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David Meenagh, Patrick Minford and Olayinka Oyekola

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Cardiff Business School Aberconway Building Colum Drive Cardiff CF10 3EU United Kingdom t: +44 (0)29 2087 4000 f: +44 (0)29 2087 4419 business.cardiff.ac.uk

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Energy Business Cycles^{*}

David Meenagh Cardiff University, UK Patrick Minford Cardiff University and CEPR, UK Olayinka Oyekola[†] Cardiff University, UK

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Abstract

We find that, when estimated, a two sector computable dynamic stochastic general equilibrium open economy model of the U.S. that formally admits energy into the production process can generate plausible parameter values that can be applied to deal with a broad range of economic issues. As a benchmark, we require that the model fits the data for output, real exchange rate, energy use, and consumption: output because it serves as a measure of a country's total income; real exchange rate because it serves as a determinant of a country's relative competitiveness; energy use because it serves as an indicator of *special* inputs into a country's production process; and consumption because it serves as a yardstick for evaluating a country's standard of living. Finally, we argue that this model, with appropriate extensions, some of which we also propose, can help future modelers to tackle other research questions.

JEL Classification: E32, D58, F41, C52, Q43.

Keywords: Two sector, US DSGE model, Oil price volatility, Open economy, Indirect inference

^{*}An online appendix containing further details is available.

[†]Correspondence to: Olayinka Oyekola, Cardiff Business School, Cardiff University, Colum Drive, Cardiff, CF10 3EU, UK. OyekolaOA1@cardiff.ac.uk

1 Introduction

... the interesting question raised by the ... model is surely not whether it can be accepted as 'true' ... Of course the model is not 'true': this much is evident from the axioms on which it is constructed. We know from the outset in an enterprise like this (I would say, in *any* effort in positive economics) that what will emerge - *at best* - is a workable approximation that is useful in answering a limited set of questions. Robert E. Lucas, Jr. (Models of Business Cycles, 1987, p. 45).

Shocks come and they go, often leading to, and leaving behind, unusual business cycle realizations. Like the hurricanes, we like to name these events: the Great Depression in the 1930s, Stagflation in the 1960s, Oil Crises in the 1970s, the Great Moderation commencing in the 1980s, Japan's lost decade for the 1990s, and the Great Recession of the late 2000s. Each of these experiences have tended to provoke intensified research efforts trying to explain what has happened, and, sometimes, to offer policy instruments for resolving the problem(s). The current paper is related to such studies, seeking to explain the causes and consequences of an adverse shock and paths to recovery. It is, however, different in one important dimension: it is a study not reacting per se to a particular oil price shock but mainly adding to the ever-growing body of work on energy economics. Meanwhile, as in Blanchard and Gali (2007), by exempting the policy implications, this work is connected to the literature on both the impact effect of energy price movements on economic activities, as put forth by Bruno and Sachs (1985), and the surprisingly small changes to economic activities over time when energy prices move.

The above raises two further points of debate. First, is that one of the important questions that have been circulating in the economics profession since the Great Moderation is, "Is the reduced influence of energy price shocks on output volatility observed in the data since the mid-1980s the new norm?" This is a legitimate concern if we consider, for instance, that the positive percentage energy price change reached a high of 145% in 2008 having been climbing from 2002 and yet the Great Recession was attributed to the demand shock of housing default and the supply shock of financial credit constraint.

To answer this question, among others, a significant strand of theoretical and empirical literature has been built around a dividing line, with many continuing to lend support to the seminal contribution of Hamilton (1983), who showed that variations in the price of oil are an important correlate to observed variations in many indicators of aggregate and sectoral economic activities [see also Loungani (1986)]. It is not surprising, however, that, with the benefits of the additional time series data that is now available, and with economic characteristics that are distinct to those that Hamilton studied, the economic effects of energy price changes on macroeconomic variables such as output, consumption, and investment appear to have been reversed in studies that spanned beyond Hamilton's sample period to, say, the late 1980s or early 1990s [see, for example, Hooker (1997)].

The second point of debate is like the first: there seems to be no agreement in outcome because of the linear structure between oil (prices) and output originally assumed in Hamilton's empirical work, which technical interpretation and specification has been carried over into theoretical modelling [see, for example, Kim and Loungani (1992), Rotemberg and Woodford (1996), and Finn (2000)]. This is a problem arising from treating energy price shocks symmetrically. Indeed, researchers of oil-macroeconomic relationships in the late 1970s and early 1980s did not face this problem because the evidence before them was that energy shocks were mainly price rises. This led Mork (1989) to advocate the need to correct for the true effects of energy price shocks by assuming asymmetry. That is, given the log of real crude oil price series as depicted in panel **a** of Figure 1, Hamilton's original approach of symmetry would admit panel b, which is equivalent to the first-difference of the log of real crude oil price, while Mork's treatment encourages splitting panel b into panels c and d, which, respectively, define the first-difference of the positive and negative regions of panel b. His point is that we should study the respective contributions towards output variations of prices rising and falling separately.

While this adaptation of the oil price series may have appeared unnecessary pre-1970, it clearly seems like a convincing experiment to carry out post-1970 as the decades of true oil volatilities were ushered in. The benchmark approach adopted here, therefore, is to treat energy price shocks symmetrically. Hooker's finding that data does not support nonlinear and asymmetric representation of the oil-macroeconomic variable interaction permits this launch pad, plus we are mainly interested in how energy price shocks impact aggregate macroeconomic variables. Moreover, on theoretical grounds, this is the right place to start given that our model may not capture the asymmetric response of macroeconomic variables.

The remainder of this paper proceeds as follows. In Section 2, we describe the main features of the two-sector model in general form. In Section 3, we provide brief discussions of the econometric method of indirect inference (II) used in estimating the model, the data serving as the empirical counterparts to model variables, and the parameter values used to initialize the starting points for the Simulated Annealing (SA) algorithm. We present the main findings in Section 4 and conclude with Section 5.





Note: Crude oil price series is from the Energy Information Administration (EIA). Shaded bars are the NBER-dated recessions for 1949-2012. **a**: log of real crude oil price scaled up by 100; **b**: first-difference of the log of real crude oil price; **c**: net oil price increases obtained by setting negative first-differences to zero; **d**: net oil price decreases obtained by setting positive first-differences to zero.

2 The Model

The model is based on Long and Plosser (1983) as augmented by the model of Kim and Loungani (1992).¹ On the supply side, there are two production sectors consisting of firms producing two types of goods with different levels of energy intensities. The firms requiring greater amounts of energy for production make up the energy intensive, e, sector producing energy intensive goods, Y_e , and the remaining firms are the non-energy intensive, n, sector producing non-energy intensive goods, Y_n .² In each sector, firms are

¹We set this up as a two-sector open economy model, which is essential to characterising the data properties of a two-sector U.S. open economy. We suppose that the finished goods of the two sectors are imperfect substitutes for identical products being produced abroad; that is, trade is assumed to be necessary and made possible by representative households in different countries who are willing to buy goods from other countries similar to those being produced in their own countries, mainly because they attribute different qualities to products based on production origin.

²We present the main functional forms in a later section, but for detailed and explicit set-up and characterization of the agents' optimization problems, the first-order conditions, and the log-linearized version, see the Appendix. Note that foreign agents' problems and solutions can be inferred from those of the domestic agents. Meanwhile, we use the following notations

supposed to employ three factors of production, namely: labor hours, capital services, and primary energy. Labor hours and capital services are assumed to be internationally immobile, but the domestic firms import their primary energy requirements.³ On the demand side, there are households who demand composite consumption goods, C, and who make decisions on investment, I, pay taxes to or receive benefits from the government, T, and supply aggregate labor hours, H. Households can invest in two types of physical capital, K_e and K_n , assumed to be subject to capital adjustment cost, and have access to domestic bonds, B. Lastly, we assume that households carry out all trades in goods and services with the rest of the world (RoW), while the firms only trade in crude oil.⁴ To simplify matters, the model economy has been described in terms of the domestic country, where all prices have been expressed relative to the general price level in the RoW, which has been chosen to be the numeraire, $\mathsf{P}^{im} = 1$.

2.1 Firms

Aggregate output of the domestic country, denoted by Y, is defined as the sum of the gross output of the two production sectors of the economy

$$Y = Y_e + Y_n \tag{1}$$

where output has been measured in volumes so as to follow standard practices in computing national income accounts. Then, we assume that sector-specific output, Y_j , is produced with a homogeneous-of-degree-one production function, which differs from the standard neo-classical production function, mainly because we have assumed that the input of primary energy, E_j , is essential to production.⁵ In sum, firms produce Y_j

³The U.S. is a net oil importer.

for convenience: UPPER-CASE letters $X' \equiv X_{t+1}$, $X \equiv X_t$, and $X_t \equiv X_{t-1}$ for dynamic variables for next, current, and lagged periods, respectively; lower-case letters, say x, to denote non-stochastic steady state variables; hatted letters, \hat{x} , to denote variables in their log-linear form; the Greek and sans serif letters to denote the exogenous state variables; and j = e, n, indexes sector-specific variables, $\Delta x (\Delta^2 x)$ denotes the first (second) derivative of x. Further, our crude assumption is that any product that is energy (non-energy) intensive in its production is likewise energy (non-energy) intensive in its consumption.

⁴Many open economy models, at least whenever countries being modeled are allowed to produce more than one good, are usually assumed to have products that are tradable and non-tradable, exportable (importable) and non-exportable (non-importable), etc. While this is a valid assumption, it is one that has been studied extensively. Furthermore, and very importantly, due to the focus of our study, we have eliminated the non-tradable (/ non-exportable/ non-importable) aspects of the model economies such that, for the purpose of our exercise, we have basically assumed that all produced goods and services are tradable between the domestic country and the rest of the world. It is meant to be heuristic and we then use this to draw attention to a four-goods world. This is supported by Engel (1999) and Chari et al. (2002): they found that variations in the relative price of non-tradable are unimportant in terms of accounting for the changes in real exchange rate. Hence, unlike in Stockman (1980), there is no complete specialization in the production of goods.

 $^{{}^{5}}$ A suggestion by McCallum (1989) re-ignited the need to study an environment where the production function is allowed to have exogenous flavours other than the unobserved technology shock, and to allow for the inclusion of energy in the production process/ function.

by using

$$Y_{j} = A_{j}F^{j}\left(\left[H_{j}, \alpha_{j}\right], \left\{\left[U_{j}K_{\prime j}, \mathsf{O}_{j}E_{j}\right], \theta_{j}, \nu_{j}\right\}\right) = D_{j}^{d} + EX_{j}$$

$$\tag{2}$$

where A_j denotes the neutral sector-specific productivity shock, H_j denotes the sector-specific demand for labor hours, $U_j K_{'j}$ denotes the sector-specific demand for capital services, and O_j denotes an exogenous sector-specific shock to the productive efficiency of energy input.⁶ The last term in Equation (2) says that the output of each sector can either be absorbed domestically, D_j^d , or exported to the rest of the world, EX_j . The parameters are α_j , a measure of labor elasticity, θ_j , the weight of capital services, and ν_j , a measure of substitution elasticity between capital services and energy.⁷ Further, the function F^j (·) is assumed to obey standard regularity conditions.

