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Zigraiova, Diana; Jakubik, Petr

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Updating the Long Term Rate in Time: A Possible Approach

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Institute of Economic Studies, Faculty of Social Sciences
Charles University in Prague

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Institute of Economic Studies,
Faculty of Social Sciences,
Charles University in Prague

[UK FSV – IES]

Opletalova 26
CZ-110 00, Prague
E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

Institut ekonomických studií
Fakulta sociálních věd
Univerzita Karlova v Praze

Opletalova 26
110 00 Praha 1

E-mail : ies@fsv.cuni.cz
http://ies.fsv.cuni.cz

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Updating the Long Term Rate in Time: A Possible Approach

Diana Zigraiova[a,b]
Petr Jakubik[a,c]

[a]Institute of Economic Studies, Faculty of Social Sciences, Charles University in Prague, Smetanovo nabrezi 6, 111 01 Prague 1, Czech Republic
[b]Czech National Bank, Na prikope 28, 115 03 Prague 1, Czech Republic
[b] European Insurance and Occupational Pensions Authority (EIOPA)
Email (corresponding author): diana.zigraiova@fsv.cuni.cz

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Abstract:
This study proposes the potential methodological approach to be utilized by regulators when setting up a Long-Term Rate (LTR) for the evaluation of insurers' liabilities beyond the last liquid point observable in the market. Our approach is based on the optimization of two contradictory aspects – stability and accuracy implied by economic fundamentals. We use U.S. Treasury term structure data over the period 1985-2015 to calibrate an algorithm that dynamically revises LTR based on the distance between the value implied by long-term growth of economic fundamentals in a given year and the regulatory value of LTR valid in a year prior. We employ both Nelson-Siegel and Svensson models to extrapolate yields over maturities of 21-30 years employing the selected value of the LTR and compare them to the observed yields using mean square error statistic. Furthermore, we optimise the parameter of the proposed LTR formula by minimising the defined loss function capturing both mentioned factors.

Keywords: Long term rate, Nelson-Siegel, Svensson, Term structure of interest rates, Extrapolation
JEL: E43, G22, L51, M21

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Introduction

The aim of this paper is to propose the methodological framework on updating the Long-Term Rate (LTR) based on the regulator's preference between stability and accuracy reflecting a theoretical value. By defining a quantitative definition on these two criteria, regulators would obtain a clear simple rule when updating the regulatory LTR value. As interest rates on investment instruments with very long maturities cannot be typically observed in the market, Long-Term Rate (LTR) is essential for valuation of long-term commitments of insurers.

The current low interest rate environment poses two types of risk for insurance companies (e.g. EIOPA Financial Stability Report, 2013). First, cashflow risks arise from a narrowing yield spread, as new premiums and returns on maturing investment are reinvested at lower yields relative to the yields that insurers have committed to pay. The available margin on this business is thus gradually eroded by a low yield environment if no action is taken to alter the underlying position. Second, valuation risks are linked to the calculation of present values of assets and liabilities of insurance companies. Under low interest rates, a decline in benchmark interest rates will be also reflected in the discount rate applied to liabilities. The fact that the duration of liabilities is typically greater than that of assets for life insurers in particular leads to the erosion of available net assets, because the present value of liabilities would increase more than that of assets. Consequently, insolvency risks of insurances are exacerbated.

At present, the LTR used for discounting insurers' long-term liabilities is not universal across countries. For instance, the European Insurance and Occupational Pensions Authority (EIOPA) recommends in its Technical Specification for the Preparatory Phase of Solvency II (2014) that the LTR (called UFR - ultimate forward rate) is set to 4.2 per cent until the end of 2016. In this specification, LTR is defined as a function of long-term expectations of the inflation rate, and of the long-term average of short-term real interest rates. Furthermore, variations in the recommended LTR are arranged for countries with different inflation expectations (EIOPA, March 2016). The LTR can either take the value of 3.2 per cent for currencies with low inflation expectations (Swiss Franc, Japanese Yen), or 4.2 per cent for EEA currencies and those non-EEA currencies that are not explicitly mentioned in any other category, or 5.2 per cent for Brazilian, Indian, Mexican, Turkish and South African currencies, for which inflation expectations are higher. In contrast, some national supervisors decided to implement their own LTR methodologies in the domestic financial markets. In this

spirit, the Swiss Financial Market Supervisory Authority (FINMA) implemented in July 2015 the LTR of 3.9 per cent while at the same time the Dutch National Bank adjusted the LTR for the Dutch pension sector. In its 2015 field testing package for the insurance capital standard, the International Association of Insurance Supervisors (IAIS) chose to apply the LTR equal to 3.5 per cent (EIOPA, April 2016). The EIOPA's LTR framework is, however, currently undergoing revisions. The new methodology for the calculation of the LTR on an ongoing basis is expected to be implemented in 2017 (EIOPA, April 2016).

With regard to how frequently the LTR should be revised, we propose in this paper a quantitative approach that reflects on two contradictory aspects – the LTR stability in time versus its distance from the derived theoretical benchmark value based on the economic fundamentals.

A Brief Literature Review

The low yield environment resulting from monetary policies followed by European central banks poses at present the most prominent risk to the insurance sector. Despite the fact that such policies contributed to financial stability in the short term (IMF Global Financial Stability Report, 2013), lower yields on corporate and sovereign bonds in many European countries have unfavourable implications for insurer companies' profitability, solvency and sustainability (EIOPA, June 2016).

Overall, insurance companies are seen as a relatively stable segment of the financial system. However, over time their interaction with other agents in the financial system, such as banks or pension funds, has intensified. The negative spill-overs and risk of bi-directional contagion led to an increased acknowledgement of the importance of the insurance sector for the overall financial stability (e.g. Bakk-Simon et al., 2012). This interconnectedness and the size of insurance segment make insurance firms important from a financial stability point of view and lay ground for further research in this area.

In terms of performance of insurance companies, there are several papers focusing on modelling their profitability. In line with research on drivers of bank profitability (e.g. Staikouras and Wood, 2004; Macit, 2012; Ameur and Mhiri, 2013, Goddard, Molyneux, and Wilson, 2004), Christophersen and Jakubik (2014) revealed a strong link between insurance companies' premiums, on one side, and economic growth and unemployment on the other side. Similarly, Nissim (2010) argues that the overall economic activity affects insurance carriers' growth, because the demand for their

products is affected by the available income. Moreover, Nissim underlines that investment income is highly sensitive to interest rates, both in the short and in the long run. D'Arcy and Gorvett (2000) argue that inflation heavily affects the liability side of property-liability insurers' balance sheets. As for insurer insolvencies, Browne et al. (1999) find a positive correlation between the number of insurers in the life-insurance industry, unemployment and stock market returns on one side and life-insurers' insolvency on the other side. Similarly, failure rate of property-liability insurers was also found to be positively correlated with the number of insurers in the industry (Browne and Hoyt, 1995).

Since interest rates were shown to affect income and profitability of insurance companies in previous research studies, we propose to further investigate in this paper the optimal time for revision of long-term interest rate used for discounting of insurance firms' long-term commitments which has substantial valuation implications.

The paper is organized as follows. Section 2 presents the term structure data used in our analysis, section 3 describes the methodology applied to the LTR setting, section 4 presents our results, section 5 discusses implications for insurance companies linked to LTR changes, and section 6 concludes.

