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# A Sharing Model of the Household: Explaining the Deaton-Paxson Paradox and Computing Household Indifference Scales\*

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## Abstract

This paper presents a new model of the household that is able to explain a variety of consumption patterns that existing models cannot describe, most notably, those associated with the Deaton and Paxson (1998) paradox. The most distinctive feature of this model is the presence of common-pool goods (rival and non-excludable) previously ignored in the literature. Under regularity conditions, the model can be interpreted as a hybrid between non-cooperative and a collective models of the household. Empirically, the paper revisits the Deaton-Paxson paradox exploiting household splits in longitudinal data and computes the elusive indifference scales coefficients.

JEL: D13, J12, O15

Keywords: sharing model, collective model, intra-household allocation, Deaton-Paxson paradox, household economies of scale, indifference scales.

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# 1 Introduction

The economic theory of individual decision-making is almost undisputed. However, theoretical models explaining household choice remain controversial. Since most people live in multi-person households, modeling group behavior correctly is crucial in many theoretical and empirical studies. Previous research indicates that ignoring the interaction of household members affects the measurement of demand elasticities (Browning and Chiappori (1998)), family labor supply (Chiappori (1988), Fortin and Lacroix (1997), Browning and Meghir (1991)) and the impact of social policies (Lundberg et al. (1997), Duflo (2003)).

This paper presents a model for the intra-household allocation of resources where each member fully controls his/her income and voluntarily shares purchased goods with other members. The main ingredient that differentiates this model, denominated here the “sharing model”, from others previously studied in the literature is the presence of common-pool goods. These largely ignored commodities have the characteristics of being rival in consumption yet non-excludable to household members, and are capable of affecting the household demand system in fundamental ways. The sharing model can be viewed as a natural generalization that nests existing theories of the household. However, it has the ability to explain several consumption regularities that have been proven incompatible with previous models, in particular those associated with the Deaton and Paxson (1998) paradox discussed below. In light of the solution proposed for this paradox, this paper derives and implements a method to compute household indifference of scale that reconciles the economic theory with the empirical evidence previously in conflict.

In the sharing model, the allocation of goods is determined by their nature and the norms in the household. In this respect, the model contemplates three types of commodities: public goods, private goods and, new in the literature, common-pool goods. These latter commodities are ignored in standard theories of the household. Common-pool goods have the characteristic of being rival and non-excludable in consumption. Thus, when a household member purchases such goods, she knows that they have to be shared with others. An example of a common-pool good is food at home that can be consumed by any household member irrespectively of who purchased it. The non-excludability condition of common-pool goods does not mean the absence of a rule to allocate them. It means that the buyer cannot fully appropriate the benefits because other members are allowed to consume them. The existence of common-pool goods is in the essence of the household. For example, the United Nations defines a multi-person household “*as a group of two or more persons living together who make **common provision for food** or other essentials for living*”(emphasis added).<sup>1</sup> Similarly, the majority of surveys defines the household as a function of common goods, usually food.<sup>2</sup>

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<sup>1</sup><http://unstats.un.org/unsd/demographic/sconcerns/fam/fammethods.htm>

<sup>2</sup>For example, the Mexican Family Life Survey used in this study defines a household as “*a person or group of people, related or unrelated by biological bonds, who usually live together in a part of, or in an entire building/ dwelling and usually consume meals provided by a common budget on the same stove/oven and may even use the same utensils for preparing meals*”

The contribution of the sharing model becomes apparent when it is contrasted with the theories of the household that are most commonly adopted in the literature: i) non-cooperative models, ii) unitary models and iii) collective models. Non-cooperative models of the household are theoretically analyzed in Chen and Woolley (2001), Lechene and Preston (2011), Lundberg and Pollak (1993) and Browning et al. (2010). Among other things, these models deal with voluntary contributions of public good. Their analysis is tightly connected with previous studies in the public finance literature on the private provision of public goods (Warr (1983), Bergstrom et al. (1986), Fraser (1992), Bergstrom et al. (1992)). All of these papers ignore common-pool goods.

Unitary models assume that households can be treated ‘as if’ they were individuals. Specifically, the mechanism governing household members’ interactions results in well-defined group preferences capable of being represented by an aggregate utility function. Unitary models accurately represent household’s choices in some specific cases, such as households where only one person makes all the decisions or households where all members have identical preferences. However, these models cannot account for the general case. Two empirical implications of unitary models have been tested in the literature: *the income pooling property* (i.e. the identity of the income earner should not affect household demands for goods) and *the symmetry of the Slutsky matrix*. The empirical evidence is now ample against the unitary model (Thomas (1990), Schultz (1990), Lundberg et al. (1997), Deaton and Paxson (2003), Browning and Chiappori (1998), Attanasio and Lechene (2002)).

Collective models, originally developed by Chiappori (1988) and later extended by Blundell et al. (1993), Browning and Chiappori (1998), Bourguignon et al. (2009) and Chiappori and Ekeland (2009) among others, are commonly used and accepted alternative to unitary models. They recognize that households are formed by interacting individuals with different preferences and incentives. However, this theory does not explicitly model the group decision process. It simply assumes that such process leads to an efficient allocation of resources in the household. Collective models are appealing because they make only one extra assumption on top of individual rationality and are compatible with a great number of mechanisms.

Theoretical papers related to collective models deal with the question of whether this theory is too general to obtain meaningful conclusions and testable implications. These papers take Pareto-optimality as given and derive the conclusions that emerge from it. On the other hand, empirical studies take the implications of the collective model to test whether this theory is a valid representation of households’ behavior. Despite the attractiveness of the collective setting, the evidence is not conclusive. While some studies cannot reject the collective model hypothesis (Browning and Chiappori (1998), Bobonis (2009), Attanasio and Lechene (2014)), others find strong evidence against Pareto-optimality within the household (Udry (1996), Duflo and Udry (2004), Dercon and Krishnan (2003)).

The existing models of the household just described have proven to be valuable devices. However, there are empirical regularities that none of them can explain. The most important one is the fact that, holding total

household per capita expenditures constant, per capita food consumption tends to decline with household size. This relationship, which was first described by Deaton and Paxson (1998), contradicts the predictions of the Barten (1964) model in relation to the presence of household economies of scale. The ample evidence of this empirical relationship across a wide range of countries and periods suggests that previous household models fall short in fully explaining intra-household allocation.<sup>3</sup> The sharing model presented in this paper provides a natural explanation.

The Deaton and Paxson (1998) paradox can be summarized as follows. Household economies of scale are thought to come from the presence of public goods (e.g. housing). Because these goods are non-rival in consumption, per capita monetary contributions to maintain a given level of consumption declines as household size increases (i.e. the effective price of public goods declines). Then, under reasonable price and income elasticities, the share of food on total expenditure is expected to increase with household size, but the opposite is observed in most of the countries. The sharing model in this paper gives a simple and intuitive explanation of this paradox. Since food is likely to be a common-pool good, as households become larger, the individual consumption of each additional unit of food purchased declines. Consequently, household members have incentives to reallocate resources away from food to private goods (e.g. shoes) for which the buyer can fully appropriate the benefits.

The sharing model generates non-monotonic economies of scale, or more precisely indifference scales, in relation to household size.<sup>4</sup> As a result, there is an economically optimal number of members in the family beyond which household division is welfare-enhancing. In contrast, previous economies of scale models (Barten (1964), Deaton and Paxson (1998)) predict that adding an extra member in the household while holding per capita income constant always (weakly) expands the consumption choice set, implicitly indicating that the economically optimal household size tends to infinity.

In addition to the fact that i) food as a share of total expenditure tends to decline with household size, the sharing model is able to explain ii) why the expenditures on food as a share of food plus housing increases with household size (Gan and Vernon (2003) explanation of the paradox) despite the observed decline in per capita food expenditures (Deaton and Paxson (2003) response to Gan and Vernon), and iii) why food consumed away from home tends to increase with household size. Moreover, if the sharing model is enriched by allowing the altruistic parameter to vary with income as the experimental evidence indicates (Chowdhury and Jeon (2014)), it can explain iv) why the Deaton-Paxson paradox is more prevalent in low-income countries and v) why the Engel's curve tends to be hump-shaped in poor regions.

An interesting feature of the sharing models is that, under certain regularity conditions, it can be written

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<sup>3</sup>Logan (2011) finds evidence of this empirical relationship as far back as 1888 in the U.S.

<sup>4</sup>Chiappori (2016) indicates the difference between economies of scale and indifference scales. The former refers to “*How much income would an individual living alone need to attain the same utility level as a family of given composition?*” and the latter to “*How much income would an individual living alone need to attain the same indifference curve over goods that this individual attains as a member of a family of given composition?*” p539.

as a hybrid between a collective model and a non-cooperative model. This hybrid model consists of two-stages. In the first stage, each family member non-cooperatively decides how much to spend on a given set of goods and how much to contribute to a common pool of resources. In the second stage, household members jointly decides how to allocate the common pool of resources following a *conditional* collective model.<sup>5</sup> The standard non-cooperative household model (Chen and Woolley (2001) and Lechene and Preston (2011)) and the pure collective model (Chiappori (1988) and Blundell et al. (1993)) are polar cases of this hybrid model. The hybrid representation of the sharing model helps understand why some statistical tests fail to reject the collective model hypothesis despite that the intra-household allocation is Pareto inefficient.

The contradictions between theoretical predictions and empirical regularities stated in the Deaton and Paxson (1998) paradox constitute a serious obstacle to recover meaningful economies of scale (or indifference scales) from the data. Using predictions from the sharing model, which proposes a solution to this paradox, the empirical section of this paper develops and implements a method to estimate household indifference scales. That is, a measure of “*how much income would an individual living alone need to attain the same indifference curve over goods that this individual attains as a member of a family of given composition*” Chiappori (2016) p539. This concept is closely related to economies of scale, but has some desirable properties. The measurement of economies of scale or indifference scales is fundamental for the analysis of individual living standards using household data. It affects all poverty and inequality indexes. Therefore, the policy implications are enormous since many social programs determine eligibility on the basis of poverty status.<sup>6</sup> Nonetheless, the estimation of economies and indifference scales has been elusive.

Recent papers in the literature of indifference scales are Browning et al. (2013) and Dunbar et al. (2013). These studies provide remarkable methodological contributions to estimating indifference scales and are likely to be valid in certain contexts. Nonetheless, both papers work under the assumption that households allocate resources in accordance to the collective model. As previously mentioned and furthered discussed elsewhere in the paper, this model cannot conceptually explain the stylized facts in Deaton and Paxson (1998).

The rest of the paper is organized as follows. Section 2 presents the theoretical model. It first describes the different types of goods consumed in the household and why the rules to allocate them differ between each other (section 2.1). Then, it presents the sharing model (section 2.2) and the necessary conditions to be interpreted as a hybrid between non-cooperative and collective models (section 2.3). Section 3 discusses the Deaton-Paxson paradox and other empirical regularities unexplained by existing models. Section 3.1 specifies a caring functional for the utility functions to obtain close form solutions and show how the sharing model is able to explain the consumption patterns described in the previous section. Section 4 presents a

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<sup>5</sup>The term *conditional* indicates that the collective model applies i) after individual voluntary contributions have been made, and ii) over subset of good that can vary to none to all commodities. The specificities will be clear in the next section.

<sup>6</sup>Most conditional cash transfer programs determine eligibility on the basis of living standards, for example *Progresa-Oportunidades* in Mexico and *Juntos* in Peru.

method to measure household indifference scales. Section 5 shows the results of the estimated Engel's curve and the indifference scales coefficients. Finally, section 6 presents the conclusions of the paper.

## 2 Theoretical Framework

This section presents a general version of the sharing model. It first classifies the goods consumed in the household in relation to their intrinsic characteristics and allocation rules. Then, it describes the components of the sharing model and its equilibrium. Finally, it shows that under certain regularity conditions the sharing model can be interpreted as a hybrid between a standard non-cooperative model and a collective model.

### 2.1 Appropriable and non-appropriable goods in the household

The literature on intra-household allocation of resources has traditionally dealt with only two types of goods consumed in the household, public goods and private goods. The distinction between them is made on the principles of *rivalry* and *excludability*. Public goods are modeled as non-rival and non-excludable in the sense that they are consumed simultaneously by all members in the household, while private goods are modeled as fully rival and excludable since they are assigned to and consumed by only one person at a time.<sup>7</sup>

Notably, previous papers on intra-household allocation of resources have ignored goods that can be consumed by only one person, but cannot be ex-ante appropriated. That is, goods that are rival in consumption yet non-excludable to household members. These 'common-pool' goods are expected to constitute an important share of the household budget. According to the United Nations:

*A multi-person household [is] defined as a group of two or more persons living together who make **common provision for food** or other essentials for living.*<sup>8</sup>(emphasis added)

Excludability within the household arises due to intrinsic characteristics of the good - e.g. items that are size specific such as shoes - or as a consequence of norms - e.g. no underage alcohol consumption. Non-excludable goods are likely to be those that are sharable by nature such as food. In this paper the terms 'common-pool goods', 'sharable goods' and 'non-appropriable goods' are used interchangeably. Non-excludability in this context does not mean a complete absence of rules to allocate common-pool goods in the household. It means that once a member purchases a quantity of such goods, he or she cannot fully appropriate it. For example, food presumably has to be shared somehow with other household members.

**Definition 1.** *There are three types of goods consumed by member  $i = 1, \dots, n$  in the household. **Private goods**, which consumption is denoted by  $x_i$ , are rival and excludable (e.g. clothing specific to each member).*

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<sup>7</sup>The literature has also covered the intermediate case of public goods with congestion where the principle of rivalry is only partially satisfied. In this case, the addition of a household member affects the quantity or quality of the public good consumed by others.

