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# Working Paper Assessing the role of interbank network structure in business and financial cycle analysis

NBB Working Paper, No. 307

**Provided in Cooperation with:** National Bank of Belgium, Brussels

*Suggested Citation:* Gnabo, Jean-Yves; Scholtes, Nicolas K. (2016) : Assessing the role of interbank network structure in business and financial cycle analysis, NBB Working Paper, No. 307, National Bank of Belgium, Brussels

This Version is available at: https://hdl.handle.net/10419/173763

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Assessing the role of interbank network structure in business and financial cycle analysis



# by Jean-Yves Gnabo and Nicolas K. Scholtes

# October 2016 No 307



# Assessing the role of interbank network structure in business and financial cycle analysis<sup>\*</sup>

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October 6, 2016

#### Abstract

We develop a DSGE model incorporating a banking sector comprising 4 banks connected in a stylised network representing their interbank exposures. The micro-founded framework allows *inter alia* for endogenous bank defaults and bank capital requirements. In addition, we introduce a central bank who intervenes directly in the interbank market through liquidity injections. Model dynamics are driven by standard productivity as well as banking sector shocks. In our simulations, we incorporate four different interbank network structures: Complete, cyclical and two variations of the core-periphery topology. Comparison of interbank market dynamics under the different topologies reveals a strong stabilising role played by the complete network while the remaining structures show a non-negligible shock propagation mechanism. Finally, we show that central bank interventions can counteract negative banking shocks with the effect depending again on the network structure.

*Keywords*: Interbank network, DSGE model, banking, liquidity injections JEL Classification: D85, E32, E44, E52, G21

<sup>&</sup>lt;sup>\*</sup>This paper has been prepared for the 2016 National Bank of Belgium conference on "*The Transmission Mechanism of New and Traditional Instruments of Monetary and Macroprudential Policy*". We thank Olivier Pierrard, Grégory de Walque and Raf Wouters for useful comments. The views expressed are those of the authors and do not necessarily reflect those of the ECB or the NBB. Financial support from the NBB and ARC grant No. 13/17-055 is gratefully acknowledged

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# 1 Introduction

In the aftermath of the global financial crisis, there was a strong drive amongst academics and policymakers alike to better understand the role of the financial sector as a source of macroeconomic fluctuations as well as the various amplification mechanisms associated to financial shocks. Moreover, there has been a renewed interest in modelling the *supply*-side of credit markets in order to highlight the important role played by banks in driving the business cycle. From a macroeconomic modelling perspective, this has led to a reappraisal of the demand-side credit market frictions developed by Kiyotaki and Moore (1997) and Bernanke et al. (1999) while recognising the caveat related to the reliance on Modigliani and Miller (1958) irrelevance in assuming away supply-side conditions revolving around bank behaviour in credit markets. Moreover, economic models post-crisis are being tailored to reflect the current economic climate, characterised by increasing financial system regulation and central bank interventions.

Parallel to these developments in macro-financial modelling, the notion that financial system *interconnectedness* can impair financial stability has opened up a research agenda seeking to apply tools from the *network theory* literature to study the threats to systemic risk posed by the various types of financial interdependence. Broadly speaking, the majority of papers in this literature aim to quantify the role of the network structure (commonly referred to as its *topology*) as a shock propagation and amplification mechanism. The seminal contribution by Allen and Gale (2000) finds that increasing the density of linkages between financial institutions has a mitigating effect on the propagation of liquidity shocks to individual banks. More recently, Elliott et al. (2014) and Acemoglu et al. (2015) develop models of cascading defaults wherein the network structure and the location of the shock<sup>1</sup> in the network determine the extent of financial contagion.

This paper provides a first attempt at reconciling these two seemingly unrelated developments by proposing a framework for combining the network structure of the interbank market with a macroeconomic mode that *inter alia* allows for interactions between the banking sector and the wider economy as well as imperfections in interbank and credit markets. To this end, we develop a microfounded framework that incorporates an active banking sector and interbank market into a dynamic stochastic general equilibrium (DSGE) model. This is combined with the network interpretation of the interbank market by treating the pattern of interlinkages between banks as given. Consequently, the manner in which banks are connected in the network is mapped into the bank microfoundations by condi-

<sup>&</sup>lt;sup>1</sup>This is known as the "*robust-yet-fragile*" (Haldane, 2013) property of financial networks. The fact that certain nodes are more susceptible to engendering widespread failure by virtue of their location of the network has been well documented. Key references include Nier et al. (2007) and Gai and Kapadia (2010).

tioning the set of variables associated to interbank transactions *viz.* lending, borrowing and endogenous default choices on the set of counterparties given by the network.

Our model is based on De Walque et al. (2010) (hereafter, WPR) who develop a DSGE model of the real business cycle (RBC)-variety while allowing for an endogenous banking sector, bank regulation (in the form of a capital requirement) and monetary policy through liquidity injections into the interbank market. Before introducing the networks dimension, we modify their construction of a heterogeneous banking sector, comprising deposit and merchant banks (interbank lenders and borrowers, respectively) by combining them into one bank who intermediates between households and firms. Following this, the key contribution of the paper is the application of the stylised four-node network methodology of Allen and Gale (2000) and Lee (2013), which we use to construct four representative interbank networks: 1) a complete network 2) a cyclical network and 3) two variations of the coreperiphery topology. The structure is thus imposed exogenously and dictates the pattern of lending and borrowing counterparties.

As stated by Goodhart et al. (2006), the interactive dimension of bank behaviour is crucial from a financial stability perspective as well as for the design and implementation of monetary policy and (macro)prudential requirements.<sup>2</sup> The manner in which networks are introduced into the DSGE methodology is flexible enough that it factors into the policy aspect as well. Specifically, our regulatory framework consists of a capital requirement combined with a linear utility term for holding a capital buffer in excess of the regulator-imposed minimum. The network is featured via the dynamic risk-weights associated to interbank lending which are driven by banks' expectations on their (network-determined) counterparties loan repayment ability. In addition, the monetary authority injects liquidity on a bilateral basis wherever there is a mismatch between supply and demand of interbank funding. Consequently, each link in the network constitutes a market (in which a price, the interbank rate, is formed). As a result, the number and structure of the links in the network affects the impact of monetary policy.

Though the paper draws primarily from WPR, there is a growing literature incorporating a banking sector into DSGE models<sup>3</sup>. Within this framework, they introduce various endogenous financial frictions and an active interbank market. Notably, Gertler and Kiyotaki (2010) introduce credit market frictions in both the retail and wholesale financial markets as well as a comprehensive analysis of Federal Reserve credit policies. Another strand of research adds a layer of realism by introducing imperfect competition in the form

 $<sup>^{2}</sup>$ From a regulatory standpoint, the Basel III reforms identify interconnectedness as one of the five categories for determining global systemically important banks (BCBS, 2010).

<sup>&</sup>lt;sup> $^{3}$ </sup>The interested reader is referred to Vlcek and Roger (2012) for a comprehensive survey of their use at various central banks.

of '*market power*' into bank behaviour. Such an approach has been applied by Gerali et al. (2010) and Dib (2010) and Pariès et al. (2011), who also incorporate regulation in the form of a capital-based measure. The role played by financial shocks in driving business cycle fluctuations is studied in Christiano et al. (2010).

Due to the growing availability of increasingly granular financial data, researchers have begun mapping real interbank networks in order to study the risks to financial stability. A common thread that has emerged across various studies is that interbank networks exhibit a *tiered* or *core-periphery* architecture wherein a small set of large 'core' banks intermediate between a larger set of smaller 'peripheral' banks who do not interact amongst themselves. The systemic implications of such a configuration are clear, as shown in the theoretical model of Freixas et al. (2000) who study the impact of central bank liquidity interventions in the presence of a large money center bank. From an empirical standpoint, various interbank markets are shown to exhibit a core-periphery structure namely, the TARGET2 (Gabrieli and Georg, 2015), Belgian (Degryse et al., 2007), UK (Langfield et al., 2014), US Fedwire (Soramäki et al., 2007), German (Craig and Von Peter, 2014), Italian e-MID (Fricke and Lux, 2015) and Dutch (van Lelyveld et al., 2014) interbank markets.

Our choice of WPR as a modelling benchmark is driven by our desire to shed light on the macroeconomic impact of the *structure* of the interbank market. By abstracting from the more complex modelling approaches and frictions mentioned above, we are able to zoom in on the network drivers of economic and financial fluctuations. Broadly speaking, our model can be summarised as follows: We introduce four 'regions' that raise liquidity on the *inter*-regional wholesale market, characterised by a specific network structure, in order to complement *intra*-regional deposits used to finance *intra*-regional credit<sup>4</sup>.

The results of our model are presented in two steps: first we compare the effect of adding a regional banking shock to a baseline scenario consisting of four productivity shocks across all regions under the different network structures. Herein, we study the impact of each imposed network on the dynamics of *interbank* variables. The second step compares the networks directly by analysing the responses of aggregated (that is, summed across all four regions) variables from the real economy. In this section, we provide a policy perspective by comparing model dynamics when banks are subject to liquidity injections by the central bank.

Our results highlight the importance of taking the network into account when studying economic fluctuations, a key reason being that local shocks are easily transmitted via banks' interlocking exposures. Thus, a bank not itself subject to a shock can still be affected

<sup>&</sup>lt;sup>4</sup>In this regard, our model is similar to the region-specific DSGE with banking sector models developed by Brzoza-Brzezina et al. (2015) Bokan et al. (2016).

through its network of counterparties (and their counterparties etc.) This is made clear in the interbank dynamics under the cyclical and core-periphery topologies. The latter is prone to large potential instability when the core bank is limited in its ability to obtain wholesale funding. By contrast, the complete network performs a stabilising, *dissipative* role due to the large number of links. Turning to the policy side, we observe that the impact of liquidity injections varies depending on the network. The complete network is again, subject to relatively smaller downturns while the remaining structures require stronger central bank intervention.

