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### Working Paper

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# DYNAMIC CAUSE – EFFECT MODELS AND THEIR SWITCHING TRENDS - SELECTED PROBLEMS (mathematical economics approach)

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**DYNAMIC CAUSE – EFFECT MODELS AND THEIR SWITCHING TRENDS**  
- **SELECTED PROBLEMS**  
**(mathematical economics approach)**

**Summary**

The paper consists of two parts devoted to the cause-effect dynamic models. In each part of the deterministic properties of the dynamic version of the model are presented. Thus, each of the considered dynamic models can be presented in the form of an equivalent for it the switching trend. Switches in this trend were exogenous variables of the dynamic model expressed in terms of the respective functions. This approach allowed:

- first, clearly define short-term and long-uncorrected (Part I) and corrected (Part II) interaction effects of exogenous variables on the endogenous variable in a dynamic model,
- second, to simulate the behavior of the endogenous variable model for complex dynamic changes of exogenous variables and present graphs switching trend with its boundary levels,
- third, check the equivalence considered causal dynamic model of the trend defined correctness of the switching and impact effects of exogenous variables on the endogenous variable.

For each version of the model a dynamic random effects impact on the formation of endogenous variable described by the trend of switching was assumed. It allowed to check the properties of estimates of causal dynamical model based on data generated by the trend in terms of stochastic switching.

Generally, the first part of the paper is devoted to the dynamic models in which the exogenous variables are no time delays, which corresponds roughly to a certain concept of partial adjustment model (PAM). The second part of the paper focuses on dynamic models in which each of the exogenous variables, accompanied by the same variable with one-period time delay. Thus, this corresponds to the concept of error correction models (ECM).

In both parts of the paper the problem of deterministic seasonality quarterly trends generated using switching was considered. Recursive set of equations was formulated so that on the basis of structural parameters of variables occurring in the zero-ones-in the dynamic model can clearly determine the effects of seasonal variations indicative of the level of endogenous variable switching trend, which is the level determined by exogenous factors model in a given period.

In conclusion, the second part of the paper presents the results of the procedure specifications and estimates of cause-effect dynamic model describing the evolution of the level of wages in Poland. The model was estimated on the basis of statistical data for the period from 1 quarter of 1996 to the 4th quarter of 2013.

**Keywords:** dynamic cause-effect model, switching trend function; error correction model (ECM), seasonality; dynamic wage model

**JEL:** C2, C5, J3

## PART I

### SIMULATION ANALYSIS ON THE BASIS OF CAUSE – EFFECT DYNAMIC MODELS TYPE ADL(1,0,i)

#### 1.1 CAUSE – EFFECT DYNAMIC MODELS TYPE ADL(1,0,i)

##### 1.1.1. INTRODUCTION

**The introducing form of the model:**

$$(1) \quad y(y_{t-1}, x_t, \varepsilon_t) : \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 x_t + \varepsilon_t, \quad (t = (0), 1, 2, 3, \dots), \quad (0 < \alpha < 1).$$

where:

- $y_t$  - endogenous variable,
- $\varepsilon_t$  - error term,
- $x_t$  - non-random exogenous variable (assumption).

**The model of the ADL(1,0) type, purified from the effect of the error term:**

$$(2) \quad \tilde{y}(\tilde{y}_{t-1}, x_t) : \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 x_t, \quad (t = (0), 1, 2, 3, \dots), \quad (0 < \alpha < 1)$$

**The differentia form of the cause-effect dynamic model and the gross effect of the increase in endogenous variable:**

$$(3) \quad \Delta\tilde{y}(\Delta\tilde{y}_{t-1}, \Delta x_t) : \quad \Delta\tilde{y}_t = \alpha \cdot \Delta\tilde{y}_{t-1} + \beta_1 \Delta x_t, \quad (t = (0), 1, 2, 3, \dots)$$

where:

$$\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}, \quad \Delta\tilde{y}_{t-1} = \tilde{y}_{t-1} - \tilde{y}_{t-2}, \quad \Delta x_t = x_t - x_{t-1}$$

##### 1.1.2. SHORT-TERM EFFECTS

**Inertial effect in the dynamic model ( $x_t = const. \rightarrow \Delta x_t = 0$ )**

$$(4) \quad Ef_{\Delta y}^{iner} : \quad \Delta\tilde{y}_t^{iner} = \alpha \cdot \Delta\tilde{y}_{t-1} \Rightarrow \alpha = \frac{\Delta\tilde{y}_t^{iner}}{\Delta\tilde{y}_{t-1}}$$

**Pure short-term effect of exogenous effects on endogenous variable – the net effect:**

$$(5) \quad Ef_t^s : \quad \Delta y_t^n = \Delta\tilde{y}_t - \alpha \cdot \Delta\tilde{y}_{t-1} = \beta_1 \Delta x_t,$$

where net increase of the endogenous variable ( $\Delta y_t^n$ ) is the difference between the increase in gross profit and increase inertial.

##### 1.1.3. LONG-TERM MULTIPLIER EFFECTS (LTME)

**The net impact of the multiplier effects of the variable  $x$  from period  $t$  in following periods  $t+i$ :**

$$(6.1) \quad \begin{aligned} &\text{for } t+1: \quad \Delta\tilde{y}_{t+1}^n = \alpha \cdot \Delta\tilde{y}_t^n = \alpha \cdot \beta_1 \Delta x_t, \quad (\Delta x_{t+1} = 0), \\ &\text{for } t+2: \quad \Delta\tilde{y}_{t+2}^n = \alpha \cdot \Delta\tilde{y}_{t+1}^n = \alpha^2 \cdot \beta_1 \Delta x_t, \quad (\Delta x_{t+2} = 0), \\ &\text{for } t+3: \quad \Delta\tilde{y}_{t+3}^n = \alpha \cdot \Delta\tilde{y}_{t+2}^n = \alpha^3 \cdot \beta_1 \Delta x_t, \quad (\Delta x_{t+3} = 0), \dots \end{aligned}$$

**Long term (net) multiplier effect resulting from the increase in the variable  $x$  by  $\Delta x_t$  in the period  $t$ :**  
(the sum of an infinite decreasing geometric progress)

$$(6.2) \quad LTME_t : \quad \Delta\tilde{y}_t^e = \lim_{i \rightarrow \infty} \sum_{i=0}^{\infty} \Delta\tilde{y}_{t+i}^n = (1 + \alpha + \alpha^2 + \alpha^3 + \dots) \cdot \beta_1 \Delta x_t = \frac{\beta_1}{1 - \alpha} \Delta x_t ,$$

where  $[\beta_1/(1-\alpha)]$  is a **long-term multiplier (LTM)**.

## 1.2 SWITCHING TREND FUNCTION OF CAUSE – EFFECT DYNAMIC MODEL TYPE ADL(1,0,1)

The form of the switching trend function of the dynamic model (2)

$$(7) \quad \tilde{y}(t, x_t) : \quad \tilde{y}_t = \frac{\beta_0}{1-\alpha} + \frac{\beta_1}{1-\alpha} x_t - \alpha^t \frac{\beta_0}{1-\alpha} - \alpha^t \frac{\beta_1}{1-\alpha} x_t + \alpha^t \cdot \tilde{y}_{0t}^1$$

where:

a) The initial value of the switching trend in time  $t$ :

$$(7.a) \quad \tilde{y}_{0t} = \tilde{y}_0 + \frac{\beta_1}{1-\alpha} \sum_{t=1}^n (1-\alpha^{1-t}) \cdot \Delta x_t$$

where for time  $t=1$  value of  $\tilde{y}$  during initialization  $t=0$  is  $\tilde{y}_0$ .

b) The limit value of the switching trend in time  $t=h$ :

$$(7.b) \quad \tilde{y}_{t=h}^e = \lim_{t \rightarrow \infty} \tilde{y}_t(t, x_t) = \frac{\beta_0}{1-\alpha} + \frac{\beta_1}{1-\alpha} x_{t=h}, \quad (x_h = x_{t \geq h} = \text{const.})$$

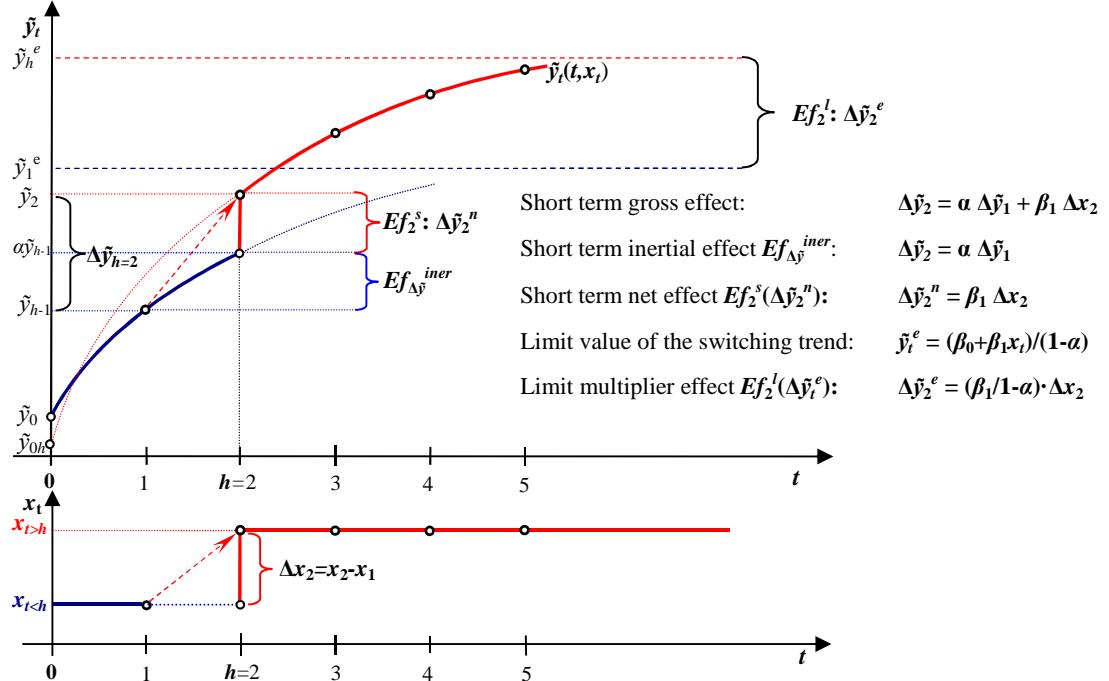
c) The limit effect of influence of the exogenous variable on the endogenous variable in time  $t=h$ :

$$(7.c) \quad Ef_h^l : \quad \Delta \tilde{y}_h^e = \frac{\beta_1}{1-\alpha} \Delta x_h, \quad (\Delta x_h = x_h - x_{h-1}, \quad t = h) \quad [\text{porównaj: (6.2)}]$$

Taking into account (6.2) and (7.c), note that:

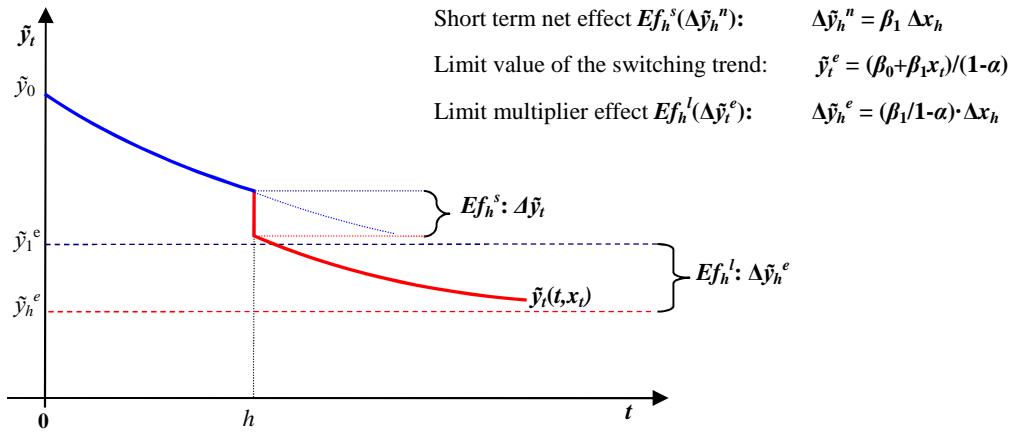
$$(7.d) \quad LTME_t = \frac{\beta_1}{1-\alpha} \Delta x_t \quad \equiv \quad Ef_t^l = \frac{\beta_1}{1-\alpha} \Delta x_t$$

## 1.3 GRAPHIC APPROACH OF THE TREND SWITCHING MODEL TYPE ADL(1,0,1)

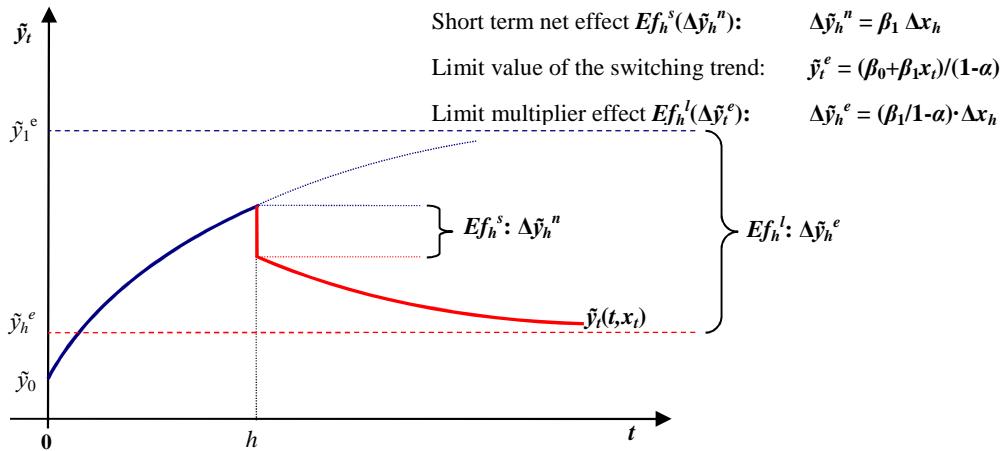


**Figure 1.A.** Switching trend function  $\tilde{y}(t, x_t)$  and short term and long term effects of changes of the endogenous variable  $\tilde{y}_t$  as a result of the increase of the exogenous variable  $x_h$  ( $\Delta x_h > 0$ ) in period  $t=h$  caused by continuous increase of the endogenous variable.

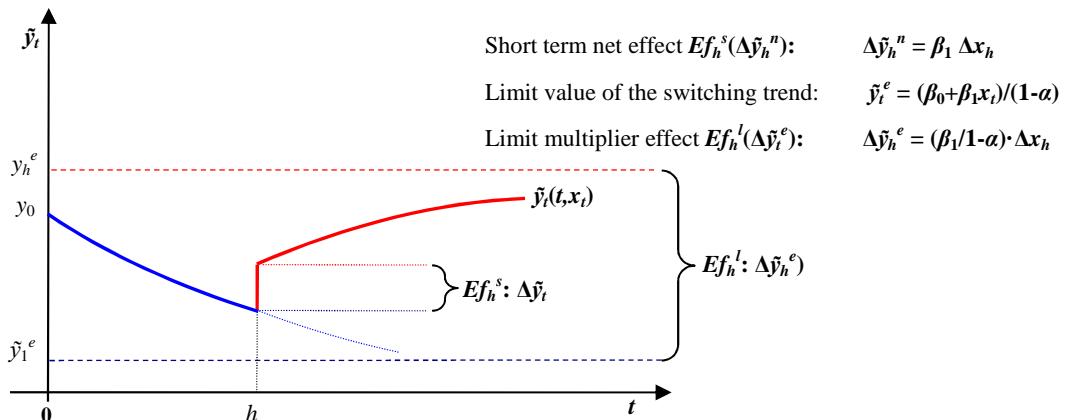
<sup>1</sup> The proposed solution is presented in [Ossowski, 2007]



**Figure 1.B.** Switching trend function  $\tilde{y}(t, x_t)$  and short term and long term effects of changes of endogenous variable  $\tilde{y}_t$  as a result of increasing of the exogenous variable  $x_h$  ( $\Delta x_h > 0$ ) in period  $t=h$  caused by the continues decreasing of the endogenous variable.



**Figure 1.C.** Trend switching function  $\tilde{y}(t, x_t)$  and short term and long term effects of changes of the endogenous variable  $\tilde{y}_t$  as a result of increasing of the exogenous variable  $x_h$  ( $\Delta x_h > 0$ ) in period  $t=h$  caused by the increase of the endogenous variable on the beginning and decrease later.



**Figure 1.D.** Switching trend function  $\tilde{y}(t, x_t)$  and short term and long term effects of changes of endogenous variable  $\tilde{y}_t$  as a results increasing of exogenous variable  $x_h$  ( $\Delta x_h > 0$ ) in period  $t=h$  caused by the decrease of the endogenous variable on the beginning and increasing later.