Data

In our analysis we use the U.S. Treasury term structure data collected by Gurkaynak et al. (2006). The advantages of using U.S. data as opposed to European data stem from the availability of long historical time series of yield curves with maturities up to 30 years. The data set is compiled on a daily basis, with the first entry in 1961 and is being regularly updated. This data set includes all U.S. Treasury bonds and notes with the exception of the following:

i. Securities with option-like features, i.e. callable bonds or flower bonds.

ii. Securities with less than three months to maturity due to a specific behaviour of yields on securities with such short residual maturities.

iii. Treasury bills that seem to be affected by segmented demand from money market funds and other short-term investors (Duffee, 1996).

iv. Twenty-year bonds in 1996 owing to their cheapness relative to ten-year notes of comparable duration.

v. Securities with maturities of two, three, four, five, seven, ten, twenty and thirty years issued in 1980 or later owing to the fact that they trade at a

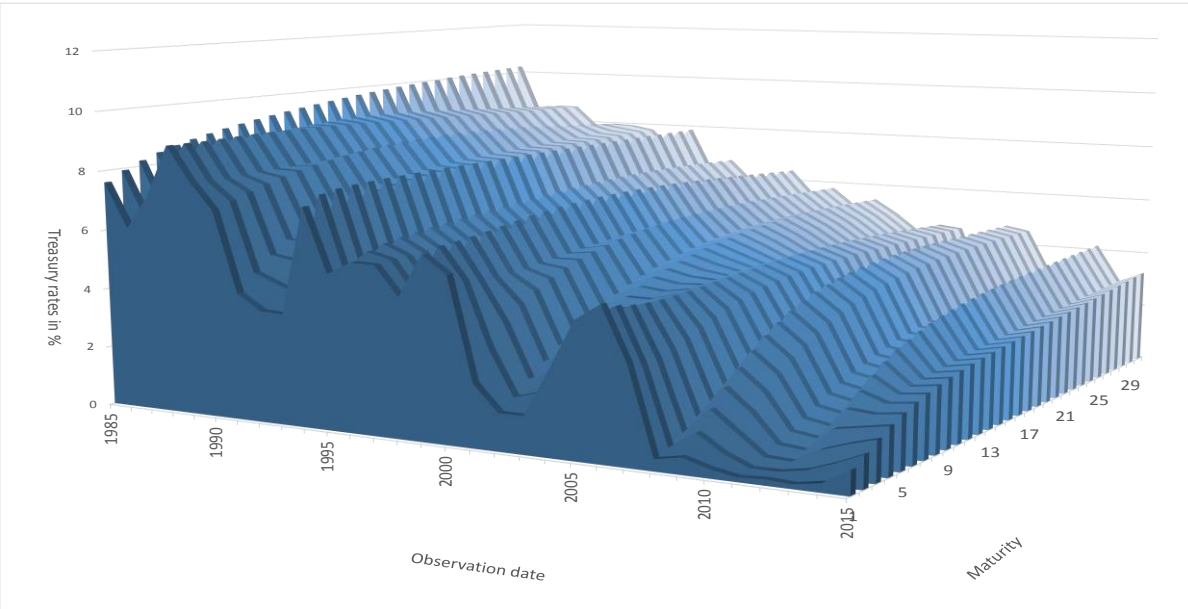
premium to other treasury securities given their greater liquidity in the repo market.

vi. Securities excluded on an ad hoc basis to deal with other data issues.

All in all, the treasury yield curve provided in this data set is estimated in a way that liquidity of the included securities is adequate and relatively uniform.

For the purposes of our analysis we extract from the data set by Gurkaynak et al. (2006) one yield curve per year from the 1985-2015 period. We opt for the last available yield curve in each calendar year, usually from December 31. Thus, our sample consists of 31 yield curves altogether. The starting date of our observed time period is conditional on the availability of Treasury zero coupon rates with maturities up to 30 years. In the data set by Gurkaynak et al. (2006) 30 years is the maximum available maturity for U.S. securities and the first year when a yield curve with this maturity becomes available is 1985 which also marks the start of our sample. Figure 1 shows the full U.S. Treasury term structure for the 1985-2015 period and maturities 1 to 30.

Figure 1: Term structure (1985-2015)



Note: X-axis shows maturities of U.S. Treasury securities, y- axis indicates the period of observation and z-axis depicts Treasury zero rates in per cent

Next, we use the historical yield curve data for the 1985-2015 period extracted from the data set by Gurkaynak et al. (2006) to calibrate a simple framework for setting up the simple rule when to revise the Long-Term Rate (LTR).

Methodology

In this section we present a framework for setting up the LTR and for providing a LTR revision mechanism using a benchmark value for the long-term rate that reflects economic conditions in the long run using extrapolation of the term structure based on two different models.

Setting the Long Term Rate

EIOPA's Technical Specification for the Preparatory Phase of Solvency II (2014) defines the LTR as the sum of the long-term average of short-term real interest rates and long-term expectations of the inflation rate, usually captured by the central bank's inflation target.

In our framework we set the benchmark for the LTR equal to the average growth of nominal U.S. GDP over the previous twenty years.[2] Hence, the benchmark for LTR reflects average long-term growth of real GDP and inflation in the U.S. We obtain the data from the Federal Reserve Bank of St. Louis and use Equation 1 to calculate the average twenty-year growth rate of nominal GDP for each year in the 1985-2015 period:

$$g_t = \left(\frac{GDP_t}{GDP_{t-20}}\right)^{1/20} - 1, \tag{1}$$

where g is the average long-term growth rate and t indicates year from the 1985-2015 period.[3]

Next, we set the initial regulatory LTR equal to the average growth rate of nominal U.S. GDP over the previous twenty years in 1984 using Equation 1. Subsequently, we calculate UFR_t for every year over the 1985-2015 period using the following equation:

$$LTR_t = f(g_t, LTR_{t-1}) + LTR_{t-1}$$
$$f(g_t, LTR_{t-1}) = \begin{Bmatrix} g_t - LTR_{t-1} & if & |g_t - LTR_{t-1}| > p \\ 0 & if & |g_t - LTR_{t-1}| \leq p \end{Bmatrix}, \tag{2}$$

where g_t is obtained from Equation 1, t indicates a year from 1985 to 2015 and p is the distance between the long-term growth rate of nominal U.S. GDP at time t and LTR from time t-1. Equation 2 thus resets LTR at time t if the distance between the

[2] There are many alternative ways to set up the benchmark for the LTR. However, the aim of this paper is to set up a framework providing a rule on the LTR revision rather than proposing the regulatory value.

[3] We opt for twenty-year average to capture the whole economic cycle and not being substantial affected by technological changes.

long-term nominal GDP growth at time t and regulatory LTR from the previous period t-1 exceeds the value given by p. As we prefer to express g_t in percentages in our analysis, the values we assign are also in percentages. Hence, p takes values of 0.1%, 0.2%, 0.3%, 0.4%, up to 3.5% and we calculate the LTR in each year of the 1985-2015 period for every assigned value of p from Equation 2.

Extrapolation of Yield Curves

The next step in our framework for setting up the LTR and its optimal adjustment frequency is extrapolation of zero rates on U.S. Treasury securities for maturities beyond twenty years. Given that EIOPA Technical Standards (2016) set the last liquid point (LLP), i.e. the maturity up to which yields on securities are quoted on the market, to 20 years, we also adopt this definition and extrapolate yields on securities with maturities from 21 to 30 years, i.e. the maximum maturity available in the data set provided by Gurkaynak et al. (2006) from 1985.

For extrapolation we use the models by Nelson and Siegel (1987) and its extension by Svensson (1994) that are frequently employed by central banks and other market participants (e.g. BIS, 2005) to fit term structures of interest rates. Furthermore, the studies by Diebold and Li (2006) and De Pooter, Ravazzolo and van Dijk (2007) provide evidence that these models are a useful tool in forecasting exercises of term structures of interest rates.

