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Medical Progress, Demand for Health Care, and Economic Performance

by
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Medical Progress, Demand for Health Care, and Economic Performance*

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Abstract

We study medical progress within an economy of overlapping generations subject to endogenous mortality. Individuals demand health care with a view to lowering mortality over their life-cycle. We characterise the individual optimum and the general equilibrium of the economy and study the impact of improvements in the effectiveness of health care. We find that general equilibrium effects dampen strongly the increase in health care usage following medical innovation. Moreover, an increase in savings offsets the negative impact on GDP per capita of a decline in the support ratio.

Keywords: life-cycle model, longevity, health care, medical innovation, overlapping generations, value of life.

JEL-Classification: D91, I11, I12, J11, J17, O31, O41.

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1 Introduction

The impact of demographic and medical change on the sustainability of health care systems and the resulting need for reform have been the subject of empirical analyses for considerable time.\(^1\) By now, a consensus has emerged that medical progress is driving both the increase in health care spending per capita or per unit GDP and the increase in longevity (e.g. Cutler 2004, Chandra and Skinner 2012, Chernew and Newhouse 2012).\(^2\) Recent analysis by Fonseca et al. (2013) shows that about 30 percent of health care spending growth in the US over the period 1965-2005 can be explained by medical progress, with improved health insurance coverage explaining 6 percent and income growth explaining 4 percent.\(^3\), \(^4\) At the same time, medical progress explains most of the increase in life expectancy over the period of observation, which in welfare terms more than offsets the greater spending. These findings echo, at aggregate level, earlier results by Cutler and Huckman (2003) and Cutler (2007) who find that the technological improvements in the treatment of heart disease over the 1980s and 1990s were generating benefits from increased survival, the value of which was more than compensating the boost to health care costs.\(^5\)

Although explaining the macro-economic implications of medical progress, the current line of inquiry remains to a large extent silent about the general equilibrium effects of this very medical progress. Indeed, there is strong evidence that medical innovations tend to boost the utilisation of health care (e.g. Baker et al. 2003; Cutler and Huckman 2003; Wong et al. 2012; Roham et al. 2014). Given that the main concern about the expanding health share in the economy lies with its absorption of resources that may be employed more productively in other sectors of the economy (Pauly and Saxena 2012, Kuhn and Prettner 2016) it is then surprising that the role of medical progress in this has not yet received more attention. An examination of this concern warrants a general equilibrium analysis that keeps track of the way in which the increase in the demand for

\(^1\)See e.g. Breyer and Felder (2006) and Breyer et al. (2015) for Germany; Dormont et al. (2006) for France; Meara et al. (2004) and Shang and Goldman (2007) for the US; Karlsson and Klohn (2014) for Sweden; Zweifel et al. (2005) for a set of OECD countries; and European Union (2015) for the then EU27. For an overview see Breyer et al. (2010).

\(^2\)Other important drivers include income (Hall and Jones 2007) and the presence of social security (Zhao 2014).

\(^3\)The analysis also reveals an important complementarity between medical progress and income, which explains 57 percent of the increase in spending.


\(^5\)Skinner et al. (2006) and Chandra and Skinner (2012) take a more nuanced view, showing that whether or not welfare gains arise from the adoption of new medical technologies depends both on the nature of technology as well as on the organisation of the health care system into which it is adopted.
health care that is induced by medical change leads to changes in the sectoral structure of the economy and of the way in which the induced price changes feed back again into the pattern of individual demand.

In this paper, we examine the impact of medical progress on individual life-cycle outcomes as well as on economic performance by analysing an OLG model, involving an endogenous demand for and supply of health care. The demand for health and health care is derived from utility maximisation within a life-cycle model with a realistic mortality pattern. Health care is provided within a medical sector, employing capital and labour, competing for resources with a final goods production sector. We characterise the optimal life-cycle allocation in terms of consumption and health care and show how it evolves with age, depending on the various prices and on the state of medical technology. As one important determinant of the demand for health care, we characterise the value individuals attach to their survival, which will prove to be an important link between macro-economic changes and their impact on the micro-decisions. Solving the profit maximisation problem of perfectly competitive providers within the final goods and health care sectors, we can characterise the optimal structure of supply and factor demand as well as the aggregate dynamics.

We then employ our model to analyse numerically the impact of medical progress on the provision of health care. Based on a steady-state benchmark scenario that is calibrated to represent the US economy in the year 2003, we illustrate the importance of the micro-macro feedback by studying the impact of a medical innovation which is either (i) unanticipated or (ii) anticipated.

In contrast to Hall and Jones (2007), Suen (2009), Fonseca et al. (2013), Zhao (2014) and Koijen et al. (2016), we do not focus so much on characterising the contribution of different factors to health care expenditure growth. We rather seek to identify and characterise in detail the mechanisms that govern the impact of medical innovations on the economy. In so doing, we adopt a quasi-experimental approach by which we study the impact of a "stylised" medical innovation on a steady state economy, tracing out the adjustment processes at micro- and macro-level that lead the economy into a new steady state. This distinguishes our model from the steady state comparisons in Suen (2009), Fonseca et al. (2013) and Zhao (2014) and balanced growth representations in Hall and Jones (2007) and Koijen et al. (2016). Here, the abstraction from interfering macroeconomic time trends allows a much clearer analytical and numerical identification of the impact of medical innovation.
Considering a medical innovation that improves the effectiveness of health care and raises life expectancy by a little more than 1 year, which is broadly consistent with the increase in life expectancy brought about by the US cardiac revolution during the 80s and 90s (Cutler 2007), our key findings include the following. Health expenditure per capita increases by some 12.2%, about 0.9 percentage point of which owing to an increase in the price for medical care, about 1.8 percentage points owing to the ageing of the population that is induced by the medical innovation, and the remaining 9.5 percentage points owing to an increase in individual demand. Although this is a substantive impact, we find that more than half of the increase in individual demand that would be obtained under a constant set of prices is absorbed by the general equilibrium increase in the price for health care. This suggests that estimations or projections of the impact of medical innovation on health care spending need to keep close track of possible general equilibrium repercussions in order to avoid strong biases. With the health expenditure share in GDP increasing by some 1.6 percentage points, it may come as a surprise perhaps, that the level of GDP per capita itself remains unaffected. This is because the drop in the employment rate that comes with a disproportionate increase in survival amongst the retired population is neutralised by the accumulation of additional wealth as is induced by the increase in longevity and the prospect for individuals to purchase more effective health care in their old age.\(^6\) Indeed, if a medical innovation is fully anticipated, individuals increase their savings prior to the innovation, triggering a temporary economic boom.\(^7\) Finally, mortality reducing medical innovations tend to come with a reduction in the value of survival over large parts of the life-course. On the one hand, this reflects a reduction in consumption levels; on the other hand, it implies that the price of medical care per life-year gained has fallen, a result that is in line with empirical evidence (Cutler et al. 1998).

Our work ties in with two lines of literatures. First, a long-standing literature on the individual demand for health and health care over the life-course (e.g. Grossman 1972; Ehrlich and Chuma 1990; Ehrlich 2000; Hall and Jones 2007; Kuhn et al. 2011, 2015; Fonseca et al. 2013; Dalgaard and Strulik 2014). While these works are providing important insights into the determinants of the demand for health and health care at the individual level, they take a partial equilibrium stance

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\(^6\)This is consistent with empirical evidence provided by De Nardi et al. (2010).

\(^7\)While a number of recent studies have empirically examined the anticipation effects related to Medicare Part D reform at microeconomic level (Hu et al. 2014; Alpert 2016, Kaplan and Zhang 2017), we are unaware of a theoretical analysis of the macroeconomic impacts of anticipated medical innovation. See Mertens and Ravn (2011) for a theoretical treatment of anticipation effects in the context of tax reform.
by assuming an exogenous set of prices. As we will see, however, a neglect of general equilibrium
effects may lead to a rather exaggerated assessment of the boost to the demand for health care
following an innovation.

Second, our work adds to an emerging literature that considers the role of health care within
a general equilibrium context. Similar to our approach, Suen (2009) considers the impact of life-
saving health care, the productivity of which is raised by medical change. However, his model differs
in important respects: In contrast to our framework, Suen (2009) considers a single sector economy
with health care spending being deducted from consumption. Partly for this reason, he does not
model an endogenous price for health care but rather imposes an exogenous price trajectory. As
our analysis shows, however, sectoral reallocation plays an important role in explaining the impact
of medical progress on the economy and GDP, while the endogenous increase in the price of medical
care plays a key role in dampening the increase in the demand for health care following medical
innovation. Zhao (2014) analyses the impact of social security on health care spending, when
the latter enhances survival and finds by way of a numerical calibration for the US economy a
substantial positive impact. He does not, however, touch on the role of medical progress. Jung and
Tran (2016) model the general impact of the US 2010 health care reform but do not consider the
role of medical progress. Koijen et al. (2016) study the interaction between financial and real health
care markets and find that the premium associated with regulatory risk for e.g. pharmaceutical
companies lowers research and development (R&D) investments by more than a half and thereby
contains growth of health care expenditure by more than 3 percent. Kuhn and Prettner (2016)
examine the impact of exogenous variations to the size of the health care sector within an R&D-
driven growth economy, where health care enhances the survival and labour market participation
of overlapping generations of individuals. They conclude that while Euro area health care systems
impose a drag on economic growth, they are typically nevertheless favourable on welfare grounds.
Schneider and Winkler (2016) study an endogenous growth economy in which overlapping cohorts
of individuals invest in health care in order to lower mortality. Comparing the balanced growth
paths associated with different states of medical technology, they find that the technology leading
to a higher life expectancy imposes a drag on economic growth but leads to a welfare gain. Finally,
Kelly (2017) studies the response of a neoclassical economy with a medical sector to changes in
total factor productivity and in the productivity of health care.\footnote{In contrast to the other approaches, the health care sector modelled in Kelly (2017) is not employing domestic production factors. Changes to the provision of health care are therefore unrelated to factor prices and final goods production.} The present work differs by the more realistic modelling of the individual life-cycle from Koijen et al. (2016) and Kelly (2017) who consider an infinitely lived representative individual; from Jung and Tran (2016) who consider overlapping generations subject to exogenous mortality; as well as from Kuhn and Prettner (2016) and Schneider and Winkler (2016) who consider Blanchard-Yaari type models with endogenous but age-inspecific mortality and perfect annuitisation. The realistic demographic modelling is important in as far as the economic impact of medical progress hinges on its impact on the age distribution of the population.\footnote{In particular, those models that assume infinitely-lived agents or an exogenous profile of mortality are abstracting altogether from a saving response to health-induced changes in longevity. As e.g. Bloom et al. (2003) and De Nardi et al. (2010) show, however, such a response is empirically relevant.} Finally, Jones (2016) studies the interaction of conventional and life-saving R&D but does so within a social planner context.

The remainder of the paper is structured as follows: The following section is devoted to a presentation of the model; Sections 3 and 4 solve for and characterise the individual life-cycle allocation and the general equilibrium of the economy, respectively; Section 5 provides an analytical assessment of the impact of medical progress; Section 6 presents the numerical analysis before Section 7 wraps up. Some of the proofs have been relegated to an Appendix.

\section{The Model}

We consider an OLG model in which individuals choose consumption and health care over their life-course. Individuals are assumed to be representative within each cohort and are indexed by their age $a$ at time $t$, with $t_0 = t - a$ denoting the birth year of an individual aged $a$ at time $t$. At each age, the representative individual is subject to a mortality risk, where $S(a, t) = \exp \left[ - \int_{0}^{a} \mu(\hat{a}, h(\hat{a}, \hat{t}), M(\hat{t})) d\hat{a} \right]$ is the survival function at $(a, t)$, with $\mu(a, h(a, t), M(t))$ denoting the force of mortality. Following Kuhn et al. (2010, 2011, 2015) we assume that mortality can be lowered by the consumption of a quantity $h(a, t)$ of health care. In addition, we assume that mortality depends on the state of the medical technology $M(t)$ at time $t$. More specifically, we
assume that the mortality rate $\mu(a, h(a, t), M(t))$ satisfies

$$
\mu(a, h(a, t), M(t)) \in (0, \tilde{\mu}(a, t)] \quad \forall (a, t) ;
$$

$$
\mu_h(\cdot) < 0, \mu_{hh}(\cdot) > 0 ;
$$

$$
\mu_h(a, 0, M(t)) = -\infty, \mu_h(a, \infty, M(t)) = 0 \quad \forall (a, t) ;
$$

where $\tilde{\mu}(a, t) = \mu(a, 0, M(t))$ is the “natural” mortality rate for an individual aged $a$ at time $t$ when no health care is consumed. By purchasing health care, the representative individual can lower the instantaneous mortality rate, and can thereby improve survival prospects, but can only do so with diminishing returns.\footnote{Zweifel et al. (2005) provide empirical evidence of decreasing returns to health expenditure in the reduction of mortality. The decreasing returns assumption is also reflected in other empirical work on the relationship between health care and mortality (e.g., Cremieux et al. 1999, Lichtenberg 2004, Hall and Jones 2007, Baltagi et al. 2012).}

In regard to medical technology, we assume the following properties

$$
\mu_M(\cdot) < 0, \mu_{MM}(\cdot) \geq 0, \mu_{hM}(\cdot) \succcurlyeq 0 \quad \forall (a, t) .
$$

Hence, medical technology contributes toward reductions in mortality ($\mu_M(\cdot) < 0$) with (weakly) decreasing returns. We leave it open, however, whether for any given positive level of health care, $h(a, t) > 0$, medical technology is complementing the consumption of health care ($\mu_{hM}(a, h(a, t), M(t)) \leq 0$) or substituting it ($\mu_{hM}(a, h(a, t), M(t)) > 0$).

