estimated is  $G+5$ . It is obvious that for  $G=1$  we obtain homogeneous dependence and for  $G=N$  we obtain heterogeneous dependence. In order to overcome the numerical challenge of optimizing for  $N \geq 50$  Oh and Patton (2016) proposed method from DCC model of Engle (2002) called "variance targeting" which has to be modified for our purposes. We may rewrite the evolution of the parameter  $\lambda_{i,t}$  from the heterogeneous dependence model as

$$\log \lambda_{i,t} = \mathbb{E}[\log \lambda_{i,t}](1 - \beta) + \beta \log \lambda_{i,t-1} + \alpha s_{i,t-1} \quad (16)$$

using the result from Creal et al. (2013) that  $\mathbb{E}_{t-1}[s_{i,t}] = 0$ , and so  $\mathbb{E}[\log \lambda_{i,t}] = \omega_i/(1 - \beta)$ .

As stated in Oh and Patton (2016), factor copula arising from equations 11 and 12 is known in analytic form only for a few choices of  $F_Z$  and  $F_\epsilon$ , for example if both distributions are normally distributed then the implied copula is also normal. In general, implied copula is not known in closed form, thus likelihood function is not known in closed form, either. For estimation purposes specific numerical procedures must be employed in order to overcome this problem. Oh and Patton (2013) proposed estimation methods for copulas which do not poses closed-form likelihood function based on simulations - simulated method of moments estimation. Unfortunately, their approach do not allow for time-varying conditional copula. To overcome this obstacle Oh and Patton (2016) propose a new approach in order to overcome this obstacle. This approach is based on numerical integration and it exploits the fact that we need only integrate out the one-dimensional common factor despite the fact that copula is  $n$ -dimensional.

Oh and Patton (2016) state the assumptions which provide a method for using sample rank correlations to obtain an estimate of  $\mathbb{E}[\log \lambda_{i,t}]$ , which eliminates the need to numerically optimize over the intercept parameters,  $\omega_i$  or  $\omega_{g(i)}$ . These assumptions enable us to treat the estimation procedure of the vector of parameters as multi-stage GMM and thus yielding consistent and asymptotically normal parameter estimates. For more details on the estimation of this model, see Oh and Patton (2016).

#### 2.4.1. Heterogeneous Dependence Model

In this section, we apply our proposed dynamic factor copula model with heterogeneous GAS dynamics (i.e. heterogeneous dependence structure) defined

earlier in equation 11. Estimated parameters of the distributions of common and idiosyncratic factors for financial and energy companies are described in Table 5. We observe higher level of persistence within the financial sector than within the energy sector with  $\beta$  parameter for financial companies being equal to 0.970 and  $\beta$  for energy companies being equal to 0.902 (see Creal et al. (2013) for explanation of the fact that  $\beta$  parameter in GAS model plays the same role  $\alpha + \beta$  in GARCH model). Inverse of degrees-of-freedom parameters for the common factor for the energy sector is smaller than for the financial sector suggesting greater departure from normality and thus higher tail dependence in the financial sector. Inverse of degrees of freedom parameter for idiosyncratic factor is slightly smaller for the financial sector than for the energy and both these values suggests non-normality of the idiosyncratic factor. This we may interpret as that in the financial sector the extreme events across companies are more driven by the common influences and in the energy sector by the idiosyncratic effects. Thus this suggest higher connectedness and potential systemic risk in the financial sector. Asymmetry parameter  $\psi_z$  for the financial sector is very close to zero, which supports hypothesis of symmetrically distributed common factor. On the other hand, asymmetry parameter within the energy sector is slightly positive suggesting positive skewness of the distribution of the common factor. That implicates that the energy companies are slightly more correlated during the bad periods as the bad periods are characterized by increase of CDS spreads. From Table 5 we also observe that  $\nu_z > \nu_\epsilon$  suggesting 0 tail dependence, for more on this topic see Oh and Patton (2012). This only means that the limit correlation of the extreme events is zero and does not say anything about "near extreme" events as discussed in Oh and Patton (2016).

Table 5: Estimated parameters from heterogeneous dependence model.

	Financial	Energy
$\bar{\omega}$	0.014	0.010
$\alpha$	0.040	0.103
$\beta$	0.970	0.902
$\nu_z^{-1}$	0.054	0.023
$\nu_\epsilon^{-1}$	0.177	0.191
$\psi_z$	-0.009	0.016

Figure 6 depicts estimated factor loadings of the model. We observe that factor loadings within the financial sector evolve "more non-stationary" than in the energy sector. Also average factor loadings is higher in the financial sector than in the energy sector suggesting higher connectedness within this sector. Joint probability of distress is then expected to be higher within the financial sector. We see that factor loadings do not dramatically spike during any period, not even in the beginning of the sovereign debt crisis in 2012. On the other hand, during this crisis 5% and 95% quantiles of the distribution of the  $\lambda$  across financial companies dramatically widened. In the energy sector we do

not observe any medium-term widening like this. In both sectors factor loadings in the end of 2014 dramatically fell down but quickly went back to the previous level and also exceeded it at the end of year 2015, which is especially present within the financial sector. This observations is in agreement with our naive analysis of the connectedness within the sectors based on the rolling window of rank correlations.

Figure 6: Estimated factor loadings from heterogeneous dependence dynamic factor copula model with GAS dynamics.

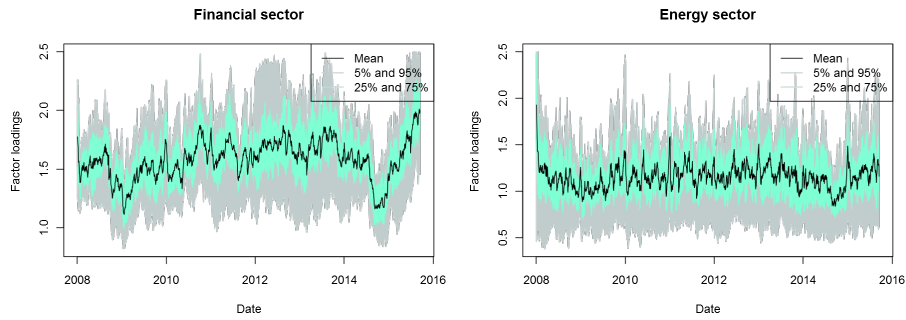
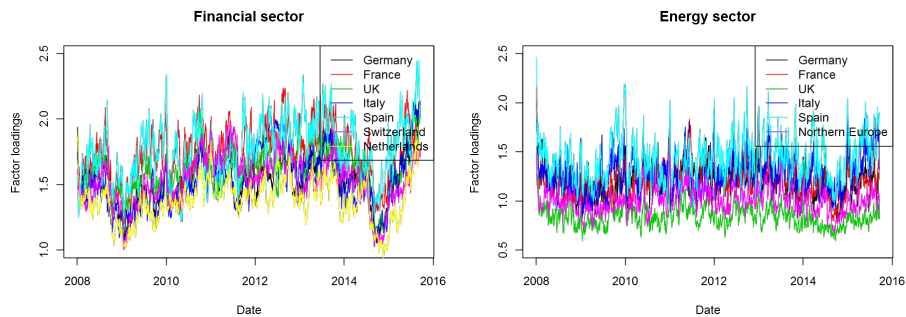


Figure 7 depicts the evolution of factor loadings from the heterogeneous dependence model averaged with respect to the nationalities of the companies, average values are then summarized in Table 6. Average factor loading within financial and energy companies is equal to 1.587 and 1.155, respectively. This clearly suggests higher rate of connectedness within the financial sector.

Figure 7: Estimated factor loadings from heterogeneous dependence dynamic factor copula model with GAS dynamics averaged across the companies from the same country.



From 6, we conclude that in both financial and energy sector the highest values of average factor loadings can be found within the Spanish companies. This suggest high vulnerability to the systemic events within these companies. We may also notice that average factor loadings averaged with respect to the nationality is higher within the financial sector than within the energy sector for every country . On the other hand, the lowest average factor loadings can

be found within companies from Netherlands in financial sector, and within companies from Northern Europe in energy sector.

Table 6: Average estimated factor loadings from heterogeneous dependence model within financial and energy sector averaged with respect to nationality.