Despite these advantages, Bjork and Christensen (1999) showed that the Nelson-Siegel model is not theoretically arbitrage-free, i.e. theoretical prices of securities resulting from the model and the actual prices observed on the market differ to such an extent that transaction costs do not prevent arbitrage. Since this condition between theoretical and observed prices is not hard-coded into the model, it was assumed that the model violates no-arbitrage condition. However, Coroneo et al. (2011) show on U.S. yield curve data from 1970 until 2000 that the Nelson-Siegel model is statistically arbitrage-free. In this sense, another popular model, Smith-Wilson (2001) model used by EIOPA to extrapolate the yield curve for very long maturities, is arbitrage-free as it fits the yield curve exactly up to LLP.

The Nelson-Siegel (1987) model models the yield curve at a point in time as follows:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right] + \beta_3 \left[\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right], \tag{3}$$

where $y(\tau)$ is the zero rate for maturity τ, parameters β_1, β_2, β_3 and λ need to be estimated. β_1 is independent of the time to maturity and as such indicates the long-term yield; β_2 exponentially decays to zero with increasing τ, thus it only influences the short end of the yield curve. β_3 function first increases then decreases with increasing τ which adds a hump to the yield curve.

The Svensson (1994) model extends the Nelson-Siegel (1987) model by adding a second hump to the yield curve:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1-\exp(-\tau/\lambda_1)}{\tau/\lambda_1}\right] + \beta_3 \left[\frac{1-\exp(-\tau/\lambda_1)}{\tau/\lambda_1} - \exp(-\tau/\lambda_1)\right] + \beta_4 \left[\frac{1-\exp(-\tau/\lambda_2)}{\tau/\lambda_2} - \exp(-\tau/\lambda_2)\right], \quad (4)$$

where $y(\tau)$ is again zero rate for maturity τ and six parameters, β_1, β_2, β_3, β_4, λ_1 and λ_2 need to be estimated. This model is able to better capture the shape of the yield curve as it allows for a second hump that usually occurs at long maturities (i.e. twenty years and more). The occurrence of the second hump can be attributed to convexity which pulls down the yields on long-term securities and as a consequence makes the yield curve's shape concave at long maturities.

In order to extrapolate U.S. Treasury yield curves for maturities 21-30 we use the R-project package "ycinterextra" by Moudiki (2013). The package allows us to extrapolate the term structure using the LTR calculated from Equation 2 for every yield curve over the 1985-2015 period and for every value of p. We thus extrapolate U.S. Treasury yields for maturities 21 to 30 using both, Nelson-Siegel and Svensson model.

Construction of a Loss Function

The last step in constructing our framework is to join the LTR setting and extrapolation of yields using the two yield curve models into a single statistic for each value of p. In particular, we take into account how stable the LTR set in the previous subsection is over the entire observed time period and how close the extrapolated yields using that particular LTR are to the actual yields at maturities 21-30. We call this aggregate statistic a loss function as it penalizes frequent changes in LTR setting and the distance of extrapolated yields from actual yields at maturities 21-30. We

calculate the loss function for every value of p, which expresses the distance between the average long-term growth of nominal GDP and the regulatory LTR from the previous period, over the 1985-2015 period.

Our proposed loss function has the following form:

$$Loss_p = w_{Prec} \times MSE_p + w_{Stab} \times \left(\min_{t \epsilon T}(MSE_{p,t}) + k \times \left(\max_{t \epsilon T}(MSE_{p,t}) - \min_{t \epsilon T}(MSE_{p,t}) \right) \right) \quad (5)$$

$$T = \langle 1985; 2015 \rangle,$$

where T is the set of the observed time period, p is the distance between long-term growth rate of nominal U.S. GDP at time t and LTR from time t-1, k is the number of LTR changes over the total number of years in the observed period of 1985-2015 for the corresponding value of p, and w_{Prec}, w_{Stab} are the weights of the two loss function components. They can take values from 0 to 1 and express a regulator's preference towards either extrapolation precision or LTR stability. It needs to hold that $w_{Prec} + w_{Stab} = 1$. Therefore, the weights set to 0.5 would indicate there is no preference towards either precision of extrapolation or LTR stability as the two components are weighed equally in the loss function. MSE_t, mean square error, is a standard statistical concept that measures the average of the squares of the errors between the yields at maturities 21-30 obtained from extrapolation using Nelson-Siegel and Svensson models for the chosen regulatory LTR, and the actual yields at these maturities. For each value of p we calculate the corresponding average mean square error MSE_p over the observed time period defined as follows:

$$MSE_p = \frac{1}{31} \times \sum_{t=1985}^{2015} MSE_t = \frac{1}{31} \times \frac{1}{10} \times \sum_{t=1985}^{2015} \sum_{i=21}^{30} \left(\widehat{y_{i,t}} - y_{i,t} \right)^2, \quad (6)$$

where i takes values of maturities 21 to 30, t indicates a year in the 1985-2015 period and $\widehat{y_{i,t}}$ stands for an estimate of the yield at maturity i and year t obtained by extrapolation from either Nelson-Siegel or Svensson model while $y_{i,t}$ is the actual yield at maturity i in year t.

As for the second component of the loss function, the LTR stability over the observed period, we approximate it with the ratio of the number of LTR changes for the corresponding p over the number of years in the period 1985-2015, i.e. 31 years. We also rescale this ratio to correspond numerically to the first component of the loss function MSE_p, as shown in Equation 5.

We are interested in the value of p that minimizes loss for the 1985-2015 period for the regulator's preferences towards extrapolation precision and LTR stability. Such a

value of p would reveal by how much the long-term nominal GDP growth rate in a given year should deviate from the regulatory LTR from the previous year to have the LTR reset to the value given by Equation 2. The loss-minimizing value of p depends on a regulator's preference towards either precision or LTR stability.

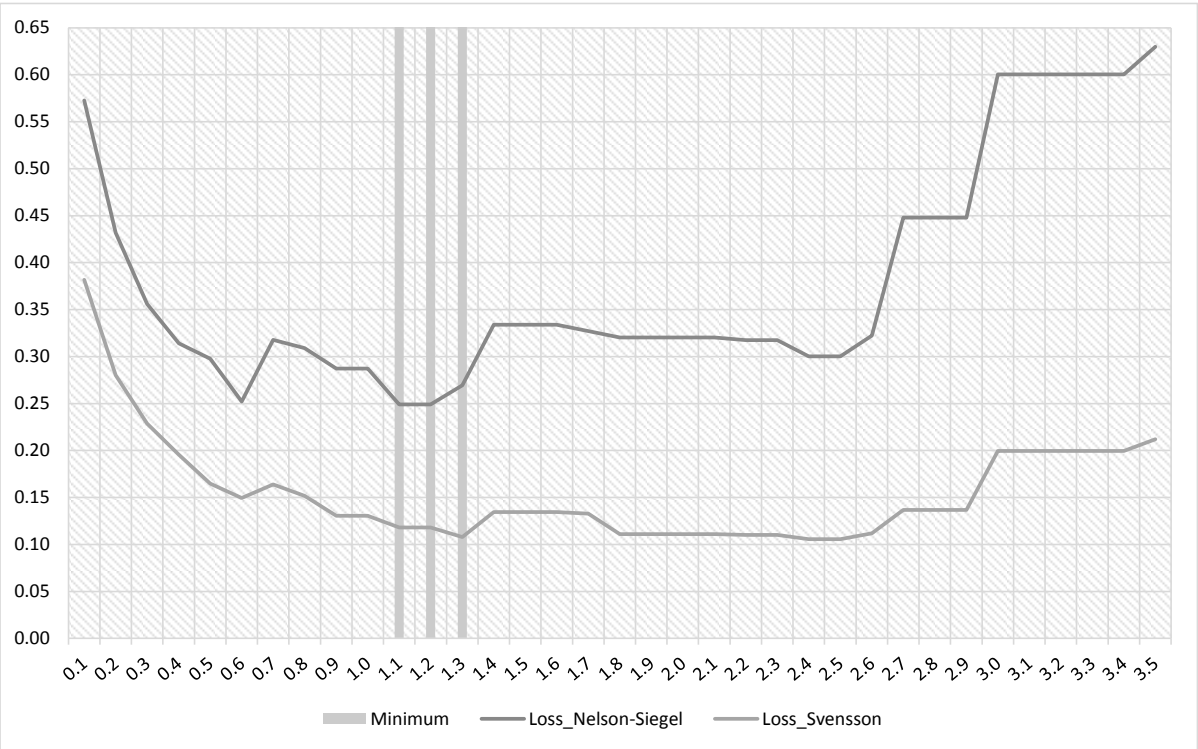
Table A1 presenting LTRs calculated in each year over the 1985-2015 period for the values of p that minimize loss under different regulator's preferences is available in the Appendix. The next section presents the results of calculation of loss for the overall period across different values of p, using Nelson-Siegel and Svensson models and different regulator's preferences.