Firms in each sector are assumed to be perfectly competitive in both the product and factor markets, maximizing their profits, which are given by

$$\Pi_j = P_j Y_j - W H_j - R_j U_j K_{\prime j} - \mathsf{Q} E_j \tag{3}$$

subject to Equation (2), where P_j denotes the relative price of sector-specific output, W denotes consumer real wage (producer real wage in each sector is given by W/P_j), R_j is a sector-specific rental rate of capital services, and Q denotes the exogenous price of primary energy determined on the world market. While this is one channel of openness and transmission of the type of macroeconomic consequences documented for the U.S., it is dependent on a positive balance sheet statement for net primary energy transactions.

The relevant first-order conditions that solve the firms' profit maximization problems are

$$\mathsf{A}_{j}F_{H_{i}}^{j}\left(H_{j}, U_{j}K_{'j}, \mathsf{O}_{j}E_{j}\right) = W/P_{j} \tag{4}$$

$$\mathsf{A}_{j}F_{U_{j}K_{\prime_{j}}}^{j}\left(H_{j}, U_{j}K_{\prime_{j}}, \mathsf{O}_{j}E_{j}\right) = R_{j}/P_{j} \tag{5}$$

and

$$\mathsf{A}_{j}\mathsf{O}_{j}F_{E_{j}}^{j}\left(H_{j},U_{j}K_{'j},\mathsf{O}_{j}E_{j}\right) = \mathsf{Q}/P_{j} \tag{6}$$

The equilibrium conditions in both sectors simply equate the marginal product of each input to their marginal cost.

⁶Energy efficiency shocks capture the productivity effect of changing the quantity/ type of energy, and/ or the impacts of developing or gaining access to a better technology for delivering the energy input into the production process.

⁷ In this explanation, we have anticipated the parameters that enter the specific functional forms assumed for the production functions when we carry out an example economy simulation below. This pattern is maintained throughout for the utility and aggregator functions below.

2.2 Households

In this sub-section, we discuss households' decisions relating to domestic variables. In particular, we assume that their lifetime utility function can be described by

$$\mathbb{E}\sum_{0}^{\infty}\beta\tau U\left(\left[C-\iota C_{\prime},\epsilon\right],\left[\zeta H,\omega\right]\right)\tag{7}$$

where \mathbb{E} is an expectations operator, C is aggregate consumption, H is the supply of labor hours, τ is the exogenous intertemporal preference shock, ζ is the exogenous labor supply shock, β is the fixed discount factor, ι is the degree of habit formation, ϵ is the elasticity of consumption, and ω is a measure of Frisch elasticity of labor supply. Further, the function $U(\cdot)$ is assumed to obey standard regularity conditions, which they are assumed to maximize, subject to the following period-by-period budget constraint

$$\mathbb{E}R'B' + C + I + T = B + WH + \sum_{j=e,n} R_j U_j K_{j} + \Pi$$
(8)

where $\mathbb{E}R' = 1/R$ is the stochastic discount factor with $\mathbb{E}R'B'$ defining period t's price of period t + 1's random payment of B', and R is the gross interest rate, B is domestic government's bonds, T is lump-sum taxes or transfers, and Π is the profit income. The budget constraint states that households' expenditure must be equated by their income.

Additionally, it is assumed that households choose physical capital, K_{j} , which are the beginning-ofthe-period t sector-specific stock of capital. Then, we assume that households have access to the following technology for altering the quantity of j capital stocks from period t - 1 to period t

$$K_{j} = \left(1 - \delta\left(\left[U_{j}, \delta_{j0}, \delta_{j1}, \mu_{j}\right]\right)\right) K_{'j} + \mathsf{Z}_{j}I_{j} - \Psi_{j}\left(\frac{K_{j}}{K_{'j}}, \psi_{j}\right)$$
(9)

where I_j is sector-specific gross investment, Z_j is sector-specific exogenous investment-specific technological shock, $\delta(\cdot)$ is sector-specific time-varying depreciation rate, assumed to possess the following properties: $0 \leq \delta(\cdot) \leq 1$, $\Delta\delta(\cdot) > 0$, and $\Delta^2\delta(\cdot) > 0$, U_j is the sector-specific index of period t utilization rate of beginning-of-the-period sector-specific capital stock, δ_{j0} is the constant portion of the steady state level of physical capital, δ_{j1} is the slope of the depreciation function, μ_j governs the elasticity of marginal depreciation with regards to capital utilization rate, and $\Psi_j(\cdot)$ denotes the assumption that changing the stocks of physical capital is subject to convex adjustment costs, which is in turn assumed to possess the following properties: $\Psi_j(\cdot) = \Delta \Psi_j(\cdot) = 0$, and $\Delta^2 \Psi_j(\cdot) > 0$ as in Baxter and Crucini (1995), with ψ_j defining the adjustment cost parameter.⁸

Aggregate investment is defined as the sum of the two sector-specific investments

$$I = I_e + I_n \tag{10}$$

Households choose sequences $\{C, H, I, I_e, I_n, B', U_e, U_n, K_e, K_n\}_0^\infty$ by maximizing the utility function (7) subject to Equations (8), (9), (10), and a borrowing constraint of the form

$$\lim_{\to\infty} \mathbb{E} \frac{B^{+z}}{\prod_{v=0}^{z}} \le 0 \tag{11}$$

The first-order necessary conditions of the households' maximization problem consists of the budget constraint (8), capital accumulation equations (9), aggregate investment (10), the borrowing constraint (11) holding with equality, and the following

$$-\frac{\zeta U_H \left(C - \iota C', \zeta H\right)}{U_C \left(C - \iota C', \zeta H\right)} = W \tag{12}$$

$$\frac{\tau U_C \left(C - \iota C', \zeta H\right)}{\beta \tau' U_C \left(C' - \iota C, \zeta' H'\right)} = R \tag{13}$$

$$R_e = \frac{\Delta\delta\left(U_e\right)}{\mathsf{Z}_e} \tag{14}$$

$$R_n = \frac{\Delta\delta\left(U_n\right)}{\mathsf{Z}_n} \tag{15}$$

$$\left(1+\Psi_e\left(\frac{K_e}{K_{\prime e}}\right)\right) = \beta \frac{\tau' U_C \left(C'-\iota C, \zeta' H'\right)}{\tau U_C \left(C-\iota C', \zeta H\right)} \frac{\mathsf{Z}_e}{\mathsf{Z}'_e} \left\{1-\Psi_e \left(\frac{K'_e}{K_e}\right) + R'_e U'_e \mathsf{Z}'_e - \delta \left(U'_e\right) + \Delta \Psi_e \left(\frac{K'_e}{K_e}\right) \frac{K'_e}{K_e}\right\}$$
(16)

and

$$\left(1+\Psi_n\left(\frac{K_n}{K_{\prime n}}\right)\right) = \beta \frac{\tau' U_C \left(C'-\iota C, \zeta' H'\right)}{\tau U_C \left(C-\iota C', \zeta H\right)} \frac{\mathsf{Z}_n}{\mathsf{Z}_n'} \left\{1-\Psi_n\left(\frac{K_n'}{K_n}\right) + R_n' U_n' \mathsf{Z}_n' - \delta\left(U_n'\right) + \Delta \Psi_n\left(\frac{K_n'}{K_n}\right) \frac{K_n'}{K_n}\right\}$$
(17)

where equilibrium condition (12) states that the marginal rate of substitution between leisure and aggregate consumption is equal to the wage rate. Equilibrium condition (13) states that the intertemporal marginal rate of substitution in the aggregate consumption is equal to the relative price of bonds, such that households are indifferent between consumption today and saving against tomorrow. Equilibrium conditions (14)-(15) equate, for each type of physical capital stocks, marginal user cost to marginal user benefits. Finally,

⁸We have followed Kose (2002) in assuming that there are capital adjustment costs for both types of capital goods.

equilibrium conditions (16)-(17) relate marginal costs and returns to optimal choices between consumption and investments in the two types of physical capital stocks.

2.3 Traders

So far we have discussed the model economy as if it was a closed one, except for the mention of the purchase of primary energy by firms on the world market. This has been done intentionally in the hope that this structure would help to make the model interactions more tractable. We are now, therefore, ready to open the model up more, both sectorally and internationally. Our working assumption is that consumption, investment, and government spending in the domestic country and the rest of the world are composites of domestic and the rest of the world's energy and non-energy intensive goods. Clearly, we have assumed away any extra costs that may arise due to the import or export of a good or service, such as transport costs. Thus, we define the composites of consumption, investment, and government spending, respectively, by

$$C = \Phi_C \left(C_e^{dp}, C_n^{dp}, C_e^{fp}, C_n^{fp} \right)$$
(18)

$$I = \Phi_I \left(I_e^{dp}, I_n^{dp}, I_e^{fp}, I_n^{fp} \right)$$
(19)

and

$$\mathsf{G} = \Phi_{\mathsf{G}} \left(\mathsf{G}_{e}^{dp}, \mathsf{G}_{n}^{dp}, \mathsf{G}_{e}^{fp}, \mathsf{G}_{n}^{fp} \right)$$
(20)

where, for f = C, I, G, the aggregator function Φ_f is supposed to be increasing and homogeneous-of-degreeone in all its arguments, and for variable V = C, I, G, superscript dp (fp) with subscript e (n) implies demand for domestically (foreign-) produced energy (non-energy) intensive goods.

Proceeding, we continue to think in terms of aggregate variables as much as possible without compromising on the research goal. Thus, defining the aggregate spending of the domestic country (or equivalently, the domestic absorption) as the sum of aggregate consumption, aggregate investment and aggregate government spending, we have that

$$D = C + I + \mathsf{G} \tag{21}$$

where, by implication, D is a composite of all the four types of goods described by

$$D = \kappa \left(D^d, IM \right) \tag{22}$$

where the aggregator function κ is supposed to be increasing and homogeneous-of-degree-one in both its

arguments, D^d is the quantity of domestically produced goods demanded by domestic residents, and IM is the total spending of domestic residents on imports.⁹ As shown by Backus et al. (1995), the above aggregator function is sufficient for use if one is modeling two countries with two goods as they did. We have a model of *two countries* and four goods, however, and therefore need further disaggregation.

