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De-Biasing Strategic Communication

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De-Biasing Strategic Communication

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Abstract

This paper studies the effect of disclosing conflicts of interest on strategic communication when the sender has lying costs. I present a simple economic mechanism under which such disclosure often leads to more informative, but at the same time also to more biased messages. This benefits rational receivers but exerts a negative externality from them on naive or delegating receivers; disclosure is thus *not* a Pareto-improvement among receivers. I identify general conditions of the information structure under which this effect manifests and show that whenever it does, full disclosure is socially inefficient. These results hold independently of the degree of receivers' risk-aversion and for an arbitrary precision of the disclosure statement.

Keywords: strategic communication, misreporting, conflict of interest, disclosure

JEL Classification: D82, D83, L51

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1 Introduction

A substantial part of the world's economic activity deals with the elicitation of information by experts and its communication to non-experts. Examples include stock analysts, researchers, consultants or managers reporting to shareholders. Too often, experts face a conflict of interest (henceforth COI) such as sale commissions or affiliations which provide an incentive to bias their reports. Inefficiencies then arise because of two main reasons: First, receivers of such information may ignore the expert's COI and make poor choices by following biased information. Second, receivers lack information about the COI, e.g. its relative magnitude and the direction of the bias it induces. Without such information, they cannot accurately correct the expert's advice. They may then rationally decide to ignore the expert's message, at least partially, such that valuable information is lost. Disclosure of COIs promises to be a simple remedy to this problem. The idea is that information about the expert's COI helps at least those receivers who can use it to correct for potential bias. It is also tempting to policy makers as it carries the, as I will show incorrect, intuition that flattening information asymmetries is always desirable and should at least not hurt anyone. From a regulatory view, disclosure is also an appealing option as it is less paternalistic and less costly to regulators than direct supervision and regulation.¹

The objective of this paper is to describe an economic mechanism which shows how such disclosure often can lead to consequences opposite to those intended. It does so by considering a communication game in which the sender's private type is two-dimensional. This type consists of the superior information an expert owns and the COI which provides an incentive to communicate this information not truthfully. The sender faces lying costs of doing so, e.g. reputational or expected legal costs which are increasing in the size of the lie – talk is thus not cheap. The model also allows some receivers to be naive towards the sender's COI while others are fully strategic and rational, in a Bayesian sense. Alternatively, naivety in this setup is equivalent to the delegation of decisions to an expert, e.g. to a managed fund. The combination of these factors then unveils a simple economic mechanism through which disclosure can lead to more biased communication which then hurts naive receivers who do not anticipate the strategic effects of disclosure.

To understand the source of this adverse effect, consider an analyst ("he") who knows a share's fundamental value but also benefits from demand for this asset, e.g. via sales commissions. When commenting on the asset, he then faces a COI to overstate its value. The magnitude of this bias is determined by equalizing the marginal costs of lying to the marginal return of doing so. The latter

¹A prominent example of such a policy is contained in the Sarbanes-Oxley-Act which was enacted in 2002 as a response to prior corporate frauds, in particular among financial analysts. Among its adopted regulations is the requirement to "[...]disclose in each research report, as applicable, conflicts of interest that are known or should have been known by the securities analysts[...]" United States Congress (2002, Sec. 501b). Other cases of disclosure rules can be found in Fung et al. (2007), along with reasons why they seem appealing and examples for their failure.

is given by the average marginal reaction receivers, i.e. demand, to the sender's message weighted with the commission's size. Now regard a client ("she") who receives a message from the sender and is aware of the potential bias. She can try to de-bias it by correcting for the bias' expected value. However, since the COI is the sender's private information, she faces uncertainty regarding the commission's actual size or even its direction. Her de-biasing of the sender's message can thus worsen things when the expected bias differs from the its actual value. Facing such strategic uncertainty, rational receivers will then form their estimate of the actual state of the world as a combination between the sender's imperfectly de-biased message and her prior about the state of the world. For this, the relative weight which a rational receiver puts on the de-biased message is inversely related to the strategic uncertainty. Disclosing the sender's COI decreases this strategic uncertainty and increases this weight; disclosure thus translates into a larger marginal reaction to the sender's message. However, as explained above, the marginal reaction of receivers scales the sender's bias which, therefore, increases with disclosure. Delegating or naive receivers who do not account for the strategic effects of biasing and de-biasing communication are then hurt by this increase.

The above reasoning combines two main insights: First, the reaction to the sender's message by rational, risk-averse receivers depends on the quality of information they can extract from it. Second, an expert who faces a COI and has lying costs biases his message in proportion to the reaction it induces. Both of these effects are simple in their economic intuition. Combined, however, they deliver the surprising result that increasing transparency can be a bad idea when the disclosed information is not used by everyone and that lying costs play a crucial role in creating this adverse effect. In particular, it disproves the idea that disclosing COIs is a Pareto-improvement among receivers, except when all of them are fully rational. Accordingly, disclosure can even decrease overall efficiency, depending on the relative share of receiver types. I model these effects in a framework which allows for arbitrary degrees of risk-aversion as well as arbitrary quality of the disclosure process. General conditions under which this effect manifests and which allow evaluating the welfare consequences of disclosure are identified. The key variable in this regard is the correlation between the expert's COI and the information on which he has superior knowledge. For example, whenever this correlation is weakly positive, disclosure backfires and full disclosure is never optimal for efficiency. I also show that there can be situations in which disclosure is a Pareto-improvement among all receivers and that only in these cases, full disclosure is efficient.

Related literature: On the empirical side, the findings Malmendier and Shanthikumar (2014) who examine the communication by financial analysts relate closely to this paper. They show that analysts *strategically* inflate their stock recommendations by tailoring it to the receivers' reactions, a feature which is maintained in the following analysis. To do so, they use data which covers a period

before and after the Sarbanes-Oxley-Act which requires financial analysts to disclose COIs. Their analysis shows that the strategic bias did not disappear after the act was put into action in 2001.² Similarly, Mullainathan et al. (2012) conducted an audit study and show that after the act came into effect, financial advice remained of poor quality. Clean, causal evidence for negative effects of disclosure comes from Cain et al. (2005): In their experiment, subjects in the role of experts had ample opportunities to study a jar filled with coins. These subjects then advised others who had to estimate the amount of money inside the jar but who could not examine it beforehand. Their results first confirm the straightforward intuition that when the experts' pay is based on the final estimates' accuracy, their advice and the clients' resulting estimates are better as when experts are paid based on how high estimates are. However, they also show that when receivers are made aware of the experts' incentive to induce a high estimate, thus when COIs are disclosed, the experts' bias *increases*, relative to when they are unaware. On average, receivers do not account for this effect and end up making worse decisions than without disclosure. This finding on the adverse effects of disclosure has also been replicated in similar setups (see Koch and Schmidt, 2010; Inderst et al., 2010; Cain et al., 2011).³ The effects identified here are in line with these results.

This paper contributes to the theoretical literature on strategic communication. In their seminal work on the topic, Crawford and Sobel (1982) characterize communication equilibria to be partitional when talk is cheap, i.e. lying costs are absent: An informed sender with a commonly known COI endogenously partitions the state space and just announces the partition which contains the actual state of the world. In consequence, information is lost in communication. This result applies independently of the specific meaning of language, i.e. how exactly states map to messages by the sender and back from messages into actions by receivers, as long as this mapping is common knowledge.⁴ Often, however, this meaning of language is determined by the circumstances. For example, if a financial analyst announces "I expect share X will pay Y this year" many people would understand its meaning to be literal, thus that Y is the share's actual performance or at least the analyst's best estimate. Studies which conclude that financial analysts' statements are often upward biased also adapt this understanding (see Hayes, 1998; Michaely and Womack, 1999; Malmendier and Shanthikumar, 2014). In contrast, the partitioning result in combination with such a literal meaning and understanding of messages implies that, on average, the message and the inferred state of the world should not differ. To reconcile a literal understanding and persistently inflated messages, one

²See Malmendier and Shanthikumar (2014), p.1298: They state that their measure of strategic bias remains sizable and positive for affiliated analysts when they split the sample by August 2001, the date when the scandals became public and which contributed to the enactment of the Sarbanes-Oxley-Act shortly afterward.

³For a further review of the failure of disclosure and psychological approaches to it, see Loewenstein et al. (2014).

⁴See Sobel (2013) for an overview of the rich literature which has utilized and extended the partitioning result. Also, see the section on pragmatics therein for a further discussion on language and its meaning in the context of strategic communication.

or both of the two crucial assumptions which underlie the partitioning result need to be changed. Addressing them, Kartik et al. (2007) and Kartik (2009) show that these assumptions are first, the boundedness of the state space and second, cheap talk, i.e. no costs of lying.⁵ Capturing these insights, this paper allows for an unbounded support, for example when the variable on which the sender has private information is normally distributed. It also allows for lying costs similar to those used by Kartik (2009), thus costs which increase in the distance of the message to the actual state of the world. These costs thus reflect a literal interpretation of language which in turns leads naive receivers to trust the sender and act based on his message's face value.

In contrast to the above works, I assume that there is strategic uncertainty, i.e. the sender's COI is not common knowledge. This relates to Morgan and Stocken (2003) who find in a cheap talk framework, absent lying costs and with a compact state space, that the messaging strategy remains partitional when the sender's COI is described by a binary random variable. Li and Madarasz (2008) show that when this variable also has an expected value of zero, disclosing its non-zero realization can decrease informational efficiency, thus the number of equilibrium partitions. Inderst and Ottaviani (2012) also look on disclosure of COIs which they model as commissions paid by producers to intermediaries who advise customers on which out of two products to choose. In their model, the state of the world and the expert's message are therefore both binary. They show that disclosing COIs reduces the provision of commissions but less so, in relative terms, for the inferior product. Consequently, the relative bias rises upon disclosure and consumers make worse decisions. None of these approaches features lying costs. I show that when communication is not partitional or binary, disclosure can be harmful too, but due to another reason for which lying costs are crucial.

This paper's framework, in which I establish these findings, describes strategic communication when both, the state of the world and the sender's COI are represented by continuous, possibly correlated, variables. The model's specific form extends a framework introduced by Fischer and Verrecchia (2000). They study a manager who gets linear utility from influencing his company's share price through his earnings announcement while facing quadratic costs of lying (this setting is, among others, also admissible here). I extend this setup to more general, elliptical distributions as recently used by Deimen and Szalay (2014) who study strategic communication when players cannot agree on the relative importance of different information they hold. This approach has recently been adapted by Kartik and Frankel (2017) as a part of their general treatment of projection-based signaling, i.e. one-dimensional signals sent by two-dimensional sender types.⁶ Comparative statics

⁵For one-dimensional sender types Kartik et al. (2007) show that under general conditions, unbounded support is sufficient for the sender's messaging strategy to be continuous and revealing; this also applies when there is a lower bound and lying costs. Kartik (2009) considers a compact state space with lying costs. Equilibria are then of ten of the "LSHP (low types separate and high types pool)"-form: An upward-biased sender exaggerates his statement by a fixed bias if the state is below a certain threshold. If it is above, messaging becomes partitional.

⁶Technically also related, Bénabou and Tirole (2006) study projection of pro-social motivation in a linear-quadratic-

in Fischer and Verrecchia (2000) show that a decrease in strategic uncertainty with regard to the sender's motives leads to an increase in his bias when strategic incentives are uncorrelated to the state of the world. Kartik and Frankel (2017) have a similar result when this correlation is non-negative. This work focuses on this effect and to study it rigorously, I extend the analysis of communication games in this class along three main dimensions:

First and foremost, the following analysis incorporates the presence of naive receivers and their strategic effect on the sender's messaging strategy. This allows to examine the welfare consequences of disclosure with regard to both, the message's informativeness which matters for rational receivers and its bias – the deviation from a honest, literally meant message on which naive receivers rely. Second, this work explicitly studies the role of correlation between the sender's COI and the state of the world, including a negative correlation. Apart from allowing to capture several realistic settings which imply a negative correlation, I show that this can crucially affect comparative statics: Only with a negative correlation, disclosure can decrease the sender's bias and only then, disclosure is a Pareto-improvement which does not hurt naive receivers. Third and closely related to the preceding point, I explicitly model disclosure through a signal of arbitrary precision. Just performing comparative statics with respect to a single parameter, i.e. the variance which describes uncertainty for the sender's COI, overlooks the fact that information on one variable also contains information on correlated variables. The approach presented here allows to analyze the effects of disclosure on the whole posterior distribution of beliefs and actions.

The next section outlines the model's structure and evaluates the assumptions made with respect to several settings of expert advice. In Section 3, I derive the equilibrium behavior of senders and receivers when the sender's COI is undisclosed. Section 4 covers the case of disclosure. Section 5 synthesizes the preceding analysis and assesses the consequences of disclosure on receivers and overall efficiency. Section 6 concludes by summarizing the main insights and discussing policy implications.

2 The model

Consider a mass of receivers. Each would like to know the state of the world, denoted by $s \in S \subseteq \mathbb{R}$, because she has to take a decision $d \in S$ whose return is dependent on the realization of s . For example, s might represent an asset's return and d the receiver's optimal position into this asset. A receiver then suffers a loss which is the greater, the more d and s are misaligned. This is captured normal framework. Kartik and Frankel (2017) use linear benefits for the sender and costs which are quadratic in an upward bias but zero for downward-biased messages. In their setup, the sender always wants to induce high beliefs such that this does not restrict results (see also their footnote 19). I do not allow such free downward deviation as in settings relevant here and laid out above, the sender might have good reasons to induce downward-biased beliefs.

by her ex-post utility

$$u^R(d, s) = L(d - s) \quad (1)$$

where L is a C^2 loss function with argument $d - s$ which is strictly concave and symmetric around its maximum. Without loss of generality, I assume $L(0) = 0$ for its bliss point's value. As the leading example, consider $L(d, s) = -\frac{1}{2}(d - s)^2$, the quadratic loss function.⁷

Receivers do not know s and refer to a risk-neutral sender who knows its value. The sender communicates via a public message $m \in M = S$ which is understood to mean the value of s . In doing so, he faces costs of lying which are increasing the size of the lie. I will henceforth assume that lying costs are quadratically increasing in the lie, i.e. they are given by $k(m - s)^2$ where $k > 0$ is a scaling factor. This functional form can capture several possible sources of lying costs, e.g. costs based on social preferences, moral concerns against lying or reputational concerns in a stage version of a repeated game.⁸

If the above were the sender's only strategic incentive, he would be honest and always send $m = s$. Receivers would then just follow the message and implement their optimal choice. However, such strong influence of the sender on the receivers' decisions can be exploited. The sender can be incentivized to induce either a high or low action among receivers, e.g. via sales commissions. Such a COI of the sender is then denoted by a variable $c \in C \subseteq \mathbb{R}$. It scales proportionally to the aggregate decision of receivers $D(m)$, e.g. expected aggregate demand, which the sender possibly influences with his message m . A positive value implies that the sender has an incentive to generate a high demand, for example in the case of sales commissions. Conversely, a negative value rewards creating a low demand, for example when the sender wants to temporarily decrease the price of an asset in which he would like to take a position. The magnitude of c then determines the strength of these incentives, relative to lying costs. By adjusting c and its distribution appropriately, one can always assume that $k = \frac{1}{2}$. The sender's expected utility is then given by

$$E[u^S(m) | s, c] = cD(m) - \frac{1}{2}(m - s)^2. \quad (2)$$

Note that the commission is proportional to the demand. This differs from other approaches

⁷This is the canonical example which is put forward by Crawford and Sobel (1982) and used in much of the literature on strategic communication. Ottaviani (2000) shows that this specific function covers the case of a receiver with exponential utility who invests d into a risky asset of which she knows its variance but not its expected value s .