Individuals enjoy period utility $u(c(a, t))$ from consumption $c(a, t)$. Period utility is increasing and concave: $u_c(\cdot) > 0, u_{cc}(\cdot) \leq 0$. In addition, we assume the Inada condition $u_c(c_0) = +\infty$ with $c_0 \geq 0$ denoting a level of subsistence consumption. Individuals maximise the present value of their expected life-cycle utility

$$
\max_{c(a, t), h(a, t)} \int_0^{\omega} e^{-\rho a} u(c(a, t)) S(a, t) da
d(1)
$$

by choosing a stream of consumption and health care on the interval $[0, \omega]$, with $\omega$ denoting the maximal possible age, with $\rho \geq 0$ denoting the rate of time preference, and with $S(a, t)$, defined above, denoting the survival function.\footnote{Note that from the individual’s perspective age and time progress simultaneously, following the identity $a \equiv t - t_0 \in [0, \omega]$ for $t \in [t_0, t_0 + \omega]$. Thus, we have $\int_{t_0}^{\omega} e^{-\rho a} u(c(a, t)) S(a, t) da = \int_{t_0}^{\omega} e^{-\rho a} u(c(a, t_0 + a)) S(a, t_0 + a) da = \int_{t_0}^{t_0 + \omega} e^{-\rho (t - t_0)} u(c(t - t_0, t)) S(t - t_0, t) dt.$}
The individual faces as constraints the dynamics of survival and the dynamics of individual assets $k(a, t)$, as described by the system\(^{13}\)

\[
\begin{align*}
\dot{S}(a, t) &= -\mu(a, h(a, t), M(t))S(a, t), \\
\dot{k}(a, t) &= r(t)k(a, t) + l(a)w(t) - c(a, t) - \phi(a, t)p_H(t)h(a, t) - \tau(a, t) + \pi(a, t) + s(t),
\end{align*}
\]

with the boundary conditions

\[
\begin{align*}
S(0, t_0) &= 1, \quad S(\omega, t_0 + \omega) = 0 \\
k(0, t_0) &= k(\omega, t_0 + \omega) = 0.
\end{align*}
\]

Here, (2) describes the reduction of survival according to the force of mortality. While for the sake of simplification we are subsequently referring to $S(a, t)$ as survival, the function may, in fact, be interpreted as a more general measure of health that is subject to depreciation over the life-course (Chandra and Skinner 2012, Kuhn et al. 2015). Indeed, (2) not only describes the mortality process, but also proxies for the gradual decline in health over the life-course, as is documented by the gradual accumulation of health deficits (e.g., Rockwood and Mitnitski 2007, Dalgaard and Strulik 2014). With our focus being on an individual representing a whole cohort, it is plausible to assume that the consumption of health care slows down the decline in health but cannot reverse it.\(^{14}\) Furthermore, assuming that utility from consumption and utility from good health are multiplicatively separable, one can easily generalise the interpretation of (1) to include not only health-dependent duration of life but also health-dependent quality of life.

According to (3) an individual’s stock of assets $k(a, t)$ (i) increases with the return on the current stock, where $r(t)$ denotes the interest rate at time $t$; (ii) increases with earnings $l(a)w(t)$.

---

\(^{13}\)In the following, we will use the () notation to indicate both the derivative $x(a, t) := x_a + x_t$ for life-cycle variables and the derivative $X(t) := X_t$ for aggregate variables. Drawing again on the identity $t \equiv t_0 + a$ from the individual’s perspective, it follows that $x(a, t)$ collapses into a single dimension.

\(^{14}\)This, in fact, is the only distinguishing feature from the modelling of the health process à la Grossman (1972), where prevention and/or treatment of specific conditions may raise the stock of health of a specific individual. We should like to stress, however, that for the purpose of analysing the impact of medical change on the aggregate demand for health care it is entirely sufficient to focus on a (cohort-)representative individual as long as the age-specific demand for health care, the resulting survival and, thus, the age-distribution of the population are reflecting the data.
where \(w(t)\) denotes the wage rate at time \(t\), and where \(l(a)\) denotes an individual’s effective age-dependent labour supply; (iii) decreases with consumption, the price of consumption goods being normalised to one; (iv) decreases with private health expenditure, \(\phi(a,t)p_H(t)h(a,t)\), where \(p_H(t)\) denotes the price for health care, and where \(\phi(a,t)\) denotes an \((a,t)\)-specific rate of coinsurance; (v) decreases with an \((a,t)\)-specific tax, \(\tau(a,t)\); (vi) increases with \((a,t)\)-specific benefits \(\pi(a,t)\); and (vii) increases with a transfer \(s(t)\) by which the government redistributes accidental bequests in a lump-sum fashion. Here, we follow Suen (2009), Ludwig et al. (2012) and Zhao (2014) by considering a setting without an annuity market.\(^{15,16}\) We assume that the survival function is bounded between 1 at birth and 0 at the maximum feasible age \(\omega\) [see (4)], and that individuals enter and leave the life-cycle without assets [see (5)].

Denoting by \(B(t-a)\) the size of the birth cohort at \(t_0 = t - a\), the cohort aged \(a\) at time \(t\) has the size

\[
N(a,t) = S(a,t)B(t-a).
\]

By aggregating over the age-groups who are alive at time \(t\) we obtain the following expressions for the population size,\(^{17}\) aggregate capital stock, aggregate effective labour supply, aggregate consumption, aggregate demand for health care, aggregate fiscal income from taxation, and aggregate

\(^{15}\)This is well in line with evidence that few individuals annuitise their wealth (e.g. Warshawsky 1988, Reichling and Smetters 2015). Hansen and Imrohoroglu (2008) show that the empirically relevant hump-shaped life-cycle profiles of consumption can be consistently explained within a life-cycle model only when assuming that annuity markets are assumed to be absent (or severely imperfect).

\(^{16}\)We have also considered a specification with imperfect annuities yielding a return \(r(t) + \theta \Pi(a,t)\), where \(\theta \in [0,1]\) and where \(\Pi(a,t) = \mu(a,h^*(a,t),M(t))\) is the expected mortality, given the equilibrium level of health care \(h^*(a,t)\). Following Heijdra and Mierau (2012) in considering a scenario with \(\theta = 0.7\), we obtain qualitatively similar results to those reported in this paper.

\(^{17}\)In a slight abuse of notation, \(N(t)\) denotes the population size at time \(t\), whereas \(N(a,t)\) represents the size of the cohort aged \(a\) at time \(t\).
transfer payments, each at time t:

\[ N(t) = \int_{0}^{\omega} N(a,t)da, \]
\[ K(t) = \int_{0}^{\omega} k(a,t)N(a,t)da, \]
\[ L(t) = \int_{0}^{\omega} l(a)N(a,t)da, \]
\[ C(t) = \int_{0}^{\omega} c(a,t)N(a,t)da, \]
\[ H(t) = \int_{0}^{\omega} h(a,t)N(a,t)da, \]
\[ \Upsilon(t) = \int_{0}^{\omega} \tau(a,t)N(a,t)da, \]
\[ \Pi(t) = \int_{0}^{\omega} \pi(a,t)N(a,t)da. \]  

(6)

The economy consists of a manufacturing sector and a health care sector. In the manufacturing sector a final good is produced by employment of capital \( K_Y(t) \) and labour \( L_Y(t) \) according to a neoclassical production function \( Y(K_Y(t), A(t) L_Y(t)) \), with \( A(t) \) measuring the state of labour augmenting technology. A manufacturer’s profit can then be written as

\[ V_Y(t) = Y(K_Y(t), A(t) L_Y(t)) - w(t)L_Y(t) - [\delta + r(t)] K_Y(t), \]  

(8)

where \( \delta \) denotes the depreciation rate of capital.

Health care goods and services are produced by employment of labour \( L_H(t) \), and capital \( K_H(t) \) according to the neoclassical production function \( F(K_H(t), L_H(t)) \). Recalling the price for health care \( p_H(t) \), the profit of a health care provider is then given by

\[ V_H(t) = p_H(t) F(K_H(t), L_H(t)) - w(t)L_H(t) \]
\[ V_H(t) = -[\delta + r(t)] K_H(t), \]  

(9)

where we assume that capital depreciates at the same rate across both sectors. Note that the presence of perfect competition together with a neoclassical production function in the two sectors implies \( V_Y(t) = V_H(t) = 0 \) in equilibrium.

The government and/or a third-party payer (e.g. a health insurer) raise taxes (or contribution
rates, e.g. insurance premiums) for the purpose of co-financing health care at the rate $1 - \phi (a, t)$ and of paying out transfer payments $\pi (a, t)$. More specifically, $\pi (a, t)$ may refer to pension benefits, implying that

$$\pi (a, t) = \begin{cases} 0 & \iff a < a_R \\ \pi \geq 0 & \iff a \geq a_R \end{cases}$$

with $\pi$ a uniform pension benefit and $a_R$ the retirement age. In such a setting we would also have

$$l (a) = \begin{cases} \hat{l}(a) \geq 0 & \iff a < a_R \\ 0 & \iff a \geq a_R \end{cases}.$$ 

Likewise, $\tau (a, t)$ are age-specific taxes. We could distinguish these into taxes used to finance health care payments (or health insurance premiums), $\tau_H (a, t)$, and social security contributions, $\tau_\Pi (a, t)$, where $\tau (a, t) = \tau_H (a, t) + \tau_\Pi (a, t)$. Furthermore, we could, in principle distinguish between lump-sum and labour income taxes, $\tau_j (a, t) = \tilde{\tau}_j (a, t) l (a) w(t)$, with $j = H, \Pi$. As long as we assume a unified government budget and an exogenous labour supply, it is sufficient to consider $\tau (a, t)$.

Assuming that the government budget must be balanced within each period $t$ we obtain the constraint

$$\int_0^\omega \left\{ [1 - \phi (a, t)] p_H (t) h(a, t) + \pi (a, t) - \tau (a, t) \right\} S(a, t) B(t-a) da = 0.$$ 

Finally, we assume that

$$s(t) = \frac{\Upsilon_B(t)}{N(t)}.$$ 

where

$$\Upsilon_B(t) = \int_0^\omega \mu(a, t) k(a, t) N(a, t) da$$

are total accidental bequests.$^{18}$

$^{18}$In order to ease on notation, we will subsequently refer to the shortcut $\mu(a, t)$ for $\mu(a, h(a, t), M(t))$. 

11
3 Individual Life-Cycle Optimum

In Appendix A1 we show that the solution to the individual life-cycle problem is described by the following two sets of conditions

\[
\begin{align*}
\frac{u_c(c(a,t))}{\exp \left\{ - \int_a^{\hat{a}} \left[ \rho + \mu(h, t + \hat{a} - a) \right] d\hat{a} \right\} u_c(c(\hat{a}, t + \hat{a} - a))} &= \exp \left[ \int_a^{\hat{a}} r \left( t + \hat{a} - a \right) d\hat{a} \right], \\
- \mu_h(a, t) \psi(a, t) &= \phi(a, t) p_H(t) \quad \forall (a, t),
\end{align*}
\]

(13)

(14)

describing the optimal pattern of consumption \(c(a, t)\) and the demand for health care \(h(a, t)\), respectively, of an individual aged \(a\) at time \(t\). Condition (13) is the well-known Euler equation, requiring that the marginal rate of intertemporal substitution between consumption at any two ages/years \((a, t)\) and \((\hat{a}, t + \hat{a} - a)\) equals the compound interest. Note that in the absence of annuity markets, the uninsured mortality risk can be interpreted as an additional factor of discounting, implying an effective discount rate \(\rho + \mu(a, t)\) at any \((a, t)\).

Condition (14) requires that at each \((a, t)\) the marginal value of health care, \(- \mu_h(a, t) \psi(a, t)\), equals its effective price, \(\phi(a, t) p_H(t)\). The marginal value of health care is given by the marginal effect of health care on mortality, \(- \mu_h(a, t)\), weighted with the private value of life (VOL). The private VOL is defined by

\[
\psi(a, t) := \int_a^\omega v(\hat{a}, t + \hat{a} - a) R(\hat{a}, a) d\hat{a},
\]

(15)

with

\[
v(a, t) := \frac{u(c(a, t))}{u_c(\cdot)},
\]

(16)

and

\[
R(\hat{a}, a) := \exp \left[ - \int_a^{\hat{a}} r \left( t + \hat{a} - a \right) d\hat{a} \right],
\]

(17)

and amounts to the discounted stream of consumer surplus, \(v = u(\cdot) / u_c(\cdot)\) taken over the expected remaining life-course \([a, \omega]\).

\(^{19}\)The VOL as we calculate it here differs from the typical representation of the value of a statistical life as e.g. in
For a given set of prices, the evolution of consumption with age is described by (for a derivation see Appendix A1)

\[
\dot{c} = \frac{u_c}{u_{cc}} (\rho - r + \mu).
\] (18)

Noting that \(u_{cc} < 0\), it is readily seen that consumption tends to increase over the life-cycle if and only if \(r - \rho > \mu\). In the absence of an annuity market, the uninsured mortality risk imposes a downward drag on consumption over the life-cycle and implies that consumption will eventually decrease with age when mortality \(\mu\) grows sufficiently high.

