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Do Corporate Environmental Contributions Justify the Public Interest Defence?

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Do Corporate Environmental Contributions Justify the Public Interest Defence?

Abstract

Corporations make significant direct contributions to environmental improvement and also indirect contributions, through expenditure on process and product innovation. We explore alternative motivations for these expenditures that look beyond the assertion that they are a consequence of business ethics. Two motives are explored: environmental improvement leading to reduced production costs, and publicized environmental expenditures boosting brand image. We analyze the equilibrium with environmental contributions and social welfare implications. These motives are then combined to determine whether environmental expenditures can justify public interest defence for the operation of a cartel. Using a variant of the Dixit-Stiglitz model we identify when reduced competition caused by a decrease in the number of active firms leads to greater environmental expenditures and higher welfare. However, allowing the operational firms to form a cartel and raise prices above Nash equilibrium levels always reduces environmental expenditures. Welfare falls, as a consequence, and the public interest defence fails.

JEL-Codes: L490, Q580.

Keywords: environment, public interest, cartel.

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1 Introduction

There is frequent media attention placed upon damage to the environment caused by the activities of firms. The Deepwater Horizon oil spill in April 2010 is an extreme case, but there are many others including the ongoing debate about the potential harm caused by fracking to extract shale gas. What receives much less attention are the resources spent by firms on improving the environment. Morgenstern et al. (2001) cite a US EPA estimate that these expenditures amount to 2 percent of GDP.1 Vernon (2000) reports that the corporate sector in Australia contributed about 40 per cent of total environmental expenditure. Corporations also contribute indirectly to environmental improvement through improvement of processes and products. For example, between 1980 and 2014 the average mileage per gallon of cars and small trucks in the US has increased from 14.9 to 21.14.2 In 2014 Toyota issued green bonds for 1.75 billion US dollars, to finance new gas-electric and other alternative fuel car production. Apple’s $1.5 billion green bond issuance, announced in 2016, will be used to fund the company’s conversion to renewable energy and use of biodegradable materials, and the projects on improvement of energy-efficiency of heating and cooling systems.3 These observations raise the question of why corporations make environmental contributions of such significance, and what consequences the contributions have.

A simple answer might be that environmental expenditures are an act of pure goodwill on the part of the corporations driven by a sense of corporate social responsibility. This interpretation is difficult to accept because any corporate environmental expenditure needs explicit approval of the company’s managers and therefore must be a deliberate act with perceived benefits for the corporation. To explain why contributions are made we must go beyond altruism to search for motives that are founded upon material benefit. Our starting point is that profit maximization is a company’s legal obligation to shareholders. Therefore, we seek to explain environmental contributions as the outcome of profit maximization, without having to resort to invoking business ethics or corporate social responsibility. We consider two potential motives for corporations environmental expenditures that have differing economic effects. For both motives we analyze the equilibrium that emerges and the social efficiency of the corporate contributions.

The first motive is based on expenditures causing a direct reduction in production costs for all active firms. What we have in mind here is that the mitigation of environmental damage will feed back into lower costs, so there is a supply-side argument for environmental expenditures.4 The assumption that environmental damage reduces the level of output is frequently invoked by integrated assessment models (IAMs) that are used to evaluate the impact of greenhouse gas emissions on the economy. These models are familiar from the review of Stern (2007) and much other research on climate damage such as the DICE model (Nordhaus, 2008) and the REMIND model (Luderer et al., 2013). What the IAM modelling has generally failed to do (partly because it is undertaken at a macro level) is to observe that corporations have an incentive to undertake acts to mitigate the damage. The interesting economic feature is that mitigation is a public good so that the benefit is shared among firms. This makes the incentive to contribute greater when the number of firms is smaller, and provides the interesting trade-off between the level of competition and efficiency in mitigation that we explore in this paper.

The second motive is driven by demand-side considerations and is based on the assumption that publicized environmental expenditures boost brand image. The many adverts that extol the environment-saving efforts of corporations demonstrate the role of brand image as a key factor in driving sales. Consequently, brand image is carefully cultivated by many corporations. It is hard to overstate the potential benefits for a corporation that is able to embed environmentalism within its brand. An Ipsos MORI poll reported in TANDBERG (2007) discovered that “More than half of global consumers interviewed said they would prefer to purchase products and services from a company with a good environmental reputation, and almost 80% of global workers believe that working for an environmentally ethical organization is important. That amounts to one billion consumers and over 700 million workers worldwide.” A YouGov (2016) survey of millenials carried out for GT Nexus supply chain management platform found that 22 per cent of respondents would switch