⁸ When L is also quadratic, this cost function captures concerns for the utility of a receiver who follow the sender's message at face value. Kartik (2009) uses this specific form of costs as a prominent example, i.e. for intrinsic costs of lying (see Erat and Gneezy, 2012; López-Pérez and Spiegelman, 2012; Abeler et al., 2014). See also Abeler et al. (2016) for a recent meta-study on the determinants of lying costs. Reputational concerns in the spirit of Sobel (1985) or Morris (2001) can also be proxied by it: If the actual value of s became knowledge ex-post, the squared distance of s and m is part of the nominator of the sender's coefficient of determination (R^2) which one obtains by regressing s on the sender's prior messages m ; his credulity is thus decreasing in this squared distance.

which assume that the sender has the same utility functions L as the receivers, only with a shifted bliss point (see Crawford and Sobel, 1982; Ottaviani and Squintani, 2006; Kartik et al., 2007). In particular, such approaches implicitly assume that the sender suffers the more a receiver's action differs in *either* direction from the sender's bliss point. Inducing a too high reaction by receivers is then as bad as inducing a too low reaction.⁹ While such an approach can be sometimes valid, an action or demand as high or low as possible is often preferred, e.g. when a sender gets a sales commission. The assumption here captures such a motive where the message is only restricted by the costs of lying and the loss of credulity due to the sender's resulting bias. Apart from accommodating such reasoning, this specification also yields an intuitive form of the sender's optimal message:

Lemma 1. *Suppose $D(m)$ is a C^2 function. A sender's optimally chosen message m^* then solves the implicit function*

$$m = s + cD'(m) \tag{3}$$

and is strictly increasing in s for all $(s, c, m) \in \mathbb{R}^3$.

Proof: See appendix.

From (3) one gets that the sender's message equals the state of the world plus a bias. The bias' size and direction are determined by the COI's value c and the marginal reaction of receivers to the sender's message. For example, if no-one listens to the sender, i.e. $D'(m) = 0$ holds, there is no point of lying and the bias equals zero. More generally, this reflects that, in the presence of lying costs, a lie should be scaled to the reaction it aims to affect. This feature will be crucial in understanding the adverse conflicts of disclosure.¹⁰

Note also that the sender's message is strictly increasing in s . If c were commonly known, a situation I will later treat as a special case, the sender's strategy would be an univariate, monotone function from M to S . As such, it would be invertible and the sender's message would therefore be revealing. Accordingly, there would be a bias but Bayesian, rational receivers would not be hurt by it.¹¹ In the following, I will relax two assumptions which ensure that messages can be biased but fully informative to everyone: First, I will allow that c is the sender's private information. The message is then a projection from $S \times C$ to M and rational receivers face strategic uncertainty when they try to infer s from it. Second, I will also allow that some receivers are naive in the sense that they

⁹Naive receivers who follow the sender's message one-to-one and overshoot are can then have the same adverse consequences for the sender as rational ones who do not react enough to his message. In these frameworks (but not the present one) naive receivers therefore have a similar effect as lying costs for the sender (see Ottaviani and Squintani, 2006; Kartik et al., 2007).

¹⁰This crucial feature is independent of the functional form of the lying costs, as long as the lying costs are differentiable in m and strictly increasing in the distance between m and s .

¹¹See Kartik et al. (2007) who establish this insight in a related but different framework. If their language is used, that is x denotes s and \hat{x} denotes $D(m)$ (the receiver action which the sender wants to influence), then their assumption A.4 on their modeling framework corresponds to $\partial^2 \mathbb{E}[u^S(m) | s, c] / (\partial s \partial D(m)) < 0$ which is easily verified to be violated. Nevertheless, messages are biased but yet revealing (w.r.t. s).

do not account for the sender's bias and follow his message's face value. Apart from the immediate effect that each single of these two features nullifies the result that a bias does not hurt receivers, their combination leads to the main result that decreasing strategic uncertainty can backfire.

Information: Both, the state of the world s and the COI c are the sender's private information. They are assumed to be drawn from an elliptical distribution F with support $K \times S = \mathbb{R}^2$. The (multivariate) normal distributions is the prime example for an elliptical distribution and whenever convenient, can be assumed in the following for F . Others elliptical distributions to which the following applies include the heavier-tailed Laplace or Student-t-distribution which are often used in financial and risk modeling or the logistic distribution which is often used to model latent processes underlying discrete outcomes. Some distributions with compact support such as the uniform distribution are also elliptical. For the sake of simplicity and without much loss of generality, I maintain the assumption of unbounded support.¹²

A specific elliptical distribution F such as the normal one can be described by $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$ where $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$ represent the vector of expected values and the variance-covariance matrix with finite, real-valued elements. Three properties of elliptical distributions will be important:

- E1) Elliptically distributed random variables are closed under linear transformations, i.e. a linear transformation of jointly F -distributed variables is itself F -distributed.
- E2) A random variable, conditional on the realization of another random variable, both being jointly elliptically distributed according to $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$, is also distributed according to F . The vector of expected values and the variance-covariance matrix of the resulting conditional distribution are linear transformations of $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$.
- E3) Elliptical distributions are symmetric around their vector of expected values.

A more formal statement of these properties, together with a precise definition of elliptical distributions and references can be found in the appendix. I will denote the elements of $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$ by

$$\boldsymbol{\eta} = \begin{bmatrix} \bar{s} \\ \bar{c} \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_s^2 & \sigma_{sc} \\ \sigma_{sc} & \sigma_c^2 \end{bmatrix}.$$

When appropriate, I will refer to the correlation of s and c instead of its covariance σ_{sc} . To make

¹²Note that by choosing the distribution's parameters appropriately, the probability that realizations of (s, c) are within some compact set can be made arbitrarily high. For an example of how the uniform distribution can be generated from the general definition of elliptical distributions see the survey by Gómez et al. (2003), pp. 359/360. For a related application of elliptical distribution with compact support, see Kartik and Frankel (2017). The additional assumptions which are, in some cases, necessary to maintain all results presented here are essentially to specify suitable out-of-equilibrium beliefs, an issue which is bypassed under unbounded support. Embrechts et al. (2002) describes how elliptical distributions can be used in financial and risk analysis, including the caveats of doing so.

things interesting, I also assume $\sigma_s \sigma_c > 0$ and $|Corr[s, c]| = \frac{|\sigma_{sc}|}{\sigma_s \sigma_c} < 1$ as otherwise, the receiver's inference problem would become effectively one-dimensional or vanish entirely.

The above information structure is suited to naturally model how players, in particular, rational receivers, arrive at their prior. First, assuming unbounded support for (s, c) means that no commonly known bounds on the state space are required. In contrast, assuming compact support implies common knowledge of such sharp bounds. Sometimes, this is straightforward and reasonable, e.g. when the sender communicates the share of one's wealth which should be invested in a certain asset. However, once leverages become available or when s reflects asset returns, appropriate bounds are not clear-cut. A solution to this is then to assume that all real values are theoretically possible while unrealistic, extreme realizations receive appropriately low probabilities by choosing the above moment parameters accordingly. Similarly, when the prior is based on the averages of historical data, e.g. when (3) forms the basis of a structural model, the resulting estimates of (s, c) are, by the central limit theorem, approximately normally distributed.¹³

As an import feature, this framework allows to handle the case of $\sigma_{sc} \neq 0$, e.g. when the COI is dependent on the state of the world. As the prime example, consider a financial analyst or a manager who reports on a share for which he holds a call option. Accordingly, he faces an incentive to shift, via his report, the share price over the option's strike price. Given some c and that lying costs determine a limit on the bias, this only works if the share's fundamental value s is high enough so that the effect of his biased message can "bridge" the distance to the strike price. Otherwise, there is effectively no COI and, thus, $\sigma_{sc} > 0$ applies. A negative correlation can also be relevant, for example when the sender owns the asset on which he has superior information. If he faces a bad outlook, he might want to sell the asset. He then faces a COI to shift demand and the sale price upwards so that $\sigma_{sc} < 0$ results.

Rational and naive receivers: From (3) one gets that COIs induce the sender to not report truthfully. How should receivers then take such a distortion into account? A receiver who is rational, in a Bayesian sense, should do so by acting on the information she can extract from the sender's message such that it maximizes her expected utility. That is, here action is given by $d_r(m) \equiv \arg \max_{d \in S} E[L(s, d)|m]$. In case that L is the quadratic loss function and s and m are jointly normally distributed, this is clearly $E[s|m]$. The following result shows that this generalizes

¹³Another scenario worth to be mentioned and which can be easily captured in this framework is scientific fraud: First, empirical research and its assessment often boils down to statistics with elliptical distributions, e.g. normally or student-t-distributed regression coefficients. Second, pressure to publish and publication bias create a COI to inflate these statistics (Simmons et al., 2011). Third, outright cheating such as making up data or more subtle techniques such as selective sampling and data mining are methods with which these test statistics can be misrepresent or manipulated at the costs of ethical, reputational and legal concerns (Fanelli, 2009; Steen, 2011; Simonsohn, 2014). Finally, disclosure policies are employed in this context, e.g. by many journals. This framework can thus be applied to study their consequences.

to any function L when m and s are jointly elliptically distributed:

Lemma 2. *If m is distributed according to F , rational receivers choose $d_r(m) = E[s|m]$.*

Proof: the proof adapts the proof of part i) of lemma 2 in Deimen and Szalay (2014) to this paper's framework and can be found in the appendix.

The optimal decision $d_r(m)$ is that of fully rational, Bayesian receiver who is capable of adjusting for the effect of the sender's COI on his message and connecting it to the common prior. While some receivers, e.g. professional ones, can act in such a manner, empirical evidence shows that many people do not anticipate and correct for others' strategic behavior or ignore strategically important information (e.g. Brown et al., 2012; Brocas et al., 2014). In line with such reasoning, Malmendier and Shanthikumar (2007) show that small investors such as private households follow analysts' optimistic recommendation more closely than bigger, institutional ones who are more cautious and adjust for biases. To capture these observations, I allow for the possibility that there are naive receivers who take the sender's signal at face value. Their action is thus given by $d_n(m) = m$. This assumption is also used by Ottaviani and Squintani (2006), Kartik et al. (2007), and Chen (2011) to model naive receivers in related settings. However, since the sender's objective is not to shift demand to a bliss point but to induce a demand as high as possible in the direction of c , naive receivers do not impose a lying cost in itself (see footnote 9). I denote the share of naive or delegating receivers by $\mu \in [0, 1]$.¹⁴ The mass of rational receivers is then given by $1 - \mu$ which yields the following expected demand function for the receiver:

$$D(m) = \mu m + (1 - \mu)d_r(m) \tag{4}$$

Note that naive receivers who directly implement the sender's message are strategically equivalent to receivers who delegate their decision d to the sender, e.g. to a managed fund. This can be either because of blind trust (Gennaioli et al., 2015) or because handling the assets for oneself and fully rationally de-biasing a sender's message creates fixed costs which are too high, relative the informational gain from acting rationally (Sims, 2003). Finally, note that the above description also captures a scenario in which a risk-neutral sender faces a single receiver but does not know whether this receiver is naive (with probability μ) or rational.

¹⁴By appropriate scaling of μ one can always account for situations where naive or delegating receivers do not react one-to-one, e.g. when $d_n(m)$ is a positive affine transformation with $d'_n(m) = r > 0$. As an example, suppose that there is a mass 0.5 of naive receivers for whom, on average, $d_n(m) = 0.6m$ holds. From the sender's point of view, this is the same as if there were a mass 0.2 of receivers who ignore him, mass 0.3 of naive receivers who follow one-to-one, and a mass 0.5 of rational receivers. Using $\mu = \frac{0.3}{0.5+0.3}$ would then be strategically equivalent.

3 Undisclosed conflicts of interest

The communication game with undisclosed commissions has the following timing:

- 1) the sender's type (s, c) is drawn from F and privately observed by the sender,
- 2) the sender sends a signal m about s ,
- 3) receivers observe m , if rational update their belief about s , then choose d ,
- 4) payoffs are realized.

I look for a Perfect Bayesian Equilibrium of this game. It consists of a pair of equilibrium strategies $m^* : S \times C \rightarrow M$ for the sender and $d_r^* : M \rightarrow S$ for a rational receiver such that each player's expected utility is maximized, given the other players' strategies when their beliefs are formed by Bayes' rule. Naive receivers are assumed to have a dominant strategy of following the sender. Therefore, their beliefs do not matter. The key equilibrium belief in this context is belief of rational receivers about s as, by Lemma 2, they choose $d_r^*(m) = E[s|m^*] \equiv E[s|m]_{m=m^*(s,c)}$.¹⁵

I will use this equilibrium concept under different settings of common knowledge, henceforth called information structures \mathcal{I} . The information structure of the game with undisclosed incentives is given by $\mathcal{I}_U = \{F(\eta, \Sigma), \mu\}$, the game's fundamental parameters and their joint distribution. If commissions are disclosed, I will denote this information structure by \mathcal{I}_D and will later specify it more exactly. Whenever I use the expectations operator or terms based on it such as variance or covariance, it is with respect to the respective information structure. For example, $E[c] = \bar{c}$ when $\mathcal{I} = \mathcal{I}_U$ but this will not hold with disclosed COIs (see next section).