Results

In this section we present the results of the loss calculation over the 1985-2015 period and different values of p using both, Nelson-Siegel and Svensson model, and different preferences, i.e. weighting schemes.

First, we assume that a regulator places equal weight on LTR stability and extrapolation precision. In this case, the following condition holds for the weights in Equation 5: $w_{Prec} = w_{Stab} = 0.5$.

Figure 2: Loss for Different Values of p (weights: 0.50, 0.50)

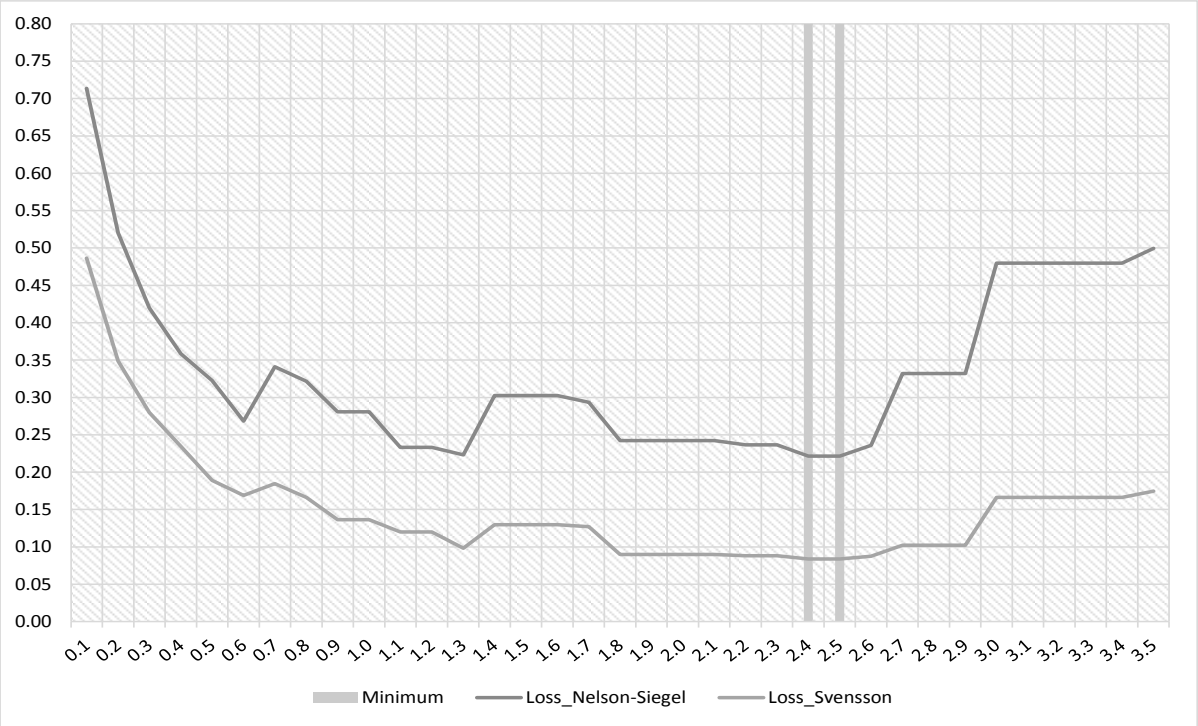


Note: The dark grey line shows loss over the 1985-2015 period for different values of p (on horizontal axis) calculated from Nelson-Siegel model while the light grey line depicts the loss from Svensson model over the same period. The light grey bars highlight those values of p that minimize the loss function for both models. The vertical axis indicates

magnitude of loss. The calculation uses equal weighting.

We can observe from the Figure 2 that the value of p equal to both 1.1% and 1.2% minimizes loss over the 1985-2015 period when yields are extrapolated using Nelson-Siegel model. For Svensson, the loss minimizing value of p equals to 1.3%. Next, we turn to alternative weighting schemes in case that a regulator considers either stability of LTR overtime more important than how closely a model can extrapolate long-term yields to their actual values (yields on Treasury securities for maturities of 21-30), and vice versa.

Figure 3: Loss for Different Values of p (weights: 0.33, 0.67)



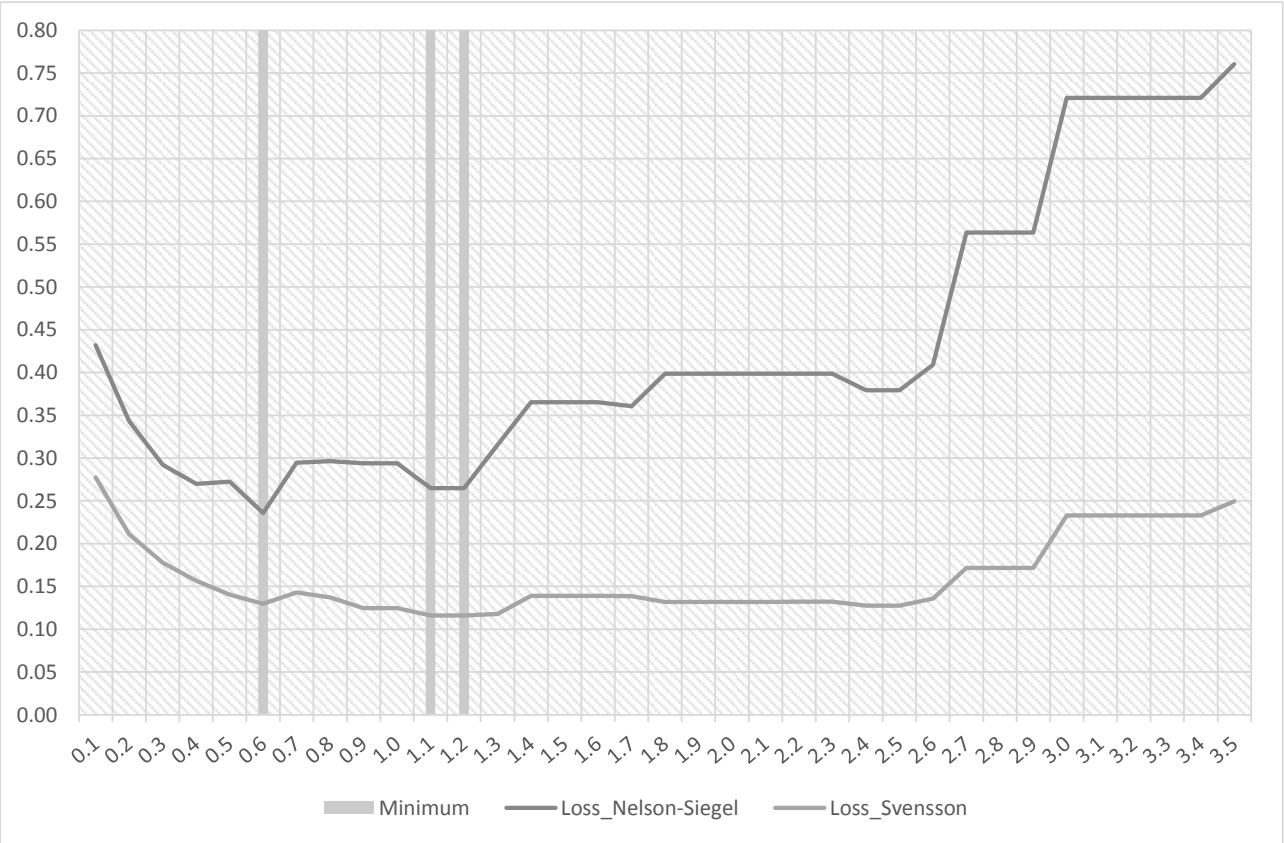
Note: The dark grey line shows loss over the 1985-2015 period for different values of p (on horizontal axis) calculated from Nelson-Siegel model while the light grey line depicts the loss from Svensson model over the same period. The light grey bars highlight those values of p that minimize the loss function for both models. The vertical axis indicates magnitude of loss. The weight of 33% is placed on extrapolation precision while 67% is placed on LTR stability