<sup>8</sup><http://unstats.un.org/unsd/demographic/sconcerns/fam/fammethods.htm>

**Public goods**, denoted by  $Q$ , are non-rival and non-excludable (e.g. housing). **Sharable goods**, non-appropriable goods or common-pool goods, which consumption is  $s_i$ , are rival but non-excludable (e.g. food).

The three types of commodities in Definition 1 are extreme cases. Goods vary continuously in terms of rivalry - i.e. different degrees of congestions - and excludability - i.e. goods that are sharable for a subset of the household such as tobacco. However, these intermediate cases add little or no insight to the model at the cost of reducing tractability. Nonetheless, the empirical section 5.4 analyzes the role of congestion in public goods.

Excludability is a characteristic that affects the mechanism to allocate goods among household members and, as a result, the individual incentives to purchase them. Unitary and collective models cannot differentiate private goods from common-pool goods because they do not explicitly model household members' interaction. Instead, they maximize the objective function of a household 'central planner', resulting in the same allocation as the underlying 'true' mechanism once certain conditions are met. However, the treatment that the central planner gives to two rival goods is identical, even if one of them is excludable and the other one not. This problem becomes evident in the next two sections.

## 2.2 A sharing model of the household

The sharing model assumes a non-cooperative behavior of household members.<sup>9</sup> Assume that each person  $i$  in a household with  $n$  members has an additively separable caring utility function (Browning et al. (2014)) that he/she maximizes.

$$\max \quad U_i(x_i, s_i, Q) + \tau_i \left( \frac{1}{n-1} \sum_{j \neq i}^n \alpha_{ij} U_j(x_j, s_j, Q) \right) \quad (1)$$

The function  $U_i(\cdot)$  is the egotistic component that maps own consumption of private goods ( $x_i$ ), public goods ( $Q$ ) and common-pool or sharable goods ( $s_i$ ) on utility. The parameter  $\tau_i \geq 0$  is the level of altruism. It indicates how much individual  $i$  values the (weighted average) utility that other members of the household derive from consumption. The weights  $\alpha_{ij}$  measure altruism heterogeneity from  $i$  to other members (e.g. a mother may care more about her children than her in-laws.). The *caring* functional form (1) is non-essential. However, it simplifies the analysis, exposition and interpretation.

The characteristics of goods and norms in the household imply that purchased and consumed quantities may differ. For private goods (e.g. shoes), the quantity consumed  $x_i$  equals the purchases  $\tilde{x}_{ji}$  made by all

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<sup>9</sup>The non-cooperative behavior is similar to that in Chen and Woolley (2001), Lechene and Preston (2011), Lundberg and Pollak (1993) and Browning et al. (2010).



the  $j = 1, \dots, n$  members in individual  $i$ 's behalf.<sup>10</sup>

$$x_i = \sum_{j=1}^n \tilde{x}_{ji} \quad (2)$$

The sharable good  $s_i$  (e.g. food) is rival but non-excludable in consumption. Thus, buyers cannot allocate it to a specific person. The purchases  $\tilde{s}_j$  made by each  $j$  member go to a common pool. Then, total household purchases  $\sum_{j=1}^n \tilde{s}_j$  are distributed to each member  $i$  using the following rule.

$$s_i = f_i \left( \sum_{j=1}^n \tilde{s}_j \right) \quad (3)$$

$$s.t. \quad \sum_{j=1}^n \tilde{s}_j = \sum_{i=1}^n f_i \left( \sum_{j=1}^n \tilde{s}_j \right) \quad (4)$$

The function  $f_i(\cdot)$  maps total household purchases of the sharable good to individual consumption  $s_i$ . For example, the allocation rule  $f_i \left( \sum_{j=1}^n \tilde{s}_j \right) = \delta_i \sum_{j=1}^n \tilde{s}_j$  indicates that member  $i$  receives a constant share  $\delta_i$  of total purchases. Alternatively, the function  $f_i \left( \sum_{j=1}^n \tilde{s}_j \right) = \tilde{s}_i$  indicates that member  $i$  receives exactly what he/she purchased, making the sharable good indistinguishable from a private good. In general, the functional form  $f_i(\cdot)$  depends on a variety factors: i) what breadwinners are supposed to provide for their families according to the society where they live, ii) the bargaining power of household members, iii) individual incomes, prices, etc. As discussed in the next section, the allocation rule (3) can be obtained from the weights attributed to household members in an embedded collective model.

Equation (4) is an accounting identity that the allocation rule of sharable goods should satisfy. It says that the sum of individual consumptions of the sharable good, i.e. the right-hand side, must be identical to total purchases, i.e. the left-hand side.

Being non-rival and non-excludable, the total consumption of the public good  $Q$  equals the sum of all members' purchases  $\tilde{Q}_j$ , for  $j = 1, \dots, n$ .

$$Q = \sum_{j=1}^n \tilde{Q}_j \quad (5)$$

Individuals are in full control of his/her resources and voluntarily decides to purchase goods. Thus, a person with income  $y_i$  facing prices  $p_x$ ,  $p_s$  and  $p_q$  have the following budget constraint.

$$p_x \sum_{j=1}^n \tilde{x}_{ij} + p_s \tilde{s}_i + p_q \tilde{Q}_i = y_i \quad (6)$$

Notice that the subscripts for the purchases of private goods are interchanged from (2) to (6). The summation in the first case indicates the goods purchased by *all household members transferred to member*

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<sup>10</sup>For example, private goods consumed by a teenager in the household could be clothing that his mother bought for him, a motorcycle his father gave him and video games he bought for himself.

$i$ , while the summation in the budget constraint refers to all the private goods purchased by member  $i$  in her's and others' behalf.

Given the quantities purchased by other household members, individual  $i$  maximizes utility (1) subject to equalities (2) to (6) and non-negative constraints. The result is a system of conditional demands that represents the best response functions in a static non-cooperative game.

$$\begin{cases} \tilde{x}_{ij}^c = g_{ij}^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^n \tilde{x}_{kr} \right\}_{r=1}^n, \sum_{k \neq i}^n \tilde{s}_k, \sum_{k \neq i}^n \tilde{Q}_k \right) \\ \tilde{s}_i^c = h_i^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^n \tilde{x}_{kr} \right\}_{r=1}^n, \sum_{k \neq i}^n \tilde{s}_k, \sum_{k \neq i}^n \tilde{Q}_k \right) \\ \tilde{Q}_i^c = l_i^c \left( y_i, p_1, p_2, p_q, n, \left\{ \sum_{k \neq i}^n \tilde{x}_{kr} \right\}_{r=1}^n, \sum_{k \neq i}^n \tilde{s}_k, \sum_{k \neq i}^n \tilde{Q}_k \right) \end{cases}$$

The Marshallian demands (7) are obtained when the household reaches a Nash equilibrium.

$$\begin{cases} \tilde{x}_{ij} &= g_{ij}(y_i, p_1, p_2, p_q, n) \\ \tilde{s}_i &= h_i(y_i, p_1, p_2, p_q, n) \\ \tilde{Q}_i &= l_i(y_i, p_1, p_2, p_q, n) \end{cases} \quad (7)$$

The conditions for the existence and uniqueness of an equilibrium are those of non-cooperative models without common-pool goods.<sup>11</sup>

Intuitively, when altruism is imperfect, i.e. *ceteris paribus* the consumption of other members is valued strictly less than own consumption, the equilibrium described by (7) is not efficient since individuals have the incentive to purchase sub-optimal quantities of sharable goods and public goods (the free rider problem) and purchase above-optimal quantities of private goods which benefits can be fully appropriated. The mis-allocation of resources occurs even when there is a single breadwinner in the household. In this case, the household head can perfectly control the consumption of the private good for each member in the household (e.g. buying clothes for each child and the spouse) but not the consumption of the sharable good (e.g. food) since everyone eats from one common pot. As a result, members with zero income will consume more food and less clothing than what they would in an efficient equilibrium.

A natural question in such situation is to what extent the breadwinner is able to perfectly control  $f_i(\cdot) \forall i = 1, \dots, n$  in equation (3), which would make sharable goods indistinguishable from private goods. There are two reasons to believe that this is not the case. First, the breadwinner is unlikely to perfectly monitor at no cost what others eat. Second and most importantly, social norms related to households and families seem to agree with the existence of common-property goods (see U.N. definition of the household in section 2.1). The allocation of such goods may not be completely determined by the breadwinner. For example, a stay-at-home mother may be the one who makes the decisions regarding food consumption.

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<sup>11</sup>These condition are in Browning et al. (2010), Bergstrom et al. (1986), Bergstrom et al. (1992), Fraser (1992) and Lechene and Preston (2005).

**Recurrence and the folk theorem** The household is a relatively stable unit where the process of allocating resources is repeated over time. Without a certain date of household dissolution, the problem given by equations (1)-(6) does not have a unique equilibrium. The game theory literature indicates that two of infinite equilibria in this household “game” of allocating resources over time are i) the **Nash equilibrium in the static game** and ii) the **subgame perfect equilibrium supporting an efficient outcome** (folk theorem).<sup>12</sup>

The collective model previously described, which imposes efficiency within the household, is conceived with the idea that the folk theorem holds, see Browning and Chiappori (1998). That is, the recurrence interaction of household members generates mechanisms to avoid any mis-allocation of resources. Although, this equilibrium is supported by problem (1)-(6) in a repeated setting, this paper states that it is not the one prevailing. Instead, the observed consumption patterns described below are highly consistent with a Nash equilibrium in the static game as that described in (7). A well-known result in the literature is that a Nash equilibrium in a static game is always a subgame perfect equilibrium in the repeated game. Then, the allocation described in the static model (1)-(6) is also an equilibrium in a dynamic setting.

### 2.3 The sharing model as a hybrid between the collective and the non-cooperative household models

Under specific conditions, the sharing model is equivalent to a hybrid where a collective model is a voluntary arrangement embedded in a non-cooperative environment. Assume the utility function (1) is separable in  $s_i$ , i.e. the egotistic part of the utility can be expressed as  $U_i(x_i, K_i(s_i), Q)$  where  $K_i(\cdot)$  is a strictly increasing real-value function independent of  $x_i$  and  $Q$  (Deaton and Muellbauer (1980), Bergstrom (2016)), then the sharing model can be written as a two-step problem. In the first stage, each individual non-cooperatively decides i) on the purchases and the allocation of a set of goods and ii) on how much to contribute to a common pool of resources. Then, in the second stage, household members jointly decides how to spend common resources.

Assume that the only goods purchased with common resources are those previously denominated sharable goods - as explained below, this is just one case of many in this two-stage procedure. Then, the first stage is identical to the sharing model described above with the caveats that i) the budget constraint (6) becomes

$$p_x \sum_{j=1}^n \tilde{x}_{ij} + p_q \tilde{Q}_i + c_i = y_i \quad (8)$$

where  $c_i$  is the voluntary contribution to the common pool of resources, and ii) the allocation rule (3) is determined endogenously by an embedded collective model in the second stage. More precisely, the common

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<sup>12</sup>Infinitely repeated games generate a continuum of equilibria. This paper focuses on the static Nash equilibrium, which is capable of explaining consumption patterns.

pool of resources is allocated in the second stage with the following procedure.

$$\max \sum_{i=1}^n \mu_i(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z}) \left[ K_i(s_i) + \tau_i \left( \frac{1}{n-1} \sum_{j \neq i}^n \alpha_{ij} K_j(s_j) \right) \right] \quad (9)$$

$$s.t. \quad p_s \sum_{i=1}^n s_i = \sum_{i=1}^n c_i \quad (10)$$

Problem (9)-(10) is a standard collective model. But, instead of maximizing the weighted sum of the entire utility of individuals - expression (1) - as is common practice, it operates exclusively with the separable components  $K(\cdot)$  in  $U_i(x_i, K_i(s_i), Q)$  that derives utility from the sharable good. Following the collective model literature, the weight functions  $\mu_i(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z})$  for  $i = 1, \dots, n$  may depend on the vectors of prices  $\mathbf{p} = (p_x, p_s, p_q)$ , incomes  $\mathbf{Y} = (y_1, \dots, y_n)$ , and variables  $\mathbf{z}$ , called ‘*distribution factors*’, which affects the negotiation process within the household but not individual preferences  $U_i(\cdot)$ . In this particular two-step model, the weights may also depend on the individual contributions to the common pool of resources  $\mathbf{c} = (c_1, \dots, c_n)$ . The solution to problem (9)-(10) is as follows.

$$s_i = f_i \left( \sum_{i=1}^n c_i, \mu(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z}) \right) \quad (11)$$

Expression (11) is the allocation rule (3), but obtained endogenously from an embedded collective model and explicitly indicating the influence of the n-vector of weights  $\mu(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z}) = (\mu_1(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z}), \dots, \mu_n(\mathbf{Y}, \mathbf{p}, \mathbf{c}, \mathbf{z}))$ .

The two-step procedure just described can be applied to any partition of goods as long the utility function is separable in the two sets: the set of goods purchased with common resources in the second stage and the set of goods purchased with individual resources in the first stage. For example, it could be reasonable to assume that public goods are also purchased with common resources in the second stage. Although, the general case given by equations (1)-(6) and the two-step procedure just described do not always give the same solution, the two approaches are identical for the model in section 3.1 used to study indifference scales (see Appendix I).<sup>13</sup>

There are two polar cases in this two-step procedure. One where all goods are purchased with individual resources in the first stage, which reduces the model to a pure non-cooperative model as those mentioned above. In this case, all goods that are rival in consumption are also excludable (i.e. no sharable goods). The other polar case is where all goods are purchased with ‘common’ resources in the second stage, which transform the model into a pure collective model. Which of the non-rival goods are shareable and allocated in the second stage and which are private and allocated in the first stage reverts to the discussion in section 2.1.