To resolve this, we could split D into a function of energy and non-energy intensive goods, and given that, we could also split IM into a function of energy and non-energy intensive goods.¹⁰ Thus, both D and IM are defined, respectively, by

$$D = \Sigma \left(D_e, D_n \right) \tag{23}$$

and

$$IM = \Xi \left(IM_e, IM_n \right) \tag{24}$$

where the aggregator functions Σ and Ξ are supposed to be increasing and homogeneous of degree one in their arguments, D_e is the total spending of domestic residents on energy intensive goods, D_n is the total spending of domestic residents on non-energy intensive goods, IM_e denotes total spending of domestic residents on imported energy intensive goods, and IM_n denotes total spending of domestic residents on imported non-energy intensive goods.

Given this background, we cast the problems facing domestic residents in the international market. There are two problems that the traders have to solve. First, they have to choose D_e , IM, and IM_e in order to minimize their expenditure on both the domestically and foreign-produced goods. Second, they have to infer what their export functions are for EX and EX_e by solving the problem of traders in the foreign country. Formally, these two problems can be summarized by the choices made on domestic goods by min $\{P^dD^d + IM - PD\}$ subject to Equation (22), where P^d is the price index for composite domestically produced goods, P is the consumer price index in the domestic country, and both prices have been defined relative to the consumer price index in the RoW.¹¹ Also, they min $\{P_eD_e + P_nD_n - PD\}$ subject to Equation (23). Moreover, relying on Walras' Law, we shut down activities in the market for the non-energy intensive goods since when the market for energy intensive goods clears, the Law implies that the market for the

⁹One way to proceed from here would be to choose a functional form for Φ_f , and given that households and the government have chosen their expenditures on C, I, and G, then their respective problems reduces to maximizing their utilities and profits by optimally allocating their aggregate expenditures among the components of each of C, I, and G. This will yield the result we are looking for, but the number of variables and expressions will increase with this approach, and needlessly so. Given that our research goal does not require this much level of disaggregation, we find it more manageable, to pursue the analysis this way in terms of aggregate demand rather than in terms of *components of* its components.

¹⁰Again, at this junction, we could proceed alternatively by defining the components of D as functions of energy and nonenergy intensive goods; that is, $D^d = \Sigma_d (D_e, D_n)$ and $IM = \Sigma_m (IM_e, IM_n)$.

¹¹Hence, P is also the real exchange rate. This is a crude appropriation of the purchasing power parity hypothesis by using the ratio of the export and import prices to proxy the exchange rate between two economies.

non-energy intensive goods also clears.¹² Lastly, they choose the share of expenditure on imported goods that must be allocated to the purchase of imported energy intensive goods by $\min\{\mathsf{P}_e^{im}IM_e + \mathsf{P}_n^{im}IM_n - IM\}$ subject to Equation (24), where P_e^{im} and P_n^{im} are prices of imported energy and non-energy intensive goods, respectively, and are treated as exogenous variables to the domestic agents.¹³

Hence, domestic agents choose sequences $\{D_e, IM, IM_e\}_0^\infty$. The first-order conditions associated with these choices are

$$\kappa_2\left(D^d, IM\right) = \frac{1}{P} \tag{25}$$

$$\Sigma_1 \left(D_e, D_n \right) = \frac{P_e}{P} \tag{26}$$

and

$$\Xi_1 \left(IM_e, IM_n \right) = \mathsf{P}_e^{im} \tag{27}$$

where for function $\lambda = \kappa, \Sigma, \Xi, \lambda_{\varrho}$ denotes a partial derivative of λ with respect to its ϱ -th argument. In the same vein, we assume that the problems of agents in the rest of the world are a mirror image to those of agents in the domestic country. Hence, we use the import functions of foreign households to infer the export functions for the domestic country. More explicitly, we want to know the quantities of aggregate exports and exports of energy intensive goods. To do this, we assume for the rest of the world that

$$\mathsf{D}^{w} = \boldsymbol{\kappa}^{w} \left(D^{f}, I M^{f} \right) \tag{28}$$

where $\mathsf{D}^w = C^w + I^w + \mathsf{G}^w$ is the aggregate world spending or demand, D^f is the rest of the world's demand for its own goods, and IM^f is the aggregate import of goods by the rest of the world from the domestic economy, which, incidentally, is equal to the export of goods to the rest of the world by the domestic country denoted by EX. Hence, we find it convenient to replace IM^f (and its components) with EX (and its components). Also, κ^w is supposed to be increasing and homogeneous-of-degree-one in both its arguments. We are concerned with the first-order condition with respect to EX that is given by

$$\kappa_2^w \left(D^f, EX \right) = P \tag{29}$$

where κ_{ϱ}^{w} denotes a partial derivative of κ^{w} with respect to its ϱ -th argument. Noting that EX is also a

¹²Clearly, $D_n = D - D_e$. Thus, having obtained aggregate domestic absorption and domestic absorption of energy intensive goods implies a solution for the non-energy intensive goods and its components.

¹³Only P_{e}^{im} appears in the model simulation because of the reason given in the previous footnote.

composite defined by

$$EX = \Xi^w \left(IM_e^f, IM_n^f \right) = \Xi^w \left(EX_e, EX_n \right)$$
(30)

where the aggregator function Ξ^w is supposed to be increasing and homogeneous-of-degree-one in both its arguments. The required first-order condition is with respect to EX_e and is given by

$$\Xi_1^w \left(EX_e, EX_n \right) = \frac{P_e}{P} \tag{31}$$

where Ξ_{ρ}^{w} denotes a partial derivative of Ξ^{w} with respect to its ρ -th argument. We conclude this sub-section by defining the real exchange rate, P, in the domestic country as

$$P = \Sigma \left(P_e, P_n \right) \tag{32}$$

2.4 Government, Market Clearing, and Equilibrium

The government is also included in the current model and assumed to face the following period-by-period budget constraint

$$\mathsf{G} + B = T + \mathbb{E}R'B' \tag{33}$$

where G denotes the exogenous government spending shock. In order to close the model, we define some aggregate variables and market-clearing conditions. Aggregate demand for labor hours by the firms is the sum of the two sectors' demand for labor hours and is equal to the total supply of labor hours by the households

$$H = H_e + H_n \tag{34}$$

Aggregate demand for primary energy input by the firms is the sum of the two sectors' demand for primary energy

$$E = E_e + E_n \tag{35}$$

We assume for feasibility that the current account constraint is satisfied in each period

$$PEX = \mathbf{Q}E + IM \tag{36}$$

That is, the value of exports equals the value of imports: foreign demand for domestically produced goods equals domestic demand for primary energy and foreign-produced goods. Given that the current account holds, total demand is, therefore, equal to total supply

$$D = Y \tag{37}$$

Additionally, the sectoral market for energy intensive goods clears

$$Y_e = D_e + EX_e - IM_e \tag{38}$$

Finally, we define the competitive equilibrium.

Definition 1 A competitive equilibrium is a set of 28 endogenous stochastic processes $\{C, I, I_e, I_n, H, H_e, H_n, K_e, K_n, U_e, U_n, E, E_e, E_n, Y, Y_e, Y_n, D, IM, IM_e, D_e, EX, EX_e, P, P_e, P_n, W, R\}_0^{\infty}$ satisfied by the solutions to the problems of the producers, consumers, and traders, given the set of 12 exogenous stochastic AR(1) processes $\{A_e, A_n, D^w, G, \zeta, O_e, O_n, P_e^{im}, Q, \tau, Z_e, Z_n\}_0^{\infty}$, the initial conditions $C_{-1}, K_{-1}^e, K_{-1}^n$ and B_0 , and the bond, labor, capital, energy, and good markets clear with the relevant transversality conditions binding.

3 Econometric Methodology and Data

In this section, we briefly discuss the method of indirect inference used to evaluate and estimate the model, and document the data used in the process.¹⁴ The empirical method of indirect inference is a classical statistical inferential approach to assessing an applied model, whether calibrated or already estimated, in which we maintain the basic concepts well-known in the evaluation of RBC models of comparing the moments generated by data simulated from the model with the observed data. Indeed, we know that using moments for the comparison gives a framework that is free of distribution. Instead, we posit a general but simple formal model (an auxiliary model, which we choose to be a VAR(1) in this case) – in effect the conditional mean of the distribution of the data – and base our comparison on features of this model estimated from simulated and actual data. When we use the method for evaluation purposes, we take as given the structural model parameters. The assumption behind this is that if the given model is correct, then it will give predictions about the features of the model, as depicted by impulse response functions and moments, that

¹⁴The method of indirect inference was developed in Le et al. (2011). An interested reader is referred to this paper, and the references therein, for details.

match those observed in the actual data. On the other hand, when we use indirect inference for estimation purposes, the task is to choose values for the structural model parameters such that when simulated, the model generates coefficients of the auxiliary model that replicates the findings in the actual data. Below, we make attempts at both evaluating and estimating the model under indirect inference. The data used are taken from Meenagh et al. (2015); a detailed description of the data sources and construction of the 28 observables (empirical counterparts to the endogenous variables) are presented in Appendix B. These are U.S. annual data covering the period 1949-2013, and are logarithmically transformed, real [using Bureau of Labor Statistics (BLS) series: consumer price index (CPI, 2009=100)] per capita [using Bureau of Labor Statistics (BLS) series: civilian non-institutionalized population over 16 years old] terms except for wage rate, interest rate, real exchange rate, relative prices, and capital utilization rates. All series are filtered following Hodrick and Prescott's (1997) procedure, setting the smoothing parameter to 400, as in Kim and Loungani (1992) and Costello (1993).

4 Results

The presentation of results in this section is divided into two parts: the calibrated version of the model is analyzed in the first sub-section, and in the second sub-section, we summarize the results from the estimated version of the model.

4.1 Empirical Analysis I

In this part, we discuss the simulation of the model, the choice of specific functional forms, the calibration of the model parameters, and quantitatively test the fit of the model to the data for a number of key macroeconomic variables.