For a given set of prices and a given state of the medical technology, the demand for health care evolves with age as described by (for a derivation see Appendix A1)

\[
\dot{h} = -\frac{1}{\mu_{hh}} \left[ \mu_{ha} + \mu_h \left( \frac{\dot{\psi}}{\psi} - \frac{\phi}{\phi} \right) \right].
\] (19)

Noting that \(\mu_{hh} > 0\), the impact of age on the consumption of health care involves three forces: (i) the changing effectiveness of health care with age \(\mu_{ha}\), a stronger (weaker) effectiveness with age, \(\mu_{ha} < 0 (> 0)\) implying an increase (decrease) in health care; (ii) the rate at which the VOL changes with age, a decrease implying a reduction in health care; and (iii) changes with age in the co-insurance rate, \(\phi\), as e.g. during a transition from private to public health insurance at the onset of retirement.

Differentiating (15) with respect to age, we obtain the dynamics of the private VOL as

\[
\dot{\psi} (a, t) = r(t) \psi (a, t) - \frac{u_c (a, t)}{u_{cc} (a, t)}. \] (20)

Thus, the private VOL increases with the interest rate and declines over time as the consumer surplus from a life-year lived is written off.

Shepard and Zeckhauser (1984), Rosen (1988), Johansson (2002), or Murphy and Topel (2006) in as far as (i) the discount factor does not include the mortality rate; and (ii) the VOL does not include the current change to the individual’s wealth, \(lw - c - h - \tau + \pi + s\). Both of these features are due to the absence of an annuity market.
4 General Equilibrium

Perfectly competitive firms in the production sector choose labour $L_Y(t)$ and capital $K_Y(t)$ so as to maximise period profit (8). The first-order conditions imply

$$r(t) = Y_{K_Y}(t) - \delta$$
$$w(t) = Y_{L_Y}(t),$$

i.e. the factor prices are equalised with their respective marginal products.

Likewise, perfectly competitive providers of health care choose labour $L_H(t)$ and capital $K_H(t)$ so as to maximise period profit (9). From the first-order condition we obtain

$$r(t) = p_H(t) F_{K_H}(t) - \delta$$
$$w(t) = p_H(t) F_{L_H}(t).$$

Combining these conditions with (21) and (22) we obtain

$$p_H(t) = \frac{Y_{L_Y}(t)}{F_{L_H}(t)} = \frac{Y_{K_Y}(t)}{F_{K_H}(t)},$$

implying that capital and labour inputs are distributed across the production and health care sector in a way that equalises the marginal rate of transformation (i.e. the relative output gain in production as compared to the output loss in health care from re-allocating one factor unit from health care into production) with the price for health care. The higher the latter, the greater the marginal rate of transformation, implying that more workers will be allocated to the health care sector. With appropriate Inada conditions, $Y_{L_Y}(K_Y,0) = Y_K = (0, AL_Y) = \infty$ and $F_{L_H}(K,0) = F_K(0,L_H) = \infty$ we always have an interior allocation with $L_H(t) = L(t) - L_Y(t) \in (0, L(t))$ and $K_H(t) = K(t) - K_Y(t) \in (0, K(t))$.

4.1 Market Clearance and General Equilibrium

Our setting involves four markets: two input markets for capital and labour, respectively; and two output markets for health care and for final goods, respectively. From the four market clearing
conditions

\[ K_Y(t) + K_H(t) = K(t), \]
\[ L_Y(t) + L_H(t) = L(t) \]
\[ F(t) = H(t), \]
\[ Y(t) = C(t) + \dot{K}(t) + \delta K(t), \]

we obtain a set of equilibrium prices \( \{ r^*(t), w^*(t), p^*_H(t) \} \) as well as the level of net capital accumulation \( \dot{K}(t) \). We provide a more detailed description of the general equilibrium structure in Appendix A2.

## 5 Impact of Medical Progress

**Demand for health care and value of life (VOL):** In Appendix A4 we show that the impact of medical progress, as measured by an increase in the level of medical technology, \( dM > 0 \), on the demand for health care at \((a, t)\) is described by

\[
\frac{dh(a, t)}{dM} = -\frac{\mu_{hM}}{\mu_{hh}} + \frac{\mu_h(a, t)}{\mu_{hh}} \left( \frac{1}{p_H(t)} \frac{dp_H(t)}{dM} - \frac{1}{\psi(a, t)} \frac{d\psi(a, t)}{dM} \right). \tag{26}
\]

Term (i) represents the effect of medical technology on the demand for health care through changes in the effectiveness of care. If technology raises the marginal effectiveness of health care \((\mu_{hM} < 0)\), term (i) is positive and more health care will be consumed at \((a, t)\) in response to medical progress. Term (ii) implies that the demand for health care tends to fall if medical progress raises the price for health care. Finally, the demand for health care changes in line with the impact of medical progress on the VOL [term (iii)].

The impact of medical progress on the VOL can be written as

\[
\frac{d\psi(a, t)}{dM} = \int_a^\omega R(\tilde{a}, a) \left( -v(\tilde{a}, t + \tilde{a} - a) \int_a^{\tilde{a}} \frac{dr(t + \tilde{a} - a)}{dM} \frac{d\tilde{a}}{dM} \left( \frac{d\psi(\tilde{a}, t + \tilde{a} - a)}{dM} \right) d\tilde{a} \right) \tag{27}
\]
where \( v(\hat{a}, t + \hat{a} - a) \) and \( R(\hat{a}, a) \) are given by ((16)) and ((17)), respectively, and where
\[
\frac{dv(\hat{a}, t + \hat{a} - a)}{dM} = \left(1 - \frac{uu_{cc}}{u^2_c}\right) \frac{dc(\hat{a}, t + \hat{a} - a)}{dM}.
\] (28)

Thus, technology bears on the VOL through two channels: through changes in the interest rate at which the monetary value of each remaining life year is discounted [term (iii.i)], and through changes in age-specific consumption over the remaining life-course [term (iii.ii) and (28)]. According to (iii.i), the VOL increases whenever improvements in medical technology reduce the interest rate, an effect that arises only in general equilibrium. Noting that \( 1 - \frac{uu_{cc}}{u^2_c} > 0 \) (see Appendix A4), term (iii.ii) implies that a positive effect of medical technology on future consumption translates into an increase in the demand for health care.

Generally, we can write
\[
c(\hat{a}, t + \hat{a} - a) = c(a, t) \exp \left[ \int_a^{\hat{a}} g_c(\hat{a}, t + \hat{a} - a) d\hat{a} \right],
\]
where \( c(a, t) \) is the initial consumption level at birth, and where
\[
g_c(\hat{a}, t + \hat{a} - a) := \frac{u_c}{u_{cc}c(\hat{a}, t + \hat{a} - a)} \left[ \rho - r(t + \hat{a} - a) + \mu(\hat{a}, t + \hat{a} - a) \right]
\]
is rate of consumption growth at \((\hat{a}, t + \hat{a} - a)\) as given by the dynamic Euler equation (18). Thus, we have
\[
\frac{dc(\hat{a}, t + \hat{a} - a)}{dM} = c(\hat{a}, t + \hat{a} - a) \left\{ \frac{dc(a, t)}{dM} + \int_a^{\hat{a}} \frac{dg_c(\hat{a}, t + \hat{a} - a)}{dM} d\hat{a} \right\},
\] (29)
according to which the impact of medical progress on consumption at \((\hat{a}, t + \hat{a} - a)\) is governed by two possibly offsetting effects: the impact on initial consumption \( c(a, t) \), which is implicitly determined through the life-cycle budget constraint, and the impact on the growth rate of consumption over the life-cycle, the latter of which depends in particular on changes in the interest rate and the mortality rate. More specifically, medical change tends to increase the growth rate of consumption at \((\hat{a}, t + \hat{a} - a)\) to the extent that it increases the spread between interest rate and mortality rate \( r(t + \hat{a} - a) - \mu(\hat{a}, t + \hat{a} - a) \), e.g. by lowering mortality.

**Prices:** Given the various offsetting effects in (26)-(29) it is difficult to arrive at a general statement about the impact of medical technology on the VOL and on the demand for health care without placing undue restrictions on the model. At this point, we therefore content ourselves with
having identified the various channels through which medical progress feeds on consumption and the demand for health care and defer a quantitative assessment of the various offsetting effects to our numerical analysis in Section 6.3.

In the following, let us assume that the production in the final goods and health care sector, respectively, is described by the set of Cobb-Douglas production functions

\[ Y(t) = K_Y(t)^\alpha \left[ A(t) L_Y(t) \right]^{1-\alpha} \]

\[ F(t) = K_H(t)^\beta \left[ L_H(t) \right]^{1-\beta}, \]

with \( \alpha, \beta \in [0,1] \). The general equilibrium feedback on the demand for health care is then driven by changes in the market interest rate. Noting from Appendix A3 that all prices in the economy can be calculated as a function of the interest rate, we show in Appendix A4 that

\[ \frac{dw(t)}{dM} = -\frac{\alpha w(t)}{1 - \alpha \frac{r(t)}{1 - \alpha}} \frac{dr(t)}{dM}, \]

\[ \frac{dp_H(t)}{dM} = p_H(t) \frac{\beta - \alpha}{\frac{r(t)}{1 - \alpha} + \delta} \frac{dr(t)}{dM}, \]

The general equilibrium impact of medical progress on the wage rate as well as on the price for health care is thus determined by its effect on the market interest rate. Most importantly, the impact of medical change on the wage rate is always opposite to its impact on the interest rate. This is because a reduction (increase) in the market interest rate leads to an increase (reduction) of capital employed in production which translates into an increase (decrease) in the marginal productivity of labour. The effect of medical progress on the price of health care is ambiguous. As equation (33) indicates, we have \( sgn\frac{dp_H(t)}{dM} = -sgn\frac{dr(t)}{dM} \) if and only if \( \beta < \alpha \), i.e. if and only if the capital elasticity is lower in the health care sector as compared to the remaining industry. In Section 6.1 we will provide empirical evidence to the effect that this is, indeed, the case. Whenever medical change induces a reduction in the interest rate, this will then lead to a corresponding boost in the wage rate, which also drives up the price for health care, the latter being produced in a relatively labour intensive way.

While we are unable to present a closed theoretical expression for the effect of medical progress on the market interest rate, \( \frac{dr(t)}{dM} \), we can draw on the mechanics of the capital market to derive
some insight into the matter. Denote by \( K^d_Y(t,r) \) and \( K^d_H(t,r) \) the capital demand functions in the final goods and health care sector, respectively. From (21) and (23) it is readily checked that ceteris paribus capital demand decreases in the interest rate \( r \) and does not directly depend on \( M \).

In contrast, the supply of capital \( K^s(t,r,M) \) can be shown ceteris paribus to increase with the interest rate and with the level of technology \( M \). Denote by \( r(t) \) the interest rate that equilibrates the capital market such that \( K^d_Y(t,r(t)) + K^d_H(t,r(t)) = K^s(t,r(t),M) \) in period \( t \) and consider now an improvement in medical technology, \( dM > 0 \). While it is difficult to assess the general equilibrium impact, it is easy to see that the instantaneous impact involves an outward shift of the capital supply function and, thus, \( K^d_Y(t,r(t)) + K^d_H(t,r(t)) < K^s(t,r(t),M + dM) \). The excess supply of capital then implies a downward pressure on the interest rate, \( \frac{dr(t)}{dM} < 0 \). But then an improvement of medical technology should also imply \( \frac{dGDP(t)}{dM} > 0 \) and \( \frac{dp_H(t)}{dM} > 0 \). This intuition is, indeed, confirmed by the numerical analysis in Section 6.3.1.

**Economic performance (GDP):** Finally, consider the impact of medical progress on the GDP per capita as a measure of economic performance. Note that in our framework GDP is defined as the sum of output in the final goods and health care sector, as measured in units of the final good, \( GDP(t) = Y(t) + p_H(t) F(t) \). Expressing GDP per capita

\[
\frac{GDP(t)}{N(t)} = \frac{L(t) \, GDP(t)}{N(t) \, L(t)}
\]

as the product of the employment rate \( \frac{L(t)}{N(t)} \) and the GDP per worker \( \frac{GDP(t)}{L(t)} \) it is easy to see that the impact of medical progress on economic performance comes (i) through a change in the employment rate; and (ii) through a change in the GDP per worker. The impact of medical innovation on the employment rate strongly depends on the age-profile of mortality rates and their dependency on medical progress. While the dependency is generally ambiguous, we would conjecture that in developed economies in which technology-related gains in survival are concentrated amongst the older population, the likely impact of medical progress on the employment rate is negative, and this is, indeed, confirmed by our numerical simulation calibrated to the US setting.