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<sup>13</sup>For example, Browning et al. (2010) show that household members jointly contribute to at most one public good in a non-cooperative two-person household model similar to that given by (1)-(6) but without common-pool goods. Their result implies a local income pooling (i.e., a small redistribution of income among members does not affect the intra-household allocation for certain range of income shares), that does not necessarily hold in the two-step procedure described above.

**Remark 1. Multiple accounts and mental accounting.** It worth mentioning that the two-step procedure just described does not necessary require a unique common pool of resources. For example, a family may have separate accounts for food and transportation. In such case, there are two parallel second stages (9)-(10), one for the allocation of each of the two categories just mentioned. Having several ‘accounts’ for different types of goods is in line with the concept of mental accounting in behavioral economics (e.g. Thaler (1985)) but applied to the household rather than the individual.

**Remark 2. The limited power of collective model tests.** The hybrid model presented above may help explain why some collective model tests fail to reject their null hypothesis despite that several other papers find the intra-household allocation of resources to be inefficient. For example, Attanasio and Lechene (2014) develop and implement a clever test for the collective model. This test exploits that the distribution factors  $\mathbf{z}$  affect the demand for goods only indirectly by modifying individual weights  $\mu_i$ , see equation (9), (Bourguignon et al. (2009)). Then, under the null hypothesis that the collective model is true, for any pair of goods  $(v, w)$  and any pair of distribution factors  $(z_1, z_2)$ , equality (12) should hold.<sup>14</sup>

$$H_0 : \frac{\frac{\partial v}{\partial z_1}}{\frac{\partial v}{\partial z_2}} = \frac{\frac{\partial w}{\partial z_1}}{\frac{\partial w}{\partial z_2}} \quad (12)$$

If the sharing model from section 2.2 is true (i.e. the collective model is false), the null hypothesis (12) should not hold for at least one pair  $(v, w)$  of goods. If  $v$  and  $w$  are taken from different commodity composites (e.g.  $v$  is a sharable good and  $w$  is a private good), equality (12) certainly fails. However, Attanasio and Lechene use only the components of total food expenditures to test (12). In such a case, when only sharable goods are used, this type of collective model tests has no power against the sharing model.

### 3 The Deaton-Paxson paradox and other unexplained regularities of the Engel’s curve

This section presents empirical regularities in consumption that are incompatible with existing theories of the household, but can be explained by the sharing model introduced above.

Deaton and Paxson (1998) seminal paper reveals inconsistencies between the predictions of micro-economic models of the household and consumption patterns observed across countries. Specifically, the existence of intra-household economies of scale implies that, as household size increases, per capita expenditure on public goods (e.g. housing) should decrease, freeing resources to increase the consumption of non-public goods (e.g. food). However, evidence from developed and developing countries consistently shows the opposite. Formally, the Deaton-Paxson model (and the Barten (1964) model) with pure private and pure

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<sup>14</sup>Attanasio and Lechene (2014) do not test directly condition (12) but the equivalent  $\mathbf{z}$ -conditional demands.

public goods is as follows.<sup>15</sup>

$$\max U(s, Q) \quad (13)$$

$$s.t. \quad p_s s + \frac{p_q}{n} Q = y \quad (14)$$

In model (13)-(14), all household members are identical, thus symmetry is pre-imposed and the variables  $s$  and  $Q$  are per capita consumptions of a rival good (e.g. food) and a public good (e.g. housing). The effect of household size is clear from the per capita budget constraint (14). Holding total per capita expenditures  $y$  constant, an extra member in the household generates a de-facto decrease in the average price of public good  $Q$  that each member faces. Then, the condition for per capita consumption of food  $s$  to increase with  $n$  is as follows:<sup>16</sup>

$$\xi_{sp_s}^c > -\xi_{sy}(1 - \omega_q) \quad (15)$$

As stated in Deaton and Paxson (1998), inequality (15) is expected to hold. The own compensated price elasticity of food  $\xi_{sp_s}^c$  is presumably small and the income elasticity of food  $\xi_{sy}$  at the equilibrium share of the public good  $\omega_q = p_q Q / \mathbf{Y}$  is expected to be relatively large, particularly in low-income countries. Nonetheless, the empirical evidence is broadly inconsistent with inequality (15). As household size increases, the share of food expenditures tends to decline in both developed and developing countries. This is commonly referred in the literature as the Deaton-Paxson paradox.

Utility (13) can be obtained as a weighted sum of identical household members' utilities given by expression (1). That is  $U(s, Q) = \sum_{i=1}^n \mu_i(\mathbf{Y}, \mathbf{p}, \mathbf{z}) \left( U(s_i, Q) + (\tau/(n-1)) \sum_{j \neq i}^n U(s_j, Q) \right)$  after pre-imposing symmetry. Thus, condition (15) can be interpreted as that obtained from either a unitary model if Pareto weights  $\mu_i$  are fixed and identical for all  $i = 1, \dots, n$  or a collective model if the functions  $\mu_i(\cdot)$  are identical for all members.

It can be easily proven (see Appendix II), that replacing model (13)-(14) by a standard non-cooperative model does not do better in explaining the Deaton-Paxson paradox. However, this and the following unexplained regularities of the Engle's curve are consistent with the sharing model.

**Consumption regularities:** *i) As household size increases, the share of food in total expenditure declines (Deaton-Paxson paradox), ii) food as a share of food plus housing increases with household size (consistent with the Barten model) despite a simultaneous decline in per capita food expenditure, iii) food consumed away from home increases with household size, iv) The Engel's curve tends to be hump-shaped in poor countries, v) The Deaton-Paxson paradox is more evident in low-income countries (i.e., the decline in food share with household size is stronger).*

<sup>15</sup>Deaton and Paxson also consider the intermediate case of partially rival goods, but not sharable goods.

<sup>16</sup>The per capita demand for food is a function  $s = s(p_x, p_s, \frac{p_q}{n}, \frac{\mathbf{Y}}{n})$ . Holding per capita income constant (i.e. increasing  $\mathbf{Y}$  in the same proportion as  $n$ ), the elasticity of  $s$  with respect to  $n$  equals the negative elasticity of  $s$  with respect to  $p_q$ . Using the Slutsky equation, inequality (15) is obtained.

Regularities i) and iii) are described in Deaton and Paxson (1998). The first part of regularity ii) is shown by Gan and Vernon (2003) in response to the paradox, claiming that the evidence is consistent with the Barten model when one includes food and a good that is clearly more public than food. Nonetheless, Deaton and Paxson (2003) reply to Gan and Vernon arguing that the puzzle is not solved since the decline in per capital food consumption remains unexplained (second part of regularity ii). Regularity iv) has received less attention, but it is equally important. For several countries, the share of food in total expenditure increases with living standards among the poor, which appears to be inconsistent with the standard micro theory. Regularity v), also described in Deaton and Paxson (1998), strengthens the paradox. In poor countries, price elasticities of food are expected to be relatively low and the income elasticity of food relatively high, making inequality (15) more likely to hold. The next section shows how the sharing model can explain all the regularities in remark 1.

### 3.1 Sharing Model: Symmetric case with Stone-Geary utilities

This section specifies the functional form of the utility function (1) to make the model tractable and obtain close form solutions that are capable of explaining the consumption patterns described in section 3. Appendix V shows the condition that the income and price elasticities should satisfy to solve the Deaton and Paxson (1998) paradox for a generic utility function.

Consider a household with  $n$  identical members in relation to preferences and endowments. There is no heterogeneity in the level of altruism ( $\alpha_{ij} = 1 \ \forall i, j$ ). Then, the utility function (1) for member 1 reduces to:

$$\max \left( \frac{x_1^{1-\sigma}}{1-\sigma} + \frac{(s_1 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right) + \tau \left( \frac{x_2^{1-\sigma}}{1-\sigma} + \frac{(s_2 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right) \quad (16)$$

The utility derived from own consumption  $U_1(\cdot)$  has a Stone-Geary shape. The constant  $\gamma$  is usually interpreted as the minimum food consumption for subsistence. While  $\gamma = 0$  simplifies the expression to a CES utility,  $\gamma > 0$  breaks the homotheticity and guarantees the 1st Engel's law to hold (i.e. the share of food declines with income). All the other  $(n - 1)$  members of the household are identical. Thus, setting the consumption of all of them to be the same imposes no constraint. The altruistic term in (1) simplifies to the utility that any of the other members derives from consumption, such as member 2.<sup>17</sup>

The rules (2)-(5) simplify. The total consumption of member 1's private good  $x_1$  equals the quantity she purchases to be consumed by her  $\tilde{x}_{11}$  plus the  $(n - 1)$  identical quantities purchased by each of the other members of the household in her's behalf  $\tilde{x}_{21}$ , equation (17). On the other hand, the private goods consumed by each of the other household members  $x_2$  equals the quantity member 1 gives to each of them  $\tilde{x}_{12}$ , the quantities they purchase for themselves  $\tilde{x}_{22}$  and the transfers/gifts made among them  $\tilde{x}_{21}$  (equation (18)),

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<sup>17</sup>In equilibrium  $x_2 = x_3 = \dots = x_n$ , thus  $\frac{1}{n-1} \sum_{j=2}^n U(x_j, s_j, Q) = U(x_2, s_2, Q) = \left( \frac{x_2^{1-\sigma}}{1-\sigma} + \frac{(s_2 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right)$

which by symmetry should be identical to the quantities given to member 1 in (17).

$$x_1 = \tilde{x}_{11} + (n-1)\tilde{x}_{21} \quad (17)$$

$$x_2 = \tilde{x}_{12} + \tilde{x}_{22} + (n-2)\tilde{x}_{21} \quad (18)$$

The purchases of sharable goods go to a common pool formed by member 1's contribution  $\tilde{s}_1$  and the  $n-1$  contributions of others. Each member consumes an equal share.

$$s_1 = s_2 = \frac{\tilde{s}_1 + (n-1)\tilde{s}_2}{n} \quad (19)$$

Finally, the consumption of public goods  $Q$  equals the contribution of member 1 ( $\tilde{Q}_1$ ) plus the contribution of the other  $(n-1)$  members in the household ( $\tilde{Q}_2$ ), equation (20).

$$Q = \tilde{Q}_1 + (n-1)\tilde{Q}_2 \quad (20)$$

The budget constraint for the symmetric case is as follows.

$$p_x(\tilde{x}_{11} + (n-1)\tilde{x}_{12}) + p_s\tilde{s}_1 + p_q\tilde{Q}_1 = y \quad (21)$$

In (21), increasing the quantity of private goods consumed by others in one unit requires member 1 to buy  $(n-1)$  goods (e.g. one pair of shoes for each child) for this reason  $\tilde{x}_{12}$  is multiplied by this constant. The variable  $y$  represents individual income and also per capita income considering that all members are identical. The Nash equilibrium after each member maximizes (16) subject to (17)-(21) and plays a simultaneous game of complete information gives the following system of equations for per capita purchases (see appendix I for derivation).

$$\left\{ \begin{array}{l} \tilde{x} \equiv \tilde{x}_{11} = \tilde{x}_{22} = \frac{y - p_s\gamma}{\mathbb{P} p_x^{1/\sigma} (1 + \tau)^{1/\sigma}} \\ \tilde{x}_{12} = \tilde{x}_{21} = 0 \\ \tilde{s} = \frac{y - p_s\gamma}{\mathbb{P} p_s^{1/\sigma} n^{1/\sigma}} + \gamma \\ \tilde{Q} = \frac{y - p_s\gamma}{\mathbb{P} p_q^{1/\sigma} n} \end{array} \right. \quad (22)$$

where

$$\mathbb{P} \equiv \left( (1 + \tau)^{-1/\sigma} p_x^{1-1/\sigma} + n^{-1/\sigma} p_s^{1-1/\sigma} + n^{-1} p_q^{1-1/\sigma} \right) \quad (23)$$

When altruism is imperfect (i.e. *ceteris paribus* the consumption of other members is valued strictly less than own consumption), each person will only purchase private goods for herself making zero transfer/gift to others in the household. All members will purchase a non-zero quantity of food  $\tilde{s}$  and public goods  $\tilde{Q}$ . Appendix I shows that the demand system (22) is identical to that obtained by the two-step procedure in section 2.3 when both food and housing are purchased with common resources.



***Explaining regularity i) As household size increases, the share of food in total expenditure declines (Deaton-Paxson paradox)***

The expression for the expenditure share of food  $\omega_s \equiv p_s \tilde{s}/y$  derived from (22) is given by

$$\omega_s = \frac{p_s^{1-1/\sigma}}{n^{1/\sigma} \mathbb{P}} \left( 1 - \frac{p_s \gamma}{y} \right) + \frac{p_s \gamma}{y} \quad (24)$$

Then, the condition for the share of food to decrease with household size as observed in the data is

$$\frac{\partial \omega_s}{\partial n} < 0 \quad \Longleftrightarrow \quad (1 + \tau)^{-1/\sigma} (p_x/p_q)^{1-1/\sigma} + n^{-1}(1 - \sigma) > 0 \quad (25)$$

$$\Longleftrightarrow \quad \sigma < 1 + (\omega_x/\omega_q) \quad s.t. \quad \omega_x \equiv p_x \tilde{x}/y \quad \omega_q \equiv p_q \tilde{Q}/y \quad (26)$$

Inequality (26) indicates that there is an upper limit in relation to the complementarity of goods for the model to be consistent with the observed consumption patterns. Intuitively, if the complementarity of a sharable good, say meat, and a private good, say wine, is too high, then the expenditure on meat will not decline (or may even increase) with household size to keep enjoying wine. On the other hand, if private goods and sharable goods are substitutes, the addition of a member to the household makes everyone reduce the expenditures on sharable goods because they have to be divided among more people, favoring the consumption of private goods which benefits are fully appropriable by the buyer. The elasticity of substitution between the composite goods  $s$ ,  $x$  and  $Q$  is expected to be relatively low (Deaton and Paxson (1998)). Inequality (26) indicates that the sharing model can explain the Deaton-Paxson paradox in the realistic case when composite goods are gross complements.