If one plugs (3) into (2) one gets that the sender's equilibrium messaging strategy $m^*(s, c)$ has to solve the first order condition

$$m = s + c(\mu + (1 - \mu)d_r^{*'}(m)) \quad (5)$$

where $d_r^{*'}(m) = \frac{\partial E[s|m]}{\partial m} \Big|_{m=m^*(s,c)}$. Thus, the sender's messaging strategy is not only determined by the bias' effect on the demand of naive receivers and lying costs. It is also based on how rational receiver react to the sender's messages formed under such a messaging strategy, as captured by $d_r^{*'}(m)$. To derive how rational receivers – and in response also senders – behave optimally, I define the "equilibrium inference coefficient" ρ^* as a measure of the equilibrium messaging strategy's informativeness. This equilibrium parameter captures how well, given a sender equilibrium messaging

¹⁵A complete belief profile over the sender's type also requires to specify an analogously-defined belief $E[c|m]_{m=m^*(s,c)}$. As it is payoff-irrelevant for either player it is omitted here.

strategy $m^*(s, c)$, variations in the resulting messages capture variations in the underlying state of the world s :

$$\rho^* \equiv \frac{Cov[s, m^*]}{Var[m^*]} \equiv \frac{Cov[s, m]|_{m=m^*(s,c)}}{Var[m]|_{m=m^*(s,c)}} \quad (6)$$

Throughout this paper, I will focus on the case that ρ^* is a real, strictly positive number. This precludes situations where the expert's message is completely uninformative ($\rho^* = 0$) or "reverted" ($\rho^* < 0$), i.e. where higher values of m are associated with lower values of s – features which are unlikely to be observed in an information market with experts, especially when this is an equilibrium feature (see below). Using this concept, one can show that, in equilibrium, the inference coefficient has to be equal to the marginal reaction of rational receivers' to the message, i.e. that $d_r^{*'}(m) = \rho^*$ holds. Proving this relationship constitutes the main building block for the following proposition which characterizes the players' equilibrium actions and the relevant equilibrium beliefs:

Proposition 1. *Every pure strategy Perfect Bayesian Equilibrium of the communication game with strategies $m^*(s, c)$ for the sender and $d_r^*(m) \in \mathcal{C}^2$ for rational receivers takes the form of*

$$m^*(s, c) = s + c(\mu + (1 - \mu)\rho^*) \quad (7)$$

$$d_r^*(m) = (1 - \rho^*)E[s] + \rho^*(m - E[c](\mu + (1 - \mu)\rho^*)). \quad (8)$$

The rational receivers equilibrium belief w.r.t. to s is given by $E[s|m^] = d_r^*(m)$.*

Proof. see appendix

The above result characterizes equilibria in which a rational receiver's reaction to the sender's message is smooth in the sense that it is twice continuously differentiable. Note from (4) that assuming a smooth strategy for rational receivers is a necessary condition for the overall demand $D(m)$ to be smooth. I focus on such smooth, pure strategy equilibria because it is sufficient but yet relatively general to demonstrate the main point of this paper, the adverse effects of disclosure. It also captures the idea that arbitrarily small changes in the sender's message should yield no large effect on demand. The sender's strategy then takes an intuitive form. He announces the state of the world and adds a bias, given by $m^*(s, c) - s = c(\mu + (1 - \mu)\rho^*)$. This bias equals $D'(m)$, the change in expected demand due to a change in the message, scaled by the sender's COI c . Part of this change $D'(m)$ is the change in the best response of rational receivers to the sender's message, given by $d_r^{*'}(m) = \rho^*$, weighed by their population share $1 - \mu$. This reflects that rational receivers' best response has two parts: The first is the receivers' prior about the true state of the world $E[s]$. It is invariant to m and its weight $1 - \rho^*$ is inversely related to the message's informativeness. The other part of their best response is $m - E[c](\mu + (1 - \mu)\rho^*)$, the received message corrected by the expected bias. Its weight on m is ρ^* , the message's informativeness which, therefore, equals $d_r^{*'}(m)$. This interplay between the sender's bias and rational receivers reaction through ρ^* will be crucial.

As a first step in the analysis of ρ^* , note that the correction by rational receivers for the expected bias is based on the expected commission. It can thus be wrong in both, direction and magnitude. This possible error in rational receiver's de-biasing of the sender's message and her risk-aversion provides the reason why she does often not react one-to-one to the corrected message. Whenever $\rho^* \in (0, 1)$, a rational receiver strategically ignores the sender's de-biased message and puts weight on her prior such that information is left unused. For illustration, consider σ_s^2 to be close to zero while σ_c^2 is sufficiently large, i.e. a situation where almost all uncertainty is not of fundamental, but of strategic nature. It follows then that $Cov[s, m^*]$ and through it, also ρ^* will be almost zero. Rational receivers then act almost entirely according to their prior $E[s]$ as almost any variation in the signal can only be due to the sender's bias. In equilibrium, the sender then takes into account the corresponding low, almost absent, reaction of rational receivers and scales his bias down. The above proposition shows that this is a general behavior: Lower values of ρ^* , thus a lower informativeness of the message, will lead to decrease in the sender's bias while higher informativeness increases it.

The underlying reasoning can also be understood by interpreting ρ^* as the coefficient from a linear regression of s on m : Both, a regression coefficient and ρ^* , describe the marginal change in the conditional expectation of a dependent variable due to a marginal change in the independent variable. The crucial difference is that in a regression, this refers to an exogenous change while here, it is the change in the endogenously determined equilibrium message. Using the functional form as stated in Proposition 1, the sender's messaging strategy can then be used to solve for this equilibrium inference coefficient:

Proposition 2. *In the above game with undisclosed commissions, the equilibrium inference coefficient $\rho_U^* = \frac{Cov[s, m^*]}{Var[m^*]}$ is a fixed point to*

$$g_U(\rho) = \frac{\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2}. \quad (9)$$

If $\rho_U^ \in (0, 1]$, such a fixed point is unique. If $\rho_U^* > 1$, such a fixed point is either unique or one of three such points.*

Proof: See appendix.

Together with Proposition 1, this result completely describes the game's equilibrium strategies and beliefs. With the parameters contained in \mathcal{I}_U , precise expressions for the players' actions and relevant beliefs can be computed. However, some characteristics of the relevant equilibrium parameters can also be derived from more general information:

Lemma 3. *A fixed point $\rho_U^* > 0$ to (9) exists if and only if $\sigma_{sc} > \tau^*$ for some $\tau^* < 0$.*

Proof: See appendix.

This lemma links the covariance between s and c , the game's fundamental random variables, to the covariance of s and actions induced in the game, the sender's message m . As a direct consequence from the above and Proposition 1, an equilibrium requires $\rho_U^* > 0$ and therefore σ_{sc} , to be sufficiently large, i.e. larger than τ^* . In the following, I will assume that this condition is met.

Proposition 2 also shows that multiple equilibria can only arise when the equilibrium inference coefficient is larger than one. The following result examines these cases in detail and provides general conditions under which ρ_U^* is above and below this threshold of one which will be important in the following analysis:

Lemma 4. *Suppose $\rho_U^* > 0$. Then $\rho_U^* \leq 1$ if and only if $\sigma_{sc} \geq -\sigma_c^2$.*

Proof: See appendix.

As with the preceding lemma, the result maps the game's parameters into behavior and back. It shows that whenever fundamental uncertainty as measured by σ_c^2 is sufficiently high, the rational receiver's demand is a unique, strictly convex combination between her prior and the sender's corrected message. Also note that since the condition is equivalent to $Corr[s, c] \geq -\frac{\sigma_c}{\sigma_s}$, strategic uncertainty exceeding fundamental uncertainty ($\frac{\sigma_c}{\sigma_s} > 1$) or a weakly positive correlation between s and c are sufficient conditions for this to happen, i.e. for ρ_U^* to be less than one.

Non-convex combinations are also possible when $\sigma_{sc} \in (\tau^*, -\sigma_c^2)$. The above lemma then implies that rational receivers "over-react" – a change in the sender's message induces a change in rational receivers' demand which is greater than that original change in the message. To understand how this can occur, note that such a sufficiently negative value of σ_{sc} implies that rational receivers expect the sender to have a strong incentive to push demand into a direction opposite to s . However, since ρ^* is positive, a higher message m does, in expectation, still reflect a higher value of s . In equilibrium, rational receivers anticipate this positive correlation between m and s by reacting very strongly, i.e. with $\rho^* > 1$, to the de-biased message. Such extreme correction is based on the expected bias. Since receivers have concave utility, the expected disutility caused by a possible mis-correction is the greater, the more likely extreme values of c are. Thus, when the COI is too unpredictable relative to fundamental uncertainty, i.e. when $\frac{\sigma_c}{\sigma_s} > 1$, such over-reaction cannot occur. The limit to such expectation-based corrections are also reached when $\sigma_{sc} \leq \tau^*$. In this case, the expected bias is so strong and opposed in direction to s that the risk of mis-correction outweighs the benefits of over-reacting and a meaningful communication equilibrium cannot be established.

Figure 1 illustrates these findings. It depicts the equilibrium inference coefficient for possible correlation between the state of the world and the sender's COI. The three lines represent different values of σ_s^2 , higher ones representing larger variance. This ordering reflects that ceteris paribus, for a given messaging strategy, higher variation in s explains more variation in the message and

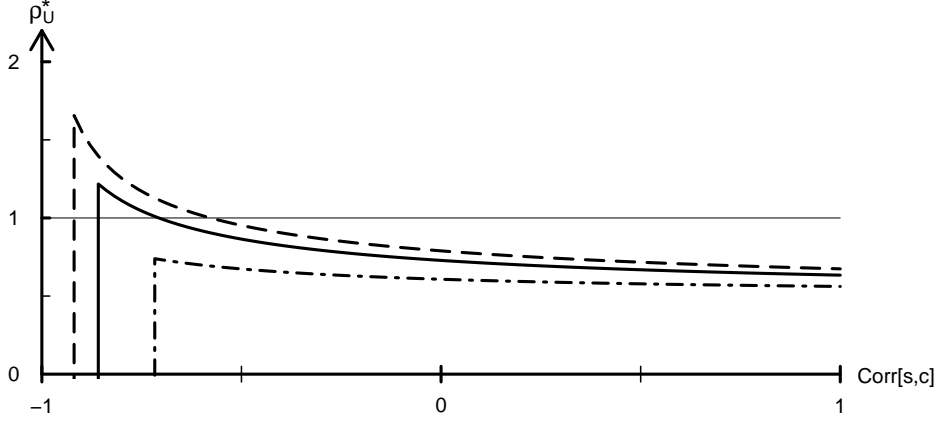


Figure 1: Stable, positive equilibrium equilibrium inference coefficients ρ_U^* over $Corr[s,c]$
Parameters: $\mu = 0.5$, $\sigma_c^2 = 1$, and $\sigma_s^2 = 1/2/3$ (bottom/middle/top line).

thereby increases its informativeness. This is then expressed in higher values for ρ_U^* . Reflecting the previous lemma, the figure also shows that for any positive correlation and whenever $\sigma_c^2 \geq \sigma_s^2$ holds, the equilibrium inference coefficient ρ_U^* is always contained in the unit interval. The left region of the upper two lines portrays parameter constellations where the (normalized) covariance σ_{sc} is sufficiently low so that ρ_U^* is larger than one. The figure also portrays the normalized cutoff value $\tau^*/(\sigma_s\sigma_c)$ as a vertical line. For correlations below it, an equilibrium with $\rho_U^* > 0$ does not exist.

4 Disclosed conflicts of interest

The above results show that, due to the sender's COI, his equilibrium message is biased. This hurts all receivers. Naive ones are hurt because they are not aware of the bias and follow the sender's biased message. Rational receivers try to correct for the bias, but as the sender's COI almost surely differs from its expected value, their following action is also sub-optimal. In addition, this strategic uncertainty often leads them to ignore valuable information. It is then tentative to demand that the sender discloses his COI so that at least rational receivers can make more use of his message.

Such disclosure of COIs will be modeled through a signal \tilde{c} about c which receivers observe before they make their choice. The signal is given by $\tilde{c} = c + \epsilon$ where the error term ϵ is uncorrelated with s or c but follows the same distribution F with support over the real numbers. It has an expected value of zero and finite variance $\sigma_\epsilon^2 \geq 0$. This variance measures the quality signal of the signal \tilde{c} . When it is zero, the signal is perfectly accurate about the sender's COI, a setting I will henceforth call "full disclosure". Conversely, "imperfect disclosure" describes settings where $\sigma_\epsilon^2 > 0$. The timing in the game with a disclosed COI is then the same as before, except that step 1 is split into two sub-parts:

- 1a) the sender's type and the signal error (s, c, ϵ) are drawn,
- 1b) $\tilde{c} = c + \epsilon$ becomes common knowledge, (s, c) is privately observed by the sender,

Step 2 through 4 remain unchanged, relative to undisclosed COIs. The only difference is thus the signal \tilde{c} which allows rational receivers to update their prior before they receive the sender's message. The procedure for this is similar to the signal extraction from the sender's message. The updated belief regarding c is a combination between the relevant prior, given by \bar{c} , and the signal \tilde{c} with weight for the latter chosen according to its precision. This weight is given by the signal's informativeness which, in analogy to the equilibrium inference coefficient, is defined as follows:

$$\rho_c \equiv \frac{\text{Cov}[c, \tilde{c}]}{\text{Var}[\tilde{c}]} = \frac{\sigma_c^2}{\sigma_c^2 + \sigma_{\tilde{c}}^2} \in (0, 1] \quad (10)$$

This coefficient reflects how much variation in c can be explained by the signal \tilde{c} about it. Accordingly, ρ_c is key for the updated prior of rational receivers:

Lemma 5. *The posterior distribution of $(s, c | \tilde{c})$ is given by $F(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\Sigma}})$ with*

$$\hat{\boldsymbol{\eta}} = \begin{bmatrix} \bar{s} + \left(\frac{\sigma_{sc}}{\sigma_c^2}\right)(\tilde{c} - \bar{s})\rho_c \\ \bar{c}(1 - \rho_c) + \tilde{c}\rho_c \end{bmatrix} \quad \text{and} \quad \hat{\boldsymbol{\Sigma}} = \begin{bmatrix} \sigma_s^2(1 - \text{Corr}[s, c]^2\rho_c) & \sigma_{sc}(1 - \rho_c) \\ \sigma_{sc}(1 - \rho_c) & \sigma_c^2(1 - \rho_c) \end{bmatrix}$$

Proof: See appendix.

The signal \tilde{c} directly affects the expected value of c which is used to de-bias the received message. Its information also reduces the strategic uncertainty σ_c^2 by a factor $1 - \rho_c$. Also, beliefs about s are also affected if this variable is correlated with c . In this case, disclosure reduces uncertainty in all dimensions, i.e. every element of $\hat{\boldsymbol{\Sigma}}$ decreases with ρ_c . These effects would be overlooked if disclosure were modeled as an univariate comparative static, e.g. with respect to σ_c^2 . Also note that full disclosure is a special case of the above. In it, $\rho_c = 1$ applies and the sender's private type becomes one-dimensional.

To see how these informational consequences of disclosure translate into strategic ones, note that this new posterior becomes common knowledge for senders and rational receivers. In the previously analysed communication game with undisclosed COIs, all results depended only on the information structure $\mathcal{I}_U = \{F(\boldsymbol{\eta}, \boldsymbol{\Sigma}), \mu\}$. After \tilde{c} has been observed and processed, the new information structure with disclosed COIs is given by $\mathcal{I}_D = \{F(\hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\Sigma}}), \tilde{c}, \mu\}$. Recall that expectations are with respect to the information structure, e.g. under \mathcal{I}_D it holds that $\mathbb{E}[c] = \bar{c}(1 - \rho_c) + \tilde{c}\rho_c$. Using this, one can proceed as before for undisclosed COI to determine equilibrium behavior by simply using \mathcal{I}_D instead of \mathcal{I}_U . In particular, Proposition 1 applies and the equilibrium messaging strategy is of the linear form where it equals the state of the world plus an endogenously determined bias. The equilibrium inference coefficient with disclosed COIs ρ_D^* can then be determined analogously to the case of undisclosed COIs. This yields the following:

Proposition 3. *There exists at least one value $\rho_D^* = \frac{\text{Cov}[s, m^*|\tilde{c}]}{\text{Var}[m^*|\tilde{c}]} > 0$. With full disclosure it is given by $\rho_D^* = 1$. With imperfect disclosure it is a fixed point to*

$$g_D(\rho) = \frac{\phi\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\phi\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \quad (11)$$

with $\phi \equiv \frac{1 - \rho_c(\text{Corr}[s, c])^2}{1 - \rho_c} > 1$.