Financial		Energy	
Nationality	Factor loading	Nationality	Factor loading
Germany	1.550	Germany	1.202
France	1.739	France	1.200
UK	1.608	UK	0.838
Italy	1.533	Italy	1.272
Spain	1.761	Spain	1.419
Switzerland	1.531	Northern Europe	1.002
Netherlands	1.386		

#### 2.4.2. Block Equidependence Model

We also perform our analysis of systemic risk using block equidependence model. One of the main advantages of this model is a lower computational burden of the estimation process. In this paper, we employ this model mainly in order to compare the estimated results from heterogeneous dependence model, i.e. we use heterogeneous dependence model as a benchmark model. A good *a priori* grouping can yield a good performance along with the economic interpretation of the model. In this model, we group the companies according to their nationality (home of registered office). This model is defined by the equation 12. We may also set the grouping criterion in some different way. For purposes of our analysis, we do not aim to come up with the best one as we try to analyze differences of systemic risk within sectors and countries. Although this grouping might not be the best one from the point of view of performance of the model, it is probably the most natural one with a satisfactory economic intuition behind it. One the the reasons might be that interbank market on a country level is expected to be very contagious. Another thing is that financial institution from the same country are expected to have in some degree homogeneous structure of their portfolios. This grouping scheme may be less intuitive for the energy sector. But both the previous arguments can be partially transferred to the energy sector, too. Energy companies are as any other company vulnerable to the economic fluctuations within the country, such as aggregate demand, political instabilities and other, so this grouping scheme can be also naturally implemented.

When employing this model, we utilize a bit different approach. As we simulate future values of the CDS spreads only every 20 days (will be explained more thoroughly later), we do not generally take into consideration development within the previous 19 days, only current state at the the staring point of the simulation. On the other hand, when we run simulation based on this equide-

pendence model, we try to compensate it by averaging  $\lambda_{i,t}$  parameter over the last 20 days and this value set as an initial value in the simulation process.

Table 7 summarizes estimated parameters from the block equidependence model. Same as in the case of the heterogeneous dependence model, level of persistence is higher within the financial sector. Also degrees of freedom parameter of common factor is lower in case of financial companies than among energy sector suggesting heavier tails of the distribution of the common factor within financial sector and related higher probability of joint extreme events. Estimated degrees of freedom of idiosyncratic factor is same as in the case of heterogeneous dependence model slightly lower across the energy sector. On the other hand, different result of block equidependence model brings estimated parameter of asymmetry of the common factor within the energy sector, which is in case of block equidependence estimated as to be negative. This can be partially compensated by the thickening the tails of the distribution of the idiosyncratic factor.

Table 7: Estimated parameters from block equidependence model.

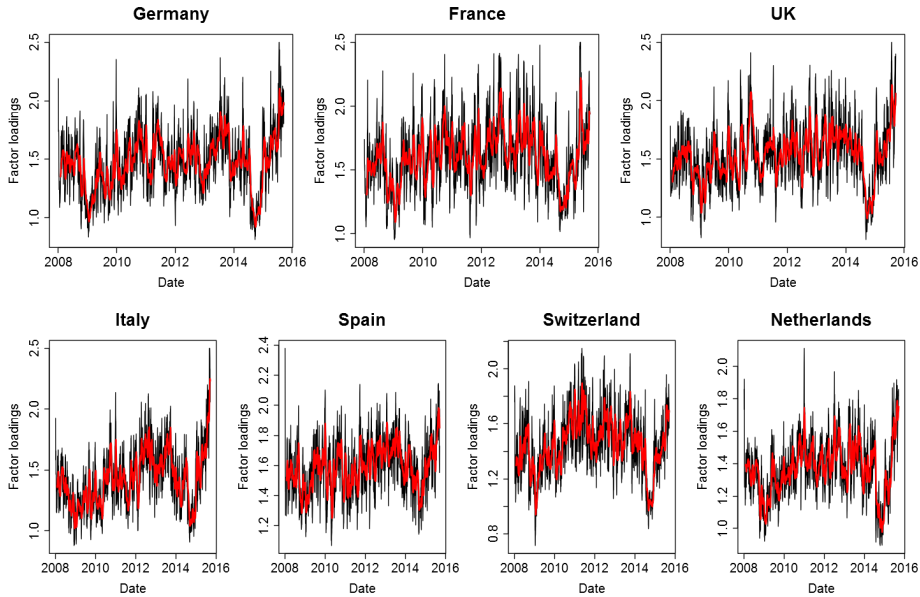
	Financial	Energy
$\bar{\omega}$	0.035	0.003
$\alpha$	0.033	0.098
$\beta$	0.907	0.758
$\nu_z^{-1}$	0.045	0.019
$\nu_\epsilon^{-1}$	0.215	0.232
$\psi_z$	-0.008	-0.014

Development of estimated factor loadings along with the 20-day rolling window of the average values for the financial and energy sector depict Figures 8 and 9. The highest number of spikes of the factor loadings greater than 2 is observed within the factor loadings for companies from France in the financial sector with 185 values exceeding this threshold (average value of the crossings within this sector is 64).

The highest number of values of the factor loadings greater than 2 in the energy sector is observed in the time series of the factor loadings for the Spain companies with total number equal to 38 (mean across the sector is equal to 9).

Table 8 summarizes average values of the factor loadings for countries within the financial and energy sector. We observe overall decrease of factor loadings estimated by the block equidependence model compared to the factor loadings estimated by heterogeneous dependence model. Another noticeable difference is that the highest average factor loadings was estimated for the companies from France within the financial sector with companies from Spain having the second highest average value. This is only a slight departure from the results estimated by heterogeneous dependence model with Spain having the highest average value of factor loadings and France having the second highest values. But differences of average factor loadings of French companies and Spanish companies is rather

Figure 8: Financial sector: estimated factor loadings from dynamic factor copula model with GAS dynamics with equidependence structure among companies from the same country. Red line represents exponentially weighted average over last 20 days.



small in both estimation cases.

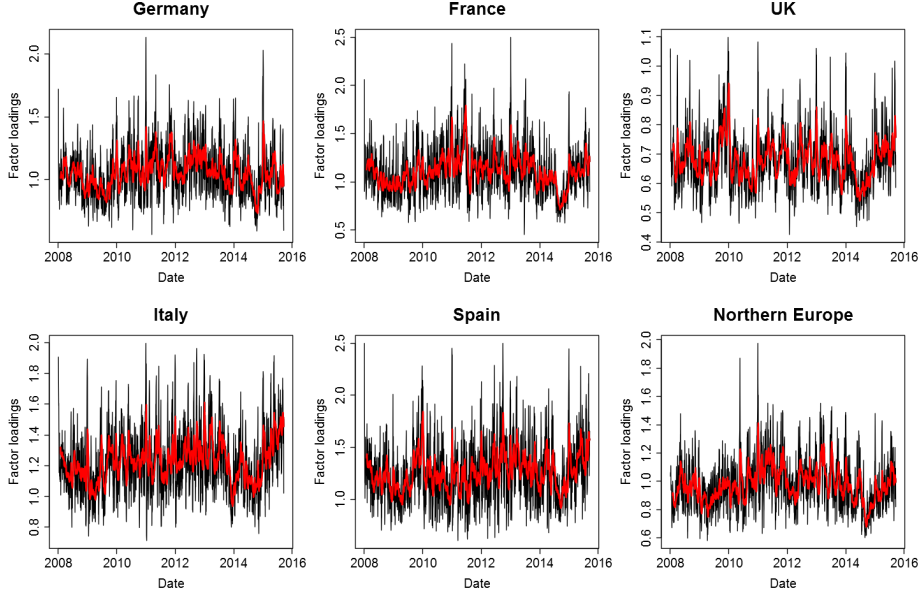
As the average values of the factor loadings estimated by block equidependence model went down in comparison with heterogeneous dependence model, we should expect that our measures of systemic risk will also go down. This is confirmed by the results summarized in Tables 10 and 10 later in the paper, but we do not observe any rapid drop in estimated values. Except this slight drop of the estimated values the results are in line with results obtained from heterogeneous dependence model. These similar results may suggest that there exists a high degree of homogeneity across companies from the same country.

### 2.5. Systemic Risk

Our estimated models for financial and energy sector enable to perform analysis of the systemic risk within the sectors based on Monte Carlo simulations. Notion of systemic risk can be described by the risk of a large proportion of market participants being in distress. Distress can be generally characterized by large negative stock return or by large positive CDS return. In the paper, we work with 2 measures of systemic risk as they are proposed in Oh and Patton (2016).