***Explaining regularity ii) food as a share of food plus housing increases with household size (consistent with Barten model) despite a decline in per capita food expenditure***

A potential explanation for the Deaton-Paxson paradox is that there are other goods consumed by household members that are more rival than food.<sup>18</sup> Thus, the share of food in food plus housing expenditures **only** should increase with household size. This idea is used by Gan and Vernon (2003) to argue that there is no contradiction between theory and empirical evidence.<sup>19</sup> The sharing model is consistent with the sign of this food share derivative but provides a different explanation.

$$\frac{p_s(\tilde{s} - \gamma)/y}{p_q \tilde{Q}/y} = \left( \frac{p_s n}{p_q} \right)^{1-1/\sigma} \Rightarrow \frac{\partial (p_s \tilde{s}/(p_q \tilde{Q} + p_s \tilde{s}))}{\partial n} > 0, \quad \text{if } \sigma > 1 \quad (27)$$

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<sup>18</sup>Both, Deaton and Paxson (1998) and Gan and Vernon (2003) mention goods that are less *public* than food rather than more *rival*. However, the model they use cannot deal with non-excludable goods (see section 2.1). Thus, rivalry is the only characteristic modeled in their papers.

<sup>19</sup>Using the same data as Deaton and Paxson (1998), Gan and Vernon estimate Engel's curves replacing the dependent variable with the share of food in food plus housing expenditures only. Their results show that the share of food in food plus housing increases with household size in accordance to economic theory.

The first term in (27) is approximately the ratio of food share to housing share. This ratio is clearly increasing in household size  $n$  for a low elasticity of substitution (i.e.  $\sigma > 1$ ), a realistic assumption considering that the model is written for composite goods (see Deaton and Paxson (1998) page 901). Since  $\tilde{Q}$  is decreasing in household size, the share of food in food plus housing increases with household size consistent with the evidence presented in Gan and Vernon (2003).<sup>20</sup>

Deaton and Paxson (2003) point out that Gan and Vernon's argument does not solve the paradox since it does not explain why per capita food consumption declines with household size. It only shows that food consumption decreases less rapidly than housing. In the sharing model, the condition for per capita food consumption to decline is given by

$$\frac{\partial \tilde{s}}{\partial n} < 0 \quad \text{if} \quad \sigma < 1 + (\omega_x/\omega_q) \quad \text{s.t.} \quad \omega_x \equiv p_x \tilde{x}/y \quad \omega_q \equiv p_q \tilde{Q}/y \quad (28)$$

which is identical to inequality (26) and the core of the paradox. While the hypothesis that other goods are more rival than food is insufficient (at least empirically) to explain the decline in per capita food consumption, the sharing model gives the theoretical conditions in (28) for this regularity.<sup>21</sup>

### ***Explaining regularity iii) food consumed away from home increases with household size***

A given explanation for the Deaton-Paxson paradox is the plausible existence of economies of scale in food preparation. However, Deaton and Paxson discard this hypothesis because the decline in per capita time required to prepare food associated with an increase in household size should induce individuals to substitute food consumed away from home for food prepared at home. But, the data tend to show the opposite.<sup>22</sup>

While this regularity may serve as evidence against the existence of strong economies of scale in food preparation, it is difficult to be explained by standard models of the household. However, the sharing model gives an intuitive explanation. Food at home is a common-pool good - rival yet non-excludable. Then, as the household becomes larger the share consumed by the person who purchases it becomes smaller. But, food away from home is a private good. If desired, it is consumed entirely by the person who purchases it. To illustrate this point, assume that food  $s$  in equation (16) is a composite of food at home  $f_h$  and food away from home  $f_a$  given by (29).

$$s = f_h + f_a^\kappa, \quad 0 < \kappa < 1 \quad (29)$$

Since food at home is a non-appropriable good, it follows allocation rule (19). But, food consumed away

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<sup>20</sup>  $\frac{p_s(\tilde{s}-\gamma)/y}{p_q \tilde{Q}/y} = \frac{p_s \tilde{s}/y}{p_q \tilde{Q}/y} - \frac{p_s \gamma/y}{p_q \tilde{Q}/y}$ , since  $\frac{\partial \tilde{Q}}{\partial n} < 0$ , then  $\frac{p_s \tilde{s}/y}{p_q \tilde{Q}/y}$  is necessarily increasing in  $n$ .

<sup>21</sup> As Deaton and Paxson (2003) indicate in relation to Gan and Vernon (2003) argument, "Gan and Vernon focus on the possibility that there are substantial economies of scale in food consumption, which, if true, would certainly help resolve the puzzle. But they generate no empirical evidence to support their contention that food has greater economies of scale than clothing or transportation" page 1362.

<sup>22</sup> Deaton and Paxson (1998) main argument against this hypothesis is that the food shares contain only food purchases not the combination of food expenditure plus the time allocated to food preparation.

from home is a private good and the allocation is given by rules (17)-(18). It can be shown that per capita demand for food consumed away from home is given by equation (30).

$$\tilde{f}_a = \left( \frac{np_h\kappa}{p_a(1+\tau)} \right)^{1/(1-\kappa)} \quad (30)$$

where  $p_h$  and  $p_a$  are the prices of food consumed at home and food consumed away from home. Consistent with the evidence presented in Deaton and Paxson (1998), food away from home is clearly increasing in household size even when food at home declines with  $n$  (see equation (28)).

The sign of  $\partial \tilde{f}_a / \partial n$  depends of the relative income elasticities of food at home and food away from home. The quasi-linearity functional form imposed in (29) is an extreme case where every additional amount of money allocated to food is spent on food at home. The demand for food away from home responds only to the price effect induced by an extra member in the household.<sup>23</sup>

***Explaining regularities iv) The Engel's curve tends to be hump-shaped in poor countries and v) The Deaton-Paxson paradox is more evident in low-income countries***

Regularity v) is documented in Deaton and Paxson (1998). The decline in food share when household size increases is more pronounced in low-income countries. This fact exacerbates the paradox. Poor people are expected to have a higher income elasticity in food expenditure making (15) more likely to hold. Regularity iv) has received less attention, but it is equally problematic to be explained by existing models of the households. The Engel's curve in low-income countries tends to be increasing at the low end of the income distribution and declining for the rest of the population. If food is the most important necessity for survival, how is it possible that the extreme poor spends a lower proportion of their income on food than households with higher living standards?

The sharing model can explain, at least partially, regularities iv) and v) by making the altruistic parameter  $\tau$  to depend on income.<sup>24</sup> The positive relationship between altruism and income has been documented by Chowdhury and Jeon (2014) in an experimental study.<sup>25</sup> Assume that  $\tau$  in the utility function (16) is heterogeneous and has the following functional form.

$$\tau = \frac{(n-1)y^\beta}{1+y^\beta} \quad (31)$$

Expression (31) allows the level of altruism  $\tau$  to depend positively on income and household size. This functional form restricts the level of altruism to be non-negative (i.e. no hate or envy among household

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<sup>23</sup>Gan and Vernon (2003) point out that consumption regularity iii) explained here does not hold for all of countries. A plausible explanation is that the income effect of food consumed away from home is relatively large in countries where this regularity is violated.

<sup>24</sup>There may be other reasons in addition to variable altruism to explain hump-shaped Engel's curves. I am not aware of any other theory explaining this regularity. More research is needed in this area.

<sup>25</sup>The same paper discusses alternative behavioral models. However, the psychological reasons linking altruism and income are beyond the scope of the sharing model. Here, the relationship between altruism and income is taken as an assumption.

members) and less than  $(n - 1)$  (i.e. utility derived from own consumption of goods is valued more than the utility obtained from the consumption of others).

$$\omega_s = \frac{p_s^{1-1/\sigma}}{n^{1/\sigma} \mathbb{P}} \left( 1 - \frac{p_s \gamma}{y} \right) + \frac{p_s \gamma}{y} \quad (32)$$

$$\text{where } \mathbb{P} \equiv \left( \left( 1 + \frac{(n-1)y^\beta}{1+y^\beta} \right)^{-1/\sigma} p_x^{1-1/\sigma} + n^{-1/\sigma} p_s^{1-1/\sigma} + n^{-1} p_q^{1-1/\sigma} \right)$$

Equation (32) is the theoretical Engel's curve obtained from equations (22) and (31). Consistent with evidence, it is non-monotonic in income. Figure 1 shows the Engel's curve (32) for a two-person household. It has a hump-shaped profile.

Expression (32) is cumbersome, but the intuition is simple. As income increases, household members become more altruistic creating incentives to contribute more to common-pool goods. But, simultaneously food share tends to decrease because it is a necessity good (1st Engel's law). For poor people, the changes in altruism may dominate, resulting in increasing levels of food consumption. Because altruism is assumed to be bounded from above (i.e. it cannot grow indefinitely), then there is an income threshold for which the first Engel's law begins to dominate.

Allowing altruism to be non-fixed as in (31) can also explain why the Deaton-Paxson paradox is stronger in low-income countries. As  $\tau$  increases, the demand system (22) approaches that obtained from the Barten Model.

$$\text{as } \tau \rightarrow (n - 1), \quad \omega_s \rightarrow \frac{p_s^{1-1/\sigma}}{\left( p_x^{1-1/\sigma} + p_s^{1-1/\sigma} + (p_q/n)^{1-1/\sigma} \right)} \left( 1 - \frac{p_s \gamma}{y} \right) + \frac{p_s \gamma}{y} \quad (33)$$

Expression (33) is exactly the share of food derived from a Barten model with individual utility function (16).<sup>26</sup>

In addition to higher income, another reason to expect relatively high levels of intra-household altruism in developed countries is that households are usually formed by nuclear families, in contrast to developing countries where households are more likely to be organized around extended families. Since altruistic behaviors are plausibly less likely to occur towards in-laws than towards spouses or children, (33) is expected to be closer to the Barten model in developed countries.

### 3.2 Non-monotonic family size effects and household division

The sharing model generates a non-monotonic relation between members' individual utility and household size (i.e., it generates both economies of scale and diseconomies of scale depending on the number of members), which naturally explains why households cannot grow indefinitely. In contrast, the Barten model can

<sup>26</sup>When  $\tau = (n - 1)$ , the collective model with identical individuals and weights  $\mu_i = 1/n$  in (??) reduces to maximizing the function  $\left( \frac{x^{1-\sigma}}{1-\sigma} + \frac{(s-\gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right)$  subject to  $p_x x + p_s s + \frac{p_q}{n} Q = y$

only generate economies of scale, implying that the larger the household is, the better their members are.

$$\mathbf{V} = \frac{1 + \tau}{1 - \sigma} (y - p_s \gamma)^{1 - \sigma} \frac{\tilde{\mathbb{P}}}{\mathbb{P}^{1 - \sigma}} \quad (34)$$

where  $\mathbb{P}$  is defined in (23) and

$$\tilde{\mathbb{P}} \equiv (1 + \tau)^{1 - 1/\sigma} p_x^{1 - 1/\sigma} + n^{1 - 1/\sigma} p_s^{1 - 1/\sigma} + p_q^{1 - 1/\sigma}$$

The indirect utility function (34) resulting from converting quantities purchased into quantities consumed in (22) and replacing them in (16) is increasing in household size when this is small and decreasing when large. The reason for a non-monotonic indirect utility function is that an additional members in the household generates two opposite effects. On the one hand, it decreases the per capita cost of public goods. On the other hand, it reduces the individual consumption share of the marginal common-pool good. When the first effect dominates the second one, there are household economies of scale, otherwise there are diseconomies of scale. Figure 2 shows the theoretical indirect utility function for different levels of altruism  $\tau$ . The optimal household size positively depends on the level of altruism. Above the maximum, members benefits by splitting the household. As indicated in the conclusions, the sharing model can be used to study the economic incentives for household formation and household dissolution together with group behaviors in an integrated framework.

## 4 Measuring indifference scales in the household

This section presents a method to estimate household indifference scales, defined as *the amount of money a person should receive to attain the same **indifference curve** as that arising from an additional household member* (Browning et al. (2013), Dunbar et al. (2013), Chiappori (2016)). The concept of indifference scales is theoretically and empirically less demanding than the traditional income-based economies of scale, which is defined as *the amount of money a person/family should receive to attain the same **utility** as that arising from an additional household member*.

The procedure in this section is derived from the sharing model introduced in section 2.2. The method imposes no functional form for the utilities  $U_i(\cdot)$ . By assuming that all household members are identical, the resulting measure should be interpreted as the monetary increment that put the reference individual in the same indifference curve as having an extra household member identical to herself/himself. Empirically, all regressions include characteristics of the household that account for observable heterogeneity in preferences.