Proof: See appendix.

Together with Proposition 1, this result completely characterizes equilibrium behavior in games with disclosed COIs. In the special case of full disclosure, $c = \tilde{c}$ and $\rho_D^* = 1$ applies. In this case, the sender's equilibrium strategy is given by $m^*(s, c) = s + c$ while rational receivers correctly infer $E[s|m, \tilde{c}] = m - \tilde{c}$. Reflecting the case of one-dimensional signaling discussed earlier, the sender's message is then biased but revealing to rational receivers.

With imperfect disclosure, the equilibrium inference coefficient ρ_D^* solves (11). The only difference of this expression relative to the case of undisclosed COIs as stated in (9) is the additional term $\phi > 1$ in the nominator and denominator of (11). Comparing $\hat{\Sigma}$ to Σ , one can see that this ratio represents how much fundamental uncertainty regarding s remains after disclosure, relative to strategic uncertainty regarding c . Since ϕ increases in the signal's informativeness ρ_c , the effect on the equilibrium inference coefficient ultimately depends on the relative size of the elements of Σ . These elements, in turn, determine whether ρ_U^* passes the threshold of one (see Lemma 4) which crucially determines the effects of disclosure.

How the effect of disclosure relates to the threshold of one can be seen easiest by recalling the effects of full disclosure. By the above proposition, $\rho_D^* = 1$ then holds after disclosure, irrespective of ρ_U^* 's prior value. Accordingly, if $\rho_U^* > 1$ held before, the coefficient shrinks while if $\rho_U^* < 1$ held, it increases upon disclosure. The following lemma generalizes this pattern to imperfect disclosure:

Lemma 6. *For any \mathcal{I}_U , the following cases can occur upon imperfect disclosure:*

- a) *When there is a solution $\rho_U^* \in (0, 1)$, it is unique and there is a unique solution $\rho_D^* \in (\rho_U^*, 1)$.*
- b) *When there is a solution $\rho_U^* = 1$, it is unique and there is a unique solution $\rho_D^* = 1$.*
- c) *When there is a unique solution $\rho_U^* > 1$, there is a unique solution $\rho_D^* \in (1, \rho_U^*)$.*
- d) *When there are three solution $\rho_{U,k}^* > 1$ with $k \in \{1, 2, 3\}$, there is either*
 - a unique solution ρ_D^* such that $1 < \rho_D^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$ or there are*
 - three such solutions $\rho_{D,k}^*$ such that $1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{D,2}^* < \rho_{D,3}^* < \rho_{U,3}^*$.*

Proof: See appendix.

Cases a) through c) show that a unique inference coefficient moves towards a value of one or, in the special case that $\rho_U^* = 1$, remains at this value. Result d) covers the case when before disclosure, there were multiple, that is three, possible equilibrium inference coefficients. After disclosure, there are then either one or three such values. If there is one, the above pattern repeats as its value is closer to one than all previous values. Only for the second sub-case when there are three solutions after disclosure, and only for the intermediate value therein, disclosure leads to an increase of the inference coefficient. All others values also move towards one, i.e. decrease, when COIs are disclosed.

It will be useful to define a comprehensive notation for an equilibrium in this communication game and introduce a stability criterion for it:

Definition. *An equilibrium of this game under information structure $\mathcal{I} = \mathcal{I}_{U(D)}$ is the collection $\mathcal{E}(\rho_k^*)$ of strategies and beliefs as specified in Proposition 1 with ρ_k^* as the k -lowest-valued fixed point to $g_{U(D)}(\rho)$ as described in proposition 2 (3).*

An equilibrium $\mathcal{E}(\rho_k^)$ is then called an asymptotically "stable equilibrium" if the corresponding fixed point ρ_k^* obeys $\frac{d}{d\rho}(g_{U(D)}(\rho) - \rho)|_{\rho=\rho_k^*} < 0$.*

The notation captures the fact than an equilibrium is given by the strategies as specified in Proposition 1 which, in turn, are affected by disclosure via its effect on the equilibrium inference coefficient. The second part of the definition introduces the notion of a stable equilibrium based on the notion of asymptotic stability (Hirsch and Smale, 1974). Although originally a dynamic concept, asymptotic stability has a long history of being used in the analysis of equilibria which originate from one-shot situations, e.g. for tâtonnement processes in (general) equilibrium and recently also in strategic communication settings (Blume and Board, 2014).¹⁶ In particular, it captures the notion that such stable equilibria converge back to their equilibrium value after any small perpetuation, are locally unique, and can be found via iterative procedures. An equilibrium which is not stable does not have these properties. As the following result shows, the notion of stability allows to order the effects of disclosure in a clear way:

Proposition 4. *In any stable equilibrium $\mathcal{E}(\rho_k^*)$ it holds for the associated equilibrium coefficient before and after disclosure that $\rho_{k,U}^* \gtrless \rho_{k,D}^* \gtrless 1$.*

Proof: See appendix.

For the remainder of this analysis, I will only focus on stable equilibria because unstable ones, as they fail to have any of the above-mentioned robustness characteristics, are unlikely to have any

¹⁶Adapting the notation and results of Hirsch and Smale (1974), pp. 185-188 to the notation of this paper, a fixed point is asymptotically stable if $f'(\rho)|_{\rho=\rho_k^*} < 0$ where $f(\rho) \equiv g(\rho) - \rho$ such that $f(\rho)|_{\rho=\rho_k^*} = 0$ holds. This then leads to the above definition. Blume and Board do so in their work to examine endogenously chosen vagueness in a one-shot communication game. They also provide references on how asymptotic stability relates more generally to one-shot situations, in particular to Samuelson's correspondence principle.

relevance. By Lemma 6, this excludes only the special, intermediate-valued equilibrium in the second sub-case under d). All other equilibria are stable. For them, disclosure has a systematic effect on the equilibrium inference coefficient and, therefore, also on the message's informativeness and bias. The following section explores this key effect of disclosure in more detail by looking at how it affects the welfare of the different receiver types and overall efficiency.

5 Consequences of disclosure

In the following, I examine how the strategic changes which disclosure of COIs brings to the above communication game affect welfare. For this, I will take an ex-ante view, thus before a draw of the sender's type takes place. I start with a view on naive receivers. They are agnostic about the sender's bias and do not account for the strategic change as for them, $d_n^*(m) = m$ holds. Recall that the utility of receivers decreases in the distance of their decision and the state of the world. In equilibrium, the sender announces s plus a bias. Since naive receivers follow this message, their utility's argument equals this bias. Their expected utility is then given by

$$E[u_n^R(\mathcal{E}(\rho_k^*))] = E[L(c(\mu + (1 - \mu)\rho^*))] = E[L(|c|(\mu + \rho_k^*(1 - \mu)))] < 0 \quad (12)$$

where the second equality follows from the fact that only c can be non-positive, while L is negatively valued and symmetric around its maximum of zero. Therefore, the expected utility of naive receivers strictly decreases in the equilibrium inference coefficient so that, by Proposition 4, this follows:

Corollary 1. *Upon disclosure of COIs, the expected utility of naive receivers decreases (increases) in every asymptotically stable equilibrium $\mathcal{E}(\rho_k^*)$ if and only if $\rho_U < (>) 1$.*

Naive receivers can only benefit from disclosure when the equilibrium inference coefficient before disclosure is larger than one, i.e. if none of the conditions stated in Lemma 4 is fulfilled. Otherwise, naive receivers suffer from disclosure.

To evaluate the overall effect of disclosure, one needs to also look at its effects on rational receivers. These receivers de-bias the message based on what they expect to be the sender's bias. How much they are hurt when the expected bias differs from the actual one does, therefore, depend on how much they rely on the corrected message and how volatile this bias is. To obtain a tractable measure for rational receivers' expected utility, one can exploit that their decision error $d_r^* - s$ is a linear combination of elliptically distributed random variables and, therefore, is itself elliptically distributed. One can then show that this is sufficient to represent rational receivers' expected utility as mean-variance preferences. As rational receivers' de-biasing is, on average, correct (i.e. has mean zero) this admits the following, single-argument representation:

Lemma 7. *The expected utility of rational receivers in equilibrium $\mathcal{E}(\rho_k^*)$ is given by*

$$\mathbb{E}[u_r^R(\mathcal{E}(\rho_k^*))] = \mathcal{L} \left(\sigma_s^2 \left[1 - (\text{Corr}[s, m]_{m=m^*(s,c)})^2 \right] \right) \leq 0$$

with $\mathcal{L}'(x) \leq 0$ for any $x \geq 0$. Furthermore, $\mathbb{E}[u_r^R(\mathcal{E}(\rho_k^*))] = 0$ if and only if there is full disclosure of the sender's COI.

Proof: The proof adapts some techniques from Meyer (1987) and can be found in the appendix. The above lemma shows that rational receivers expected utility can be expressed as a function which is decreasing in σ_s^2 . This argument is scaled down by the square of one minus the correlation between about the state of the world s and the sender's equilibrium message m about it. This alternative measure of the message's informativeness also connects to the previously indicated regression analogy. Its empirical counterpart is the coefficient of determination (the R^2) one would obtain if one regressed past values of s on the corresponding messages by the sender.

This formulation of the expected utility for rational receivers helps in analyzing the opposing effects of disclosure: If ρ^* increases upon disclosure, this reflects a greater informativeness of the sender's message. However, the sender strategically anticipates this and in turn, also increases the bias' magnitude which might make the rational receivers' inference more complicated again. Conversely, when ρ^* decreases, so does the bias. But does such a decrease in the inference coefficient then not imply that also the message's informativeness and with it, also the rational receivers' utility, decreases? Using the above lemma, the following result shows that in both scenarios, the net effect of disclosure on rational receivers' expected utility is positive:

Proposition 5. *Upon disclosure of COIs, the expected utility of rational receivers increases in every stable equilibrium $\mathcal{E}(\rho_k^*)$.*

Proof: See appendix.

While this is good news from the perspective of rational receivers, naive ones are often hurt by disclosure. The following results immediately follows from the preceding ones:

Corollary 2. *In any stable equilibrium $\mathcal{E}(\rho_k^*)$, disclosure of COIs is a Pareto-improvement among all receivers if and only if $\rho_U^* \geq 1$.*

The above shows that only when the inference coefficient is at least one, then all receivers benefit from disclosure. If this is not the case, thus if any of the conditions in Lemma 4 apply, naive receivers will suffer from disclosure. Based on a Pareto-criterion, disclosure is then not optimal. In this case, a policy maker who can influence disclosure and the quality of disclosed information might want to resort to other criteria such as overall efficiency. I capture such a criterion by assuming the following

welfare function with weights w_n , w_r , and w_k such that $w_r > 0$ and $\max\{w_n, w_k\} > 0$:

$$W(\mathcal{E}(\rho_k^*)) = w_n \cdot \mathbb{E}[u_n^R(\mathcal{E}(\rho_k^*))] + w_r \cdot \mathbb{E}[u_r^R(\mathcal{E}(\rho_k^*))] - w_k \cdot \mathbb{E}[(c(\mu + \rho_k^*(1 - \mu)))^2] \quad (13)$$

In the above, the first two terms capture the expected utility of naive and rational receivers. The third term allows to capture the sender's expected cost of lying, i.e. the squared bias. The sender's COI such as commissions or prices earned on options do not appear in the above as these are essentially transfers. Choosing appropriate weights can capture several efficiency notions. For example, when the sender's lying costs are immaterial and a planner only regards material consequences, then $w_k = 0$ follows and the planner might assign weights based on population shares, i.e. $w_n = \mu$ and $w_r = 1 - \mu$. If Bayesian inference and the steps involved therein are costly for rational receivers, a fully informative but biased message is not optimal. In face of such de-biasing costs, only a truthful, unbiased message would be optimal. A higher value for w_n would reflect the relative importance of these costs, relative to the pure informational content of the sender's message. Setting $w_k > 0$ allows capturing situations when the sender's expected costs of lying are also relevant.¹⁷

Generally, the exact decision of whether disclosure is efficient and how precise it should be can only be answered when specific utility functions and parameters are assumed. However, policy-relevant statements with respect to the effect of disclosure on W can be made even when such exact parameters are unknown:

Proposition 6. *If $\rho_U^* < (\geq) 1$, full disclosure never (always) maximizes welfare W in any stable equilibrium $\mathcal{E}(\rho_k^*)$.*

Proof: See appendix.

The above shows that when $\rho_U^* < 1$ holds, imperfect disclosure is always more efficient than full disclosure. This follows from the fact that receivers have strictly concave utility. Rational receivers can achieve maximum utility only with full disclosure (see Lemma 7). When near to this optimum, some sufficiently low noise is added to perfect signal \tilde{c} , the resulting loss can then be made arbitrarily small, relative to the gain in expected utility which the associated decrease in ρ^* brings to naive receivers. Note that while full disclosure is often not optimal, the reverse reasoning does not work: No or imperfect disclosure can be optimal, in a second-best sense. Determining precise criteria for this, however, requires more specific assumptions on the informational environment and preferences. An example where no disclosure at all is efficient is contained in the appendix.

¹⁷Apart from a welfare function which takes into account the lying costs per se, such costs may also matter from a policy point of view when they represent a reduced form of the sender's reputation. This becomes economically relevant when the loss in the sender's credulity impedes his economic function of information elicitation and dissemination. Alternatively, if the sender's cost of lying come from a situation where he has to invest own funds, e.g. own equity, according to his advice, these costs reflect the cost of such mis-allocated resources.

The preceding results show that the consequences of disclosing COIs depend crucially on whether ρ_U^* is above or below the threshold of one. Besides computing this value via (9), Lemma 4 allows to determine this directly from the second moments of the prior distribution. However, not even knowledge of these parameters is always necessary for an outside observer to yield testable predictions and to make informed decisions. One only needs to observe how receivers react to messages by the sender: As naive receivers follow the receiver literally, one gets from combining (4) and (8) that $\rho_D^* > 1$ is equivalent to $D'(m) > 1$. Thus, when receivers react, on average, stronger than one-to-one to a change in the sender's message, it follows from the second case in Proposition 6 that full disclosure is optimal as it benefits all receivers. Conversely, observing $D'(m) < 1$, i.e. a less than one-to-one reaction implies that upon disclosure, naive receivers will be hurt. Full disclosure is then not optimal from the point of maximizing welfare.

Finally, note that the above welfare results do not necessarily require the presence of naive receivers. This follows from the fact that the sender's expected lying costs are a special, parametrized case of the naive receivers' expected utility. For both, the sender's bias is the relevant argument. Therefore, the above results with regard to welfare also hold if only one of the two respective weights is positive, e.g. if naive receivers are absent ($\mu = w_n = 0$) or do not matter for other reasons while the sender's lying costs are relevant ($w_k > 0$). By the same reasoning, Corollary 1 can then be adapted such that then, net of transfers, disclosure of COIs is a Pareto-improvement among senders and rational receivers if and only if $\rho_U^* \geq 1$.

