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A contribution to the theory of fertility and economic development

Luca Gori* • Mauro Sodini†

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Abstract

The aim of this research is to build on a theory for explaining economic development in a (neoclassical) growth model with endogenous fertility. The economy is comprised of overlapping generations of rational and identical individuals and identical competitive firms producing with a constant-returns-to-scale technology with no externalities. From a theoretical perspective, the distinguishing feature of this work is that endogenous fertility *per se* is able to explain the existence of low and high development regimes. It provides alternative reasons (history driven or expectations driven) why some countries enter development trajectories with high GDP and low fertility and others experience under-performances with low GDP and high fertility. The model is also able to reproduce fertility fluctuations and explain the baby busts and baby booms observed in the last century in some developed countries.

Keywords Economic development; Endogenous fertility; Local and global indeterminacy; OLG model

JEL Classification C61; C62; J1, J22; O41

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1 Introduction

"...if the theory is complicated, it's wrong." Richard P. Feynman

The present research explains the reasons why some countries develop whilst others remain entrapped in stagnation or poverty by using a neoclassical growth model with overlapping generations (OLG) and endogenous fertility (child quantity). The work enters the debate about history versus self-fulfilling expectations as in the seminal articles of Krugman (1991) and Matsuyama (1991). It introduces a novel *utility-driven* mechanism describing the existence of different convergence groups of countries and confirming the empirical findings of Mankiw et al. (1992) and the related literature. It also provides a new theoretical reason why economies starting from similar initial conditions experience different development trajectories or initially poorer (resp. richer) economies enter a phase of sustained development (resp. under-development) with larger values of GDP and lower fertility rates (resp. smaller values of GDP and higher fertility rates), as several European, Latin American and East Asian countries show by considering the end of World War II as a starting point. From a theoretical perspective, this work complements the continuous-time optimal growth model (encompassing the child quantity-quality trade-off) of Palivos (1995), where multiple steady states and problems of coordination failures were *production-driven*. The work now proceeds by discussing the main motivations and the links to the related literature.

Demographic variables were recognised to play a preeminent role as determinants of long-term macroeconomic outcomes of nations. The interactions between demographics and economics are the object of a growing body of theoretical and empirical studies [see, for instance, Fogel (2004); Galor (2011); Cervellati and Sunde (2013)]. Human beings in Western countries have experienced tremendous improvements in both the standard of living and quality of life in the past two centuries, although there is still no clear consensus on which were the main sources of this development (e.g., human capital accumulation [Glaeser et al. (2004)] and/or quality of institutions [Acemoglu et al. (2001)]). The influence of longevity and fertility on economic growth (a concept referred to the growth of an economic variable such as GDP per capita) and development (a multi-dimensional phenomenon related - amongst other things - to fertility, life expectancy, poverty, quality of institutions, the distribution of income and so on) has led several economists to consider them as endogenous variables and tackle this issue in models that - since the pioneering work of Becker (1960) - have originated the Unified Growth Theory (UGT) [Galor and Weil (2000); Galor and Moav (2002, 2004); Galor (2011)]. This theory aims at explaining the process of economic development based on the interaction between endogenous technological progress and human capital formation by showing that endogenous fertility [Galor and Weil (2000)] and endogenous human evolution [Galor and Moav (2002)] are relevant causes of the demographic transition (that is, the transition from stagnation to growth is accompanied by a demographic shift from high to low birth and death rates [Jones and Ter-

tilt (2006])). A distinctive feature of this literature is the building of a conceptual framework to explain the process of development across nations and continents. This process is usually divided in three phases: 1) Malthusian epoch (a long period that ends up almost at 1750 AD). 2) Post-Malthusian regime (1750-1870). 3) Modern growth regime (1870-today). The works belonging to this literature have substantially modified the standard OLG model by including some additional ingredients, such as human capital accumulation, child and adult mortality and so on. Except some of the seminal contributions of Galor and his co-authors, which tend to emphasise the importance of fertility and child mortality as determinants of the economic and demographic transition, the mechanisms leading from a phase of pre-industrialisation to a phase of intensive industrial production are technology-and-mortality-driven [Cervellati and Sunde (2005, 2015); Fiaschi and Fioroni (2014)]. In these works, fertility is introduced to explain the phases of the demographic transition without representing, however, the triggering factor amongst the various stages of development.

There are several contributions analysing problems of economic development in growth models with endogenous fertility that do not strictly belong to the UGT. These works aim at explaining the reasons why some countries achieve high values of GDP and low fertility rates whereas others remain entrapped in a situation where GDP is low and fertility is high. In most cases, scholars use the OLG framework. This is because it represents a natural basis where including demographic variables, although there exist some works in a continuous-time set up with infinite horizon optimising agents [Wang et al. (1994); Palivos (1995); Palivos et al. (1997)]. Within the class of OLG models with finite lived individuals, we mention here the works of Galor and Weil (1996), Blackburn and Cipriani (2002) and Fanti and Gori (2014) that consider fertility as a consumption good and come to light several distinct reasons why multiple stationary equilibria can exist. All these models share the same characteristic, i.e. they include substantial changes to the standard OLG framework. In particular, Galor and Weil (1996) examine the relationship between fertility and economic growth by including *gender differences*. The decrease in fertility and the increase in output growth is due to a threefold reason: the increase in capital per worker and women's relative wages; the resulting increase in the opportunity cost of children that contributes to reduce fertility; the decline in fertility eventually favours an additional the increase in capital per worker. Multiple development regimes were possible because of the positive effect on the rate of output growth caused by women joining the labour force. Differently, the existence of multiple steady states in the models of Blackburn and Cipriani (2002) and Fanti and Gori (2014) is due to (endogenous) adult mortality changes driven, respectively, by education and public health expenditure. The main findings are that poor countries tend to have high fertility and mortality rates together with a low level of GDP per capita, according to the empirical evidence on the demographic transition.

The present article adds to the literature an explanation (history or expectations driven) for the fertility transition within the wider phenomenon of the demographic transition based on

the existence of a high regime of development (the accumulation of capital is high and fertility is low), which resembles the Modern growth regime, and a low regime of development (the accumulation of capital is low and fertility is high), which resembles the Malthusian regime. These results were obtained in a very basic framework with *homogeneous* agents and no externalities in production. This is worth to be mentioned as the model is able to give rise to coordination failures: agents know that there exist multiple equilibria but do not know how to coordinate themselves to avoid Pareto dominated outcomes. This is uncommon in the absence of externalities and, to the best of our knowledge, it is the first time that indeterminacy is fertility-driven instead of labour-supply-driven in an OLG model.

The rest of the article proceeds as follows. Section 2 develops a simplified version of the model of child quantity and time cost of children of Galor and Weil (1996). Section 3 characterises the conditions for the existence of stationary equilibria and studies equilibrium dynamics by clarifying the main theoretical results with simulative exercises (global analysis). Section 4 outlines the conclusions. An Appendix provides some mathematical results.