The rest of the paper is structured as follows. Section 2 develops the model's microfoundations, section 3 outlines the imposed network structures, section 4 provides the calibrations used, section 5 reports the results and section 6 concludes.

# 2 The Model Economy

Following Allen and Gale (2000), the economy is divided into four ex-ante identical regions  $i = \{A, B, C, D\}$ . Each region consists of a household and firm who act as lenders and borrowers of funds, respectively. In addition, we introduce a regional banking sector which finances lending to firms through access to two markets. In the retail financial market, banks obtain deposits from households within the same region while the wholesale financial market allows banks across regions to raise funds by borrowing and lending amongst themselves.

The microfoundations in our model are based primarily on De Walque et al. (2010). However, in addition to the aforementioned regional structure, our approach differs in a number of ways. Firstly, we depart from the notion that banks perform a specialised function as either originators or receivers of funding stemming from household deposit supply and firm credit demand, respectively. In this setup, the interbank market arises to restore equilibrium between banks with a liquidity surplus and those in deficit. By contrast, the banks in our approach perform a dual role by intermediating between regional households and firms and lending/borrowing on the wider interbank market. As such, each regional bank is subject to counterparty default on credit markets (by firms for whom the repayment rate is endogenous) as well as on the interbank market, where borrowing banks feature an endogenous repayment rate on past interbank loans. Other features of the banking sector include own funds commitment, insurance funds and portfolio diversification.

The second major contribution is our use of a stylised, tractable framework for modelling bank interconnectedness across regions. As detailed in section 3, the manner in which banks can exchange liquidity is constrained by the network structure. In this regard, two banks i, j can be connected such that i can only borrow from j or vice versa.

The economy is represented schematically in Figure 1. In addition to the three types of agents, all flows between them as well as the relevant interest rates are reported. Since our approach allows banks to both lend and borrow on the interbank market simultaneously<sup>5</sup>, we compile bank *i*'s lending and borrowing choice vis-à-vis counterparty *j* into the parameter  $\Xi^{b,ij} = \{L^{b,ij}, B^{b,ij}\}$  for ease of exposition.<sup>6</sup>



Figure 1: Flows between agents within and across regions

<sup>&</sup>lt;sup>5</sup>The concept that banks enter into long and short interbank positions has been observed empirically, using German balance sheet data, by Bluhm et al. (2016)

<sup>&</sup>lt;sup>6</sup>Note that the interbank trading parameters given by  $\Xi$  capture the maximum number of exposures between banks as represented by the 'complete' network in Figure 3(a).

#### 2.1 Households

The household in region *i* chooses consumption  $C_t^{h,i}$  and deposit supply to its local bank  $D_t^{h,i}$  to maximise a logarithmic utility function comprising a quadratic disutility term for deposits. This represents households' preference for a stable level of deposits around their long-run optimum.

$$\max_{\left\{C_{t}^{h,i}, D_{t}^{h,i}\right\}} \sum_{s=0}^{\infty} \beta^{s} E_{t} \left[ \ln\left(C_{t+s}^{h,i}\right) - \frac{\chi}{2} \left(\frac{D_{t+s}^{h,i}}{1+r_{t}^{d,i}} - \frac{\bar{D}^{h}}{1+r^{d}}\right)^{2} \right]$$
(1)

The household budget constraint is given by

$$T_t + C_t^{h,i} + \frac{D_t^{h,i}}{1 + r_t^{d,i}} = w_t N_t + D_{t-1}^{h,i}$$
(2)

where  $T_t$  denotes a lump-sum tax levied on households to finance both liquidity injections into the banking sector by the central bank and an insurance fund that allows banks to recover a fraction of non-performing loans on the interbank and credit market. Finally, we impose an exogenous labour supply  $N_t = \bar{N}$ . Solving the dynamic problem yields the following Euler equation for consumption (augmented with the deposit target term):

$$\frac{1}{C_t^{h,i}} = \beta E_t \left[ \frac{1}{C_{t+1}^{h,i}} \right] - \frac{\chi}{1 + r_t^{d,i}} \left( \frac{D_{t+s}^{h,i}}{1 + r_t^{d,i}} - \frac{\bar{D}^h}{1 + r^d} \right)$$
(3)

### 2.2 Banks

The primary function of bank *i* in each region  $i = \{A, B, C, D\}$  is the intermediation of funds between depositors (households) and ultimate borrowers (firms). This comprises the retail market of the national financial market. In addition, we allow banks to obtain and provide wholesale funding on the interbank market. Counterparty information is provided by the sets  $S_i$  (suppliers) and  $\mathcal{D}_i$  (demanders) which determine from whom *i* can obtain funds and to whom *i* can provide funds, respectively. The expected payoff function of bank *i* consists of a concave representation of profits,  $\pi_t^{b,i}$  less a non-pecuniary disutility cost  $d_b$ associated to the endogenous default decision vis-à-vis its interbank creditors,  $j \in S_i$ . On the interbank market, the set of endogenous variables for each *i* thus consists of the default rate  $\delta_t^{b,ij}$  on past borrowing as well as current borrowing,  $B_t^{b,ij}$  from creditors  $j \in S_i$  and bilateral lending,  $L_t^{b,ij}$  to debtor banks  $j \in \mathcal{D}_i$ . In their role as financial intermediaries, banks choose fund allocation from amongst deposits from households  $\mathcal{D}_t^{b,i}$  and credit to firms in region i,  $X_t^{h,i}$  as well as investment in risky securities,  $S_t^{b,i}$ . Finally, banks are subjected to a positive linear utility  $d_{F^b}$  for the buffer of own funds chosen by the bank  $F_t^{b,i}$  above the minimum regulatory capital requirement represented by the coverage ratio k as well as the respective risk weights  $w_t^{f,i}$ ,  $\left\{w_t^{b,ij}\right\}_{j\in\mathcal{D}_i}$ ,  $w_t^S$  on firm credit, interbank loans and the bank's securities portfolio. The balance sheet of each bank can thus be represented as:



Figure 2: Bank balance sheet

Putting these together yields the following bank maximisation programme:

$$\max_{\left\{\left\{\delta_{t}^{b,ij},B_{t}^{b,ij}\right\}_{j\in\mathcal{S}_{i}},\left\{L_{t}^{b,ij}\right\}_{j\in\mathcal{D}_{i}},s=0}^{\infty} E_{t}\left\{\beta^{s}\left[\ln\left(\pi_{t+s}^{b,i}\right)-d_{b}\sum_{j\in\mathcal{S}_{i}}\left(1-\delta_{t+s}^{b,ij}\right)\right]^{2}\right\} + d_{F^{b}}\left(F_{t+s}^{b,i}-k\left(w_{t+s}^{f,i}X_{t+s}^{b,i}+\sum_{j\in\mathcal{D}_{i}}w_{t+s}^{b,ij}L_{t+s}^{b,ij}+w_{t}^{S}S_{t+s}^{b,i}\right)\right)\right\},$$

$$(4)$$

subject to constraints on the profit function, the law of motion of own funds and the dynamic evolution of the capital requirement risk weights:

$$\begin{aligned} \pi_{t}^{b,i} &= \frac{D_{t}^{b,i}}{1+r_{t}^{d,i}} - \frac{X_{t}^{b,i}}{1+r_{t}^{l,i}} + (1+\Gamma_{t}) S_{t-1}^{b,i} - S_{t}^{b,i} \\ &+ \sum_{j \in \mathcal{S}_{i}} \frac{B_{t}^{b,ij}}{1+r_{t}^{b,ij}} + \sum_{j \in \mathcal{D}_{i}} \delta_{t}^{bji} L_{t-1}^{b,ij} + \zeta_{b} \sum_{j \in \mathcal{D}_{i}} \left(1-\delta_{t-1}^{bji}\right) L_{t-2}^{b,ij} + \alpha_{t}^{f,i} X_{t-1}^{b,i} + \zeta_{f} \left(1-\alpha_{t-1}^{f,i}\right) X_{t-2}^{b,i} \\ &- \left[D_{t-1}^{b,i} + \sum_{j \in \mathcal{D}_{i}} \frac{L_{t}^{b,ij}}{1+r_{t}^{b,ij}} + \sum_{j \in \mathcal{S}_{i}} \delta_{t}^{b,ij} B_{t-1}^{b,ij} + \frac{\omega_{b}}{2} \left(\sum_{j \in \mathcal{S}_{i}} \left(1-\delta_{t-1}^{b,ij}\right) B_{t-2}^{b,ij}\right)^{2}\right] \end{aligned}$$
(5)

$$F_t^{b,i} = (1 - \xi_b) F_{t-1}^{b,i} + \nu_b \pi_t^{b,i}$$
(6)

$$w_t^{f,i} = \tilde{w}^f \left[ \left( \frac{\alpha}{\alpha_{t+1}^{f,i}} \right)^{\eta_f} \right] \tag{7}$$

$$w_t^{b,ij} = \tilde{w}^b \left[ \left( \frac{\delta}{\delta_{t+1}^{b,ij}} \right)^{\eta_b} \right] \text{ for each } j \in \mathcal{D}_i$$
(8)

Equation 5 defines the period profits of bank i. The first line collects the bank's activity on the retail and credit market namely, deposits remunerated at rate  $r_t^{d,i}$  less credit to firms at the rate  $r_t^{l,i}$  as well as the stochastic return on past securities investment, denoted by  $\Gamma_t$  which follows an AR(1) process, less current purchases. The second line collects the parameters pertaining to an *inflow* of funds in period t. The first two terms outline interbank borrowing from all creditors  $j \in S_i$  and repayment by debtor banks  $j \in D_i$  on past interbank loans. Note that this includes the repayment rate  $\delta_t^{b,ji}$  which is featured in counterparty j's optimisation programme and thus, taken as given by i. The insurance fund allows banks to recover a fraction  $\zeta_b$  of each borrowing counterparty's default on the interbank market. Similarly, each bank is subject to the default of regional firms on credit extended in the previous period. Similar to interbank lending, the repayment rate is exogenous to the bank while an insurance fund allows it to recover a fraction  $\zeta_f$  of the firm's defaulted amount. Finally, the third line collects all outflows of funds. This includes lending on the interbank market as well as repayment on past interbank loans. The latter features the repayment rate parameter  $\delta_t^{b,ij}$  which is now endogenous to *i*. The last term represents the quadratic pecuniary penalty associated to interbank loan default and reflects *inter alia* the reputation costs of defaulting.