The Figure 3 shows the loss minimizing value of p when weight of 33% is put on extrapolation precision and double of that is placed on LTR stability increases to 2.4% and 2.5% which is approximately double of the value of p that minimizes loss under equal weighting. All in all, the loss is minimized at p=2.4% and p=2.5% for both models under the given preferences.

Next, we choose to favour extrapolation precision over LTR stability in our calculation. We put weight of 67% on the first component of the loss function in Equation 5 and half of that weight on how stable LTR is in time.

In this case, the loss minimizing value of p drops to 0.6% when extrapolation is performed using Nelson-Siegel model. As for Svensson, the loss minimizing p equals to 1.1% and 1.2% under these preferences, which is quite close to the optimal value of p under equal weighting. Figure 4 presents the results.

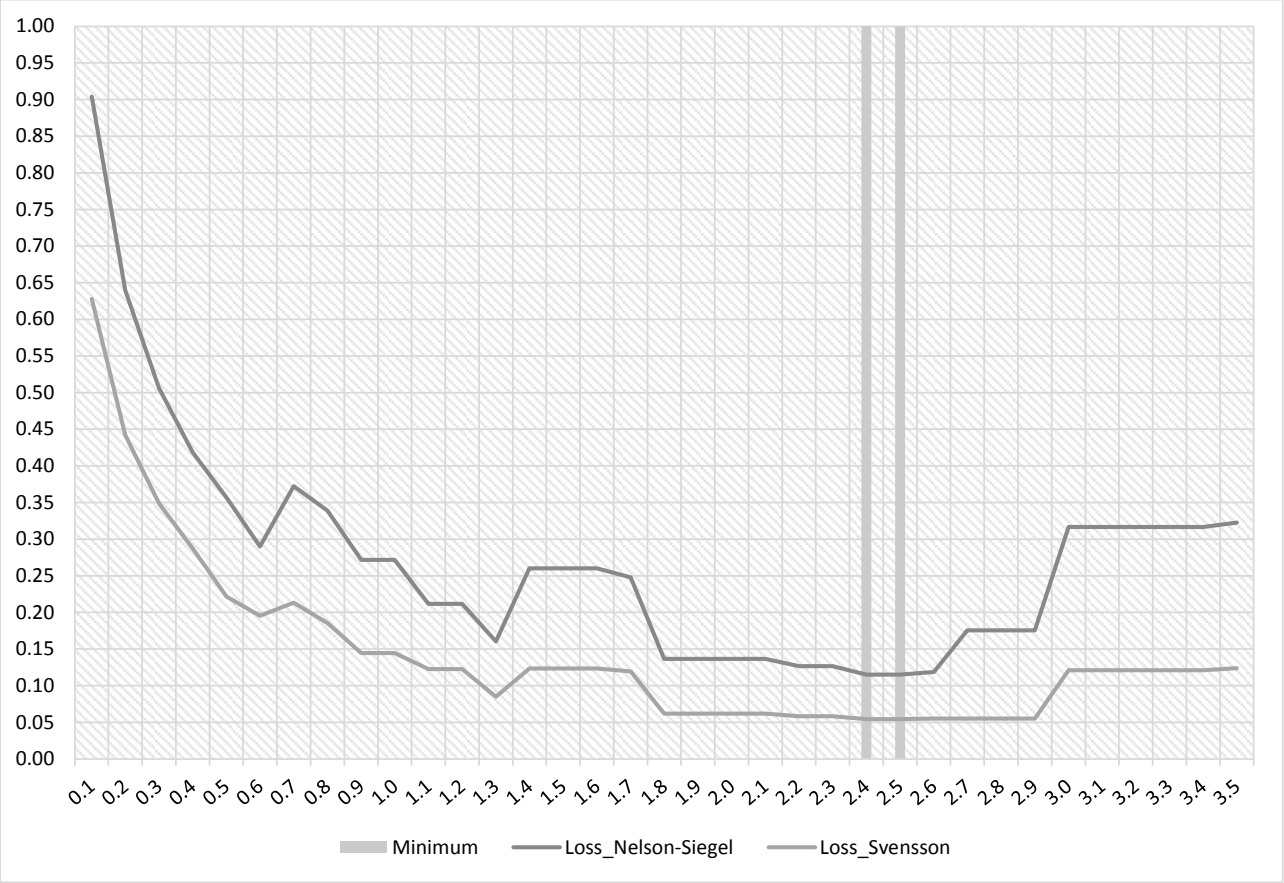
Figure 4: Loss for Different Values of p (weights: 0.67, 0.33)



Note: The dark grey line shows loss over the 1985-2015 period for different values of p (on horizontal axis) calculated from Nelson-Siegel model while the light grey line depicts the loss from Svensson model over the same period. The light grey bars highlight those values of p that minimize the loss function for both models. The vertical axis indicates magnitude of loss. The weight of 67% is placed on extrapolation precision while 33% is placed on LTR stability.

For the last two weighting schemes, we suppose that a regulator cares very little about one component of the loss function, either MSE or LTR stability, while the other aspect is found to be crucial. Figure 5 presents the results.