2 The model

This section builds on a simplified version of the model of Galor and Weil (1996). The OLG (general equilibrium) closed economy is populated by a continuum of (perfectly) rational and identical individuals of measure N_t per generation ($t = 0, 1, 2, \dots$). The life of the typical agent is divided into childhood and adulthood. As a child, an individual does not make economic decisions and spends time in the parent's household by consuming resources directly from him. As an adult, an individual is economically active, works and takes care of children when he is young, and retires when he is old. The N_t young members of generation t overlap for one period (youth) with N_{t-1} old individuals belonging to generation $t - 1$ and for one period (old-age) with N_{t+1} young individuals belonging to generation $t + 1$. When young, an individual is endowed with 2 units of time.¹ We assume that raising children is a purely time consuming activity [see Guryan et al. (2008) for empirical evidence]. The child rearing technology requires an exogenous fraction $q < 2$ of the parent's time endowment to raise a child that represents parent's foregone earnings. Then, by letting $n_t > 0$ be the number of children at time t , qn_t is the time needed to care for n_t descendants of a parent that belongs to generation t . This implies that the time required to care for children cannot be spent working and the marginal time cost of children is constant, i.e. the opportunity cost of children is proportional to the wage rate. The remaining share $\ell_t = 2 - qn_t > 0$ of time is supplied to firms in exchange for wage w_t per unit of labour. Individuals consume only in the second period of their life [Woodford (1984); Reichlin (1986); Galor and Weil (1996)]. The budget constraint of the young individual

¹Considering a time endowment larger than one allows obtaining a growth factor of population larger than, smaller than or equal to one, i.e. a population that grows, decreases or it is stationary over time.

representative of generation t is $s_t = w_t \ell_t$, i.e. labour income is entirely saved (s_t) to consume one period later. When old, an individual retire and consumption (C_{t+1}) is determined by the amount of resources saved when young plus expected interest accrued from time t to time $t + 1$, so that $C_{t+1} = R_{t+1}^e s_t$ where R_{t+1}^e is the expected interest factor, which will become the realised interest factor at time $t + 1$. Therefore, the lifetime budget constraint can be expressed as follows:

$$C_{t+1} = R_{t+1}^e w_t (2 - qn_t), \quad (1)$$

where $qn_t < 2$ must hold to satisfy the constraint dictated by the individual time endowment.

An adult individual of generation t has preferences towards the number of children and material consumption. Parents are selfish and give birth to children not for being supported when they will be old or enjoying their well-being but exclusively to increase their own utility. The lifetime utility index of the individual representative of generation t is described by the following additively separable function:

$$U_t(n_t, C_{t+1}) = \frac{n_t^{1-\gamma}}{1-\gamma} + \frac{C_{t+1}^{1-\sigma}}{1-\sigma}, \quad (2)$$

where $\gamma > 0$ ($\gamma \neq 1$) and $\sigma > 0$ ($\sigma \neq 1$) measure the constant elasticity of marginal utility with respect to fertility and consumption, respectively. This functional form of a utility function is aimed for generality. In the particular case $\gamma = 1$ and $\sigma = 1$, the expression in (2) boils down to $U_t(n_t, C_{t+1}) = \ln(n_t) + \ln(C_{t+1})$. The formulation for lifetime utility as those expressed in (2) is crucial for the results shown later in this article and, in the case of no young material consumption, $1/\sigma$ (resp. $1/\gamma$) may be interpreted as a proxy for measuring the (constant) elasticity of substitution in consumption (resp. children). An increase in σ (resp. γ) causes a decline in the marginal utility of material consumption (resp. fertility) when C_{t+1} (resp. n_t) increases. Empirical evidence [Hall (1988); Jones and Schoonbroodt (2010); Havranek et al. (2015)] finds that the elasticity of substitution in consumption is consistently smaller than one ($\sigma > 1$). Differently, $1/\gamma$ can be interpreted as a measure for the elasticity of (inter-generational) substitution between consumption and children. In a recent work, Córdoba and Ripoll (2016) find that this index is significantly larger than one ($\gamma < 1$), meaning that material goods and children tend to be substitutes over time.

By substituting out (1) in (2) and taking factor prices as given, the maximisation problem of the individual representative of generation t is:

$$\max_{n_t \in (0, 2/q)} \left\{ \frac{n_t^{1-\gamma}}{1-\gamma} + \frac{[R_{t+1}^e w_t (2 - qn_t)]^{1-\sigma}}{1-\sigma} \right\}. \quad (3)$$

Therefore, the first order condition is given by:

$$n_t^{-\gamma} = q(R_{t+1}^e w_t)^{1-\sigma} (2 - qn_t)^{-\sigma}. \quad (4)$$

Eq. (4) implies that the marginal utility of an extra child should be equal to the (indirect) marginal utility of consumption. It tells us how much consumption to give up when old to

consume one more child when young by keeping utility unaltered. The expression in (4) can also be rewritten as follows:

$$\frac{n_t^\gamma}{(2 - qn_t)^\sigma} = \frac{(R_{t+1}^e w_t)^{\sigma-1}}{q}. \quad (5)$$

This formulation allows clarifying the effects (at the individual level) of a change in wage income on the demand for children. In particular, the left-hand side of (5) is an increasing function of n_t . An increase in the wage causes a twofold effect. On the one hand, it implies that children become more costly relative to material consumption. Then, at the optimum, an individual wants to substitute the consumption of children when young for the consumption of material goods when old (substitution effect). On the other hand, an individual gets richer as the value of his overall time endowment increases (income effect). As he offers a positive amount of his time endowment to firms (labour supply), what eventually determines the sign of the change in the demand for children following a wage increase is the sign of the income effect. When $\sigma > 1$ (resp. $\sigma < 1$) the income effect is positive (resp. negative) and children are a normal (resp. inferior) good. Therefore, an increase in wage income increases (resp. reduces) the demand for children through this channel. When $\sigma = 1$ children are neither normal nor inferior goods (the substitution and income effects cancel each other out in that case) and the demand for children is independent of the (capitalised) wage income. Definitely, when children are a normal good ($\sigma > 1$) the substitution effect and the income effect are of opposite sign and the final effect of a change in wage income on the demand for children is a priori uncertain. In contrast, when children are an inferior good ($\sigma < 1$) the substitution effect and the income effect are both negative so that the demand for children reduces when the wage increases (this is in accord with the Beckerian tradition).² Therefore, an additively separable (non-log) utility function is able per se to provide a reason why fertility reacts differently to a change in wage income. This adds a novel utility-driven mechanism that can potentially explain the historical pattern of the fertility transition within the wider phenomenon of the demographic transition. The analysis of the relationship between the number of children and GDP per young person will be clarified later in this article in both cases $\sigma > 1$ and $\sigma < 1$ when we will account for the macroeconomic (general equilibrium) effects of the model.