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1See US Environmental Protection Agency (1990).
3Data on labelled green bonds are tracked by Climate Bonds Initiative, a UK charity. See https://www.climatebonds.net/cbi/pub/data/bonds.
4This should be distinguished from the assumption that a reduction in output reduces the emission of harmful by-products (see, e.g. André et al., 2009, Maloney and McCormick, 1982).
brand if products were not environmentally friendly. Reinhardt (2008) argues that a firm aiming to develop an environmental image “must discover or create a willingness in consumers to pay for public goods; they must overcome barriers to the dissemination of credible information about the environmental attributes of their products; and they must defend themselves against imitation.” This captures the fundamental problem for a firm: development of a brand image is costly and can reduce profitability unless it creates a sufficient increase in demand.

Recent developments in European Union (EU) competition law have allowed non-competition interests to be taken into account in cartel cases. For example, the Netherlands explicitly allows the “public interest” argument of sustainability to enter a cartel defence. This defence is founded on the profitability of the cartel allowing the firms involved to make socially-useful expenditures (such as the support of environmental projects) that would not be possible if the cartel were dismantled. The defence is accepted if the social benefits of the expenditures can be demonstrated to exceed the losses through reduced competition then the cartel is permitted. We construct a model that combines both motives for corporate environmental contributions, and then consider the effect of cartelization. In particular, we explore whether there are circumstances in which the formation of a cartel can increase public welfare because of its impact on the provision of environmental expenditures.

The extent to which national competition authorities can take account of non-competition interests is under debate in the EU. The European Commission itself holds the view that competition authorities should consider only arguments directly concerning competition. A counter-argument is that the position of the Commission is inconsistent with EU Treaties and case-law of the European courts. The basis of this argument is that the Treaty on the Functioning of the European Union imposes a duty on the European Commission and the competition authorities to consider public interest arguments. The Dutch competition authority (ACM) is at the forefront of arguing for the integration of the public interest argument of sustainability in cartel cases. A position paper (ACM, 2013) states that ACM will accept the argument of collusive production of public interests as a defence. The criteria include the contribution of cartel’s activities to “improving the production or distribution of goods or to promoting technical or economic progress, while allowing consumers a fair share of the resulting benefit.” The logic of the argument is that public interests – such as environmental protection – may not be supplied in a competitive environment. A cartel agreement to restrict the supply of a good can provide the coordination that is needed to supply the public interest.

Our results show that if firms make environmental expenditures, which are a public good, then it is possible for consumer utility to be lower when the number of active firms is greater despite consumers valuing product variety. This is a consequence of lower environmental expenditures, higher costs, and higher equilibrium prices. Welfare can also exhibit non-monotonic behavior and, in some instances, it is optimal to have a restricted number of firms. When environmental expenditures are a private good the results on profit are more conventional but welfare can be maximized for a finite number of firms. Hence, both motives for firms to make environmental expenditures can result in situations in which it is socially efficient to limit competition. These results are interesting and in sharp contrast to standard results, but they do not directly address the public interest defence. That argument concerns the consequences of giving a fixed number of firms the right to act as a cartel with additional marker power. When we analyze this situation we can show that an increase in cartel power will always reduce environmental contributions and will lead to a lower level of welfare. No case is found in which the public interest defence can be sustained.

This conclusion should be contrasted to that of Schinkel and Spiegel (2017) who show, in a duopoly model with linear demand, that the validity of the public interest defence depends on the order of moves in the strategic game between firms. In their model, each firm chooses output and the sustainability attribute of its product. The order in which these are chosen determines whether the chosen sustainability attribute is higher or lower with a cartel. Our analysis is more general in three dimensions. First, we allow any number of firms to be active and give considerable attention to how the number of firms affects welfare. Second, we adopt a preference system that permits the elasticity of demand to be parametrically varied. Third, and most importantly, we consider three different forms of environmental expenditures. Two of the three are different forms of expenditure on brand image. One is “nominal” expenditure - such as advertising - that promotes image without substantive change to the product. The other - “real” expenditure - improves the product as well as image and is similar to the sustainability attribute of Schinkel and Spiegel (2017). The third form of environmental expenditure has the characteristics of a public good for the firms. This introduces entirely new issues into the analysis and an additional set of strategic considerations.
Our paper contributes to the literature on corporate environmentalism and the voluntary approach to environmental protection (Khanna, 2001, Lyon and Maxwell, 2004, Segerson 2013). We explore profit-seeking motives for green behaviour, with a specific focus on the interaction between competition regulation and voluntary environmental contributions, abstracting from the impact of mandatory environmental programmes and policies. The novelty of our work is the use of a theoretical framework that allows for the analysis of strategic behavior in a market with an arbitrary degree of market power, and the simultaneous effect of voluntary environmental contributions on the supply and demand sides of the market. This general approach reveals potential non-monotonicity in the effect of corporate environmentalism on social welfare, and addresses the viability of an important policy measure debated by European policy-makers.