6 Conclusion

This paper describes a setting where a sender communicates the value of a random variable of interest to uninformed receivers. The sender does so while he faces a conflict of interest to manipulate the receivers' actions but, at the same time, also faces costs of lying. I show how, in equilibrium, rational receivers discount for the resulting bias in the sender's report, even when they do not know the bias' magnitude or direction. I also show how, in turn, the sender adjusts to this correction and that, despite the bias, his messages positively covary with the state of the world they refer to. Crucially, the model also allows for the co-presence of naive or delegating receivers who are agnostic about the bias and how they affect the sender's messaging and the rational receivers' de-biasing strategy. This provides a parsimonious framework for studying the effect of disclosing the sender's COI via a signal of arbitrary precision within a wide class of distributions and for general levels of the receivers' risk aversion.

I find that disclosure fulfills the aim of informing rational receivers: Information about the sender's COI helps them to infer more from the sender's biased message and to adjust their actions more

closely to the actual state of the world. On the downside, this paper's core result shows that exactly this desired effect of disclosure backfires on naive or delegating receivers. It does so because, in equilibrium, the average reaction to the biased signal and the sender's bias are mutually dependent. Upon disclosure, as rational receivers get helpful information to de-bias the sender's message, their reaction to the sender's message often increases. With this reaction, the bias contained in the sender's message also increases. Naive receivers who do not account for the strategic aspect of communication are then hurt by disclosure. Disclosure therefore often amplifies a negative externality which rational receivers exert on their naive peers; it then hurts those who are most vulnerable to communication biases.

I also determine more precisely when and how these adverse effects of disclosure manifest. In terms of economic fundamentals, this is always the case when the state of the world and the sender's conflict of interest are weakly positively correlated. Another sufficient condition for disclosure to backfire is when strategic uncertainty regarding the sender's bias exceeds fundamental uncertainty regarding the state of the world. In terms of observed behavior, this happens when an expert's message does not induce at least a one-to-one average reaction among receivers. Only when they react stronger than one-to-one to changes in the sender's message, then disclosure is an improvement among all, rational *and* naive, receivers. This is also the only case when full disclosure is optimal from an efficiency point of view. In all other cases, a less than perfect signal about the sender's COI, potentially even an uninformative one, is optimal for maximizing efficiency.

The results presented here show that when some people do not have the ability or the information and time to act in a completely Bayesian and rational manner, disclosure often hurts. Merely communicating an expert's conflict of interest does often *not* solve the problems which arise – it rather increases its negative effects. In consequence, disclosure is not the regulatory panacea it promises to be. This suggests that eliminating conflicts of interest promises much better outcomes than just announcing them.

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Appendix

Features of elliptically distributed random variables

The following definition is obtained from definition 1 in the survey on elliptical distributions by Gómez et al. (2003) and Theorem 4 ii) therein:

Definition. A random vector $\mathbf{x} \in \mathbb{R}^n$ is elliptically distributed with expected value $\boldsymbol{\eta} \in \mathbb{R}^n$, positive definite variance-covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{n \times n}$, and the Lebesgue-measurable function $g : [0, \infty) \rightarrow [0, \infty)$ s.t. $\int_0^\infty t^{\frac{n}{2}-1} g(t) dt < \infty$ as parameters if it has the density function

$$f(\mathbf{x}; \boldsymbol{\eta}, \tilde{\boldsymbol{\Sigma}}, g) = c_n |\tilde{\boldsymbol{\Sigma}}|^{-\frac{1}{2}} g \left((\mathbf{x} - \boldsymbol{\eta})^T \tilde{\boldsymbol{\Sigma}}^{-1} (\mathbf{x} - \boldsymbol{\eta}) \right)$$

where $c_n = \Gamma(\frac{n}{2}) / \left(\phi^{\frac{n}{2}} \int_0^\infty t^{\frac{n}{2}-1} g(t) dt \right)$ and $\tilde{\boldsymbol{\Sigma}} \propto \boldsymbol{\Sigma}$.

Therefore, the exact form of the distribution depends on the density generator g . In the context of this paper, it is assumed to be implicitly defined by the specific elliptical distribution F which is used. The generic example is when F denotes the (multivariate) normal distribution which would imply that $g(t) = \exp(-\frac{1}{2}t)$. Other examples include the multivariate logistic, student-t or power exponential families of distributions.

The results in this paper do not depend on the specific distribution F as long as it is elliptical, but just on its first two moments, $\boldsymbol{\eta}$ and $\boldsymbol{\Sigma}$. To illustrate them, consider a random vector $\mathbf{x} \in \mathbb{R}^n$ with $n \geq 2$ which is elliptically distributed according to $F(\boldsymbol{\eta}, \boldsymbol{\Sigma})$. Also consider two non-empty partitions $[\mathbf{x}_1, \mathbf{x}_2]$ of this vector. Partition analogously the corresponding vector of expected values as $\boldsymbol{\eta} = (\boldsymbol{\eta}_1, \boldsymbol{\eta}_2)$ and the variance-covariance matrix $\boldsymbol{\Sigma}$ into blocks $(\boldsymbol{\Sigma}_{11}, \boldsymbol{\Sigma}_{12}, \boldsymbol{\Sigma}_{21}, \boldsymbol{\Sigma}_{22})$. Then, the following properties, which reflect properties E1 through E3 in the main text and which will be used in the following proofs, hold for \mathbf{x} :

E1': linear combinations of elements of \mathbf{x} are distributed according to F

E2': $(\mathbf{x}_2 | \mathbf{x}_1)$ is distributed according to

$$F(\boldsymbol{\eta}_2 + \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} (\mathbf{x}_1 - \boldsymbol{\eta}_1), \boldsymbol{\Sigma}_{22} - \boldsymbol{\Sigma}_{21} \boldsymbol{\Sigma}_{11}^{-1} \boldsymbol{\Sigma}_{12})$$

E3': \mathbf{x} is symmetrically distributed around $\boldsymbol{\eta}$

E3' follows from the above density function. Properties E1' and E2' are consequences of Theorem 5 and Theorem 8, respectively in Gómez et al. (2003) which also contains further references on the original research establishing these properties for elliptical distributions. It will be useful to note that for the special case that $(\mathbf{x}_2 | \mathbf{x}_1) \in \mathbb{R}^2$, E2 implies that $(x_2 | x_1)$ is distributed according to

$$F \left(\mathbb{E}[x_2] + (x_1 - \mathbb{E}[x_1]) \frac{\text{Cov}[x_1, x_2]}{\text{Var}[x_1]}, \text{Var}[x_2] (1 - (\text{Corr}[x_1, x_2])^2) \right).$$

Proof of Lemma 1

The implicit function $R(s, m, c) = cD'(m) + s - m = 0$, stated as (3), follows directly from the first-order condition for maximizing (2). Note that the associated second-order-condition for an optimum implies $\frac{\partial R(s, m, c)}{\partial m} = cD''(m) - 1 < 0$ at each optimally chosen m . Using the implicit function theorem, one then gets that

$$\frac{dm}{ds} = - \frac{\partial R(s, m, c)}{\partial s} / \frac{\partial R(s, m, c)}{\partial m} = \frac{1}{1 - cD''(m)} > 0$$

for each such optimally chosen message m . □

Proof of Lemma 2

When m is distributed according to F , it is jointly elliptically distributed with s . By E2', the resulting distribution of s conditional on m , denoted by its pdf $f(s|m)$, is then also elliptical. Furthermore, E3' implies that $f(s|m)$ is symmetric around $E[s|m]$. By definition, it then has to hold that $d_r = \operatorname{argmax}_{d \in S} \int_{\mathbb{R}} L(d-s)f(s|m)ds$. The necessary FOC for a candidate solution $d_r = E[s|m]$ is given by

$$0 = \int_S L'(d_r - s)f(s|m)ds = \int_{-\infty}^{+\infty} L'(E[s|m] - s)f(s|m)ds$$

and is also sufficient as L is strictly concave. To verify that this FOC applies for this candidate solution note that by being strictly concave, L is single peaked and symmetric around its bliss point s . Let $\Delta \leq 0$ be the absolute deviation of the candidate solution from the optimal choice, i.e. $\Delta = |d_r - s|$. By symmetry of L around zero it holds that $L'(\Delta) = -L'(-\Delta)$. Since $f(s|m)$ is symmetric around $E[s|m] = d_r$ it then follows that

$$L'(\Delta)f(d_r - \Delta|m) = -L'(-\Delta)f(d_r + \Delta|m) \leq 0$$

applies for any $\Delta \geq 0$. Integrating over all $\Delta \in \mathbb{R}_+$ then validates that the above FOC actually holds. Since L is single-peaked, it is also the only solution. \square

Proof of Proposition 1

The following proof constructs a pure strategy equilibrium when the implied demand $D(m)$ is twice continuously differentiable. To do so, it proceeds in three steps. Step 1 solves the rational receiver's problem to choose his optimal action, given that the sender's message contains information about s . Step 2 determines how such signal extraction by rational receivers manifests in equilibrium when the sender anticipates this process. Step 3 combines these results to obtain equilibrium actions and beliefs.

Step 1: Consider a candidate equilibrium messaging strategy $\tilde{m}(s, c)$ such that $D(m) = \mu m + (1 - \mu)\tilde{d}_r(m)$ with $\tilde{d}_r(m) \equiv \operatorname{argmax}_{d \in S} E[L(d-s)|m]_{m=\tilde{m}(s,c)}$ exists and is twice differentiable w.r.t. m . By (3), \tilde{m} has to solve $\tilde{m} = s + c(\mu + (1 - \mu)\tilde{d}_r^{*'}(\tilde{m}))$. Note that given, the message \tilde{m} and the candidate equilibrium messaging function \tilde{d}_r , $\tilde{d}_r^{*'}(\tilde{m})$ is a non-random image of the function $\tilde{d}_r'(m)$, i.e. a constant. The associated messaging function which yielded \tilde{m} is thus given by

$$\tilde{m}(s, c) = s + c(\mu + (1 - \mu)\tilde{d}_r^{*'}(\tilde{m})).$$

Therefore, $\tilde{m}(s, c)$ is a linear combination of s and c and, by E1', distributed according to F . Lemma 2 then implies that $\tilde{d}_r(m) = E[s|m]_{m=\tilde{m}(s,c)} \equiv E[s|\tilde{m}]$. Using E2' then yields that for a given equilibrium message \tilde{m} , it has to hold that

$$\tilde{d}_r(m) = E[s] + \left(m - E[s] - E[c](\mu + (1 - \mu)\tilde{d}_r^{*'}(m)) \right) \frac{\operatorname{Cov}[s, m]_{m=\tilde{m}(s,c)}}{\operatorname{Var}[m]_{m=\tilde{m}(s,c)}} \quad (14)$$

where $E[m]_{m=\tilde{m}(s,c)} = E[s] + E[c](\mu + (1 - \mu)\tilde{d}_r^{*'}(m))$ has been used.

Step 2: In the candidate equilibrium with the associated equilibrium coefficient $\tilde{\rho} \equiv \frac{\operatorname{Cov}[s, m]_{m=\tilde{m}(s,c)}}{\operatorname{Var}[m]_{m=\tilde{m}(s,c)}}$ the function $\tilde{d}_r^{*'}(m)$ then has to solve the first-order linear differential equation

$$\tilde{d}_r^{*'}(m) = \left(1 - E[c](1 - \mu)\tilde{d}_r^{*''}(m) \right) \tilde{\rho}.$$

When $\tilde{d}_r^{*'}(m) = 0$, it follows that $\tilde{d}_r^{*''}(m) = \tilde{\rho} = 0$. Similarly, if $E[c] = 0$, then $\tilde{d}_r^{*'}(m) = \tilde{\rho}$. Now suppose that $\tilde{\rho}E[c] \neq 0$. One then gets $\tilde{d}_r^{*'}(m)$ as the solution to the above differential equation,

given by

$$\tilde{d}_r^l(m) = \rho + \xi \cdot \exp\left(-\frac{m}{(1-\mu)\mathbb{E}[c]\tilde{\rho}}\right),$$

where ξ is an integration factor. To determine its value, integrate the obtained $\tilde{d}_r^l(m)$ over $M = \mathbb{R}$:

$$\mathbb{E}[s|\tilde{m}] = \int_{-\infty}^{+\infty} \tilde{d}_r^l(m) dm = m\tilde{\rho} - \xi(1-\mu)\mathbb{E}[c]\tilde{\rho} \cdot \exp\left(-\frac{m}{(1-\mu)\mathbb{E}[c]\tilde{\rho}}\right) + \tilde{K} \quad (15)$$

In the above, \tilde{K} is a constant of integration. This can expression be plugged into the sender's expected utility to obtain $\mathbb{E}[U^S(s, c, m)|_{m=\tilde{m}(s,c)}] =$

$$c\mu m + c(1-\mu) \left[m\tilde{\rho} - \xi(1-\mu)\mathbb{E}[c]\tilde{\rho} \cdot \exp\left(-\frac{m}{(1-\mu)\mathbb{E}[c]\tilde{\rho}}\right) + \tilde{K} \right] - \frac{1}{2}(m-s)^2 \quad (16)$$

To determine ξ , I start with the case that $c > 0$. In this case, $U^S(s, c, m)$ is increasing in $\mathbb{E}[s|\tilde{m}^*]$, the term above in square brackets. If $\tilde{\rho}\mathbb{E}[c] > 0$, the sender's expected utility decreases exponentially in m while all other terms involving m are either linear or quadratic. If $\xi < 0$, the sender would then maximize his expected utility by choosing $m \rightarrow -\infty$ and there is no equilibrium. Therefore, $\xi \geq 0$ has to hold in this case for any equilibrium. For any $\xi > 0$, however, $U^S(s, c, m)$ would be lower than with $\xi = 0$. Since ξ is part of the endogenous inference of the sender's signal, he will not send a signal which allows such an inference. It follows that with $c > 0$ and $\rho\mathbb{E}[c] > 0$, only $\xi = 0$ can be the equilibrium integration factor.

Continue to suppose that $c > 0$ but now $\rho\mathbb{E}[c] < 0$ holds. Inverse to the the preceding reasoning, $\mathbb{E}^*[s|\tilde{m}]$ now increases exponentially in m which implies a global maximum of the sender's expected utility at $m \rightarrow +\infty$ whenever $\xi > 0$. Thus, for an equilibrium, $\xi \leq 0$ has to hold. Also inverse to the above, any $\xi < 0$ would decrease the sender's expected utility so that $\xi = 0$ is chosen in any equilibrium with $c > 0$ and $\rho\mathbb{E}[c] < 0$.

For the case that $c < 0$, $U^S(s, c, m)$ is decreasing in $\mathbb{E}[s|\tilde{m}]$. The same reasoning as for the case of $c > 0$, but with reversed signs can then be repeated which rules out any $\xi \neq 0$ in an equilibrium with $c < 0$ and $\tilde{\rho}\mathbb{E}[c] \neq 0$ holds.

Eventually, when $c = 0$ the inference $\mathbb{E}[s|m]$ does not enter $U^S(s, c, m)$; it is equivalent to $\xi = 0$. Thus, $\xi = 0$ is the unique integration factor and $\tilde{d}_r^*(m) = \tilde{\rho}$ holds.