Firms are identical and act competitively on the market. At time t , the representative firm produce a homogeneous good (Y_t) by combining capital (K_t) and labour (L_t) by means of the neoclassical technology:

$$Y_t = AF(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}, \quad (6)$$

where $0 < \alpha < 1$ is the output elasticity of capital and $A > 0$ is the (constant) total factor pro-

²This result can be ascertained by imposing the restriction $\gamma = \sigma$. In this case, in fact, from (5) one can get the unique closed-form expression for n_t . However, this simplification does not represent a good approximation to characterise all the development scenarios the model is able to generate. Then, given also the different empirical estimates on γ and σ , we will continue studying the model by keeping these parameters at different values.

ductivity. Profits are given by $AK_t^\alpha L_t^{1-\alpha} - w_t L_t - R_t K_t$. The temporary equilibrium condition in the labour market is given by $L_t = \ell_t N_t = (2 - qn_t)N_t$, i.e. the amount of labour hired by firms is equal to the mass of young individuals of generation t times the fraction of time they spend working. By taking factor prices as given, profit maximisation gives:

$$w_t = (1 - \alpha)Ak_t^\alpha(2 - qn_t)^{-\alpha}, \quad (7)$$

$$R_t = \alpha Ak_t^{\alpha-1}(2 - qn_t)^{1-\alpha}, \quad (8)$$

where $k_t := K_t/N_t$ is the stock of capital per young person.

The market-clearing condition in the capital market is $K_{t+1} = S_t = s_t N_t$. As $N_{t+1} = n_t N_t$ determines the evolution of fertility, equilibrium implies:

$$k_{t+1} = \frac{s_t}{n_t}, \quad (9)$$

where $s_t = w_t(2 - qn_t)$ and n_t is determined by the individual first order conditions. By using (4), (7), (8), (9) and knowing that individuals have perfect foresight, so that $R_{t+1} = \alpha Ak_{t+1}^{\alpha-1}(2 - qn_{t+1})^{1-\alpha}$,³ the dynamics of the economy is characterised by the two-dimensional map:

$$M : \begin{cases} k_{t+1} &= Q_1(k_t, n_t) := \frac{A(1 - \alpha)k_t^\alpha(2 - qn_t)^{1-\alpha}}{n_t} \\ n_{t+1} &= Q_2(k_t, n_t) := \frac{1}{q} \left(2 - k_t^{\frac{-\alpha}{1-\alpha}} (2 - qn_t)^{-\alpha + \frac{1}{(1-\sigma)(1-\alpha)}} n_t^{-1 - \frac{\gamma}{(1-\sigma)(1-\alpha)}} B \right) \end{cases}, \quad (10)$$

where capital per young person (resp. fertility) is a state (resp. control) variable and

$$B := A^{\frac{-(1+\alpha)}{1-\alpha}} (1 - \alpha)^{\frac{-\alpha}{1-\alpha}} \alpha^{\frac{-1}{1-\alpha}} q^{-\frac{1}{(1-\sigma)(1-\alpha)}}. \quad (11)$$

We note that the initial value of the control variable (n_0) does not affect the initial value of the state variable $k_0 := K_0/N_0$. The assumptions of additively separable preferences and perfect foresight are crucial and contribute to determine a dynamic expression for the number of children, which is absent in the case of log-utility (in which it is not necessary to specify any expectations formation mechanisms about the future interest factor). In fact, if $\gamma = 1$ and $\sigma = 1$ fertility is constant and given by $n = 1/q$ so that the dynamics of the economy is characterised by the uni-dimensional map $k_{t+1} = q(1 - \alpha)Ak_t^\alpha$, from which one can get the unique (globally asymptotically stable) stationary equilibrium $k^* = [q(1 - \alpha)A]^{\frac{1}{1-\alpha}}$.

³This is an important assumption allowing to get a link between k and n at time $t + 1$.

3 Existence of stationary equilibria and equilibrium dynamics

The analysis begins with by identifying the stationary equilibria of the map. By the first equation of map M we have that at the stationary state it must hold

$$k = h(n) := (2 - qn) \left(\frac{A(1 - \alpha)}{n} \right)^{\frac{1}{1-\alpha}}. \quad (12)$$

The steady-state values of the fertility rate are solutions of the following equation:

$$g(n) := \frac{1}{q} \left(2 - n^{\frac{2\alpha-1}{(1-\alpha)^2} + \frac{\gamma}{(\sigma-1)(1-\alpha)}} (2 - qn)^{-\frac{\alpha}{1-\alpha} - \frac{1}{(\sigma-1)(1-\alpha)}} D \right) = n, \quad (13)$$

where $D := B[(1 - \alpha)A]^{-\frac{\alpha^2}{(1-\alpha)^2}}$. From the expression in (12) it follows a negative relationship between n and k at the stationary state. This implies that larger values of the capital stock are related to lower values of fertility. Then, under the assumption that children are a normal good ($\sigma > 1$) the model overcomes the paradox between individual choices and macro behaviour. In fact, the empirical evidence shows the existence of a positive relationship between wage income and the number of children (positive income effect) at the individual level, whereas larger values of GDP are associated with lower fertility rates in the last stages of the economic and demographic transition at an aggregate level. Although assessing the effects of changes in fertility on growth is not an easy task (this is because population variables change endogenously along the process of development), there exists evidence confirming the importance of fertility declines for explaining GDP growth [Jones and Tertilt (2006); Ashraf et al. (2013)].

In order to characterise the number of equilibria, we now study the behaviour of g when $n \rightarrow 0^+$ and when $n \rightarrow (2/q)^-$.

Lemma 1 *Let*

$$\tilde{\gamma} := \frac{(1 - 2\alpha)(\sigma - 1)}{1 - \alpha}, \quad (14)$$

be a threshold value of γ . (1) If $\sigma > 1$ and $\gamma > \tilde{\gamma}$ or if $\sigma < 1$ and $\gamma < \tilde{\gamma}$ then $\lim_{n \rightarrow 0^+} g(n) = 2/q$. (2) If $\sigma > 1$ and $\gamma < \tilde{\gamma}$ or if $\sigma < 1$ and $\gamma > \tilde{\gamma}$ then $\lim_{n \rightarrow 0^+} g(n) = -\infty$. (3) If $\sigma > 1$ then $\lim_{n \rightarrow (2/q)^-} g(n) = -\infty$. (4) If $\sigma < 1$ then $\lim_{n \rightarrow (2/q)^-} g(n) = 2/q$. (5) In addition, g has a critical point

$$n_{crit} := \frac{(4\sigma - 2\gamma - 4)\alpha - 2\sigma + 2\gamma + 2}{[\alpha^2(\sigma - 1) + (\sigma - \gamma)\alpha - \sigma + \gamma]q}, \quad (15)$$

in the interval $(0, 2/q)$ if and only if $\gamma < \tilde{\gamma}$. (6) If

$$[(1 - \alpha)\gamma + 2\alpha\sigma - 2\alpha - \sigma + 1][(1 - \alpha)\gamma + 3\alpha\sigma - 2\alpha - 2\sigma + 1](\sigma - 1) > 0 \quad (16)$$

then one or two inflection points for g can exist in the interval $(0, 2/q)$.