In Appendix 4 we show that for the Cobb-Douglas functions in (30) and (31) we can write the equilibrium level of GDP per worker as a function of the employment share \( \lambda(t) := \frac{L_Y(t)}{L(t)} \).
and the aggregate capital intensity $K(t)/L(t)$

$$\frac{GDP(t)}{L(t)} = \frac{Y(t) + p_H(t)F(t)}{L(t)} = \left[1 + \frac{p_H(t)F(t)}{Y(t)}\right]Y(t)L(t)$$

$$= \frac{1 - \alpha + (\alpha - \beta)\lambda(t)}{1 - \beta} \Lambda(t)^{1-\alpha} \left[\frac{\alpha(1 - \beta)}{\beta(1 - \alpha) + (\alpha - \beta)\lambda(t)}\right]^\alpha \left(\frac{K(t)}{L(t)}\right)^\alpha. \quad (34)$$

Taking the total differential of this expression with respect to $M$ we can then show that (see Appendix A4)

$$d\left(\frac{GDP(t)}{L(t)}\right) = \frac{-(1 - \alpha)(\alpha - \beta)^2[1 - \lambda(t)]}{[1 - \alpha + (\alpha - \beta)\lambda(t)][\beta(1 - \alpha) + (\alpha - \beta)\lambda(t)]} \frac{GDP(t)}{L(t)} d\lambda(t)$$

$$+ \alpha \frac{GDP(t)}{K(t)} \frac{d}{dM} \left(\frac{K(t)}{L(t)}\right). \quad (35)$$

As is readily verified we have that $\frac{d}{dM} \left(\frac{GDP(t)}{L(t)}\right) > 0$ holds if $\frac{d\lambda(t)}{dM} \leq 0$ and $\frac{d}{dM} \left(\frac{K(t)}{L(t)}\right) \geq 0$. Thus, medical progress tends to raise GDP per worker if (i), for a given structure of the economy as described by the employment share $\lambda(t)$, it leads to capital deepening, i.e. to an increase in the economy-wide capital intensity $\frac{K(t)}{L(t)}$; and (ii) it induces a shift in resources to the more labour intensive health care sector, as measured by a decline in the employment share of final goods production $\lambda(t)$. 20 Our numerical analysis in Section 6.3.1 shows that, indeed, medical innovation triggers both an increase in the aggregate capital stock per worker and a reduction in final goods employment. Thus, its impact on the GDP per worker is unambiguously positive. Whether or not this induces an increase in GDP per capita then depends on the extent to which the the employment rate $L(t)/N(t)$ is curbed by medical progress. For the US health care context studied in Section 6.3.1, we find the increase in the GDP per worker to be the (weakly) dominating effect.

6 Numerical Analysis

Following a description of our numerical analysis, we present the outcomes for three scenarios, consisting of a benchmark and two numerical experiments. The benchmark features a realistic economy calibrated to US data, reflecting the year 2003. The experiments involve (i) the impact of an unanticipated medical advance, leading to a reduction in mortality; and (ii) the impact of the

\[20\text{It is easy to verify that a decline in the employment share } \lambda(t) \text{ will in optimum be accompanied by a decline in } K_Y(t)/K(t).\]
same advance when it is anticipated.

6.1 Specification of the Numerical Analysis

The main components of our numerical model are specified as follows.

Demography

With model time progressing in single years, individuals enter the model economy at age 20 and can live up to a maximum age 100. In our model, a "birth" at age 20 implies that $\omega = 80$. Population growth is partly endogenous due to endogenous mortality and partly exogenous due to a fixed growth rate of "births" $\nu = 0.013$, which is calibrated to match the elderly share of the adult (20 years and older) US population, equalling 17.6% according to the decennial census in the US in 2000. Due to the exogenous path of births, our results will not be confounded by a variation in birth numbers across the experiments.

Mortality

The force of mortality $\mu$ is endogenously determined in the model, depending on health care, $h$, as a decision variable; an exogenous level of medical technology, $M$; and an exogenous age-dependent base mortality, $\tilde{\mu}(a)$. As not all reductions in mortality can be attributed to health expenses or technological progress (see e.g. Hall and Jones 2007), we introduce an exogenous factor $I(a)$ that captures changes in age-dependent mortality rates due to exogenous circumstances. Generalising Kuhn et al. (2011, 2015) we formulate

$$\mu(a,t) = \tilde{\mu}(a) \cdot \left( I(a) - \eta(a) [h(a,t) \cdot M(t)]^{\epsilon(a)} \right),$$

where $\eta(a)$ and $\epsilon(a)$ are parametric functions that reflect decreasing efficiency of health care with age, where $\epsilon(a)$ reflect the age-specific elasticity of mortality with respect to health care demand as reported in Hall and Jones (2007). The base mortality $\tilde{\mu}(a)$ reflects a mortality profile that is higher in level (to a sufficient extent) than the US mortality in the year 2003, which we aim to replicate in the calibration. For this purpose we employ for $\tilde{\mu}(a)$ single year mortality rates for

\footnote{We follow the bulk of the literature and neglect life-cycle decisions during childhood.}
the year 1950 in the US, as reported in the Human Mortality Database (HMD) (see Figure 1a). The age-dependent parametric functions $\eta(a)$ and $I(a)$ are then chosen to approximate age-specific health expenditures and mortality $\mu(a, t)$ in the year 2003.\footnote{Note that $I(a)$ only influences mortality (because $\mu_h$ is independent of $I$), whereas $\eta(a)$ also influences the demand for health care. The 2003 mortality rates are again taken from HMD. Due to limited data availability, we use health expenditure data for the year 2000, as provided in Meara et al. (2004).} We normalise the state of medical technology to the year 2003 and, thus, set $M(t) \equiv 1$ in the benchmark case.

![Figure 1: Mortality and labor employment age-profiles](image)

(a) US 1950 and 2003 Force of mortality (HMD)  
(b) Age-specific labour employment schedule

Utility

We assume instantaneous utility to be given by

$$u(a, t) = b + \frac{(c(a, t) - c_0)^{1-\sigma}}{1 - \sigma},$$

where $c_0 = $11000 is an exogenous minimal consumption level.\footnote{Dollar values are to be interpreted as year 2003 Dollars throughout.} We choose the inverse of the elasticity of intertemporal substitution to be $\sigma = 1.75$ which is within the range of empirically consistent values, as suggested by Chetty (2006). Setting $b = 8$ then guarantees that $u(a, t) \geq 0$ throughout. Furthermore, $b = 8$ generates a VOL that lies within the range of plausible estimates, as suggested in Viscusi and Aldy (2003). Finally, we assume a rate of time preference $\rho = 0.02$.\footnote{We use the minimum consumption for reasons of improving the fit of the consumption profile. While the minimum level is never hit in optimum, it helps to avoid an unrealistically sharp drop in consumption and consequently debt repayment during the oldest ages. The level of the minimum consumption profile is thus set such that assets of the elderly never fall below zero.}
Effective labour supply and income

We proxy the effective supply of labour $\hat{l}(a)$ by an age-specific income schedule (see Figure 1b), constructed from 2003 earnings data, as contained in the Current Population Survey (CPS) provided by the Bureau of Labor Statistics (BLS). We rescale the schedule such that the employment-population ratio $L(t)/N(t)$ matches the empirical value of 62% for the US in 2003 as reported by the BLS. Individuals at the age 65 or older are assumed to have no income from labour but receive a fixed social security pension for the remainder of their lifetime, as detailed further on below.

Production

There are two production functions in the model. Production of the final good is described by

$$Y(t) = K_Y(t)^\alpha (A(t)L_Y(t))^{1-\alpha},$$

where $K_Y(t)$ and $L_Y(t)$ denote capital and labour in final good production, where $L_Y(t)$ is the workforce working in this sector, and where $A(t)$ is an exogenous technology index. $A(t)$ is calibrated so that $l(50)w(t)$ matches the average earnings of a 50-year old in 2003; the elasticity of capital $\alpha$ is chosen to be 1/3.

The health care sector produces medical goods and services that individuals purchase with a view to lowering their mortality. Its production is given by

$$F(t) = K_H(t)^\beta (L_H(t))^{1-\beta},$$

where $K_H(t)$ and $L_H(t)$ denote capital and labour in this sector. For the production elasticity of capital in the health care sector we take an estimate from Acemoglu and Guerrieri (2008) and set $\beta = 1/5$. Finally, we assume a rate of capital depreciation equal to $\delta = 0.05$.

Health Insurance, Medicare and Social Security

Health expenditures are subsidised through two different sources: (a) private health insurance with coinsurance rate $\phi_P$ and (b) Medicare for the elderly (available after retirement) with coinsurance rate $\phi_{MC}$. Private health insurance is financed through a "risk-adequate" premium equal to the
expected health expenditure covered by the insurance for an individual at a given time and age. It is thus given by \( \tau_P = [1 - \phi_P(a,t)] p_H(t) h^*(a,t) \), where \( h^*(a,t) \) denotes the equilibrium demand for health care at \((a,t)\). Following Zhao (2014) we assume that 70\% of the US workforce is health insured, with 70\% of expenses being covered (in 2000). Thus, we assume that 51\% of health expenditures are paid out-of-pocket on average among the working population. Zhao (2014) states that 35\% of the elderly have health insurance with a coverage of 30\%, leading to average health insurance subsidies of 10.5\%. Medicare is financed through a payroll tax, with the rate \( \hat{\tau}_{MC} \) being endogenously determined such that the Medicare budget constraint holds. We assume that Medicare covers 38\% of the health expenses of the elderly\(^{25}\). This results in 51.5\% out-of-pocket expenditures for the elderly. In total, the out-of-pocket share of health expenses paid by the individual is

\[
\phi = \begin{cases} 
0.51 & \text{if } a < a_R \\
0.515 & \text{if } a \geq a_R
\end{cases}
\]

where \( a_R \) is the mandatory age of retirement. The budget-constraint for Medicare is given as follows:

\[
\int_{a_R}^{\omega} [1 - \phi_{MC}(a,t)] p_H(t) h(a,t) N(a,t) da = \hat{\tau}_{MC}(t) w(t) L(t),
\]

where \( 1 - \phi_{MC}(a,t) \) is the share of health expenditures paid by Medicare and where \( \hat{\tau}_{MC} \) is the payroll tax for Medicare.

Social security, received by retirees, is financed through a payroll tax which is determined endogenously from the social security budget constraint:

\[
\int_{a_R}^{\omega} \pi(a,t) N(a,t) da = \hat{\tau}_\Pi(t) w(t) L(t),
\]

where \( \pi(a,t) \) is the social security pension and \( \hat{\tau}_\Pi \) the payroll tax devoted to social security. We assume social security benefits to be exogenous and use the CPS Annual Social and Economic Supplement data for the year 2003 which states an approximately $10300 mean social security

\(^{25}\)This value was calculated based on the following data of the US economy in 2003: Share of the elderly in total health spending =40\% (NHEA); health share in the GDP =15\% (NHEA); Medicare share in the GDP =2.3\% (Zhao, 2014).
income for individuals aged 65 years or older in 2003. Thus, we set $\pi(a, t) = 10300$ for $a \geq a_R$ and otherwise to zero.

Altogether, individuals face the following taxes (including the premium for the private health insurance):

$$
\tau(a, t) = \pi_{\Pi}(a, t) l(a) w(t) + \pi_{MC}(a, t) l(a) w(t) + [1 - \phi P(a, t)] p_H(t) h^*(a, t).
$$

**Overview of Functional Forms and Parameters**

Table 1 summarises the functional forms and parameters we are employing. Table 2 shows further parameters and functional forms that are used in the calibration to match various empirical moments. The $\equiv$ symbol denotes that the function is assumed to be constant in all arguments.

<table>
<thead>
<tr>
<th>Parameter &amp; Functional Forms</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = 80$</td>
<td>life span</td>
</tr>
<tr>
<td>$t_0 = 120$</td>
<td>entry time of focal cohort, year 2003</td>
</tr>
<tr>
<td>$\rho = 2%$</td>
<td>pure rate of time preference</td>
</tr>
<tr>
<td>$\sigma = 1.75$</td>
<td>inverse elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$c_0 = $11000</td>
<td>subsistence minimum</td>
</tr>
<tr>
<td>$a_R = 65$</td>
<td>mandatory retirement age</td>
</tr>
<tr>
<td>$\delta = 5%$</td>
<td>rate of depreciation</td>
</tr>
<tr>
<td>$\alpha = 1/3$</td>
<td>elasticity of capital in $Y$</td>
</tr>
<tr>
<td>$\beta = 1/5$</td>
<td>elasticity of capital in $F$</td>
</tr>
<tr>
<td>$u(a, t) = b + \frac{(c(a, t) - c_0)^{(1-\sigma)}}{1-\sigma}$</td>
<td>instantaneous utility function</td>
</tr>
<tr>
<td>$B(t) = B_0 \exp[\nu t]$</td>
<td>number of births</td>
</tr>
<tr>
<td>$s(t) = \frac{T_B(t)}{N(t)}$</td>
<td>transfer from accidental inheritances</td>
</tr>
<tr>
<td>$Y(t) = K_Y(t)^{\alpha}(A(t)Y(t))^{(1-\alpha)}$</td>
<td>production in manufacturing sector</td>
</tr>
<tr>
<td>$F(t) = K_H(t)^{\beta}(L_H(t))^{1-\beta}$</td>
<td>production in health sector</td>
</tr>
<tr>
<td>$\mu(a, t) = \mu(a) \left( I(a) - \eta(a) [h(a, t)M(t)]^{\alpha} \right)$</td>
<td>age-time specific mortality rate</td>
</tr>
<tr>
<td>$\phi(a, t) = {0.51$ if $a &lt; a_R$, $0.515$ if $a \geq a_R}$</td>
<td>age-specific total coinsurance</td>
</tr>
</tbody>
</table>

Table 1: Parameters and functional forms
Table 2: Moments to match

In the following, we will present the numerical results (see Appendix A5 for details on the solution of the numerical problem) for the benchmark case and two numerical experiments. We focus on a selection of the most salient outcomes.26

6.2 Benchmark

In order to economise on space we illustrate the benchmark allocation in the same graphs as our first experiment: unanticipated medical advance (see Figures 3-5). The benchmark allocation is depicted by blue, solid plots throughout, whereas the experiments are depicted by green, dashed plots. Some figures also contain red, dotted plots, which refer to a partial equilibrium allocation.