4.1.1 Functional Forms, Calibration, and Model Simulation

Following Christiano (1988) and King et al. (1988a, b), we solve the model by first obtaining the equilibrium conditions based on the chosen functional forms, adding the market-clearing conditions and the assumed laws of motion for the model's structural errors. Then, we derive the deterministic version of the model where all the standard deviations of innovations are identically equal to zero, and it is around these values that we express the decision variables of the model as a linear approximation using a Taylor-series

Table 1: Functional for	\mathbf{ns}
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Function	Form
Utility	$U = \left(\left(C - \iota C_{-1} \right)^{1-\epsilon} / (1-\epsilon) - \zeta H^{1+\omega} / (1+\omega) \right)$
Energy sector production function	$Y_{e} = A_{e} (H_{e})^{1-\alpha_{e}} \left(\theta_{e} (U_{e}K_{e,-1})^{-\nu_{e}} + (1-\theta_{e}) (O_{e}E_{e})^{-\nu_{e}}\right)^{-\alpha_{e}/\nu_{e}}$
Non-energy sector production function	$Y_n = A_n (H_n)^{1-\alpha_n} \left(\theta_n (U_n K_{n,-1})^{-\nu_n} + (1-\theta_n) (O_n E_n)^{-\nu_n} \right)^{-\alpha_n/\nu_n}$
Energy goods time-varying depreciation rate	$\delta\left(U_e\right) = \delta_{e0} + \delta_{e1} \left(U_e\right)^{\mu_e} / \mu_e$
Non-energy goods time-varying depreciation rate	$\delta\left(U_{n}\right) = \delta_{n0} + \delta_{n1}\left(U_{n}\right)^{\mu_{n}}/\mu_{n}$
Energy goods adjustment cost function	$\Psi_e \left(K_e / K_{e,-1} \right) = \psi_e \left(K_e / K_{e,-1} - 1 \right)^2 K_{e,-1} / 2$
Non-energy goods adjustment cost function	$\Psi_n \left(K_n / K_{n,-1} \right) = \psi_n \left(K_n / K_{n,-1} - 1 \right)^2 K_{n,-1} / 2$
Domestic absorption in terms of goods' origin	$D = \left(\kappa^{1/\phi} \left(D^d\right)^{(\phi-1)/\phi} + (1-\kappa)^{1/\phi} \left(IM\right)^{(\phi-1)/\phi}\right)^{\phi/(\phi-1)}$
Domestic absorption in terms of goods' type	$D = \left(\sigma^{1/\varsigma} (D_e)^{(\varsigma-1)/\varsigma} + (1-\sigma)^{1/\varsigma} (D_n)^{(\varsigma-1)/\varsigma}\right)^{\varsigma/(\varsigma-1)}$
Imports in terms of goods' type	$IM = \left(\chi^{1/\eta} \left(IM_e\right)^{(\eta-1)/\eta} + (1-\chi)^{1/\eta} \left(IM_n\right)^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}$
World demand in terms of goods' origin	$D^{w} = \left(\kappa_{w}^{1/\phi_{w}} \left(D^{f}\right)^{(\phi_{w}-1)/\phi_{w}} + (1-\kappa_{w})^{1/\phi_{w}} \left(EX\right)^{(\phi_{w}-1)/\phi_{w}}\right)^{\phi_{w}/(\phi_{w}-1)}$
Exports in terms of goods' type	$EX = \left(\chi_w^{1/\eta_w} \left(EX_e\right)^{(\eta_w - 1)/\eta_w} + (1 - \chi_w)^{1/\eta_w} \left(EX_n\right)^{(\eta_w - 1)/\eta_w}\right)^{\eta_w/(\eta_w - 1)}$

expansion. This result gives a solution that permits a state-space representation of the model's endogenous variables in a way that allows for a possible matching of a set of observables.

The assessment of the quantitative workings of the model can only begin when one has chosen values for the model parameters such that one is able to simulate the model. Thus, we discuss next the chosen functional forms for preferences, technologies, time-varying depreciation rates, capital adjustment costs, and the aggregator functions. The forms employed for the quantitative exploration exercises in this section are listed in Table 1.

Table 2 documents the calibrated parameter values. Notably, we follow the procedure of Iacoviello et al. (2011) in providing parameters with numerical values.¹⁵ Their approach requires that the parameters be divided into three groups, namely: (1) Parameters whose values are fixed throughout the exercise; (2) Parameters whose values are estimated but for which we must provide initial values as suggestions for the Simulated Annealing (SA) search algorithm during the estimation process;¹⁶ and (3) Parameters whose values are derived from the values of parameters in groups one and two, and the average values of some observed data ratios.

In the first group are the discount factor, β , which we fix at 0.96 suggesting that we have taken the annual real rate of interest to be 4%, which is consistent with the average post-WWII interest rate for the U.S., and the steady state of the depreciation functions for the two types of investment goods, δu^e and δu^n , which are set to be equal to the long-run average data values of investment-capital ratios for the energy and non-energy intensive goods. Then, all twelve shocks are normalized to unity in steady state such that we

¹⁵The classic references for calibration remain, of course, Kydland and Prescott (1982) and Prescott (1986).

¹⁶Note that, unlike Iacoviello et al. (2011), who estimated their model using Bayesian techniques, we have adopted the method of indirect inference discussed in the previous section. In a way, however, the initial parameters may be likened to Bayesian priors.

Description	Symbol	Values
Discount factor	β	0.96
Frisch elasticity	ω	5
Consumption elasticity	ε	2
Habit formation parameter	L	0.7
e share of capital and energy	α_e	0.43
n share of capital and energy	α_n	0.28
e elasticity of substitution between capital and energy	ν_e	0.7
n elasticity of substitution between capital and energy	$ u_n$	0.7
e marginal cost of capital utilization	$\delta_{e1} u^{\mu_e}$	0.132
e depreciation elasticity of capital utilization	μ_{e}	1.463
n marginal cost of capital utilization	$\delta_{n1} u^{\mu_n}$	0.102
n depreciation elasticity of capital utilization	μ_n	1.694
e adjustment cost parameter	$\psi_{m{e}}$	0.001
n adjustment cost parameter	ψ_n	0.001
Substitution elasticity, $d_t^d - im_t$ goods	ϕ	1.5
Substitution elasticity, $d_t^f - ex_t$ goods	ϕ_{w}	1.5
Substitution elasticity, $im_t^e - im_t^n$ goods	η	0.44
Substitution elasticity, $ex_t^e - ex_t^n$ goods	η_w	0.44
Weight of d_t^e goods	σ	0.55
Substitution elasticity, $d_t^e - d_t^n$ goods	κ	0.7
Substitution elasticity, $d_t^e - d_t^n$ goods	ς	0.9
e goods steady state depreciation	δu^e	0.09
n goods steady state depreciation	δu^n	0.06
e weight on capital services	θ_e	0.9903
n weight on capital services	θ_n	0.9961

 Table 2: Calibration

Note: e is energy intensive, n is non-energy intensive.

can proceed to the second group.

The second group is made up of twelve autocorrelation parameters, twelve standard deviations of innovations, and twenty deep structural parameters. To calibrate the shocks, we assume that the twelve exogenous processes follow AR(1) stationary processes in logarithm. Further, by supposing that the innovations are serially uncorrelated, their 24 parameters can be calculated based on twelve derived series.¹⁷ Nine of these equations are without expectations such that the structural errors are backed out directly as residuals. For the last three (intertemporal preference, and energy and non-energy intensive investment-specific technology shocks) that are with expectations, the residuals are derived using the instrumental variable method recommended by McCallum (1976) and Wickens (1982), where the instruments are the lagged values of the endogenous variables. We then fit a univariate, AR(1), model to each of the calculated series for the shocks - Table 3 documents the results.

Here we calibrate the starting values of the remaining twenty parameters that are estimated later in

¹⁷See Appendix A for details.

Shocks	ρ_s	σ_s
Energy intensive sector productivity, \widehat{A}_e	0.4930	0.0203
Non-energy intensive sector productivity, \widehat{A}_n	0.2594	0.0285
World demand, \widehat{D}^w	0.6144	0.1968
Government spending, \widehat{G}	0.4076	0.0572
Labor supply, $\widehat{\zeta}$	0.4888	0.1235
Energy intensive sector energy efficiency, \widehat{O}_e	0.5881	0.5161
Non-energy intensive sector energy efficiency, \widehat{O}_n	0.5873	0.5470
Price of imported energy intensive goods, \widehat{P}_{e}^{im}	0.5572	0.0405
Energy price, \widehat{Q}	0.5366	0.3127
Intertemporal preference, $\hat{\tau}$	0.5760	1.1988
Energy intensive goods investment-specific technology, \widehat{Z}_{e}	0.2996	0.0733
Non-energy intensive goods investment-specific technology, \widehat{Z}_n	0.2990	0.0730
<i>Note:</i> ρ and σ are respectively the persistence parameters	and the	standard

Table 3: Driving Processes

Note: ρ_s and σ_s are, respectively, the persistence parameters and the standard deviations of each shock, s.

the process. We set the elasticity of labor supply, ω , as equal to 5, fix consumption elasticity, ϵ , at 2, and preserve the CES form of the production functions by setting the respective sector's elasticity of substitution between capital services and efficient energy use, ν_e and ν_n , as equal to 0.7.¹⁸ In setting ι value to 0.7, we suppose that there is a high degree of habit formation for agents in this model, which is in line with previous estimates in the literature for a developed country such as the U.S. A very small value of 0.001 is chosen for the parameters that relate to the adjustment costs of capital, ψ_e and ψ_n , following a popular practice in the literature.

Moving on to the component parameters of the two depreciation functions, the steady state implies that $\delta u^j = \delta_{j0} + \delta_{j1} (u^j)^{\mu_j} / \mu_j$ for j = e, n, for which we note that the only four of their six parameters that needed identifying are δ_{e0} , δ_{n0} , μ_e and μ_n . So, conditional on the values of the discount factor and the real rental rates, we calibrate the parameters governing the elasticities of marginal depreciations with respect to capital utilization rates using $\mu_e = \beta \delta_{e1} u^{\mu_e} / (\beta (1 + \delta_{e1} u^{\mu_e}) - 1) = 1.463$ and $\mu_n = \beta \delta_{n1} u^{\mu_n} / (\beta (1 + \delta_{n1} u^{\mu_n}) - 1) = 1.694$, which are reasonably located in the range found in the literature.

Moreover, with no loss of generality, we fix the values for δ_{e1} and δ_{n1} at unity.¹⁹ The idea is that δ_{j1} and u^j are admitted into the model only jointly as $\delta_{j1} (u^j)^{\mu_j}$ such that $\delta_{j1} = 1$ has a trivial implication that $\delta_{j1} (u^j)^{\mu_j} = (u^j)^{\mu_j}$. In addition, using households' optimality conditions with regards to capital utilization

¹⁸Kim and Loungani (Table 2, p. 180) provide a justification for using this value. They also considered a value of 0.001 suggesting a Cobb-Douglas form and high elasticity of substitution between capital services and energy use. We, however, stick to the parameter value that preserves the general form of specification and leave the optimal choice of parameter value to the estimation stage later on.