Proof. Results (1)-(4) are obtained by studying the sign of the exponents of the terms n and $2 - qn$ in the expression $g(n)$. Results (5) and (6) are obtained by noting that the sign of $g'(n)$ coincides with the sign of the first degree polynomial:

$$p(n) := q \left[\sigma(\alpha^2 + \alpha - 1) - (\alpha - 1)\gamma - \alpha^2 \right] n + (2\sigma - 1)(1 - 2\alpha) - 2\gamma(1 - \alpha), \quad (17)$$

and the sign of $g''(n)$ coincides with the sign of the second degree polynomial:

$$P(n) := P_2 n^2 + P_1 n + P_0, \quad (18)$$

where

$$P_2 := \left[\left(\sigma - \frac{1}{3}\gamma - \frac{2}{3} \right) \alpha - \frac{2}{3}\sigma + \frac{1}{3}\gamma + \frac{1}{3} \right] \left[(\sigma - 1)\alpha^2 + (\sigma - \gamma)\alpha - \sigma + \gamma \right] q^2, \quad (19)$$

$$P_1 := \left[\frac{4}{3}q(\alpha\gamma - 2\alpha\sigma + 2\alpha - \gamma + \sigma - 1) \right] [\alpha\gamma - 3\alpha\sigma + 2\alpha - \gamma + 2\sigma - 1], \quad (20)$$

$$P_0 := - \left\{ \frac{8}{3} \left[(\sigma - 1)\alpha^2 + (-4\sigma + \gamma + 4)\alpha + 2\sigma - \gamma - 2 \right] \right\} \left[\left(\sigma - \frac{1}{2}\gamma - 1 \right) \alpha - \frac{1}{2}\sigma + \frac{1}{2}\gamma + \frac{1}{2} \right]. \quad (21)$$

■

Remark 2 *From an empirical point of view, $1 - 2\alpha > 0$ as $\alpha < 0.5$ generally holds [Krueger (1999); Gollin (2002); Jones (2004)]. Values of the capital share in income larger than 0.5 may make sense by broadening the concept of capital including human components [see Chakraborty (2004) and the literature cited therein]. However, all numerical simulations presented in this work adopt the usual notion of physical capital and make use of a standard value around 0.33, which is consistent with the data.*

From Lemma (1) we get the proposition characterising the existence and number of stationary equilibria of map M .

Proposition 3 *[Existence and number of stationary equilibria]. (1) If $\sigma > 1$ and $\gamma > \tilde{\gamma}$ then there exists a unique interior fixed point. (2) If $\sigma > 1$ and $\gamma < \tilde{\gamma}$ [this case is meaningful only when $\alpha < 1/2$] then there exists a threshold value $\tilde{A} > 0$ such that for $A < \tilde{A}$ there are no interior fixed points and for $A > \tilde{A}$ there exist two interior fixed points. (3) If $\sigma < 1$ and $\gamma > \tilde{\gamma}$ then there exists a unique interior fixed point. (4) If $\sigma < 1$ and $\gamma < \tilde{\gamma}$ [this case is meaningful only when $\alpha > 1/2$] then there exists a threshold value $\bar{A} > 0$ such that for $A < \bar{A}$ there exist two interior fixed points and for $A > \bar{A}$ there are no interior fixed points.*

Proof. We separate the proof with respect to the cases introduced in the statement of the proposition. (1) By Lemma 1, $g'(n)$ has constant and negative sign in the interval $(0, 2/q)$. In fact, given $\hat{\gamma} := [\alpha^2(\sigma - 1) + \sigma(\alpha - 1)]/(\alpha - 1)$ we have that if $\gamma < \hat{\gamma}$ then $p(n)$ defines a negatively sloped linear function that vanishes at a point $n < 0$, whereas if $\gamma > \hat{\gamma}$ then $p(n)$ defines a positively sloped linear function that vanishes at a point $n > 2/q$. (2) By Lemma 1, g has an interior maximum point $n_{\max} := n_{crit}$ by the study of $g''(n)$ it follows that no inflection points do exist and then g is always concave. (3) It is easy to show that g is increasing in the interval $(0, 2/q)$ and $\lim_{n \rightarrow 0^+} g(n) = -\infty$, and $g(2/q) = 2/q$ and $g'(2/q) = 0$ (then, the the graph of g lies above the 45 degree line). To verify that g is concave we consider two cases. If $\tilde{\gamma} < \gamma < \frac{(3\sigma-2)\alpha+1-2\sigma}{\alpha-1}$ then the discriminant of the polynomial in (18) is negative so that there do not exist interior inflection points and $g''(n) \leq 0$ in the interval $(0, 2/q)$. If $\gamma > \frac{(3\sigma-2)\alpha+1-2\sigma}{\alpha-1}$ then the roots of the polynomial in (18) are located outside the interval $(0, 2/q)$ and $g''(n) \leq 0$ in the interval $(0, 2/q)$. (4) Function g always admits a minimum point n_{\min} in the interval $(0, 2/q)$. First, define the following threshold value of γ : $\bar{\gamma} := (\alpha^2 - 4\alpha + 2)(\sigma - 1)/(1 - \alpha)$, where $\bar{\gamma} < \tilde{\gamma}$. Second, to inquire about the number of fixed points, it is convenient to distinguish between two cases. If $\bar{\gamma} < \gamma < \tilde{\gamma}$ then g is decreasing and convex in the interval $(0, n_{\min})$, where $n_{\min} := n_{crit}$; it is increasing and convex in the interval $(n_{\min}, 2/q)$; it is increasing and concave in the interval $(f_1, 2/q)$, where $f_1 \in (n_{\min}, 2/q)$ is the unique inflection point; it eventually ends up at point $2/q$ with $g(2/q) = 2/q$ and $g'(2/q) = 0$. If $\gamma < \bar{\gamma}$ then g has two inflection points f_1 and f_2 in the interval $(0, 2/q)$. Function g is decreasing and concave in the interval $(0, f_1)$; it is decreasing and convex in the interval (f_1, n_{\min}) ; it is increasing and convex in the interval (n_{\min}, f_2) ; it is increasing and concave in the interval $(f_2, 2/q)$ with $g(2/q) = 2/q$ and $g'(2/q) = 0$. ■

The geometry of existence and number of stationary states of map M outlined in Proposition 3 is illustrated in Panels (a)-(d) of Figure 1. The stationary states are the intersection points of $g(n)$ with the 45° degree line. Depending on parameter configurations, there exist either uniqueness (Panels (a) and (c)) or multiplicity (Panels (b) and (d)). In the case of multiple equilibria, the Total Factor Productivity parameter plays a crucial role in determining the gap between the two states. This is in line with the result of the endogenous lifetime model of Chakraborty (2004).