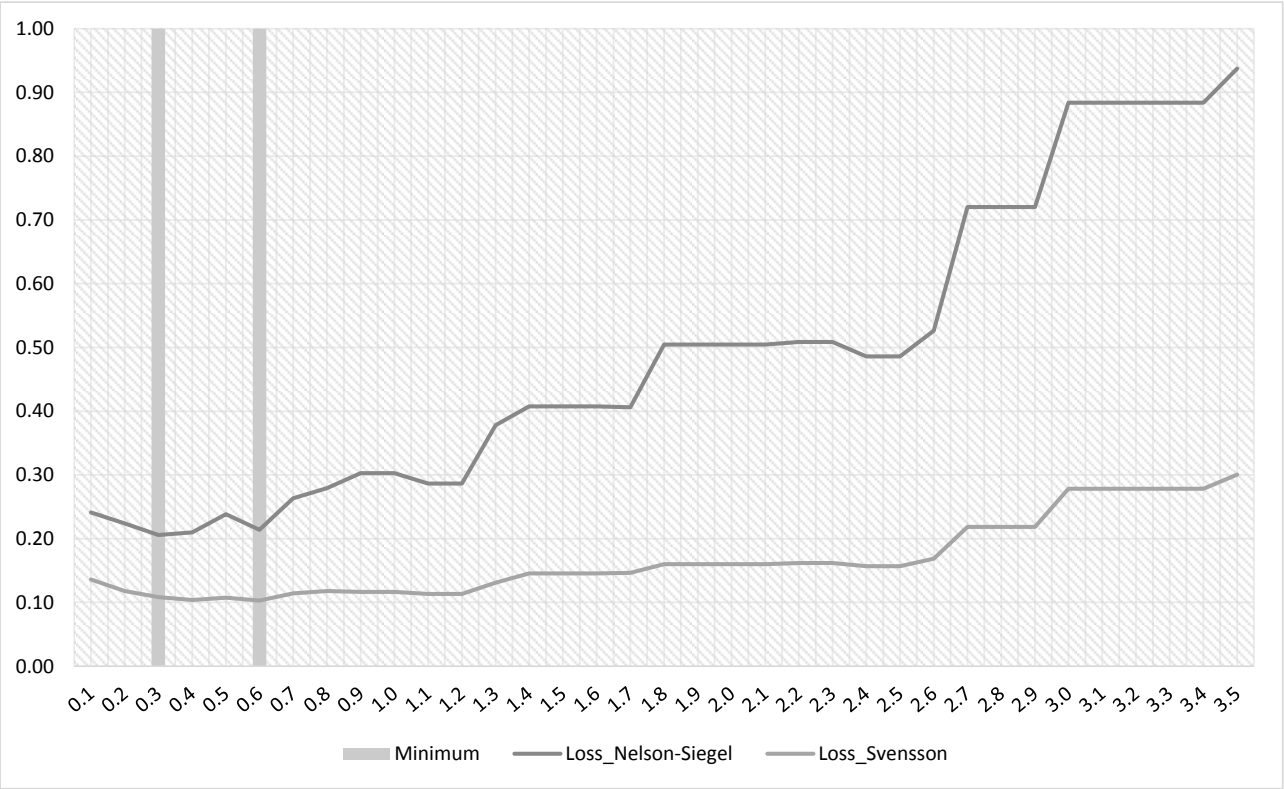
Figure 5: Loss for Different Values of p (weights: 0.10, 0.90)



Note: The dark grey line shows loss over the 1985-2015 period for different values of p (on horizontal axis) calculated from Nelson-Siegel model while the light grey line depicts the loss from Svensson model over the same period. The light grey bars highlight those values of p that minimize the loss function for both models. The vertical axis indicates magnitude of loss. The weight of 10% is placed on extrapolation precision while 90% is placed on LTR stability.

The Figure 5 presents the loss minimizing values of p when the weight of only 10% is placed on extrapolation precision as opposed to the weight of 90% put on stability of LTR. For yield extrapolation by both Nelson-Siegel and Svensson model the optimal value of p is equal to 2.4% and 2.5%, which is the same as under the weighing scheme of 33% placed on extrapolation precision and 67% placed on LTR stability.

Figure 6: Loss for Different Values of p (weights: 0.90, 0.10)



The dark grey line shows loss over the 1985-2015 period for different values of p (on horizontal axis) calculated from Nelson-Siegel model while the light grey line depicts the loss from Svensson model over the same period. The light grey bars highlight those values of p that minimize the loss function for both models. The vertical axis indicates magnitude of loss. The weight of 90% is placed on extrapolation precision while 10% is placed on LTR stability.

In case of the reversed weighing of 90% for the precision component of the loss function and 10% for LTR (Figure 6), the stability of the value of p that minimizes loss under the Nelson-Siegel extrapolation drops to 0.3% while p equal to 0.6% is optimal for Svensson model (as shown in the Figure above).

All in all, under equal regulator's preferences, it appears that if the difference between the long term rate measured by average twenty-year growth of nominal GDP at time t and LTR valid in period t-1 exceeds 1.2% when Nelson-Siegel model is used for extrapolation, LTR at time t should be adjusted to reflect long-term average growth of nominal GDP at time t. This difference slightly increases to 1.3% for Svensson model. The optimal value of p equal to 1.1% and 1.2% for Nelson-Siegel model amounts to the total of three LTR adjustments over the 1985-2015 period while the optimal p=1.3% for Svensson model implies only two adjustments, as shown in Table A1.

The loss minimizing value of p either rises or drops in response to changing regulator's preferences. With the regulator in favour of LTR stability overtime by at

least two thirds compared to the MSE component, the distance indicative of resetting LTR increases to 2.5%. On the other hand, the regulator caring very little about LTR stability would lean towards more frequent revisions of LTR. This is reflected by the optimal distance between economic fundamentals and the regulatory LTR as small as 0.3% and 0.6% under Nelson-Siegel and Svensson model, respectively.

Next, we use an insurer's hypothetical portfolio of liabilities to demonstrate valuation effects of changes in LTR.

Policy Implications

Under a low yield regime, a decline in benchmark interest rates translates into the reduced discount rate applied in an insurer's liabilities valuation overall. This in turn leads to a steeper increase in the present value of liabilities over assets, eroding an insurer's surplus and exacerbating insolvency risk of insurance entities. While actual market interest rates are applied in valuation of liabilities with short maturities, the long term interest rate is used for discounting liabilities with long maturities. In line with our assumption that LLP is set to 20 years, changes in LTR affect value of only those liabilities with maturities greater than 20 years. [4]

In this section we illustrate on long-term liabilities of different duration within a hypothetical insurer's portfolio how their present value changes in response to changes in long-term interest rate within the proposed framework. We take as the LTR benchmark value the long-term U.S. nominal GDP growth at reference year 2005. We assume LTR has been constant since then, i.e. fixed to 5.39% in 2015. We calculate alternative LTRs in 2015 from the formula given in Equation 2.

We choose those LTRs that correspond to a loss-minimizing value of p under different regulatory preferences from the previous section. Table 1 shows changes in the present value of long-term liabilities of different duration within a hypothetical portfolio given different regulatory preferences towards the LTR setting, and using both Nelson-Siegel and the Svensson model. We calculate the change in the present value of an insurance's long-term liabilities due to changes in LTR for average long-term maturities of 21, 22, 25, 28 and 30 years using the standard definition of modified duration:

$$\Delta PV_\tau = -\Delta IR_\tau \times MD \tag{7}$$

$$\tau = \{21,22,25,28,30\},$$

[4] This is in line with the EIOPA Technical Standards (March 2016).

where ΔPV_τ indicates a change in the present value of liabilities with average maturity τ, ΔIR_τ expresses change in discount rate of liabilities with average maturity τ with respect to difference between LTR setting and its benchmark, and MD stands for modified duration, i.e. the corresponding maturity bracket from the set τ.

Table 1: Impact of different regulatory preferences on the long-term liabilities within a portfolio

Nelson-Siegel Extrapolation

Preferences	benchmark	w_{Stab}=0.10	w_{Stab}=0.33	w_{Stab}=0.67
LTR value in 2015	5.39%	4.22%	4.56%	5.39%
AVERAGE modified duration of liabilities (in years)	21	10.18%	7.22%	0%
	22	11.32%	8.03%	0%
	25	14.79%	10.49%	0%
	28	18.28%	12.97%	0%
	30	20.61%	14.62%	0%