We may realize that good indicator of a firm being in distress is given by its CDS spread being very high. This implies that protection buyer has to pay more for their protection against the exposure against given firm. We define firm being in distress in the 5-months (100-days) horizon as a dummy variable

Figure 9: Energy sector: estimated factor loadings from dynamic factor copula model with GAS dynamics with equidependence structure among companies from the same country. Red line represents exponentially weighted average over last 20 days.



that assigns 1 when CDS spread in 100 days is higher than some threshold and 0 otherwise, formally

$$D_{i,t+100} \equiv \mathbf{I}\{S_{i,t+100} > c_{i,t+100}^*\}. \quad (17)$$

We utilize this definition for both measures of systemic risk.

Now, we define our first measure of systemic risk. We define joint probability of distress (JPD) as a probability that number of firms in distress is greater than some limit, which is formally given by

$$JPD_{t,k} \equiv \text{P}_t \left[ \sum_{i=1}^n D_{i,t+100} \geq k \right], \quad (18)$$

where  $k$  is some predefined number of firms standing for the notion of "large number of firms". In our empirical investigation we work with  $k$  such that  $k = \frac{1}{3}N$ , where  $N$  is number of all market participants within a given sector (9 within financial sector, and 6 within energy sector). Our empirical analysis also contains brief summary of the results for different values of  $k$ . Thus joint distress occurs when at least one third of the companies from the sector is in distress in 100-days ahead horizon.

Another measure utilized in the exploring the systemic risk is expected proportion firms in distress (EPD) conditioned on a given firm being in distress,

Table 8: Average estimated factor loadings from block equidependence model within financial and energy sector.

Financial		Energy	
Nationality	Factor loading	Nationality	Factor loading
Germany	1.471	Germany	1.053
France	1.607	France	1.109
UK	1.535	UK	0.679
Italy	1.425	Italy	1.228
Spain	1.587	Spain	1.260
Switzerland	1.446	Northern Europe	0.978
Netherlands	1.368		

i.e.

$$EPD_{i,t} \equiv \mathbb{E}_t \left[ \frac{1}{n} \sum_{j=1}^n D_{j,t+100} | D_{i,t+100} = 1 \right]. \quad (19)$$

This measure captures very well the severity of possible contagion among subjects and serves as a complement to the first systemic risk measure.

As the threshold value,  $c_{i,t+100}^*$ , we choose 95% conditional quantile of the distribution of 100-days ahead future values of CDS spreads obtained via the simulation process. Formally, it is defined as

$$c_{i,t+100}^* \equiv q_\alpha(S_{i,t+100} | \mathcal{F}_t), \quad (20)$$

where we set  $\alpha = 0.95$  if not stated otherwise. Higher values of  $\alpha$  does not bring any qualitatively different results of our analysis. Brief summary of the results for  $\alpha = 0.99$  is also featured. This definition of threshold value captures the time-varying character of the dependence structure between the subjects.

Future values are obtained via numerical simulation following the specification of conditional mean and variance models and also factor copula models with GAS dynamics defined earlier. Because of the computational complexity of these simulations, we simulate future values every 20 days (approximately 1 time every month). Number of simulations that we perform in these days is set equal to 500.

Unconditional average 95% quantile among financial companies is 281 bps and average 95% quantile among energy companies is 185 bps. If not stated otherwise, we work with  $k$  such that proportion of firms in distress must be at least one third, that implies that  $k = 9$  for the financial sector, and  $k = 6$  for the energy sector.

### 2.5.1. Results from the Heterogeneous Dependence Model

Figure 10 depicts development of JPD in both examined sectors. We observe that both series seem to be stationary suggesting that the series oscillate around some long-term mean value. Mean value of JPD for financial sector is 0.0323



and 0.021 for energy sector corresponding to the average 3.23% chance of at least one third of the companies being in distress in future 100-days ahead time point in financial sector and average 2.08% chance in the energy sector. This implies about 46% higher probability of the occurrence of the systemic event in the financial sector than in the energy sector. We observe one peak of the JPD within the financial sector in year 2012 when the JPD was equal to 4.2%, energy sector reaches its peaks during the years 2008, 2009, and 2012 when JPD reaches value equal to 3.4%. This peaks may be also partially caused by the overall increase of the volatility in these time points.

From Table 9, we see that when we tighten the criteria of the definition of JPD (increase of  $\alpha$  and  $k$ ), probabilities go substantially lower and differences between JPD within financial sector and energy sector further widens. For example, average probability that at least half of the companies within the sector will be in distress is more than twice as large across the financial sector than in the energy sector.

Table 9: Simulated values of JPD from heterogeneous dependence model within financial and energy sectors for various values of  $\alpha$  and  $k$ .

	$\alpha = 0.95$			$\alpha = 0.99$		
Financial sector	$k = 5$	$k = 9$	$k = 13$	$k = 5$	$k = 9$	$k = 13$
Mean JPD	7.509%	3.228%	1.378%	0.925%	0.214%	0.061%
Energy sector	$k = 3$	$k = 6$	$k = 9$	$k = 3$	$k = 6$	$k = 9$
Mean JPD	7.372%	2.083%	0.529%	0.691%	0.091%	0.012%

Figure 10: Simulated JPD from heterogeneous dependence model, distress defined as 95% conditional quantile and proportion of firms in distress is 1/3.

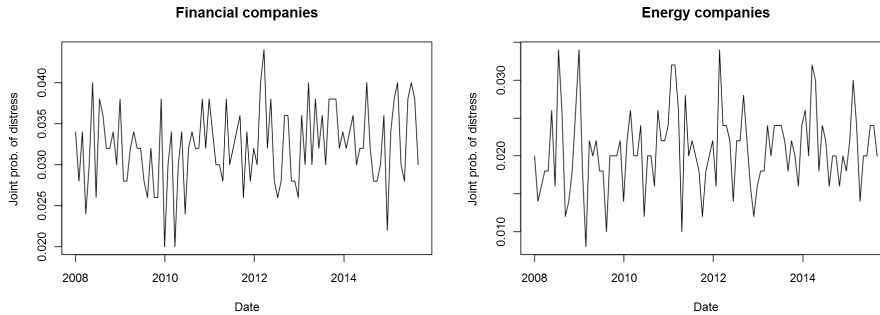
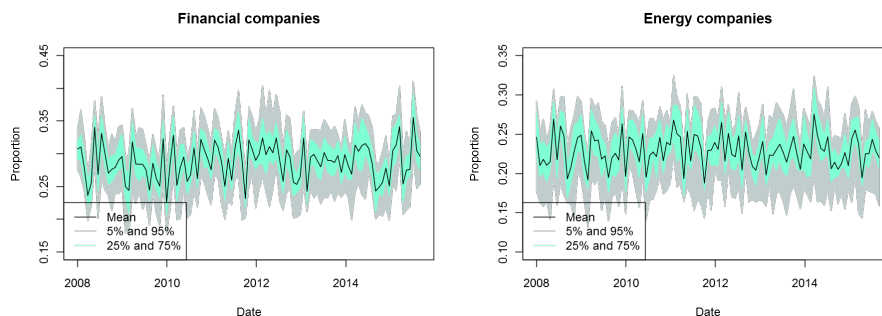


Figure 11 plots evolution of average EPD with 5%-95% and 25%-75% confidence bands in both sectors. Same as in the case of JPD, series are stationary oscillating around some constant value. For financial sector, mean value during the observation period is 0.288, and 0.229 within the energy sector meaning that the average proportion of companies being in distress given one company being in distress is 29% for financial sector and 23% for energy sector, which means

approximately 26% higher risk in the financial sector. Unlike in the case of JPD, we do not observe any steep peaks of the average EPD, neither widening the confidence bands. The series of EPD within both sectors oscillates around their mean values in quite narrow areas. In Appendix B in Table B.21 and B.22 can be found average and basic descriptive statistics of EPD for every company we have taken into consideration from both financial and energy sector. Company with high EPD is expected to be more dependent on the systemic events in the market and conversely the market environment is more dependent on financial health of this company. On the other hand, company with a low EPD is more likely to be dependent on its idiosyncratic component of the risk and market is not so strongly correlated with this company, i.e. distress of this company is not an indicator of market distress. Company with the highest average EPD within the financial sector is Unicredit SpA (Italy) with average EPD being equal to 0.310, closely followed by Intesa Sanpaolo (Italy) with average EPD 0.309, and then by Credit Agricole SA (France) with average EPD 0.305, and BNP Paribas SA (France) and Banco Bilbao Vizcaya Argentaria (Spain) both with average EPD 0.304. On the other side of the spectrum is Mediobanca SpA (Italy) with by far the lowest average JPD equal to 0.196. The second lowest EPD poses Rabobank (Netherlands) with average EPD 0.266. Notice that company with the lowest and the highest EPD are from the same country, Italy. This may undermine our assumption from the next section when we estimate block equidependence model and we group the companies according to their nationality. On the other hand, in the rest of the cases variances of the average EPD within the companies from the same country are pretty low. In the energy sector company with the lowest average EPD is Royal Dutch Shell PLC (UK) with average EPD 0.154 and Statoil ASA (Northern Europe) with 0.188. The highest average value of EPD within energy sector has Iberdrola SA (Spain) with value 0.252 and Enel SpA (Italy) with value 0.250.