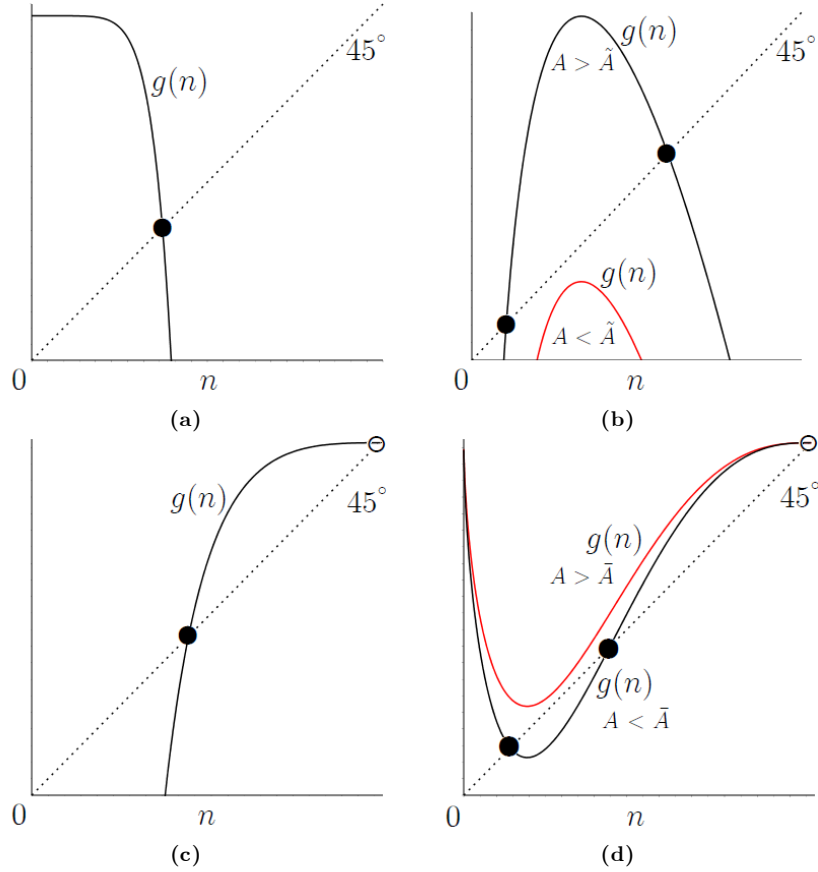


Figure 1. Geometry of existence and number of stationary states (denoted by the black point) of map M as detailed in Proposition 3. (a) Case 1: $\sigma > 1$ and $\gamma > \tilde{\gamma}$. There is a unique stationary state. (b) Case 2: $\sigma > 1$ and $\gamma < \tilde{\gamma}$ [this case is meaningful only when $\alpha < 1/2$]. If $A < \tilde{A}$ there are no stationary states (red curve). If $A > \tilde{A}$ there are two stationary states (black curve) (c) Case 3: $\sigma < 1$ and $\gamma > \tilde{\gamma}$. There is a unique stationary state. (d) Case 4: $\sigma < 1$ and $\gamma < \tilde{\gamma}$ [this case is meaningful only when $\alpha > 1/2$]. If $A < \bar{A}$ there are two stationary states (black curve). If $A > \bar{A}$ there are no stationary states (red curve). In Case 3 and Case 4, $n = 2/q$ (denoted by the empty circle) is not a stationary state of the map. However, it can play an important role for the dynamics of the model, as is shown later in this article.

Let us compare now a situation where there exist two stationary states under the assumption that children are a normal good ($\sigma > 1$). In the long term, workers will get a lower wage at a stationary state with a low stock of capital (less developed economy) in comparison with the wage will be obtained at a stationary state where capital is larger (developed economy). Given the same interest factor (partial equilibrium), this means that individuals choose to have less children in a context of underdevelopment than in richer countries. However, the model also implies that the interest factor is higher where the capital stock is lower. As children are a normal good and the interest factor is an element that helps capitalising wage income over time, in the long term (general equilibrium) fertility increases when the capital stock reduces. Then, this theory is able to explain the empirical behaviour on the demand for children both

at microeconomic and macroeconomic levels.

Of course, the existence of one or more stationary states is economically meaningful only whether there exist trajectories leading to convergence. The following results provide a classification of the equilibrium dynamic properties of map M .

Lemma 4 *If (a) $\sigma < 1$ and $\gamma > \sigma$ or (b) if $\sigma > 1$ and $\gamma > \frac{\sigma(2-qn_{ss})}{2-qn_{ss}}$ then the determinant of the Jacobian matrix of map M is positive, where n_{ss} is the generic stationary state value of n .*

Proposition 5 *[Local stability of stationary equilibria]. Under the hypotheses of Lemma 4, if the graph of g at n_{ss} intersects the 45° lines from below, the point (k_{ss}, n_{ss}) is a saddle, where k_{ss} is the generic stationary state value of k obtained by the expression in (12).*

Proposition 6 *Let $\sigma < 1$ and $\gamma > -\frac{(1-\sigma)(\alpha^2qn_{ss}-2\alpha+2)+qn_{ss}\sigma}{2-qn_{ss}}$. If the graph of g at n_{ss} intersects the 45° lines from above, the point (k_{ss}, n_{ss}) is not a saddle.*

Proof. The proof of Lemma 4 and Propositions 5 and 6 are in the Appendix. ■

In the light of the previous results, we can give an insight about the stability of equilibrium points in the different cases outlined in Figure 1. Specifically, under the assumptions introduced in the propositions, the left-located stationary state in Panel (b) identifies a saddle point. This means that given an initial condition on the stock of capital, there exists a unique choice on the control variable allowing the economy to lie on the trajectory converging towards it. The same result holds for the unique stationary state identified in Panel (c) and for the right-located one of Panel (d). Instead, nothing can be said in the case detailed in Panel (a) as equilibrium dynamics can have different properties. In order to clarify the outcome in this case, Figure 2 shows the possibility that the equilibrium is locally indeterminate. Although this is the unique stationary state of the model, there exist infinite choices on the control variable (fertility) that generating trajectories that lead towards it for a given initial condition of the state variable (capital). These trajectories are characterised by different values of capital accumulation and fertility.