Consider the indirect utility function obtained from problem (1)-(6)

$$\mathbf{V}(p_x, p_s, p_q, y, n)$$

The derivative of the indirect utility function with respect to household size  $n$  can be written as a function

of marginal utilities evaluated at the optimum and changes in quantities demanded with respect to  $n$ .

$$\frac{\partial \mathbf{V}}{\partial n} = \frac{\partial U^*}{\partial x} \frac{\partial x}{\partial n} + \frac{\partial U^*}{\partial s} \frac{\partial s}{\partial n} + \frac{\partial U^*}{\partial Q} \frac{\partial Q}{\partial n} \quad (35)$$

Expression (35) is the increase in ‘utils’ as a result of having an extra member in the household. The first order conditions in the maximization problem and the symmetry obtained from identical individuals generate the following expressions for the Lagrange multiplier  $\lambda$  (see Appendix I).

$$\lambda = \frac{\partial U^*}{\partial x} \frac{1}{p_x} = \frac{\partial U^*}{\partial s} \frac{(1+\tau)}{np_s} = \frac{\partial U^*}{\partial Q} \frac{(1+\tau)}{p_q} \quad (36)$$

Using (36) and the fact that in equilibrium the relationship between per capita consumed and per capita purchased goods are:  $s = \tilde{s}$ ,  $x = \tilde{x}$  and  $Q = n\tilde{Q}$ , then (35) simplifies to the following expression:<sup>27</sup>

$$\frac{\partial \mathbf{V}}{\partial n} / \lambda = p_x \frac{\partial \tilde{x}}{\partial n} + \frac{np_s}{(1+\tau)} \frac{\partial \tilde{s}}{\partial n} + \frac{np_q}{(1+\tau)} \frac{\partial \tilde{Q}}{\partial n} + \frac{p_q \tilde{Q}}{(1+\tau)} \quad (37)$$

$$= \frac{y}{(1+\tau)} \left( \frac{(1+\tau)}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q \right) \quad (38)$$

Usually, the Lagrange multiplier in a utility maximization problem is equal to the marginal utility of income. But in this case,  $\lambda$  is the marginal utility of income *conditional on the purchases of other members to be constant*. Instead, the quantity needed is the marginal utility of income after arriving to the new Nash equilibrium. It can be shown (see appendix III) that (38) can be adjusted in a simple way:

$$\frac{\partial \mathbf{V}}{\partial n} / \lambda \mathbf{C} = \frac{y}{\mathbf{C}(1+\tau)} \left( \frac{(1+\tau)}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q \right) \quad (39)$$

where

$$\mathbf{C} = \left[ 1 + \tau \left( \frac{\partial \omega_x}{\partial \log(y)} + \omega_x \right) + (n-1) \left( \frac{\partial \omega_s}{\partial \log(y)} + \omega_s \right) + (n-1) \left( \frac{\partial \omega_q}{\partial \log(y)} + \omega_q \right) \right] \quad (40)$$

The left hand side of (39) is the indifference scale coefficient. It is the amount of money that a person living in a household with  $n$  members should receive to attain the same indifference curve as himself/herself living in an household with  $n+1$  members holding the same per capita income in both cases. The right hand side of (39) is formed by quantities that can be estimated with a standard Engel’s curve, with the exception of the altruistic parameter. Section 5.3 estimates indifference scales coefficients for different values of  $\tau$ .<sup>28</sup>

## 5 Empirical Analysis

### 5.1 Data

The dataset used in the analysis is the Mexican Family Life Survey (MxFLS), Rubalcava and Teruel (2006, 2013). It is a multi-thematic nationally representative longitudinal survey. Individuals are followed over time

<sup>27</sup>Equation (38) is obtained using the fact that the change in the budget share  $\omega_h$  of good  $h \in \{s, x, Q\}$  with respect to household size  $n$  is  $\partial \omega_h / \partial n = (p_n / y)(\partial h / \partial n)$ .

<sup>28</sup>The fact that  $\tau$  is not identified is not a drawback of this paper. Other studies in the literature of economies of scale or indifference scales simply compute their results assuming implicitly or explicitly zero altruism.

regardless of changes in residence and household division. This characteristic makes the data particularly suitable to study indifference scales and resource allocation within the household. There are currently three rounds available. The first round took place in 2002, the second round in 2005-2006 and the third round in 2009-2010. All the rounds contain information about consumption and income. Consumption expenditures are measured at the household level and include a detailed enumeration of food and non-food items. Labor income is reported at the individual level and non-labor income at both, the individual level and the household level. The sample used in this study includes all households with non-zero income and non-zero food expenditure. The last row of Table 2 shows the total number of households with these characteristics disaggregated by year.

**The family and the household** The analysis exploits the distinction between the family and the household. This is new in this literature. The definition of the household is taken from the survey. It consists of people who live under the same roof and share resources.<sup>29</sup> The family is defined as a group of people that belonged to the same household in 2002 regardless of whether they moved and formed a new household in subsequent years. This definition is purely instrumental for the econometric analysis. It does not try to follow any sociological or anthropological definition. Figure 3 shows diagrammatically the relationship between households and families. For example, in the first round of the survey, two identical households are observed, say the father, the mother and one son. In both households, the son gets married and has a baby between the first and the second rounds. However, in one case the son and his nuclear family move to a new house and in the other case the son and his nuclear family stay in the same house. Family A and family B are treated as identical in all dimensions except that A is considered to be *split* (i.e. living in two houses rather than in one) in the last two rounds.

The longitudinal structure of the data allows the empirical analysis to follow closely the ideal situation to study indifference scales. According to the Barten model, when a household splits their members are on average worse off because they have to duplicate the expenditure on public goods to obtain the same per capita consumption. For example, if all members in families A and B in Figure 3 want a new refrigerator in 2005, then family A has to reduce more the consumption of other goods to release resources to buy two new refrigerators, one for each house, and obtain the same per capita fridge consumption as family B. Table 2 shows the total number of families defined as in Figure 3. In 2002, the number of households and the number of families are the same by construction.

**Consumption shares and rent imputation** The largest component of household public goods is presumably housing. However, most of the household heads report paying no rent or mortgage. Ignoring

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<sup>29</sup>The survey defines the household as “A person or group of people, related or unrelated by biological bonds, who usually live together in a part of or in an entire building/dwelling and usually consume meals prepared with a common budget on the same stove/oven and even use the same tools for preparing the meals.”

housing in total consumption underestimates the share of public goods and overestimates the shares of food and other non-food items in total expenditure. A solution to this problem is imputing the rent. Since the 2005-2006 round, interviewees were asked about the hypothetical rent that they would pay for the house they live in. Although this measure is expected to be noisy, on average it is assumed to be correct. The rent imputation procedure is based on the following hedonic regression.

$$\log(h_{jls}) = x'_{il}\alpha + d_s + u_l + \epsilon_{jl} \quad (41)$$

The log reported hypothetical rent  $h_{jls}$  for house  $j$  in administrative unit (state)  $s$  in locality  $l$  is regressed on characteristics of the house  $x_{jls}$ , dummy variables for the state  $d_s$  where the house is located and dummy variables for the population size of the locality  $u_l$ . The house characteristics included in the regression are the materials of the roof, walls and floors, the access to drinkable water inside the house, the number of rooms, sewage, whether the house has a kitchen and the fuel used to cook. Regression (41) is computed for round 2005-2006 when the hypothetical rent is available.<sup>30</sup> Since the regressors are available in all periods, the imputed rent (42) for each household in each period  $t$  is computed taking the exponential of the predictions.

$$irent_{jlst} = \exp(x'_{ilst}\hat{\alpha} + \hat{d}_s + \hat{u}_l) \quad t = 2002, 2005, 2009 \quad (42)$$

The total consumption is computed by adding the imputed rent and the reported consumption expenditures of other goods. Results without imputed rent are also reported. All values are adjusted by inflation. Following the theory from previous section, goods are classified into: i) sharable goods (food), ii) public goods (housing and durables) and iii) private goods (the rest of the goods). The shares are reported in Table 2 with and without imputed rent. A detailed list of the components of each commodity composite is in Table 1.

## 5.2 Empirical strategy

The first part of the empirical section revisits the Deaton-Paxson paradox enhancing the econometric method commonly used to measure household economies of scale (or indifference scales). In addition to analyzing how food share correlates with household size, the specification exploits household splits in longitudinal data.

$$\omega_{ft}^s = \beta_1^s \log(y_{ft}) + \beta_2^s \log(n_{ft}) + \beta_3^s split_{ft} + z'_{jt}\beta_4^s + \zeta_f^s + \psi_t^s + \epsilon_{ft}^s \quad (43)$$

The estimating equation (43) is an augmented Engel's curve where the share of food in total expenditure  $\omega_{ft}^s$  for family  $f$  in period  $t$  is regressed on (log) family *per capita* expenditures  $y_{ft}$ . As is common practice, the size of the family  $n_{ft}$  in the regression is intended to measure the existence of economies of scale (or more recently indifference scales). In the presence of public goods,  $\beta_2$  should be positive to agree with

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<sup>30</sup>The round 2009-2010 also contains information about hypothetical rent. Results are almost identical when this round is used in the hedonic regression. The correlation of imputed rents using the 2005-2006 and the 2009-2010 rounds is above 0.9.



the prediction of the Barten model. Regression (43) also includes the variable *split*. It is an indicator that takes the value one if the family lives in two houses and zero if it lives in one house (see Figure 3). Conditional on family size,  $\beta_3$  is expected to be negative in the presence of positive indifference scales. The coefficients  $\beta_2$  and  $\beta_3$  measures the same underlying phenomenon. So, they have to agree on the conclusions. If the Deaton-Paxson paradox is genuinely a behavioral fact contradicting existing theoretical models of the household rather than the result of an econometric misspecification, not only should  $\beta_2$  be negative, but also  $\beta_3$  positive.

A potential econometric problem generally not addressed in the estimation of Engel's curves is the endogeneity of family size. The desire for a relatively large family may be associated with the composition of goods consumed. Although heterogeneous preferences are partially controlled for with the inclusion of demographic variables  $z_{ft}$ , age and sex may not be sufficient statistics. To the extent that preferences are stable over time, the inclusion of family fixed effects  $\zeta_f$  in longitudinal data solves the problem.<sup>31</sup>

Previous studies on Engel's curves focus on the consequences of a potential misspecified functional forms and the presence of measurement error. The evidence indicates that, even though the relationship between per capita household expenditure and the share of food consumed may not be log linear as in (43), allowing more flexibility in the functional form does not alter the conclusions in relation to indifference scales. Potentially more problematic is the measurement error in total family expenditures because it affects simultaneously the right hand side and the denominator of the left hand side of (43). Following previous studies, (log) family per capita expenditures is instrumented with (log) family per capita income. The measurement errors between the instrument and the instrumented variables are likely uncorrelated. Finally,  $\psi_t$  in (43) is a set of survey round fixed effects and  $\epsilon_{ft}$  is a disturbance term.

**Indifference scales** Equation (43), which is estimated for the share of food, can also be computed for the share of public goods  $Q$  and private goods  $x$  as defined in the previous section. Then, the estimated coefficients can be used to construct a plug-in estimator of indifference scales by taking the sample counterpart of equations (39) and (40).

$$\widehat{scale}(n, y, \tau) = \frac{1}{\widehat{C}(1 + \tau)} \left( \frac{(1 + \tau)}{n} \widehat{\beta}_2^x + \widehat{\beta}_2^s + \widehat{\beta}_2^q + \widehat{\omega}_q(y) \right) \quad (44)$$

where

$$\widehat{C} = \left[ 1 + \tau \left( \widehat{\beta}_1^x + \widehat{\omega}_x(y) \right) + (n - 1) \left( \widehat{\beta}_1^s + \widehat{\omega}_s(y) \right) + (n - 1) \left( \widehat{\beta}_1^q + \widehat{\omega}_q(y) \right) \right] \quad (45)$$

Equation (44) differs from (39) in that the right hand side is divided by family per capita expenditures. Then, the scale coefficient is now interpreted as the equivalent *percentage change* in total expenditure

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<sup>31</sup>There are no clear instruments in the literature for household size. Studies about fertility and child investment use *twining* as a natural experiment. However, twins's datasets containing detailed information about income and consumption are rare. Even when such data are obtained, changes household size cannot be disentangled from changes in household composition.

corresponding to an additional member in the family. The coefficient depends on family size, per capita expenditures and level of altruism. Two of these variables are observed,  $\tau$  has to be assumed.

The coefficient superscripts in (44)-(45) indicate the dependent variable used to estimate (43) ( $s$ : food,  $x$ : private goods and  $q$ : public goods). The linearity in (43) guarantees that the estimated elasticities sum zero, so there is no need to impose that  $\beta_m^s + \beta_m^x + \beta_m^q = 0$  for  $m = 1, 2$ . The non-homotheticity of preferences implies that the shares depend on household per capita expenditures. The next section shows results for the mean and quartiles of the distribution.

### 5.3 Results

Table 3 shows results from estimating Engel's curves. Panel A columns 1 to 3 use the standard method where the unit of observation is the household and the presence of indifference scales is identified with the coefficient associated with log number of members. Column 1 shows the result of estimating equation (43) by OLS after pooling the three rounds of the survey. Column 2 instruments household per capita expenditures with income. Column 3 exploits the panel structure of the data by including household fixed effects in addition to instrumenting household per capita income. In these first three regressions, the computation of household expenditures *does not* include imputed rent. The specifications used in columns 4 and 5 are identical to those used in columns 2 and 3 but including imputed rent in the denominator of food shares and in total log household per capita expenditures.