We start with an overview of the Dixit-Stiglitz (1977) model of monopolistic competition, and its generalizations, which serves as the basic framework for our analysis. Section 3 considers the provision of environmental expenditures when they directly reduce production cost but have a beneficial impact for all firms. Section 4 analyses the determination of environmental expenditure when they are used to improve brand image. Section 6 combines the analysis of the previous sections to address the issue of public interest and cartel formation. Conclusions are given in Section 7.

2 Preferences and Elasticities

We build our analysis on extensions of the Dixit-Stiglitz (1977) model of monopolistic competition. d’Aspremont et al. (1996) distinguish between three versions of the Dixit-Stiglitz model, according to the assumption under which the demand elasticities are calculated. In the first version, as developed in the original paper of Dixit and Stiglitz (1977), it is assumed that the firms ignore the effect of their pricing decisions on the aggregate price index. This assumption is most plausible when the number of firms is “large”. The second version, due to Yang and Heijdra (1993), takes into account the price index effect, but ignores the indirect income-feedback effect. Finally, the third version, developed by d’Aspremont et al. (1996), considers the effect of pricing decisions on consumer income, which is referred to as the Ford effect, in a model where the numeraire good is interpreted as leisure (the firms employ the consumers as workers).

We choose to model the environmental contributions of firms in the Yang and Heijdra version of the Dixit-Stiglitz model, ignoring the Ford effect. This is because we wish to consider cases with a small number of firms which makes the price-index effect important. However, we view the industry as a small part of a larger world, so feel it is safe to set aside the income-feedback effect. This partial equilibrium approach focusses on the production of an imperfectly competitive polluting industry, on the interaction between competition and contributions to environmental improvement, and on the welfare implications. The choice of the second version is supported by assuming that the utility function of the representative consumer is separable in the goods produced by the industry under analysis and all other goods, and that the expenditure on the former is fixed.

Denote the number of active firms by \( N \) and consumption of their outputs by \( \{q_1, \ldots, q_N\} \). It is assumed that the goods are substitutes and that utility has the CES form, so

\[
U(q_1, \ldots, q_N) = \left[ \sum_{j=1}^{N} q_j^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)},
\]

with \( \sigma > 1 \). It is also assumed that expenditure, \( I = \sum_{j=1}^{N} p_j q_j \), on these goods is fixed, where \( p_j \) denotes the price of good \( j \).\(^5\) The demand function that results from maximization of utility in (1) is given by

\[
q_j = \frac{I}{NP} \left( \frac{p_j}{P} \right)^{-\sigma},
\]

\(^5\)This, for example, would be implied by a Cobb-Douglas utility, \( u(q_0, q_1, \ldots, q_N) = q_0^{1-\alpha} [U(q_1, \ldots, q_N)]^{\alpha} \) where \( q_0 \) is the numeraire good.
where the price index, $P$, is defined by
\[ P = \left( \frac{1}{N} \sum_{j=1}^{N} p_j^{1-\sigma} \right)^{1/(1-\sigma)}. \] (3)

The elasticity of the price index with respect to the price of good $j$ is
\[ \varepsilon_{Pj} = \frac{p_j}{P} \frac{\partial P}{\partial p_j} = \frac{1}{N} \left[ \frac{P}{p_j} \right]^{\sigma-1}. \] (4)

Clearly, $\lim_{N \to \infty} \varepsilon_{Pj} = 0$, and in the symmetric equilibrium ($p_j = P$ for all $j$) $\varepsilon_{Pj} = 1/N$. Note that firm $j$’s market share is
\[ s_j = \frac{p_j q_j}{I} = \frac{p_j}{I N P} \left( \frac{P}{P} \right)^{-\sigma} = \frac{1}{N} \left[ \frac{P}{p_j} \right]^{\sigma-1} = \varepsilon_{Pj}, \] (5)

which gives another interpretation of the elasticity of the price index.