Step 3: Given the above, one can determine the integration constant

$$\tilde{K} = \mathbb{E}[s] - (\mathbb{E}[s] + \mathbb{E}[c](\mu + (1-\mu)\tilde{\rho}))\tilde{\rho}$$

by combining (14) and (15). Using $\xi = 0$ then allows to write (16) as

$$U^S(s, c, m) = mc(\mu + (1-\mu)) - \frac{1}{2}(m-s)^2 + c(1-\mu)K$$

It is then easily verified that the unique message which maximizes the above expression is given by $m = s + c(\mu + (1-\mu))\tilde{\rho}$. In equilibrium, it has to hold that $m^*(s, c) = s + c(\mu + (1-\mu))\tilde{\rho}^*$ with $\rho^* = \tilde{\rho} = d_r^{*'}(m) = \frac{Cov[s, m]_{m=m^*(s,c)}}{Var[m]_{m=m^*(s,c)}}$, as stated in (7). Using $\tilde{\rho} = \rho^*$, $\xi = 0$, and the above expression for \tilde{K} on (15) then yields the rational receivers belief and strategy as stated in (8). \square

Proof of Proposition 2, part 1

By using $m^*(s, c) = s + c(\mu + (1 - \mu)\rho)$ from Proposition 1 and the definition of $\rho^* = \frac{Cov[s, m^*]}{Var[m^*]}$, the latter must be a solution to

$$\begin{aligned} \rho &= \frac{Cov[s, m^*]}{Var[m^*]} = \frac{Cov[s, m]_{m=m^*(s,c)}}{Var[m]_{m=m^*(s,c)}} = \frac{E[(s - E[s])((s - E[s]) + (\mu + (1 - \mu)\rho)(c - E[c]))]}{E[((s - E[s]) + (\mu + (1 - \mu)\rho)(c - E[c]))^2]} \\ &= \frac{\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \end{aligned} \quad (17)$$

which yields (9). It will be useful to denote the above nominator and denominator by the following functions:

$$\begin{aligned} N(\rho) &\equiv \sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc} \\ D(\rho) &\equiv \sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2 \end{aligned}$$

For the rational function (17), expressed as $g_U(\rho) \equiv N(\rho)/D(\rho)$, the following properties hold:

Property a) $N(\rho)/D(\rho)$ is continuous with $D(\rho) > 0$ for all $\rho \in \mathbb{R}$.

Proof: Since both, $D(\rho)$ and $N(\rho)$ are continuous in ρ , it is sufficient to show that $D(\rho) > 0$ always holds. Suppose to the contrary it would not. Rearranging $D(\rho)$, this would require that $\rho^2 + a\rho + b = 0$ with

$$a = \frac{2(\sigma_{sc} + \mu\sigma_c^2)}{(1 - \mu)\sigma_c^2} \quad b = \frac{\sigma_s^2 + 2\mu\sigma_{sc} + \mu^2\sigma_c^2}{(1 - \mu)^2\sigma_c^2}$$

has at least one real solution, thus that $(a/2)^2 - b \geq 0$ holds. Plugging in and rearranging, this yields $(\sigma_{sc}/(\sigma_c\sigma_s))^2 \geq 1 - a$ a contraction to $|Corr[s, c]| < 1$. \square

Property b) $\lim_{\rho \rightarrow +\infty} \left(\frac{N(\rho)}{D(\rho)}\right) = 0^-$ if $\sigma_{sc} < 0$ and $\lim_{\rho \rightarrow +\infty} \left(\frac{N(\rho)}{D(\rho)}\right) = 0^+$ if $\sigma_{sc} \geq 0$

Proof: $N(\rho)$ strictly decreases (weakly increases) linearly in ρ when $\sigma_{sc} < 0$ ($\sigma_{sc} \geq 0$) and attains negative (positive) values for ρ large enough. From a), $D(\rho)$ is strictly positive and it grows quadratically in ρ . Therefore, for large values of ρ , the ratio $N(\rho)/D(\rho)$ is negative (positive) and arbitrarily close to zero. \square

Property c) $\frac{N(\rho)}{D(\rho)}$ has at most two extreme points.

Proof: Any extreme point has to set the first derivative

$$\begin{aligned} \left(\frac{N(\rho)}{D(\rho)}\right)' &= \frac{(1 - \mu)\sigma_{sc}D(\rho) - 2(1 - \mu)N(\rho)(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2)}{(D(\rho))^2} \\ &= \frac{(1 - \mu)}{D(\rho)} \cdot \left(\sigma_{sc} - \frac{N(\rho)}{D(\rho)} \cdot 2(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2)\right) \end{aligned} \quad (18)$$

equal to zero. By a) and $\mu \in [0, 1)$, the first factor is non-zero. Extreme points therefore have to solve

$$\sigma_{sc}D(\rho) = N(\rho) \cdot 2(\sigma_{sc} + (\mu + (1 - \mu)\rho)\sigma_c^2)$$

Plugging in the functions for $N(\rho)$ and $D(\rho)$ yields an equation which is quadratic in ρ and, therefore, has at most two real solutions. \square

Before I continue with the remaining part of the proof of Proposition 2, I first proof Lemma 3 and 4. While they are stated in the main text after Proposition 2 for reasons of better exposure, they only rely on properties proved so far.

Proof of Lemma 3

Necessity: By property a) as derived above, it follows that for a fixed point ρ_U^* which solves $g_U(\rho_U^*) = N(\rho_U^*)/D(\rho_U^*) > 0$, $N(\rho_U^*) > 0$ has to hold. This is equivalent to $\sigma_{sc} > \tau(\rho_U^*)$ where $\tau(\rho) = -\sigma_s^2/(\mu + (1-\mu)\rho) < 0$ is defined for any $\rho > 0 \geq -\mu/(1-\mu)$ and for which $\tau'(\rho) > 0$ holds whenever $\rho \geq 0$. For $\rho_U^* > 0$ it therefore has to hold that $\sigma_{sc} > \tau^* \equiv \tau(\rho_U^*)$ with $\tau^* < 0$.

Sufficiency: To see that $\sigma_{sc} > \tau^*$ is also sufficient for (9) to have a solution $\rho_U^* > 0$, note that by the above reasoning $\sigma_{sc} > \tau(\rho^*) > \tau(0)$ holds and therefore, $Cov[s, m^*]|_{\rho=0} = N(0) > 0$ applies. Since $Var[m^*]_{\rho=0} = D(0) > 0$, it then follows that $g_U(0) = N(0)/D(0) > 0$. Together with continuity and a limit of zero of $g_U(\rho) = N(\rho)/D(\rho)$ as derived in properties a) and b) above, this means that there has to be at least one fixed point, i.e. at least one intersection of $g_U(\rho)$ with the 45-degree line over \mathbb{R}_{++} . \square

Proof of Lemma 4

Necessity: By proposition (17) in the hitherto part of the proof of Proposition 2, $\rho_U^* \leq 1$, conditional on $\rho_U^* > 0$, holds if and only if

$$\frac{\sigma_s^2 + (\mu + (1-\mu)\rho)\sigma_{sc}}{\sigma_s^2 + 2(\mu + (1-\mu)\rho)\sigma_{sc} + (\mu + (1-\mu)\rho)^2\sigma_c^2} \Big|_{\rho=\rho_U^*} = \rho_U^* \leq 1 \quad (19)$$

This condition simplifies to $\sigma_{sc} \geq -(\mu + (1-\mu)\rho_U^*)\sigma_c^2$. This inequality becomes slacker for higher, positive values of ρ_U^* . Inserting $\rho_U^* = 1$, the upper bound on the desired value range, then yields $\sigma_{sc} \geq -\sigma_c^2$ as a necessary condition.

Sufficiency: To see that that this condition is also sufficient first note from (19) that, for any $\sigma_{sc} \geq 0$, $\rho_U^* \in (0, 1)$ follows immediately. Second, from the above reasoning it also follows immediately that $\sigma_{sc} = \sigma_c^2$ implies $\rho_U^* = 1$. Now suppose $\sigma_{sc} \in (-\sigma_c^2, 0)$, i.e. $\sigma_c^2 = -\frac{\sigma_{sc}}{\lambda}$ for some $\lambda \in (0, 1)$. To show that then, $\rho_U^* < 1$ follows, suppose the opposite and substitute into (19) to get

$$\frac{\sigma_s^2 + (\mu + (1-\mu)\rho)\sigma_{sc}}{\sigma_s^2 + (\mu + (1-\mu)\rho_U^*)\sigma_{sc} \cdot \left(2 - \frac{\mu + (1-\mu)\rho_U^*}{\lambda}\right)} \Big|_{\rho=\rho_U^*} \geq 1.$$

Since the above denominator represents, in equilibrium, $Var[m^*] = D(\rho^*) > 0$ (see property a) in the preceding proof), this simplifies to

$$0 \geq (\mu + (1-\mu)\rho)\sigma_{sc} \cdot \left(1 - \frac{\mu + (1-\mu)\rho}{\lambda}\right) \Big|_{\rho=\rho_U^*}$$

Clearly, this is a contradiction as, with $\sigma_{sc} < 0$, $\rho = \rho_U^* \geq 1$ and $\lambda \in (0, 1)$, both of the above RHS's factors will be strictly negative. \square

Proof of Proposition 2, part 2

A solution ρ_U^* to (9) requires an intersection of the 45-degree line and $N(\rho)/D(\rho)$. Note that every such fixed point has to be a root of $k(\rho) = \rho D(\rho) - N(\rho)$, i.e. the following cubic equation:

$$k(\rho) = \underbrace{(1-\mu)^2\sigma_c^2}_{\equiv A} \cdot \rho^3 + \underbrace{2(1-\mu)(\sigma_{sc} + \mu\sigma_c^2)}_{\equiv B} \cdot \rho^2 + \underbrace{\sigma_s^2 + \mu^2\sigma_c^2 + (3\mu-1)\sigma_{sc}}_{\equiv C} \cdot \rho - \underbrace{\sigma_s^2 - \mu\sigma_{sc}}_{\equiv D} \quad (20)$$

To examine multiplicity of roots to $k(\rho)$, I use the following result:

Theorem. (Descarte's rule of signs) Consider a n -degree polynomial $p(x) = \sum_{d=0}^n c_d \cdot x^d$ with real coefficients. Order the non-zero coefficients c_k in an descending order of the exponent d . The number of positive, real roots of the polynomial is less by an even number or equal to the number of sign changes between successive coefficients in this ordering.

It always holds that $A > 0$. Furthermore, by the proof of Lemma 3, a solution $\rho_U > 0$ implies $D < 0$ because this is equivalent to $\sigma_{sc} > \tau(0)$ and $\tau(0) > \tau(\rho_U^*) = \tau^*$. By the sign rule, the only configuration for more than one sign change, given that $A > 0 > D$, is $C > 0 > B$. Thus, there are either one or three positive roots, which correspond to fixed points of $g_U(\rho)$.

Multiple fixed points require $B < 0$ and, therefore, $\sigma_{sc} < 0$. Suppose they exist. By property a) and b) derived in the first part of this proof, this means that $g_U(\rho) = N(\rho)/D(\rho)$ continuously approaches zero from below when ρ becomes large enough. Also, it has been shown that $N(0)/D(0) > 0$ (see proof of Lemma 3). Together, this implies that g_U has to have a negatively valued local minimum on \mathbb{R}_{++} denoted by ρ_- , i.e. $g_U(\rho_-) < 0$. If ρ_- is the only extreme value over \mathbb{R}_{++} this implies only one intersection with the 45-degree line, thus a unique fixed point. If it is not the unique extreme value then, by property c) derived in the first part of the proof, there is exactly one other extreme value of g_U over \mathbb{R}_{++} . Given that ρ_- is a local minimum, it has to be a local maximum and is denoted by ρ_+ . It then follows from $\lim_{\rho \rightarrow +\infty} g_U(\rho) = 0^-$ that $0 < \rho_+ < \rho_+$ and $g_U(\rho_-) < 0 < g_U(\rho_+)$ have to hold. Accordingly, $g_U(\rho) = N(\rho)/D(\rho)$ is non-increasing on $[\rho^+, \rho^-]$. This, together with $N(0)/D(0) > 0$ implies that $\rho_U(\rho)$ cuts the 45-degree line exactly only once within this interval and never on $(\rho^-, +\infty)$ such that the unique fixed-point in (ρ^+, ρ^-) is the highest-valued one. As multiple, positively valued fixed points of $g_U(\rho)$ number to three, their coordinates can be denoted w.l.o.g. by $0 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$. It then has to hold that

$$0 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_+ < \rho_{U,3}^* < \rho_-$$

As there is no further extreme point over $[\rho_{U,1}^*, \rho_{U,2}^*] \subset (0, \rho_+)$ while $0 < g_U(0) < g_U(\rho_+)$ holds, it follows that $g_U(\rho)$ is non-decreasing on $[\rho_{U,1}^*, \rho_{U,2}^*]$. Three fixed points of $g_U(\rho)$ at $\rho_{U,1}^* < \rho_{U,2}^*$ and $\rho_{U,3}^* \in (\rho_+, \rho_-)$ then imply that $g_U(\rho) = N(\rho)/D(\rho)$ cuts the 45-degree line (which has slope 1) thrice: First from above at $\rho_{U,1}^*$, then from below at $\rho_{U,2}^*$ (which requires a slope larger than 1), and then from above at $\rho_{U,3}^*$. It thus holds that

$$g_U(\rho)'|_{\rho=\rho_{U,3}^*} < 0 < g_U(\rho)'|_{\rho=\rho_{U,1}^*} < 1 < g_U(\rho)'|_{\rho=\rho_{U,2}^*} \quad (21)$$

Using the fact that if this indeed an equilibrium, $\rho_{U,1}^* = g_U(\rho_{U,1}^*) = N(\rho_{U,1}^*)/D(\rho_{U,1}^*)$ has to hold, the requirement of a positive slope greater at $\rho_{U,1}^* > 0$ translates via (18) into

$$\left(\frac{N(\rho)}{D(\rho)} \right)' \Big|_{\rho=\rho_{U,1}^*} = \frac{(1-\mu)}{D(\rho_{U,1}^*)} \cdot (\sigma_{sc} - 2\rho_{U,1}^*(\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)\sigma_c^2)) > 0$$

For this to hold, $\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)\sigma_c^2 < 0$ is a necessary condition as $\rho_{U,1}^* > 0 > \sigma_{sc}$. Multiplying by $(\mu + (1-\mu)\rho_{U,1}^*) > 0$ yields the equivalent necessary condition

$$(\mu + (1-\mu)\rho_{U,1}^*)\sigma_{sc} + (\mu + (1-\mu)\rho_{U,1}^*)^2\sigma_c^2 = D(\rho_{U,1}^*) - N(\rho_{U,1}^*) < 0$$

Rearranging this inequality then yields

$$1 < N(\rho_{U,1}^*)/D(\rho_{U,1}^*) = \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$$

as a necessary condition for multiple fixed points ρ_U^* . □

Proof of Lemma 5

The assumptions on ϵ as stated in the main text can be re-stated as follows:

$$\begin{pmatrix} s \\ c \\ \epsilon \end{pmatrix} \sim F \left(\begin{bmatrix} \bar{s} \\ \bar{c} \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_s^2 & \sigma_{sc} & 0 \\ \sigma_{sc} & \sigma_c^2 & 0 \\ 0 & 0 & \sigma_\epsilon^2 \end{bmatrix} \right)$$