In all the regression in panel A, the coefficient associated with household size is negative and statistically different from zero. The results contradict the Barten model and are in line with the Deaton-Paxson paradox described in other countries.

In Table 3 panel B, the unit of observation is the family instead of the household. Then, it is possible to measure the impact that living in two houses ( $split = 1$ ) rather than in one ( $split = 0$ ) has on the share of food. Other than this, the specifications in panel B are identical to those in panel A. Per capita expenditures on public goods, such as housing, appliances, furniture and house decoration, are expected to increase when households split. As a result, the share of food should decline according to the Barten model. However, all the regressions in panel B show the opposite. Holding family size and total family per capita consumption constant, the share of food increases when households split. These results lead to the same conclusions obtained in panel A and reinforces the Deaton-Paxson observation that the empirical evidence contradicts the standard theory.

The enhanced method to estimate Engel's curve suggests that the lack of agreement between theory and empirics is not the result of econometric problems. Instead, it is the theory that should be revised to reconcile it with the facts. The sharing model introduced in section 2.2 is a plausible explanation of this old puzzle.

Gan and Vernon (2003) suggest that the Deaton-Paxson paradox is not such. They claim that there are

goods consumed in the household that are more rival than food.<sup>32</sup> When they consider a basket of goods containing only food and goods that are presumably less rival than food (e.g. housing), the predictions of the Barten model holds. That is, the evidence shows that the share of food on food plus housing increases with household size.

Table 4 shows the result of estimating (quasi) Engel's curves where the dependent variable is the share of food in the consumption of food and housing. Columns 1 and 2 follow the standard method. As in Gan and Vernon (2003), the share of food in food plus rent increases with household size. This result is maintained in columns 3 and 4. As predicted by the Barten model, families that live in two houses rather than in one (*split* = 1) consume a smaller share of food in food plus housing.

Despite that results in Table 4 agree with the Barten model, Deaton and Paxson (2003) argue that the paradox is not solved because the fact that, at a given level of total per capita consumption, per capita food consumption (not the food share) tends to decline as households become larger remain unexplained. In the presence of public goods, the monetary contribution each member has to make to obtain a given level of consumption declines as household size increases. Then, the freed resources can be used to consume more. Being food a normal good, the Barten model predicts that its consumption should increase.

Table 5 shows the demand for food. The specifications are identical to those in Table 3 but with log per capital food consumption as a dependent variable instead of the food share. Deaton and Paxson (2003) critique is evident. Panel A shows that per capita food consumption declines with household size. This fact cannot be explained by arguing that other goods are less rival than food as in Gan and Vernon (2003). Panel B corroborates the results in panel A. Families that liven in two houses rather that in one, consume on average *more* food per capita at a given level of total per capita expenditures.

The sharing model presented in section 2.2 is capable of explaining the consumption patterns in Tables 3-5. If food is a common-pool good but still rival in consumption, then the larger the household is, the smallest the portion a member gets from her marginal contribution to the common pot. Thus, as households become larger, their members have more incentives to reallocate resources from sharable goods (e.g. food) to private goods (e.g. clothing).

**Economies of scale coefficients, a traditional approach** The Engel's curve specification used to compute scale coefficients in this section corresponds to the pooled instrumental variable as shown in column 4 Table 3. This specification generates similar results to panel-fixed-effects, but allows the computation of scale coefficients for different quartiles of the distribution.<sup>33</sup>

Table 6 shows estimated household indifference scales using formula (44)-(45). In this case, the public

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<sup>32</sup>This paper as well as others in the the literature refer as goods that are more *private* rather than more *rival*. However, the only characteristic in these models that distinguishes private and public goods is rivalry. They cannot incorporate excludability (see discussion in section ??)

<sup>33</sup>Some families change quartiles from one round to another creating problems in partitioning the sample by living standards.

goods, private goods, and food shares are evaluated at the sample mean. Each column is generated using a different assumption about the level of altruism in the family. The results indicate, for example, that a two-person household with no altruism ( $\tau = 0$ ) would need 12.4% more income to obtain the same utility level as adding an extra member and maintaining the same **per capital** income.

The last row of Table 6 shows the single adjustment factor that best fits the results. Inequality and poverty studies that attempt to adjust per capita consumption/income by traditional economies of scale usually rely on the following single parameter formula (Deaton (1997)).

$$y^{adj} = \frac{\mathbf{Y}}{n^\xi}, \quad 0 \leq \xi \leq 1 \quad (46)$$

Adjusted household per capita income/consumption  $y^{adj}$  is computed dividing total income/consumption  $\mathbf{Y}$  by household size adjusted by  $\xi$ . Depending on the level of altruism assumed, the estimated adjustment factor  $\hat{\xi}$  ranges from 0.7 to 0.84. Figure 4 plots two of the columns in Table 6 (solid lines) and the implied coefficients computed with formula (46) and the estimated  $\xi$  at the bottom of the Table (dotted lines). The estimated values and the fitted lines using (46) are remarkably similar.

For comparison reasons, the last column of Table 6 reports similar statistics than previous columns but computed with scale parameters used by the OECD, see Cowell (2011). The OECD assumes significantly stronger economies of scale from a one-person household to a two-person households.<sup>34</sup>

Table 7 shows the scale coefficients for different quartiles of the total family expenditures distribution. The adjustment for indifference scales is not monotonic along the income distribution because the slope of the Engel's curve is highest (in absolute terms) for the second quartile.

The method used in this section to measure indifference scales in the family is easily implemented and does not require panel data. It can be computed with any cross-section data containing information about household consumption.

## 5.4 Public goods with congestion

For the computation of indifference scales, goods are considered extreme cases in relation to rivalry and excludability (e.g. public goods are considered fully non-rival and non-excludable). This section presents a sensitivity analysis changing these assumptions.

When congestion affect public goods, the sharing rule (20) should be modified.

$$Q = \frac{\tilde{Q}_1 + (n-1)\tilde{Q}_2}{n^\phi}, \quad 0 \leq \phi \leq 1 \quad (47)$$

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<sup>34</sup>The formula used by the OECD to adjust family income is  $y^{adj} = \frac{\mathbf{Y}}{\sum_i^n d_i}$ , where  $\mathbf{Y}$  is total family income and  $d_i$  is a value assigned to household member  $i = 1, \dots, n$ . The values are  $d_i = 0.67$  for the first adult in the household  $d_i = 0.33$  for each of the other adults and  $d_i = 0.2$  for each child under 14 years old (Cowell (2011) page 105). Last column in Table 6 assumes that all members are adults.

The parameter  $\phi$  controls the degree of congestion. When  $\phi = 0$ , the good is completely non-rival, when  $\phi = 1$  the good is fully rival and the allocation rule becomes identical to that for sharing goods in equation (19). It can be shown that congestion affects the measurement of indifference scales by adjusting the last term of (39). This expression becomes (see appendix IV)

$$\frac{\partial \mathbf{V}}{\partial n} / \lambda \mathbf{C} = \frac{y}{\mathbf{C}(1 + \tau)} \left( \frac{(1 + \tau)}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q(1 - \phi) \right) \quad (48)$$

Table 8 shows indifference scales coefficients for congestion levels  $\phi = 0.1$  and  $\phi = 0.3$  (low and medium levels of altruism). The values in this Table are lower in relation to those in Table 6. For relatively high congestion  $\phi = 3$  the scale coefficients decline approximately one third.

## 6 Summary and conclusions

This paper presents a new model for the allocation of resources in the household where individuals are assumed to control their own income and voluntarily share purchased goods with other members of the family. The allocation rule for each good depends on the intrinsic characteristic of the commodity and on the norms governing the interaction of household members. In this respect, in addition to public goods and private goods commonly included in other models, this paper recognizes the existence of common-pool goods. Common-pool goods are rival in consumption, but contrary to private goods, they are non-excludable. These goods cannot be fully appropriated by the buyer because they are supposed to be shared with other members in the household. Food is argued to be a common-pool good. The ‘*common provision of food*’ is considered a distinctive characteristic in many definitions of the household.

The model in the paper challenges the assumption made by the collective model that the allocation of resources in the household is efficient. A Pareto-efficient allocation is incompatible with the consumption patterns observed in the data, but not with the Nash equilibrium described in the paper. More specifically, the model is able to explain why per capita food consumption tends to decline as household size increases. This fact is considered an unresolved puzzle (Deaton-Paxson paradox). Previous models of the household are not capable of explaining this consumption patterns.

Under certain conditions, the sharing model can be interpreted as a hybrid between a traditional non-cooperative model and a pure collective model. This characteristic of the model can explain why some empirical test of collective models fail to reject their null hypothesis while at the same time other papers reject the Pareto-efficient allocation implied by such models.

The first part of the empirical analysis revisits the Deaton-Paxson paradox exploiting household divisions in longitudinal data. Results using the new approach reinforces the conclusions found in Deaton and Paxson (1998). The second part of the empirical section derives and implements a method to compute household indifference scales. The estimation of indifference scales coefficients has been elusive because of the unresolved

Deaton-Paxson paradox. The correct computation of indifference scales has enormous policy implications since it affects how to measure inequality and poverty.

The empirical papers in the literature rejecting intra-household Pareto-optimality do not usually model households' behavior to explain their findings. Udry (1996) suggests that more theoretical research is needed to explain why households do not reach the utility-possibility frontier as described in his paper. The sharing model is a simple and tractable theory that breaks the tradition of imposing intrahousehold efficiency. It is grounded on the reasonable idea that allocation rules differ by good type. The ability of the sharing model to explain consumption regularities that other theories cannot describe suggests that the model deserves more examination.

The sharing model presented in this paper can be used to study a variety of topics. Ongoing research includes the unification of theories about household formation, household dissolution and intrahousehold allocation of resources, as well as the study of joint labor supply.

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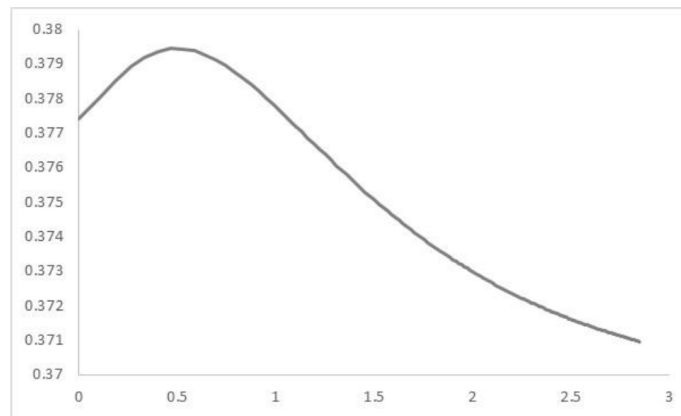
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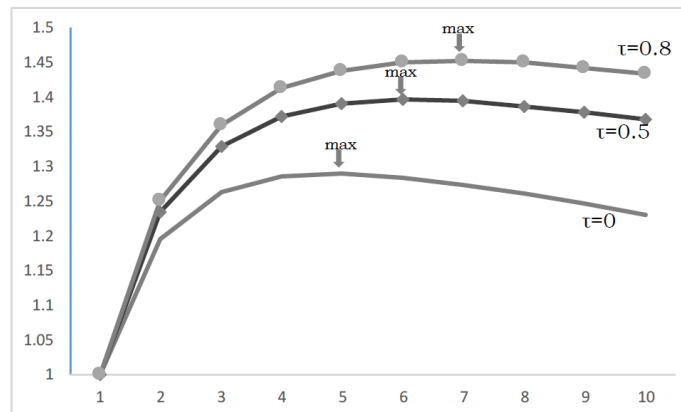
## 7 Figures and tables

Figure 1: Hump-shaped Engel's curve ( $n=2$ ):  
Food share as a function of log per capita income



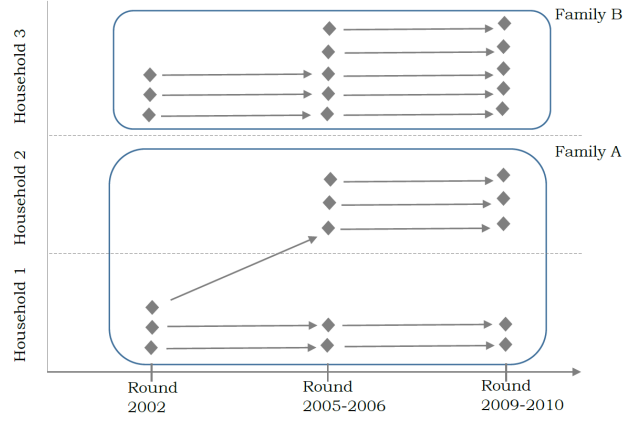
The parameter values are:  $\sigma = 2$ ,  $\beta = 3$ ,  $\gamma = 0.043$ . The prices of all goods are set to 1.

Figure 2: Indirect utility as a function of household size:  
Different levels of altruism



The parameter values are:  $\sigma = 2$ ,  $\gamma = 0.043$ . Per capita expenditures are set to 10. The three indirect utilities are rescaled to equal one for  $n = 1$ .

Figure 3: ‘Empirical’ definition of households and families



Note: Each diamond represents a person-round observation. The arrows follow persons across the longitudinal dataset rounds.