## 1.4 SIMULATION MODEL ASSUMPTIONS FOR TYPE ADL(1,0,1) AND SIMULATION RESULTS

Assumed values of parameters:

$$\alpha = 0,75$$

$$\beta_0 = 5$$

$$\beta_1 = 10$$

$$\tilde{y}_0 = 50$$

$$\text{It means that: } \frac{\beta_0}{1-\alpha} = \frac{5}{1-0,75} = 20, \quad \frac{\beta_1}{1-\alpha} = \frac{10}{1-0,75} = 40$$

The limit values of the switching trend in the period  $t$ :

$$\tilde{y}_t^e = \frac{\beta_0}{1-\alpha} + \frac{\beta_1}{1-\alpha} x_t = 20 + 40 \cdot x_t$$

Assumed values of  $x_t$  for given periods:

$$\text{from } t=1 \text{ to } t=6: \quad x_t = 10.$$

$$\text{from } t=7 \text{ to } t=12: \quad x_t = 14$$

$$\text{from } t=13 \text{ to } t=16: \quad x_t = 16$$

$$\text{from } t=17 \text{ to } t=25: \quad x_t = 17, 18, 19, 20, 21, 22, 23, 24, 25 \text{ (trend)}$$

$$\text{from } t=26 \text{ to } t=32: \quad x_t = 23, 22, 24, 26, 25, 28, 29 \text{ (irregular changes)}$$

$$\text{from } t=33 \text{ to } t=36: \quad x_t = 30$$

$$\text{from } t=37 \text{ to } t=40: \quad x_t = 20$$

Symbols for the software system MICROFIT:

The switching trend of the depended variable:  $\tilde{y}_t(t, x_t) = YX$

Estimates in the MICROFIT software:

The trend of switching:

$$\tilde{y}_t(t, x_t)$$

$$YX = 20 - 20 * 0.75^{1-t} + 40 * x - 40 * x * 0.75^{1-t} + y_0 x * 0.75^{1-t}$$

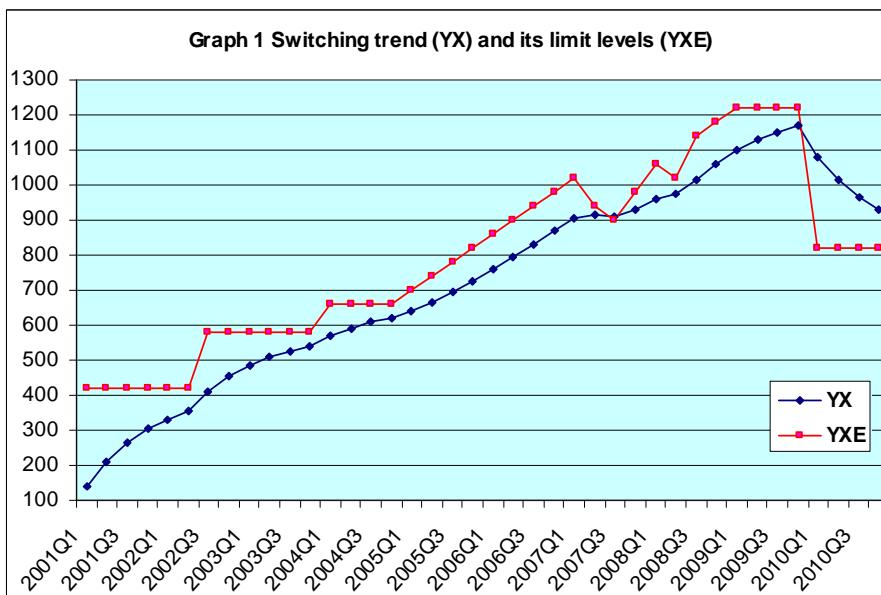
where:

$$1) (1 - 0,75^{1-t}) : \quad POM = 1 - 0,75^{1-t}$$

$$2) (1 - 0,75^{1-t}) \cdot \Delta x_t : \quad POMDX = pom * dx$$

$$3) \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t : \quad CSUMPOMDX = csum(pomdx)$$

$$4) \tilde{y}_{0t} = \tilde{y}_0 + \frac{\beta_1}{1-\alpha} \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t : \quad Y0X = 50 + 40 * csumpomdx$$



**Table 1 The simulation results of the switching trend for ADL(1,0,1) model**

<b>Period</b>	(t)	$x_t$	$x_{t-1}$	$\Delta x_t$	$\tilde{y}_t = \tilde{y}(x_t, t)$	$\Delta \tilde{y}_t$	$\Delta \tilde{y}_t^{iner} = \alpha \Delta \tilde{y}_{t-1}$	$\Delta \tilde{y}_t^n$	$\tilde{y}_t^e$	$\Delta \tilde{y}_t^e$
	T	X	X(-1)	DX	YX	DYX	INER1	DYXN	YXE	DYXE
2001Q1	1	10	10	0	142,5	0	0	0	420	0
2001Q2	2	10	10	0	211,875	69,375	0	0	420	0
2001Q3	3	10	10	0	263,9062	52,0313	52,0312	0	420	0
2001Q4	4	10	10	0	302,9297	39,0234	39,0234	0	420	0
2002Q1	5	10	10	0	332,1973	29,2676	29,2676	0	420	0
2002Q2	6	10	10	0	354,1479	21,9507	21,9507	0	420	0
2002Q3	7	14	10	4	410,611	56,463	16,463	40	580	160
2002Q4	8	14	14	0	452,9582	42,3473	42,3473	0	580	0
2003Q1	9	14	14	0	484,7187	31,7604	31,7604	0	580	0
2003Q2	10	14	14	0	508,539	23,8203	23,8203	0	580	0
2003Q3	11	14	14	0	526,4042	17,8653	17,8653	0	580	0
2003Q4	12	14	14	0	539,8032	13,3989	13,3989	0	580	0
2004Q1	13	16	14	2	569,8524	30,0492	10,0492	20	660	80
2004Q2	14	16	16	0	592,3893	22,5369	22,5369	0	660	0
2004Q3	15	16	16	0	609,292	16,9027	16,9027	0	660	0
2004Q4	16	16	16	0	621,969	12,677	12,677	0	660	0
2005Q1	17	17	16	1	641,4767	19,5078	9,5078	10	700	40
2005Q2	18	18	17	1	666,1075	24,6308	14,6308	10	740	40
2005Q3	19	19	18	1	694,5807	28,4731	18,4731	10	780	40
2005Q4	20	20	19	1	725,9355	31,3548	21,3548	10	820	40
2006Q1	21	21	20	1	759,4516	33,5161	23,5161	10	860	40
2006Q2	22	22	21	1	794,5887	35,1371	25,1371	10	900	40
2006Q3	23	23	22	1	830,9415	36,3528	26,3528	10	940	40
2006Q4	24	24	23	1	868,2062	37,2646	27,2646	10	980	40
2007Q1	25	25	24	1	906,1546	37,9485	27,9485	10	1020	40
2007Q2	26	23	25	-2	914,616	8,4613	28,4613	-20	940	-80
2007Q3	27	22	23	-1	910,962	-3,654	6,346	-10	900	-40
2007Q4	28	24	22	2	928,2215	17,2595	-2,7405	20	980	80
2008Q1	29	26	24	2	961,1661	32,9446	12,9446	20	1060	80
2008Q2	30	25	26	-1	975,8746	14,7085	24,7085	-10	1020	-40
2008Q3	31	28	25	3	1016,9	41,0314	11,0314	30	1140	120
2008Q4	32	29	28	1	1057,7	40,7735	30,7735	10	1180	40
2009Q1	33	30	29	1	1098,3	40,5801	30,5801	10	1220	40
2009Q2	34	30	30	0	1128,7	30,4351	30,4351	0	1220	0
2009Q3	35	30	30	0	1151,5	22,8263	22,8263	0	1220	0
2009Q4	36	30	30	0	1168,6	17,1197	17,1197	0	1220	0
2010Q1	37	20	30	-10	1081,5	-87,1602	12,8398	-100	820	-400
2010Q2	38	20	20	0	1016,1	-65,3701	-65,3701	0	820	0
2010Q3	39	20	20	0	967,0828	-49,0276	-49,0276	0	820	0
2010Q4	40	20	20	0	930,3121	-36,7707	-36,7707	0	820	0

$\tilde{y}_t = \tilde{y}(x_t, t)$

- the trend of the switching function,

$\Delta \tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$

- the short term increase in gross variable,

$\Delta \tilde{y}_t^{iner} = \alpha \Delta \tilde{y}_{t-1}$

-the short term inertial effect,

$\Delta \tilde{y}_t^n = \Delta \tilde{y}_t - \alpha \Delta \tilde{y}_{t-1}$

- the short term increase in net variable  $\tilde{y}_t$  (where:  $\alpha=0,75$ ), equivalently:  $\Delta \tilde{y}_t^k = \beta_1 \Delta x_t$  (where:  $\beta_1=10$ )

$\tilde{y}_t^e = \lim \tilde{y}(x_t, t)$

- the limit of the switching trend in condition when  $x_t=const.$  and  $t \rightarrow \infty$

$\Delta \tilde{y}_t^e$

-increase of the limit switching trend caused by the change of  $x_t$  in time  $t$ , equivalently: Long term multiplier effect ( $LTME_t$ ), which is the expected increase in the level of endogenous variable due to the increase in the variable  $x_t$ .

## 1.5 RESULTS OF ESTIMATION OF THE DYNAMIC MODEL TYP ADL(1,0) BASED ON SWITCHING TREND SIMULATED (validation of simulation)

The form of the simulated model::

$$(8.a) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 \cdot x_t$$

```

Ordinary Least Squares Estimation
*****
Dependent variable is YX
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  5.0000          0.00            *NONE*
YX(-1)              .75000          0.00            *NONE*
X                   10.0000         0.00            *NONE*
*****
R-Squared           1.0000          R-Bar-Squared    1.0000
S.E. of Regression .0000          F-stat.        F( 2, 36)   *NONE*
*****
```

Results of estimated simulation model:

$$(8.b) \quad \tilde{y}_t = 5 + 0,75 \cdot \tilde{y}_{t-1} + 10 \cdot x_t$$

## 1.6 RESULTS OF A DYNAMIC MODEL ESTIMATES BASED ON TREND SIMULATED SWITCHING IN THE CASE OF STOCHASTIC CHANGES OF ENDOGENOUS VARIABLE

The form of the switching stochastic trend (7):

$$(9) \quad y(t, x_t, u_t): \quad y_t = \tilde{y}(t, x_t) + u_t,$$

where  $u_t$  is the normally distributed generated error term..

### Characteristic of the distribution of error terms $u_t$ .

		(where: $u_t = U$ )
		Sample period : 2001Q1 to 2010Q4
Variable(s)	:	U
Maximum	:	4.9794
Minimum	:	-4.3530
Mean	:	-.12534
Std. Deviation	:	2.3659
Skewness	:	.0068260
Kurtosis - 3	:	-.55216
Coef of Variation:	:	18.8758

Order	Autocorrelation Coefficient	Standard Error	Sample from 2001Q1 to 2010Q4	
			Box-Pierce Statistic	Ljung-Box Statistic
1	.35954	.15811	5.1707[.023]	5.5685[.018]
2	-.089111	.17738	5.4884[.064]	5.9196[.052]
3	-.042156	.17850	5.5595[.135]	6.0003[.112]
4	-.16714	.17874	6.6769[.154]	7.3039[.121]
5	-.19483	.18261	8.1952[.146]	9.1259[.104]
6	.11540	.18773	8.7279[.189]	9.7839[.134]
7	.14582	.18950	9.5784[.214]	10.8664[.145]
8	-.17060	.19228	10.7425[.217]	12.3943[.134]
9	-.23951	.19603	13.0371[.161]	15.5031[.078]
10	-.0070926	.20322	13.0391[.221]	15.5059[.115]
11	-.041921	.20322	13.1094[.286]	15.6078[.156]
12	-.081504	.20344	13.3752[.342]	16.0063[.191]
13	-.060013	.20425	13.5192[.409]	16.2304[.237]

Estimation of the dynamic model [see (1)]:

$$(10.a) \quad y(y_{t-1}, x_t, \varepsilon_t): \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 x_t + \varepsilon_t, \quad [\varepsilon_t = f(u_t, u_{t-1}, \tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_t^e)]$$

where in the MICROFIT program report:  $y_t = YXU$ ,  $x_t = X$

```
Ordinary Least Squares Estimation
*****
Dependent variable is YXU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressors          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.6526          1.3514            3.4429[.001]
YXU(-1)             .75122          .0032095        234.0590[.000]
X                   9.9680          .14874           67.0187[.000]
*****
R-Squared           .99992         R-Bar-Squared       .99992
S.E. of Regression   2.4621          F-stat. F( 2, 36) 236118.6[.000]
Mean of Dependent Variable 742.8488 S.D. of Dependent Variable 274.4845
Residual Sum of Squares 218.2371 Equation Log-likelihood      -88.9180
Akaike Info. Criterion -91.9180 Schwarz Bayesian Criterion     -94.4133
DW-statistic          2.2250          Durbin's h-statistic    -.70258[.482]
*****
Diagnostic Tests
*****
* Test Statistics * LM Version * F Version *
*****
* A:Serial Correlation*CHSQ( 4)= 8.6313[.071]*F( 4, 32)= 2.2737[.083]*
* B:Functional Form  *CHSQ( 1)= .021838[.883]*F( 1, 35)= .019609[.889]*
* C:Normality        *CHSQ( 2)= 1.5274[.466]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= 1.0297[.310]*F( 1, 37)= 1.0034[.323]*
*****
```

Model estimated by the OLS method:

$$(10.b) \quad \hat{y}_t = 4,653 + 0,751 \cdot y_{t-1} + 9,968 \cdot x_t \quad [\text{comp.: (8.b)}]$$

Estimated limit function:

$$(10.c) \quad \hat{y}_t^e = \frac{\hat{\beta}_0}{1-\hat{\alpha}} + \frac{\hat{\beta}_1}{1-\hat{\alpha}} x_t = 18,69 + 40,03 x_t$$

## 1.7 THE DETERMINISTIC SEASONALITY IN THE DYNAMIC MODEL TYPE ADL(1,0,1)<sup>2</sup>

The deterministic switching trend with quarterly seasonality:

$$(11) \quad \tilde{s}\tilde{y}_t(t, x_t, v_{ij}): \quad \tilde{s}\tilde{y}_t = \tilde{y}_t(t, x_t) + e_1 \cdot v_{t1} + e_2 \cdot v_{t2} + e_3 \cdot v_{t3}, \quad [e_4 = -(e_1 + e_2 + e_3)]$$

where:

$$v_{ij} = x_{ij} - x_{i4}, \quad j \neq 4$$

$$x_{ij} = \begin{cases} 1 & \text{in } j\text{'th quarter} \\ 0 & \text{in other quarters} \end{cases}$$

We can say that, in every  $j$ 'th quarter endogenous variable  $\tilde{s}\tilde{y}_t$  deviates from the switching trend  $\tilde{y}(t, x_t)$  by  $e_j$  units.

Purified from the error terms model type ARDL(1,0) with quarterly seasonality:

$$(12) \quad \tilde{s}\tilde{y}(\tilde{y}_{t-1}, x_t, v_{ij}): \quad \tilde{s}\tilde{y}_t = \beta_0 + \alpha \cdot \tilde{s}\tilde{y}_{t-1} + \beta_1 x_t + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3}, \quad [\gamma_4 = -(\gamma_1 + \gamma_2 + \gamma_3)]$$

---

<sup>2</sup> The proposed solution is presented in [Ossowski, 1997] and [Ossowski, 2007].

Equations for identification of parameters  $e_j$  on the basis of parameters  $\gamma_j$ :

$$(13.1) \quad \begin{aligned} e_1 &= \alpha \cdot e_4 + \gamma_1 \\ e_2 &= \alpha \cdot e_1 + \gamma_2 \\ e_3 &= \alpha \cdot e_2 + \gamma_3 \\ e_4 &= \alpha \cdot e_3 + \gamma_4 \end{aligned}$$

where:

$$(13.2) \quad e_4 = (\alpha^3 \cdot \gamma_1 + \alpha^2 \cdot \gamma_2 + \alpha \cdot \gamma_3 + \gamma_4) / (1 - \alpha^4)$$

## 1.8 ASSUMPTIONS OF THE SIMULATED MODEL IN THE CASE OF THE DETERMINISTIC SEASONALITY

Assumed values of parameters of seasonal effects:

$$e_1 = -30, \quad e_2 = 20, \quad e_3 = -20, \quad e_4 = 30$$

Assumed notation of the software system MICROFIT:

$$\tilde{s}\tilde{y}_t = \tilde{y}_t(t, x_t) - 30 \cdot v_{t1} + 20 \cdot v_{t2} - 20 \cdot v_{t3}, : \quad SYX = yx - 30 * v1 + 20 * v2 - 20 * v3$$

Results of estimation of the dynamic model based on the simulated switching trend, with the deterministic trend  
*(validation of simulation)*

The form of the simulated model [see: (12)]

$$(14.a) \quad \tilde{s}\tilde{y}_t = \beta_0 + \alpha \cdot \tilde{s}\tilde{y}_{t-1} + \beta_1 x_t + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3},$$

```
Ordinary Least Squares Estimation
*****
Dependent variable is SYX
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  5.0000         0.00           *NONE*
SYX(-1)            .75000         0.00           *NONE*
X                   10.0000        0.00           *NONE*
V1                 -52.5000        0.00           *NONE*
V2                 42.5000         0.00           *NONE*
V3                 -35.0000        0.00           *NONE*
*****
R-Squared           1.0000       R-Bar-Squared      1.0000
S.E. of Regression .0000       F-stat.      F( 5, 33)    *NONE*
*****
```

Estimated model:

$$(14.b) \quad \hat{\tilde{s}\tilde{y}}_t = 5 + 0,75 \cdot \tilde{y}_{t-1} + 10 \cdot x_t - 52,5 \cdot v_{t1} + 42,5 \cdot v_{t2} - 35 \cdot v_{t3}$$

It means that:

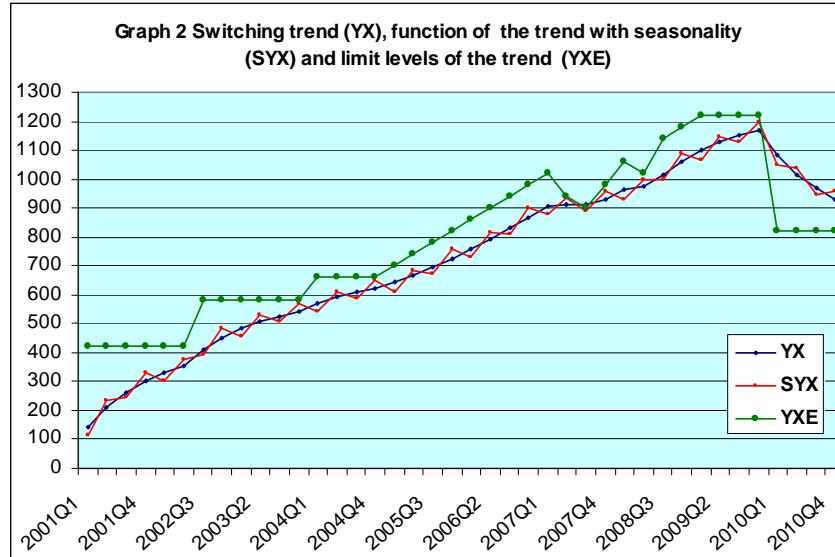
$$\gamma_1 = -52,5, \quad \gamma_2 = 42,5, \quad \gamma_3 = -35, \quad \Rightarrow \quad \gamma_4 = 45$$

Identification of parameters  $e_j$  on the basis of parameters  $\gamma_j$ :

$$\begin{aligned} e_4 &= (\alpha^3 \cdot \gamma_1 + \alpha^2 \cdot \gamma_2 + \alpha \cdot \gamma_3 + \gamma_4) / (1 - \alpha^4) = \\ &= (-0,75^3 \cdot 52,5 + 0,75^2 \cdot 42,5 - 0,75 \cdot 35 + 45) / (1 - 0,75^4) = \\ &= 20,5078 / 0,6836 = 30,0 \end{aligned}$$

so from (13.1) we get:

$$\begin{aligned}
 e_1 &= \alpha \cdot e_4 + \gamma_1 = 0,75 \cdot 30 - 52,5 = -30 \\
 e_2 &= \alpha \cdot e_1 + \gamma_2 = -0,75 \cdot 30 + 42,5 = 20 \\
 e_3 &= \alpha \cdot e_2 + \gamma_3 = 0,75 \cdot 20 - 35 = -20 \\
 e_4 &= \alpha \cdot e_3 + \gamma_4 = -0,75 \cdot 20 + 45 = 30
 \end{aligned}$$



### 1.9 RESULTS OF ESTIMATION OF THE DYNAMIC MODEL BASED ON SIMULATED SWICHING TREND WITH THE DETERMINISTIC SEASONALITY IN THE CASE OF STOCHASTIC CHANGES OF THE ENDOGENOUS VARIABLE

The form of the stochastic switching trend with seasonality (7):

$$(15) \quad sy(y_{t-1}, x_t, v_{ij}, \varepsilon_t) : \quad sy_t = \beta_0 + \alpha \cdot sy_{t-1} + \beta_1 x_t + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t, \quad [\varepsilon_t = f(u_t, u_{t-1}, \tilde{y}_t, \dots)]$$

where  $u_t$  is the normally distributed generated error term (see: part 1.6 and model 10.a).

**Estimated dynamic model (15):**

where in the MICROFIT program report:  $sy_t = SYXU$ ,  $x_t = X$ ,  $v_{ij} = V_j$

```

Ordinary Least Squares Estimation
*****
Dependent variable is SYXU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.5990         1.3818          3.3283[.002]
SYXU(-1)           .75101        .0032738        229.4010[.000]
X                   9.9793         .15193          65.6822[.000]
V1                 -51.8470        .72407         -71.6050[.000]
V2                 42.8744         .69564          61.6328[.000]
V3                 -35.4741        .69137         -51.3097[.000]
*****
R-Squared           .99993        R-Bar-Squared       .99992
S.E. of Regression 2.5079        F-stat.   F( 5, 33)  91580.41[.000]
Mean of Dependent Variable 743.6180    S.D. of Dependent Variable 275.3045
Residual Sum of Squares 207.5489    Equation Log-likelihood     -87.9388
Akaike Info. Criterion -93.9388    Schwarz Bayesian Criterion -98.9295
DW-statistic         2.2451        Durbin's h-statistic     -.76547[.444]
*****
*      Test Statistics *      LM Version      *      F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 9.1011[.059]*F( 4, 29)= 2.2069[.093]*
* B:Functional Form   *CHSQ( 1)= .17473[.676]*F( 1, 32)= .14402[.707]*
* C:Normality        *CHSQ( 2)= 1.4841[.476]* Not applicable   *
* D:Heteroscedasticity*CHSQ( 1)= 2.0352[.154]*F( 1, 37)= 2.0372[.162]*
*****

```

**Estimated model:**

$$(16.a) \quad \hat{y}_t = 4,6 + 0,751 \cdot sy_{t-1} + 9,98 x_t - 51,85 v_{t1} + 42,87 v_{t2} - 35,47 v_{t3}$$

It means that:

$$\hat{\gamma}_1 = -51,85 \quad \hat{\gamma}_2 = 42,87 \quad \hat{\gamma}_3 = -35,47 \Rightarrow \gamma_4 = 44,45$$

**Identification of the estimated parameters  $e_j$  on the basis of the estimated parameters  $\gamma_j$ :**

$$\begin{aligned} \hat{e}_4 &= (\alpha^3 \cdot \hat{\gamma}_1 + \alpha^2 \cdot \hat{\gamma}_2 + \alpha \cdot \hat{\gamma}_3 + \hat{\gamma}_4) / (1 - \hat{\alpha}^4) = \\ &= (-0,75^3 \cdot 51,85 + 0,75^2 \cdot 42,87 - 0,75 \cdot 35,47 + 44,45) / (1 - 0,75^4) =, \\ &= 20,0877 / 0,6836 = 29,39 \quad [e_4 = 30] \end{aligned}$$

and from (13.1) we have:

$$\begin{aligned} \hat{e}_1 &= \hat{\alpha} \cdot \hat{e}_4 + \hat{\gamma}_1 = 0,75 \cdot 29,39 - 51,85 = -29,81 \quad [e_1 = -30] \\ \hat{e}_2 &= \hat{\alpha} \cdot \hat{e}_1 + \hat{\gamma}_2 = -0,75 \cdot 29,81 + 42,87 = 20,51 \quad [e_2 = 20] \\ \hat{e}_3 &= \hat{\alpha} \cdot \hat{e}_2 + \hat{\gamma}_3 = 0,75 \cdot 20,51 - 35,47 = -20,08 \quad [e_3 = -20] \\ \hat{e}_4 &= \hat{\alpha} \cdot \hat{e}_3 + \hat{\gamma}_4 = -0,75 \cdot 20,08 + 44,45 = 29,39 \quad [e_4 = 30] \end{aligned}$$

**Estimated limit function:**

$$(16.b) \quad \hat{y}_t^e = \frac{\hat{\beta}_0}{1 - \hat{\alpha}} + \frac{\hat{\beta}_1}{1 - \hat{\alpha}} x_t = 18,47 + 40,08 x_t$$

## 1.10 CAUSAL DYNAMIC MODEL ADL(1,0,2) TYPE (the case of two exogenous variables)

**The introducing form of the model**

$$(17) \quad y(y_{t-1}, x_t, z_t, \varepsilon_t) : \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 x_t + \beta_2 z_t + \varepsilon_t, \quad (t = (0), 1, 2, 3, \dots), \quad (0 < \alpha < 1).$$

**Purified from the error terms model (17):**

$$(18) \quad \tilde{y}(\tilde{y}_{t-1}, x_t) : \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 x_t + \beta_2 z_t, \quad (t = (0), 1, 2, 3, \dots), \quad (0 < \alpha < 1)$$

**The differential form of the cause-effect dynamic model (18)**  
- the gross effect of the increase in endogenous variable:

$$(19) \quad \Delta \tilde{y}(\Delta \tilde{y}_{t-1}, \Delta x_t, \Delta z_t) : \quad \Delta \tilde{y}_t = \alpha \cdot \Delta \tilde{y}_{t-1} + \beta_1 \Delta x_t + \beta_2 \Delta z_t, \quad (t = (0), 1, 2, 3, \dots)$$

**Pure short-term effect of the impact of exogenous variables on the endogenous variable**  
- the net effect:

$$(20) \quad Ef_t^s : \quad \Delta \tilde{y}_t^n = \Delta \tilde{y}_t - \alpha \cdot \Delta \tilde{y}_{t-1} = \beta_1 \Delta x_t + \beta_2 \Delta z_t,$$

where net effect of the endogenous variable ( $\Delta \tilde{y}_t^n$ ) is the difference between the increase in gross profit and increase inertial growth.

## 1.11 SWICHING TREND FUNCTION OF THE CAUSAL DYNAMIC MODEL ADL(1,0,2) TYPE (the case of two exogenous variables)

**The form of the switching trend function of the considered dynamic model**

$$(21) \quad \tilde{y}(t, x_t, z_t) : \quad \tilde{y}_t = \frac{\beta_0}{1-\alpha} + \frac{\beta_1}{1-\alpha} x_t + \frac{\beta_2}{1-\alpha} z_t - \alpha^t \frac{\beta_0}{1-\alpha} - \alpha^t \frac{\beta_1}{1-\alpha} x_t - \alpha^t \frac{\beta_2}{1-\alpha} z_t + \alpha^t \cdot \tilde{y}_{0t}$$

where:

a) The initial value of the switching trend in time  $t$ :

$$(21.a) \quad \tilde{y}_{0t} = \tilde{y}_0 + \frac{\beta_1}{1-\alpha} \sum_{t=1}^n (1-\alpha^{1-t}) \cdot \Delta x_t + \frac{\beta_2}{1-\alpha} \sum_{t=1}^n (1-\alpha^{1-t}) \cdot \Delta z_t$$

where for period  $t=1$  value of  $y$  in initial period  $t=0$  is  $\tilde{y}_0$ .

b) Limit of the switching trend in time  $t=h$ :

$$(21.b) \quad \tilde{y}_{t=h}^e = \lim_{t \rightarrow \infty} \tilde{y}_t(t, x_t, z_t) = \frac{\beta_0}{1-\alpha} + \frac{\beta_1}{1-\alpha} x_{t=h} + \frac{\beta_2}{1-\alpha} z_{t=h}, \quad (x_{t \geq h}, z_{t \geq h} = \text{const. dla } h = 1, 2, 3, \dots)$$

c) Limit impact of the partial effects of exogenous variables  
on the endogenous variable in time  $t=h$ :

$$(21.c.1) \quad Ef_h^I(x) : \quad \Delta \tilde{y}_h^e(x) = \frac{\beta_1}{1-\alpha} \Delta x_h, \quad (\Delta x_h = x_h - x_{h-1}, \quad \Delta z_h = 0 \quad t = h)$$

$$(21.c.2) \quad Ef_h^I(z) : \quad \Delta \tilde{y}_h^e(z) = \frac{\beta_2}{1-\alpha} \Delta z_h, \quad (\Delta z_h = z_h - z_{h-1}, \quad \Delta x_h = 0 \quad t = h)$$

d) Cumulative effect of the long-term effects of exogenous variables  
on the endogenous variable in time  $t=h$ :

$$(21.d) \quad Ef_h^I(x, z) : \quad \Delta \tilde{y}_h^e = \frac{\beta_1}{1-\alpha} \Delta x_h + \frac{\beta_2}{1-\alpha} \Delta z_h,$$

## PART II

### SIMULATION ANALYSIS ON THE BASIS OF THE CAUSAL DYNAMIC MODEL TYPE ADL(1,1,1-2)

#### 2.1 CAUSAL ADL(1,1,1) MODEL – ERROR CORRECTION MODEL (ECM)

##### 2.1.1. INTRODUCTION

The expected (desired) level of endogenous variable:

$$(22) \quad \tilde{y}_t^e(x_t) : \quad \tilde{y}_t^e = b_0 + b_1 x_t$$

Effect of exogenous variable on the expected level of endogenous variable in period  $t$ :  
(long-term effect)

$$(23) \quad Ef_t^d : \quad \Delta\tilde{y}_t^e = b_1 \Delta x_t$$

Error Correction Model (ECM) – generalized version of the Partial Adjustment Model (PAM)<sup>3</sup>:

$$(24) \quad \underbrace{\tilde{y}_t - \tilde{y}_{t-1}}_{\Delta\tilde{y}_t : \text{short term gross increase of } \tilde{y} \text{ in period } t} = \underbrace{\lambda(\tilde{y}_t^e - \tilde{y}_{t-1}^e)}_{\text{change in the desired values}} + \underbrace{\alpha'(\tilde{y}_{t-1}^e - \tilde{y}_{t-1})}_{\text{past period's disequilibrium}}, \quad (0 < \lambda, \alpha < 1), (\alpha' = 1 - \alpha).$$

It follows that the increase ( $\Delta\tilde{y}_t$ ) in the variable  $\tilde{y}$  in period  $t$  is the result of changes in the level of the target for the period  $t$  one hand, and changes in the level of imbalance in the period  $(t-1)$  on the other.

Introducing (22) into (24) we get:

$$(25) \quad \Delta\tilde{y}_t = \lambda(b_0 + b_1 x_t - b_0 - b_1 x_{t-1}) + \alpha'(b_0 + b_1 x_{t-1} - \tilde{y}_{t-1}),$$

which gives, after transformation:

$$(26) \quad \Delta\tilde{y}_t = \alpha' b_0 - \alpha' \cdot \tilde{y}_{t-1} + \lambda b_1 \cdot \Delta x_t + \alpha' b_1 \cdot x_{t-1},$$

or:

$$(27) \quad \Delta\tilde{y}_t = (1 - \alpha)b_0 + (\alpha - 1) \cdot \tilde{y}_{t-1} + \lambda b_1 \cdot \Delta x_t + (1 - \alpha)b_1 \cdot x_{t-1}$$

because:

$$\alpha' = 1 - \alpha \iff -\alpha' = \alpha - 1$$

It means that:

$$(28) \quad \tilde{y}_t = (1 - \alpha)b_0 + \alpha \cdot \tilde{y}_{t-1} + \lambda b_1 \cdot x_t + (1 - \alpha - \lambda)b_1 \cdot x_{t-1}$$

Which can be written as the ADL(1,1,1) type model:

$$(29) \quad \tilde{y}(y_{t-1}, x_t, x_{t-1}) : \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 \cdot x_t + \beta_2 \cdot x_{t-1}$$

where:

$$(30.1) \quad \alpha = 1 - \alpha'$$

$$(30.2) \quad \beta_0 = \alpha' \cdot b_0$$

$$(30.3) \quad \beta_1 = \lambda \cdot b_1$$

$$(30.4) \quad \beta_2 = (\alpha' - \lambda) \cdot b_1$$

---

<sup>3</sup> See: [Maddala, 2001, pp. 405-409, 564-569].

### 2.1.2. SHORT TERM EFFECTS IN THE CASE OF THE ECM MODEL (see: Figure 2 and Table 2)

a) Short term gross increase of the endogenous variable in time  $t$ :

$$(31.a) \quad \Delta\tilde{y}_t = \alpha \cdot \Delta\tilde{y}_{t-1} + \beta_1 \cdot \Delta x_t + \beta_2 \cdot \Delta x_{t-1}$$

b) Pure inertial effect in time  $t$ :

$$(31.b) \quad Ef_{\Delta y}^{iner} : \quad \Delta\tilde{y}_t^{iner} = \alpha \cdot \Delta\tilde{y}_{t-1},$$

c) Short term net increase ( $\Delta\tilde{y}_t^n$ ) of the endogenous variable in time  $t$ :

$$(31.c) \quad \Delta\tilde{y}_t^n = \Delta\tilde{y}_t - \Delta\tilde{y}_t^{iner} = \beta_1 \cdot \Delta x_t + \beta_2 \cdot \Delta x_{t-1},$$

d) Short term direct (uncorrected) effect of influence  $\Delta x$  on  $\Delta\tilde{y}$  in time  $t$  ( $\Delta\tilde{y}_t^{nk}$ ):

$$(31.d) \quad Ef_t^{nk} : \quad \Delta\tilde{y}_t^{nk} = \beta_1 \cdot \Delta x_t$$

e) Correction in the interaction of  $\Delta x$  on  $\Delta\tilde{y}$  in time  $t$  ( $cr_t = \Delta\tilde{y}_t^{cr}$ ):

$$(31.e) \quad cr_t : \quad \Delta\tilde{y}_t^{cr} = \Delta\tilde{y}_t^n - \Delta\tilde{y}_t^{nk} = \beta_2 \cdot \Delta x_{t-1}$$

We can notice that the net increase of ( $\Delta\tilde{y}_t^n$ ) given in (31.c) because of:

$\Delta x_t = \Delta x_t - \Delta x_{t-1} \Rightarrow \Delta x_t = \Delta x_{t-1} + \Delta\Delta x_t$ , can be written as:

$$(31.f) \quad \Delta\tilde{y}_t^n = \beta_1 \cdot (\Delta x_{t-1} + \Delta\Delta x_t) + \beta_2 \cdot \Delta x_{t-1} = (\beta_1 + \beta_2) \cdot \Delta x_{t-1} + \beta_1 \cdot \Delta\Delta x_t :$$

g) Neutralizing factor of the direct effect

$$(31.g) \quad nutr_t : \quad \Delta\tilde{y}_t^{nutr} = \beta_1 \cdot \Delta\Delta x_t = \beta_1 \cdot \Delta x_t - \beta_1 \cdot \Delta x_{t-1}$$

h) Short term (net) corrected effect of the interaction of  $\Delta x$  from time  $(t-1)$  to  $\Delta\tilde{y}$  in time  $t$  ( $\Delta\tilde{y}_t^s$ ):

$$(31.h) \quad Ef_t^s : \quad \Delta\tilde{y}_t^s = \Delta\tilde{y}_t^n - \Delta\tilde{y}_t^{nutr} = (\beta_1 + \beta_2) \cdot \Delta x_{t-1},$$

So consequently we can say that:

If a variable  $x$  in a given period will increase by one unit then the adjusted net gain of variable  $\tilde{y}$  in the next period will be  $(\beta_1 + \beta_2)$  units. The increase is the sum of gross and net increase in inertial effect.