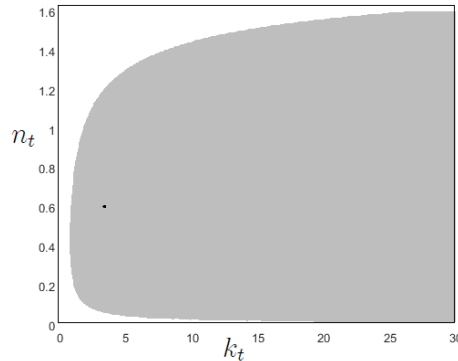


Figure 2. Parameter set: $\alpha = 0.354$, $\sigma = 4.01993$, $A = 1.61$, $q = 0.94$ and $\gamma = 1.4485$. Local indeterminacy of the unique attractor of the map. The grey-coloured region is the basin of attraction of the attractor. The white area is the region of unfeasible trajectories.

When the hypotheses of the previous results are violated, the classification of the stationary states of map M is quite cumbersome from an analytical point of view, with conditions that are very difficult to be interpreted economically. More details are available in the Appendix.

As is shown in Proposition 3, there exist cases with respect to which there are multiple stationary states. This makes it possible to have also global indeterminacy. This implies that the model is able to generate distinct development trajectories leading to different long-term values of capital per young person (state variable) and fertility (choice variable). For instance, with the following parameter values (which are plausible values also from an empirical point of view) $\alpha = 0.33$, $\sigma = 4.7$, $q = 0.53$ (which represents almost the 30 per cent of the time endowment of parents for the caring of children), $A = 1.545$, $\gamma = 0.183$ we obtain two stationary states. One of these two states represents the under-development outcome (low GDP and high fertility), i.e. the low development regime, and its coordinate values are given by $(k^*, n^*) = (1.23, 1.13)$. The other, instead, represents the paradigm for developed countries (high GDP and low fertility), i.e. the high development regime, and its coordinate values are given by $(k^{**}, n^{**}) = (1.96, 0.88)$. The long-term low development regime is a locally indeterminate fixed point, whereas the long-term high development regime is a saddle (Figure 3).

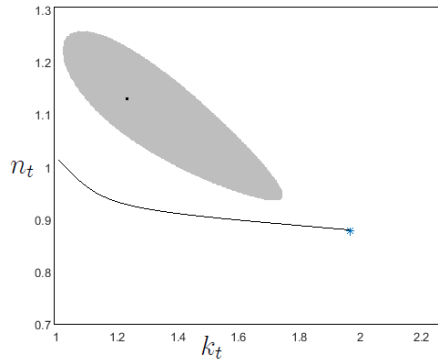


Figure 3. Parameter set: $\alpha = 0.33$, $\sigma = 4.7$, $q = 0.53$, $A = 1.545$, $\gamma = 0.183$. Global indeterminacy. The grey-coloured region is the basin of attraction of (k^*, n^*) . The black line represents an approximation of a branch of the stable manifold of the saddle (k^{**}, n^{**}) on which an economy converges towards the developed state.

On the existence of feasible trajectories with $k_t \rightarrow 0$ and $n_t \rightarrow 2/q$ when $\sigma < 1$. By exploring the two equations of map M , it is not possible to have feasible trajectories such that $k_t \rightarrow 0$ and $n_t \rightarrow 2/q$ when $\sigma < 1$. In this case, point $(0, 2/q)$ is an attractor of the system even though

the map is not defined on such a point. This event is shown through numerical simulations in the example of Figure 4. Panel (a) depicts the (grey-coloured) basin of attraction of $(0, 2/q)$, depicted in red in the figure, whereas the boundary of the basin is defined by the stable manifold of the interior stationary state, i.e. the saddle (black) point (k^*, n^*) . The white region represents the space of initial conditions for which trajectories become unfeasible after a finite number of iterations. From Figure 4(a), one may conclude that the system is globally indeterminate as there exist two distinct long-term outcomes (only one of them is an interior fixed point) that can be achieved given the same initial condition. Specifically, there exist (1) an infinite number of trajectories leading to the locally indeterminate state $(0, 2/q)$, which represents a poverty trap scenario with a low level of capital and high fertility, and (2) a unique (saddle) path on which the economy converges to the interior stationary state, which represents a paradigm of developed countries with a high level of capital and low fertility. Converging towards one of these long-term state is a matter of individuals' choices about fertility. This is a typical expectations-driven outcome leading to coordination failures. In fact, U evaluated at $(0, 2/q)$ is smaller than U evaluated at (k^*, n^*) meaning that (k^*, n^*) Pareto dominates $(0, 2/q)$ but individuals can choose to coordinate themselves on the Pareto dominated equilibrium. This holds because individuals (by expecting a low return on capital) tend to increase the amount of time devoted to child rearing. Then, fertility increases approaching its upper bound $(2/q)$ and capital increasingly reduces. Panel (b) of Figure 4 shows two typical trajectories leading to the poverty trap outcome.

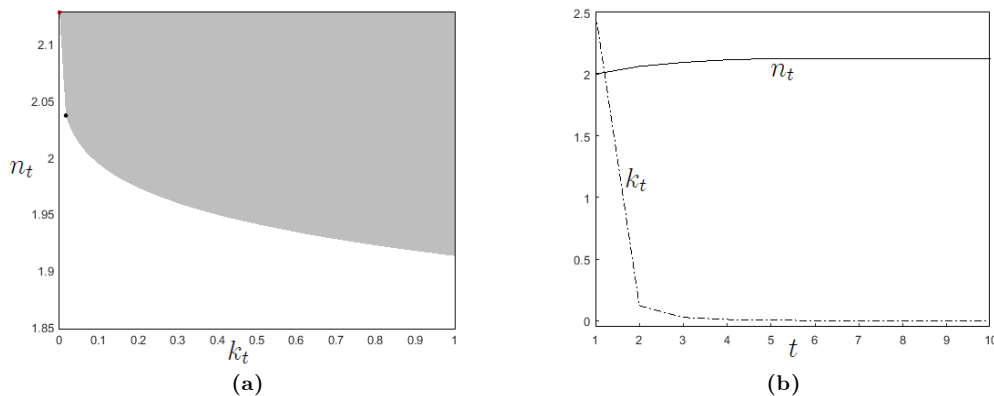


Figure 4. Parameter set: $\alpha = 0.354$, $\sigma = 1.3$, $A = 1.07$, $q = 0.94$, $\gamma = 0.004$. (a) Basin of attraction (depicted in grey) of $(0, 2/q)$ (the red point) and the boundary of the basin that defines the stable manifold of the saddle (black) point (k^*, n^*) . (b) Time series of n_t and k_t of a trajectory approaching towards $(0, 2/q)$.