Equation 6 provides the law of motion of own funds. This consists of contributions to the insurance fund managed by the government (represented by the parameter  $\xi_b$ , this is used to recover losses from counterparty defaults) and an exogenous fraction of profits  $\nu_b$ redirected towards own funds. Equations 7 and 8 reflect the risk-sensitive credit weights on loans to firms and banks, respectively. Set by the supervisory authority, the risk weights increase as expectations of default increase.

A key feature of our microfounded network model is that the pattern of interlinkages between banks maps into the *number* and *composition* of the policy functions. To be precise, given that each banks' optimisation programme features three bilateral interbank variables (those superscripted by *ij* namely, lending, borrowing and default), each additional interbank link thus gives rise to three additional first-order conditions.

The above characterises the relationship between the network and the number of policy functions. The difference in composition occurs when imposing an *asymmetric*<sup>7</sup>network structure where banks differ in the manner in which they're connected to their counterparties. For example, if bank *i* only lends to bank *j*, this is reflected in the interbank components of *i* profit function comprising only  $L^{b,ij}$  while removing the borrowing variables  $B^{b,ij}$  and  $\delta^{b,ij}$  (with the inverse holding for bank *j*). Another bank in the same network that both lends and borrows would feature all interbank variables.

For the sake of brevity, we compute the first-order conditions assuming the (symmetric) cyclical network topology shown in Figure 3(b) wherein each bank *i* has two distinct counterparties, one from which it borrows (indexed by j) and another to which it lends (indexed by k):<sup>8</sup>

$$\lambda_t^{\pi} B_{t-1}^{b,ij} = E_t \left[ \beta \lambda_{t+1}^{\pi} \omega_b \left( 1 - \delta_t^{b,ij} \right) \left( B_{t-1}^{b,ij} \right)^2 \right] + d_b \tag{9}$$

$$\frac{\lambda_t^{\pi}}{1+r_t^{b,ik}} = E_t \left[ \beta \lambda_{t+1}^{\pi} \delta_{t+1}^{b,ik} + \beta^2 \lambda_{t+2}^{\pi} \zeta_b \left( 1 - \delta_{t+1}^{b,ki} \right) \right] - d_{F_b} k \bar{w}_t^{b,ik} E_t \left[ \left( \frac{\delta}{\delta_{t+1}^{b,ki}} \right)^{B} \right]$$
(10)

$$\frac{\lambda_t^{\pi}}{1+r_t^{b,ij}} = E_t \left[ \beta \lambda_{t+1}^{\pi} \delta_{t+1}^{b,ij} + \beta^2 \lambda_{t+2}^{\pi} \omega_b \left( 1 - \delta_{t+1}^{b,ij} \right)^2 B_t^{b,ij} \right]$$
(11)

$$\frac{\lambda_t^{\pi}}{1+r_t^{l,i}} = E_t \left[ \beta \lambda_{t+1}^{\pi} \alpha_{t+1}^{f,i} + \beta^2 \lambda_{t+2}^{\pi} \zeta_f \left( 1 - \alpha_{t+1}^{f,i} \right) \right] - d_{F_b} k \bar{w}_t^{f,i} E_t \left[ \left( \frac{\alpha}{\alpha_{t+1}^{f,i}} \right)^{\eta_f} \right]$$
(12)

$$\frac{\lambda_t^{\pi}}{1+r_t^{d,i}} = E_t \left[\beta \lambda_{t+1}^{\pi}\right] \tag{13}$$

$$d_{F_b}\nu_b = \left(\lambda_t^{\pi} - \frac{1}{\lambda_t^{\pi}}\right) - E_t \left[\beta \left(1 - \xi_b\right) \left(\lambda_{t+1}^{\pi} - \frac{1}{\lambda_{t+1}^{\pi}}\right)\right]$$
(14)

where  $\lambda_t^{\pi}$  is the Lagrange multiplier associated to bank profits and Equations 9-12 are the Euler equations for interbank lending, borrowing and firm lending respectively.

<sup>&</sup>lt;sup>7</sup>Throughout the paper, we refer to symmetric networks as those in which each bank features the same number of incoming and outgoing links. In this case, the choice of optimising bank is arbitrary as their interbank policy functions will be identical albeit with different counterparty superscripts.

<sup>&</sup>lt;sup>8</sup>Appendix A reports the relevant equations for all networks.

# 2.3 Firms

Each firm maximises the discounted sum of expected payoffs by choosing employment, borrowing from its regional bank in the current period and the repayment rate on previous period borrowing,  $\alpha_t^{f,i}$  (the default rate from the point of view of the bank). Similar to the bank, defaulters are subject to both a linear disutility cost,  $d_f$  as well as a quadratic pecuniary cost on profits, represented by the parameter  $\omega_f$ . The firm maximisation programme is then given by

$$\max_{\left\{N_{t}, X_{t}^{f,i}, \alpha_{t}^{f,i}, K_{t}, \right\}} \sum_{s=0}^{\infty} E_{t} \left\{ \beta^{s} \left[ \pi_{t+s}^{f,i} - d_{f} \left( 1 - \alpha_{t+s}^{f,i} \right) \right] \right\}$$
(15)

subject to the following constraints

$$\pi_t^{f,i} = A_t Y_t^i - w_t^i N_t^i - \alpha_t^{f,i} X_{t-1}^{f,i} - \frac{\omega_f}{2} \left[ (1 - \alpha_{t-1}^{f,i}) X_{t-2}^{f,i} \right]^2$$
(16)

$$K_t^i = (1-\tau)K_{t-1}^i + \frac{X_t^{j,i}}{1+r_t^{l,i}}$$
(17)

where Equation 16 represents period profits of the firm.  $A_t$  is a stochastic AR(1) total factor productivity shock. Each firm produces output using an identical CRS Cobb-Douglas production function with capital and labour as inputs,  $K_t^{\mu} N_t^{1-\mu}$ . Equation 17 is the law of motion of capital with depreciation rate  $\tau$  and expansion of capital stock financed by firm borrowing from its regional bank. The first-order conditions are given by:

$$w_t^i = (1-\mu)A_t \left(\frac{K_t^i}{N_t^i}\right)^{\mu}$$
(18)

$$\frac{\lambda_t^K}{1+r_t^{l,i}} = E_t \left[ \beta \alpha_{t+1}^{f,i} + \beta^2 \omega_f \left(1-\alpha_{t+1}^{f,i}\right)^2 X_t^{f,i} \right]$$
(19)

$$X_{t-1}^{f,i} = \beta \omega_f \left(1 - \alpha_t^{f,i}\right) \left(X_{t-1}^{f,i}\right)^2 + d_f$$

$$(20)$$

$$\mu A_t \left(\frac{K_t^i}{N_t^i}\right)^{\mu-1} = \lambda_t^K - E_t \left[\beta \lambda_{t+1}^K (1-\tau)\right]$$
(21)

### 2.4 Central bank and government

#### 2.4.1 Government

As mentioned, the government levies a lump-sum tax on households which is used to fund the insurance scheme (in addition to the period contributions into the fund by banks and firms) against interbank counterparty and regional firm default. We assume that central bank money creation is not financed by the immediate lump-sum tax on households. Rather, we assume that the central bank is not balance-sheet constrained and can thus create real cash balances by itself.

In addition to the standard Ricardian equivalence assumption of fiscal policy, we also assume that each of the four regions is responsible for its own taxation scheme. For the interbank market, this implies that each regional government provides insurance to its own local bank against default to its counterparties in other regions. Thus, each regional government is tasked with minimising outgoing spillovers due to local financial strains. Finally, by treating taxation in a disaggregated manner, we abstract from consideration of the redistributive effects of taxation. The government budget constraint thus comprises four equations (one for each region) of the form:

$$T_{t} = \zeta_{b} \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{D}_{i}} \left( 1 - \delta_{t-1}^{b,ij} \right) B_{t-2}^{b,ij} + \sum_{i \in \mathcal{N}} \left( 1 - \alpha_{t-1}^{f,i} \right) X_{t-2}^{b,i} - \xi_{b} \sum_{i \in \mathcal{N}} F_{t-1}^{b,i}$$
(22)

The first two terms in Equation 22 collect all payments to banks out of the insurance fund. The last term (in parentheses) denotes payments into the insurance fund (taken out of banks' own funds).