The salient features of the benchmark allocation can be summarised as follows. Consumption of the focal cohort, entering at $t_0 = 120$ (when they are 20 years old), is hump-shaped (see Figure 3). The fact that the interest rate (approx. 4.3%) lies above the rate of time preference (2%) implies a rising consumption until around age 70. Due to missing annuity markets, consumption falls, however, at higher ages as implied by the individual Euler equation (18). Individual health expenditures follow a hump-shaped pattern (Figure 3). While the demand for care grows very moderately up to age 40, it exhibits from then on a strong increase up to age 80 before dropping again for the highest ages. Figure 2 illustrates our model fit with respect to age-specific health care expenditures27. Similar to the simulation in Hall and Jones (2007), we underestimate health care

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26 A full set of outcomes is available from the authors on request.
expenditures until approximately age 40 and overestimate them until the peak at approximately age 80. This is likely due to our focus on health care expenditures affecting survival, as opposed to, for instance, costs caused by pregnancy. Nevertheless, we match age-specific health expenditures by Meara et. al (2004) until age 80 within a reasonable margin of error. While health care expenditures do not fall in Meara et. al (2004), who use an open age interval for all ages 80 years and older, our result of falling health expenditures after age 80 is in line with the simulation in Hall and Jones (2007) and the qualitative life-cycle pattern observed in Martini et al. (2007).28

\[
\text{Health care expenditure over lifetime: } p_H h(t)
\]

Figure 2: Health care expenditure over lifetime from the simulation in Hall and Jones (2007) (blue, solid line), the empirical data in Meara et. al (2004) (green, dashed line) and the MEDPRO Simulation (red, dotted line)

The value of life (VOL) peaks at approx. age 50 (Figure 3), which is consistent with empirical evidence on the value of a statistical life in Aldy and Viscusi (2008). In our model, the hump-shaped age-profile of the VOL follows the equally hump-shaped age profile of individual consumption. In line with (20), the VOL increases during early life where consumption levels are low (yet) such that the value of life years written off falls short of a (yet) high return on the VOL. This relationship reverses in old age. The remaining life expectancy at age 20 is 58.0 years in the benchmark case and, thus, matches the empirical value for the US in 2003 (58.1 years, HMD) very closely.

---

28Indeed, the averaging of health care expenditures across the highest age groups is prone to mask an ultimate decline with age as the population shares used for the weighting are rapidly declining, too. Furthermore, a hump-shaped pattern is not inconsistent with the finding that health care utilisation/expenditure increases with the closeness to death (e.g. Zweifel et al. 1999). This is because the ”cost of dying” itself is declining with age for the highest ages (e.g. Cutler 2007).
It is worth of note that given our assumption of constant $A$, $M$ and $\nu$, prices and per-capita quantities are constant in the benchmark scenario. Thus, a steady state appears to exist although we are not imposing it. In the benchmark model GDP per capital amounts to $39700$ [$39700 according to Table 1.5.5 of the revised National Income and Product Accounts of the Bureau of Economic Analysis (BEA), 2003], and health expenditures per capita to $5720$ [$5750 according to NHEA, 2003]. The health share (in GDP) in the benchmark case is 14.4% and matches the data from the National Health Expenditure Accounts provided by CMS. Furthermore, the benchmark model features a Medicare share of 2.3% [2.3% according to Zhao (2014)]. A summary of the model’s fit is provided in Table 3.

<table>
<thead>
<tr>
<th>Name</th>
<th>Data</th>
<th>Benchmark</th>
<th>Medical advance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital-output ratio</td>
<td>3.1</td>
<td>3.3</td>
<td>3.5</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>$39700</td>
<td>$39700</td>
<td>$40000</td>
</tr>
<tr>
<td>Health spending per capita</td>
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<td>$5720</td>
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<td>Health spending (% of GDP)</td>
<td>14.4%</td>
<td>14.4%</td>
<td>16.0%</td>
</tr>
<tr>
<td>Life expectancy at age 20</td>
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<td>58.0</td>
<td>59.5</td>
</tr>
<tr>
<td>Medicare payroll tax rate, $\hat{\tau}_{MC}$</td>
<td>2.9 %</td>
<td>3.4 %</td>
<td>3.8 %</td>
</tr>
<tr>
<td>Medicare expenditures (% of GDP)</td>
<td>2.3%</td>
<td>2.3 %</td>
<td>2.7 %</td>
</tr>
<tr>
<td>Population share 65 years and older</td>
<td>17.6 %</td>
<td>17.5 %</td>
<td>18.4 %</td>
</tr>
<tr>
<td>Employment-Population ratio</td>
<td>62 %</td>
<td>62 %</td>
<td>61.5 %</td>
</tr>
</tbody>
</table>

Table 3: Fit of the benchmark model (data provided for the year 2003) and outcomes for an unanticipated medical advance

Before setting out on the experiments a clarifying remark is warranted on the purpose and design of our numerical analysis. The main objective of our analysis lies in an analytical and quantitative understanding of the mechanisms which are underlying the macro-economic impacts of medical change. In order to avoid that these impacts are confounded by other sources of change, we have structured our numerical analysis in a way that the economy is “quasi-stationary” in the

29 GDP and the health share are calculated as \( GDP(t) = p_H(t)H(t) + Y(t) \) and \( \frac{p_H(t)H(t)}{GDP(t)} \), respectively.

30 The capital-output ratio was calculated as the ratio of the capital stock and the gross domestic product as provided in the National Income and Production Accounts of the Bureau of Economic Analysis (BEA) in 2003. In the model it is calculated as \( K(t)/GDP(t) \).

31 Note, that the population share of individuals aged 65 or older as well as the employment-population ratio refers to the total population aged 20 or older.
years surrounding the shock. This is why we are abstracting from time-trends in the states of technology, $A(t)$ and $M(t)$ as well as in the birth rate $\nu$, the appropriate calibration of which would have allowed us to arrive at a more realistic dynamic representation of the economy.\footnote{For instance, we could match both, the age-structure and the rate of population growth in 2003 (0.9\%) by assuming an appropriate time-profile of the birth rate $\nu$ prior to the year 2003. While this would give us a (more) realistic description of the demographic change following the year 2003, the impact of this on the economy would interfere with our experiments.} This notwithstanding, we have calibrated the model to the US economy in the year 2003 in order to provide a realistic static backdrop for our experiments.

6.3 Medical Advance

6.3.1 Unanticipated Medical Advance

We consider here an unanticipated increase in the state of the medical technology from $M(t) = 1$ for $t \leq 150$ to $M(t) = 2$ for $t > 150$. The advance of medical technology renders the use of health care in lowering mortality more effective.\footnote{To see this note that} The timing implies that the focal cohort, entering the model at $t_0 = 120$, is aged 50 at the point of the innovation.

Based on a comparison of steady-state values, we find that the innovation raises the remaining life-expectancy of a 50 year old by some 1.1 years and induces additional (discounted) expenditures of about $19000 over the remaining life-course. These magnitudes are broadly in line with evidence provided by Cutler (2007) on the impact of revascularisation, as was introduced into the US during the late 1980s. Cutler finds that for a patient with myocardial infarction, revascularisation would raise life-expectancy by about 1 year and induce about $40000 in additional expenditure. While the impact of innovation in our model is, thus, comparable in the order of magnitude, it should be borne in mind that the figures are not directly comparable, as in Cutler (2007) the values apply (ex-post) to individuals who have had a heart attack, whereas in our model they apply (ex-ante) to the representative agent on whom we are building our macroeconomic analysis.

At the level of the representative individual, we find the following effects of an unanticipated
medical advance: As Figure 3 illustrates, and as one would expect, the innovation induces individuals at age 50 to reallocate expenditure from consumption to health care. Indeed, the drop in consumption is persistent over the remaining life-cycle but the highest ages, where the increase in survival chances induces individuals to raise consumption. When it comes to the impact of the innovation on the demand for health care (as measured by individual health expenditure), a more ambiguous picture emerges in Figure 3: For a given set of prices, the expenses for medical care would increase for all age groups by a substantive amount (see the red, dotted plot). However, such a partial equilibrium take is inappropriate, as the general equilibrium impact of the innovation on the underlying demand and supply system needs to be taken into account. Once we do this, much of the demand expansion vanishes (see green, dashed plot). This notwithstanding, the medical innovation raises remaining life-expectancy at age 20 from 58.0 to 59.1 years for a member of the focal cohort. Notably, the strong increase in demand for a constant set of prices would induce an additional gain of only 0.35 life years.

Equation (26) affords some insight into the demand response of individual health care to medical
progress. Obviously, the increased marginal effectiveness of health care through medical progress \((\mu_{h,M} < 0)\) boosts demand, an effect that is consistent with the empirical evidence in Baker et al. (2003), Cutler and Huckman (2003), Wong et al. (2012) and Roham et al. (2014).\(^{34}\) The effect is dampened, however, by the ensuing reduction in consumption over the remaining life-time, which tends to diminish the VOL (but within the highest age groups) and, thus, the individual’s willingness to pay for health care. Notably, the consumption level tends to drop because a greater part of the life-cycle budget is allocated to health care and because the remaining budget now needs to be spread over a longer life-time. According to Equation (29), however, improved survival chances also induce individuals to shift consumption into higher age classes, a force that leads to increasing consumption at the highest ages.

Overall, the reallocation of resources from consumption to health care in response to medical progress tends to be substantive in a partial equilibrium context. In general equilibrium, it is subject, however, to additional impacts from the price changes induced. Most notably, medical progress triggers a reduction in the market interest rate \(r\) and an increase in the price for health care \(p_H\) (which will be discussed later).\(^{35}\) While the reduction in the market interest rate works to increase the value of life and, thus, boosts health demand, the negative impact of a rising price of health care is dominating. Hence, in the general equilibrium scenario health demand is dampened compared to the partial equilibrium case due to the price increase for health care. We find that while per capita health care expenditure would increase by some 30 percent in partial equilibrium, in general equilibrium they increase by only 12.2 percent, and, thus, by less than a half.\(^{36}\)

Although per capita demand for health care and the associated expenditure, \(p_H(t)H(t)/N(t)\), have increased after the innovation, (see Figure 4 ) the magnitude of the effect varies across age-groups. Specifically, those over 80 exhibit a very modest demand increase in spite of the innovation. For these cohorts the willingness to pay for care, as measured by the VOL, is so low that the value of...
the survival gains from the innovation barely outweighs the price increase. Finally, and strikingly, the medical innovation leads to a reduction in the VOL of the focal individual past age 50 at which the innovation has become available (see Figure 3). At face value, the lower willingness to pay for survival follows from the reduction in consumption over the remaining life-course.

However, a different interpretation can be attached to it in light of the fact that the demand of health care is non-decreasing in response to the medical innovation over the full life-cycle. Rewriting the first-order condition for the demand of health care (14) to
\[ \psi(a,t) = -\phi(a,t) p_H(t) \mu_h^{-1}, \]
we find that the VOL is equated to the effective (or quality-adjusted) price of medical care \(-\phi(a,t) p_H(t) \mu_h^{-1}\), the latter depending on both the market price and the marginal impact on mortality of health care, \(-\mu_h\). Recalling that \(\mu_{hh} > 0\), an increasing demand for care would ceteris paribus imply a greater effective price. But then it must be true that the medical innovation has lowered the effective price for medical care (recall that \(\mu_{hh} < 0\)) to an extent that it over-compensates the increase in the market price, \(p_H(t)\). Notably this finding is consistent with evidence produced by Cutler et al. (1998) who find that while the price for heart attack treatments, as measured by a Service Price Index, was increasing over the time span 1983-1994, the quality-adjusted price was effectively declining. From this perspective, the decline in the VOL following the medical innovation can be interpreted in terms of basic consumption theory: An optimal choice between the two goods, survival and consumption, is given if the marginal rate of substitution between survival and consumption, i.e. the VOL, equals the price of survival in terms of consumption goods, i.e. the effective price of medical care. But then a decrease in the price of survival triggers a reallocation from consumption to survival (through the purchase of additional health care), implying a decline in the marginal rate of substitution and, thus, in the VOL.