¹⁹See, for example, Burnside and Eichenbaum (1996), Boileau and Normandin (1999), King and Rebelo (1999), and Leduc and Sill (2004).

rates conditioned on the values for the respective sector's rental rate of capital in the steady state, one can show that $\delta_{j1} (u^j)^{\mu_j} = r_j = 1/\beta - (1 - \delta (u^j))$, which simplifies to give the values reported in the table for $\delta_{e1} u^{\mu_e}$ and $\delta_{n1} u^{\mu_n}$ of 0.132 and 0.102, respectively.

Parameters governing the elasticities of labor hours in the energy and non-energy intensive sectors, α_e and α_n , are found to be 0.43 and 0.28, respectively, being calibrated to match the respective sector's capitaloutput ratios. The values chosen for the elasticities of substitution parameters in the aggregator functions are all standard in the trade literature: $\phi = \phi_w = 1.5$, $\eta = \eta_w = 0.44$, and $\varsigma = 0.99$ [see, for example, Stern and Schumacher (1976), Whalley (1985), and Shiells and Reinert (1993)].

Finally, the parameters in the third group (θ_e, θ_n) are calibrated given the fixed and/ or estimated parameters and an array of target steady state ratios of the model. These weight parameters are pinned down by the respective sector's average energy-capital ratio over the sample period, given values of some other underlying structural parameters using the expression, $\theta_j = 1/(1 + (e_j/k_j)^{1+\nu_j}/\delta_{j1}u^{\mu_j})$. The values mainly change with the parameters ν_j . The initial values chosen for the bias parameters, σ and κ , are 0.55 and 0.7, respectively.²⁰

4.1.2 Assessing the Fit of the Calibrated Model Using Directed Wald Test

Now, we would like to know if the model can be fitted to the data for the macroeconomic variables we are most interested in. Davidson et al. (2010) showed that the greater the number of variables included the more difficult it becomes for the model to fit the data, and also that the joint distribution of variables of interest depends more on the covariances between the VAR coefficients and not necessarily by the individual cross-correlation of variable coefficients.²¹ A similar idea was put forth by Le et al. (2011) and we have therefore restricted the number of variable combinations and/ or lag order to a maximum of four for the auxiliary VAR(1) model for which we present results, since a more rigorous pass criteria is set the higher the number of variables and/ or lags. The importance of this Directed Wald test is that it helps to narrow down the economic questions that can be addressed using the constructed economic model.

Hence, the primary goal is the result from the joint distribution. As a benchmark, therefore, we would particularly like to fit the model to the data on aggregate output, which serves as a measure of domestic country's total income, the real exchange rate, which serves as a measure of the domestic country's compet-

 $^{^{20}}$ We provide more interpretation for parameters and the implications of their values below when we discuss the estimated model.

²¹This was already implied by Long and Plosser (1983, p. 39) in the introduction to Real Business Cycles when they wrote that the "... term business cycles refers to the joint time-series behavior of a wide range of economic variables ...".

Coefficients	Actual	Lower Bound	Upper Bound	Mean
$\rho_{\widehat{Y},\widehat{Y}}$	0.4913	-0.2048	1.2775	0.5753
$ ho_{\widehat{P},\widehat{Y}}$	-0.0076	-0.8295	0.9103	0.0572
$ ho_{\widehat{E},\widehat{Y}}$	-0.0354	-0.1424	0.1600	0.0121
$\rho_{\widehat{C},\widehat{Y}}^{-,-}$	0.1080	-0.1121	0.1875	0.0255
$\rho_{\widehat{Y},\widehat{P}}$	0.0047	-0.6448	0.3742	-0.1102
$\rho_{\widehat{P},\widehat{P}}$	0.6797	-0.1720	0.9948	0.4269
$\rho_{\widehat{E},\widehat{P}}$	0.1341^{\P}	-0.1331	0.0861	-0.0226
$\rho_{\widehat{C},\widehat{P}}^{-,-}$	0.4435^{\P}	-0.1358	0.0862	-0.0207
$\rho_{\widehat{Y},\widehat{E}}$	0.4621	-1.6359	1.7728	0.1506
$\rho_{\widehat{P},\widehat{E}}$	-0.1641	-2.3913	1.9571	-0.0564
$\rho_{\widehat{E},\widehat{E}}$	0.4959	0.1970	0.8475	0.5445
$\rho_{\widehat{C},\widehat{E}}^{-,-}$	-1.2027^{\P}	-0.3194	0.4214	0.0378
$\rho_{\widehat{Y},\widehat{C}}$	0.0697	-1.3391	0.7529	-0.2228
$\rho_{\widehat{P},\widehat{C}}$	0.0041	-1.3542	0.9729	-0.1666
$ ho_{\widehat{E},\widehat{C}}$	-0.0198	-0.1893	0.2235	0.0173
$\rho_{\widehat{C},\widehat{C}}^{-,\circ}$	0.5978	0.5854	0.9816	0.8255
$\sigma_{\widehat{V}}^2$	0.0009 ^{\$}	0.0139	0.0369	0.0241
$\sigma^{2}_{\widehat{P}}$	$0.0314^{ analytic}$	0.0077	0.0199	0.0128
$\sigma_{\widehat{E}}^{2}$	$0.0413^{ analytic}$	0.0778	0.2410	0.1514
$\sigma_{\widehat{C}}^{\widehat{Z}}$	$0.0004^{ atural}$	0.0485	0.2361	0.1230
	Dynamics+Volatilities	Dynamics	Volatilit	ies
Wald $(\%)$	100	100	100	
TMD	40.14	39.33	14.21	

Table 4: VAR Results for the Calibrated Model

Note: $\rho_{i,j}$ denotes the VAR coefficient of a lagged variable *i* on a variable *j*; σ_j^2 denotes the variance of variable *j*; \hat{Y} denotes aggregate output; \hat{P} denotes the real exchange rate; \hat{E} denotes aggregate energy use; \hat{C} denotes consumption; TMD denotes transformed Mahalanobis distance. The VAR coefficients that lie outside of the 95% confidence bounds; ^{\(\beta\)} the data variances of the variables that lie outside of the 95% confidence bounds.

itiveness against the rest of the world, energy use since this is an energy real business cycle (ERBC) model and because it is an indicator of inputs into the production process, and possibly consumption, which may serve as a measure of the agents' standard of living.

Now, one can assess Table 4 for the results of the model's predictions of individual and joint distributions of output, real exchange rate, energy use, and consumption. The model is rejected for the joint distribution test of the dynamics plus volatilities, and for separate tests on the dynamics and volatilities, where for the three auxiliary models, the Wald statistic is 100, and the respective transformed Mahalanobis distances are 40.14, 39.33, and 14.21. This can be expected as the model was only able to replicate the individual distribution of thirteen of the twenty VAR coefficients. Particularly, it failed to capture the effects of lagged energy use on current real exchange rate, the effects of lagged consumption on current real exchange rate, and the effects of lagged consumption on current energy use. In two of the three instances, the model under-predicted the co-movements observed in the data, but over-predicted it in one. More interestingly, the dynamic coefficients that lie outside the 95% bounds relate to the cross-effects of consumption and real exchange rate, with each appearing twice. Also, all the four data variances lie outside of the 95th percentile; specifically, the model over-projects the volatilities of output, consumption, and energy use, but under-projects the variability in real exchange rate.

4.2 Empirical Analysis II

4.2.1 Parameter Estimates

We begin the discussion of the empirical results by considering the estimated parameters in Table 5 where the calibrated values from Table 2 serve as the initial values for the SA algorithm. Three of the parameters are kept fixed throughout the exercise: β , δu^e , and δu^n (the last two are indeed the steady state representations of the depreciation functions), and two are derived from the calibrated/ estimated parameters: θ_e and θ_n . Some of the estimates have values that are not too far from the suggested initial values, while others have values that are quite far off. In the first category are the values for ω , ε , ι , α_e , α_n , $\delta_{e1}u^{\mu_e}$, $\delta_{n1}u^{\mu_n}$, μ_e , μ_n , ψ_e , ψ_n , ϕ_w , η , η_w , κ , and σ , whose values changed by less than 100% in absolute terms relative to the initial values; in the second category are ν_e , ν_n , ϕ , and ς , whose values changed by more than 100% in absolute terms relative to the starting values.

Consumption elasticity is one of the highly difficult parameters to pin down in economics. The estimate of its inverse is 5.59, which is arguably within the range of estimates found in many DSGE models [Hall (1988) and Smets and Wouters (2003, 2005)]. The consumption elasticity of 0.18 found here implies that households are less willing to smooth consumption across time in response to a change in the real interest rate.²² Compared to the initial value, the estimated value of the habit formation parameter, interestingly, is suggesting that households' utility in period t is less dependent on past consumption. Specifically, the estimated value of roughly 0.4 reduces the level of consumption inertia to transitory shocks; this should perhaps remove the hump-shaped nature of consumption impulse response functions [see Fuhrer (2000)].

²²This implies that agents would respond less to substitution effect than to wealth effect. A value of between zero and one is uncontroversial in the literature [see, for example, Kocherlakota (1988) advocated for a value close to zero]. Meanwhile, Campbell (1994) used values ranging from zero to infinity in his analysis. For more on the range of values and other estimates of consumption elasticity, see, for example, Nelson and Nikolov (2002), Bergin (2003), and Cromb and Fernandez-Corugedo (2004).

Description	Symbol	Initial Values [#]	Estimation
Discount factor*	β	0.96	0.96
Frisch elasticity	ω	5	8.64
Consumption elasticity	ε	2	5.59
Habit formation parameter	ι	0.7	0.38
e share of capital and energy	α_e	0.43	0.43
n share of capital and energy	α_n	0.28	0.39
\boldsymbol{e} elasticity of substitution between capital and energy	ν_e	0.7	0.12
\boldsymbol{n} elasticity of substitution between capital and energy	ν_n	0.7	0.07
e marginal cost of capital utilization	$\delta_{e1} u^{\mu_e}$	0.132	0.32
e depreciation elasticity of capital utilization	μ_e	1.463	5.51
n marginal cost of capital utilization	$\delta_{n1} u^{\mu_n}$	0.102	0.44
n depreciation elasticity of capital utilization	μ_n	1.694	4.32
e adjustment cost parameter	ψ_e	0.001	0.003
n adjustment cost parameter	ψ_n	0.001	0.002
Substitution elasticity, $d_t^d - im_t$ goods	ϕ	1.5	0.16
Substitution elasticity, $d_t^f - ex_t$ goods	ϕ_w	1.5	34.8
Substitution elasticity, $im_t^e - im_t^n$ goods	η	0.44	0.51
Substitution elasticity, $ex_t^e - ex_t^n$ goods	η_w	0.44	2.12
Weight of d_t^e goods	σ	0.55	0.48
Substitution elasticity, $d_t^e - d_t^n$ goods	κ	0.7	0.68
Substitution elasticity, $d_t^e - d_t^n$ goods	ς	0.9	0.27
e goods steady state depreciation ^{\mathbf{x}}	δu^e	0.09	0.09
n goods steady state depreciation ^{X}	δu^n	0.06	0.06
e weight on capital services [¶]	θ_e	0.9902	0.9631
n weight on capital services ¶	θ_n	0.9961	0.9833

Table 5: Estimation

Note: e is energy intensive, n is non-energy intensive. [#]Calibrated values are used to initialize the simulated annealing search algorithm; [#] parameters that are fixed throughout the exercise; [¶] parameters that are derived based on fixed and estimated parameters.