#### 2.4.2 Central bank

Since we restrict our attention to a purely real model, the standard approach to shortterm nominal rate setting via a Taylor policy-rule does not apply in this context. In our framework, the central bank injects liquidity into the banking system in order to equalise supply and demand of interbank funding between each pair of connected banks<sup>9</sup>. The general form of central bank liquidity injections is thus given by

$$M_t^{ij} = B_t^{ij} - L_t^{ji}, \quad \forall i, j \in \mathcal{E}$$

$$\tag{23}$$

where  $M_t^{ij} > (<)0$  represents an injection (withdrawal) of liquidity by the central bank

 $<sup>^{9}</sup>$ The identity of connected banks depends on the particular network structure applied to represent the interbank market, as will be made clear in Section 3

into (from) the bilateral transfer between banks i and j. We assume no liquidity injections at steady state,  $M_t^{ij} = 0, \forall i, j \in \mathcal{N}$ .

While the above equation signifies how the monetary policy instrument in our setup features in banking sector dynamics, it does not provide the main objective of monetary policy. Following WPR, liquidity injections serve to smooth interbank rate fluctuations relative to their long run value<sup>10</sup> via the following rule:

$$M_t^D = \nu \left( \bar{r}^b - r^b \right) \tag{24}$$

As  $M_t$  is also driven by autoregressive shocks, we use the superscript D to denote the deterministic component of liquidity injections. The variable  $\bar{r}^b$  denotes the average interbank rate. From a network perspective, each directed edge between banks constitutes a market for which a price is determined either purely through market forces (setting  $\nu = 0$ ) or through a combination of market forces and central bank liquidity injections. In this case, the value of  $\nu > 0$  represents the responsiveness of the central bank to deviations in the average interbank rate from its steady state value.

### 2.5 Closing the model

### 2.5.1 Structural shocks

The model features three types of autoregressive shocks, total factor productivity shocks, shocks to bank profits (represented by an unexpected change in the market book) and a liquidity shock . All three follow stochastic AR(1) processes. The productivity shock is applied to firms in all four regions and is given by:

$$A_t^i = \left(A_{t-1}^i\right)^{\rho_A} \exp\left(\varepsilon_t^A\right) \tag{25}$$

where  $i = \{A, B, C, D\}$ . In the first section of our impulse response analysis (see section 5.1), we treat the aggregate productivity shock scenario as a benchmark against which we study the *additional* impact of a banking shock on variable dynamics<sup>11</sup>. The banking shock is applied on a *regional* rather than aggregate basis in order to highlight how the shock is transmitted across the network via banks' interconnected interbank asset and liability

<sup>&</sup>lt;sup>10</sup>Though highly stylised, this methodology closely mirrors central banks' use of the overnight rate to signal the policy stance and launch the monetary policy transmission mechanism. An early study by Bernanke and Blinder (1992) confirms the importance of the US federal funds rate as an indicator for monetary policy. More recently, Linzert and Schmidt (2011) study the drivers of the widening spread between the EONIA and ECB MRO rate.

<sup>&</sup>lt;sup>11</sup>From a modelling perspective, this implies taking advantage of the linearity of the policy functions to add the impulse responses under the two shocks.

structures. The banking shock specification is as follows:

$$\Gamma_t^A = \left(\bar{\Gamma}\right)^{1-\rho_{\Gamma}} (\Gamma_{t-1})^{\rho_{\Gamma}} \exp\left(\varepsilon_t^{\Gamma}\right)$$
(26)

where  $\overline{\Gamma} > 0$  is the (calibrated) average market book return of banks. In this case, the shock is applied only to bank A. As will be made clear in section 3, the choice of A as the target for a banking shock is arbitrary for two of the structures, the complete and cyclical networks, due to their symmetry. By contrast, for the (asymmetric) core-periphery topologies, the *location* of the shock has a marked impact on its propagation dynamics.

Similar to the above scenario in which a regional banking shock is compared to a benchmark comprising an aggregate productivity shock, section 5.2 maintains the banking and productivity shock scenario as a benchmark against which to compare the impact of central bank liquidity injections. Recall that these consist of a deterministic component given by Equation 24 as well as a stochastic AR(1) component given by:

$$M_t^S = \rho_M M_{t-1} + \exp\left(\varepsilon_t^M\right) \tag{27}$$

The central bank policy rule is then obtained by adding the stochastic and deterministic (i.e. liquidity injection) components:

$$M_t = M_t^S + M_t^D \tag{28}$$

Finally, all innovations are assumed to be i.i.d-normally distributed i.e.  $\varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2)$ where  $Z = \{A, \Gamma, M\}$ .

### 2.5.2 Market clearing

Note that the central bank liquidity injections given in Equation 23 provide the clearing conditions for the interbank market. In order to bridge the gap between the policy rule (Equation 28) and central bank interventions, we simply divide  $M_t$  by the number of links of the network being considered. This yields the individual  $M_t^{ij}$  values, thereby assuming equal treatment by the central bank of the interbank market constituents.<sup>12</sup>

In the simulations that do not feature liquidity injections, interbank market clearing is simply given by:

$$B_t^{ij} = L_t^{ji}, \quad \forall i, j \in \mathcal{E}$$

$$\tag{29}$$

<sup>&</sup>lt;sup>12</sup>Though this stylised approach is an abstraction from reality, it bears some similarity to the pre-crisis allotment policies of the major central banks who fix the aggregate volume of open market operations (OMOs) at their discretion (Blenck et al., 2001).

# 3 Interbank network structures

An interbank network consists of a set of banks connected by interbank claims on one another. In section 2, we outlined banks' optimisation programmes subject to the optimising behaviour of regional households and firms as well as other banks to whom they are connected (which constitutes the interbank market). The latter revolves around the lending and borrowing counterparty sets  $S_i$  and  $\mathcal{D}_i$  respectively which constrain to/from whom banks can provide/request liquidity. Up to now, these sets remain undefined. In this section, we construct several stylised network structures based on completeness and interconnectedness. Consequently, the interbank network provides the foundation on which the DSGE model is superimposed.

# 3.1 Complete and cyclical topologies

In the complete (i.e. perfectly interconnected) network, each bank has exposures to all other banks in the system. Though unrealistic, the complete network structure allows us to study the case where banks are maximally connected. In our stylised model of four banks, each bank thus lends to and borrows from the three remaining banks corresponding to six directed edges per banks for a total of 12 edges in the network. This is captured in Figure 3(a) below<sup>13</sup>:



Figure 3: (a) Complete and (b) Cyclical network topologies

The cyclical structure in Figure 3(b) assumes that each bank has one borrowing and one lending relationship to two distinct adjacent banks. Notice that both topologies are symmetric across banks. Depending on the interbank network structure in place, the support of the bank optimisation programme given in Equation 4 consisting of interbank market transactions (lending, borrowing and defaults) will vary. For example, under a cyclical

<sup>&</sup>lt;sup>13</sup>For simplicity, we combine the ingoing and outgoing edges between bank pairs. A double-headed arrow between nodes i and j indicates that i both lends to and borrows from j and vice versa.

network, the set of bilateral transactions consists of the following set of optimal lending and borrowing choices and the interbank rate that clears each market:<sup>14</sup>

$$\Xi = \left\{ \left\{ L^{b,AC}, B^{b,CA} \right\}, \left\{ L^{b,CD}, B^{b,DC} \right\}, \left\{ L^{b,DB}, B^{b,BD} \right\}, \left\{ L^{b,BA}, B^{b,AB} \right\} \right\}$$

$$R = \left\{ r^{b,AC}, r^{b,CD}, r^{b,DB}, r^{b,BA} \right\}$$

# 3.2 Core-periphery topologies

Though the aforementioned network topologies are interesting cases for the study of the implications of financial interconnectedness, they are somewhat unrealistic and cannot be seen to represent the structure of real interbank networks. By contrast, mounting empirical evidence points towards the tiered/core-periphery topology as representative of the *form* and *function* of the interbank market. Here, the form refers to the densely interconnected core connected to a sparsely interconnected periphery while the function refers to the fact that larger core banks act as money centre banks who intermediate between smaller peripheral banks who do not exchange liquidity amongst themselves (Craig and Von Peter, 2014).

While maintaining the same stylised 4-bank structure, we provide the following two core-periphery configurations:



Figure 4: Core-periphery network with (a) core bank as a net borrower and (b) core as a net lender

Unlike the previous setup, the core-periphery topology features more distinct roles for banks vis-à-vis each other. Under both configurations, bank A acts as a market-maker, redistributing liquidity across the banking system. However, in configuration (a), the core

<sup>&</sup>lt;sup>14</sup>For bilateral rates  $r^{b,ij}$ , we adopt the following convention: The first superscript *i* denotes the lending counterparty (located at the tail of the link connecting them in the network) while the second corresponds to the interbank borrower.

relies on two banks (C and D) for wholesale funding compared to one (bank B) in configuration (b).

This asymmetry between banks depending on their position in the network implies that the location of the banking shock is no longer arbitrary as in the complete and cyclical frameworks. To account for this in our analysis, we target the banking shock according to the following three configurations:



Figure 5: Shock configurations under core-periphery net borrower network.

In configuration (a), the core bank is subject to a market book shock. By virtue of its centrality in the network, the shock is then transmitted to *all* remaining banks due to the core optimally changing its portfolio composition in response to the shock. Note however, that banks B and C will be impacted differently due to their different roles as core borrower and core lender, respectively. By contrast, D will face the same impact as C. In the results, we thus only report the impulse responses for C for the sake of brevity.

The same set of shocks are applied to the net-lender setup given in Figure 4(b). In this case, a shock to bank B (Figure 5(b)) will have a different impact due to B's role as the sole provider/recipient of interbank funding.