The innovation at \(t = 150\) induces an increase in the health expenditure share of the GDP by some 1.6 percentage points (Figure 4, left panel; and Table 3). Underlying this increase in the health share is a strong increase in per capita health expenditure by some 12.2 percent (in the new steady state). The right panel in Figure 4 decomposes the increase in per capita health expenditure into an increase in individual demand at each given age, \(h(a,t)\), given the pre-innovation age-structure and price for health care (corresponding to the cyan, dashed-dotted line), the additional impact of a changing age-structure, as measured by the age-shares \(N(a,t)/N(t)\) (corresponding to the distance between the cyan, dashed dotted and the red, dotted lines), and the increase in the price.
Figure 4: Health expenditure share of GDP (left panel) and health expenditure per capita (right panel) for the benchmark (blue, solid line) and for the unanticipated increase in M in general equilibrium (green, dashed line). The cyan, dashed-dotted line indicates the pure shift in individual demand, $h(a,t)$, holding the population shares, $N(a,t)/N(t)$, and the price of medical care, $p_H(t)$, constant. The red, dotted line denotes the effect holding only $p_H(t)$ constant.

for health care, $p_H(t)$ (corresponding to the distance between the red, dotted and the green, dashed line). Overall, the instantaneous boost to demand amounts to a 6.7 percent increase in medical expenditure per capita (=55 percent of the total increase), with a further 2.8 percent increase following during the adjustment process (=23 percent of the total effect). The reason for why individual demand increases over and above the instantaneous impact lies with the fact that later born cohorts have been able to accumulate additional savings for the purchase of health care. The shift in the population structure toward higher ages with more intensive health care needs amounts to an expenditure increase by 1.8 percent (=15 percent of the total effect), with the price increase adding another 0.9 percent (=7 percent of the total effect). While a total of 78 percent of the increase in per capita health expenditure following the innovation is, thus, explained by the boost to individual demand, induced population ageing and price inflation play a significant part over the transition phase.

The shift from final goods production to health care that is following the innovation leads to a reduction of the employment share in the manufacturing sector, a reduction in the interest rate and an increase in the wage rate (see Figure 5). The change in the factor prices comes with an increase in the price of health care,37 which is underlying the dampening of the demand response to innovation.38 Furthermore, the social security payroll tax rises, following the pronounced increase

37According to Equations (32) and (33) the increase in the wage rate and in the price of health care is directly linked to the lower market interest rate.
38A partial equilibrium perturbation of $p_H$ enables us to determine the price elasticity of per-capita health care expenditures for the benchmark calibration. We find a price elasticity of $-0.3$, which is close to the estimated mean elasticity of $-0.2$ determined in the RAND Health Insurance Experiment (Manning et. al. 1987).
Figure 5: Market prices, employment share and taxes

in longevity, despite the simultaneous increase in the gross wage. Similarly, Medicare payroll taxes increase as a consequence of both greater health spending and the boost in longevity.

These sectoral and price adjustments notwithstanding, the medical advance has very little impact on GDP per capita (see Table 3). The survival gains induced by the innovation are greatest among older cohorts and, for a fixed retirement age, lead to a 1 percent reduction in the employment-population ratio, $L(t)/N(t)$.$^{39}$ At the same time, however, the expansion of the expected retirement period and the prospect of greater health expenditures in the presence of a more effective medical technology trigger additional savings, translating into a 4 percent increase in the

$^{39}$The medical innovation raises the remaining life expectancy at age 20 by 1.0 years from 58.04 years (and, thus, by 1.3 percent) and remaining life expectancy at age 65 by .81 years from 18.02 years (and, thus, by 4.5 percent).
capital stock per capita, \( K(t)/N(t) \). Overall, this leads to capital deepening, i.e. to a higher \( K(t)/L(t) \), which in optimum induces a shift of resources to the more labour intensive health care sector. As we have shown in Section 5, both the increase in \( K(t)/L(t) \) and the shift in resources to the health care sector lead to an increase in GDP per worker. Our numerical analysis shows that for the US context we are studying, this effect is strong enough to compensate (even mildly over-compensate) the decline in the employment rate.

Thus, we can summarise the following set of insights.

**Result 1** (i) Medical innovation leads to a reallocation of consumption to health care expenditures for all but the highest ages, and to a reallocation of consumption to higher ages. (ii) The general equilibrium impact of a mortality reducing medical innovation on the demand for health care tends to be dampened by an associated price increase. (iii) About 78 percent of the increase in per capita health care expenditure following a medical innovation are due to an increase in individual demand, about 15 percent are due to induced population ageing, and 7 percent are due to a price increase. (iv) Medical innovation leads to a reduction in the VOL and in the effective (quality-adjusted) price for medical care. (v) Medical innovation tends to stimulate additional saving. (vi) The ensuing increase in the economy-wide capital intensity, combined with the shift of employment into the health-care sector increase the economy-wide productivity, i.e. GDP per worker by enough to compensate the reduction of the employment-population ratio, leading to little impact on GDP per capita.

It is worth noting that the transitional dynamics following a medical innovation tie in closely with recent findings about the impact of capital deepening on the structural composition of an economy. Specifically, Acemoglu and Guerrieri (2008) show for a two-sector economy that capital deepening, i.e. an increase in the economy-wide capital intensity tends to raise the output share of the capital-intensive sector but at the same time induces a shift of both labour and capital inputs into the labour intensive sector. These shifts are accompanied by an increase in the wage rate, as is the case in our model. Acemoglu and Guerrieri (2008) go on to show that the same process is underlying unbalanced growth whenever productivity growth is larger in the capital-intensive sector.

\[ \text{Indeed, these channels have been confirmed empirically by Bloom et al. (2003) and De Nardi et al. (2010).} \]

\[ \text{As we have seen already, these shifts in quantities are accompanied by an increase in the wage rate, the latter inducing an increase in the price for health care.} \]
sector (see also Baumol 1967).

While the transition to a new equilibrium that is following a medical innovation in our model follows a similar process of unbalanced growth à la Baumol (1967), this is for rather different reasons. First, technical progress occurs in the health care sector; second, and importantly, medical progress works through the household side of the economy: Through its impact on survival and the consequent shift of the age-structure toward older cohorts, medical progress triggers an increase in savings, and, thus, in the per capita supply of capital while at the same time reducing the per capita supply of labour. Notably, this impact is present even when holding the aggregate demand for health care fixed. As we have seen, capital deepening and the sectoral shift combine to render the overall economy more productive, as measured by GDP per worker.

6.3.2 Anticipated Medical Advance

In many instances, medical advances do not arrive as "shocks", but they are anticipated in terms of prior medical research and/or the clinical trials leading to the admission of new medical technologies or pharmaceuticals. Thus, it is appropriate to take into account consumers’ anticipation of such innovations. In the following, we consider once again a medical innovation from $M(t) = 1$ to $M(t) = 2$, but assume now that it is fully anticipated. In order to gain a better understanding of the anticipation effect we assume that the innovation is taking place at $t = 200$, with the focal cohort entering at $t_0 = 170$.

To study the role of anticipation in modulating the impacts of medical innovation, it is instructive to focus on macroeconomic variables.\footnote{As compared to the previous case of a non-anticipated medical innovation, anticipation does not vastly alter the life-cycle allocation of the focal cohort. One distinction is that consumption is reduced smoothly over the full life-cycle, allowing the individual to avoid the utility loss from a sudden drop in consumption at the arrival of the innovation.} Figure 6 plots how the health share of GDP, the health care expenditures per capita, and the employment share in the production sector, $L_Y(t)/L(t)$ respectively, develop over time when individuals are anticipating the innovation. For the moment, we focus on the blue, solid line, representing the benchmark scenario as well as on the green, dashed line representing the anticipated advance in technology. Each of the three quantities exhibits a particular pattern, reflecting the impact of anticipation at aggregate level. Reading the figures backwards in time, the innovation at $t = 200$ eventually leads to the expected increase in the health share and
in the per capita expenses on health care over and above their respective benchmark levels, as well as to a corresponding shift of employment from production to the health care sector.\footnote{GDP per capita exceeds the benchmark level by a small amount, reflecting the steady-state increase in financial wealth and the capital stock due to higher longevity after the innovation.}

Notably, however, for a time span of about 30 years before the innovation, health expenditures (and consequently the health share) fall below their benchmark levels. This amounts to an anticipation effect, where individuals postpone the consumption of care to wait for the innovation to occur.\footnote{Such a demand-reducing anticipation effect has been identified empirically in regard to the consumption of pharmaceuticals prior to the Medicare D reform aimed at including pharmaceutical expenditure into the coverage (Hu et al. 2014; Alpert 2016, Kaplan and Zhang 2017).} The corresponding shrinking of the health care sector is reflected in a temporary boost to the employment share in final goods production.\footnote{A close-up look shows that the anticipation-related slump in the demand for health care itself is, in turn, anticipated in as far as prior to the slump, the demand for health care and the employment share in health care are slightly elevated over and above their benchmark levels. Overall, this amounts to an anticipation wave, akin to the one described by Feichtinger et al. (2006) for the impact of technological progress on capital accumulation.}

Figure 7 plots the development of the capital per capita, $K(t)/N(t)$, the market interest rate, $r(t)$, the wage rate, $w(t)$ and the price for health care, $p_H(t)$. The paths show a pattern that...
differs distinctly from the one arising in the case of an unanticipated shock (recall Figure 5). The postponement of health expenditures over the anticipation period translates into higher saving, an effect that is complemented by an anticipative reduction in per capita consumption below its benchmark (not shown here). The resulting boost to the capital held by individuals triggers a decline in the interest rate and a boost to the wage rate. With the health care sector being relatively labour intensive, the increase in the wage rate drives up the price for health care despite the deferral of demand. At the arrival of the medical innovation, individuals begin to dissave in order to purchase greater quantities of what is more effective health care now, and over time capital per capita falls back to its new steady-state level, which nevertheless lies above the benchmark. The factor prices and the price for health care do not return to their initial levels either. The reason for this lies with the post-innovation shift of economic activity towards the more labour intensive health care sector. Hence, while prices are driven by the supply-side over the anticipation period, they tend to be determined by the demand-side after the innovation. Finally, the boost in capital per capita over the anticipation period translates into a temporary boom of the economy, as measured by GDP per capita (see Figure 6).
We conclude this second experiment by isolating the drivers behind the changes in the level of per capita health expenditure. Figure 8a decomposes the change in health expenditure from the benchmark (blue, solid line) to the outcome under the anticipated medical advance (cyan, dotted line) into two partial effects: a price effect (red, dashed-dotted line), holding constant per capita demand $H(t)/N(t)$ at the benchmark level; and a demand effect (green, dashed line), keeping the price at the benchmark level. The overall impact of the price change is relatively small, accounting for roughly 8% of the overall increase in per capita expenditure at the point of innovation. Figure 8b decomposes the changes in the per capita demand for health care (blue, solid line = baseline; cyan, dotted line = experiment) into a component that reflects changes in the levels of individual demand, $h(a,t)$, for the baseline age-structure of the population (red, dashed-dotted line); and a component that reflects changes in the age-structure for the baseline age-profile of individual demand (green, dashed line). Similar to the case of an unanticipated innovation, the increase in individual demand levels is the dominant driver. Notably, there is an over-shooting of individual demand at the point of innovation, reflecting the short-run economic boom. The subsequent downward adjustment in the per capita demand for health care toward the new steady state is dampened, however, by the shift towards an older population with its higher demand for health care.

We can summarise as follows.

**Result 2** The anticipation of a mortality reducing innovation leads to (i) the contraction of the demand and supply for health care to a level below the benchmark for a period prior to the
innovation; (ii) the accumulation of extra capital prior to the innovation and for a certain period, following the innovation; and (iii) to a concomitant reduction (increase) in the interest rate (wage rate and price for health care) prior to the innovation. (iv) By inducing extra saving, anticipation generates a temporary economic boom. (v) The changes in health expenditure per capita before and after an anticipated innovation are predominantly demand driven rather than price driven, with a peak in demand arising at the point of innovation.

One could argue that the reduction in health care in anticipation of an innovation lacks realism in as far as health care bears on survival. We do not wish to imply that individuals facing life-threatening conditions are deferring treatments. However, anticipatory adjustments are quite probable in regard to the intensity of given treatments such as e.g. drug prescriptions (Alpert 2016; Kaplan and Zhang 2017). They are also conceivable in as far as the utilisation of distinct treatments with different intensities respond to current and expected prices and benefits (e.g. Cutler and Huckman 2003 for treatments of coronary disease). For our representative consumer approach, changes in the distribution of treatments across the patient population translate into adjustments in the intensity of care.

These arguments notwithstanding, we have studied an alternative scenario in which the demand for health care is fixed to the benchmark level before the medical innovation materialises. Although individuals continue to fully anticipate the advance, they are now restricted in their response to changes in their saving behaviour. In Figures 6 and 7 this scenario is represented by the red, dotted lines. In Figure 6, we observe that although the demand for health care is fixed before the shock, expenditures increase due to an increase in the price for health care. Importantly, however, in this scenario, too, individuals forego consumption and increase savings in anticipation of the innovation. The impact is strong enough to trigger a temporary boom similar to the one observed in the scenario without restriction. Notably the accumulation of additional capital and the associated boost to GDP sets in even earlier when individuals are not allowed to change their demand for health care in an anticipative way. Prices react almost identically in comparison to the previous experiment (see Figure 7). We can, thus, conclude that while changes to the health care sector in anticipation of a medical innovation are somewhat difficult to predict, the anticipatory boost to savings and,
consequently, to GDP appears to be a robust result.\footnote{In a further robustness check, that we do not present here, we assume that the technological advance is fully anticipated but evolves over a period of 30 years. Again, we observe the same anticipation pattern, albeit smaller in magnitude, as in the scenario where the advance arrives as a shock.}

7 Conclusion

We have set out an OLG model built around the endogenous demand and supply of health care. In contrast to much of the received macro-economic literature on health and health care, our model involves a rich model of the life-cycle, based on a realistic pattern of mortality. This allows us to characterise in detail the individual life-cycle allocation of consumption and health care, and to construct macro-economic aggregates that are based on a realistic age-structure of the population. At the micro-economic level, we can study in detail how the demand for health care responds to medical progress, taking into account induced price changes and changes in the willingness-to-pay for health care, as summarised by the value of life.