Figure 11: Simulated EPD from heterogeneous dependence model, distress defined as 95% conditional quantile.



So, to conclude, results from the heterogeneous dependence model strongly suggest higher degree of systemic risk in the financial sector than in the energy sector. This statement is supported by the average estimated values of the factor

loadings and by the estimated risk measures. Another result from this model is that companies from the Spain are the most systemic in both financial and energy sector, which is based on the the estimated factor loadings and estimated values of EPD.

### 2.5.2. Results from the Block Equidependence Model

Table 10 contains estimated values of JPD for various values of  $\alpha$  and  $k$ . The main departure from the results obtained from the block equidependence model against the results from the heterogeneous dependence model is substantially lower estimated probability of JPD in the energy sector for  $\alpha = 0.99$  and  $k = 9$ , which is equal to 0.012% from heterogeneous dependence model and 0.004% from block equidependence model. Probability of the equivalent event in the financial sector estimated by the block equidependence model is 10 times greater.

Table 10: Simulated values of JPD from block equidependence model within financial and energy sectors for various values of  $\alpha$  and  $k$ .

	$\alpha = 0.95$			$\alpha = 0.99$		
Financial sector	$k = 5$	$k = 9$	$k = 13$	$k = 5$	$k = 9$	$k = 13$
Mean JPD	7.313%	2.927%	1.109%	0.838%	0.176%	0.042%
Energy sector	$k = 3$	$k = 6$	$k = 9$	$k = 3$	$k = 6$	$k = 9$
Mean JPD	7.194%	1.770%	0.370%	0.610%	0.075%	0.004%

Figure 12 depicts evolution of JPD for both examined sectors. Same as in the case of heterogeneous dependence model, we observe one peak during the year 2012 within the financial sector, moreover we also observe 2 other peaks in year 2014 and 2015, which are less noticeable from the heterogeneous dependence model. Within the energy sector, the results from block equidependence model differ more than in the case of financial sector. We observe one peak during year 2011 and few other peaks which were more noticeable from heterogeneous dependence model. This differences may be caused by the different methodology of the simulation process (averaging starting value of  $\lambda$ ) or by greater heterogeneity across the energy companies from the same country. On the other hand, these differences are overall not so significant and we can conclude that grouping with respect to the nationality of the companies delivers reasonable results.

Series of EPD among both sectors depicts Figure 13. Series are also stationary. Overall, we observe that EPD systemic risk measure in both cases (heterogeneous and block equidependence model) does not possess any noticeable peaks.

To summarize the results from the block equidependence model, we should point out that the results are qualitatively similar to the results obtained from the heterogeneous dependence model. Estimated dependencies and related systemic risk for both sectors are slightly smaller but not considerable different. These results support the proposed grouping according to the company's nationality suggesting substantial dependence homogeneity across the companies from the same country.

Figure 12: Simulated JPD from block equidependence model, distress defined as 95% conditional quantile and proportion of firms in distress is 1/3.

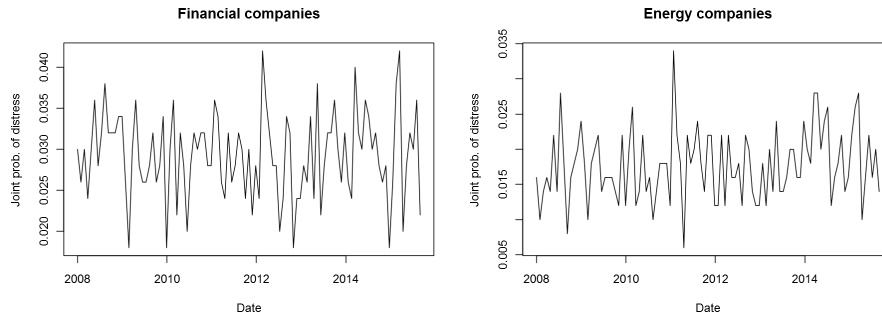
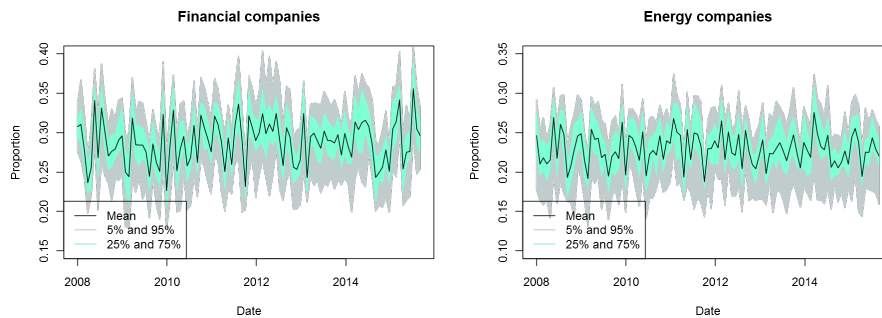


Figure 13: Simulated EPD from block equidependence model, distress defined as 95% conditional quantile.



### 3. Conclusion

In this paper, we investigated and compared systemic risk within European financial and energy sector using factor copula model and CDS daily time series data between January 2008 and September 2015, which has not been done yet before to the best of our knowledge. Our results aim to enrich ongoing empirical discussion about systemic risk in the Europe, and provide some background for regulatory purposes, portfolio selection, and others. Model that we employ is capable of capturing various important features present in the data, such as asymmetric and tail dependence among subjects, which is essential for studying dependencies during extreme events. The main reason for choosing CDS data for purposes of our analysis is the fact that CDS spreads contain information about the probability of default of the reference entity perceived by the market participants, and thus they are appropriate for investigating systemic risk.

We compared financial and energy sectors with respect to their connectedness and measures of systemic risk. We made use of two different measures of systemic risk that can fully exploit possibilities of the proposed models. One of them is the joint probability of distress (JPD), and the second measure is the

expected proportion of firms in distress conditioned on a given firm being in distress (EPD)

We estimated 2 specifications of the dynamic factor copula model. First, we estimated heterogeneous dependence version of the model. Then, we utilized Monte Carlo simulations to obtain estimates of the systemic risk. We conclude that systemic risk within the financial sector is noticeable higher than within the energy sector as the average values of JPD are about 50% higher within the financial sector. The main peak of the systemic risk estimated from the heterogeneous dependence model is in year 2012 with estimated value of probability of at least one third of companies being in distress equal to 4.2% within the financial sector, and within the energy sector we observe 3 main peaks in years 2008, 2009, and 2012 with estimated probability equal to 3.4%. We also estimated average values of EPD for financial and energy sector. Beside that, we include in Appendix B descriptive statistics of EPD for every company from both sectors that we took into consideration in the analysis. We also conclude that the most systemic companies from both sectors are companies from Spain.

Second, we estimated block equidependence version of the model, which groups the companies according to their nationality and model dependence on the common factor homogeneously for all the subjects within the same group. Results from the block equidependence model are qualitatively the same as the results from the heterogeneous dependence model. Dependencies, and thus systemic risk estimated by this model, are only slightly smaller. These results support our notion of high degree of homogeneity across the companies from the same country and validates our grouping scheme.