Theoretically, there is no judgement against this result, but quantitatively, it is too low relative to evidence of past estimates for a developed country [see Boldrin et al. (2001), Smets and Wouters (2004), and Christiano et al. (2005); however, low habit persistence has mainly been found in studies relating to emerging countries - see Uribe and Yue (2006)].

The high value for Frisch elasticity $(1/\omega = 0.12)$ says that labor hours react more to changes in real wages. The estimated values for the share of capital services and energy in production in both sectors are a bit low but not unreasonable. $1 + \nu_e (1 + \nu_n)$ is a measure of the inverse elasticity of substitution between capital services and primary energy use in the energy (non-energy) intensive sector such that the values of $\nu_e = 0.12$ and $\nu_n = 0.07$ imply that the two production functions are more Cobb-Douglas than the assumed general CES form, particularly in the non-energy intensive sector. Both estimates for the adjustment cost parameters are close to zero, although they have different quantitative impacts on the model. As it is, the value of 0.003 for the energy intensive capital adjustment cost parameter, compared to the 0.002 for the equivalent non-energy intensive parameter, means that the marginal user cost in the energy intensive sector responds by 50% more to changes in interest rate than does the marginal user cost in the accumulation of non-energy intensive capital goods.

Further, the estimate of the marginal costs of capital utilization is 32% in the energy intensive sector while it is 44% in the non-energy intensive sector. These different estimates simply indicate that return to investment/ marginal product of capital services in the non-energy intensive sector is higher. Greenwood et al. (1988) and related literature [see, for example, Finn (1991, 1996), Baxter and Farr (2005), and Leduc and Sill (2004)] suggest that there is yet to be an empirical guide on the choice of a value (magnitude) for the elasticity of capital utilization rate, with Basu and Kimball (1997) admitting that it is a problematic parameter to value correctly. Our estimates here are 5.51 and 4.32, respectively, for the energy and non-energy elasticities of capital utilization rates. Lastly, all the elasticity of substitution parameters in the aggregator functions are sensibly in the ballpark of other estimates found in the literature.

4.2.2 Error Properties of the Estimated Model

Using the parameter estimates reported in Table 5, the functional forms provided in the previous section, the relevant log-linearized equilibrium conditions, and the actual time series discussed in Section 3, we extract the structural errors from the model for the twelve exogenous variables. The estimated AR(1) equations are given by

$$\widehat{\mathsf{A}}_{e} = 0.4641 \widehat{\mathsf{A}}_{e,-1} + \varepsilon^{\mathsf{a}^{e}}, \ \varepsilon^{\mathsf{a}_{e}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.0174^{2}\right)$$
(39)

$$\widehat{\mathsf{A}}_{n} = 0.2171 \widehat{\mathsf{A}}_{n,-1} + \varepsilon^{\mathsf{a}^{n}}, \varepsilon^{\mathsf{a}_{e}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.0402^{2}\right)$$

$$\tag{40}$$

$$\widehat{\mathsf{D}}^{w} = 0.6625 \widehat{\mathsf{D}}_{-1}^{w} + \varepsilon^{\mathsf{d}^{w}}, \ \varepsilon^{\mathsf{d}^{w}} \stackrel{i.i.d.}{\backsim} N\left(0, 4.6214^{2}\right)$$

$$\tag{41}$$

$$\widehat{\mathsf{G}} = 0.4076\widehat{\mathsf{G}}_{-1} + \varepsilon^{\mathsf{g}}, \, \varepsilon^{\mathsf{g}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.0572^2\right) \tag{42}$$

$$\widehat{\zeta} = 0.5750\widehat{\zeta}_{-1} + \varepsilon^{\zeta}, \ \varepsilon^{\zeta} \stackrel{i.i.d.}{\backsim} N\left(0, 0.2157^2\right)$$
(43)

$$\widehat{\mathsf{O}}_e = 0.6009 \widehat{\mathsf{O}}_{e,-1} + \varepsilon^{\mathsf{o}_e}, \ \varepsilon^{\mathsf{o}_e} \stackrel{i.i.d.}{\backsim} N\left(0, 2.0775^2\right)$$
(44)

$$\widehat{\mathsf{O}}_{n} = 0.6025 \widehat{\mathsf{O}}_{n,-1} + \varepsilon^{\mathsf{o}_{n}}, \ \varepsilon^{\mathsf{o}_{n}} \stackrel{i.i.d.}{\backsim} N\left(0, 3.9343^{2}\right)$$

$$\tag{45}$$

$$\widehat{\mathsf{P}}_{e}^{im} = 0.5572 \widehat{\mathsf{P}}_{e,-1}^{im} + \varepsilon^{\mathsf{p}_{e}^{im}}, \ \varepsilon^{\mathsf{p}_{e}^{im}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.0353^{2}\right)$$
(46)

$$\widehat{\mathsf{Q}} = 0.4353\widehat{\mathsf{Q}}_{-1} + \varepsilon^{\mathsf{q}}, \, \varepsilon^{\mathsf{q}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.2959^2\right) \tag{47}$$



Figure 2: Residuals from the estimated model.

$$\hat{\tau} = 0.5759\hat{\tau}_{-1} + \varepsilon^{\tau}, \, \varepsilon^{\tau} \stackrel{i.i.d.}{\backsim} N\left(0, 1.2076^2\right) \tag{48}$$

$$\widehat{\mathsf{Z}}_{e} = 0.4585 \widehat{\mathsf{Z}}_{e,-1} + \varepsilon^{\mathsf{z}_{e}}, \ \varepsilon^{\mathsf{z}_{e}} \stackrel{i.i.d.}{\backsim} N\left(0, 0.0875^{2}\right)$$

$$\tag{49}$$

$$\widehat{\mathsf{Z}}_n = 0.5708\widehat{\mathsf{Z}}_{n,-1} + \varepsilon^{\mathsf{z}_n}, \ \varepsilon^{\mathsf{z}_n} \stackrel{i.i.d.}{\backsim} N\left(0, 0.1074^2\right)$$
(50)

Figures 2 and 3 plot the residuals (actual errors/ shocks) and innovations respectively of the estimated model. We observe that all the shocks are mildly persistent, with the highest AR coefficient being that of the world demand shock and the lowest that of non-energy intensive sector productivity shock. It is also observed that world demand, labor supply, energy intensive sector energy efficiency, non-energy intensive sector energy efficiency, non-energy intensive sector energy efficiency, energy price, and intertemporal preference shocks are the most volatile.

4.2.3 Assessing the Fit of the Estimated Model Using Directed Wald

Table 6 documents the estimates of the VAR coefficients given the estimated structural errors. Compared to the results reported in Table 4 for calibrated parameter values, the present results are a big improvement for the tests of individual and joint distributions of the model vis-à-vis the data, but especially regarding their joint distribution. More specifically, evaluating the estimated model against the data for output, real exchange rate, energy use, and consumption, the model fits the data jointly well having acceptance of roughly 93% level when both the dynamics and volatilities of the four variables are included. The transformed



Figure 3: Innovations of the estimated model.

Mahalanobis distance (TMD) is 1.46. The dynamic fit of the model to the data passes the Wald test slightly better at the 91.7% level with an associated TMD of 1.3. This suggests a strong causality between the four VAR components. The model continues to fail massively in capturing the data variances for both the individual and joint distributions, however; in fact, there is evidence that the model's predictions of these variables have a deteriorating effect on the model fit. Evidently, the model is predicting the size of the joint data variances incorrectly.

Figure 4 lends support to these findings. The estimated model matches the persistence of each of the four macroeconomic variables well - plotted on the diagonal. Only real exchange rate, at lags 6-7, failed to fall within the 95% bound. We plot the cross-correlations of the four variables on the off-diagonal points of the graph. This replicates qualitatively the quantitative results discussed above, i.e. that the model is unable to match the individual cross-correlations found in the data. The model appears to capture well the negative and positive cross-effects at lags and leads of 2, with the best fits involving real exchange rate correlations. On all occasions, the simulated and actual data diverge on contemporaneous cross-effects between these variables.

Regarding the individual distributions of the VAR coefficients, all but one of the sixteen dynamic VAR parameters measuring autocorrelations and cross-correlations lie within the 95% confidence bounds, which is also an improvement over the results obtained for the calibrated parameter values. The only cross-effects that lies outside of the 95% confidence bounds is that from consumption to energy use. Meanwhile, all four data variances are now over-predicted by the model. In particular, the calibrated and estimated model differ

Coefficients	Actual	Lower Bound	Upper Bound	Mean
$\rho_{\widehat{Y},\widehat{Y}}$	0.4913	0.0316	1.0204	0.5756
$ ho_{\widehat{P},\widehat{Y}}$	-0.0076	-0.0628	0.0982	0.0132
$ ho_{\widehat{E},\widehat{Y}}$	-0.0354	-0.0720	0.0646	-0.0039
$\rho_{\widehat{C},\widehat{Y}}^{-,-}$	0.1080	-0.0767	0.1086	0.0107
$\rho_{\widehat{Y},\widehat{P}}$	0.0047	-2.2335	3.3631	0.5176
$\rho_{\widehat{P},\widehat{P}}$	0.6797	0.0987	0.8771	0.5107
$\rho_{\widehat{E},\widehat{P}}$	0.1341	-0.3020	0.4652	0.0780
$\rho_{\widehat{C},\widehat{P}}^{-,-}$	0.4435	-0.4135	0.5990	0.0769
$\rho_{\widehat{Y},\widehat{E}}$	0.4621	-5.1792	4.1954	-0.6893
$\rho_{\widehat{P},\widehat{E}}$	-0.1641	-0.8366	0.6663	-0.0840
$ ho_{\widehat{E},\widehat{E}}$	0.4959	-0.2948	1.0473	0.3765
$\rho_{\widehat{C},\widehat{E}}$	-1.2027^{\P}	-1.0638	0.7703	-0.0884
$\rho_{\widehat{Y},\widehat{C}}$	0.0697	-0.6549	1.8098	0.5547
$\rho_{\widehat{P},\widehat{C}}$	0.0041	-0.2242	0.1765	-0.0225
$\rho_{\widehat{E},\widehat{C}}$	-0.0198	-0.1247	0.2351	0.0556
$\rho_{\widehat{C},\widehat{C}}^{-,\circ}$	0.5978	0.4833	0.9146	0.7138
$\sigma_{\widehat{V}}^2$	0.0009 ^{\$}	0.0019	0.0068	0.0040
$\sigma^{2}_{\widehat{P}}$	$0.0314^{ a}$	0.0364	0.2030	0.0971
$\sigma^2_{\widehat{F}}$	$0.0413^{ a}$	0.1614	0.4604	0.2851
$\sigma_{\widehat{C}}^{\widetilde{Z}}$	0.0004^{\natural}	0.0198	0.0633	0.0374
	Dynamics+Volatilities	Dynamics	Volatilit	ies
Wald (%)	93.1	91.7	98.1	
TMD	1.46	1.30	2.61	