Another result the model is able to reproduce is given by (endogenous) fertility fluctuations that are in line with the baby busts and baby booms observed the last century in some developed countries. There are two recent articles analysing the reasons why fertility fluctuates over time. We refer to the works of Doepke et al. (2015) and Jones and Schoonbroodt (2016). The

former wishes to assess the effects of the shock of World War II on subsequent baby boom in U.S. (following the historical decrease in fertility due to working of demographic transition forces). The author consider a model where women can choose labour supply, the number of children and when having children and there exists an interaction amongst subsequent cohorts. Then, they perform quantity experiments to explain the post-war increase in fertility on the basis of a drop in labour force participation of young women (whose wages declined in that period) because of the increase in competition caused by the higher participation of older women (and the persisted high demand of female labour after the end of World War II). Then, younger women exited the labour market and started having children. The latter work, instead, considers a general equilibrium model with endogenous fertility and dynastic altruism showing that fertility and the opportunity cost of children in U.S. are pro-cyclical. Our theory simplifies the framework substantially and it is able to give an explanation of these fluctuations within a typical neoclassical set up. The main economic mechanism behind fertility fluctuations is similar to the one developed by Doepke et al. (2015) (see Figure 5 for a simulative exercise). This is because in our model an increase in fertility directly reduces both the labour supply and labour productivity.

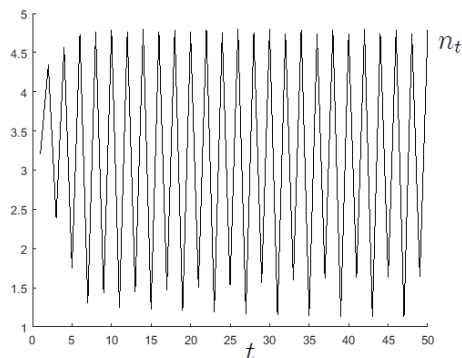


Figure 5. Parameter set: $\alpha = 0.3$, $\sigma = 1.956$, $A = 1$, $q = 0.3$, $\gamma = 0.34$. Fertility fluctuations.

4 Conclusions

For a very long time in human history, the number of births and deaths were almost equivalent and the world total population was relatively stable (fluctuating around a low level). During this extended period, income per person remained quite constant or it grew slowly [Galor and Weil (2000)]. After the process commonly known as Industrial Revolution, mortality started declining in newly industrialised countries and after an initial stage of stability, fertility followed a declining trend as well. In these phases, total population started increasing together with income. Then, Europe faced a long age of dramatic social, political and institutional changes that subsequently spread to other countries all over the world.

Economic development is a long-term involved phenomenon that includes social, institutional, economic and demographic changes across nations and continents. The present work treated development on the side of economic and demographic transition. Why do some countries achieve high levels of GDP and low fertility and others low levels of GDP and high fertility? Standard one-sector models of neoclassical growth often conclude that economies with similar technologies will converge towards a common stationary-state equilibrium even if the initial conditions are very different. This is the main result of the Solow-type growth set up implying that poorer countries will grow faster than richer countries. This finding is also shared by several works in the (OLG and continuous-time optimal) growth literature in both cases of exogenous fertility and endogenous fertility. However, it was widely accepted that there exist persistent differences in the level of real activity and fertility rates amongst (distinct groups of) countries [e.g., Mankiw et al. (1992); Palivos (1995); Hall and Jones (1999); Jones and Tertilt (2006)]. This kind of models, therefore, cannot explain these macroeconomic and demographic differences, so that the above as well as other similar questions are likely to remain unanswered within both the basic neoclassical growth set up and endogenous growth one. This unsatisfactory result has led several economists to modify these frameworks in several ways trying to building on more suitable theories with exogenous fertility [Azariadis and Drazen (1990)] or endogenous fertility [Galor and Weil (1996)]. The literature has then grown rapidly leading to what is commonly known as the UGT [Galor (2011)], where the main factors explaining the demographic and economic transitions were generally child mortality/fertility (surviving children), adult mortality, human capital accumulation and structural changes. However, it is still difficult to find theories where fertility per se represents the triggering factor for the transition amongst the various stages of development. The present article used a basic OLG model of neoclassical growth to fill that gap. The works most closely related to the present one are Palivos (1995) and Galor and Weil (1996). The former introduced endogenous fertility (child quantity and child quality) in a continuous-time neoclassical optimal growth set up with infinite lived individuals, finding a *production-driven* channel through which fertility choices may be a source of multiple steady states and coordination failures. The latter emphasised the importance of *gender differences* in wage income for the existence of multiple paths of economic development in an OLG model with child quantity. The present work showed that multiple paths of economic development can exist also in a simplified version of Galor and Weil (1996) with *homogeneous* agents. It also introduced a new *utility-driven* mechanism through which fertility is a source of global indeterminacy. The model entered the debate about history versus self-fulfilling expectations and provided reasons why economies with different initial conditions (history) or, alternatively, similar or the same initial condition(s) (expectations) converge towards different long-term equilibria.

The work had the ambition of giving an answer to the question raised by Jones et al. (2008): "Fertility Theories: Can They Explain the Negative Fertility-Income Relationship?" Of course, we are aware that this is toy model and preferences may depend on culture, beliefs and social

norms specifically related to institutions or ethnic groups (often followed by linguistic and religious contours that also affect choices about contraception), and that these elements should therefore be included as endogenous variables in the analysis. However, the main goal of the work was to keep the model as simple as possible to bring to light some theoretical outcomes that remained until now unexplored within the standard economic theory of fertility.

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Conflict of Interest The authors declare that they have no conflict of interest.

Appendix

For the sake of completeness, we analyse here some mathematical details of map M not discussed in the main text.

On the feasible region of map M . Map M is defined on a sub set of the non-negative orthant. In fact, given $n_t > 0$ and $k_t > 0$, in order to have $n_{t+1} > 0$ and $k_{t+1} > 0$ it must hold that

$$k_t > \left[\frac{(2 - qn_t)^{-\alpha - \frac{1}{(\sigma-1)(1-\alpha)}} n_t^{-1 + \frac{\gamma}{(\sigma-1)(1-\alpha)}}}{2B} \right]^{\frac{1-\alpha}{\alpha^2}}. \quad (22)$$

Depending on the parameter setting we have three different cases.

1) If $\sigma > 1$ and $\gamma > (\sigma - 1)(1 - \alpha)$ then the region defined by the inequality in (22) is described by the grey area in Panel (a) of Figure A.1.

2) If $\sigma > 1$ and $\gamma < (\sigma - 1)(1 - \alpha)$ then the region defined by the inequality in (22) is described by the grey area in Panel (b) Figure A.1.

3) If $\sigma < 1$ then the region defined by the inequality in (22) is described by the grey area in Panel (c) of Figure A.1.