## References

- Adrian, T. and Brunnermeier, M. K. (2009). Covar: a method for macroprudential regulation. *Federal Reserve Bank of New York Staff Report*, 348.
- Brownlees, T. and Engle, R. F. (2015). Srisk: A conditional capital shortfall index for systemic risk measurement. *Department of Finance, New York University*.
- Carr, P. and Wu, L. (2010). A simple robust link between american puts and credit protection. *Review of Financial Studies*, page hhq129.
- Cherubini, U., Luciano, E., and Vecchiato, W. (2004). *Copula methods in finance*. John Wiley & Sons.
- Choroś, B., Ibragimov, R., and Permiakova, E. (2010). Copula estimation. In *Copula theory and its applications*, pages 77–91. Springer.
- Christoffersen, P., Jacobs, K., Jin, X., and Langlois, H. (2014). Dynamic dependence and diversification in corporate credit. In *The 12th International Paris Finance Meeting*.
- Creal, D., Koopman, S. J., and Lucas, A. (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics*, 28(5):777–795.
- Creal, D. D. and Tsay, R. S. (2015). High dimensional dynamic stochastic copula models. *Journal of Econometrics*.
- Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, 20(3):339–350.
- Hansen, B. E. (1994). Autoregressive conditional density estimation. *International Economic Review*, pages 705–730.
- Kleinow, J. and Moreira, F. (2015). Systemic risk among european banks: A copula approach.
- Liu, X. (2016). Measuring systemic risk with regime switching in tails. *Economic Modelling*.
- Lucas, A., Schwaab, B., and Zhang, X. (2014). Conditional euro area sovereign default risk. *Journal of Business & Economic Statistics*, 32(2):271–284.
- Manner, H. and Reznikova, O. (2012). A survey on time-varying copulas: Specification, simulations, and application. *Econometric Reviews*, 31(6):654–687.
- McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative Risk Management: Concepts, Techniques and Tools: Concepts, Techniques and Tools*. Princeton university press.

- Oh, D. H. and Patton, A. J. (2012). Modelling dependence in high dimensions with factor copulas. *Manuscript, Duke University*.
- Oh, D. H. and Patton, A. J. (2013). Simulated method of moments estimation for copula-based multivariate models. *Journal of the American Statistical Association*, 108(502):689–700.
- Oh, D. H. and Patton, A. J. (2016). Time-varying systemic risk: Evidence from adynamic copula model of cds spreads. *Journal of Business & Economic Statistics*, (just-accepted):1–47.
- Patton, A. (2012a). Copula methods for forecasting multivariate time series. *Handbook of economic forecasting*, 2:899–960.
- Patton, A. J. (2006). Modelling asymmetric exchange rate dependence\*. *International economic review*, 47(2):527–556.
- Patton, A. J. (2009). Copula-based models for financial time series. In *Handbook of financial time series*, pages 767–785. Springer.
- Patton, A. J. (2012b). A review of copula models for economic time series. *Journal of Multivariate Analysis*, 110:4–18.
- Pourkhanali, A., Kim, J.-M., Tafakori, L., and Fard, F. A. (2016). Measuring systemic risk using vine-copula. *Economic Modelling*, 53:63–74.
- Sklar, M. (1959). *Fonctions de répartition à n dimensions et leurs marges*. Université Paris 8.

## Appendix A. CDS Spreads

### Appendix A.1. CDS Spreads of Financial Companies

Table A.11: Cross-sectional distributuin of CDS spreads of financial companies.

	Mean	Min	25%	Median	75%	Max%
Mean	130.65	58.40	106.04	132.38	159.46	202.38
Std. dev	70.17	16.95	39.97	64.67	92.69	134.49
1 <sup>st</sup> order autocorr.	0.995	0.985	0.993	0.995	0.997	0.998
Skewness	1.05	0.03	0.79	1.03	1.29	2.76
Kurtosis	0.92	-0.97	-0.15	0.30	1.29	7.70
Min	38.25	24.52	31.90	39.59	43.55	59.50
25%	77.62	43.70	69.41	77.54	86.05	110.45
Median	111.50	56.51	95.34	109.71	130.95	167.57
75%	167.19	70.94	124.57	161.54	207.92	296.49
Max	390.80	128.24	266.53	378.89	488.69	841.62

Table A.12: Cross-sectional distribution of log-differences of CDS spreads of financial companies in basis points.

	Mean	Min	25%	Median	75%	Max%
Mean	3.03	-0.99	1.82	3.14	4.61	6.80
Std. dev	421.26	309.23	389.51	428.17	448.57	476.01
1 <sup>st</sup> order autocorr.	0.157	-0.063	0.149	0.162	0.196	0.224
Skewness	0.05	-2.12	-0.22	0.02	0.20	3.71
Kurtosis	11.31	3.67	5.66	6.60	9.93	86.22
Min	-3295	-7238	-3693	-3130	-2664	-2076
25%	-192.67	-233.44	-210.57	-197.53	-182.57	-57.13
Median	-1.89	-9.08	-2.61	-1.19	-0.05	1.83
75%	195.30	59.06	183.34	199.94	213.41	241.59
Max	3246	2223	2818	3119	3579	6177

Table A.13: Financial companies and descriptive statistics of their CDS spreads in basis points.

Company	Nationality	Mean	5%	25%	Median	75%	95%
Allianz SE	Germany	77.34	34.11	52.31	76.00	95.68	133.31
Commerzbank AG	Germany	132.38	65.25	83.25	109.71	165.74	274.78
Deutsche Bank AG	Germany	108.97	64.33	81.87	98.20	122.74	193.61
Hannover Rueck SE	Germany	84.57	44.19	57.26	85.41	106.54	136.20
Munich Re	Germany	58.40	35.83	43.70	56.51	70.94	86.76
AXA SA	France	140.94	56.09	77.11	126.26	176.29	319.70
BNP Paribas SA	France	110.24	52.16	65.28	86.46	129.75	259.13
Credit Agricole SA	France	134.52	62.47	78.58	110.66	165.15	289.42
Societe Generale SA	France	141.59	70.06	84.59	108.61	166.86	332.95
Aviva PLC	UK	131.75	62.24	87.52	123.89	161.54	223.32
Barclays Bank PLC	UK	124.23	54.58	81.76	119.35	153.30	226.50
HSBC Bank PLC	UK	84.87	45.75	62.73	77.89	103.30	143.74
Lloyds Bank PLC	UK	147.61	50.90	75.83	137.43	188.76	321.65
The Royal Bank of Scotland PLC	UK	163.08	63.66	105.35	152.67	201.50	331.01
Assicurazioni Generali SpA	Italy	155.84	58.92	77.54	120.95	224.53	365.65
Intesa Sanpaolo SpA	Italy	176.24	50.16	80.45	124.52	271.63	464.83
Mediobanca SpA	Italy	171.84	52.96	93.75	135.80	214.35	427.77
UniCredit SpA	Italy	202.38	69.72	105.06	144.52	296.49	502.34
Banco Bilbao Vizcaya Argentaria SA	Spain	184.76	69.33	89.29	138.35	267.97	408.52
Banco Santander SA	Spain	176.34	69.91	88.07	135.64	253.49	387.57
Credit Suisse Group AG	Switzerland	103.11	54.10	72.93	92.94	126.40	184.36
Swiss Reinsurance Co Ltd	Switzerland	138.02	38.03	71.49	107.87	137.27	490.43
UBS AG	Switzerland	112.30	45.52	71.03	97.73	144.06	213.15
Zurich Insurance Co Ltd	Switzerland	95.61	41.74	68.91	98.48	118.52	149.59
Aegon NV	Netherlands	176.35	70.08	110.45	167.57	214.51	354.35
Rabobank	Netherlands	82.33	43.30	59.67	73.89	99.57	146.66
ING Bank NV	Netherlands	112.05	52.59	69.92	103.16	137.41	220.80



Table A.14: Cross-sectional distribution of CDS spreads of energy companies.

	Mean	Min	25%	Median	75%	Max
Mean	94.36	55.84	65.98	78.24	108.43	179.19
Std. dev	45.30	13.55	21.51	30.82	59.42	113.11
1 <sup>st</sup> order autocorr.	0.995	0.991	0.995	0.996	0.996	0.997
Skewness	1.42	0.67	0.99	1.12	1.52	4.52
Kurtosis	3.08	-0.20	0.44	1.34	2.78	26.59
Min	34.48	20.76	23.74	31.59	40.64	60.80
25%	61.06	41.28	50.64	54.94	71.10	100.06
Median	83.52	50.27	60.60	69.50	94.28	161.31
75%	114.52	63.19	74.68	88.80	140.03	241.42
Max	293.84	109.89	156.51	190.67	480.21	637.90

Table A.15: Cross-sectional distribution of log-differences of CDS spreads of energy companies in basis points.

	Mean	Min	25%	Median	75%	Max
Mean	3.57	0.04	2.03	3.99	4.45	7.24
Std. dev	346.22	282.71	313.64	333.05	377.44	434.03
1 <sup>st</sup> order autocorr.	0.098	-0.209	0.065	0.113	0.153	0.210
Skewness	0.82	-0.12	0.29	0.47	0.74	3.55
Kurtosis	17.08	5.52	7.99	11.02	16.83	70.24
Min	-2676	-3926	-3030	-2548	-2324	-1717
25%	-136.60	-199.45	-153.65	-134.88	-121.01	-0.47
Median	-2.87	-9.91	-5.19	-1.51	-0.18	0.00
75%	125.87	0.00	114.56	125.56	138.13	182.19
Max	3287	1907	2570	2890	3567	6638

*Appendix A.2. Energy Companies*

Table A.16: Energy companies and descriptive statistics of their CDS spreads in basis points.