# 4 Calibration

We calibrate our model following WPR who use average historical real quarterly US data from 1985Q1 to 2008Q2. However, our modelling methodology differs along two dimensions: (i) we combine their heterogeneous banking sector into one representative, regional bank who intermediates directly between households and firm in the same region and (ii) the interbank market is represented as a form of intra-regional transfers of funds between banks. Consequently, the parameter values *inferred* at steady state will not only differ from those in their paper (due to the difference in modelling framework) but will also differ across the imposed network structures in our own approach. Relative to WPR, we maintain the same base weight values on risky asset exposures. However in their approach, bank exposures are *either* to the interbank market (in the case of the deposit bank) or to the credit market (in the case of the merchant bank) while our intermediary is exposed to firm as well as interbank default risk on the asset side of its balance sheet. In addition, the steady states values for various flow variables obey the following ratios as per WPR: L = 0.5X, B = L and D = 2X while the steady state interbank and firm repayment rates are given by  $\delta = 0.99$  and  $\alpha = 0.95$ , respectively. Table 1 below reports the remaining banking sector calibrations:

Table 1:	Parameter	calibration:	Banks
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Parameter	Definitions	Value
	$Capital\ requirement$	
k	Minimum own funds ratio	0.08
$ ilde{w}^{f,i}$	Risk weight: loans to firms	0.8
$\tilde{w}^{b,ij}$	Risk weight: interbank loans	0.05
$ ilde{w}^S$	Risk weight: market book	1.20
	Insurance fund	
$\zeta_b$	Insurance coverage: interbank default	0.80
$\zeta_f$	Insurance coverage: firm default	0.80
$\vartheta_b$	Insurance fund contributions from profits	0.5

We collect the parameters implied at steady state in Tables 2 and 3 in order to study how they differ depending on the network structure. The first table provides the inferred values for the complete and cyclical networks. The three rates were set in order to minimise the differences between the two sets of parameters.

Table 2: Inferred parameters: Banks (Symmetric networks)

Parameter	Definition	Network structure	
		Complete	Cyclical
$r^d$	Deposit rate	0.5%	0.5%
$r^l$	Prime lending rate	0.1%	0.5%
$r^b$	Interbank rate	1.2%	1%
$d_b$	Interbank default disutility	3773	3642
$d_{F^b}$	Own funds utility	7849	9148
$\xi_b$	Insurance fund contribution	0.0548	0.0640
$\omega_b$	Interbank default cost	326	532

As shown in Appendix A, the manner in which nodes are connected in the core-periphery structure imposes a degree of asymmetry which affects the set of variables in banks' optimisation programmes (depending on their location in the network).

Paramete	Parameter Definition Network structure						
			CP-nb			CP-nl	
		A	В	C/D	A	В	C/D
$r^d$	Deposit rate	"	0.05%	"	"	0.05%	"
$r^l$	Prime lending rate	"	0.04%	"		0.04%	"
$r^b$	Interbank rate	"	0.09%	"	"	0.09%	"
$d_b^i$	Interbank default disutility	3760	5233	-	2325	-	5233
$d_{F^b}^i$	Own funds utility	10462	12458	9290	8131	150150	12458
$\xi_b^i$	Insurance fund contribution	0.0480	0.0403	0.0540	0.0617	0.0540	0.0403
$\omega_b$	Interbank default cost		637			637	

Table 3: Inferred parameters: Banks (Asymmetric networks)

Tables 4 and 5 report the parameters associated to the real economy (firms and households) and the stochastic processes specified in the model.

Parameter	Definitions	Value
	Households	
$\frac{\chi}{\bar{N}}\\\bar{D}^h$	Deposit gap disutility Labour supply Deposit target	0.01 0.20 0.38
	Firms	
$egin{array}{c} d_f \ \omega_f \ \mu \  au \end{array}$	Firm default disutility Firm default cost Capital share Capital depreciation rate	$\begin{array}{c} 0.163 \\ 15 \\ 0.333 \\ 0.03 \end{array}$

Table 4: Parameter calibration: Real economy

### Table 5: Parameter calibration: Exogenous processes

Parameter	Definition	Value
$\rho_A$	AR parameter: productivity shock	0.95
$\sigma_A$	Standard deviation: productivity shock	0.1
$ ho_{\Gamma}$	AR parameter: banking shock	0.5
$\sigma_{\Gamma}$	Standard deviation: banking shock	0.1
$ ho_M$	AR parameter: liquidity shock	0.5
$\sigma_M$	Standard deviation: liquidity shock	0.1

# 5 Results

In this section, we discuss the results of the model. In order to analyse how the network structure contributes to financial stability, we provide two simulation studies.<sup>15</sup> Both take the form of a '*crisis simulation*' whereby the innovations of the relevant stochastic processes are set to one negative standard deviation. All impulse responses are reported as variations from the steady state, in % points for the repayments and in % for the other variables.

# 5.1 Responses to a banking shock

In the first simulation, we focus on the effect of adding a banking shock to the baseline productivity shock as outlined in Section 2.5. Given that all banks are ex-ante homogenous and the regional productivity shocks are identical, impulse responses across the four banks in the benchmark scenario will not vary. The banking shock introduces ex-post heterogeneity into the system. As this is calibrated to hit only one of the four banks, analysing the impact on the remaining banks via the different network structures (i.e. those in regions only hit by productivity shocks to firms) will provide an indication of its shock transmission properties. To this effect, our impulse response analysis in this section focuses specifically on the evolution of banking sector variables namely, interbank rates, lending and borrowing and bank repayment rates.

#### 5.1.1 Cyclical network

Recall that the cyclical network given in Figure 3(b) entails one interbank lending and one borrowing relationship for each bank. In order to simplify the interpretation of the bilateral variables, the region to which a particular IRF is associated is given in bold font above the y-axis. Figures 6-7 below report the impulse responses of banking sector variables under the cyclical network topology.

As shown in Figure 6, interbank volumes experience a small increase on impact following the baseline aggregate productivity shock. This is immediately followed by a largermagnitude decrease wherein the exposures for all banks decrease relative to the steady state before gradually converging 20 periods into the simulation.

<sup>&</sup>lt;sup>15</sup>We are grateful to the authors of De Walque et al. (2010) for providing us with their Dynare codes.



Figure 6: Interbank lending and borrowing volumes - Cyclical network

By contrast, the addition of a regional banking shock is not only larger in magnitude for all exposures, but exhibits more persistent dynamics as well. For example, bank A's lending to  $C (L^{b,AC})^{16}$  shows an initial decrease of approximately 4% followed by an increase to a peak of 6% (with the transition from negative to positive occurring eight periods into the simulation).

Comparing this with  $(L^{b,BA})$  i.e. the volume of interbank liquidity borrowed by A, we see that the dynamics (and corresponding magnitudes) are inverted but identical. Thus changes in the shocked bank's lending behaviour are offset by changes in its borrowing behaviour. The interbank repayment rates, given in Figure 7 below further highlight this symmetry between the shocked bank's *immediate* counterparties. Namely, A's repayment rate on borrowing from B,  $\delta^{b,AB}$  features the same offsetting effect relative to C's repayment on borrowing from A,  $\delta^{b,CA}$ . Furthermore, comparing the loan volumes to their corresponding repayment rate<sup>17</sup> reveals that the initial decrease in lending from A to C results in a corresponding decrease in C's repayment to A and vice versa. This quid pro quo mechanism also applies to B's initial increase in lending to A.

Network effects come into play when observing the same intermediary dynamics for nonshocked banks as any banking dynamics herein occur solely due to the outward propagation from the source through the network. As before, analysis of  $L^{b,DB}$  and  $L^{b,CD}$  wherein the intermediary D is not subject to a banking shock, reveals a similar but *imbalanced* offsetting effect, with the trough in D's lending being smaller than the corresponding peak in borrowing.

<sup>&</sup>lt;sup>16</sup>Due to the market clearing conditions under no liquidity injections, studying both the lending and borrowing components of a bilateral exposure is redundant. We thus restrict our focus to interbank lending.

<sup>&</sup>lt;sup>17</sup>Note that insofar as the network links represent a flow of liquidity from a lender to a borrower, repayment rates flow in the opposite direction as they are undertaken by borrowers towards their lenders.



Figure 7: Interbank repayment - Cyclical network

The same asymmetry is present in the corresponding repayment rates (though the two still follow the same general trajectory as before). We explain these dynamics within the context of the network as follows: A's financial distress is propagated to B through increased loan delinquencies which then drives B to increase its borrowing and repayment from D. Given that the initial default is only transmitted in this direction due to the network (and not from A to C, as this entails a lending relationship), this accounts for the observed asymmetry.

The notion that the cyclical network transmits banking shocks outwards is further corroborated through analysis of the interbank rates. We begin by reporting the spread between the market rate on each bank's lending subtracted by the market rate of borrowing in Figure 8 below



Figure 8: Interbank rate spreads - Cyclical network

At first glance, it is apparent that strains in the regional productive sector do not affect (what can loosely be interpreted as) the bid-offer spread on wholesale funding. By contrast, the banking shock produces persistent dynamics across all banks' spreads. The leftmost figure reports the spread between the rate at which A provides liquidity (to C) and the rate at which it receives liquidity (from B). The initial widening on impact is contrasted by the

remaining spreads which experience a small decline followed by an increase in their lending relative to their borrowing rate. Analysis of the superscripts in panels B and C reveal that only one of the variables in the spreads stems from the shocked bank, which explains their smaller magnitudes. Panel D reports the spread for interbank rates not *directly* connected to A. As expected, the magnitude is smaller compared to the previous cases, indicating that while the shock does propagate through the network, its impact dissipates as proximity to the source decreases. Though less indicative than spreads, Appendix B provides a further decomposition of interbank rates. Specifically, Figure B.1 shows that the dynamics of standalone rates are largely driven by the decline due to the aggregate productivity shock. Isolation of the banking shock in Figure B.2 shows an initial increase in interbank rates across all regions. The magnitudes follow the cyclical structure with the initial rate increase being the largest for  $r^{AC}$  followed by  $r^{CD}$ ,  $r^{DB}$  and finally,  $r^{BA}$ .