Based on a calibration of the model to the US economy in the year 2003, our numerical analysis is designed to provide a quasi-experimental identification of the channels through which changes in medical technology are transmitted between individual choices and macro-economic dynamics. Our numerical experiments yield a number of policy relevant, and potentially challenging, insights.

First, we find that a medical innovation that increases the remaining life expectancy at age 20 by some 1.1 years, boosts health expenditure per capita by some 12.2 percent, with 0.9 percentage point owing to price inflation, 1.8 percentage points owing to a shift in the age-structure towards older individuals with greater consumption of health care, and 9.5 percentage points owing to an increase in individual demand. Our finding that the expansion in health expenditure is mostly driven by an increase in utilisation is well in line with recent evidence (Bundorf et al. 2009, Chernew and Newhouse 2012). However, our model also suggests that in spite of its modest contribution to expenditure growth in accounting terms, the increase in the price for health care has a significant impact on demand as described in the following.

Second, more than half of the partial equilibrium impact on the individual demand for health care of a mortality reducing innovation is neutralized in general equilibrium by an increase in the price for medical care. This result indicates a need for a general equilibrium framework when it
comes to assessing the impact of medical change on health care expenditure, as otherwise findings may be biased.

Third, for an economy with social security and health care organised in similarity to the US (as of 2003), a costless medical innovation does not have a negative impact on economic performance, as measured by GDP. This is despite a reduction in the employment rate due to a growing population of pensioners. The main mitigating channel is the accumulation of additional savings/capital for the purpose of financing consumption over an extended life-course and purchasing more effective health care at a higher price. Indeed, this channel is very much in line with evidence for the US on savings related to health expenditures in old age (e.g. De Nardi et al. 2010). Overall, the capital deepening of the economy always combines with the shift in economic activity to the health care sector in raising GDP per worker. As it turns out for our calibration, this effect more than compensates the decline in the employment rate. Two caveats are worth of note here: The cost of medical innovation, e.g. through the absorption of production factors within a medical R&D sector may after all induce a drag on economic growth (Jones 2016). In addition, the question as to whether additional savings are induced in the wake of a medical innovation is likely to depend on the particular design of the social security system. To the extent that expenditures during retirement are financed through public transfers, the savings response is prone to be weaker, implying that the reduction in the employment rate is not sufficiently offset through the accumulation of capital. Additional offsetting impacts arise if health improvements not only translate into lower mortality but also into a greater propensity to provide labour into older ages (Kuhn and Prettner 2016).

Fourth, mortality reducing medical innovations tend to come with a reduction in the value of life over large parts of the life-course. This finding has two interesting ramifications. At face value, the reduction in the value of life arises from a reallocation by the individual of resources from consumption to health care. While per se, this is reflecting an efficient response by the individual to the availability of more effective health care, it also implies that individuals may be less willing to prevent risks to their life. Thus, some of the benefits of medical innovations in terms of improved survival prospects may well be offset by the adoption of less healthy life-styles. As we have shown, the reduction in the value of life also implies a reduction in the effective (quality-adjusted) price of

\footnote{Note, however, that within a decentralised economy with R&D-driven growth a la Romer (1990) the increase in the capital intensity of final goods production that follows the absorption of (relatively more) labour by a growing health care sector, provides a stimulus for conventional R&D (Kuhn and Prettner 2016).}
medical care as triggered by the innovation. This is in line with evidence for the US, as provided in Cutler et al. (1998) and suggests that in settings in which individuals choose the demand for health care, the value of life can be interpreted as a marginal rate of substitution, the decline of which reflects a shift in consumption toward survival by means of (additional) health care.

Fifth, anticipation of a medical innovation may come with a deferral in the demand for medical care prior to the innovation with consequences for the sectoral structure and the price structure. Furthermore, individuals always reduce consumption and boost their saving in anticipation of the advance, inducing a boost to the capital stock per capita which is strong enough to trigger a temporary economic boom. The boom is accompanied by a peak in the nominal price for medical care at the point of innovation, leading to a dampening of the impact of medical innovation on the effective price of care. While these effects are only temporary and vanish over the transition to the long-run steady state, they suggest that care needs to be taken about possible anticipation effects when assessing the impacts of medical innovation on economic and health outcomes. While we are unaware of empirical evidence on anticipation effects in the context of medical innovation, their empirical relevance has been established in the context of health policy reform (Hu et al. 2014, Alpert 2016, Kaplan and Zhang 2017) and strikes us as conceivable in the innovation context, too, certainly in regard to the anticipatory boost in savings.

In the present work, we have abstracted from long-run trends to productivity and population in order to avoid that these trends obfuscate the identification of the transmission channels of medical progress that were at the heart of this paper. Based on the insights of the present analysis, we will in future work include more realistic dynamics in regard to productivity growth as well as background trends of medical progress and population in order to arrive at a quantitatively more precise assessment of the role of medical change. Work in progress also involves the explicit modelling of a medical R&D sector in order to analyse the joint dynamics within the nexus of health expenditure, longevity expansion and medical progress.
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8 Appendix

A1: Optimal Solution to the Individual Life-Cycle Problem

The individual’s life-cycle problem, i.e. the maximisation of (1) subject to (2) and (3) can be expressed by the Hamiltonian

\[ H = uS - \lambda S \mu S + \lambda_k (r k + l w - c - \phi p H) - \tau + \pi + s, \]

leading to the first-order conditions

\[ \mathcal{H}_c = u_c S - \lambda_k = 0, \quad (36) \]
\[ \mathcal{H}_h = -\lambda S \mu_h S - \lambda_k \phi p H = 0, \quad (37) \]

and the adjoint equations

\[ \dot{\lambda}_S = (\rho + \mu) \lambda_S - u, \quad (38) \]
\[ \dot{\lambda}_k = (\rho - r) \lambda_k. \quad (39) \]

Optimality conditions (13) and (14): Evaluating (36) at two different ages/years \((a, t)\) and \((\hat{a}, t + \hat{a} - a)\), equating the terms and rearranging gives us

\[ \frac{u_c (\hat{a}, t + \hat{a} - a)}{u_c (a, t)} = \frac{\lambda_k (\hat{a}, t + \hat{a} - a) S (a, t)}{\lambda_k (a, t) S (\hat{a}, t + \hat{a} - a)} \]
\[ = \exp \left\{ \int_a^{\hat{a}} \left[ \rho + \mu (\hat{a}, t + \hat{a} - a) - r \left( t + \hat{a} - a \right) \right] d\hat{a} \right\}. \quad (40) \]

which is readily transformed into the Euler equation (13) as given in the main body of the paper.

Inserting (36) into (37) allows to rewrite the first-order condition for health care as

\[ -\mu_h (a, t) \frac{\lambda_S (a, t)}{u_c (\cdot)} = \phi (a, t) p_H (t). \quad (41) \]

Integrating (38) we obtain

\[ \lambda_S (a, t) = \int_a^\omega u (\hat{a}, t + \hat{a} - a) \exp \left[ - \int_a^{\hat{a}} (\rho + \mu) d\hat{a} \right] d\hat{a}. \]

Using this, we can express the private VOL as

\[ \psi (a, t) := \frac{\lambda_S (a, t)}{u_c (a, t)} = \int_a^\omega \frac{u_c (\hat{a}, t + \hat{a} - a)}{u_c (a, t)} \frac{u (\hat{a}, t + \hat{a} - a)}{u_c (\hat{a}, t + \hat{a} - a)} \exp \left[ - \int_a^{\hat{a}} (\rho + \mu) d\hat{a} \right] d\hat{a}. \]

Substituting from (40) and rearranging we obtain (15) as given in the main body of the paper. Inserting this into (41) gives condition (14) in the main body of the paper.
Dynamics (18) and (19): Total differentiation of (36) with respect to age gives
\[\frac{\partial}{\partial \lambda k} S \cdot c + \frac{\partial}{\partial \lambda k} S - \lambda k = \frac{\partial}{\partial \mu} S \cdot c - (\rho - r + \mu) u_c S = 0.\]
From this we obtain the consumption dynamics (18) as given in the main body of the paper.

Holding prices and the state of medical technology constant, total differentiation of
\[-\mu h (a,t) \psi (a,t) - \phi (a,t) p_H (t) = 0\]
with respect to age gives
\[-(\mu hh + \mu ha) \psi - \mu h \psi - p_H \phi = 0.\]
Substituting \(p_H = -\mu h \psi \phi^{-1}\) from (14) and rearranging, we obtain the dynamics for health care as reported in (19) within the main body of the paper.

A2: Characterisation of General Equilibrium
For each period \(t\) we have the following unknown variables:
- inputs \(\{K_Y (t), K_H (t), L_Y (t), L_H (t)\}\),
- prices \(\{r (t), w (t), p_H (t)\}\),
- aggregate demand \(\{C (t), H (t)\}\),
- aggregate net saving, equivalent to the change in the capital stock \(\dot{K} (t)\),
summing up to 10 variables. These are determined through
- 4 first-order conditions on factor inputs (21)-(24), which give the factor demand functions \(\{K_Y^d (r, w; A, M, B), K_H^d (r, w, p_H; M, B), L_Y^d (r, w; A, M, B), L_H^d (r, w, p_H; M, B)\}\), depending on prices as well as on technology and population \(\{A, M, B\}\);\(^{48}\)
- a set of first-order conditions (13) and (14) for \(a \in [0, \omega]\), which together with the individual’s life-cycle budget constraint determine the age-specific levels of consumption \(c (a, t)\) and health care \(h (a, t)\). Aggregation according to (6) and (7) gives the demand for consumption \(C (r, w, p_H; M, B, \phi)\) and health care \(H^d (p_H; M, B, \phi)\), depending on the three prices as well as on technology, population and the vector of co-insurance rates;\(^{39}\)

\(^{48}\)Note here that \(K_Y^d (r, w; A, M)\) and \(L_Y^d (r, w; A, M)\) may vary with \(M\) and \(B\) through its impact on the aggregate supply of effective labour \(L\).

\(^{39}\)Through the life-cycle budget constraint and the individual Euler equation the demand function \(C (\cdot)\) is also contingent on the expectation about future prices over the remaining life-course. The same applies to the demand function \(H^d (\cdot)\) for which the future price paths filter in through the VOL.
• 4 market clearing conditions

\[
\begin{align*}
K_Y^d (r, w; A, M, B) + K_H^d (r, w, p_H; M, B) &= K, \\
L_Y^d (r, w; A, M, B) + L_H^d (r, w, p_H; M, B) &= L(M, B), \\
F(K_H^d (r, w, p_H; M, B), L_H^d (r, w, p_H; M, B)) &= H^d (p_H; M, B, \phi), \\
Y(K_Y^d (r, w; A, M, B), AL_Y^d (r, w; A, M, B)) &= C(r, w, p_H; M, B, \phi) + \hat{K} + \delta K,
\end{align*}
\]

which determine the set of equilibrium prices \( \{ r^* (A, M, B, \phi, \hat{K}) \} \) and aggregate net saving, as captured by \( \hat{K} \).

**A3: Equilibrium Relationships with Cobb-Douglas Technologies**

Consider the Cobb-Douglas specifications in (30) and (31). From the first-order conditions (21), (22), (23) and (24) we then obtain the (implicit) factor demand functions

\[
\begin{align*}
K_Y^d (t) &= \frac{\alpha Y(t)}{r(t) + \delta}, \\
L_Y^d (t) &= \frac{(1 - \alpha) Y(t)}{w(t)}, \\
K_H^d (t) &= \frac{\beta p_H(t) F(t)}{r(t) + \delta}, \\
L_H^d (t) &= \frac{(1 - \beta) p_H(t) F(t)}{w(t)}.
\end{align*}
\]

Combining (42) with (43) and (44) with (45) we obtain the equilibrium capital intensity

\[
\begin{align*}
k_Y^*(t) := \frac{K_Y^d (t)}{L_Y^d (t)} &= \frac{\alpha}{1 - \alpha \frac{w(t)}{r(t) + \delta}}, \\
k_H^*(t) := \frac{K_H^d (t)}{L_H^d (t)} &= \frac{\beta}{1 - \beta \frac{w(t)}{r(t) + \delta}}.
\end{align*}
\]

and, thus, \( K_Y^d (t) = k_Y^*(t) L_Y^d (t) \). Using \( k_Y^*(t) \) in (30) to rewrite \( Y(t) = L_Y^d (t) A(t)^{1-\alpha} (k_Y^*)^\alpha \) and inserting this in (43) we can solve for the equilibrium wage as a function of the interest rate

\[
w^* (t) = \tilde{w} (r(t); A(t)) = (1 - \alpha) A(t) \left[ \frac{\alpha}{r(t) + \delta} \right]^{\frac{1}{1-\alpha}}.
\]

This, in turn, determines the capital intensities \( k_Y^*(t) = \tilde{k}_Y (r(t); A(t)) \) and \( k_H^*(t) = \tilde{k}_H (r(t); A(t)) \). Using the market clearing condition \( F(p_H^d (t); K_H^*(t), L_H^d (t)) = H^d (p_H^* (t); M(t), B(t)) \) and (44) and (45) we obtain the general equilibrium price for health care as

\[
\begin{align*}
p_H^*(t) &= \tilde{p}_H (r(t), w^* (t), H^*_3 (t)) \\
&= \tilde{p}_H (r(t); A(t), M(t), B(t)) \\
&= \frac{(r + \delta)^\beta w^{1-\beta}}{\beta^\beta (1 - \beta)^{1-\beta}}.
\end{align*}
\]
Reinserting this, we obtain the equilibrium utilisation of health care, as
\[ H^d(p_H^t(t); M(t), B(t)) = \hat{H}(r(t); A(t), M(t), B(t)). \] Using (45) we can determine now \( L_H^*(t) = \hat{L}_H(p_H^t(t), w^*(t), H'_a(t)) = \hat{L}_H(r(t); A(t), M(t), B(t)). \) The labour market equilibrium then determines
\[ L_H^*(t) = L(t) - L_H^*(t), \]
where \( L(t) = \hat{L}(r(t); A(t), M(t), B(t)). \) This implies the restriction
\[ \hat{L}(r(t); A(t), M(t), B(t)) \geq \hat{L}_H(r(t); A(t), M(t), B(t)). \]
Given this is satisfied, we now have all inputs and outputs as functions of \( r(t) \) and the states \( \{A(t), M(t), B(t)\} \).