Company	Nationality	Mean	5%	25%	Median	75%	95%
E.ON SE	Germany	75.72	52.54	60.44	72.32	83.38	125.82
EnBW	Germany	70.64	40.30	54.37	66.05	84.29	125.46
RWE AG	Germany	78.55	43.31	61.76	76.44	92.10	130.61
Electricite de France SA	France	77.94	39.50	55.48	68.00	95.63	147.05
Engie	France	72.61	39.69	50.31	61.43	91.83	134.05
TOTAL SA	France	55.84	30.39	41.28	50.27	66.42	93.44
Veolia Environnement SA	France	114.24	54.90	74.21	99.34	148.43	204.97
BP PLC	UK	80.50	35.17	49.99	68.08	85.76	146.29
National Grid PLC	UK	80.61	43.93	54.62	70.93	85.59	192.80
Royal Dutch Shell PLC	UK	56.43	31.48	43.47	56.91	63.19	92.75
Enel SpA	Italy	179.19	74.23	88.50	140.29	241.42	411.02
Eni SpA	Italy	91.00	40.80	55.26	79.10	114.86	190.79
Gas Natural SDG SA	Spain	170.20	51.92	81.46	161.31	223.65	354.60
Iberdrola SA	Spain	151.96	66.52	81.86	128.15	196.35	312.43
Repsol SA	Spain	158.64	71.48	100.06	129.23	180.58	373.40
Fortum OYJ	Northern Europe	64.42	46.24	53.04	59.57	69.71	103.80
Statoil ASA	Northern Europe	59.24	30.76	41.31	55.61	71.78	102.81
Vattenfall AB	Northern Europe	60.83	40.48	51.63	60.33	66.31	89.96

## Appendix B. Estimation Results

### Appendix B.1. ARMA-GARCH models

Table B.17: Estimated ARMA-GARCH models of financial companies.

Company	Nationality	$\phi_0 \times 10^4$	$\phi_1$	$\phi_m$	$\omega \times 10^4$	$\alpha$	$\beta$	$\delta$	$\nu$	$\psi$
Allianz SE	Germany	-2.110 (9.035)	-0.051 (0.032)	0.326 (0.037)	1.218 (0.215)	0.222 (0.037)	0.733 (0.028)	-0.005 (0.042)	5.038	-0.044
Commerzbank AG	Germany	3.695 (10.520)	0.087 (0.037)	0.168 (0.045)	0.451 (0.138)	0.075 (0.024)	0.870 (0.026)	0.060 (0.021)	5.212	-0.051
Deutsche Bank AG	Germany	3.469 (10.476)	0.078 (0.037)	0.189 (0.045)	0.383 (0.093)	0.091 (0.019)	0.872 (0.017)	0.038 (0.021)	5.185	-0.023
Hannover Rueck SE	Germany	1.413 (9.284)	-0.001 (0.032)	0.259 (0.037)	0.816 (0.176)	0.124 (0.023)	0.799 (0.026)	0.088 (0.035)	4.772	-0.043
Munich Re	Germany	-0.264 (9.061)	-0.120 (0.028)	0.339 (0.035)	1.030 (0.203)	0.123 (0.023)	0.821 (0.021)	0.025 (0.028)	4.418	-0.052
AXA SA	France	-0.171 (8.175)	0.000 (0.033)	0.275 (0.034)	0.985 (0.176)	0.100 (0.020)	0.796 (0.024)	0.062 (0.028)	4.558	-0.028
BNP Paribas SA	France	3.760 (10.587)	0.019 (0.035)	0.219 (0.045)	0.479 (0.154)	0.069 (0.020)	0.882 (0.024)	0.055 (0.020)	5.634	-0.021
Credit Agricole SA	France	1.947 (9.940)	0.044 (0.037)	0.233 (0.044)	0.500 (0.159)	0.064 (0.019)	0.877 (0.025)	0.064 (0.020)	5.193	-0.038
Societe Generale SA	France	4.164 (9.905)	0.023 (0.036)	0.248 (0.044)	0.592 (0.200)	0.067 (0.019)	0.858 (0.030)	0.085 (0.023)	5.686	-0.034
Aviva PLC	UK	0.753 (7.897)	-0.052 (0.032)	0.291 (0.032)	0.654 (0.134)	0.129 (0.023)	0.818 (0.022)	0.022 (0.027)	4.904	-0.022
Barclays Bank PLC	UK	1.018 (10.116)	0.033 (0.038)	0.217 (0.047)	0.581 (0.132)	0.116 (0.022)	0.841 (0.020)	0.025 (0.025)	5.473	0.001
HSBC Bank PLC	UK	2.376 (8.467)	-0.081 (0.031)	0.301 (0.034)	0.613 (0.142)	0.106 (0.020)	0.834 (0.023)	0.050 (0.027)	4.985	-0.029
Lloyds Bank PLC	UK	2.520 (8.760)	-0.059 (0.033)	0.318 (0.038)	0.475 (0.111)	0.113 (0.022)	0.838 (0.022)	0.046 (0.026)	5.787	0.017
The Royal Bank of Scotland PLC	UK	0.719 (9.799)	-0.005 (0.034)	0.262 (0.042)	0.496 (0.105)	0.145 (0.026)	0.817 (0.021)	0.034 (0.028)	5.052	0.001
Assicurazioni Generali SpA	Italy	2.816 (9.388)	0.015 (0.036)	0.295 (0.041)	0.793 (0.159)	0.139 (0.026)	0.810 (0.021)	0.033 (0.030)	4.505	-0.055
Intesa Sanpaolo SpA	Italy	4.367 (12.099)	0.146 (0.038)	0.130 (0.051)	0.968 (0.282)	0.095 (0.022)	0.840 (0.034)	0.034 (0.024)	5.023	-0.033
Mediobanca SpA	Italy	5.726 (6.539)	-0.251 (0.024)	0.329 (0.023)	0.318 (0.066)	0.114 (0.018)	0.847 (0.017)	0.035 (0.023)	3.696	-0.037
UniCredit SpA	Italy	4.587 (10.550)	0.031 (0.038)	0.244 (0.049)	0.926 (0.197)	0.137 (0.023)	0.807 (0.025)	0.024 (0.026)	5.265	-0.017
Banco Bilbao Vizcaya Argentaria SA	Spain	4.912 (11.694)	0.149 (0.037)	0.117 (0.048)	0.639 (0.159)	0.109 (0.021)	0.843 (0.023)	0.032 (0.022)	6.298	-0.016
Banco Santander SA	Spain	4.641 (11.548)	0.113 (0.039)	0.116 (0.052)	0.692 (0.164)	0.112 (0.020)	0.838 (0.020)	0.033 (0.023)	6.485	-0.007
Credit Suisse Group AG	Switzerland	1.804 (8.575)	-0.056 (0.031)	0.248 (0.035)	0.467 (0.087)	0.197 (0.029)	0.790 (0.020)	0.001 (0.031)	4.269	-0.018
Swiss Reinsurance Co Ltd	Switzerland	-2.119 (8.247)	-0.025 (0.033)	0.248 (0.034)	1.169 (0.234)	0.117 (0.025)	0.774 (0.033)	0.069 (0.032)	4.333	-0.020
UBS AG	Switzerland	0.242 (8.640)	-0.054 (0.033)	0.293 (0.037)	0.569 (0.124)	0.075 (0.020)	0.846 (0.023)	0.088 (0.024)	4.763	-0.013
Zurich Insurance Co Ltd	Switzerland	-0.972 (8.308)	-0.018 (0.034)	0.271 (0.036)	1.207 (0.269)	0.103 (0.025)	0.800 (0.034)	0.027 (0.029)	4.834	-0.022
Aegon NV	Netherlands	1.546 (9.267)	0.156 (0.032)	0.094 (0.032)	0.581 (0.112)	0.123 (0.024)	0.808 (0.022)	0.052 (0.027)	5.053	-0.011
Rabobank	Netherlands	3.356 (6.848)	-0.136 (0.028)	0.327 (0.026)	0.496 (0.117)	0.098 (0.022)	0.841 (0.026)	0.038 (0.024)	4.980	-0.047
ING Bank NV	Netherlands	1.080 (7.681)	-0.105 (0.030)	0.350 (0.031)	0.434 (0.108)	0.088 (0.019)	0.867 (0.022)	0.032 (0.020)	4.532	-0.031

Table B.18: Estimated ARMA-GARCH models of energy companies.