Figure 4: Percentage increase in per capita income equiv. to adding a household member (by family size)

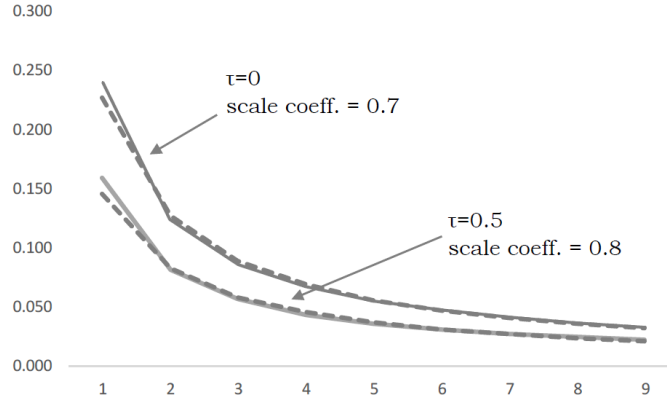


Table 1: Consumption components

Commodity composite	Goods included	List
Sharable goods	food except meals consumed away from home	vegetables, fruits, cereals, grains, meats, industrially-processed food
Public goods	housing, durables and school fees	imputed rent, TV sets, radios, cameras, washing machines, refrigerators, furniture, school tuition and fees, etc.
Private goods	clothing, tobacco, transportation, hygiene, etc.	clothing, toys, medicines, doctor's visits lotions, deodorants, magazines, etc.

Note: Some studies consider children as household public goods, this is why school tuitions and fees are included in this category. Results are very similar when these items are considered private goods.

Table 2: Summary statistics

	Survey year					
	2002		2005-2006		2009-2010	
	mean	s.d.	mean	s.d.	mean	s.d.
family size	4.419	1.982	4.939	2.343	5.699	2.878
split	0.000	0.000	0.082	0.275	0.182	0.386
shares w/imputed rent						
share food	0.427	0.163	0.456	0.155	0.465	0.151
share private	0.329	0.153	0.307	0.155	0.311	0.147
share public	0.245	0.127	0.237	0.120	0.224	0.111
shares without imp. rent						
share food	0.520	0.196	0.564	0.194	0.569	0.184
share private	0.394	0.172	0.369	0.174	0.373	0.166
share public	0.086	0.123	0.067	0.104	0.058	0.092
(log) per capita expend.	7.356	0.833	7.298	0.814	7.275	0.752
(log) per capita income	7.244	1.232	7.239	1.117	7.200	1.085
families in sample	5,304		4,840		4,465	
households in sample	5,304		5,205		5,165	

Table 3: Engel's curve

	Food share (excl. rent)			Food share (w/imp. rent)	
	OLS (1)	IV-GMM (2)	IV-FE (3)	IV-GMM (4)	IV-FE (5)
Panel A: Households					
log household per capita exp. <sup>+</sup>	-0.132*** (0.00169)	-0.175*** (0.00468)	-0.174*** (0.0287)	-0.137*** (0.00453)	-0.124*** (0.0288)
log household size	-0.0644*** (0.00341)	-0.0980*** (0.00518)	-0.0999*** (0.0206)	-0.0641*** (0.00492)	-0.0538** (0.0209)
Constant	1.553*** (0.0155)	1.913*** (0.0402)	1.897*** (0.232)	1.523*** (0.0398)	1.401*** (0.239)
Observations	15,639	15,639	15,639	15,674	15,674
Panel B: Families					
log family per capita exp. <sup>++</sup>	-0.132*** (0.00173)	-0.173*** (0.00479)	-0.170*** (0.0272)	-0.135*** (0.00457)	-0.116*** (0.0266)
log family size	-0.0679*** (0.00362)	-0.0998*** (0.00542)	-0.0911*** (0.0208)	-0.0682*** (0.00510)	-0.0459** (0.0205)
split	0.0336*** (0.00543)	0.0529*** (0.00527)	0.0539*** (0.0150)	0.0381*** (0.00486)	0.0310** (0.0145)
Constant	1.562*** (0.0159)	1.902*** (0.0412)	1.854*** (0.220)	1.517*** (0.0403)	1.325*** (0.221)
Observations	14,609	14,609	14,609	14,609	14,609

Standard errors in parentheses

\*\*\* p&lt;0.01, \*\* p&lt;0.05, \* p&lt;0.1

All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender

<sup>+</sup> instrumented with log household per capita income in columns 2 to 5<sup>++</sup> instrumented with log family per capita income in columns 2 to 5

Table 4: (quasi) Engel's curve:  
Dep. var.: share of food in food plus imputed rent

	Households		Families	
	IV-GMM (1)	IV-FE (2)	IV-GMM (3)	IV-FE (4)
log household per capita exp. <sup>+</sup>	-0.0211*** (0.00421)	0.0698*** (0.0204)		
log family per capita exp. <sup>++</sup>			-0.0189*** (0.00419)	0.0735*** (0.0189)
log household size	0.0341*** (0.00457)	0.101*** (0.0149)		
log family size			0.0301*** (0.00469)	0.0947*** (0.0146)
split			-0.0159*** (0.00455)	-0.0605*** (0.0103)
Constant	0.808*** (0.0366)	0.0509 (0.169)	0.797*** (0.0367)	0.0283 (0.157)
Observations	15,633	15,633	14,609	14,609

Standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender

<sup>+</sup> instrumented with log household per capita income

<sup>++</sup> instrumented with log family per capita income

Table 5: Demand for food  
Dep. var. log per capita food consumption

	excl. imputed rent			w/imputed rent	
	OLS (1)	IV-GMM (2)	IV-FE (3)	IV-GMM (4)	IV-FE (5)
Panel A: Households					
log household per capita exp. <sup>+</sup>	0.688*** (0.00431)	0.590*** (0.0123)	0.584*** (0.0707)	0.620*** (0.0134)	0.651*** (0.0813)
log household size	-0.121*** (0.00868)	-0.197*** (0.0133)	-0.228*** (0.0516)	-0.158*** (0.0145)	-0.162*** (0.0606)
Constant	1.633*** (0.0394)	2.445*** (0.104)	2.517*** (0.572)	2.054*** (0.116)	1.828*** (0.676)
Observations	15,630	15,630	15,630	15,630	15,630
Panel B: Families					
log family per capita exp. <sup>++</sup>	0.687*** (0.00442)	0.595*** (0.0126)	0.598*** (0.0674)	0.625*** (0.0137)	0.667*** (0.0776)
log family size	-0.123*** (0.00924)	-0.195*** (0.0141)	-0.184*** (0.0514)	-0.156*** (0.0153)	-0.116* (0.0597)
split	0.0960*** (0.0139)	0.140*** (0.0123)	0.150*** (0.0371)	0.119*** (0.0131)	0.111*** (0.0423)
Constant	1.645*** (0.0405)	2.415*** (0.107)	2.388*** (0.545)	2.020*** (0.119)	1.673*** (0.645)
Observations	14,609	14,609	14,609	14,609	14,609

Standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

All regressions include survey round fixed effects and dummies for the share of household members with ages 0-5, 6-11, 12-17, 18-64 and 65+ by gender

<sup>+</sup> instrumented with log household per capita income in columns 2 to 5

<sup>++</sup> instrumented with log family per capita income in columns 2 to 5

Table 6: Perc. increase in per capita income equiv.  
to adding a household member

family size	altruism				OECD <sup>‡</sup>
	$\tau = 0$	$\tau = 0.2$	$\tau = 0.5$	$\tau = 0.8$	
1	0.241	0.200	0.159	0.132	0.340
2	0.124	0.103	0.081	0.067	0.128
3	0.086	0.071	0.056	0.046	0.068
4	0.067	0.055	0.043	0.036	0.037
5	0.055	0.045	0.036	0.029	0.030
6	0.047	0.039	0.030	0.025	0.022
7	0.041	0.034	0.027	0.022	0.013
8	0.036	0.030	0.024	0.019	0.014
9	0.033	0.027	0.021	0.018	0.011
$\hat{\xi}$	<b>0.704</b>	<b>0.753</b>	<b>0.803</b>	<b>0.837</b>	<b>0.752</b>

In all cases shares are evaluated at sample mean

<sup>‡</sup> Computed using modified-OECD equivalence scales

Table 7: Economies of scale by income quartile, altruism and family size

family size	low altruism ( $\tau = 0$ )				medium altruism ( $\tau = 0.5$ )			
	q1	q2	q3	q4	q1	q2	q3	q4
1	0.295	0.257	0.248	0.219	0.204	0.244	0.149	0.133
2	0.135	0.089	0.147	0.139	0.091	0.069	0.087	0.084
3	0.087	0.052	0.112	0.108	0.059	0.038	0.065	0.065
4	0.064	0.036	0.093	0.090	0.043	0.026	0.054	0.055
5	0.051	0.028	0.082	0.078	0.034	0.020	0.048	0.048
6	0.042	0.022	0.074	0.069	0.028	0.016	0.043	0.043
7	0.036	0.019	0.068	0.063	0.024	0.013	0.040	0.039
8	0.031	0.016	0.064	0.058	0.021	0.011	0.037	0.036
9	0.027	0.014	0.061	0.054	0.018	0.010	0.035	0.033
$\hat{\xi}$	<b>0.696</b>	<b>0.796</b>	<b>0.614</b>	<b>0.638</b>	<b>0.791</b>	<b>0.833</b>	<b>0.768</b>	<b>0.774</b>



Table 8: Economies of scale in the presence of congestion

family	$\tau = 0$		$\tau = 0.5$	
size	$\phi = 0.1$	$\phi = 0.3$	$\phi = 0.1$	$\phi = 0.3$
1	0.217	0.169	0.145	0.117
2	0.110	0.080	0.072	0.055
3	0.075	0.054	0.049	0.036
4	0.058	0.041	0.038	0.027
5	0.048	0.033	0.031	0.022
6	0.040	0.028	0.026	0.019
7	0.035	0.024	0.023	0.016
8	0.031	0.021	0.021	0.014
9	0.028	0.019	0.019	0.013
$\hat{\xi}$	<b>0.739</b>	<b>0.808</b>	<b>0.825</b>	<b>0.869</b>

## Appendix I: Derivation of demand system

The Kuhn-Tucker conditions in problem (16)-(21) obtained from increasing in one unit the quantity purchased of each good are:

$$\begin{aligned}
x_1^{-\sigma} &\leq \lambda p_x \quad , & \tilde{x}_{11} &\geq 0 \quad , & \tilde{x}_{11} (x_1^{-\sigma} - \lambda p_x) &= 0 \\
\tau x_2^{-\sigma} &\leq \lambda(n-1)p_x \quad , & \tilde{x}_{12} &\geq 0 \quad , & \tilde{x}_{12} (\tau x_2^{-\sigma} - \lambda(n-1)p_x) &= 0 \\
(s_1 - \gamma)^{-\sigma} + \tau(s_2 - \gamma)^{-\sigma} &\leq \lambda n p_s \quad , & \tilde{s}_1 &\geq 0 \quad , & \tilde{s}_1 ((s_1 - \gamma)^{-\sigma} + \tau(s_2 - \gamma)^{-\sigma} - \lambda n p_s) &= 0 \\
Q^{-\sigma} + \tau Q^{-\sigma} &\leq \lambda p_q \quad , & \tilde{Q}_1 &\geq 0 \quad , & \tilde{Q}_1 (Q^{-\sigma} + \tau Q^{-\sigma} - \lambda p_q) &= 0
\end{aligned}$$

In the symmetric case  $x_1 = x_2$ , which implies that the second equation holds with strict inequality when the first one holds with equality for  $\tau < (n-1)$ . This means that there will be zero gift/transfer of private goods to other members ( $\tilde{x}_{12} = 0$ ). The symmetry of the game implies that  $\tilde{x}_{11} = \tilde{x}_{22}$ ,  $\tilde{x}_{12} = \tilde{x}_{21} = 0$ ,  $\tilde{s}_1 = \tilde{s}_2$  and  $\tilde{Q}_1 = \tilde{Q}_2$ . Then, using conditions (17)-(20), the Kuhn-Tucker conditions simplifies to

$$\tilde{x}^{-\sigma} = \lambda p_x \quad (49)$$

$$(1 + \tau)(\tilde{s} - \gamma)^{-\sigma} = \lambda n p_s \quad (50)$$

$$(1 + \tau)(n\tilde{Q})^{-\sigma} = \lambda p_q \quad (51)$$

where  $\tilde{x}$ ,  $\tilde{s}$  and  $\tilde{Q}$  are per capita expenditures on each of the composite goods. These equalities together with the budget constraint  $p_x \tilde{x} + p_x \tilde{s} + p_q \tilde{Q} = y$  determine the per capita demand system (22).

### Equivalence with two-step procedure

Section 2.3 presented a two-step procedure. In the first stage, individuals decides on the purchases of a set of goods and on the contribution to a common pool of resources. In the second stage, members of the family jointly decide how to use the ‘common’ pool of resources and how to allocate individual consumptions.