**A4: Impact of Medical Technology**

**Impact on the demand for health care and on the VOL:** Totally differentiating the first-order condition for individual health demand, \(-\phi(a,t)p_H(t) - \mu_h(a,t)\psi(a,t) = 0\), with respect to the state of technology \( M(t) \) gives
\[ -\phi dp_H - (\mu_{hh}dM + \mu_{hM}dM)\psi - \mu_h d\psi = 0, \]
which transforms to
\[
\frac{dh(a,t)}{dM(t)} = -\frac{1}{\mu_{hh}} \left[ \mu_{hM} + \frac{1}{\psi(a,t)} \left( \phi \frac{dp_H(t)}{dM(t)} + \mu_h(a,t) \frac{d\psi(a,t)}{dM(t)} \right) \right]. \tag{50}
\]
The impact of technology on the private value of life, as defined in (15), is given by
\[
\frac{dv(a,t)}{dM(t)} = \int_{a}^{\omega} \frac{dv(\hat{a},t+\hat{a}-a)}{dM(t)} R(\hat{a},a) + v(\hat{a},t+\hat{a}-a) \frac{dR(\hat{a},a)}{dM(t)} d\hat{a} = \int_{a}^{\omega} \frac{dv(\hat{a},t+\hat{a}-a)}{dM(t)} R(\hat{a},a) - v(\hat{a},t+\hat{a}-a) R(\hat{a},a) \int_{a}^{\hat{a}} \frac{dr(t+\hat{a}-a)}{dM(t)} d\hat{a} \tag{51}
\]
where
\[
\frac{dv(a,t)}{dM(t)} = \left( \frac{u_c - u_{cc}}{u_c^2} \right) \frac{dc(a,t)}{dM(t)} = \left( 1 - \frac{u_{cc}}{u_c} \right) \frac{dc(a,t)}{dM(t)}. \]

Note, that \((1 - \frac{u_{cc}}{u_c})\) is always positive: Assuming \( b \) is sufficiently large and \( c > c_0 \), \( u(c) = b + \frac{(c-c_0)^{1-\sigma}}{1-\sigma} > 0 \), \( u_c = (c-c_0)^{-\sigma} > 0 \) and \( u_{cc} = -\sigma(c-c_0)^{-\sigma-1} < 0 \). Equation (26) is then obtained by inserting (51) into (50).

**Impact on the the wage rate and price for health care:**\(^{51}\) In the following we derive

\(^{50}\) Note that through the impact of the demand for health care on the pattern of survival, labour supply becomes a function of the prices and the states of the economy.

\(^{51}\) In the following, we drop the time index for notational convenience.
equation (32) and (33). We use equation (48) from Appendix A3 and obtain

\[
\frac{dw}{dM} = -A\alpha \left( \frac{1}{1-\alpha} \right) (r+\delta)^{\frac{1}{1-\alpha}} \frac{dr}{dM}
\]

\[
\frac{dw}{dM} = -A \left( \frac{\alpha}{r+\delta} \right) \frac{dr}{dM}
\]

\[
\frac{dw}{dM} = -\frac{\alpha}{1 - \alpha} w \cdot \frac{dr}{dM}.
\]

Hence, given equation (49), it then holds, that

\[
\frac{dp_H}{dM} = 1 \beta \left( \frac{1 - \beta}{1 - \beta} \right) A \left( \frac{1}{1-\alpha} \right) (r+\delta)^{\frac{1}{1-\alpha}} \frac{dr}{dM}
\]

\[
\frac{dp_H}{dM} = \frac{1}{\beta(1-\beta)^{1-\beta}} \frac{dr}{dM} (r+\delta)^{-1} \alpha \left( \frac{\alpha}{1-\beta} \right)
\]

\[
\frac{dp_H}{dM} = \frac{p_H \beta - \alpha \beta r + \delta}{1 - \alpha \beta dM}.
\]

Impact on the GDP per worker: In the following we will assume Cobb-Douglas specifications (30) and (31) of the production functions, see Appendix A3 for details. The GDP is defined as the sum of output value in the health care sector, \(p_H F\), and in the final good sector, \(Y\). Hence, GDP per unit of labour is given by

\[
\frac{GDP}{L} = 1 \left( \frac{p_H F + Y}{Y} \right) = \frac{Y}{L} \left( \frac{p_H F}{Y} + 1 \right).
\]

Defining the employment share of the final goods sector as \(\lambda := \frac{L_Y}{L}\) one can then show that

\[
\frac{GDP}{L} = \left[ \frac{1 - \alpha 1 - \lambda}{1 - \beta \lambda} + 1 \right] A^{1-\alpha} \left( \frac{K_Y}{K_L} \right)^\alpha \lambda \left( \frac{K}{L} \right)^\alpha
\]

where we used equation (30) together with

\[
\frac{p_H F}{Y} = \frac{1 - \alpha 1 - \lambda}{1 - \beta \lambda}
\]

which follows from dividing equation (45) by (43) and rearranging The economy-wide capital-intensity can be expressed as

\[
\frac{K}{L} = \frac{K_Y + K_H}{L_Y + L_H} = \frac{\alpha Y + \beta p_H F}{(1 - \alpha)Y + (1 - \beta)p_H F} \frac{w}{r + \delta} = \frac{(1 - \alpha)\beta + (\alpha - \beta)\lambda}{(1 - \alpha)(1 - \beta)} \frac{w}{r + \delta}
\]

where equation (53) was employed. Using in addition (46) we can write

\[
\frac{K_Y/L_Y}{K/L} = \frac{\alpha}{(1 - \beta)^{1-\beta} + (\alpha - \beta)\lambda}
\]

Substituting this into (52) and rearranging we obtain

\[
\frac{GDP}{L} = \frac{1 - \alpha + (\alpha - \beta)\lambda}{1 - \beta} A^{1-\alpha} \left[ \frac{\alpha}{\beta(1 - \alpha) + (\alpha - \beta)\lambda} \right] \left( \frac{K}{L} \right)^\alpha
\]
as reported in equation (34). Taking the total derivative with respect to medical technology then yields

\[
\frac{d}{dM} \left( \frac{\text{GDP}}{L} \right) = (\alpha - \beta) \frac{\text{GDP}}{L} \left[ \frac{1}{1 - \alpha + (\alpha - \beta) \lambda} - \frac{\alpha}{\beta (1 - \alpha) + (\alpha - \beta) \lambda} \right] \frac{d\lambda}{dM}
\]

\[
= \alpha \frac{\text{GDP}/L}{K/L} \frac{d}{dM} \left( \frac{K}{L} \right)
\]

\[
= \frac{- (1 - \alpha) (\alpha - \beta)^2 (1 - \lambda)}{[1 - \alpha + (\alpha - \beta) \lambda][\beta (1 - \alpha) + (\alpha - \beta) \lambda]} \frac{\text{GDP}}{L} \frac{d\lambda}{dM}
\]

\[
+ \alpha \frac{\text{GDP}}{K} \frac{d}{dM} \left( \frac{K}{L} \right),
\]

as reported in equation (35) in the main body of the paper. Note, that the denominator \([1 - \alpha + (\alpha - \beta) \lambda][\beta (1 - \alpha) + (\alpha - \beta) \lambda]\) is positive as follows from equation (54).

A5: Solving the Numerical Problem

We pursue the following steps towards tracing out the numerical solution, sketched here for the benchmark scenario, using the specific functional forms presented in section 6:

1. We derive from the first-order condition for consumption (13) the relationship

\[
[c(a, t_0 + a) - c_0]^{-\sigma} = [c(0, t_0) - c_0]^{-\sigma} \exp \left\{ \int_0^a [\rho - r(t_0 + \hat{a}) + \mu(\hat{a})] d\hat{a} \right\}.
\]

(55)

2. We derive the life-cycle budget constraint

\[
\int_0^\infty \left[ \begin{array}{c}
w(t_0 + a) l(a) - c(a, t_0 + a) + \pi(a, t) \\
-\phi(a, t)p_H(t_0 + a) h(a, t_0 + a) - \tau(a, t) + s(t_0 + a)
\end{array} \right] R(a, 0) da = 0,
\]

with \(R(a, 0)\) as given by (17). We then insert (55) and obtain the consumption level

\[
c(0, t_0) - c_0 = \int_0^\infty \left[ \begin{array}{c}
w(t_0 + a) l(a) - c_0 + \pi(a, t) \\
-\phi(a, t)p_H(t_0 + a) h(a, t_0 + a) - \tau(a, t) + s(t_0 + a)
\end{array} \right] R(a, 0) da \]

\[
\int_0^\infty \exp \left\{ \int_0^a \left[ \frac{1 - \sigma}{\sigma} r(t_0 + \hat{a}) - \frac{\rho + \mu(\hat{a})}{\sigma} \right] d\hat{a} \right\} da
\]

(56)

for an individual born at \(t_0\), contingent on the stream of health care, \(h(a, t_0 + a)\), and the set of prices \(\{w(t_0 + a), r(t_0 + a), p_H(t_0 + a)\}\) over the interval \([t_0, t_0 + \omega]\).

3. We derive from the first-order condition for health care (14) a vector of age-specific demand levels

\[
h(a, t_0 + a) = \left( \lambda_s(a, t_0 + a) [c(a, t_0 + a) - c_0]^{\sigma} \tilde{\mu}(a) \eta(a) c(a) M(t_0 + a)^{\gamma(a)} \right)^{1/\gamma(a)}
\]

(57)

for all \(a \in [0, \omega]\).

4. We show in Appendix A3 that the set of prices \(\{w(t_0 + a), p_H(t_0 + a)\}\) as well as all input and output quantities can be expressed in terms of the interest rate \(r(t_0 + a)\) alone.
5. Using (55) together with (57) we can calculate the life-cycle allocation for consumption, \( c(a, t_0 + a) \), depending on the allocation for health expenditures, \( h(a, t_0 + a) \), \( \forall a \in [0, \omega] \) and on the set of prices \( \{ w(t_0 + a), r(t_0 + a), p_H(t_0 + a) \} \) over the interval \([t_0, t_0 + \omega]\). Vice versa, the allocation of health expenditures can be calculated from the allocation of consumption and the macroeconomic prices.

6. We apply these calculations on initial guesses of \( c \) and \( h \) iteratively. We then use the results as an initial guess to the age-structured optimal control algorithm, as presented in Veliov (2003). This yields an optimal allocation of individual consumption and health expenditures contingent on an initially assumed \( r(t_0 + a) \).

7. Drawing on this, we apply the following recursive approximation algorithm: (i) Guess an initial interest rate \( r(t_0 + a) \) and derive the optimal life-cycle allocation. (ii) Based on this, calculate the market interest rate \( r^*(t_0 + a) \) from the capital market equilibrium \( K^d(r(t_0 + a), \hat{w}(r(t_0 + a))) = K^s(r(t_0 + a)) \). (iii) Adjust the initial interest rate, so that it approaches \( r^*(t_0 + a) \), e.g. by setting \( r_1(t_0 + a) := r_0(t_0 + a) + \epsilon(r^*(t_0 + a) - r_0(t_0 + a)) \), \( \epsilon \in (0, 1] \). The process converges to an interest rate for which households optimise and capital demand equals capital supply. The output market clearing condition, \( Y(t_0 + a) = C(t_0 + a) + \dot{K}(t_0 + a) + \delta K(t_0 + a) \) then determines the dynamics of the capital stock to the next period. (iv) This process is reiterated in a recursive way, employing a solution algorithm based on Newton’s method. Equations (55)-(57) allow us to verify ex-post an optimum life-cycle allocation for the focal cohort born at \( t_0 \). While the numerical algorithm cannot determine in a precise way the optimal allocation for other cohorts, it nevertheless structures the allocation in a way that approximates the optimum for all cohorts.
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