### 2.1.3. LONG TERM CORRECTED MULTIPLIER EFFECTS (LTCME)

The starting assumption:  $z_t = x_{t-1} \Rightarrow \Delta z_t = \Delta x_{t-1} \neq 0$

The net impact multiplier effects of growth of the variable  $z$  from period  $t$  to following periods  $t+i$ :

From (31.h) we can say that:

$$(31.A) \quad \begin{aligned} \text{dla } t+1: \quad \Delta\tilde{y}_{t+1}^n &= \alpha \cdot \Delta\tilde{y}_t^n = \alpha \cdot (\beta_1 + \beta_2) \Delta z_t, \quad (\Delta z_{t+1} = 0), \\ \text{dla } t+2: \quad \Delta\tilde{y}_{t+2}^n &= \alpha \cdot \Delta\tilde{y}_{t+1}^n = \alpha^2 \cdot (\beta_1 + \beta_2) \Delta z_t, \quad (\Delta z_{t+2} = 0), \\ \text{dla } t+3: \quad \Delta\tilde{y}_{t+3}^n &= \alpha \cdot \Delta\tilde{y}_{t+2}^n = \alpha^3 \cdot (\beta_1 + \beta_2) \Delta z_t, \quad (\Delta z_{t+3} = 0), \dots \end{aligned}$$

Long term corrected multiplier net effect resulting from the increase of the variable  $z$  by  $\Delta z_t$  in time  $t$ :  
(sum of infinitely decreasing geometric progression)

$$(31.B) \quad LTCME_t : \quad \Delta\tilde{y}_t^e = \lim_{i \rightarrow \infty} \sum_{i=0} \Delta\tilde{y}_{t+i}^n = (1 + \alpha + \alpha^2 + \alpha^3 + \dots) \cdot (\beta_1 + \beta_2) \Delta z_t = \frac{\beta_1 + \beta_2}{1 - \alpha} \Delta z_t$$

Using the starting assumption, based on (31.B) we can conclude that:

$$(31.C) \quad LTCME_t : \quad \Delta\tilde{y}_t^e = \frac{\beta_1 + \beta_2}{1 - \alpha} \Delta x_{t-1} \quad \text{in long term} \Rightarrow \Delta\tilde{y}_t^e = \frac{\beta_1 + \beta_2}{1 - \alpha} \Delta x_t \quad (\text{for limit values}).$$

where  $[(\beta_1 + \beta_2)/(1-\alpha)]$  is a long-term corrected multiplier (LTCM).

## 2.2 IDENTIFICATION OF THE LIMIT PARAMETRS OF THE ENDOGENOUS VARIABLE AND THE IMBALANCE PARAMETERS BASED ON THE ADL(1.1.1)

Note that the transforming expression (29) as follows:

$$\tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 \cdot x_t - \beta_1 \cdot x_{t-1} + \beta_1 \cdot x_{t-1} + \beta_2 \cdot x_{t-1},$$

We can write them in the following form:

$$(32) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 \cdot \Delta x_t + (\beta_1 + \beta_2) \cdot x_{t-1}$$

Transforming (32) as follows:

$$\tilde{y}_t - \tilde{y}_{t-1} = \beta_0 + \alpha \cdot \tilde{y}_{t-1} - \tilde{y}_{t-1} + \beta_1 \cdot \Delta x_t + (\beta_1 + \beta_2) \cdot x_{t-1}$$

Finally we have:

$$(33) \quad \Delta\tilde{y}_t = \beta_0 + (\alpha - 1) \cdot \tilde{y}_{t-1} + \beta_1 \cdot \Delta x_t + (\beta_1 + \beta_2) \cdot x_{t-1}$$

After further transformations of the above expression gives:

$$(34) \quad \Delta\tilde{y}_t = (1 - \alpha) \left[ \left( \frac{\beta_0}{1 - \alpha} + \frac{\beta_1 + \beta_2}{1 - \alpha} x_{t-1} \right) - \tilde{y}_{t-1} \right] + \beta_1 \cdot \Delta x_t$$

Where, according to (30.3) and (30.4), we have:

$$(35) \quad \beta_1 + \beta_2 = \lambda \cdot b_1 + \alpha' \cdot b_1 - \lambda \cdot b_1 = \alpha' \cdot b_1$$

Assuming that we know the parameters in the model (29) and thus (33), on the basis of (30.1) - (30.4) and (35), we find that:

$$(36.1) \quad \alpha' = 1 - \alpha$$

$$(36.2) \quad b_0 = \frac{\beta_0}{1 - \alpha}$$

$$(36.3) \quad b_1 = \frac{\beta_1 + \beta_2}{1 - \alpha}$$

$$(36.4) \quad \lambda = \frac{\beta_1}{\beta_1 + \beta_2} \cdot (1 - \alpha)$$

Using the above equations the model (34) we write as follows:

$$(37) \quad \Delta\tilde{y}_t = (1 - \alpha) [(b_0 + b_1 x_{t-1}) - \tilde{y}_{t-1}] + \beta_1 \cdot \Delta x_t$$

Using (22) we can write the following form [which incidentally can be derived directly from (22) and (24)]:

$$(38) \quad \Delta\tilde{y}_t = (1 - \alpha) \cdot (\tilde{y}_{t-1}^e - \tilde{y}_{t-1}) + \beta_1 \cdot \Delta x_t$$

Note that the first term of expression (term in brackets) in the forms (34), (37) and (38) of the affected model - according to (24) - determines the level of imbalance in the previous period (t-1). Thus, the impact of the increase in the variable  $x$  in period  $t$  ( $\beta_1 \Delta x_t$ ) to increase the variable  $\tilde{y}$  ( $\Delta\tilde{y}$ ) in period  $t$  imbalance is

**corrected level of the previous period. This means that the expression  $(\beta_1 \Delta x_t)$ , we can determine the level of factor corrected by imbalances in the previous period (see Table 3)**

On the other hand, recognizing the equivalence (24) and (38) it is clear that:

$$(38.1) \quad \lambda \Delta \tilde{y}_t^e = \beta_1 \Delta x_t \Rightarrow \Delta \tilde{y}_t^e = \frac{\beta_1}{\lambda} \Delta x_t, \quad (\Delta \tilde{y}_t^e = \tilde{y}_t^e - y_{t-1}^e)$$

Now introducing the above expressions defined in (36.4), the parameter  $\lambda$  finally get:

$$(38.2) \quad \Delta \tilde{y}_t^e = \frac{\beta_1 + \beta_2}{1 - \alpha} \Delta x_t,$$

Based on the above, we can say that:

**if the variable  $x$  in period  $t$  will increase by one unit, then in the same period, the increase in limit values of the explanatory variable  $\tilde{y}^e$  will be  $[(\beta_1 + \beta_2)/(1-\alpha)]$  units, which is equivalent to an increase of  $(\beta_1/\lambda)$  units.** It follows that each equivalent expression:  $\{(\beta_1/\lambda)\Delta x_t\}$ ,  $\{[(\beta_1 + \beta_2)/(1-\alpha)]\Delta x_t\}$  may be called the **long-term corrected multiplier effect (LTCME)<sub>t</sub>**.

Note, that from (30.3), we have:

$$(39) \quad \beta_1 \equiv \beta_1 + \beta_2 \Rightarrow \beta_2 \equiv 0 \Rightarrow \lambda \equiv \alpha'$$

**In above situation the error correction model (24) can be reduced to the partial adjustment model, taking the following form:**

$$(40) \quad \tilde{y}_t - \tilde{y}_{t-1} = \alpha' (\tilde{y}_t^e - \tilde{y}_{t-1}^e), \quad (\alpha' = 1 - \alpha)$$

In terms of cause-effect model ADL(1,1,1) written as (29) is reduced to the ADL(1,0,1) model of the form:

$$(41) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_1 \cdot x_t, \quad (\alpha = 1 - \alpha')$$

This model corresponds exactly to the model (2) explained in part I of this paper.

### 2.3 SWICHING TREND FUNCTION OF THE CAUSAL DYNAMIC MODEL TYPE ADL(1,1,1):

Using the switching trend model for the two variables (21), and recognizing that the variable  $z_t$  is equal to the lagged variable  $x_t$  (that is:  $z_t = x_{t-1}$ ) model for the case of the trend of switching autoregressive model (29) we write as follows:

$$(42) \quad \tilde{y}(t, x_t, x_{t-1}) : \tilde{y}_t = \frac{\beta_0}{1 - \alpha} + \frac{\beta_1}{1 - \alpha} x_t + \frac{\beta_2}{1 - \alpha} x_{t-1} - \alpha' \frac{\beta_0}{1 - \alpha} - \alpha' \frac{\beta_1}{1 - \alpha} x_t - \alpha' \frac{\beta_2}{1 - \alpha} x_{t-1} + \alpha' \cdot \tilde{y}_{0t}$$

where:

a) **Initial value of the trend switching in time  $t=0$ :**

$$(43) \quad \tilde{y}_{0t} = \tilde{y}_0 + \frac{\beta_1}{1 - \alpha} \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t + \frac{\beta_2}{1 - \alpha} \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_{t-1}$$

Where for period  $t=1$  value of  $y$  in initial period  $t=0$  is  $\tilde{y}_0$ .

b) **Limit value of the trend switching in time  $t=h$ :**

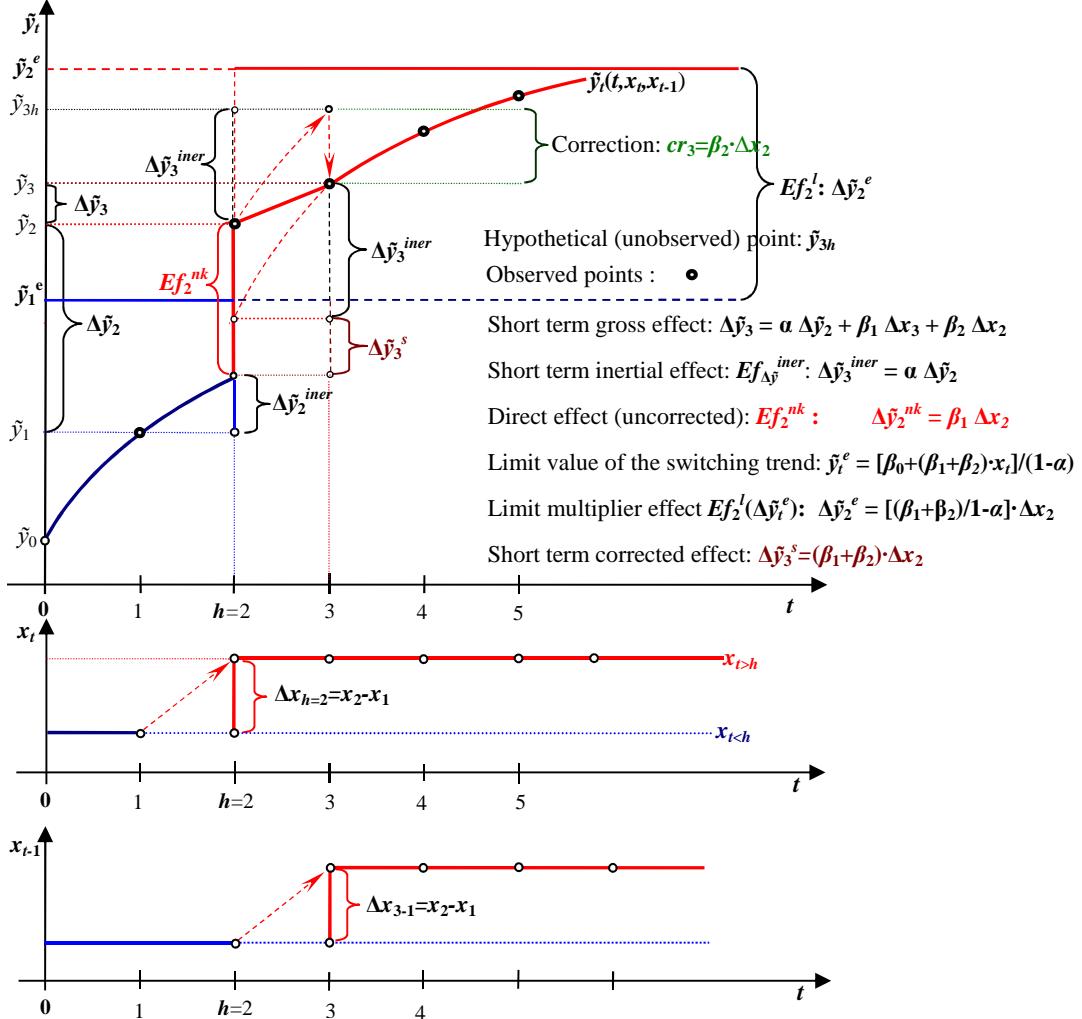
$$(44) \quad \tilde{y}_{t=h}^e = \lim_{t \rightarrow \infty} \tilde{y}_t(t, x_t, x_{t-1}) = \frac{\beta_0}{1 - \alpha} + \frac{\beta_1 + \beta_2}{1 - \alpha} x_{t=h}, \quad (x_{t>h} = x_{t=h} = \text{const.})$$

because:  $x_{t=h} = x_h = \text{const.} \Rightarrow \lim_{t \rightarrow \infty} x_{t \geq h} = \lim_{t \rightarrow \infty} x_{t-1} = x_h$

c) **Limit effect of the influence of the exogenous variable on the endogenous variable in time  $t=h$ :**

$$(45) \quad Ef_t^I : \Delta \tilde{y}_t^e = \frac{\beta_1 + \beta_2}{1 - \alpha} \Delta x_t = b_1 \cdot \Delta x_t, \text{ according to (23).}$$

## 2.4 GRAPHIC APPROACH TO THE FUNCTION OF SWITCHING TREND MODEL TYPE ADL(1,1,1) – ERROR CORRECTION MECHANISM



**Figure 2.** Switching trend function  $\tilde{y}(t, x_t, x_{t-1})$  and short term and long term effects of changes of the endogenous variable  $\tilde{y}_t$  as a result of increase of the exogenous variable  $x_h$  ( $\Delta x_h > 0$ ) in period  $t=h$  – short term effect decomposition in the case of the error correction mechanism.

## 2.5. SIMULATION MODEL ASSUMPTIONS FOR TYPE ADL(1,1,1) AND SIMULATION RESULTS:

Assumed values of parameters:

$$\begin{aligned}\alpha &= 0,75 \\ \beta_0 &= 5 \\ \beta_1 &= 10 \\ \beta_2 &= -6 \\ \tilde{y}_0 &= 50\end{aligned}$$

Which means that:

$$b_0 = \frac{\beta_0}{1-\alpha} = \frac{5}{1-0,75} = 20, \quad b_1 = \frac{\beta_1 + \beta_2}{1-\alpha} = \frac{4}{1-0,75} = 16, \quad \lambda = \frac{\beta_1}{\beta_1 + \beta_2} \cdot (1-\alpha) = \frac{10}{4} \cdot 0,25 = 0,625$$

The limit values of the switching trend in time  $t$ :

$$\tilde{y}_t^e = \frac{\beta_0}{1-\alpha} + \frac{\beta_1 + \beta_2}{1-\alpha} x_t = 20 + 16 \cdot x_t$$

The additional values of the trend of switching:

$$\frac{\beta_1}{1-\alpha} = \frac{10}{1-0,75} = 40 \quad \frac{\beta_2}{1-\alpha} = \frac{-6}{1-0,75} = -24$$

Assumed values of the variable  $x_t$  for periods:

from $t=1$ to $t=6$ :	$x_t = 10$
from $t=7$ to $t=12$ :	$x_t = 14$
from $t=13$ to $t=16$ :	$x_t = 16$
from $t=17$ to $t=25$ :	$x_t = 17,18,19,20,21,22,23,24,25$ (trend)
from $t=26$ to $t=32$ :	$x_t = 23,22,24,26,25,28,29$ (irregular changes)
from $t=33$ to $t=36$ :	$x_t = 30$
from $t=37$ to $t=40$ :	$x_t = 20$

Assumed notation of the software system MICROFIT:

The switching trend of the dependent variable:  
where:

$$\begin{aligned}\tilde{y}_t(t, x_t, x_{t-1}) &= YXZ \\ x_t &= X \\ x_{t-1} &= Z\end{aligned}$$

The calculations in MICROFIT:

The trend of switching:

$$\tilde{y}_t(t, x_t, x_{t-1}) : YXZ = 20 - 20 * 0.75^{1-t} + 40 * x_t - 40 * x_t * 0.75^{1-t} - 24 * z + 24 * z * 0.75^{1-t} + y_0 x_t * 0.75^{1-t}$$

where:

- 1)  $(1 - 0,75^{1-t}) :$   $POM = 1 - 0,75^{1-t}$
- 2)  $(1 - 0,75^{1-t}) \cdot \Delta x_t :$   $POMDX = pom * dx$
- 3)  $(1 - 0,75^{1-t}) \cdot \Delta x_{t-1} :$   $POMDZ = pom * dz$
- 4)  $\sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t :$   $CSUMPOMDX = csum(pomdx)$
- 5)  $\sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t :$   $CSUMPOMDZ = csum(pomdz)$
- 6)  $\tilde{y}_{0t} = \tilde{y}_0 + \frac{\beta_1}{1-\alpha} \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_t + \frac{\beta_2}{1-\alpha} \sum_{t=1}^n (1 - \alpha^{1-t}) \cdot \Delta x_{t-1} :$   $Y0XZ = 50 + 40 * csumpomdx - 24 * csumpomdz$