Table 6: VAR Results for Estimated Model

Note: $\rho_{i,j}$ denotes the VAR coefficient of a lagged variable *i* on a variable *j*; σ_j^2 denotes the variance of variable *j*; \hat{Y} denotes aggregate output; \hat{P} denotes the real exchange rate; \hat{E} denotes aggregate energy use; \hat{C} denotes consumption; TMD denotes transformed Mahalanobis distance. The VAR coefficients that lie outside of the 95% confidence bounds; ^{\$\$} the data variances of the variables that lie outside of the 95% confidence bounds.

in their predictions of the variances of the real exchange rate, with the data variance for real exchange rate closer to the lower bound of the acceptance interval for the estimated model. The result is opposite for the calibrated model.

We turn next to the VAR impulse response functions of output, real exchange rate, energy use, and consumption to the twelve shocks as a way to gleaning more insights into the dynamic behavior of the estimated model economy. These are shown in Figure 5, where the VAR shocks have been identified using the structural model. There appears to be congruence in the responses of both the model and the data to all the shocks for output, real exchange rate, and energy use, with their responses placed inside the 95% bounds both in the short- and the long-term. This is an interesting result yielding confidence in respect



Figure 4: Cross-correlations: data vs. estimated model

Note: The variables included are output (GDP(t)), real exchange rate (RXR(t)), energy use (E(t)) and consumption (C(t)). The dashed lines are the 95% bound of the cross-correlations observed in the data.

to the possible usability of the model for policy-related work by adapting the structural impulse response functions to determine the influences of shocks and in creating appropriate policy responses [see Christiano et al. (2005)].

We cannot say the same, however, for consumption, since there are consistent short-run differences between the model and the data, especially for non-energy intensive sector productivity, energy intensive sector energy efficiency, non-energy intensive sector energy efficiency, imported price of energy intensive goods, and energy price shocks. We have not included all the other shocks notably the energy intensive sector productivity, labor supply, and non-energy intensive investment-specific technology shocks, chiefly because there is little difference between borderline non-rejection/ rejection given the very stringent econometric procedure

Statistic	$\frac{\sigma(Y)}{\sigma(Y)}$	$\frac{\sigma(RXR)}{\sigma(Y)}$	$\frac{\sigma(E)}{\sigma(Y)}$	$\frac{\sigma(C)}{\sigma(Y)}$
Data	1.00	5.86	6.72	0.65
Model	1.00	4.76	8.49	3.10
	$\frac{\sigma(Y_{DATA})}{\sigma(Y_{MODEL})}$	$\frac{\sigma(RXR_{DATA})}{\sigma(RXR_{MODEL})}$	$\frac{\sigma(E_{DATA})}{\sigma(E_{MODEI})}$	$\frac{\sigma(C_{DATA})}{\sigma(C_{MODEL})}$
$\sigma(* DATA)$	(MODEL)	(MODEL)	(MODEL)	(*MODEL)
$\frac{\sigma(*_{MODEL})}{\sigma(*_{MODEL})}$	0.48	0.60	0.38	0.10
(MODEL)				

 Table 7: Business Cycle Statistics

Note: * = Y (output), RXR (real exchange rate), E (energy use), C (consumption), with σ denoting standard deviation.

we have applied [see Davidson et al. (2010)].

4.2.4 Accounting for the U.S. Business Cycles Post-WW II

To further study the business cycle implications of the estimated model, we report in Table 7 the standard deviations of real exchange rate, energy use, and consumption relative to that of output, and plot the model's prediction of recession compared to that of the data in panel **a** of Figure 6. Given the identified recessions in both data and model, we conclude this sub-section by providing a ranking of the behavior of the four macroeconomic time series included in the estimation following the realization of the shock in panel **b** of Figure 6. Before generating the model statistics and the plots, we simulate 1000 artificial economies each with same length as the actual data observations. The model statistics and the plots are averages of the 1000 simulations.

It is reported in Table 7 that the model predicts values of the relative volatilities of real exchange rate and energy use that are quite similar to that found in the data. On the other hand, it failed to capture the relative volatility observed for consumption with massive over-prediction. All in all, the model generates higher volatilities for the four time series compared to the data. Further, the model matches the timing and persistence of recessions very well, but it is less successful in replicating the recovery rate. The economy is normally back to its pre-recession level of output 3 years after the shock according to the data, whereas the model requires a much longer time.

Overall, it can be seen in panel **b** of Figure 6 that the model is able to preserve the after-shock business cycle behavior of real exchange rate in relation to output, but not much that of energy use and consumption. In the data, energy use relative to output does not move much, but the model picks up a decline in demand for energy given a rise in its price. Lastly, perhaps due to the lower consumption inertia estimated, there is little consumption smoothing in the model such that it drops instantly in response to the shock.



Figure 5: VAR Impulse Response Functions

Note: The endogenous variables are output (*GDP*, column 1), real exchange rate (*RXR*, column 2), energy use (*EGY*, column 3), and consumption (*CON*, column 4); the exogenous variables are energy intensive sector productivity (A_e , row 1), non-energy intensive sector productivity (A_n , row 2), world demand (D^w , row 3), government spending (G, row 4), labor supply (ζ , row 5), energy intensive sector energy efficiency (O_e , row 6), non-energy intensive sector energy efficiency (O_n , row 7), price of imported energy intensive goods (P_e^{im} , row 8), energy price (Q, row 9), intertemporal preference (τ , row 10), energy intensive goods investment-specific technology (Z_e , row 11), and non-energy intensive goods investment-specific technology (Z_n , row 12).



a. Prediction of recession by energy real business cycle model



b. Business cycle ranking of data and model





5 Concluding Remarks

The model developed in this paper has worked reasonably well both in fitting the model to the data for our benchmark macroeconomic variables of output, real exchange rate, energy use, and consumption and also in capturing some salient facts regarding the macroeconomic-oil relationship and some fundamental business cycle dynamics. Clearly, the model was only able to explain our selected features leaving many questions unanswered. To investigate whether this is a model problem, or just a circumstance of the parameters in use, we carried out a further Directed Wald test. The main outcomes include that (1) if estimated using the SA algorithm, the model appears capable of fitting the data for several variable combinations, (2) when any parameter set (that fits the model to the data) for *certain* variable combination is used as a benchmark on which other different variable combinations are evaluated, we find that the model can only explain along some lines and fails to fit the data for the remainder, suggesting that the parameters are not globally useful within the context of this model. This implies that, as for most research, our model requires a focused, definitive question of enquiry to be posed a priori. Given this, a model can then be applied to answer the proposed question. It is, therefore, a matter for future research to extend this model to deal with economic issues along other dimensions, but it is our hope that this model will be adjusted appropriately to answer many other macroeconomic questions, particularly those that concern energy issues and their policy implications.

We speculate on some of these possible future extensions to the model. First, a useful extension for model implications regarding energy policies and welfare evaluation would be to consider an economy with money (M/P) and energy use (E^h) by the household,²³ which can be achieved by increasing the arguments in the utility function (7)

$$\mathbb{E}\sum_{0}^{\infty}\beta\tau U\left(C-\iota C_{-1},\zeta H,\mathsf{j}E^{h},\mathsf{v}\frac{M}{P}\right)$$
(51)

where j and v are optional exogenous disturbances to energy use and real money balances, respectively. Then, depending on if we are interested in considering the extent of price flexibility in the economy, quadratic adjustment costs for nominal wage and price rigidities can be specified [see, for example, Hairault and Portier (1993) and Kim (2000)]. Moreover, explicit modelling of the four goods may be important. We are inclined to think that the most relevant contribution for this approach may be pursued by allowing traded goods to be used not only for consumption/ investment purposes, but to be translated into the production process, since not all exchanged goods are completely finished goods [see, for example, Goldberg and Campa (2010),

 $^{^{23}}$ As an example, see Dhawan and Jeske (2008) and Blanchard and Gali (2007) who modeled energy use by households providing stylized facts on the share of household energy use in total GDP in the vicinity of that reported for the firms.

Bergholt and Sveen (2014), and Eyquem and Kamber (2014)].²⁴ This particular extension may have the following added advantages: (1) Beyond incorporating the level of energy intensity, this should help to admit the different levels of trade intensities of the two sectors; and (2) We may also be able to examine the terms of trade for goods of both sectors with the addition of extra cost channels via trade in intermediate goods - this would be an extension to many open economy models that only analyze aggregate terms of trade [see for example Backus et al. (1993), Mendoza (1995), and Backus and Crucini (2000)].

Secondly, the complete inter-country effects through (in)complete risk sharing and uncovered interest parity can be studied by allowing both the domestic and foreign residents investment opportunities in both the domestic and foreign bonds [see, for example, Gali and Monacelli (2005) and Corsetti et al. (2008)]. Lastly, there are two more issues to contend with. The first is on the theoretical side and involves the noninclusion of the non-traded sector, which Bruno (1976) refers to as an "unimportant appendage" perhaps included just for completeness. It has, however, since been seen to be an important sector of the economy [see, for example, Tesar (1989), Stockman (1990), Stockman and Tesar (1995)]. In fact, Stockman (1990) wrote that "Both theory and evidence on open economies suggest the inclusion of non-traded goods and multiple traded goods", which in that sense could further improve the model presented in this paper. The second is an empirical problem and relates to the treatment of the data. An important and new effort could then be applied to work with non-stationary data set in this model context.

To kick-start a putative agenda to address some of these issues, Meenagh et al. (2015) introduced capital accounts into the model developed here, increasing both the number of domestic and foreign shocks, and estimating the model on non-stationary data. Their results suggest that the above proposals could yet prove to be more fruitful in future research endeavors.