The regions detailed above ensure the possibility of computing a single iterate. However, in order to have well-defined forward dynamics, the trajectory generated by a generic initial condition must be bounded in this region for every iterate. For this reason, the economically meaningful trajectories analysed in the main text actually lie on in a smaller region than the one shown in the three panels of Figure A.1.

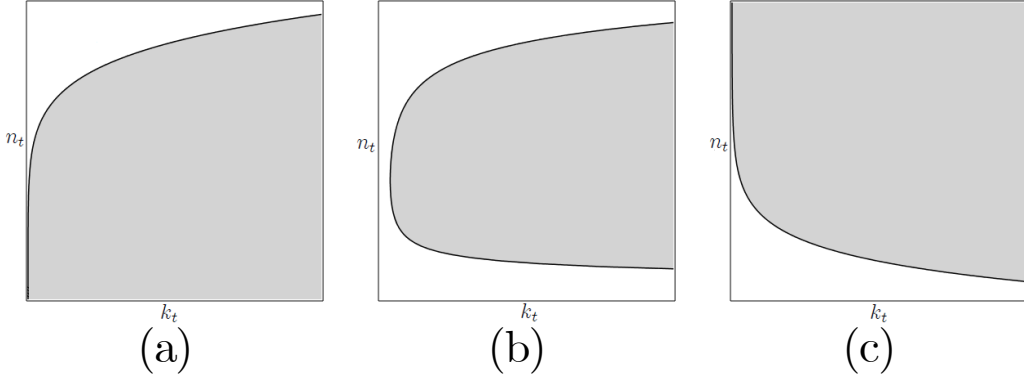


Figure A.1. Feasible region (grey-coloured) of map M depending on the parameter setting.

On the non-existence of feasible trajectories with vanishing n_t . If $\sigma > 1$ and $\gamma > (\sigma - 1)(1 - \alpha)$ then we can rule out the existence of feasible trajectories with $n_t \rightarrow 0$. In fact, if there were feasible trajectories such that $n_t \rightarrow 0$ then $k_t \rightarrow +\infty$, as can be ascertained by looking at the first equation of map (10). However, this would not be consistent with the second equation of that system, which describes the dynamics of fertility, as $k_t \rightarrow +\infty$ would imply $n_t \rightarrow 2/q$.

On the non-existence of feasible trajectories with an unbounded growth of k_t . If $\sigma > 1$ and $\gamma > (\sigma - 1)(1 - \alpha)$ then we can rule out the existence of feasible trajectories with $k_t \rightarrow +\infty$. In fact, if there were feasible trajectories such that $k_t \rightarrow +\infty$ then $n_t \rightarrow 2/q$, as can be ascertained by looking at the second equation of map (10). However, this would not be consistent with the first equation of that system, which describes the dynamics of the stock of capital, as when $n_t \rightarrow 2/q$ for a high enough value of k_t one would get $k_{t+1} < k_t$.

On the non-existence of feasible trajectories with an unbounded growth of k_t and $n_t \rightarrow 2/q$. Consider a feasible trajectory with $n_t \rightarrow 2/q$. Then, for a value of t sufficiently large it holds that $k_{t+1} = A(1 - \alpha)k_t^\alpha(2 - qn_t)^{1-\alpha} < A(1 - \alpha)k_t^\alpha \rightarrow k^* = [(1 - \alpha)A]^{\frac{1}{1-\alpha}}$.

On the non-existence of feasible trajectories with $n_t \rightarrow 2/q$ when $\sigma > 1$. We can rule out the existence of feasible trajectories with $n_t \rightarrow 2/q$ in the case $\sigma > 1$. If there were feasible trajectories such that $n_t \rightarrow 2/q$ then by the first equation in (10) $k_t \rightarrow 0$. However, this would not be consistent with the second equation in (10) as $k_t \rightarrow 0$ would imply $n_{t+1} < 0$.

Proof of Lemma 4 and Propositions 5 and 6. Results about stability of the stationary states follows by the study of Jacobian matrix evaluated at a generic state (k_{ss}, n_{ss}) . The Jacobian matrix is the following:

$$J(k_{ss}, n_{ss}) := \begin{pmatrix} J_{1,1} & J_{1,2} \\ J_{2,1} & J_{2,2} \end{pmatrix}, \quad (23)$$

where

$$J_{11} := \alpha > 0, \quad (24)$$

$$J_{1,2} := -[(1 - \alpha)A]^{\frac{1}{1-\alpha}} n^{\frac{\alpha-2}{1-\alpha}} (2 - \alpha q n_{ss}) < 0, \quad (25)$$

$$J_{2,1} := \frac{\alpha^2 k_{ss}^{\frac{\alpha^2}{\alpha-1}} (2 - q n_{ss})^{\frac{(\alpha^2 - \alpha)(1-\sigma)+1}{(1-\sigma)(1-\alpha)}} n_{ss}^{\frac{(1-\alpha)(\sigma-1)-\gamma}{(1-\sigma)(1-\alpha)}} B}{q(1 - \alpha)k_{ss}} > 0, \quad (26)$$

$$J_{2,2} := \frac{J_{2,1} k_{ss} \{[\alpha^2(\sigma - 1) - \sigma] q n_{ss} + 2(\sigma - 1)(1 + \alpha) - (2 - q n_{ss})\gamma\}}{\alpha^2(\sigma - 1)(2 - q n_{ss})n_{ss}}. \quad (27)$$

In particular, we note that $g(n) = v(h(n), n)$ from which $g'(n) = v'_k(h(n), n)h'_n(n) + v'_n(h(n), n)$. At a stationary state such expression becomes:

$$v'_k(k_{ss}, n_{ss}) \frac{\frac{\partial Q_1}{\partial n} \big|_{(k,n)=(k_{ss},n_{ss})}}{1 - \frac{\partial Q_1}{\partial k} \big|_{(k,n)=(k_{ss},n_{ss})}} + v'_n(k_{ss}, n_{ss}). \quad (28)$$

Corresponding to an intersection from below (resp. above) of the graph of g with the 45° line, the expression in (28) is greater (resp. smaller) than one. Rearranging terms, we have that such an expression identifies the condition for which $Det(J(k_{ss}, n_{ss})) - Tr(J(k_{ss}, n_{ss})) + 1 < 0$. Results follows from the sign of $Det(J(k_{ss}, n_{ss}))$, where

$$sgn \{Det(J(k_{ss}, n_{ss}))\} = sgn \left\{ \frac{2 + (\gamma - \sigma)(2 - qn)}{q(2 - qn)(1 - \sigma)} \right\} \quad (29)$$

and by identifying a sufficient condition for which $J_{22} > 0$ (that guarantees $Tr(J(k_{ss}, n_{ss})) > 0$).

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