The solution is obtained by solving the second stage given the set of contributions and then solve the non-cooperative first stage. The second stage for the symmetric case is,

$$\max \left[ \frac{(s_1 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right] + (n-1) \left[ \frac{(s_2 - \gamma)^{1-\sigma}}{1-\sigma} + \frac{Q^{1-\sigma}}{1-\sigma} \right] \quad (52)$$

$$s.t. \quad p_s(s_1 + (n-1)s_2) + p_q Q = c_1 + (n-1)c_2 \quad (53)$$

The solution to this problem is as follows.

$$s_1 = s_2 = \delta \frac{(c_1 + (n-1)c_2)}{p_s n} + (1-\delta)\gamma \quad (54)$$

$$Q = (1-\delta) \frac{(c_1 + (n-1)c_2)}{p_q} - \frac{p_s n}{p_q} \gamma \quad (55)$$

where

$$\delta = \frac{p_s^{\frac{\sigma-1}{\sigma}}}{p_s^{\frac{\sigma-1}{\sigma}} + (p_q/n)^{\frac{\sigma-1}{\sigma}}} \quad (56)$$

In the first stage, household members maximize their utility knowing what the allocation of the second step will be. That is, individuals maximize his/her utility (1) subject to (54), (55), the allocation rule for private goods (17) and (18) and the budget constraint.

$$p_x(\tilde{x}_{11} + (n-1)\tilde{x}_{12}) + c_1 = y \quad (57)$$

The first order conditions after imposing the symmetry condition  $s_1 = s_2$  are as follows.

$$x_1^{-\sigma} = \lambda p_x \quad (58)$$

$$(1 + \tau) \left( \frac{\delta}{p_s n} (s_1 - \gamma)^{-\sigma} + \frac{1 - \delta}{p_q} Q^{-\sigma} \right) = \lambda \quad (59)$$

$$\frac{Q}{s_1 - \gamma} = \frac{(1 - \delta)p_s n}{\delta p_q} \quad (60)$$

where equality (60) is obtained from (54) and (55). Given that  $x_1 = \tilde{x}$ ,  $s_1 = \tilde{s}$ ,  $Q = n\tilde{Q}$ ,  $c_1 = p_s \tilde{s} + p_q \tilde{Q}$  and that the expression for  $\delta$  is (56), then the system (58)-(60) is identical to the system (49)-(51), and therefore their solutions.

## Appendix II: Comparison of Deaton-Paxson model and non-cooperative model

Consider a household which members behave in accordance to a non-cooperative model rather than to the collective model (13)-(14). Then, each household member' problem becomes as follows.

$$\max U(s_1, Q) + \tau U(s_2, Q) \quad (61)$$

$$s.t. \quad p_s(\tilde{s}_{11} + (n-1)\tilde{s}_{12}) + p_q \tilde{Q} = y \quad (62)$$

$$s_1 = \tilde{s}_{11} + (n-1)\tilde{s}_{21} \quad (63)$$

$$s_2 = \tilde{s}_{12} + \tilde{s}_{22} + (n-2)\tilde{s}_{21} \quad (64)$$

$$Q = \tilde{Q}_1 + (n-1)\tilde{Q}_2 \quad (65)$$

Similarly to utility (13), the objective function (61) can be obtained from (1) when all individuals are identical. That is,  $U(s_1, Q) + \tau U(s_2, Q) = U(s_i, Q) + (\tau/(n-1)) \sum_{j \neq i}^n U(s_j, Q)$  after re-labeling the problem maximizer, i.e.  $s_i = s_1$  and pre-imposing identical allocation to the rest of the members, i.e.,  $s_j = s_2$  for all  $j = 2, \dots, n$ . Equality (63) indicates that the total consumption of member 1's food  $s_1$  equals the quantity she purchases to be consumed by her  $\tilde{s}_{11}$  plus the  $(n-1)$  identical quantities purchased by each of the other

members of the household in her's behalf  $\tilde{s}_{21}$ . On the other hand, equality (64) the amount of food consumed by each of the other household members  $s_2$  equals the quantity member 1 gives to each of them  $\tilde{s}_{12}$ , the quantities they purchase for themselves  $\tilde{s}_{22}$  and the transfers/gifts made among them  $\tilde{s}_{21}$ . Equality (65) show that the consumption of public goods  $Q$  equals the contribution of member 1 ( $\tilde{Q}_1$ ) plus the contribution of the other  $(n - 1)$  members in the household ( $\tilde{Q}_2$ ). Finally, the budget constraint for the symmetric case is given by (62).

Model (61)-(65) is identical to that given by equations (16)-(21) but ignoring the presence of sharable goods and assuming a generic caring utility function. Thus, the details of the maximization are those in appendix I, which results indicate that the first order conditions of (61)-(65) after imposing symmetry are as follows.

$$\frac{U_s}{U_q} = \frac{p_s(1 + \tau)}{p_q} \quad (66)$$

$$y = p_s s_1 + \frac{p_q}{n} Q \quad (67)$$

On the other hand, the first order conditions of the Deaton-Paxson model (13)-(14) are:

$$\frac{U_s}{U_q} = \frac{p_s n}{p_q} \quad (68)$$

$$y = p_s s_1 + \frac{p_q}{n} Q \quad (69)$$

The comparison of conditions (66) and (68) reveals that an increase in household size  $n$  induces a substitution towards the consumption of the public good in the latter case that is not present in the former. Thus, the change in food share as household size increases is larger in the non-cooperative model than in the collective model. As a result, if model (13)-(14) is inconsistent with facts, then the non-cooperative model (61)-(65) is also inconsistent.

## Appendix III: Marginal utility of income

The Lagrange function evaluated at the optimum is

$$\mathcal{L}^* = (1 + \tau)U(\tilde{x}^*, \tilde{s}^*, n\tilde{Q}^*) + \lambda \left[ y - p_x \tilde{x} - p_s \tilde{s} - p_q \tilde{Q} \right] \quad (70)$$

The marginal utility of per capita income (after arriving to a new Nash equilibrium) is

$$\frac{\partial \mathcal{L}^*}{\partial y} = (1 + \tau) \left[ U'_x \frac{\partial \tilde{x}^*}{\partial y} + U'_s \frac{\partial \tilde{s}^*}{\partial y} + n U'_Q \frac{\partial \tilde{Q}^*}{\partial y} \right] + \lambda \left[ 1 - p_x \frac{\partial \tilde{x}^*}{\partial y} - p_s \frac{\partial \tilde{s}^*}{\partial y} - p_q \frac{\partial \tilde{Q}^*}{\partial y} \right] \quad (71)$$

$$= \lambda \left[ 1 + \tau p_x \frac{\partial \tilde{x}^*}{\partial y} + (n - 1) p_s \frac{\partial \tilde{s}^*}{\partial y} + (n - 1) p_q \frac{\partial \tilde{Q}^*}{\partial y} \right] \quad (72)$$

$$= \lambda \left[ 1 + \tau \left( \frac{\partial \omega_x}{\partial \log(y)} + \omega_x \right) + (n - 1) \left( \frac{\partial \omega_s}{\partial \log(y)} + \omega_s \right) + (n - 1) \left( \frac{\partial \omega_q}{\partial \log(y)} + \omega_q \right) \right] \quad (73)$$

Using the FOC (49)-(51) expression (71) becomes (72). Then, the fact that

$$\frac{\partial \omega_x}{\partial \log(y)} = p_x \frac{\partial \tilde{x}^*}{\partial y} - \frac{p_x \tilde{x}^*}{y}$$

is used to obtain (73)

## Appendix IV: Measuring indifference scales in the presence of public goods with congestion

Consider the sharing rule (47). The three expressions for the Lagrange multiplier in (36) become:

$$\lambda = \frac{\partial U^*}{\partial x} \frac{1}{p_x} = \frac{\partial U^*}{\partial s} \frac{(1+\tau)}{np_s} = \frac{\partial U^*}{\partial Q} \frac{(1+\tau)}{n^\phi p_q} \quad (74)$$

Replacing (74) in (35) and using the fact than in equilibrium the relationship between per capita consumed and per capita purchased goods are:  $s = \tilde{s}$ ,  $x = \tilde{x}$  and  $Q = n^{(1-\phi)} \tilde{Q}$ , then (39) simplifies to the following expression:

$$\frac{\partial \mathbf{V}}{\partial n} / \lambda \mathbf{C} = p_x \frac{\partial \tilde{x}}{\partial n} + \frac{np_s}{(1+\tau)} \frac{\partial \tilde{s}}{\partial n} + \frac{np_q}{(1+\tau)} \frac{\partial \tilde{Q}}{\partial n} + \frac{p_q \tilde{Q}(1-\phi)}{(1+\tau)} \quad (75)$$

$$= \frac{y}{\mathbf{C}(1+\tau)} \left( \frac{(1+\tau)}{n} \frac{\partial \omega_x}{\partial \log(n)} + \frac{\partial \omega_s}{\partial \log(n)} + \frac{\partial \omega_q}{\partial \log(n)} + \omega_q(1-\phi) \right) \quad (76)$$

## Appendix V: Generic utility function (on-line material)

Equations (16)-(21) describes the problem for a household with  $n$  identical members and Stone-Gery utilities.

Equations (78)-(82) reproduces the same problem but with generic utility functions.

$$\max U(x_1, s_1, x_2, s_2, Q) \quad (77)$$

$$s.t. \quad p_x(x_{11} + (n-1)x_{12}) + p_s \tilde{s} + p_q \tilde{Q} = y \quad (78)$$

$$x_1 = x_{11} + (n-1)x_{21} \quad (79)$$

$$x_2 = \tilde{x}_{12} + \tilde{x}_{22} + (n-2)\tilde{x}_{21} \quad (80)$$

$$s_1 = s_2 = \frac{\tilde{s}_1 + (n-1)\tilde{s}_2}{n} \quad (81)$$

$$Q = \tilde{Q}_1 + (n-1)\tilde{Q}_2 \quad (82)$$

Using simple algebra and the fact that no member in the household will transfer private good when  $\tau < 1$  (i.e.,  $x_{ij} = 0$  for  $i \neq j$ , see section 3.1), the model with identical individuals can be re-written as

$$\max U(x_1, s_1, Q) + \tau U(x_2, s_1, Q) \quad (83)$$

$$s.t. \quad p_x x_1 + p_s n s + p_q Q = y + p_s(n-1)\tilde{s}_2 + p_q(n-1)\tilde{Q}_2 \quad (84)$$

Conditional on the purchases of other members in the household problem (83)-(84) gives standard demand functions for own consumption.

$$x_1 = g(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_2 + p_q(n-1)\tilde{Q}_2) \quad (85)$$

$$s_1 = h(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_2 + p_q(n-1)\tilde{Q}_2) \quad (86)$$

$$Q = l(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_2 + p_q(n-1)\tilde{Q}_2) \quad (87)$$

In equilibrium  $s_1 = \tilde{s}_1 = \tilde{s}_2$  and  $\tilde{Q}_1 = \tilde{Q}_2 = Q/n$ , then the demand system in term of purchases rather than consumptions is as follows.

$$\tilde{x}_1 = g(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_1 + p_q(n-1)\tilde{Q}_1) \quad (88)$$

$$\tilde{s}_1 = h(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_1 + p_q(n-1)\tilde{Q}_1) \quad (89)$$

$$\tilde{Q}_1 n = l(p_x, p_s n, p_q, y + p_s(n-1)\tilde{s}_1 + p_q(n-1)\tilde{Q}_1) \quad (90)$$

Equations (88)-(90) form a system with three equations and three unknowns,  $\tilde{x}_1$ ,  $\tilde{s}_1$  and  $\tilde{Q}_1$ , that can be solved recursively. First, equations (89) and (90) jointly determine an expression for  $\tilde{s}_1$  and  $\tilde{Q}_1$ . Second, the resulting expressions can be replaced on the right-hand side of (88) to solve for  $\tilde{x}_1$ .

Applying the implicit function theorem over (89) and (90) and doing the corresponding algebra, the expressions for the elasticities of  $\tilde{s}_1$  and  $\tilde{Q}_1$  with respect to household size (i.e.  $\phi_{\tilde{s}n} = \frac{\partial \tilde{s}_1}{\partial n} \frac{n}{\tilde{s}}$  and  $\phi_{\tilde{Q}n} = \frac{\partial \tilde{Q}_1}{\partial n} \frac{n}{\tilde{Q}}$ ) are given by the following system.

$$\begin{pmatrix} (1 - \eta_{sy}\omega_s(n-1)) & (-\eta_{sy}\omega_q(n-1)) \\ (-\eta_{qy}\omega_s(n-1)) & (1 - \eta_{qy}\omega_q(n-1)) \end{pmatrix} \begin{pmatrix} \phi_{\tilde{s}n} \\ \phi_{\tilde{Q}n} \end{pmatrix} = \begin{pmatrix} \eta_{\tilde{s}p_s} + \eta_{\tilde{s}y}(\omega_{\tilde{s}} + \omega_{\tilde{q}})n \\ \eta_{\tilde{Q}p_s} + \eta_{\tilde{Q}y}(\omega_{\tilde{s}} + \omega_{\tilde{q}})n - 1 \end{pmatrix}$$

where  $\omega_{\tilde{q}} = \frac{p_q \tilde{Q}_1}{y}$ ,  $\omega_{\tilde{s}} = \frac{p_s \tilde{s}_1}{y}$  are expenditure shares, and  $\eta_{sy}$ ,  $\eta_{qy}$ ,  $\eta_{sp_s}$  and  $\eta_{qp_s}$  are the usual uncompensated income and price elasticities. These elasticities correspond to system (85)-(87), holding other members' purchases as given, while  $\phi_{\tilde{s}n}$  and  $\phi_{\tilde{Q}n}$  are elasticities after the Nash equilibrium is reached. Applying Cramer's rule, the expression for  $\phi_{\tilde{s}n} > 0$  that would solve the Deaton and Paxson (1998) paradox is as follow.

$$\phi_{\tilde{s}n} = \frac{\eta_{\tilde{s}p_s}(1 - \eta_{\tilde{Q}y}\omega_q(n-1)) + \eta_{\tilde{s}y}((\omega_s + \omega_q)n - \omega_q(n-1)(1 - \eta_{\tilde{Q}p_s}))}{1 - (n-1)(\eta_{\tilde{Q}y}\omega_q + \eta_{\tilde{s}y}\omega_s)} > 0 \quad (91)$$