Svensson Extrapolation

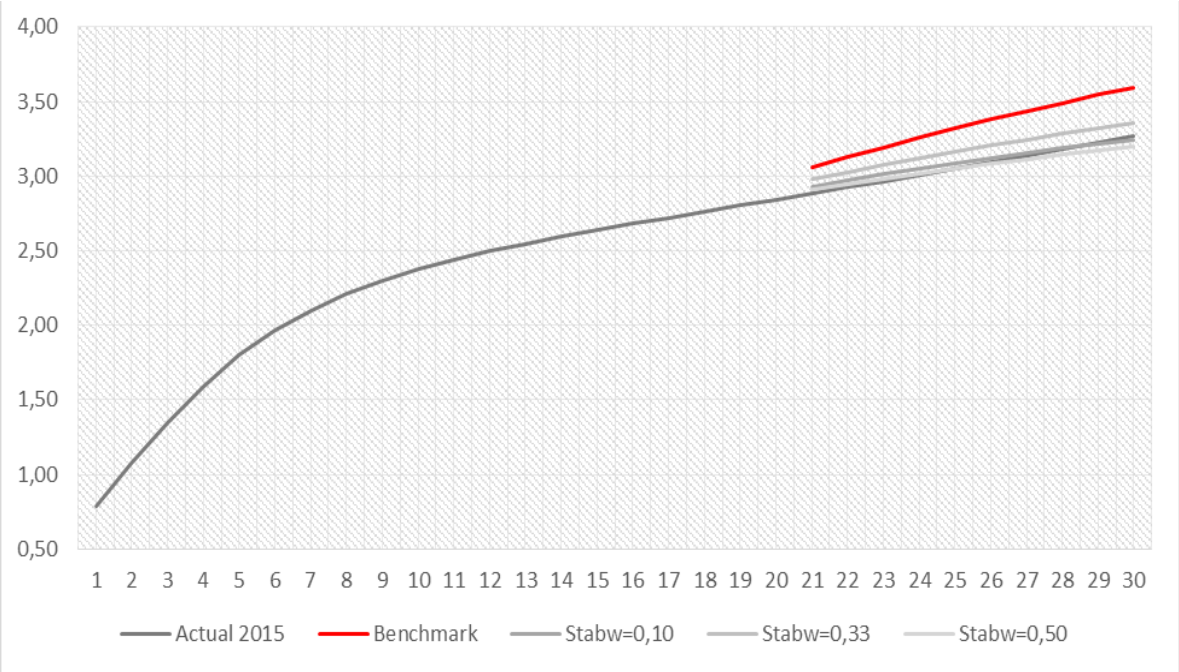
Preferences	benchmark	w_{Stab}=0.10	w_{Stab}=0.33	w_{Stab}=0.67
LTR value in 2015	5.39%	4.22%	4.56%	5.39%
AVERAGE modified duration of liabilities (in years)	21	2.72%	1.74%	0%
	22	3.42%	2.22%	0%
	25	5.82%	3.86%	0%
	28	8.55%	5.77%	0%
	30	10.52%	7.14%	0%

Note: The impact of deviations of the long-term interest rate from the benchmark given the different regulatory preferences on the present value of an insurer's long-term liabilities of different duration. The first row indicates preference of the regulator towards LTR stability. The second row states the corresponding LTR in 2015 calculated from Equation 2.

Overall, we observe a higher sensitivity of present value of liabilities with longer durations to changes in LTR. The greater the decrease in LTR with regards to the benchmark, the greater the increase in the present value of liabilities across different average durations. Therefore, for insurance firms whose portfolio consists of very long-term liabilities, such as life-insurers, a relatively small decline in the discount

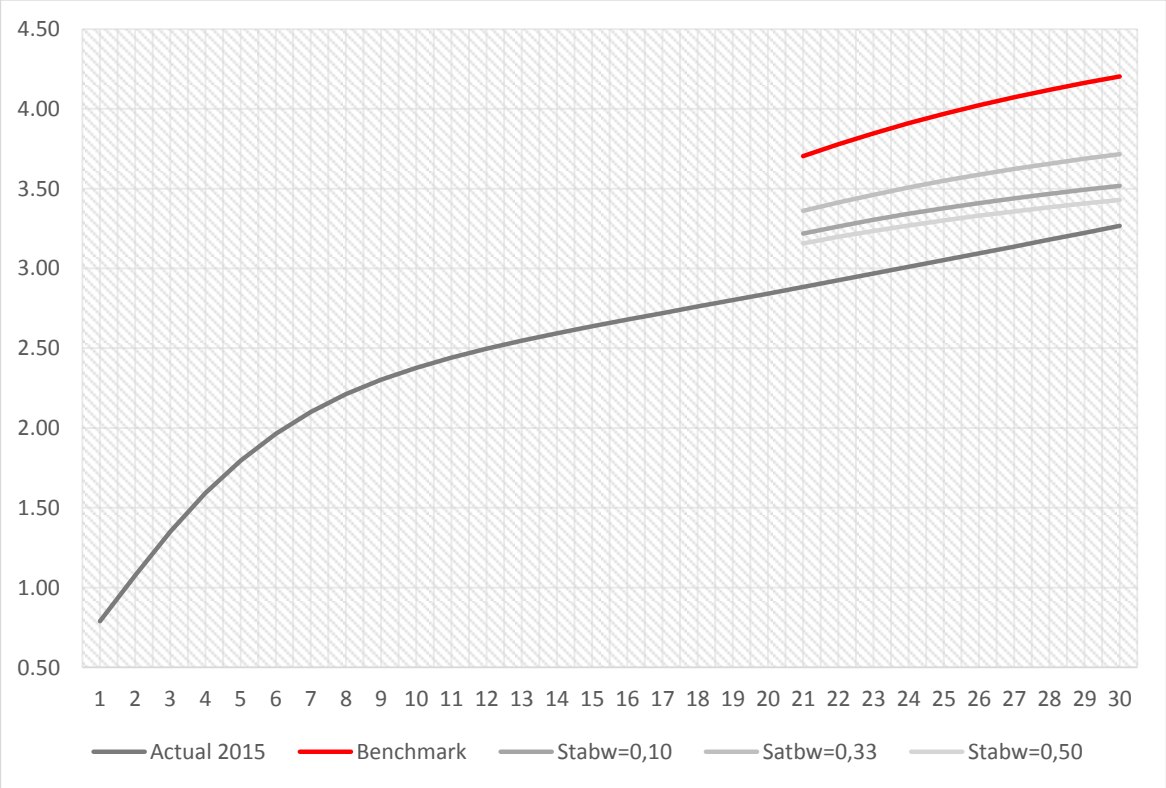
rate of -0.83% to LTR=4.56% would result in an increase in the value of long-term liabilities with average duration of 30 years by more than 14% under Nelson-Siegel extrapolation and 7% under Svensson. The smaller impact on present value when Svensson model is used for extrapolation can be attributed to smaller deviations of spot yields under different regulatory LTRs from spot yields under benchmark LTR compared to Nelson-Siegel model. Figures 7 and 8 depict extrapolated spot yields versus benchmark for Svensson and Nelson-Siegel model, respectively.

Figure 7: Svensson Extrapolation under different preferences



Note: The Figure shows the actual yield curve as of December 31, 2015 over maturities 1 to 30 years and extrapolated spot yields for maturities 21 to 30 years under LTR with different regulatory preferences, and benchmark using Nelson-Siegel model. Benchmark corresponds to extrapolation with LTR equal to average twenty-year U.S. nominal GDP growth in 2005. Vertical axis shows spot yields in percentages, horizontal axis indicates maturities in years.

Figure 8: Nelson-Siegel Extrapolation under different preferences



Note: The Figure shows the actual yield curve as of December 31, 2015 over maturities 1 to 30 years and extrapolated spot yields for maturities 21 to 30 years under LTR with different regulatory preferences, and benchmark using Nelson-Siegel model. Benchmark corresponds to extrapolation with LTR equal to average twenty-year U.S. nominal GDP growth in 2005. Vertical axis shows spot yields in percentages, horizontal axis indicates maturities in years.

Figures 7 and 8 show that extrapolated spot yields under regulatory LTRs with different preferences towards LTR stability are lower than the spot yields under constant LTR scenario (in red) for both models. However, the proximity of extrapolated yields under different regulatory preferences to the benchmark yields as well as to actual yields is greater for Svensson model. Therefore, under Svensson model changes in LTR affect present value of long-term liabilities in an insurer's portfolio to a lesser degree.

Conclusion

As liability side of insurer companies' balance sheets is typically formed by commitments with very long maturities. Hence, they need to be discounted by a corresponding long-term interest rate for valuation purposes. However, interest rates over very long maturities are seldom observable in the market. As a result, Long-Term Rate (LTR) needs to be estimated in order to evaluate such long-term contracts. Consequently, changes in LTR have valuation effects for insurers.