**Table 2. The simulation results of the short-term effects based on the switching trend in the case of a dynamic model of the type of ADL(1,1,1)**

Period	<i>t</i>	$x_t$	$x_{t-1}$	$\Delta x_t$	$\Delta x_{t-1}$	$\Delta \Delta x_t$	$\tilde{y}_t = \tilde{y}(x_t, x_{t-1}, t)$	$\Delta \tilde{y}_t$	$\Delta \tilde{y}_t^{iner}$	$\Delta \tilde{y}_t^n$	$nutr_t$	$\Delta \tilde{y}_t^s$	$\Delta \tilde{y}_t^{nk}$	$\Delta \tilde{y}_t^{cr}$
	T	X	X(-1)	DX	DX(-1)	DDX	YXZ	DYXZ	INER2	DYXZN	NUTR	DYXZK	DYXZNK	CR
2001Q1	1	10	10	0	0	0	82,5	32,5	-	0	0	0	0	0
2001Q2	2	10	10	0	0	0	106,875	24,375	24,375	0	0	0	0	0
2001Q3	3	10	10	0	0	0	125,156	18,2813	18,2813	0	0	0	0	0
2001Q4	4	10	10	0	0	0	138,867	13,7109	13,7109	0	0	0	0	0
2002Q1	5	10	10	0	0	0	149,15	10,2832	10,2832	0	0	0	0	0
2002Q2	6	10	10	0	0	0	156,863	7,7124	7,7124	0	0	0	0	0
2002Q3	7	14	10	4	0	4	202,647	45,7843	5,7843	40	40	0	40	0
2002Q4	8	14	14	0	4	-4	212,985	10,3382	34,3382	-24	-40	16	0	-24
2003Q1	9	14	14	0	0	0	220,739	7,7537	7,7537	0	0	0	0	0
2003Q2	10	14	14	0	0	0	226,554	5,8153	5,8153	0	0	0	0	0
2003Q3	11	14	14	0	0	0	230,916	4,3614	4,3614	0	0	0	0	0
2003Q4	12	14	14	0	0	0	234,187	3,2711	3,2711	0	0	0	0	0
2004Q1	13	16	14	2	0	2	256,64	22,4533	2,4533	20	20	0	20	0
2004Q2	14	16	16	0	2	-2	261,48	4,84	16,84	-12	-20	8	0	-12
2004Q3	15	16	16	0	0	0	265,11	3,63	3,63	0	0	0	0	0
2004Q4	16	16	16	0	0	0	267,833	2,7225	2,7225	0	0	0	0	0
2005Q1	17	17	16	1	0	1	279,874	12,0419	2,0419	10	10	0	10	0
2005Q2	18	18	17	1	1	0	292,906	13,0314	9,0314	4	0	4	10	-6
2005Q3	19	19	18	1	1	0	306,679	13,7736	9,7736	4	0	4	10	-6
2005Q4	20	20	19	1	1	0	321,01	14,3302	10,3302	4	0	4	10	-6
2006Q1	21	21	20	1	1	0	335,757	14,7476	10,7476	4	0	4	10	-6
2006Q2	22	22	21	1	1	0	350,818	15,0607	11,0607	4	0	4	10	-6
2006Q3	23	23	22	1	1	0	366,113	15,2955	11,2955	4	0	4	10	-6
2006Q4	24	24	23	1	1	0	381,585	15,4717	11,4717	4	0	4	10	-6
2007Q1	25	25	24	1	1	0	397,189	15,6037	11,6037	4	0	4	10	-6
2007Q2	26	23	25	-2	1	-3	382,892	-14,297	11,7028	-26	-30	4	-20	-6
2007Q3	27	22	23	-1	-2	1	374,169	-8,7229	-10,723	2	10	-8	-10	12
2007Q4	28	24	22	2	-1	3	393,627	19,4578	-6,5422	26	30	-4	20	6
2008Q1	29	26	24	2	2	0	416,22	22,5934	14,5934	8	0	8	20	-12
2008Q2	30	25	26	-1	2	-3	411,165	-5,055	16,945	-22	-30	8	-10	-12
2008Q3	31	28	25	3	-1	4	443,374	32,2088	-3,7912	36	40	-4	30	6
2008Q4	32	29	28	1	3	-2	459,53	16,1566	24,1566	-8	-20	12	10	-18
2009Q1	33	30	29	1	1	0	475,648	16,1174	12,1174	4	0	4	10	-6
2009Q2	34	30	30	0	1	-1	481,736	6,0881	12,0881	-6	-10	4	0	-6
2009Q3	35	30	30	0	0	0	486,302	4,5661	4,5661	0	0	0	0	0
2009Q4	36	30	30	0	0	0	489,726	3,4245	3,4245	0	0	0	0	0
2010Q1	37	20	30	-10	0	-10	392,295	-97,432	2,5684	-100	-100	0	-100	0
2010Q2	38	20	20	0	-10	10	379,221	-13,074	-73,074	60	100	-40	0	60
2010Q3	39	20	20	0	0	0	369,416	-9,8053	-9,8053	0	0	0	0	0
2010Q4	40	20	20	0	0	0	362,062	-7,354	-7,354	0	0	0	0	0

- $\tilde{y}_t = \tilde{y}(x_t, x_{t-1}, t)$  - the trend of switching function,
- $\Delta \tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$  - the short term increase in gross variable  $\tilde{y}_t$ ,
- $\Delta \tilde{y}_t^n = \Delta \tilde{y}_t - \alpha \Delta \tilde{y}_{t-1}$  - the short term increase in net variable  $\tilde{y}_t$  (where:  $\alpha=0,75$ )
- $\Delta \tilde{y}_t^s = (\beta_1 + \beta_2) \Delta x_{t-1}$  - the short term corrected effect of influence of the variable  $\Delta x$  from period  $(t-1)$  on  $\Delta \tilde{y}$  in period  $t$ , where:  $\beta_1=10$ ,  $\beta_2=-6$
- $\Delta \tilde{y}_t^{iner} = \alpha \Delta \tilde{y}_{t-1}$  - inertial effect ( $E_{\Delta \tilde{y}_t^{iner}}$ ) (where:  $\alpha=0,75$ )
- $nutr_t = \beta_1 \Delta x_t$  - direct effect of neutralizing factor in the impact of the variable  $x$  to the variable  $\tilde{y}$ ,
- $\Delta \tilde{y}_t^{nk} = \beta_1 \Delta x_t$  - direct short term effect of the influence of  $\Delta x$  from period  $t$  to  $\Delta \tilde{y}$  in period  $t$
- $cr_t: \Delta \tilde{y}_t^{cr} = \beta_1 \Delta x_{t-1}$  - correction in the interaction of  $\Delta x$  from period  $(t-1)$  to  $\Delta \tilde{y}$  in period  $t$ .

**Table 3 Simulation results demonstrate the long-term imbalance correction effect on the increase in the explanatory variable in the short term for the ADL(1,1,1) model**

Period	nr (t)	$\tilde{y}_t$	$\Delta\tilde{y}_t$	$\tilde{y}_t^e$	$\Delta\tilde{y}_t^e$	$\tilde{y}_t^e - \tilde{y}_t$	$\tilde{y}_{t-1}^e - \tilde{y}_{t-1}$	$\alpha'(\tilde{y}_{t-1}^e - \tilde{y}_{t-1})$	$\Delta\tilde{y}_t^{nk} = \beta_1 \Delta x_t$	LTCME <sub>t</sub>
	T	YXZ	DYXZ	YXZE	DYXZE	DYEY	DYEY(-1)	ADYEY(-1)	DYXZNK	LTCME
2001Q1	1	82,5	32,5	180	0	97,5			0	0
2001Q2	2	106,875	24,375	180	0	73,125	97,5	24,375	0	0
2001Q3	3	125,1563	18,2813	180	0	54,8438	73,125	18,2813	0	0
2001Q4	4	138,8672	13,7109	180	0	41,1328	54,8438	13,7109	0	0
2002Q1	5	149,1504	10,2832	180	0	30,8496	41,1328	10,2832	0	0
2002Q2	6	156,8628	7,7124	180	0	23,1372	30,8496	7,7124	0	0
2002Q3	7	202,6471	45,7843	244	64	41,3529	23,1372	5,7843	40	64
2002Q4	8	212,9853	10,3382	244	0	31,0147	41,3529	10,3382	0	0
2003Q1	9	220,739	7,7537	244	0	23,261	31,0147	7,7537	0	0
2003Q2	10	226,5542	5,8153	244	0	17,4458	23,261	5,8153	0	0
2003Q3	11	230,9157	4,3614	244	0	13,0843	17,4458	4,3614	0	0
2003Q4	12	234,1868	3,2711	244	0	9,8132	13,0843	3,2711	0	0
2004Q1	13	256,6401	22,4533	276	32	19,3599	9,8132	2,4533	20	32
2004Q2	14	261,4801	4,84	276	0	14,5199	19,3599	4,84	0	0
2004Q3	15	265,11	3,63	276	0	10,89	14,5199	3,63	0	0
2004Q4	16	267,8325	2,7225	276	0	8,1675	10,89	2,7225	0	0
2005Q1	17	279,8744	12,0419	292	16	12,1256	8,1675	2,0419	10	16
2005Q2	18	292,9058	13,0314	308	16	15,0942	12,1256	3,0314	10	16
2005Q3	19	306,6793	13,7736	324	16	17,3207	15,0942	3,7736	10	16
2005Q4	20	321,0095	14,3302	340	16	18,9905	17,3207	4,3302	10	16
2006Q1	21	335,7571	14,7476	356	16	20,2429	18,9905	4,7476	10	16
2006Q2	22	350,8179	15,0607	372	16	21,1821	20,2429	5,0607	10	16
2006Q3	23	366,1134	15,2955	388	16	21,8866	21,1821	5,2955	10	16
2006Q4	24	381,585	15,4717	404	16	22,415	21,8866	5,4717	10	16
2007Q1	25	397,1888	15,6037	420	16	22,8112	22,415	5,6037	10	16
2007Q2	26	382,8916	-14,2972	388	-32	5,1084	22,8112	5,7028	-20	-32
2007Q3	27	374,1687	-8,7229	372	-16	-2,1687	5,1084	1,2771	-10	-16
2007Q4	28	393,6265	19,4578	404	32	10,3735	-2,1687	-0,54217	20	32
2008Q1	29	416,2199	22,5934	436	32	19,7801	10,3735	2,5934	20	32
2008Q2	30	411,1649	-5,055	420	-16	8,8351	19,7801	4,945	-10	-16
2008Q3	31	443,3737	32,2088	468	48	24,6263	8,8351	2,2088	30	48
2008Q4	32	459,5303	16,1566	484	16	24,4697	24,6263	6,1566	10	16
2009Q1	33	475,6477	16,1174	500	16	24,3523	24,4697	6,1174	10	16
2009Q2	34	481,7358	6,0881	500	0	18,2642	24,3523	6,0881	0	0
2009Q3	35	486,3018	4,5661	500	0	13,6982	18,2642	4,5661	0	0
2009Q4	36	489,7264	3,4245	500	0	10,2736	13,6982	3,4245	0	0
2010Q1	37	392,2948	-97,4316	340	-160	-52,2948	10,2736	2,5684	-100	-160
2010Q2	38	379,2211	-13,0737	340	0	-39,2211	-52,2948	-13,0737	0	0
2010Q3	39	369,4158	-9,8053	340	0	-29,4158	-39,2211	-9,8053	0	0
2010Q4	40	362,0619	-7,354	340	0	-22,0619	-29,4158	-7,354	0	0

$\tilde{y}_t = \tilde{y}(x_t, x_{t-1}, t)$

- the switching trend function,

$\Delta\tilde{y}_t = \tilde{y}_t - \tilde{y}_{t-1}$

- the short term increase in gross variable  $\tilde{y}_t$ ,

$\tilde{y}_t^e$

- equilibrium level in the period  $t$ ,

$\Delta\tilde{y}_t^e$

- increase of the level of endogenous variable due to the increase of the variable  $x_t$ ,

$\tilde{y}_t^e - \tilde{y}_t$

- equivalently:  $\Delta\tilde{y}_t^e = LTCME_t = (\beta_1/\lambda)\Delta x_t$ , where  $\lambda = 0.625$ ,  $\beta_1 = 10$

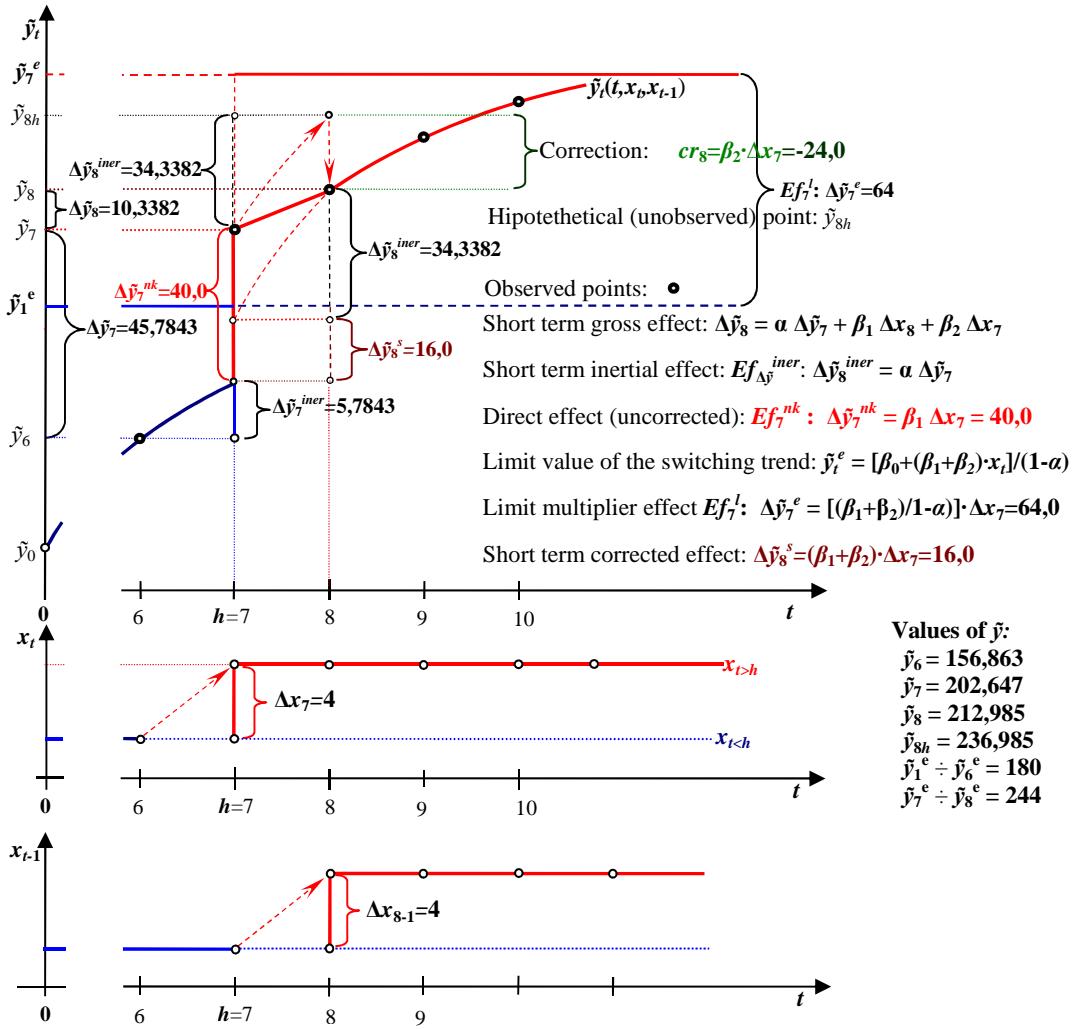
- the level of imbalance in period  $t$

$\alpha'(\tilde{y}_{t-1}^e - \tilde{y}_{t-1})$

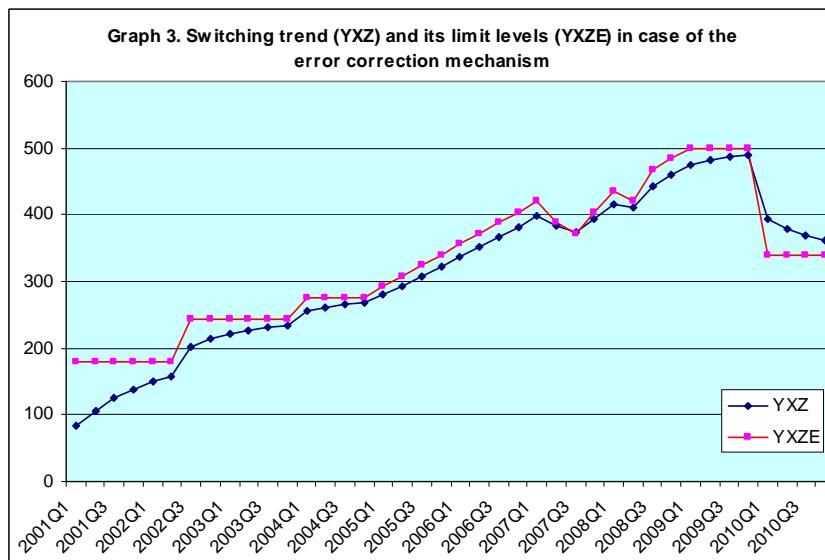
- the effect of the impact level of imbalance of the period ( $t-1$ ) to increase the variable  $\tilde{y}$  in period  $t$ ,

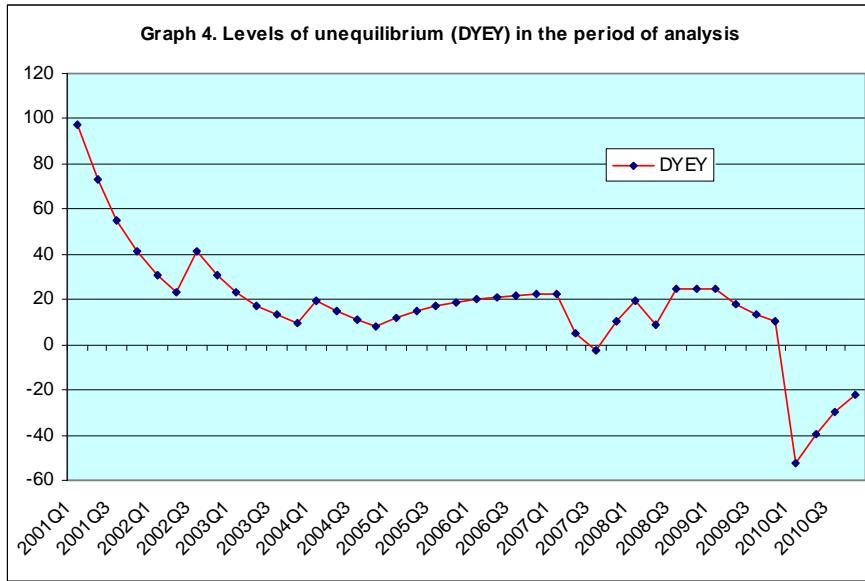
$LTCME_t$

- the long term corrected multiplier effect:  $\Delta\tilde{y}_t^e = lcy_t^e = (\beta_1/\lambda)\Delta x_t$ , gdzie  $\lambda = 0.625$ ,  $\beta_1 = 10$



**Figure 3.** Simulation of the switching trend function  $\tilde{y}(t, x_b, x_{t-1})$  and short term and long term effects of changes of the endogenous variable  $\tilde{y}_t$  as a result of increase of the exogenous variable  $x_t$  in period from  $t=6$  to  $t=8$  and short term effect decomposition in the case of the error correction mechanism based on the information included in the Table 2 and 3.