Using E1' establishes that $\tilde{c} = c + \epsilon$ is distributed according to F and so is the random vector (\tilde{c}, s, c) . Note that because ϵ is not correlated with s or c and has an expected value of zero, $Cov[s, \tilde{c}] = E[(s - E[s])(c + \epsilon - E[c])] = E[(s - \bar{s})(c - \bar{c})] = \sigma_{sc}$, $Var[\tilde{c}] = E[(c + \epsilon - E[c])^2] = E[(c + \epsilon - \bar{c})^2] = \sigma_c^2 + \sigma_\epsilon^2$, and $Cov[c, \tilde{c}] = E[(c - E[c])(c + \epsilon - \bar{c})] = E[(c - \bar{c})(c - \bar{c})] = \sigma_c^2$. Therefore, it holds that

$$\begin{pmatrix} \tilde{c} \\ s \\ c \end{pmatrix} \sim F \left(\begin{bmatrix} \bar{c} \\ \bar{s} \\ \bar{c} \end{bmatrix}, \begin{bmatrix} \sigma_c^2 + \sigma_\epsilon^2 & \sigma_{sc} & \sigma_c^2 \\ \sigma_{sc} & \sigma_s^2 & \sigma_{sc} \\ \sigma_c^2 & \sigma_{sc} & \sigma_c^2 \end{bmatrix} \right).$$

Using E2' with the parameters from the above distribution then yields, after some rearranging, the stated conditional moments for $(s, c | \tilde{c})$. \square

Proof of Proposition 3

Using again the equilibrium mapping $m^*(s, c)$ from Proposition 1, now with information structure \mathcal{I}_D and the associated conditional distribution of $(s, c | \tilde{c})$, means that with disclosed COIs an inference coefficient ρ_D^* has to solve the following expression:

$$\begin{aligned} \rho &= \frac{Cov[s, m^* | \tilde{c}]}{Var[m^* | \tilde{c}]} \\ &= \frac{(1 - \rho_c(Corr[s, c])^2) \sigma_s^2 + (\mu + (1 - \mu)\rho)(1 - \rho_c)\sigma_{sc}}{(1 - \rho_c(Corr[s, c])^2) \sigma_s^2 + 2(\mu + (1 - \mu)\rho)(1 - \rho_c)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2(1 - \rho_c)\sigma_c^2} \\ &= \frac{\left(\frac{1 - \rho_c(Corr[s, c])^2}{1 - \rho_c}\right) \sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc}}{\left(\frac{1 - \rho_c(Corr[s, c])^2}{1 - \rho_c}\right) \sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2} \end{aligned}$$

Under full disclosure, $\sigma_\epsilon^2 = 0$ holds and $\rho_c = 1$ applies such that a unique solution $\rho = \rho_D^* = 1$ follows from the second line. The last transformation assumes $\rho_c \neq 1$, i.e. $\sigma_\epsilon^2 > 0$. Substituting ϕ for $(1 - \rho_c(Corr[s, c])^2)/(1 - \rho_c)$ then yields (11). \square

Proof of Lemma 6

Analogously to the first part of the proof of Proposition 2, define $g_D(\rho) = \tilde{N}(\rho)/\tilde{D}(\rho)$ with

$$\tilde{N}(\rho) \equiv \phi\sigma_s^2 + (\mu + (1 - \mu)\rho)\sigma_{sc} > N(\rho)$$

$$\tilde{D}(\rho) \equiv \phi\sigma_s^2 + 2(\mu + (1 - \mu)\rho)\sigma_{sc} + (\mu + (1 - \mu)\rho)^2\sigma_c^2 > D(\rho)$$

to express (11). It is easily verified that $\tilde{N}(\rho)/\tilde{D}(\rho)$ inherits the properties a) through c) of $N(\rho)/D(\rho)$ as stated in the proof of Proposition 2.

Similar to the second part of that proof, where fixed points to $g_U(\rho)$ were expressed as roots to $k(\rho) = \rho D(\rho) - N(\rho)$, one can define and solve for the cubic function $\tilde{k}(\rho) = \rho \tilde{D}(\rho) - \tilde{N}(\rho)$:

$$\tilde{k}(\rho) = \underbrace{(1 - \mu)^2 \sigma_c^2}_{\equiv \tilde{A}} \cdot \rho^3 + \underbrace{2(1 - \mu)(\sigma_{sc} + \mu\sigma_c^2)}_{\equiv \tilde{B}} \cdot \rho^2 + \underbrace{\phi\sigma_s^2 + \mu^2\sigma_c^2 + (3\mu - 1)\sigma_{sc}}_{\equiv \tilde{C}} \cdot \rho - \underbrace{\phi\sigma_s^2 - \mu\sigma_{sc}}_{\equiv \tilde{D}} \quad (22)$$

Comparing these coefficients to those of $k(\rho)$ as stated in (20), one then gets $\tilde{A} = A > 0$, $\tilde{B} = B$, $\tilde{C} > C$, and $\tilde{D} < D < 0$. Applying the sign rule again implies that there are either one or three roots to $\tilde{k}(\rho)$, and therefore to $g_D(\rho)$.

This proof uses this cubic function \tilde{k} with coefficients \tilde{A} through \tilde{D} whose roots denote fixed points to $g_D(\rho)$. It also uses the cubic function k as stated in (20) with coefficients A through D and whose roots denote fixed points to $g_U(\rho)$ as stated in (9). Recall from Proposition 2 that this function g_U has either one or three fixed points, with any solution $\rho_U^* \in (0, 1]$ being unique.

Given these prerequisites, note from (20) and (22) that these function relate to each other as follows: $\tilde{k}(0) = \tilde{D} < k(0) = D < 0$ and $\tilde{k}'(\rho) = 3\tilde{A}\rho^2 + \tilde{B}\rho + \tilde{C} > k'(\rho) = 3A\rho^2 + B\rho + C$ for all $\rho \in \mathbb{R}_+$. Furthermore, $\tilde{k}(\rho) = k(\rho)$ if and only if $(\tilde{D} - D)/(C - \tilde{C}) = \rho$ which is easily verified to be equivalent to $\rho = 1$. It then holds that $k(\rho) > \tilde{k}(\rho)$ if $\rho \in (0, 1)$ and $k(\rho) < \tilde{k}(\rho)$ if $\rho > 1$.

Taken together, the above means that if there is a (unique) root $\rho_U^* \in (0, 1)$ of k , there must be a unique root \tilde{k} on $(\rho_U^*, 1)$ and if $\rho_U^* = 1$, $\rho_D^* = 1$ applies. To see that a root $\rho_D^* < 1$ is unique, one can repeat the same reasoning as in the second part of the proof of Proposition 2 to show that multiple solutions require all of them to have a value larger than one. This proves cases a) and b).

For case c), thus when there is a unique $\rho_U^* > 1$ it has to hold that $\tilde{k}(1) = k(1) < 0$ by the above reasoning. A unique root of k at $\rho_U^* > 1$ implies that g_U never cuts the real line again on $(\rho_U^*, +\infty)$. Neither does then \tilde{k} since $\tilde{k}(\rho) > k(\rho)$ for $\rho > 1$. This, in addition with $\tilde{k}(1) = k(1) < 0$, means that \tilde{k} cuts the real line once over $(1, \rho_U^*)$ which proves the case.

Now consider case d), i.e that there are three positively-valued fixed points to $g_U(\rho)$. By Proposition 2, their coordinates have to obey $1 < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$. The continuous, cubic function k obeys $k(0) = D < 0$ (see the second part of the proof of Proposition 2). This implies that k cuts the real line from below at $\rho_{U,1}^*$, from above at $\rho_{U,2}^*$, and again from below at $\rho_{U,3}^*$. Since it is a continuous polynomial, it has to have a local maximum and minimum in between these points. They are denoted by ρ_-^k and ρ_+^k , respectively so that

$$1 < \rho_{U,1}^* < \rho_+^k < \rho_{U,2}^* < \rho_-^k < \rho_{U,3}^*$$

holds. If \tilde{k} also has three roots, denoted by $\rho_{D,1}^* < \rho_{D,2}^* < \rho_{D,3}^*$, it is a similarly-shaped polynomial by analogous reasoning. Therefore, \tilde{k} cuts the real line from below at $\rho_{D,1}^*$, from above at $\rho_{D,2}^*$, and from below at $\rho_{D,3}^*$. From $\tilde{k}(1) = k(1) < 0$ and $\tilde{k}(\rho) > k(\rho)$ for $\rho > 1$, it follows that when \tilde{k} cuts the real line from below (above), it has to do so at lower (higher) values than k . For three roots of \tilde{k} , this implies that

$$1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{D,2}^* < \rho_{D,3}^* < \rho_{U,3}^*$$

which proves the second part of case d). If \tilde{k} has only one root (two have been ruled out by the sign rule), $\tilde{k}(1) = k(1) < 0$ and $\tilde{k}(\rho) > k(\rho)$ again imply that it cuts the real line from below, i.e. at a lower value of ρ than for k . It follows that $1 < \rho_{D,1}^* < \rho_{U,1}^* < \rho_{U,2}^* < \rho_{U,3}^*$ which proves the first part of case d). \square

Proof of Proposition 4

Fixed points to $g_U(\rho)$ can be found as roots to $f(\rho) = g_U(\rho) - \rho$. Such a fixed point ρ_U^* is then (asymptotically) stable if $f'(\rho)|_{\rho=\rho_U^*} < 0$ (see Hirsch and Smale, 1974, pp. 185-188 and footnote 16). From the first part of the proof of Proposition 2 one gets that $f(0) = g_U(0) = N(0)/D(0) > 0$ holds. It follows that for f to have three roots, it has to cut the real line from above at $\rho_{U,1}^*$, from below at $\rho_{U,2}^*$, and again from above at $\rho_{U,3}^*$. This implies $f'(\rho_{U,1}^*) < 0$, $f'(\rho_{U,2}^*) > 0$, and $f'(\rho_{U,3}^*) < 0$ which proves stability of $\rho_{U,1}^*$ and $\rho_{U,3}^*$, and that $\rho_{U,2}^*$ is not stable. By the same reasoning, a unique root ρ_U^* has to obey $f'(\rho_U^*) < 0$ and is thus stable.

For the case of undisclosed commissions one can repeat the above procedure by using $g_D(\rho) = \tilde{N}(\rho)/\tilde{D}(\rho)$ with $\tilde{N}(\rho)$ and $\tilde{D}(\rho)$ as defined in the proof of Proposition 3. In particular, when $\rho_U^* > 0$

exists, $\tilde{N}(\rho) > N(\rho) > 0$ holds for all $\rho > 0$. By the same reasoning, $\tilde{D}(\rho) > D(\rho) > 0$ applies such that $g_D(0) = \tilde{N}(0)/\tilde{D}(0) > 0$. Preceding as above, one can then define $\tilde{f}(\rho) = g_D(\rho) - \rho$ whose roots correspond to fixed points of $g_D(\rho)$ and for which $\tilde{f}(0) = \tilde{N}(0)/\tilde{D}(0) > 0$ applies. The above reasoning regarding the shape of f then applies analogously for \tilde{f} , i.e. $\rho_{D,1}^*$, $\rho_{D,3}^*$, and unique fixed points ρ_D^* are stable while $\rho_{D,2}^*$ is not stable. The proposition is then obtained by connecting these findings to the inequalities stated in Lemma 6. \square

Proof of Lemma 7

The argument of the sender's (expected) utility is given by

$$\begin{aligned} z &\equiv d_r^*(m) - s = (1 - \rho^*)\mathbb{E}[s] + \rho^* [m^*(s, c) - \mathbb{E}[c] (\mu + (1 - \mu)\rho^*)] - s \\ &= -(s - \mathbb{E}[s]) + \rho^* [m^*(s, c) - \mathbb{E}[s] - \mathbb{E}[c] (\mu + (1 - \mu)\rho^*)] \\ &= -(s - \mathbb{E}[s]) + (m^*(s, c) - \mathbb{E}[m^*(s, c)]) \rho^* n \end{aligned} \quad (23)$$

Note that by (7), $m^*(s, c)$ is a linear transformation of the vector (s, c) . By E1', it is therefore distributed according to F . Similarly, s is also distributed according to F . By the same argument, z is then distributed according to $F(\mathbb{E}[z], \text{Var}[z])$. One can normalize z via the linear transformation $\hat{z}(z) = z/\sqrt{\text{Var}[z]} - \mathbb{E}[z]$ such that \hat{z} follows $F(0, 1)$. The associated probability density function will be denoted $\psi(\hat{z})$. The expected utility of rational receivers can then be expressed as

$$\mathbb{E}[L(z)] = \int_{-\infty}^{+\infty} L\left(\mathbb{E}[z] + \hat{z}\sqrt{\text{Var}[z]}\right) \psi(\hat{z}) d\hat{z} \equiv V\left(\mathbb{E}[z], \sqrt{\text{Var}[z]}\right) \leq 0.$$

From (23) it follows that $\mathbb{E}[z] = 0$. Therefore, one can define the univariate function $\mathcal{L}(\text{Var}[y]) \equiv V(0, \text{Var}[z]) \leq 0$ which denotes a rational receiver's expected utility. This function's derivate is then given by

$$\mathcal{L}'(\text{Var}[z]) = \frac{\partial V\left(\mathbb{E}[z], \sqrt{\text{Var}[z]}\right)}{\partial \text{Var}[z]} \Big|_{\mathbb{E}[z]=0} = \frac{1}{2\sqrt{\text{Var}[z]}} \cdot \int_{-\infty}^{+\infty} \left[\hat{z} \cdot L'\left(\hat{z}\sqrt{\text{Var}[z]}\right)\right] \psi(\hat{z}) d\hat{z}.$$

Because L is strictly concave and symmetric around zero, $\text{sgn}[\hat{z}] = -\text{sgn}\left[L'\left(\hat{z}\sqrt{\text{Var}[z]}\right)\right]$ and therefore, the above expression is non-positive. Since $\psi(\hat{z})$ is symmetric around zero $\mathcal{L}'(\text{Var}[z]) = 0$ holds if and only if $\text{Var}[z] = 0$. It has been shown in the main text that under full disclosure, $u_r^R(\rho^*, \cdot) = 0$ holds since rational receivers can then fully extract s from the message and implement their optimal choice. Full disclosure therefore implies $\mathcal{L}(0) = 0$.

To see that full disclosure is also necessary for $\mathcal{L}(0) = 0$ to hold, note from the above that this requires $\text{Var}[z] = 0$ and therefore $d_r^*(m) = s$. Suppose that this held under imperfect disclosure. For $d_r^*(m) = s$ to apply in this case, (8) requires both, $\rho^* = 1$ and $c = \mathbb{E}[c]$ to hold simultaneously for *any* realization (s, c) . This is a contradiction to the fact that under imperfect disclosure with $\rho_c \in (0, 1)$, $\text{Var}[c|\tilde{c}] > 0$ and $\text{Var}[s|\tilde{c}] > 0$ applies (see Lemma 5).