In this paper we show a possible approach for updating the interest rate for long-term contracts (LTR) in a dynamic way using long-term developments of economic fundamentals as a benchmark for LTR. In addition, our approach proposes a loss function that weighs two LTR aspects, estimation precision and LTR stability.

We propose an algorithm of LTR setting that compares by how much long-term economic fundamentals measured by average twenty-year nominal GDP growth in a given year differ from regulatory LTR from the previous year. If this difference is greater than some threshold value p LTR for this period is set to the value given by economic fundamentals. A difference smaller than the threshold makes regulatory LTR from the prior year also valid in a given year.

Next, we extrapolate yields over maturities of 21-30 years using Nelson-Siegel and Svensson models and compare them to the actual yields from U.S. Treasury term structure data over the period of 1985-2015 using mean square error (MSE) statistic.

We combine the two aspects, LTR stability (the ratio of changes in LTR over the observed period) and extrapolation precision (distance between actual and extrapolated yields) into a loss function. A preference for each component of the loss function is expressed by assigned weights.

We search for such p (distance between long-term growth of economic fundamentals and LTR set in previous period) that minimizes our proposed loss function.

Finally, we find that once the distance between average twenty-year growth of nominal GDP in a given year and regulatory LTR from the previous year exceeds 1.2% and 1.3% under preference neutrality for Nelson-Siegel and Svensson model, respectively, the LTR should be adjusted. This result changes in response to a regulator's preferences. When the preference towards LTR stability dominates, the distance for resetting LTR increases implying fewer changes to LTR over the period under investigation, and vice versa.

Finally, we illustrate the impact of changes in the long-term interest rates on insurance companies by means of a hypothetical portfolio of long-term liabilities. We show that extrapolated spot yields under regulatory LTRs with different preferences towards LTR stability are lower than the spot yields generated under the assumption of constant LTR fixed to average long-term GDP growth levels.

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Appendix

Table A1: Calculated LTR values

	p	***0.30***	***0.60***	***1.10***	***1.20***	***1.30***	***2.40***	***2.50***
Year	**Nominal GDP growth (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**	**UFR (%)**
1985	8.77	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1986	8.58	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1987	8.60	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1988	8.52	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1989	8.50	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1990	8.51	8.64	8.64	8.00	8.00	8.00	8.00	8.00
1991	8.25	8.25	8.64	8.00	8.00	8.00	8.00	8.00
1992	8.07	8.25	8.64	8.00	8.00	8.00	8.00	8.00
1993	7.77	7.77	7.77	8.00	8.00	8.00	8.00	8.00
1994	7.67	7.77	7.77	8.00	8.00	8.00	8.00	8.00
1995	7.47	7.47	7.77	8.00	8.00	8.00	8.00	8.00
1996	7.21	7.47	7.77	8.00	8.00	8.00	8.00	8.00
1997	6.98	6.98	6.98	8.00	8.00	8.00	8.00	8.00
1998	6.64	6.64	6.98	6.64	6.64	6.64	8.00	8.00
1999	6.39	6.64	6.98	6.64	6.64	6.64	8.00	8.00
2000	6.28	6.28	6.28	6.64	6.64	6.64	8.00	8.00
2001	5.86	5.86	6.28	6.64	6.64	6.64	8.00	8.00
2002	5.82	5.86	6.28	6.64	6.64	6.64	8.00	8.00
2003	5.64	5.86	5.64	6.64	6.64	6.64	8.00	8.00
2004	5.43	5.43	5.64	5.43	5.43	6.64	5.43	5.43
2005	5.39	5.43	5.64	5.43	5.43	6.64	5.43	5.43
2006	5.40	5.43	5.64	5.43	5.43	6.64	5.43	5.43

2007	5.32	5.43	5.64	5.43	5.43	5.32	5.43	5.43
2008	5.03	5.03	5.03	5.43	5.43	5.32	5.43	5.43
2009	4.56	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2010	4.47	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2011	4.49	4.56	5.03	5.43	5.43	5.32	5.43	5.43
2012	4.40	4.56	4.40	5.43	5.43	5.32	5.43	5.43
2013	4.33	4.56	4.40	5.43	5.43	5.32	5.43	5.43
2014	4.22	4.22	4.40	4.22	4.22	5.32	5.43	5.43
2015	4.07	4.22	4.40	4.22	4.22	5.32	5.43	5.43
No. of UFR changes		11	6	3	3	2	1	1

Note: Table A1 shows LTR values calculated from Equation 2 for those values of p *that minimize loss under individual preference scenarios, and for each year in the 1985-2015 period. The column "nominal GDP growth" shows the average twenty-year growth of the nominal U.S. GDP in each year and serves as the benchmark for LTR calculation in Equation 2. Table A1 also shows the frequency of LTR changes for selected values of* p *over the observed period.*

Figure A1: Average Yield Curves (1985-2015)

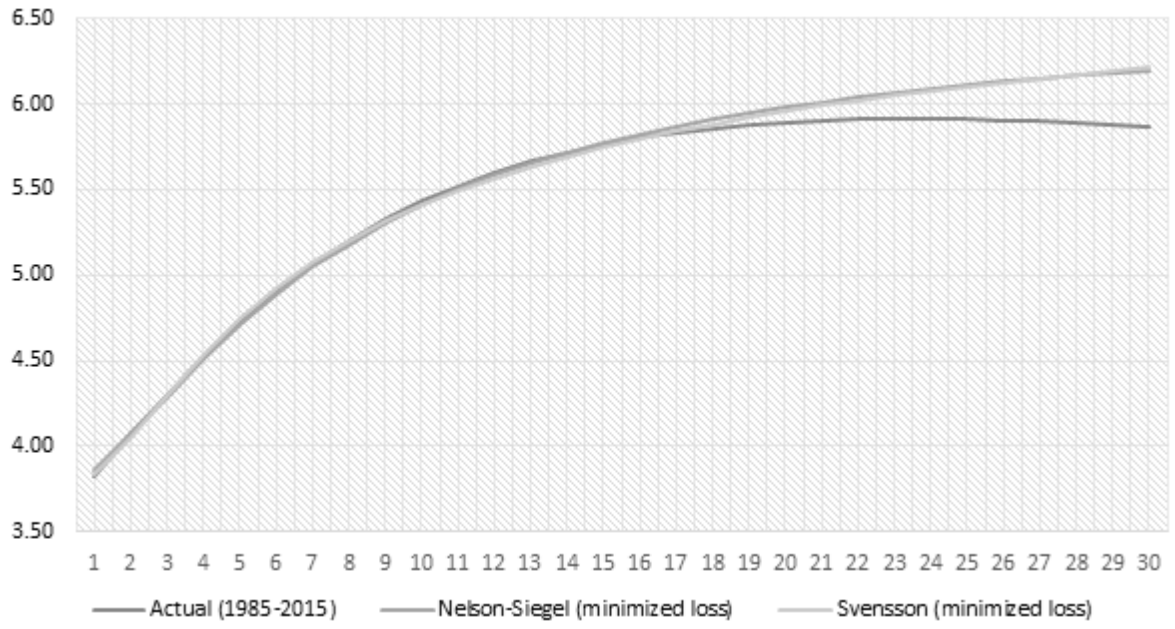

Figure A1 depicts average actual US Treasury yield curve (dark grey), average yield curve obtained from Nelson-Siegel extrapolation (medium grey) for p=1.1%/1.2% that minimizes loss under equal weighting and average yield curve from Svensson extrapolation (light grey) for p=1.3% minimizing loss under equal weighting over the 1985-2015 period. Maturities 1-30 are depicted on the horizontal axis while yields in percentages are shown on the vertical axis.

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