## 2.6 RESULTS OF ESTIMATION OF THE DYNAMIC MODEL TYPE ADL(1,1,1) BASED ON SIMULATED DETERMINISTIC SWITCHING TREND FROM THE TABLE 2 (validation of simulation)

a) The case of the model (*data from the Table 2*):

(46.a)	$\tilde{y}_t = \beta_0 + \alpha \tilde{y}_{t-1} + \beta_1 \cdot x_t - \beta_1 \cdot x_{t-1}$		
Ordinary Least Squares Estimation			
*****			
Dependent variable is YXZ			
39 observations used for estimation from 2001Q2 to 2010Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	5.0000	0.00	*NONE*
YXZ(-1)	.75000	0.00	*NONE*
X	10.0000	0.00	*NONE*
X(-1)	-6.0000	0.00	*NONE*
*****			

Estimated model:

$$(46.a') \quad \tilde{y}_t = 5 + 0,75 \tilde{y}_{t-1} + 10 \cdot x_t - 6 \cdot x_{t-1}$$

b) The case of the model (*data from the Table 2*):

$$(46.b) \quad \Delta \tilde{y}_t = \beta_0 + \alpha \tilde{y}_{t-1} + \beta_1 \cdot \Delta x_t - \beta' \cdot x_{t-1}, \quad (\beta' = \beta_1 + \beta_2)$$

Ordinary Least Squares Estimation			
*****			
Dependent variable is DYXZ			
39 observations used for estimation from 2001Q2 to 2010Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	5.0000	0.00	*NONE*
YXZ(-1)	-.25000	0.00	*NONE*
DX	10.0000	0.00	*NONE*
X(-1)	4.0000	0.00	*NONE*
*****			

Estimated model:

$$(46.b') \quad \Delta \tilde{y}_t = 5 - 0,25 \tilde{y}_{t-1} + 10 \cdot \Delta x_t + 4 \cdot x_{t-1}$$

**2.7 RESULTS OF ESTIMATION OF THE DYNAMIC MODEL TYPE ADL(1,1,1)  
BASED ON SIMULATED SWITCHING TREND  
IN THE CASE OF THE STOCHASTIC CHANGES OF THE ENDOGENOUS VARIABLE**

Form of the switching stochastic trend (7):

$$(47) \quad y(t, x_t, x_{t-1}, u_t) : \quad y_t = \tilde{y}(t, x_t, x_{t-1}) + u_t,$$

where  $u_t$  is the normally distributed error term (see: section 1.6.)

Estimation of the dynamic model of the form [see: (29)]

$$(48) \quad y(y_{t-1}, x_t, x_{t-1}, \varepsilon_t) : \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t, \quad [\varepsilon_t = f(u_t, u_{t-1}, \tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_t^e)]$$

where in the MICROFIT program report:  $y_t = \text{YXZU}$ ,  $x_t = \text{X}$ ,  $x_{t-1} = \text{X}(-1)$

**Results of estimation:**

```
Ordinary Least Squares Estimation
*****
Dependent variable is YXZU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.4000         1.4005            3.1417[.003]
YXZU(-1)           .74639        .016606           44.9476[.000]
X                   9.8261         .20762            47.3283[.000]
X(-1)              -5.7395        .33830            -16.9657[.000]
*****
R-Squared           .99953        R-Bar-Squared       .99948
S.E. of Regression 2.4653        F-stat.   F( 3, 35)  24564.0[.000]
Mean of Dependent Variable 317.9449    S.D. of Dependent Variable 108.5894
Residual Sum of Squares 212.7160    Equation Log-likelihood     -88.4183
Akaike Info. Criterion -92.4183    Schwarz Bayesian Criterion -95.7455
DW-statistic         2.2110        Durbin's h-statistic     -.66253[.508]
*****
* Test Statistics *      LM Version      *      F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 8.8650[.065]*F( 4, 31)= 2.2799[.083]*
* B:Functional Form   *CHSQ( 1)= .57984[.446]*F( 1, 34)= .51313[.479]*
* C:Normality         *CHSQ( 2)= 1.5477[.461]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .37197[.542]*F( 1, 37)= .35630[.554]*
*****
```

**Estimated model using the OLS:**

$$(49.a) \quad \hat{y}_t = 4,4 + 0,746 \cdot y_{t-1} + 9,826 \cdot x_t - 5,74 \cdot x_{t-1},$$

which gives:

$$(49.b) \quad \hat{b}_0 = \frac{\hat{\beta}_0}{1 - \hat{\alpha}} = \frac{4,4}{0,254} = 17,35 < b_0 = 20 \quad \hat{b}_1 = \frac{\hat{\beta}_0 + \hat{\beta}_1}{1 - \hat{\alpha}} = \frac{9,826 - 5,74}{0,254} = 16,11 > b_1 = 16$$

Estimation of the dynamic model of the form [see:(32)]:

$$(50) \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 \cdot \Delta x_t + \beta' \cdot x_{t-1} + \varepsilon_t$$

where:  $\beta' = (\beta_1 + \beta_2)$

where in the MICROFIT program report:  $y_t = \text{YXZU}$ ,  $y_{t-1} = \text{YXZU}(-1)$ ,  $\Delta x_t = \text{DX}$ ,  $x_{t-1} = \text{X}(-1)$

### Estimation results:

```

Ordinary Least Squares Estimation
*****
Dependent variable is YXZU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.4000         1.4005            3.1417[.003]
YXZU(-1)           .74639          .016606          44.9476[.000]
DX                  9.8261          .20762            47.3283[.000]
X(-1)               4.0866          .29834            13.6977[.000]
*****
R-Squared           .99953          R-Bar-Squared       .99948
S.E. of Regression  2.4653          F-stat.   F( 3, 35) 24564.01[.000]
Mean of Dependent Variable 317.9449          S.D. of Dependent Variable 108.5894
Residual Sum of Squares 212.7160          Equation Log-likelihood     -88.4183
Akaike Info. Criterion -92.4183          Schwarz Bayesian Criterion -95.7455
DW-statistic         2.2110          Durbin's h-statistic    -.66253[.508]
*****
* Test Statistics *      LM Version      *      F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 8.8650[.065]*F( 4, 31)= 2.2799[.083]*
* B:Functional Form    *CHSQ( 1)= .57984[.446]*F( 1, 34)= .51313[.479]*
* C:Normality          *CHSQ( 2)= 1.5477[.461]*      Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .37197[.542]*F( 1, 37)= .35630[.554]*
*****

```

### Estimation of the dynamic model of the form [see:(33)]:

$$(51) \quad \Delta y_t = \beta_0 + \alpha' \cdot y_{t-1} + \beta_1 \cdot \Delta x_t + \beta' \cdot x_{t-1} + \varepsilon_t,$$

where:  $\alpha' = 1 - \alpha$ ,  $\beta' = (\beta_1 + \beta_2)$

and in the MICROFIT program report:  $y_t = \text{YXZU}$ ,  $\Delta y_t = \text{DYXZU}$ ,  $\Delta x_t = \text{DX}$ ,  $x_{t-1} = \text{X}(-1)$

```

Ordinary Least Squares Estimation
*****
Dependent variable is DYXZU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.4000         1.4005            3.1417[.003]
YXZU(-1)           -.25361          .016606          -15.2723[.000]
DX                  9.8261          .20762            47.3283[.000]
X(-1)               4.0866          .29834            13.6977[.000]
*****
R-Squared           .98706          R-Bar-Squared       .98595
S.E. of Regression  2.4653          F-stat.   F( 3, 35) 889.8744[.000]
Mean of Dependent Variable 7.0921          S.D. of Dependent Variable 20.7983
Residual Sum of Squares 212.7160          Equation Log-likelihood     -88.4183
Akaike Info. Criterion -92.4183          Schwarz Bayesian Criterion -95.7455
DW-statistic         2.2110
*****
* Test Statistics *      LM Version      *      F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 8.8650[.065]*F( 4, 31)= 2.2799[.083]*
* B:Functional Form    *CHSQ( 1)= 1.5539[.213]*F( 1, 34)= 1.4109[.243]*
* C:Normality          *CHSQ( 2)= 1.5477[.461]*      Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .20035[.654]*F( 1, 37)= .19106[.665]*
*****
```

The case of estimated model type ADL(1,1,1) based on the simulated information in the Table 1.  
*An example of an incorrect application of the ECM procedure*

Estimation of the dynamic model of the form [see:(33)]:

$$(52) \quad \Delta y_t = \beta_0 + \alpha' \cdot y_{t-1} + \beta_1 \cdot \Delta x_t + \beta' \cdot x_{t-1} + \varepsilon_t, \quad \text{gdzie: } \alpha' = 1 - \alpha, \beta' = (\beta_1 + \beta_2)$$

and in the MICROFIT program report:  $y_t = YXU$ ,  $\Delta y_t = DYXU$ ,  $\Delta x_t = DX$ ,  $x_{t-1} = X(-1)$

```
Ordinary Least Squares Estimation
*****
Dependent variable is DYXU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.3467          1.3859          3.1363[.003]
YXU(-1)            -.25152         .0042280        -59.4898[.000]
DX                  9.8251          .20667          47.5388[.000]
X(-1)              10.0883         .19153          52.6718[.000]
*****
R-Squared           .99425          R-Bar-Squared       .99376
S.E. of Regression   2.4624          F-stat.   F( 3, 35)    2018.21[.000]
Mean of Dependent Variable 20.1241          S.D. of Dependent Variable 31.1715
Residual Sum of Squares 212.2171          Equation Log-likelihood   -88.3725
Akaike Info. Criterion   -92.3725         Schwarz Bayesian Criterion -95.6997
DW-statistic          2.2241
*****
* Test Statistics * LM Version      * F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 8.8594[.065]*F( 4, 31)= 2.2780[.083]*
* B:Functional Form *CHSQ( 1)= .55054[.458]*F( 1, 34)= .48683[.490]*
* C:Normality       *CHSQ( 2)= 1.5610[.458]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .20236[.653]*F( 1, 37)= .19298[.663]*
*****
```

**Conclusion:**

because the parameters occurring at variable  $\Delta x_t$  and the  $x_{t-1}$  to be regarded as statistically insignificantly different from one another , so as to condition (39) occurring at a variable parameter  $x_{t-1}$  in the model (29) should be regarded as statistically not significantly differing form zero. Error correction model is no longer valid. This should be confirmed in the estimation of the model version (29).

Estimation of the dynamic model of the form [see: (29)]

$$(53) \quad y(y_{t-1}, x_t, x_{t-1}, \varepsilon_t): \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t,$$

where in the MICROFIT program report:  $y_t = YXU$ ,  $x_t = X$ ,  $x_{t-1} = X(-1)$

```
Ordinary Least Squares Estimation
*****
Dependent variable is YXU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.3467          1.3859          3.1363[.003]
YXU(-1)            .74848         .0042280        177.0291[.000]
X                   9.8251          .20667          47.5388[.000]
X(-1)              .26319          .26414          .99642[.326]
*****
R-Squared           .99993          R-Bar-Squared       .99992
S.E. of Regression   2.4624          F-stat.   F( 3, 35)    157381.51[.000]
Mean of Dependent Variable 742.8488          S.D. of Dependent Variable 274.4845
Residual Sum of Squares 212.2171          Equation Log-likelihood   -88.3725
Akaike Info. Criterion   -92.3725         Schwarz Bayesian Criterion -95.6997
DW-statistic          2.2241          Durbin's h-statistic   -.69997[.484]
*****
* Test Statistics * LM Version      * F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 8.8594[.065]*F( 4, 31)= 2.2780[.083]*
* B:Functional Form *CHSQ( 1)= .31206[.576]*F( 1, 34)= .27425[.604]*
* C:Normality       *CHSQ( 2)= 1.5610[.458]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .14329[.705]*F( 1, 37)= .13645[.714]*
*****
```

## 2.8 SEASONALITY IN THE DYNAMIC MODELS TYPE ADL(1,1,1+3)

The deterministic trend of switching with quarterly seasonality:

$$(54) \quad \tilde{s}\tilde{y}_t(t, x_t, x_{t-1}, v_{ij}): \quad \tilde{s}\tilde{y}_t = \tilde{y}_t(t, x_t, x_{t-1}) + e_1 \cdot v_{t1} + e_2 \cdot v_{t2} + e_3 \cdot v_{t3}, \quad [e_4 = -(e_1 + e_2 + e_3)]$$

where:

$$v_{ij} = x_{ij} - x_{t4}, \quad j \neq 4$$

$$x_{ij} = \begin{cases} 1 & \text{in } j\text{'th quartile} \\ 0 & \text{in other quartiles} \end{cases}$$

We can say:

in every  $j$ -th quartile of the endogenous variable  $s\tilde{y}_t$  deviates from its trend of switching  $\tilde{y}(t, x_t)$  by  $e_j$  units.

Purified from the error term influence the model type ARDL(1,1) with quarterly seasonality:

$$(55) \quad \tilde{s}\tilde{y}(\tilde{y}_{t-1}, x_t, x_{t-1}, v_{ij}): \quad \tilde{s}\tilde{y}_t = \beta_0 + \alpha \cdot \tilde{s}\tilde{y}_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3}, \quad [\gamma_4 = -(\gamma_1 + \gamma_2 + \gamma_3)]$$

Equations that identify parameters  $e_j$  on the basis of parameters  $\gamma_j$ : [see:(13.1) i (13.2)]

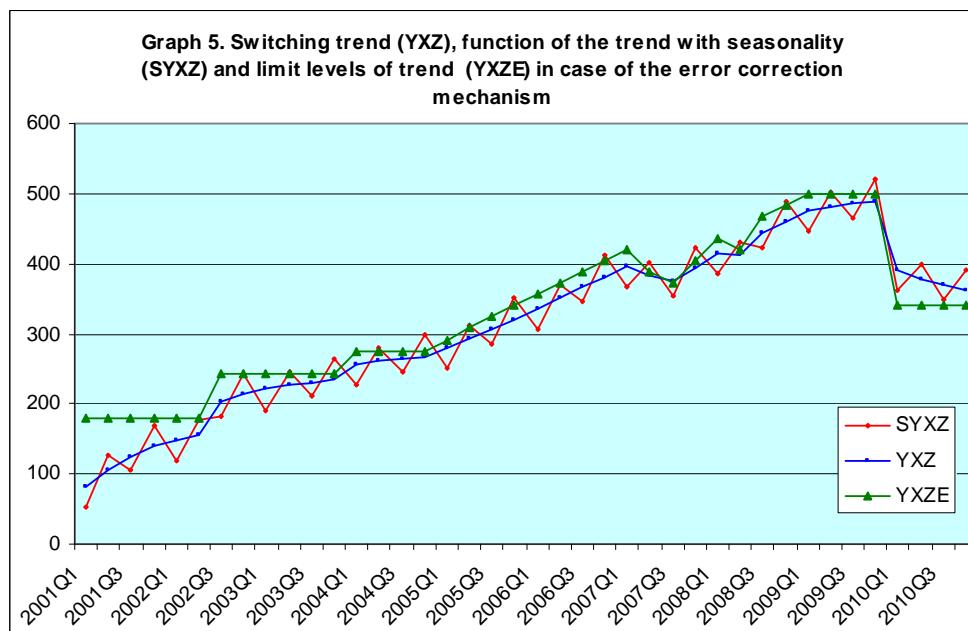
## 2.9 SIMULATION MODEL ASSUMPTIONS IN THE CASE OF DETERMINISTIC SEASONALITY FOR THE MODEL TYPE ARDL(1,1):

Assumed values of parameters for determining the seasonal effects:

$$e_1 = -30, \quad e_2 = 20, \quad e_3 = -20, \quad e_4 = 30$$

Assumed notation of the software system MICROFIT:

$$\tilde{s}\tilde{y}_t = \tilde{y}_t(t, x_t, x_{t-1}) - 30 \cdot v_{t1} + 20 \cdot v_{t2} - 20 \cdot v_{t3}, : \quad SYXZ = yxz - 30*v1 + 20*v2 - 20*v3$$



**Estimation results of the dynamic model based on simulated trend of switching deterministic trend in the conditions generated ECM mechanism**

**A. The deterministic dynamic model  
(validation of simulation)**

Results of estimation of the model (55):

Ordinary Least Squares Estimation			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	5.0000	0.00	*NONE*
SYXZ(-1)	.75000	0.00	*NONE*
X	10.0000	0.00	*NONE*
X(-1)	-6.0000	0.00	*NONE*
V1	-52.5000	0.00	*NONE*
V2	42.5000	0.00	*NONE*
V3	-35.0000	0.00	*NONE*
R-Squared	1.0000	R-Bar-Squared	1.0000
S.E. of Regression	.0000	F-stat.	F( 6, 32) *NONE*

Estimated model:

$$(56) \quad \hat{sy}_t = 5 + 0,75\tilde{sy}_{t-1} + 10x_t - 6x_{t-1} - 52,5v_{t1} + 42,5v_{t2} - 35v_{t3}$$

It means that:

$$\gamma_1 = -52,5, \quad \gamma_2 = 42,5, \quad \gamma_3 = -35, \quad \Rightarrow \quad \gamma_4 = 45$$

Identification of the parameters  $e_j$  on the basis of parameters  $\gamma_j$  correspond to the identification presented in the case of equations (14.a) i (14.b)

**Estimation results of the dynamic model based on simulated trend of switching deterministic trend in the conditions generated ECM mechanism**

**B. Stochastic form of the dynamic model**

$$(57) \quad sy(y_{t-1}, x_t, x_{t-1}, v_{tj}, \varepsilon_t) : \quad sy_t = \beta_0 + \alpha \cdot sy_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

where  $u_t$  is the normally distributed generated error term (see: 1.6).