# 5.1.2 Complete network

The complete network is characterised by a high density of linkages which makes a marketby-market analysis similar to the above more complex due to the large number of endogenous variables associated to each (interbank) link. However, the symmetry of the complete network (combined with the asymmetry introduced due to the localised banking shock) allows us to restrict our attention to a few key cases. These are provided above the relevant plots as 2- or 3-node schematics to indicate the counterparties being analysed. As before, we begin by looking at the impulse responses of interbank lending and borrowing volumes:



Figure 9: Interbank lending and borrowing volumes - Complete network

where the three scenarios correspond to (i) shocked bank lending to a non-shocked bank, (ii) non-shocked bank lending to a shocked bank and (ii) exposure between two non-shocked banks. Initial comparison with the interbank exposures under cyclicality provided in Figure 6 shows a smaller impact of the baseline under completeness. The same applies when a banking shock is incorporated. This highlights the dissipative characteristics of complete networks, as documented by Allen and Gale (2000). Under this structure, banks are maximally exposed to the wholesale market, allowing them to more efficiently redistribute risk across counterparties and limit individual exposures.<sup>18</sup> Analysis of the rightmost panel indicates that as the shock propagates outwards from A, the impact on lending between non-shocked banks is minimal, having almost the same trajectory as the baseline.

The interbank repayments behave in a similar manner, as shown in Figure 10 below with a smaller increase in banks' defaults under completeness than their cyclical counterparts. Comparing A's repayment on borrowing from B and B's default on borrowing from D, we obtain the intuitive result that the former exhibits a higher default rate due to the shock than the latter.



Figure 10: Interbank repayments - Complete network

We now proceed to the analysis of interest rate spreads. These involve three banks of which the intermediary is located in the middle of the three-node linear networks given below:

<sup>&</sup>lt;sup>18</sup>A counterargument in which the same links can exacerbate interbank tensions by acting as a channel for financial contagion for large shocks is well documented in the literature (Acemoglu et al., 2015). However, such mechanisms are not included in our theoretical model and thus, are outside the scope of this paper.



Figure 11: Interbank rate spreads - Complete network

As before, aggregate productivity shocks have minimal impact on interbank spreads. The largest swings occur when A is the intermediary whereas the smallest occurs when none of the banks involved is shocked, both of which are intuitive results. Moreover, there is a substantial difference in magnitudes (but not dynamics) between the scenarios where A is the first bank in the intermediation chain (third panel) compared to when it is the last (rightmost panel). Specifically, the magnitudes of the initial decline and peak 20 periods into the situation are both smaller when A is the terminal node in the chain compared to when it is the initial node, which further highlights the ability of the network to transmit local shocks beyond their immediate vicinity.

Another interesting case study involves the degree of *reciprocity* between bank pairs who simultaneously lend to and borrow from one another. This is captured in Figure 12 for the cases when one of the two banks or neither is subject to a banking shock:



Figure 12: Interbank rate spreads (reciprocity) - Complete network

As expected, the spread is negligible in the absence of a shock to either of the banks. By contrast, the lack of reciprocity is manifested in the left panel through an increase in A's lending rate to B relative to its borrowing rate. We end our discussion of interbank dynamics under the complete network by referring to Figures B.3 and B.4 in Appendix B which, unlike the cyclical dynamics, do not show any marked heterogeneity in interbank rates. This further exemplifies the stabilising role of the complete network.

# 5.1.3 Core-periphery networks

We end this section by reporting the IRFs under two variations of the core-periphery topology: one in which the core is a *net borrower* and one in which the core is a *net lender* on the interbank market as outlined in Figure 4. Given the inherent asymmetry of the structure, we expand the set of banking shock configurations from one to three in order to show how the *location* of the shock can affect inter-bank dynamics (see Figure 5).

**Core bank as net borrower** As in the previous cases, we begin our treatment by reporting the response of interbank volumes to an aggregate negative productivity shock and regional negative banking shock.



Figure 13: Interbank volumes under core-periphery network - net borrower case.

As expected, a negative shock to the core bank A results in the largest and most persistent fluctuations by virtue of its high centrality in the system. Upon impact, Areduces lending to B (its only borrower in this setup) while increasing its borrowing from C and D. Another intuitive result arises when C is subject to the banking shock: given its role as one of the main providers of funding for A and ultimately B, we observe a large decrease in lending from C to A followed by a slightly smaller decrease in lending from Ato B. In this case, A's role as an inter-bank intermediary dampens the pass-through of the shock. Interestingly, the same shock also produces a small reduction in D's lending to A. This shows that shock propagation in a core-periphery network is not necessarily linear and can impact banks off the direct transmission path. Finally, we observe that the shock to Bproduces (relative to the other configurations) subdued dynamics due to its relatively less important role in the interbank market compared to the intermediary and initial providers of funding.



Figure 14: Interbank repayment rates under core-periphery network - net borrower case.

As in the previous cases, repayment dynamics closely mirror interbank volumes. Closer observation of the impulse response magnitudes reveals that the initial increase in B's default when A is shocked is similar to A's default rate when C is shocked. This highlights that shocks to the 'periphery' can also drive interbank market tensions (compared to core shocks) when core banks are dependent on them for funding.

Due to its structure, the core-periphery network only permits two spreads, reported in Figure 15 which are symmetric around A for most of the shock specifications the exception being the shock to C which has a much smaller (but not zero) impact on the spread between B and D via bank A compared to the case given in the left figure.



Figure 15: Interbank rate spreads under core-periphery network - net borrower case.

The isolated banking shock reported in the Appendix (Figure B.6) shows an across the board increase in interbank rates. However, unlike the previous cases (complete and cyclical), where the initial increases were between 0.2 and 0.3 percentage points, several of the initial rate spikes were between 0.1 and 0.15 p.p. Closer analysis reveals that the shock to B results in (comparatively) smaller increases due to its non-central role in the network. By contrast, the core shock in the first row is followed by increases greater than 0.2 p.p with the largest occurring on A's lending to B. **Core bank as net lender** Under this setup, the core bank now has two banks that depend on it for interbank lending along with one sole source of wholesale funding.



Figure 16: Interbank volumes under core-periphery network - net lender case.

Despite having the same base-structure as the net-borrower case, the IRFs follow different trajectories with large differences in magnitude. For example, a banking shock to Bresults in a 20% decline in lending to A which then translates (via A) to a 10% decrease in lending to both C and D. This contrasts with the much smaller decline in volumes due to a shock to B under the net borrower case. As mentioned, the core-periphery specification imbues the network with a function in addition to the basic form. Under the net lender case, A is reliant on B for funding followed by both C and D being reliant on A. Thus a negative shock to the only source of funds will have a more pronounced effect than a shock to the ultimate recipient of funds. Similar to lending volumes, interbank repayments fare much worse under the net lender case than their net borrower counterparts



Figure 17: Interbank repayment rates under core-periphery network - net lender case.

The shock to B also has a strong impact on the interbank rate spreads, showing a relative increase in A's cost of borrowing. Interestingly, the shock to A exhibits much more subdued dynamics than its counterpart. We posit that this occurs due to the interbank market dynamics playing out in strong fluctuations in volumes and repayment rates rather

than in the interest rate. This is justified by Figure B.8 wherein the rate increase in the first column is smaller in magnitude than the second and third. The higher spike due to the B shock is consistent with the dynamics explored thus far.



Figure 18: Interbank rate spreads under core-periphery network - net lender case.

The main takeaway from the core-periphery analysis is that a small-variation in the pattern of linkages can have wide-ranging effects on the system's ability to withstand certain shocks. We have shown, in a stylised manner, that when a core bank is limited in its ability to obtain funding, this is easily transmitted to downstream banks who are themselves dependent on the core.

# 5.2 Network structure comparison: Impact on the real economy

The second simulation study provides the policy dimension of the paper. Specifically, we analyse how central bank liquidity injections (given by Equation 24) can alleviate strains to the *real economy* brought on by the banking shock and its transmission through the network. We now treat the second scenario from the first simulation (aggregate productivity and regional banking shock<sup>19</sup>) as the benchmark and toggle on the central bank policy function.

<sup>&</sup>lt;sup>19</sup>In the case of the core-periphery networks, we restrict our attention to the first configuration in which the core bank A is subject to the banking shock.

We begin by comparing the total volume of liquidity injections (m, whose dynamic)process is given in Equation 28) across the different network structures. In order to provide a basis for comparison, we normalise m by dividing by the number of links which yields:



Figure 19: Total normalised central bank liquidity injections

As expected, our crisis simulation prompts the central bank to action, injecting liquidity into the banking sector across all imposed networks. However, despite the same shock calibration, the central bank response varies widely across networks. As has been shown in the paper, the complete network is highly stable due to the shock-dissipative effect of its high interconnectedness. Interestingly, the core-periphery network in which the core bank is a net borrower requires the second lowest aggregate central bank intervention over time. We posit that this occurs due to the relative stability that having two sources of funding and only one source of default risk can provide. This argument is strengthened by the relatively inferior performance of the net lender case, in which the central bank had to intervene more aggressively. In this case, the core is subject to two sources of default risk and only one source of retail funding. As a result, it increases its reliance on wholesale funding.