Company	Nationality	$\phi_0 \times 10^4$	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_m$	$\omega \times 10^4$	$\alpha$	$\beta$	$\delta$	$\nu$	$\psi$
E.ON SE	Germany	3.130	-0.059	-0.047	-0.042	0.320	0.236	0.134	0.825	0.043	4.559	-0.086
		(5.817)	(0.035)	(0.024)	(0.023)	(0.040)	(0.048)	(0.022)	(0.019)	(0.025)		
EnBW	Germany	1.863	-0.111	-0.034	-0.121	0.337	0.010	0.051	0.922	0.008	3.277	-0.026
		(5.616)	(0.029)	(0.024)	(0.022)	(0.036)	(0.003)	(0.000)	(0.000)	(0.000)		
RWE AG	Germany	6.385	-0.016	0.006	-0.087	0.277	0.334	0.113	0.816	0.090	4.156	-0.046
		(6.499)	(0.034)	(0.023)	(0.022)	(0.043)	(0.066)	(0.021)	(0.021)	(0.027)		
Electricite de France SA	France	2.778	-0.208	-0.056	-0.033	0.457	0.486	0.099	0.808	0.104	4.056	-0.048
		(5.855)	(0.030)	(0.026)	(0.023)	(0.039)	(0.083)	(0.022)	(0.023)	(0.028)		
Engie	France	0.413	-0.164	0.027	-0.018	0.385	0.521	0.114	0.796	0.075	3.900	-0.035
		(6.106)	(0.031)	(0.025)	(0.022)	(0.038)	(0.078)	(0.021)	(0.022)	(0.027)		
TOTAL SA	France	3.274	-0.162	-0.024	-0.047	0.377	0.249	0.133	0.848	0.035	3.379	-0.048
		(5.696)	(0.028)	(0.025)	(0.023)	(0.034)	(0.053)	(0.026)	(0.015)	(0.028)		
Veolia Environnement SA	France	-0.782	-0.069	0.040	-0.041	0.329	0.434	0.115	0.805	0.094	4.236	-0.041
		(6.876)	(0.033)	(0.024)	(0.022)	(0.043)	(0.091)	(0.023)	(0.027)	(0.032)		
BP PLC	UK	6.258	0.074	-0.030	-0.003	0.235	0.364	0.139	0.814	0.045	4.065	-0.078
		(8.799)	(0.027)	(0.022)	(0.022)	(0.041)	(0.079)	(0.028)	(0.026)	(0.028)		
National Grid PLC	UK	-1.333	-0.147	-0.037	-0.062	0.360	0.127	0.128	0.831	0.075	4.528	0.010
		(4.916)	(0.033)	(0.025)	(0.023)	(0.034)	(0.044)	(0.036)	(0.013)	(0.029)		
Royal Dutch Shell PLC	UK	5.411	-0.036	0.007	0.022	0.153	0.010	0.047	0.937	-0.010	2.312	-0.044
		(6.818)	(0.024)	(0.023)	(0.022)	(0.029)	(0.003)	(0.000)	(0.000)	(0.000)		
Enel SpA	Italy	1.066	0.230	-0.023	-0.050	-0.030	0.840	0.103	0.796	0.111	4.459	-0.036
		(10.646)	(0.038)	(0.023)	(0.022)	(0.062)	(0.151)	(0.020)	(0.022)	(0.027)		
Eni SpA	Italy	2.505	-0.231	-0.075	-0.067	0.497	0.571	0.131	0.793	0.085	3.555	-0.032
		(5.401)	(0.030)	(0.026)	(0.023)	(0.038)	(0.100)	(0.023)	(0.022)	(0.030)		
Gas Natural SDG SA	Spain	3.902	0.144	-0.027	-0.034	0.114	1.341	0.102	0.796	0.050	3.755	-0.039
		(10.337)	(0.036)	(0.023)	(0.022)	(0.061)	(0.236)	(0.021)	(0.028)	(0.024)		
Iberdrola SA	Spain	1.481	0.220	-0.028	-0.078	-0.018	0.655	0.118	0.818	0.048	4.889	-0.045
		(9.826)	(0.041)	(0.023)	(0.022)	(0.065)	(0.137)	(0.022)	(0.023)	(0.025)		
Repsol SA	Spain	3.990	0.048	0.055	-0.029	0.204	0.878	0.183	0.729	0.106	4.361	-0.055
		(9.617)	(0.035)	(0.023)	(0.022)	(0.055)	(0.185)	(0.031)	(0.030)	(0.037)		
Fortum OYJ	Northern Europe	1.757	-0.082	-0.019	-0.020	0.300	0.271	0.214	0.764	0.042	3.861	-0.028
		(5.934)	(0.032)	(0.024)	(0.023)	(0.037)	(0.046)	(0.030)	(0.019)	(0.036)		
Statoil ASA	Northern Europe	2.330	-0.466	-0.232	-0.100	0.456	0.320	0.200	0.751	0.060	4.309	-0.071
		(4.004)	(0.025)	(0.026)	(0.023)	(0.026)	(0.070)	(0.034)	(0.031)	(0.037)		
Vattenfall AB	Northern Europe	2.212	-0.131	-0.060	-0.093	0.342	0.341	0.161	0.736	0.203	3.831	-0.041
		(5.146)	(0.031)	(0.024)	(0.022)	(0.035)	(0.060)	(0.024)	(0.022)	(0.038)		

*Appendix B.2. Estimation results - Heterogeneous dependence model*

Table B.19: Estimated  $\lambda$  parameters from heterogeneous dependence model for financial companies.

Company	Nationality	Mean	5%	25%	Median	75%	95%
Allianz SE	Germany	1.60	1.19	1.46	1.61	1.75	1.96
Commerzbank AG	Germany	1.71	1.34	1.54	1.68	1.85	2.15
Deutsche Bank AG	Germany	1.65	1.29	1.52	1.66	1.78	2.01
Hannover Rueck SE	Germany	1.38	1.02	1.20	1.37	1.54	1.79
Munich Re	Germany	1.40	1.01	1.24	1.40	1.54	1.85
AXA SA	France	1.61	1.16	1.42	1.61	1.79	2.07
BNP Paribas SA	France	1.79	1.42	1.61	1.77	1.94	2.18
Credit Agricole SA	France	1.80	1.39	1.63	1.80	1.96	2.22
Societe Generale SA	France	1.76	1.38	1.59	1.74	1.91	2.22
Aviva PLC	UK	1.61	1.22	1.44	1.61	1.77	2.02
Barclays Bank PLC	UK	1.73	1.32	1.58	1.72	1.88	2.15
HSBC Bank PLC	UK	1.39	1.04	1.25	1.39	1.53	1.75
Lloyds Bank PLC	UK	1.63	1.24	1.50	1.63	1.78	2.01
The Royal Bank of Scotland PLC	UK	1.67	1.25	1.53	1.68	1.82	2.05
Assicurazioni Generali SpA	Italy	1.68	1.27	1.50	1.67	1.85	2.14
Intesa Sanpaolo SpA	Italy	1.85	1.40	1.60	1.79	2.03	2.50
Mediobanca SpA	Italy	0.72	0.56	0.63	0.69	0.78	0.96
UniCredit SpA	Italy	1.88	1.43	1.63	1.82	2.07	2.50
Banco Bilbao Vizcaya Argentaria.SA	Spain	1.76	1.39	1.59	1.75	1.92	2.14
Banco Santander SA	Spain	1.77	1.38	1.62	1.76	1.93	2.16
Credit Suisse Group AG	Switzerland	1.42	1.10	1.29	1.42	1.56	1.72
Swiss Reinsurance Co Ltd	Switzerland	1.57	1.13	1.42	1.58	1.72	1.96
UBS AG	Switzerland	1.54	1.14	1.39	1.56	1.70	1.88
Zurich Insurance Co Ltd	Switzerland	1.59	1.14	1.43	1.62	1.75	1.99
Aegon NV	Netherlands	1.64	1.22	1.43	1.62	1.85	2.10
Rabobank	Netherlands	1.21	0.94	1.11	1.20	1.31	1.49
ING Bank NV	Netherlands	1.31	1.03	1.20	1.31	1.42	1.58

Table B.20: Estimated  $\lambda$  parameters from heterogeneous dependence model for energy companies.