Estimation of the dynamic model in from (57):

where in the form of a software system MICROFIT:  $sy_t=SYXZU$ ,  $x_t=X$ ,  $x_{t-1}=X(-1)$ ,  $v_{tj}=V_j$

Ordinary Least Squares Estimation			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	4.4243	1.4411	3.0701[.004]
SYXZU(-1)	.74806	.017121	43.6935[.000]
X	9.8716	.21746	45.3946[.000]
X(-1)	-5.8128	.35463	-16.3914[.000]
V1	-51.8317	.86692	-59.7886[.000]
V2	42.7390	.89085	47.9754[.000]
V3	-35.3289	.81240	-43.4872[.000]
R-Squared	.99956	R-Bar-Squared	.99948
S.E. of Regression	2.5287	F-stat.	F( 6, 32) 12226.11[.000]
Mean of Dependent Variable	318.7142	S.D. of Dependent Variable	111.1262
Residual Sum of Squares	204.6151	Equation Log-likelihood	-87.6612
Akaike Info. Criterion	-94.6612	Schwarz Bayesian Criterion	-100.4837
DW-statistic	2.2301	Durbin's h-statistic	-.72254[.470]

```
*****
* Test Statistics * LM Version * F Version *
*****
* A:Serial Correlation*CHSQ( 4)= 9.5719 [.048]*F( 4, 28)= 2.2768 [.086]*
* B:Functional Form *CHSQ( 1)= 1.3221 [.250]*F( 1, 31)= 1.0878 [.305]*
* C:Normality *CHSQ( 2)= 1.4797 [.477]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= 1.8857 [.170]*F( 1, 37)= 1.8799 [.179]*
*****
```

### The form of estimated model:

$$(58) \quad \hat{sy}_t = 4,42 + 0,748 \cdot sy_{t-1} + 9,872 x_t - 5,813 x_{t-1} - 51,83 v_{t1} + 42,74 v_{t2} - 35,33 v_{t3}$$

that gives:

$$(58.a) \quad \hat{b}_0 = \frac{\hat{\beta}_0}{1-\hat{\alpha}} = \frac{4,42}{0,252} = 17,54 < b_0 = 20 \quad \hat{b}_1 = \frac{\hat{\beta}_0 + \hat{\beta}_1}{1-\hat{\alpha}} = \frac{9,872 - 5,813}{0,252} = 16,11 > b_1 = 16$$

And

$$\hat{\gamma}_1 = -51,83 \quad \hat{\gamma}_2 = 42,74 \quad \hat{\gamma}_3 = -35,33 \Rightarrow \gamma_4 = 44,42$$

Identification of parameters estimates  $e_j$  on the basis of parameters  $\gamma_j$ :

$$\begin{aligned} \hat{e}_4 &= (\alpha^3 \cdot \hat{\gamma}_1 + \alpha^2 \cdot \hat{\gamma}_2 + \alpha \cdot \hat{\gamma}_3 + \hat{\gamma}_4) / (1 - \hat{\alpha}^4) = \\ &= (-0,748^3 \cdot 51,83 + 0,748^2 \cdot 42,74 - 0,748 \cdot 35,33 + 44,42) / (1 - 0,748^4) =, \\ &= / 686955 = 29,43 \quad [e_4 = 30] \end{aligned}$$

so from (13.1) we get:

$$\begin{aligned} \hat{e}_1 &= \hat{\alpha} \cdot \hat{e}_4 + \hat{\gamma}_1 = 0,748 \cdot 29,43 - 51,83 = -29,82 \quad [e_1 = -30] \\ \hat{e}_2 &= \hat{\alpha} \cdot \hat{e}_1 + \hat{\gamma}_2 = -0,748 \cdot 29,82 + 42,74 = 20,43 \quad [e_2 = 20] \\ \hat{e}_3 &= \hat{\alpha} \cdot \hat{e}_2 + \hat{\gamma}_3 = 0,748 \cdot 20,43 - 35,33 = -20,04 \quad [e_3 = -20] \\ \hat{e}_4 &= \hat{\alpha} \cdot \hat{e}_3 + \hat{\gamma}_4 = -0,748 \cdot 20,04 + 44,42 = 29,43 \quad [e_4 = 30] \end{aligned}$$

Estimation of the dynamic model with deterministic seasonality [see:(32)]:

$$(59) \quad sy_t = \beta_0 + \alpha \cdot sy_{t-1} + \beta_1 \cdot \Delta x_t + \beta' \cdot x_{t-1} + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

where:  $\beta' = (\beta_1 + \beta_2)$

and in the form of a software system MICROFIT:  $sy_t = SYXZU$ ,  $sy_{t-1} = SYXZU(-1)$ ,  $\Delta x_t = DX$ ,  $x_{t-1} = X(-1)$

### Results of estimation:

```
Ordinary Least Squares Estimation
*****
Dependent variable is SYXZU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.4243           1.4411            3.0701 [.004]
SYXZU(-1)          .74806           .017121           43.6935 [.000]
DX                  9.8716           .21746             45.3946 [.000]
X(-1)              4.0587           .30748             13.2002 [.000]
V1                 -51.8317          .86692            -59.7886 [.000]
V2                 42.7390           .89085             47.9754 [.000]
V3                 -35.3289          .81240            -43.4872 [.000]
*****
R-Squared           .99956          R-Bar-Squared       .99948
S.E. of Regression   2.5287          F-stat.    F( 6, 32) 12226.11 [.000]
Mean of Dependent Variable 318.7142          S.D. of Dependent Variable 111.1262
Residual Sum of Squares 204.6151          Equation Log-likelihood     -87.6612
Akaike Info. Criterion   -94.6612          Schwarz Bayesian Criterion -100.4837
DW-statistic          2.2301          Durbin's h-statistic     -.72254 [.470]
*****
```

Estimation of the dynamic model with deterministic seasonality [see:(33)]:

$$(60) \quad \Delta sy_t = \beta_0 + \alpha' \cdot sy_{t-1} + \beta_1 \cdot \Delta x_t + \beta' \cdot x_{t-1} + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

where:  $\alpha' = 1 - \alpha$ ,  $\beta' = (\beta_1 + \beta_2)$

and in the MICROFIT program report:  $sy_t = SYXZU$ ,  $\Delta sy_t = DSYXZU$ ,  $\Delta x_t = DX$ ,  $x_{t-1} = X(-1)$

Results of estimation:

```
Ordinary Least Squares Estimation
*****
Dependent variable is DSYXZU
39 observations used for estimation from 2001Q2 to 2010Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.4243         1.4411           3.0701[.004]
SYXZU(-1)          -.25194        .017121          -14.7156[.000]
DX                  9.8716          .21746            45.3946[.000]
X(-1)               4.0587          .30748            13.2002[.000]
V1                 -51.8317         .86692           -59.7886[.000]
V2                  42.7390          .89085            47.9754[.000]
V3                 -35.3289         .81240           -43.4872[.000]
*****
R-Squared           .99821        R-Bar-Squared     .99788
S.E. of Regression   2.5287       F-stat. F( 6, 32) 2975.11[.000]
Mean of Dependent Variable 8.6305       S.D. of Dependent Variable 54.8555
Residual Sum of Squares 204.6151      Equation Log-likelihood -87.6612
Akaike Info. Criterion -94.6612      Schwarz Bayesian Criterion -100.4837
DW-statistic         2.2301
*****
```

Conclusion:

In any version that are equivalent models (57), (59) and (60), the assessment of the parameters defining the seasonal effects are exactly equal and evaluation of directional parameters are exactly equal to the transformed functions of the output parameters of the model (57).

## 2.9 CASUAL DYNAMIC MODEL TYPE ADL(1,1,2): (the case of two exogenous variables)

The expected (desired) level of endogenous variable:

$$(61) \quad \tilde{y}^e(x_t): \quad \tilde{y}_t^e = b_0 + b_1 x_t + b_2 z_t$$

Effect of exogenous variables on the expected level of endogenous variable in period t:  
(conditional long-term effects)

$$(62.1) \quad Ef_t^d(x): \quad \Delta \tilde{y}_{(x)t}^e = b_1 \Delta x_t, \quad \Delta z_t = 0$$

$$(62.2) \quad Ef_t^d(z): \quad \Delta \tilde{y}_{(z)t}^e = b_2 \Delta z_t, \quad \Delta x_t = 0$$

Error correction model for the two exogenous variables:

$$(63) \quad \underbrace{\tilde{y}_t - \tilde{y}_{t-1}}_{\Delta \tilde{y}_t: \text{short term gross increase of } \tilde{y} \text{ in period } t} = \underbrace{\lambda_1 [\tilde{y}_t^e(x) - \tilde{y}_{t-1}^e]}_{\text{conditional change in the desired values}} + \underbrace{\lambda_2 [\tilde{y}_t^e(z) - \tilde{y}_{t-1}^e]}_{\text{past periods disequilibrium}} + \alpha' (\tilde{y}_{t-1}^e - \tilde{y}_{t-1}), \quad \text{where:}$$

a) conditional target level of the variable  $\tilde{y}$  in time  $t$  caused by the increase of  $x$ ,

$$(64.a) \quad \tilde{y}_t^e(x) = b_0 + b_1 x_t + b_2 (z_t = z_{t-1}), \quad (\Delta z_t = 0)$$

b) conditional target level of the variable  $\tilde{y}$  in time  $t$  caused by the increase of  $z$ ,

$$(64.b) \quad \tilde{y}_t^e(z) = b_0 + b_1 (x_t = x_{t-1}) + b_2 z_t, \quad (\Delta x_t = 0)$$

Introducing (64.a) and (64.b) into the model, with functions (61) to (63) we have:

$$(65) \quad \begin{aligned} \Delta\tilde{y}_t = & \lambda_1 \cdot [(b_0 + b_1x_t + b_2z_{t-1}) - (b_0 + b_1x_{t-1} + b_2z_{t-1})] + \\ & + \lambda_2 \cdot [(b_0 + b_1x_{t-1} + b_2z_t) - (b_0 + b_1x_{t-1} + b_2z_{t-1})] + \\ & + \alpha'(b_0 + b_1x_{t-1} + b_2z_{t-1} - \tilde{y}_{t-1}) \end{aligned}$$

which after conversion, we write the following:

$$(66) \quad \Delta\tilde{y}_t = \alpha'b_0 - \alpha'\tilde{y}_{t-1} + \lambda_1b_1 \cdot \Delta x_t + \alpha'b_1 \cdot x_{t-1} + \lambda_2b_2 \cdot \Delta z_t + \alpha'b_2 \cdot z_{t-1},$$

or:

$$(67) \quad \tilde{y}_t = \alpha'b_0 + (1 - \alpha') \cdot \tilde{y}_{t-1} + \lambda_1b_1 \cdot \Delta x_t + \alpha'b_1 \cdot x_{t-1} + \lambda_2b_2 \cdot \Delta z_t + \alpha'b_2 \cdot z_{t-1}$$

Converting (67) we get:

$$(68) \quad \tilde{y}_t = \alpha'b_0 + (1 - \alpha') \cdot \tilde{y}_{t-1} + \lambda_1b_1 \cdot x_t + (\alpha' - \lambda_1)b_1 \cdot x_{t-1} + \lambda_2b_2 \cdot z_t + (\alpha' - \lambda_2)b_2 \cdot z_{t-1}$$

That can be written as ARDL(1,1,2) type mode:

$$(69) \quad \tilde{y}(y_{t-1}, x_t, x_{t-1}, z_t, z_{t-1}): \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_{x0} \cdot x_t + \beta_{x1} \cdot x_{t-1} + \beta_{z0} \cdot z_t + \beta_{z1} \cdot z_{t-1},$$

where:

$$(70.1) \quad \alpha = 1 - \alpha' \Rightarrow \alpha' = 1 - \alpha$$

$$(70.2) \quad \beta_0 = \alpha' \cdot b_0 \Rightarrow b_0 = \frac{\beta_0}{1 - \alpha}$$

$$(70.3) \quad \beta_{x0} = \lambda_1 \cdot b_1$$

$$(70.4) \quad \beta_{x1} = (\alpha' - \lambda_1) \cdot b_1$$

$$(70.5) \quad \beta_{z0} = \lambda_2 \cdot b_2$$

$$(70.6) \quad \beta_{z1} = (\alpha' - \lambda_2) \cdot b_2$$

Model (69) can be converted to a form which is equivalent to the original model (66):

$$(71) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_{x0} \cdot \Delta x_t + (\beta_{x0} + \beta_{x1}) \cdot x_{t-1} + \beta_{z0} \cdot \Delta z_t + (\beta_{z0} + \beta_{z1}) \cdot z_{t-1}$$

Note that [what is clear both from the comparison of model parameters (66) and (71) as well as from the system of equations (70.1) - (70.6)]:

$$(72.1) \quad \beta_{x0} + \beta_{x1} = \alpha' b_1 \Rightarrow b_1 = \frac{\beta_{x0} + \beta_{x1}}{1 - \alpha}$$

$$(72.2) \quad \beta_{z0} + \beta_{z1} = \alpha' b_2 \Rightarrow b_2 = \frac{\beta_{z0} + \beta_{z1}}{1 - \alpha}$$

To identify other parameters confront error correction model (70.3) with (72.1) and (70.5) with (72.2). The result is:

$$(73.1) \quad \lambda_1 = \frac{\beta_{x0}}{\beta_{x0} + \beta_{x1}} (1 - \alpha)$$

$$(73.2) \quad \lambda_2 = \frac{\beta_{z0}}{\beta_{z0} + \beta_{z1}} (1 - \alpha)$$

From above:

$$(74.1) \quad \beta_{x0} \cong \beta_{x0} + \beta_{x1} \Rightarrow \beta_{x1} \cong 0 \Rightarrow \lambda_1 \cong \alpha'$$

$$(74.2) \quad \beta_{z0} \cong \beta_{z0} + \beta_{z1} \Rightarrow \beta_{z1} \cong 0 \Rightarrow \lambda_2 \cong \alpha'$$

the model (69) takes a form:

$$(75) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_{x0} \cdot x_t + \beta_{z0} \cdot z_t$$

On the other hand, in the case where

$$(76) \quad \beta_{x1} \neq 0 \wedge \beta_{z1} \cong 0 \Rightarrow \lambda_1 \neq \alpha' \wedge \lambda_2 \cong \alpha'$$

the dynamic model (69) takes the form of a mixed type of ECM-PAM:

$$(77) \quad \tilde{y}_t = \beta_0 + \alpha \cdot \tilde{y}_{t-1} + \beta_{x0} \cdot x_t + \beta_{x1} \cdot x_{t-1} + \beta_{z0} \cdot z_t$$

For the purpose of the proper specification, the final form of the model should has the following form:

$$(78) \quad y_t = \beta_0 + \alpha \cdot y_{t-1} + \beta_{x,s} \cdot x_{t-s} + \beta_{x,1+s} \cdot x_{t-(1+s)} + \beta_{z,s} \cdot z_{t-s} + \beta_{z,1+s} \cdot z_{t-(1+s)} + \varepsilon_t,$$

#### The algorithm for the identification of the proceedings of the final form of the model under the ECM procedure

At the starting point, we recognise that in both cases  $s=0$ . After estimation, we examine the significance of the parameters. In the case of insignificance of the variable parameter with less delay, delaying the process must continue until such a level at which:

- parameters of the variables at a lower and longer delay will be significantly different from zero,
  - parameter of the variable with lower latency will be statistically different from zero, and the parameter of the variable with a longer delay will be considered to be statistically insignificant.
- In the first case we have to deal with the type of ECM model, and the second type of model PAM. For more variables exogenous model may be a mixed type of ECM-PAM.