Having compared the effect of interbank network structure on the dynamics of interbank variables in section 5.1, we now analyse how a negative banking shock, within the context of our model, affects total credit to firms and total output. Note that the former provides the link by which interbank market tensions are transmitted to the real economy namely, through a reduction in *credit availability*:



Figure 20: Total credit

The figure above confirms that the initial banking shock and its transmission through the network results in a reduction in credit provision to firms. The stability-enhancing properties of the complete network are again evident given the lower reduction in credit it exhibits. However, we notice that the increase in total output following central bank interventions is comparatively smaller. Thus, the positive impact of liquidity injections are also subject to dissipative effects, resulting in a lower total impact.

Though difficult to observe, the CP-nl network exhibits a slightly smaller decrease in credit than the cyclical and CP-nb cases. This puzzling feature seems to contradict the relative instability of this network observed in the previous section along with the larger central bank interventions reported above.



Figure 21: Total output

We end our analysis with the decline in total output across the four networks. It is evident that liquidity injections into the banking sector thus play a positive role in stabilising economic output. Again, the complete network features the smallest decrease while the remaining structures are more difficult to distinguish.

# 6 Concluding remarks

In this paper, we develop a novel methodology for taking into account financial system *in-terconnectedness* within the framework of a DSGE model. The RBC-DSGE model we use as a benchmark is part of a recent but growing literature that recognises the importance of the banking sector for the transmission and propagation of shocks. Before developing the crux of our modelling framework, we modify the bank microfoundations in our benchmark model by allowing for *one* regional, representative bank intermediating funds between households and firms. We then assume four regions, each comprising the three agents just mentioned.

Our main contribution arises in the manner in which these four regions interact. Specifically, we assume that banks are connected in an interbank *network* which provides them access to inter-regional wholesale funding to complement intra-regional retail funding from households. We vary the structure of this network across a set of stylised but suitably varied configurations namely, the complete, cyclical and core-periphery network topologies. This structure is then imposed on the microfounded model and the effect of the network is analysed using two simulation studies.

In the first simulation, we study how interbank market dynamics are driven by the network structure. We do this by first establishing a benchmark consisting only of aggregate productivity shocks. Since transmission through the interbank network is minimal (occurring only *indirectly* through changes in firm credit demand), it provides a basis for comparing the propagation of a regional banking shock which is transmitted directly through the network. Following this, we study how central bank liquidity injections directly into the banking sector can alleviate the spillover of banking sector shocks to the real economy via decreases in credit provision to firms.

Our results highlight the important role played by the network structure in driving economic fluctuations. We show that the complete network acts as a system stabiliser by allowing shocks to dissipate across the large number of linkages. Further evidence of this is provided when analysing central bank liquidity injections and the real economy.

Unlike the complete network, the cyclical setup allows for a more in-depth analysis of the transmission of the banking shock through the network since it features nodes that are not *directly* connected to the source of the shock. A recurring result is that IRF magnitude decreases as distance from the source of the shock increases. However this decrease is never negligible which highlights the importance of accounting for interconnectedness.

The core-periphery topology introduces a degree of reality into the model given that real interbank networks are known to exhibit such a structure. Moreover, from a modelling perspective, it allows for heterogeneity in banks' microfoundations and a more complete view of how shock *location* (which was redundant in the symmetric structures treated above) can drive system dynamics.

We end our discussion by pointing out a number of caveats in our modelling framework along with potential directions for future research. As mentioned, this paper is to our knowledge, the first to combine networks and DSGE modelling in this way. As such, we have relied on very simple models along both fronts. Our use of WPR as a benchmark model was driven primarily by our desire to focus on the role played by the *network* in transmitting and propagating shocks while abstracting from the more complex frictions present in New-Keynesian DSGE models à-la Smets and Wouters (2003). Moreover, we believe that the banking sector developed by WPR includes a number of realistic features (heterogeneity, endogenous defaults, adherence to capital requirements etc.) and thus, provides an ideal benchmark.

Having said that, our approach for incorporating the network is flexible enough to incorporate an alternative set of microfoundations. One possibility would thus be to study how the network could be included in the canonical BGG financial accelerator model and whether interesting insights could be gained from their interaction.

The model features a number of parameters whose further study could provide relevant insights into the tools available to policymakers. For example, combining our core-periphery network study with a more in-depth treatment of the capital requirements would provide an interesting analysis of capital surcharges for banks deemed 'too big to fail'.

Turning now to the network, it is clear that four nodes is the minimum required to provide a reasonably varied set of structures on which to conduct our analysis. However, this belies the complexity of real financial networks. A next step would be to increase the number of nodes and links, allowing for example for an interbank core comprising more than one bank. This would then allow researchers to apply tools from network science such as the analysis of network centrality and distribution.

Though numerous extensions are possible, we believe that our approach provides a promising and intuitive benchmark for considering how banks' behaviour is affected by the structure of the system in which they reside. The intuitive aspect is largely due to the highly-visual nature of networks and their ability to allow for a holistic view of the financial system and the manner in which institutions are connected.

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# A Computations

# A.1 Cyclical network

In the cyclical network topology of Figure 3(b), each bank  $i = \{A, B, C, D\}$  lends to and borrows from two distinct banks. Thus, this network is *symmetric* and the interbank components of the profits function are identical across banks. We thus report one bank profit function. Interconnectedness is captured in Table A.1:

i	$j \in \mathcal{S}_i$	$k\in\mathcal{D}_i$
A	В	С
В	D	А
$\mathbf{C}$	А	D
D	$\mathbf{C}$	В

Table A.1: Counterparties of i in cyclical network topology

where i denotes the bank of interest with lending and borrowing counterparties given by j and k, respectively. With respect to bank i's optimisation programme given in Section 2.2, Figure A.2 below provides the basis for the first-order conditions given in Equations 9 - 14:



Figure A.1: Local interbank market for each bank i under cyclical network

We begin with the observation that while the interbank component of bank profits vary depending on the network structure, the remaining components namely deposits, lending and market book exposures remain constant. In order to focus on cross-network heterogeneities, we extract the following 'constant' (C) terms from banks' profit function:

$$\pi_t^{b,i}(C) = \frac{D_t^{b,i}}{1 + r_t^{d,i}} - D_{t-1}^{b,i} - \frac{X_t^{b,i}}{1 + r_t^{l,i}} + \alpha_t^{f,i} X_{t-1}^{b,i} + \zeta_f \left(1 - \alpha_{t-1}^{f,i}\right) X_{t-2}^{b,i} + (1 + \Gamma_t) S_{t-1}^{b,i} - S_t^{b,i}$$

The remaining terms in  $\pi_t^{b,i}$  comprise the interbank (IB) components of bank profits under cyclicality:

$$\pi_t^{b,i}(IB) = \frac{B_t^{b,ij}}{1 + r_t^{b,ji}} - \delta_t^{b,ij} B_{t-1}^{b,ij} - \frac{L_t^{b,ik}}{1 + r_t^{b,ik}} + \delta_t^{b,ki} L_{t-1}^{b,ik} + \zeta_b \left(1 - \delta_{t-1}^{b,ki}\right) L_{t-2}^{b,ik} - \frac{\omega_b}{2} \left( \left(1 - \delta_{t-1}^{b,ij}\right) B_{t-2}^{b,ij} \right)^2 + \delta_t^{b,ik} L_{t-1}^{b,ik} + \zeta_b \left(1 - \delta_{t-1}^{b,ik}\right) L_{t-2}^{b,ik} - \frac{\omega_b}{2} \left( \left(1 - \delta_{t-1}^{b,ij}\right) B_{t-2}^{b,ij} \right)^2 + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{t-1}^{b,ik} + \delta_t^{b,ik} L_{t-1}^{b,ik} L_{$$

At the steady state,  $\pi^{b,i}(C)$  is also constant across the different networks while  $\pi^{b,i}(IB)$  will vary depending on the number of borrowing and lending counterparties. Under cyclicality, we obtain:

$$\pi^{b,i}(IB) = \left(\frac{1}{1+r^b} - \delta^b\right) B^b + \left(\delta^b + \zeta_b \left(1-\delta\right) - \frac{1}{1+r^b}\right) L^b - \frac{\omega_b}{2} \left[\left(\left(1-\delta^b\right) B^b\right)^2\right]$$

Recall that B = L at steady state. Plugging this in and simplifying terms, the steady state interbank component of bank profits under cyclicality is given by:

$$\pi^{b,i}(IB) = \zeta_b \left(1 - \delta\right) L^b + \frac{\omega_b}{2} \left[ \left( \left(1 - \delta^b \right) L^b \right)^2 \right]$$

# A.2 Complete network

In the complete network topology of Figure 3(a), each bank  $i = \{A, B, C, D\}$  lends to and borrows from the three remaining banks in the economy. Despite the increasing complexity, the network remains symmetric vis-à-vis the number of counterparties (six for each bank as opposed to two under the cyclical topology). Similar to the above example, we represent bank profits for an arbitrary bank *i* and its lending and borrowing counterparties,  $\{j, k, l\}$ :

Table A.2: Counterparties of i in complete network topology

i	$j \in \mathcal{S}_i = \mathcal{D}_i$	$k \in \mathcal{S}_i = \mathcal{D}_i$	$l \in \mathcal{S}_i = \mathcal{D}_i$
А	В	С	D
В	А	$\mathbf{C}$	D
$\mathbf{C}$	А	В	D
D	А	В	С

Figure A.2 below represents the table in (local) network form:



Figure A.2: Local interbank market for each bank i under cyclical network

Derivation of the interbank component of bank i's profits under completeness gives:

$$\begin{split} \pi_t^{b,i}(IB) &= \frac{B_t^{b,ij}}{1+r_t^{b,ji}} + \frac{B_t^{b,ik}}{1+r_t^{b,ki}} + \frac{B_t^{b,il}}{1+r_t^{b,li}} - \delta_t^{b,ij}B_{t-1}^{b,ij} - \delta_t^{b,ik}B_{t-1}^{b,ik} - \delta_t^{b,il}B_{t-1}^{b,il} \\ &- \frac{L_t^{b,ij}}{1+r_t^{b,ij}} - \frac{L_t^{b,ik}}{1+r_t^{b,ik}} - \frac{L_t^{b,il}}{1+r_t^{b,il}} + \delta_t^{b,ji}L_{t-1}^{b,ij} + \delta_t^{b,ki}L_{t-1}^{b,ik} + \delta_t^{b,li}L_{t-1}^{b,il} \\ &+ \zeta_b \left[ \left( 1 - \delta_{t-1}^{b,ji} \right) L_{t-2}^{b,ij} + \left( 1 - \delta_{t-1}^{b,ki} \right) L_{t-2}^{b,ik} + \left( 1 - \delta_{t-1}^{b,il} \right) L_{t-2}^{b,il} \right] \\ &- \frac{\omega_b}{2} \left[ \left( \left( 1 - \delta_{t-1}^{b,ij} \right) B_{t-2}^{b,ij} \right)^2 + \left( \left( 1 - \delta_{t-1}^{b,ik} \right) B_{t-2}^{b,ik} \right)^2 + \left( \left( 1 - \delta_{t-1}^{b,il} \right) B_{t-2}^{b,il} \right)^2 \right], \end{split}$$

which, at steady state, is equal to:

$$\pi^{b,i}(IB) = 3\left(\frac{1}{1+r^b} - \delta^b\right)B^b + 3\left(\delta^b + \zeta_b\left(1-\delta\right) - \frac{1}{1+r^b}\right)L^b - \frac{3\omega_b}{2}\left[\left(\left(1-\delta^b\right)B^b\right)^2\right].$$

Simplifying terms as before, we arrive at:

$$\pi^{b,i}(IB) = 3\zeta_b \left(1 - \delta\right) L^b + \frac{3\omega_b}{2} \left[ \left( \left(1 - \delta^b \right) L^b \right)^2 \right]$$

# A.3 Core-periphery networks

The link asymmetry inherent in the core-periphery network precludes the kind of symmetric analysis (i.e.  $\pi_t^{b,i}(IB) \forall i \in \mathcal{N}$ ) undertaken above. Moreover, there are substantial differences between the two core-periphery networks considered.

### A.3.1 Net-borrower case

Beginning with the case where i is a net *borrower* of funds, we provide the set of counterparties for each bank in the table below:

Table A.3: Counterparties of i in core-periphery network topology (net borrower case)

i	$j\in\mathcal{S}_i$	$k\in\mathcal{D}_i$
A	C,D	В
В	А	Ø
$\mathbf{C}$	Ø	А
D	Ø	А

where  $\emptyset$  indicates that the bank does not have any counterparties with whom it has a lending or a borrowing relationship. As a result, the profit equation will vary across banks:

$$\begin{aligned} \pi_t^{b,A}(IB) &= \frac{B_t^{b,AC}}{1+r_t^{b,CA}} + \frac{B_t^{b,AD}}{1+r_t^{b,DA}} - \delta_t^{b,AC} B_{t-1}^{b,AC} - \delta_t^{b,AD} B_{t-1}^{b,AD} - \frac{L_t^{b,AB}}{1+r_t^{b,AB}} + \delta_t^{b,BA} L_{t-1}^{b,AB} \\ &+ \zeta_b \left(1 - \delta_{t-1}^{b,BA}\right) L_{t-2}^{b,AB} - \frac{\omega_b}{2} \left(\left(1 - \delta_{t-1}^{b,AC}\right) B_{t-2}^{b,AC}\right)^2 - \frac{\omega_b}{2} \left(\left(1 - \delta_{t-1}^{b,AD}\right) B_{t-2}^{b,AD}\right)^2 \\ \pi_t^{b,B}(IB) &= \frac{B_t^{b,BA}}{1+r_t^{b,AB}} - \delta_t^{b,BA} B_{t-1}^{b,BA} - \frac{\omega_b}{2} \left(\left(1 - \delta_{t-1}^{b,BA}\right) B_{t-2}^{b,BA}\right)^2 \\ \pi_t^{b,C}(IB) &= -\frac{L_t^{b,CA}}{1+r_t^{b,CA}} + \delta_t^{b,AC} L_{t-1}^{b,AC} + \zeta_b \left(1 - \delta_{t-1}^{b,AC}\right) L_{t-2}^{b,CA} \end{aligned}$$

We omit the equation for bank D as this will have the same form as that for bank C. Depending on the interbank role played by each bank, certain components of the general specification do not appear. For example, C and D lend exclusively to A and do not borrow on the interbank market. Consequently, the pecuniary cost of default (parametrised by  $\omega_b$ ) does not appear in their profit function. From this, it is apparent that the core-periphery network will feature heterogenous profits at steady state which, after simplification, are given by:

$$\begin{aligned} \pi^{b,A}(IB) &= \left(\frac{1}{1+r^b} - \delta^b + \zeta_b \left(1-\delta\right)\right) L^b - \omega_b \left[\left(\left(1-\delta^b\right) L^b\right)^2\right] \\ \pi^{b,B}(IB) &= \left(\frac{1}{1+r^b} - \delta^b\right) L^b - \frac{\omega_b}{2} \left[\left(\left(1-\delta^b\right) L^b\right)^2\right] \\ \pi^{b,C}(IB) &= \left(\delta^b + \zeta_b \left(1-\delta\right) - \frac{1}{1+r^b}\right) L^b \end{aligned}$$

# A.3.2 Net-lender case

The set of counterparties for each i when A is a net *lender* of funds is given by:

Table A.4: Counterparties of i in core-periphery network topology (net lender case)

i	$j\in\mathcal{S}_i$	$k\in \mathcal{D}_i$
А	В	C,D
В	Ø	А
$\mathbf{C}$	А	Ø
D	А	Ø

Which gives rise to the following profit equations:

$$\begin{aligned} \pi_t^{b,A}(IB) &= \frac{B_t^{b,AB}}{1+r_t^{b,BA}} - \delta_t^{b,AB} B_{t-1}^{b,AB} - \frac{L_t^{b,AC}}{1+r_t^{b,AC}} - \frac{L_t^{b,AD}}{1+r_t^{b,AD}} + \delta_t^{b,CA} L_{t-1}^{b,AC} + \delta_t^{b,DA} L_{t-1}^{b,AD} \\ &+ \zeta_b \left(1 - \delta_{t-1}^{b,CA}\right) L_{t-2}^{b,AC} + \zeta_b \left(1 - \delta_{t-1}^{b,DA}\right) L_{t-2}^{b,AD} - \frac{\omega_b}{2} \left(\left(1 - \delta_{t-1}^{b,AB}\right) B_{t-2}^{b,AB}\right)^2 \\ \pi_t^{b,B}(IB) &= -\frac{L_t^{b,BA}}{1+r_t^{b,BA}} + \delta_t^{b,AB} L_{t-1}^{b,BA} + \zeta_b \left(1 - \delta_{t-1}^{b,AB}\right) L_{t-2}^{b,BA} \\ \pi_t^{b,C}(IB) &= \frac{B_t^{b,CA}}{1+r_t^{b,AC}} - \delta_t^{b,CA} B_{t-1}^{b,CA} - \frac{\omega_b}{2} \left(\left(1 - \delta_{t-1}^{b,CA}\right) B_{t-2}^{b,CA}\right)^2 \end{aligned}$$

At steady state, these reduce to:

$$\begin{aligned} \pi^{b,A}(IB) &= \left(\frac{1}{1+r^b} - \delta^b + \zeta_b \left(1-\delta\right)\right) L^b - \omega_b \left[\left(\left(1-\delta^b\right) L^b\right)^2\right] \\ \pi^{b,B}(IB) &= \left(\frac{1}{1+r^b} - \delta^b\right) L^b - \frac{\omega_b}{2} \left[\left(\left(1-\delta^b\right) L^b\right)^2\right] \\ \pi^{b,C}(IB) &= \left(\delta^b + \zeta_b \left(1-\delta\right) - \frac{1}{1+r^b}\right) L^b \end{aligned}$$

# **B** Figures

#### $r^{\,b}$ , A C $r^{\,b}$ , $C \, D$ $r^{\,b\,,\,D\,B}$ $r^{b, BA}$ 0.5 0.5 م 0.5 0.5 0 0 0 0 -0.5 -0.5 -0.5 -0.5 -1 -1 -1 -1 -1.5 -1.5 0 10 20 30 40 50 10 20 30 40 50 10 20 30 40 50 10 20 30 40 50 0 0 0 - Agg. productivity shock - + banking shock (A) \_ \_ \_

# B.1 Additional impulse response functions: Cyclical network

Figure B.1: Interbank rates - Cyclical network



Figure B.2: Interbank rates (Isolated banking shock) - Cyclical network

# B.2 Additional impulse response functions: Complete network



Figure B.3: Interbank rates - Complete network



Figure B.4: Interbank rates (Isolated banking shock) - Complete network

# B.3 Additional impulse response functions: Core-periphery networks

# B.3.1 Net borrower case



Figure B.5: Interbank rates - CP network (net borrower case)



Figure B.6: Interbank rates (Isolated banking shock) - CP network (net borrower case)

# B.3.2 Net lender case



Figure B.7: Interbank rates - CP network (net lender case)



Figure B.8: Interbank rates (Isolated banking shock) - CP network (net lender case)

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Layout: Analysis and Research Group Cover: NBB AG – Prepress & Image

Published in October 2016