Company	Nationality	Mean	5%	25%	Median	75%	95%
E.ON SE	Germany	1.37	0.97	1.18	1.35	1.53	1.85
EnBW Energie Baden Wuerttemberg AG	Germany	1.01	0.75	0.89	1.00	1.13	1.33
RWE AG	Germany	1.22	0.91	1.07	1.21	1.36	1.55
Electricite de France.SA	France	1.29	0.92	1.11	1.28	1.44	1.69
Engie	France	1.31	0.97	1.14	1.28	1.45	1.73
TOTAL SA	France	0.83	0.64	0.74	0.83	0.92	1.04
Veolia Environnement SA	France	1.37	0.98	1.18	1.35	1.53	1.82
BP PLC	UK	0.83	0.63	0.73	0.82	0.92	1.06
National Grid PLC	UK	1.30	0.97	1.12	1.27	1.44	1.71
Royal Dutch Shell PLC	UK	0.39	0.31	0.35	0.39	0.42	0.47
Enel SpA	Italy	1.47	1.06	1.27	1.44	1.65	1.97
Eni SpA	Italy	1.07	0.80	0.94	1.06	1.19	1.42
Gas Natural SDG SA	Spain	1.40	0.99	1.19	1.36	1.57	1.91
Iberdrola SA	Spain	1.52	1.08	1.29	1.49	1.71	2.05
Repsol SA	Spain	1.34	0.98	1.17	1.32	1.50	1.77
Fortum OYJ	Northern Europe	1.20	0.88	1.06	1.19	1.33	1.56
Statoil ASA	Northern Europe	0.64	0.49	0.57	0.64	0.70	0.78
Vattenfall AB	Northern Europe	1.17	0.87	1.03	1.15	1.29	1.52

Table B.21: Estimated EPD for every financial company from heterogeneous dependence model.

Company	Nationality	$\alpha = 0.95$				$\alpha = 0.99$			
		Mean	25%	Median	75%	Mean	25%	Median	75%
Allianz SE	Germany	0.284	0.250	0.289	0.310	0.159	0.111	0.148	0.200
Commerzbank AG	Germany	0.295	0.274	0.296	0.319	0.172	0.119	0.163	0.215
Deutsche Bank AG	Germany	0.285	0.256	0.281	0.314	0.169	0.126	0.156	0.207
Hannover Rueck SE	Germany	0.274	0.246	0.276	0.302	0.163	0.111	0.148	0.200
Munich Re	Germany	0.273	0.247	0.277	0.301	0.154	0.111	0.141	0.193
AXA SA	France	0.290	0.262	0.293	0.314	0.159	0.111	0.148	0.200
BNP Paribas SA	France	0.304	0.277	0.305	0.329	0.171	0.133	0.156	0.207
Credit Agricole SA	France	0.305	0.279	0.310	0.329	0.174	0.133	0.170	0.207
Societe Generale SA	France	0.302	0.279	0.298	0.329	0.175	0.133	0.170	0.215
Aviva PLC	UK	0.295	0.264	0.295	0.321	0.162	0.111	0.156	0.200
Barclays Bank PLC	UK	0.299	0.265	0.305	0.329	0.173	0.133	0.170	0.207
HSBC Bank PLC	UK	0.281	0.250	0.277	0.310	0.161	0.119	0.156	0.200
Lloyds Bank PLC	UK	0.296	0.267	0.295	0.324	0.159	0.119	0.148	0.193
The Royal Bank of Scotland PLC	UK	0.295	0.270	0.292	0.320	0.168	0.126	0.163	0.207
Assicurazioni Generali SpA	Italy	0.293	0.262	0.299	0.321	0.165	0.126	0.163	0.200
Intesa Sanpaolo SpA	Italy	0.309	0.284	0.302	0.338	0.180	0.133	0.170	0.215
Mediobanca SpA	Italy	0.196	0.173	0.197	0.215	0.108	0.067	0.096	0.141
UniCredit SpA	Italy	0.310	0.279	0.304	0.336	0.185	0.148	0.178	0.237
Banco Bilbao Vizcaya Argentaria SA	Spain	0.304	0.277	0.304	0.327	0.174	0.133	0.163	0.215
Banco Santander SA	Spain	0.302	0.273	0.301	0.332	0.176	0.126	0.178	0.215
Credit Suisse Group AG	Switzerland	0.275	0.252	0.274	0.301	0.149	0.104	0.141	0.178
Swiss Reinsurance Co Ltd	Switzerland	0.290	0.264	0.292	0.316	0.164	0.104	0.156	0.215
UBS AG	Switzerland	0.284	0.255	0.283	0.316	0.161	0.111	0.141	0.207
Zurich Insurance Co Ltd	Switzerland	0.293	0.268	0.290	0.324	0.166	0.119	0.156	0.200
Aegon NV	Netherlands	0.293	0.267	0.293	0.323	0.165	0.126	0.156	0.207
Rabobank	Netherlands	0.266	0.240	0.273	0.298	0.136	0.096	0.133	0.170
ING Bank NV	Netherlands	0.273	0.241	0.273	0.298	0.153	0.111	0.141	0.185

Table B.22: Estimated EPD for every energy company from heterogeneous dependence model.

Company	Nationality	$\alpha = 0.95$				$\alpha = 0.99$			
		Mean	25%	Median	75%	Mean	25%	Median	75%
E.ON SE	Germany	0.245	0.224	0.242	0.269	0.143	0.111	0.133	0.178
EnBW Energie Baden Wuerttemberg AG	Germany	0.226	0.200	0.224	0.253	0.129	0.100	0.122	0.156
RWE AG	Germany	0.235	0.213	0.236	0.258	0.137	0.100	0.133	0.167
Electricite de France SA	France	0.241	0.220	0.236	0.264	0.138	0.100	0.133	0.167
Engie	France	0.244	0.222	0.244	0.267	0.139	0.111	0.133	0.167
TOTAL SA	France	0.208	0.187	0.204	0.229	0.115	0.089	0.100	0.144
Veolia Environnement SA	France	0.245	0.220	0.247	0.273	0.149	0.111	0.144	0.178
BP PLC	UK	0.206	0.182	0.204	0.229	0.116	0.089	0.111	0.144
National Grid PLC	UK	0.245	0.220	0.242	0.271	0.134	0.100	0.133	0.167
Royal Dutch Shell PLC	UK	0.154	0.136	0.153	0.169	0.090	0.067	0.078	0.100
Enel SpA	Italy	0.250	0.227	0.249	0.271	0.144	0.111	0.133	0.178
Eni SpA	Italy	0.228	0.209	0.229	0.247	0.129	0.089	0.122	0.156
Gas Natural SDG SA	Spain	0.248	0.227	0.247	0.271	0.145	0.111	0.133	0.178
Iberdrola SA	Spain	0.252	0.231	0.253	0.267	0.145	0.111	0.144	0.167
Repsol SA	Spain	0.242	0.222	0.240	0.262	0.138	0.100	0.144	0.167
Fortum OYJ	Northern Europe	0.232	0.204	0.231	0.258	0.130	0.100	0.122	0.156
Statoil ASA	Northern Europe	0.188	0.164	0.187	0.207	0.101	0.067	0.089	0.122
Vattenfall AB	Northern Europe	0.227	0.204	0.227	0.244	0.126	0.089	0.122	0.144

*Appendix B.3. Estimation results - Block equidependence model*

Table B.23: Estimated  $\lambda$  parameters from block equidependence model in financial sector

Nationality	Mean	5%	25%	Median	75%	95%
Germany	1.47	1.04	1.28	1.47	1.65	1.92
France	1.61	1.18	1.41	1.59	1.77	2.11
UK	1.54	1.12	1.36	1.52	1.70	2.01
Italy	1.42	1.06	1.23	1.40	1.59	1.87
Spain	1.59	1.29	1.45	1.59	1.72	1.89
Switzerland	1.45	1.05	1.29	1.44	1.59	1.84
Netherlands	1.37	1.05	1.24	1.36	1.49	1.69

Table B.24: Estimated  $\lambda$  parameters from block equidependence model in energy sector

Nationality	Mean	5%	25%	Median	75%	95%
Germany	1.05	0.77	0.90	1.03	1.18	1.43
France	1.11	0.75	0.93	1.08	1.25	1.56
UK	0.68	0.54	0.61	0.67	0.73	0.86
Italy	1.23	0.91	1.08	1.20	1.35	1.62
Spain	1.26	0.83	1.04	1.23	1.43	1.82
Northern Europe	0.98	0.72	0.85	0.95	1.08	1.32

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