## 2.10. CAUSAL DYNAMIC MODEL OF THE LEVEL OF WAGE<sup>4</sup>

### 2.10.1. Estimation of the wage model with constant elasticity of factors

$$(80) \quad \ln WN_t = \beta_0 + \alpha \ln WN_{t-1} + \beta_{P0} \ln PI_t + \beta_{P1} \ln PI_{t-1} + \beta_{A0} \ln iAP_t + \beta_{A1} \ln iAP_{t-1} + \beta_{U1} UR_{t-1} + \beta_{U2} UR_{t-2} + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t \quad (v_{ij} = x_{ij} - x_{i4}; \quad j \neq 4)$$

where:

$t=1 \equiv 1995Q1$ ,  $WN$  – nominal wage in Poland,  $PI$  – consumer price index,  
 $iAP$  – real wage productivity index ( $iAP_1=1,00$ ),  $UR$  – unemployment rate (fraction, end of period).,

**Estimated model in the form of a software system MICROFIT:**

$$(81) \quad LWN = \beta_0 \cdot X0 + \alpha \cdot LWN(-1) + \beta_{P0} LPI + \beta_{P1} LPI(-1) + \beta_{A0} LIAP + \beta_{A1} LIAP(-1) + \beta_{U1} UR(-1) + \beta_{U2} UR(-2) + \gamma_1 V1 + \gamma_2 V2 + \gamma_3 V3 + \varepsilon,$$

In the first step, it is considered that due to the fact that the unemployment rate is measured as at the end of the year, so the initial version of the model takes into account the state at the end of last UR (-1), and coupled with her unemployment delayed by two periods of UR (-2).

---

<sup>4</sup> The section presents the results of estimates of cause-effect model, the estimated performance and discussed in the book [Ossowski, 2013]. In the book a quarterly data from 1995Q1 to 2010Q4 were used. It was found that the stabilizing period of wage model was the interval between 1996Q1 and 2010Q4. The models estimations made here for the updated period covering the time period from 1996Q1 to 2013Q4.

## Computer printouts from MICROFIT of the estimated model (80)

### Estimates of future versions of a model for the period from Q1 1997 to Q4 2010

#### step A 1

##### Ordinary Least Squares Estimation

*****			
Dependent variable is LWN			
72 observations used for estimation from 1996Q1 to 2013Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	4.0394	.66436	6.0801[.000]
LWN(-1)	.41783	.098360	4.2480[.000]
LPI	-.12894	.21280	-.60592[.547]
LPI(-1)	.67905	.18804	3.6111[.001]
LIAP	.41398	.19029	2.1755[.033]
LIAP(-1)	-.042283	.17358	-.24359[.808]
UR(-1)	.22790	.35206	.64732[.520]
UR(-2)	-.80406	.36649	-2.1940[.032]
V1	.034854	.023261	1.4984[.139]
V2	-.025005	.0092253	-2.7104[.009]
V3	-.023032	.0097388	-2.3649[.021]
*****			
R-Squared	.99890	R-Bar-Squared	.99872
S.E. of Regression	.012313	F-stat.	F( 10, 61) 5548.6[.000]
Mean of Dependent Variable	7.7507	S.D. of Dependent Variable	.34439
Residual Sum of Squares	.0092478	Equation Log-likelihood	220.3978
Akaike Info. Criterion	209.3978	Schwarz Bayesian Criterion	196.8761
DW-statistic	1.9752	Durbin's h-statistic	.19095[.849]
*****			

In the next step must change model consisting of:

- removal of LPI, leaving LPI (-1), and the addition of LPI (-2)
- leaving the LAPL LAPL and the removal of (-1)
- removal of the UR (-1), leaving UR (-2) and the addition of UR (-3)

#### step A 2

##### Ordinary Least Squares Estimation

*****			
Dependent variable is LWN			
72 observations used for estimation from 1996Q1 to 2013Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	3.9938	.70027	5.7032[.000]
LWN(-1)	.42128	.10202	4.1294[.000]
LPI(-1)	.77948	.19156	4.0691[.000]
LPI(-2)	-.20287	.18604	-1.0905[.280]
LIAP	.35239	.076557	4.6030[.000]
UR(-2)	-.29971	.31060	-.96494[.338]
UR(-3)	-.24894	.33764	-.73731[.464]
V1	.029151	.0084252	3.4599[.001]
V2	-.023981	.0035592	-6.7377[.000]
V3	-.024536	.0058256	-4.2117[.000]
*****			
R-Squared	.99893	R-Bar-Squared	.99877
S.E. of Regression	.012079	F-stat.	F( 9, 62) 6405.8[.000]
Mean of Dependent Variable	7.7507	S.D. of Dependent Variable	.34439
Residual Sum of Squares	.0090464	Equation Log-likelihood	221.1903
Akaike Info. Criterion	211.1903	Schwarz Bayesian Criterion	199.8070
DW-statistic	2.0079	Durbin's h-statistic	-.067203[.946]
*****			

In the next step must change model consisting of:

- leaving the LPI (-1) and the removal of LPI (-2)
- leaving UR (-3) and the introduction of UR (-4)

### step A 3

```

Ordinary Least Squares Estimation
*****
Dependent variable is LWN
72 observations used for estimation from 1996Q1 to 2013Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                 4.0148          .58530           6.8594[.000]
LWN(-1)            .41991          .086116          4.8761[.000]
LPI(-1)            .57164          .095507          5.9853[.000]
LIAP               .34996          .057403          6.0966[.000]
UR(-2)             -.56998          .080385          -7.0906[.000]
V1                 .029118          .0064591          4.5081[.000]
V2                 -.022239          .0033352          -6.6679[.000]
V3                 -.021888          .0041696          -5.2493[.000]
*****
R-Squared          .99889          R-Bar-Squared       .99877
S.E. of Regression .012102          F-stat.   F( 7, 64)    8205.51[.000]
Mean of Dependent Variable 7.7507          S.D. of Dependent Variable .34439
Residual Sum of Squares .0093726         Equation Log-likelihood     219.9152
Akaike Info. Criterion 211.9152          Schwarz Bayesian Criterion 202.8085
DW-statistic        2.0330          Durbin's h-statistic     -.20522[.837]
*****
* Test Statistics * LM Version      * F Version      *
*****
* A:Serial Correlation*CHSQ( 4)= 9.5081[.050]*F( 4, 60)= 2.2823[.071]*
* B:Functional Form *CHSQ( 1)= .3551E-3[.985]*F( 1, 63)= .3107E-3[.986]*
* C:Normality      *CHSQ( 2)= 68.9080[.000]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .57441[.449]*F( 1, 70)= .56294[.456]*
*****
```

Above estimated model can be initially recognized only partially satisfactory. Estimation results cast doubt on the two main reasons. Firstly, due to decomposition residues of the model indicating the hypothesis that the normal distribution of error terms to be rejected. Secondly, because of the problems of interpretation. On the basis of the current model we can say that of increase in real wages in terms of constancy of productivity, determines the increase in the unemployment rate. Unemployment rate would not have a material impact on the impact of performance on the real wage

#### 2.10.2. Estimation of the model with variable elasticity of wages due to the productivity

Output model of the form (80) can be corrected by variation of the parameter for labor productivity (IAP) in finding that the wage level of flexibility in terms of performance is a function of the unemployment rate<sup>5</sup>.

$$(82) \quad \ln WN_t = \beta_0 + \alpha \ln WN_{t-1} + \beta_1 \ln PI_{t-1} + \beta_2 \ln iAP_t + \beta_3 (UR_{t-2} \cdot \ln iAP_t) + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

Note that on the basis of the model (82), short-term and long-term wage elasticity in terms of productivity, we define as follows:

$$(83.1) \quad E_{WN(APL/UR)}^k = \beta_2 + \beta_3 UR_{t-2} > 0 \quad (\beta_2 > 0, \beta_3 < 0)$$

$$(83.2) \quad E_{WN(APL/UR)}^d = \frac{\beta_2}{1-\alpha} + \frac{\beta_3}{1-\alpha} UR_{t-2} > 0 \quad (\beta_2 > 0, \beta_3 < 0)$$

With this solution, we recognize that the increase in wages resulting from the increased efficiency is corrected the situation on the labor market. The higher the unemployment rate, the lower the wage flexibility in terms of performance and, conversely, the lower the unemployment rate, the greater the increase in wages resulting from an increase in productivity. This also means that the relative change in the level of wages depends on the unemployment rate, and not only from the increase in this rate.

---

<sup>5</sup> Conditions related to the correctness of the variation of the parameters in the case of multiplicative models presented in the book [Ossowski, 2013, pp. 291-295].

### Estimated model in the form of a software system MICROFIT:

$$(84) \quad LWN = \beta_0 \cdot X0 + \alpha LWN(-1) + \beta_1 LPI(-1) + \beta_2 LIAP + \beta_3 LIAPUR2 + \gamma_1 V1 + \gamma_2 V2 + \gamma_3 V3 + \varepsilon,$$

where:  $LAPLUR2 = LAPL * UR(-2)$

#### Computer printouts from MICROFIT of the estimated model (84)

```

Ordinary Least Squares Estimation
*****
Dependent variable is LWN
72 observations used for estimation from 1996Q1 to 2013Q4
*****
Regressor          Coefficient      Standard Error      T-Ratio[Prob]
X0                  4.5811          .62667           7.3102[.000]
LWN(-1)             .32217          .093814          3.4341[.001]
LPI(-1)              .70828          .10732           6.5999[.000]
LIAP                 .57206          .078355          7.3008[.000]
LIAPUR2            -.014617         .0019656          -7.4365[.000]
V1                  .032191          .0065289          4.9306[.000]
V2                  -.020576         .0033627          -6.1187[.000]
V3                  -.023832          .0042000          -5.6742[.000]
*****
R-Squared           .99893          R-Bar-Squared     .99882
S.E. of Regression   .011844          F-stat.    F( 7, 64)    8566.81[.000]
Mean of Dependent Variable 7.7507          S.D. of Dependent Variable .34439
Residual Sum of Squares .0089778        Equation Log-likelihood    221.4646
Akaike Info. Criterion 213.4646        Schwarz Bayesian Criterion 204.3579
DW-statistic         2.0624          Durbin's h-statistic   -.437391[.662]
*****
* Test Statistics * LM Version * F Version *
*****
* A:Serial Correlation*CHSQ( 4)= 9.5069[.050]*F( 4, 60)= 2.2819[.071]*
* B:Functional Form *CHSQ( 1)= 3.9672[.046]*F( 1, 63)= 3.6738[.060]*
* C:Normality       *CHSQ( 2)= 5.6559[.059]* Not applicable *
* D:Heteroscedasticity*CHSQ( 1)= .0039855[.950]*F( 1, 70)= .0038750[.951]*
*****
```

Above the estimated model can be considered satisfactory, both in terms of economic and statistical. Noteworthy - shown below - short-term and long-term estimated wage elasticity with respect to the productivity, decreasing with the increase in the unemployment rate.

Unemployment rate (%)	UR%	0	5	7,5	10	12,5	15	20	25
Short term wage elasticity with respect to the productivity	$E^k_{WN(APL/UR)}$	0,572	0,499	0,463	0,426	0,390	0,353	0,280	0,207
Long term wage elasticity with respect to the productivity	$E^d_{WN(APL/UR)}$	0,844	0,734	0,679	0,624	0,569	0,514	0,404	0,294

The above results are slightly different from the results presented in the book [Ossowski, 2013, pp. 255-260], in which the above model was estimated for the period from Q1 1996 to Q4 2010. In that model, statistical considerations decided that it was appropriate delay one of the periodical with the unemployment rate UR.

#### 2.10.3. Incorrect specification of the model using the procedure of ECM

Consider the model of the form of the original:

$$(85.a) \quad \ln WN_t = \beta_0 + \alpha \ln WN_{t-1} + \beta_{11} \ln PI_{t-1} + \beta_{12} \ln PI_{t-2} + \\ + \beta_2 \ln APL_t + \beta_3 (UR_{t-1} \cdot \ln APL_t) + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

Above model, according to the ECM procedure, can be converted to the following equivalent form:

$$(85.b) \quad \Delta \ln WN_t = \beta_0 + (1-\alpha) \ln WN_{t-1} + \beta_{11} \Delta \ln PI_{t-1} + \beta_{11}^* \ln PI_{t-2} + \\ + \beta_2 \ln APL_t + \beta_3 (UR_{t-1} \cdot \ln APL_t) + \gamma_1 v_{t1} + \gamma_2 v_{t2} + \gamma_3 v_{t3} + \varepsilon_t$$

where:

$$(85.c) \quad \beta_{11}^* = \beta_{11} + \beta_{12}$$

Consider now the results of estimates of both the considered versions that are equivalent form of that same model. First, let us look at the model estimates (85.b).

### Results of the OLS estimation of the model (85.b):

*****			
Dependent variable is DLWN			
60 observations used for estimation from 1996Q1 to 2010Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	4.3802	.69995	6.2579[.000]
LWN(-1)	-.64936	.10418	-6.2334[.000]
DLPI(-1)	.79142	.22009	3.5958[.001]
LPI(-2)	.70453	.11751	5.9954[.000]
LAPL	.53698	.089906	5.9727[.000]
LAPLUR1	-.014917	.0025169	-5.9267[.000]
V1	.029671	.0076383	3.8845[.000]
V2	-.018734	.0043074	-4.3493[.000]
V3	-.029177	.0050742	-5.7500[.000]
*****			
R-Squared	.89942	R-Bar-Squared	.88365
S.E. of Regression	.012276	F-stat.	F( 8, 51) 57.0099[.000]
Mean of Dependent Variable	.019997	S.D. of Dependent Variable	.035990
Residual Sum of Squares	.0076863	Equation Log-likelihood	183.7434
Akaike Info. Criterion	174.7434	Schwarz Bayesian Criterion	165.3188
DW-statistic	2.0906		
*****			
* Test Statistics *	LM Version	* F Version *	
*****			
* A:Serial Correlation*CHSQ( 4)=	7.5660 [.109]*F( 4, 47)=	1.6955 [.167]*	
* B:Functional Form *CHSQ( 1)=	.0012400 [.972]*F( 1, 50)=	.0010333 [.974]*	
* C:Normality *CHSQ( 2)=	5.9191 [.052]*	Not applicable	*
* D:Heteroscedasticity*CHSQ( 1)=	.081593 [.775]*F( 1, 58)=	.078981 [.780]*	
*****			

Let us to compare results of estimation of the model (85.b) with results of the model (85.a).

### Results of the OLS estimation of the model (85.a):

*****			
Ordinary Least Squares Estimation			
*****			
Dependent variable is LWN			
60 observations used for estimation from 1996Q1 to 2010Q4			
*****			
Regressor	Coefficient	Standard Error	T-Ratio[Prob]
X0	4.3802	.69995	6.2579[.000]
LWN(-1)	.35064	.10418	3.3658[.001]
LEI(-1)	.79142	.22009	3.5958[.001]
LPI(-2)	-.086890	.22119	-.39283 [.696]
LAPL	.53698	.089906	5.9727[.000]
LAPLUR1	-.014917	.0025169	-5.9267[.000]
V1	.029671	.0076383	3.8845[.000]
V2	-.018734	.0043074	-4.3493[.000]
V3	-.029177	.0050742	-5.7500[.000]
*****			
R-Squared	.99865	R-Bar-Squared	.99844
S.E. of Regression	.012276	F-stat.	F( 8, 51) 4727.0[.000]
Mean of Dependent Variable	7.6641	S.D. of Dependent Variable	.31101
Residual Sum of Squares	.0076863	Equation Log-likelihood	183.7434
Akaike Info. Criterion	174.7434	Schwarz Bayesian Criterion	165.3188
DW-statistic	2.0906	Durbin's h-statistic	-.59429 [.552]
*****			
* Test Statistics *	LM Version	* F Version *	
*****			
* A:Serial Correlation*CHSQ( 4)=	7.5660 [.109]*F( 4, 47)=	1.6955 [.167]*	
* B:Functional Form *CHSQ( 1)=	.039124 [.843]*F( 1, 50)=	.032625 [.857]*	
* C:Normality *CHSQ( 2)=	5.9191 [.052]*	Not applicable	*
* D:Heteroscedasticity*CHSQ( 1)=	.60848 [.435]*F( 1, 58)=	.59422 [.444]*	
*****			

Note that estimating a model of the form (85.b) is necessary to verify the null hypothesis ( $H_0$ ) in which it is assumed that the parameters  $\beta_{11}$  and  $\beta_{11}^*$  are equal to the alternative ( $H_1$ ) declaring that these parameters are parameters that differ from each other, that is:

$$H_0 : \beta_{11} = \beta_{11}^* \quad (\text{gdzie: } \beta_{11}^* = \beta_{11} + \beta_{12})$$

$$H_1 : \beta_{11} \neq \beta_{11}^*$$

In practice, there is no need to verify the above hypothesis. Just to the estimation of the model form (85.a) - being the equivalent form of the model (85.b) - subject to verification the null hypothesis ( $H_0$ ), assuming zero value  $\beta_{12}$  for declaring the alternative hypothesis differs from the zero value of the same parameter, that is:

$$H_0 : \beta_{12} = 0$$

$$H_1 : \beta_{12} \neq 0$$

In the case that we do not have evidence to reject the null hypothesis, we have strong grounds to conclude that the parameters  $\beta_{11}$  and  $\beta_{11}^*$  are equal. This is understandable, since it:  $\beta_{11}^* = \beta_{11} + \beta_{12}$ . In this case, the error correction model is no longer valid. When we reject the null hypothesis in favor of the alternative hypothesis, we can conclude that in the form of the model (85.b) there is a condition, according to which:  $\beta_{11}^* \neq \beta_{11} + \beta_{12}$ , the same reasoning based on it is correct.

At the end, note that comparing the estimate models (85.a) and (85.b), we see that practically speaking they do not differ from each other. However, the estimates of the form (85.a) explicitly indicate the need to remove from the set of explanatory variables variable *LPI(-2)*, thereby recognising the incorrectness of the model (85.b). As a result, should be considered as a valid form of the estimated model (82).

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