 $^{^{24}}$ A simplifying assumption in the model presented in this paper is that imported/ exported goods can directly be consumed or invested on arrival without any need for added value. This is rarely the case.

References

- Backus, D. K., & Crucini, M. J. (2000). Oil prices and the terms of trade. Journal of international Economics, 50(1), 185-213.
- Backus, D., Kehoe, P. J., & Kydland, F. E. (1993). International business cycles: theory and evidence (No. w4493). National Bureau of Economic Research.
- [3] Basu, S., and Kimball, M. S. (1997). Cyclical productivity with unobserved input variation (No. w5915).National Bureau of Economic Research.
- [4] Baxter, M. and Crucini, M. J. (1995). Business Cycles and the Asset Structure of Foreign Trade. International Economic Review, 36(4), 821-854.
- [5] Baxter, M., & Farr, D. D. (2005). Variable capital utilization and international business cycles. Journal of International Economics, 65(2), 335-347.
- [6] Bergin, P. R. (2003). Putting the 'new open economy macroeconomics' to a test. Journal of International Economics, 60(1), 3-34.
- [7] Bergholt, D., & Sveen, T. (2014). Sectoral interdependence and business cycle synchronization in small open economies.
- [8] Blanchard, Olivier J., and Jordi Gali. The Macroeconomic Effects of Oil Shocks: Why are the 2000s so different from the 1970s?. No. w13368. National Bureau of Economic Research, 2007.
- [9] Boileau, M., & Normandin, M. (1999). Capacity utilization and the dynamics of business cycle fluctuations. Centre de recherche sur l'emploi et les fluctuations économiques, Université du Québec à Montréal (CREFÉ)= Center for Research on Economic Fluctuations and Employment (CREFE).
- [10] Boldrin, M., Christiano, L. J., & Fisher, J. D. (2001). Habit persistence, asset returns, and the business cycle. American Economic Review, 149-166.
- [11] Bruno, M. (1976). The two-sector open economy and the real exchange rate. The American Economic Review, 566-577.
- [12] Bruno, M., & Sachs, J. D. (1985). Economics of worldwide stagflation. NBER Books.
- [13] Burnside, C. & Eichenbaum, M. (1996). Factor hoarding and the propagation of business cycle shocks. American Economic Review.

- [14] Campbell, J. Y. (1994). Inspecting the mechanism: An analytical approach to the stochastic growth model. Journal of Monetary Economics, 33(3), 463-506.
- [15] Chari, V. V., Kehoe, P. J., & McGrattan, E. R. (2002). Can sticky price models generate volatile and persistent real exchange rates?. The Review of Economic Studies, 69(3), 533-563.
- [16] Christiano, L. J. (1988). Why does inventory investment fluctuate so much?. Journal of Monetary Economics, 21(2), 247-280.
- [17] Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of political Economy, 113(1), 1-45.
- [18] Corsetti, G., Dedola, L., & Leduc, S. (2008). International risk sharing and the transmission of productivity shocks. The Review of Economic Studies, 75(2), 443-473.
- [19] Cromb, R., & Corugedo, E. F. (2004). Long-term interest rates, wealth and consumption. Bank of England Working Paper No 243.
- [20] Costello, D. (1993). A cross-country, cross-industry comparison of the behavior of Solow residuals. Journal of Political Economy, 101, 207-22.
- [21] Davidson, J., Meenagh, D., Minford, P., & Wickens, M. R. (2010). Why crises happen-nonstationary macroeconomics.
- [22] Dhawan, R., & Jeske, K. (2008). Energy price shocks and the macroeconomy: the role of consumer durables. Journal of Money, Credit and Banking, 40(7), 1357-1377.
- [23] Engel, C. (1999). Accounting for US Real Exchange Rates. The Journal of Political Economy, 107, 507-538.
- [24] Eyquem, A., & Kamber, G. (2014). A note on the business cycle implications of trade in intermediate goods. Macroeconomic Dynamics, 18(05), 1172-1186.
- [25] Finn, M. G. (1991). Energy price shocks, capacity utilization and business cycle fluctuations. Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis.
- [26] Finn, M. G. (1996). A theory of the capacity utilization/inflation relationship. FRB Richmond Economic Quarterly, 82(3), 67-86.

- [27] Finn, M. G. (2000). Perfect competition and the effects of energy price increases on economic activity. Journal of Money, Credit and banking, 32(3), 400-416.
- [28] Fuhrer, J. C. (2000). Habit formation in consumption and its implications for monetary-policy models. American Economic Review, 367-390.
- [29] Gali, J., & Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. The Review of Economic Studies, 72(3), 707-734.
- [30] Goldberg, L. S., & Campa, J. M. (2010). The sensitivity of the CPI to exchange rates: Distribution margins, imported inputs, and trade exposure. The Review of Economics and Statistics, 92(2), 392-407.
- [31] Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. The American Economic Review, 402-417.
- [32] Hairault, J. O., & Portier, F. (1993). Money, New-Keynesian macroeconomics and the business cycle. European Economic Review, 37(8), 1533-1568.
- [33] Hall, R. E. (1988). Intertemporal Substitution in Consumption. The Journal of Political Economy, 96, 339.
- [34] Hamilton, J. D. (1983). Oil and the macroeconomy since World War II. The Journal of Political Economy, 228-248.
- [35] Hodrick, R. J., & Prescott, E. C. (1997). Postwar US business cycles: an empirical investigation. Journal of Money, credit, and Banking, 1-16.
- [36] Hooker, M. A. (1997). Exploring the robustness of the oil price-macroeconomy relationship. Division of Research and Statistics, Division of Monetary Affairs, Federal Reserve Board.
- [37] Iacoviello, M., Schiantarelli, F., & Schuh, S. (2011). Input and Output Inventories in General Equilibrium. International Economic Review, 52(4), 1179-1213.
- [38] Kim, J. (2000). Constructing and estimating a realistic optimizing model of monetary policy. Journal of Monetary Economics, 45(2), 329-359.
- [39] Kim, I. M., & Loungani, P. (1992). The role of energy in real business cycle models. Journal of Monetary Economics, 29(2), 173-189.

- [40] King, R. G., & Rebelo, S. T. (1999). Resuscitating real business cycles. Handbook of macroeconomics, 1, 927-1007.
- [41] King, R. G., Plosser, C. I., & Rebelo, S. T. (1988a). Production, growth and business cycles: I. The basic neoclassical model. Journal of monetary Economics, 21(2), 195-232.
- [42] King, R. G., Plosser, C. I., & Rebelo, S. T. (1988b). Production, growth and business cycles: II. New directions. Journal of Monetary Economics, 21(2), 309-341.
- [43] Kocherlakota, N. (1988). In defense of the time and state separable utility-based asset pricing model. Manuscript (Northwestern University, Evanston, IL).
- [44] Kose, M. A. (2002). Explaining business cycles in small open economies: 'How much do world prices matter?'. Journal of International Economics, 56(2), 299-327.
- [45] Kydland, F. E., & Prescott, E. C. (1982). Time to build and aggregate fluctuations. Econometrica: Journal of the Econometric Society, 1345-1370.
- [46] Le, V. P. M., Meenagh, D., Minford, P., & Wickens, M. (2011). How much nominal rigidity is there in the US economy? Testing a New Keynesian DSGE Model using indirect inference. Journal of Economic Dynamics and Control, 35(12), 2078-2104.
- [47] Leduc, S., & Sill, K. (2004). A quantitative analysis of oil-price shocks, systematic monetary policy, and economic downturns. Journal of Monetary Economics, 51(4), 781-808.
- [48] Long Jr, J. B., & Plosser, C. I. (1983). Real business cycles. The Journal of Political Economy, 39-69.
- [49] Loungani, P. (1986). Oil price shocks and the dispersion hypothesis. The Review of Economics and Statistics, 536-539.
- [50] McCallum, B.T., (1976). Rational expectations and the natural rate hypothesis: some consistent estimates. Econometrica 44, 4-52.
- [51] McCallum, B. T. (1989). Monetary economics: Theory and policy. Prentice Hall.
- [52] Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. International Economic Review, 101-137.
- [53] Mork, K. A. (1989). Oil and the macroeconomy when prices go up and down: an extension of Hamilton's results. Journal of Political Economy, 740-744.

- [54] Nelson, E., & Nikolov, K. (2004). Monetary Policy and Stagflation in the UK. Journal of Money, Credit, and Banking, 36(3), 293-318.
- [55] Prescott, E. C. (1986, November). Theory ahead of business-cycle measurement. In Carnegie-Rochester Conference Series on Public Policy (Vol. 25, pp. 11-44). North-Holland.
- [56] Rotemberg, J. J., & Woodford, M. (1996). Imperfect competition and the effects of energy price increases on economic activity. Journal of Money, Credit and Banking, 28(4), 549-577.
- [57] Shiells, C. R., & Reinert, K. A. (1993). Armington models and terms-of-trade effects: some econometric evidence for North America. Canadian Journal of Economics, 299-316.
- [58] Sims, Christopher A. Comments on "On the Fit of New Keynesian Models," Journal of Business & Economic Statistics, 2007, 25, 152-154.
- [59] Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. Journal of the European economic association, 1(5), 1123-1175.
- [60] Smets, F., & Wouters, R. (2004). Forecasting with a Bayesian DSGE model: An application to the Euro area. National Bank of Belgium Working Paper, (60).
- [61] Smets, F., & Wouters, R. (2005). Bayesian new neoclassical synthesis (NNS) models: Modern tools for central banks. Journal of the European Economic Association, 3(2-3), 422-433.
- [62] Stern, R. M., Francis, J., & Schumacher, B. (1976). Price elasticities in international trade: An annotated bibliography. London: Macmillan.
- [63] Stockman, A. C. (1980). A theory of exchange rate determination. The Journal of Political Economy, 673-698.
- [64] Stockman, A. C., & Tesar, L. L. (1990). Tastes and technology in a two-country model of the business cycle: Explaining international comovements (No. w3566). National Bureau of Economic Research.
- [65] Stockman, A. C., & Tesar, L. L. (1995). Tastes and technology in a two-country model of the business cycle: Explaining international comovements. American Economic Review, 85(1), 168-185.
- [66] Tesar, L. L. (1991). Savings, investment and international capital flows. Journal of International economics, 31(1), 55-78.

- [67] Uribe, M., & Yue, V. Z. (2006). Country spreads and emerging countries: Who drives whom?. Journal of international Economics, 69(1), 6-36.
- [68] Whalley, J. (1985). Trade liberalization among major world trading areas. MIT Press.
- [69] Wickens, M.R., 1982. The efficient estimation of econometric models with rational expectations. Review of Economic Studies 49, 55-67.