To see the alternative representation of the argument $\text{Var}[z]$ of \mathcal{L} , note that by using the definition of ρ^* , one gets the following:

$$\begin{aligned} \text{Var}[z] &= \text{Var}[d_r^*(m) - s] \\ &= \mathbb{E}[(-(s - \mathbb{E}[s]) + (m^*(s, c) - \mathbb{E}[m^*(s, c)])\rho^*)^2] \\ &= (\sigma_s^2 - 2\rho^* \text{Cov}[s, m^*] + (\rho^*)^2 \text{Var}[m^*]) \\ &= \sigma_s^2 - \rho^* \text{Cov}[s, m^*] \end{aligned}$$

From the law of total variance and using again the definition of ρ^* , it also holds that

$$\begin{aligned}
\mathbb{E}[Var[s|m^*]] &= Var[s] - Var[\mathbb{E}[s|m^*]] \\
&= \sigma_s^2 - \mathbb{E}[(d_r^*(m) - \mathbb{E}[s])^2] \\
&= \sigma_s^2 - \mathbb{E}[(m^* - \mathbb{E}[m^*])\rho^*]^2 \\
&= \sigma_s^2 - (\rho^*)^2 Var[m^*] \\
&= Var[z] = \sigma_s^2 - \rho^* Cov[s, m^*] \\
&= \sigma_s^2 - \frac{Cov[s, m^*]^2}{Var[m^*]} \\
&= \sigma_s^2 (1 - Corr[s, m^*]^2) \geq 0
\end{aligned}$$

where $Corr[s, m^*] = Corr[s, m]_{m=m^*(s,c)} = Cov[s, m^*]/(\sigma_s \sqrt{Var[m^*]})$. \square

Proof of Proposition 5

Lemma 7 shows that the expected utility of rational receivers strictly increases in $Corr[s, m^*]^2$. For equilibria with $\rho^* > 0$, and therefore $Cov[s, m^*] > 0$, it is then sufficient to show that $Corr[s, m^*] > 0$ increases upon disclosure. For this note that

$$Corr[s, m^*] = \frac{Cov[s, m^*]}{Var[m^*]} \cdot \frac{\sqrt{Var[m^*]}}{\sigma_s} = \rho^* \cdot \frac{\sqrt{Var[m^*]}}{\sigma_s}. \quad (24)$$

I first consider the case that $1 \geq \rho_D^* > \rho_U^* > 0$. According to (7) one gets

$$D(\rho^*) = Var[m^*] = Var[\sigma_s^2 + 2(\mu + (1 - \mu)\rho^*)\sigma_{sc} + (\mu + (1 - \mu)\rho^*)^2\sigma_c^2]. \quad (25)$$

Since the first factor on the RHS of (24) increases upon disclosure, it is then sufficient to show that also $D(\rho_D^*) > D(\rho_U^*)$ holds. From the fact that $\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2 > 0$ is a necessary and sufficient condition for $\rho_U^* \in (0, 1)$ (see proof of Lemma 4) this then follows from

$$D'(\rho^*)|_{\rho^*=\rho_U^*} = 2(1 - \mu) \cdot (\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2) > 0. \quad (26)$$

Now consider the case of asymptotically stable equilibria with $\rho_U^* > 1$. From Lemma 6 and 4 it then follows that for such equilibria, $1 < \rho_D^* < \rho_U^*$ holds, thus disclosure *decreases* ρ^* . To show that $Corr[s, m^*]$ does also increase upon disclosure in this case, I will use again that in equilibrium $\rho^* = N(\rho^*)/D(\rho^*)$ holds with

$$N(\rho^*) = Cov[s, m^*(s, c)] = Cov[s, s + c(\mu + (1 - \mu)\rho^*)] = \sigma_s^2 + (\mu + (1 - \mu)\rho^*)\sigma_{sc} \quad (27)$$

and $D(\rho^*) = Var[m^*(s, c)]$ as defined in (25). From (24) one gets that

$$\left. \frac{Corr[s, m^*(s, c)]}{\partial \rho^*} \right|_{\rho^*=\rho_U^*} = \left[\left(\frac{N(\rho^*)}{D(\rho^*)} \right)' \cdot \left(\frac{\sqrt{D(\rho^*)}}{\sigma_s} \right) + \left(\frac{N(\rho^*)}{D(\rho^*)} \right) \cdot \left(\frac{D(\rho^*)'}{2\sigma_s \sqrt{D(\rho^*)}} \right) \right] \Big|_{\rho^*=\rho_U^*} \quad (28)$$

Multiplying the above with $\sqrt{D(\rho^*)} > 0$ and using $\rho^* = N(\rho^*)/D(\rho^*)$ again yields for its sign

$$\text{sgn} \left[N(\rho^*)'|_{\rho^*=\rho_U^*} - \rho^* \cdot \frac{D(\rho^*)'}{2} \Big|_{\rho^*=\rho_U^*} \right] = \text{sgn} [\sigma_{sc} - \rho_U^*(\sigma_{sc} + (\mu + (1 - \mu)\rho_U^*)\sigma_c^2)].$$

Substituting the above RHS with $\rho_U^* = N(\rho_U^*)/D(\rho_U^*)$ and this again with (25) and (27) then yields,

after multiplying it with $D(\rho_U^*) > 0$ (and some transformations), that the sign of the above equals

$$\text{sgn}[(\sigma_{sc}^2 - \sigma_c^2 \sigma_s^2)] = \text{sgn}[(\text{Corr}[s, c]^2 - 1)] < 0.$$

In consequence, a decrease in $\rho^* > 1$ upon disclosure increases $\text{Corr}[s, m^*(s, c)]$.

Finally, consider the case of $\rho_U^* = 1$. By case c) in Lemma 6, the inference coefficient then remains constant upon disclosure. Furthermore, by Lemma 4, it has to apply that $\sigma_c^2 = -\sigma_{sc}$. Proposition 1 then implies that $\text{E}[s|m^*] = m^*(s, c) - \text{E}[c] = s + c - \text{E}[c]$. From Lemma 7 and its proof, $\sigma_s^2(1 - \text{Corr}[s, m^*]^2) = \text{E}[\text{Var}[s|m^*]]$ is the argument of the loss function \mathcal{L} which describes that rational receivers expected utility. Applying the law of total variance again yields

$$\begin{aligned} \text{E}[\text{Var}[s|m^*]] &= \text{Var}[s] - \text{Var}[\text{E}[s|m^*]] \\ &= \text{Var}[s] - \text{E}[(s + c - \text{E}[c] - \text{E}[s + c - \text{E}[c]])^2] \\ &= -2\text{Cov}[s, c] - \text{Var}[c] = \sigma_c^2 \end{aligned}$$

with undisclosed COIs. By analogous reasoning and using the posteriors from Lemma 5 one obtains for disclosed COIs, after the signal \tilde{c} has been obtained, $\text{E}[\text{Var}[s|m^*, \tilde{c}]] = (1 - \rho_c)\sigma_c^2$. As with full or imperfect disclosure, $\rho_c \in (0, 1]$ applies so that \mathcal{L} strictly increases upon disclosure. \square

Proof of Proposition 6

I start with $w_k = 0$ and denote, with slight abuse of notation, $W(\sigma_\epsilon^2) \equiv W(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))$ via the analogously defined $\text{E}[u_r^R(\sigma_\epsilon^2)] \equiv \text{E}[u_r^R(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))]$ and $\text{E}[u_n^R(\sigma_\epsilon^2)] \equiv \text{E}[u_n^R(\mathcal{E}(\rho_D^*(\sigma_\epsilon^2, \cdot)))]$. This reflects that in the case of disclosed COIs, the coefficient ρ_D^* is the only term which contains σ_ϵ^2 via the ratio ϕ (see Proposition 3) which, in turn, contains $\rho_c = \frac{\sigma_c}{\sigma_c^2 + \sigma_\epsilon^2}$. Using $\tilde{N}(\rho)$ and $\tilde{D}(\rho)$ as defined in the proof of Proposition 3, together with $\rho_D^* = \tilde{N}(\rho)/\tilde{D}(\rho)$ then yields

$$\frac{\partial \rho_D^*}{\partial \phi} = \frac{\partial (\tilde{N}(\rho_D^*)/\tilde{D}(\rho_D^*))}{\partial \phi} = \frac{\sigma_s^2 \tilde{D}(\rho_D^*) - \tilde{N}(\rho_D^*) \sigma_s^2}{(\tilde{D}(\rho_D^*))^2} = \frac{(1 - \rho_D^*) \sigma_s^2}{\tilde{D}(\rho_D^*)}$$

From $\partial \phi / \partial \sigma_\epsilon^2 = (\partial \phi / \partial \rho_c) \cdot (\partial \rho_c / \partial \sigma_\epsilon^2) < 0$ and Lemma 6 it follows that

$$\text{sgn} \left[\frac{\partial \rho_D^*}{\partial \sigma_\epsilon^2} \right] = - \text{sgn} \left[\frac{\partial \rho_D^*}{\partial \phi} \right] = \text{sgn} [\rho_D^* - 1] = \text{sgn} [\rho_U^* - 1].$$

Since $\frac{\text{E}[u_n^R(\rho^*)]}{\partial \rho^*}$ is positive (negative) if and only if $\rho_U^* > 1$ ($\rho_U^* < 1$) one then gets from (12) and the above for any $\sigma_\epsilon^2 \geq 0$ the following:

$$\text{sgn} \left[\frac{\text{E}[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \right] = \text{sgn} \left[\frac{\text{E}[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \right] = \text{sgn} \left[\frac{\text{E}[u_n^R(\rho^*)]}{\partial \rho^*} \Big|_{\rho^* = \rho_D^*} \cdot \frac{\partial \rho_D^*}{\partial \sigma_\epsilon^2} \right] = - \text{sgn} [\rho_U^* - 1] \quad (29)$$

When $\rho_U^* \in (0, 1)$, every decrease in σ_ϵ^2 therefore hurts naive receivers. In contrast, it has been shown in the main text that when there is full disclosure, i.e. $\sigma_\epsilon^2 = 0$, rational receivers achieve their maximum utility, thus that $\text{E}[u_r^R(0)]' = 0$ holds.

The first part of the proposition (that full disclosure, i.e. setting $\sigma_\epsilon^2 = 0$, is never optimal) can then be established by showing the following: When $\rho_U^* \in (0, 1)$, there exists a $\Delta > 0$ such that starting from full disclosure with $\sigma_\epsilon^2 = 0$, the gradual increase of this variance to $\sigma_\epsilon^2 = \Delta$ increases $W(\sigma_\epsilon^2) = w_r \cdot \text{E}[u_r^R(\sigma_\epsilon^2)] + w_n \cdot \text{E}[u_n^R(\sigma_\epsilon^2)]$. This is equivalent to showing that

$\lim_{\Delta \rightarrow 0^+} (W(\Delta) - W(0))$ is positive, i.e. that

$$\begin{aligned} \operatorname{sgn} \left[\lim_{\Delta \rightarrow 0^+} \left(\frac{W(\Delta) - W(0)}{\Delta} \right) \right] &= \operatorname{sgn} \left[\sum_{j=r,n} w_j \cdot \lim_{\Delta \rightarrow 0^+} \left(\frac{\mathbb{E}[u_j^R(\Delta)] - \mathbb{E}[u_j^R(0)]}{\Delta} \right) \right] \\ &= \operatorname{sgn} \left[w_r \cdot \frac{\partial \mathbb{E}[u_r^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \Big|_{\sigma_\epsilon^2=0} + w_n \cdot \frac{\partial \mathbb{E}[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \Big|_{\sigma_\epsilon^2=0} \right] \\ &= \operatorname{sgn} \left[w_n \cdot \frac{\partial \mathbb{E}[u_n^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \Big|_{\sigma_\epsilon^2=0} \right] = -\operatorname{sgn} [\rho_U^* - 1] > 0 \end{aligned}$$

holds. The second-last equality in the above follows from the fact that by Lemma 7, rational receivers expected utility w.r.t to σ_ϵ^2 is maximized under full disclosure, i.e. $\frac{\partial \mathbb{E}[u_r^R(\sigma_\epsilon^2)]}{\partial \sigma_\epsilon^2} \Big|_{\sigma_\epsilon^2=0} = 0$, while the last one follows from (29).

For the case that $w_k > 0$, note that the above proof applies for any loss function $u_n^R(\cdot) = L(\cdot)$ which is strictly concave and symmetric around zero. It therefore also holds when in addition to $\mathbb{E}[u_n^R(\sigma_\epsilon^2)]$, positive weight is assigned to $-\mathbb{E}[c(\mu + \rho^*(\sigma_\epsilon^2, \cdot)(1 - \mu))^2]$. This then yield the first part of the proposition.

The second statement is an immediate consequence of the fact that when $\rho_U^* > 1$, according to (29), increasing the signal precision (decreasing any $\sigma_\epsilon^2 > 0$) helps naive receivers and that full disclosure maximizes the utility of rational receivers (see Lemma 7). \square

Example for non-disclosure to be optimal

As a concrete example for a scenario where non-disclosure is optimal, consider the parameters $\sigma_s^2 = \sigma_c^2 = 1$, $\bar{s} = \bar{c} = \sigma_{sc} = 0$, together with $\mu = w_n = w_r = 0.5$, $w_k = 0$, and the loss function $L(d - s) = -(d - s)^2$. Plugging these parameters into (9) and solving yields $\rho_U^* \approx 0.6$. Following Lemma 6, disclosure then increases the inference coefficient. Using Proposition 1 and (13) yields

$$\begin{aligned} W(\mathcal{E}(\rho_k^*)) &= -0.5 \left(\mathbb{E}[(\rho[m^*(s, c) - \bar{c}(\mu + (1 - \mu)\rho)] + (1 - \rho)\bar{s} - s)^2] + \mathbb{E}[(m^*(s, c) - s)^2] \right) \Big|_{\rho=\rho_k^*} \\ &= -0.5 \left(\mathbb{E}[(\rho m^*(s, c) - s)^2] + \mathbb{E}[(m^*(s, c) - s)^2] \right) \Big|_{\rho=\rho_k^*} \\ &= -0.5 \left(\mathbb{E}[(s(\rho - 1) + c\rho(0.5 + 0.5\rho))^2] + \mathbb{E}[(c(0.5 + 0.5\rho))^2] \right) \Big|_{\rho=\rho_k^*} \\ &= -0.5 \left((\rho - 1)^2 \mathbb{E}[s^2] + 2(\rho - 1)\rho(0.5 + 0.5\rho)\mathbb{E}[sc] + (\rho^2 + 1)(0.5 + 0.5\rho)^2 \mathbb{E}[c^2] \right) \Big|_{\rho=\rho_k^*} \\ &= -0.5 \left((\rho - 1)^2 + (\rho^2 + 1)(0.5 + 0.5\rho)^2 \right) \Big|_{\rho=\rho_k^*} \end{aligned}$$

which is easily verified to be strictly decreasing in ρ when $\rho > 0.4$. Therefore, non-disclosure maximizes W .