The Dixit-Stiglitz version of the model assumes $\varepsilon_{Pj} = 0$, leading to the the elasticity of demand $\varepsilon_{j}^{DS} = 0$, so the demand for product $j$ becomes more elastic for given prices as $\sigma$ increases. In the Yang-Heijdra version adopted in this paper the elasticity of demand contains an additional term due to the price index effect:
\[ \varepsilon_{j}^{YH} = -\frac{p_j}{q_j} \frac{\partial q_j}{\partial p_j} \bigg|_P = -\frac{p_j}{q_j} \left[ \frac{\partial q_j}{\partial p_j} \bigg|_P + \frac{\partial q_j}{\partial P} \bigg|_{p_j} \frac{\partial P}{\partial p_j} \bigg] \]
\[ = \sigma - \frac{P}{q_j} \frac{\partial q_j}{\partial P} \bigg|_{p_j} = \sigma - (\sigma - 1) \varepsilon_{Pj} \]
\[ = \sigma - (\sigma - 1) s_j. \] (7)

The Yang-Heijdra version is equivalent to assuming strategic price-setting behavior of the firms, whereby each firm takes into account how its pricing decision affects the aggregate price index and, thus, the prices set by other firms. In a symmetric equilibrium, where $s_j = \frac{1}{N}$ all $j$,
\[ \varepsilon_{YH}^{YH}(N) = \sigma - \frac{\sigma - 1}{N}. \] (8)

It can be seen directly that in the monopoly case $\varepsilon_{j}^{YH} = 1$ since the fixed expenditure implies $q = I/p$.

The profit-maximization problem in this case is not well-defined, so in the analysis we assume that $N \geq 2$. 7

One can see that the elasticity increases with $N$. Although $N$ is integer, for the purpose of the comparative statics analysis we will treat $N$ as a continuous variable and $\varepsilon_{YH}^{YH}(N)$ as a differentiable function, with
\[ \frac{d \varepsilon_{YH}^{YH}}{dN} = \frac{\sigma - 1}{N^2} > 0. \] (9)

In the following sections of the paper we first extend this standard model of competition in prices to include strategic interaction between firms in expenditure on environmental improvement. We then introduce competition between firms in the perceived quality (or “environmental friendliness”) of products. The two extensions are then combined, and the model is applied to address the public interest defence.

\[ ^{6}\text{The associated quantity index is } Q = \left( \frac{1}{N} \sum_{j=1}^{N} q_j^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \text{ so that } I = N Q. \]

\[ ^{7}\text{Relaxing the fixed expenditure assumption to allow monopoly to be incorporated would considerably complicate the analysis. As will become clear, it would not add anything of substance to our very definite final conclusion.} \]


\section{Environment and Costs}

Atmospheric greenhouse gas accumulation is modelled in Stern (2007), and many other IAMs, as a public bad that reduces the aggregate level of output. The incentives for firms to mitigate the damage are rarely explored, nor can IAMs built on an aggregate production function model what happens at the individual firm level. We model the impact of environment on firms by assuming that a reduction in environmental quality increases each firm’s cost of production and, hence, will reduce the profit-maximizing output level if all else is constant. We also assume that firms can mitigate this effect by making contributions to environmental improvement. Pollution abatement activities can directly reduce the production cost for the firms if, for example, they allow recycling of some inputs (such as water), or reduce health damage for the labor force that otherwise would lead to lower productivity. This structure allows us to model the impact of environmental quality on production and the resulting mitigation activities of firms.

The link between environment and production cost causes the firms to be linked strategically in two dimensions. The first dimension is the standard form of price competition on the final product market. The second dimension is through the impact of environmental contributions on production costs. Environmental improvement is a public good because it reduces the production cost of all firms, so the considerations of free-riding apply (Cornes and Sandler, 1996). These features combine to make it socially beneficial to restrict competition in some cases.

Consider the typical firm, \( j \). The level of profit of the firm is

\[ \pi_j = [p_j - c_j(e)]q_j - E_j, \]

(10)

where \( c_j(e) \) is the marginal cost of production when environment quality is at level \( e \), and \( E_j \) is the contribution to environmental improvement. An improvement in environmental quality reduces the production cost for all firms, so that \( c_j' (e) < 0 \). The quality of the environment is determined by the aggregate of environmental improvement contributions from firms, so \( e = \sum_{i=1}^{N} E_j \). We assume that each firm takes the value of other firms’ contributions to the environment as given when it maximizes profit. The first-order condition with respect to price gives the standard markup condition

\[ \frac{p_j - c_j(e)}{p_j} = \frac{1}{\varepsilon_j^{YH}}. \]

(11)

The profit-maximization condition for the choice of environmental contribution, combined with the markup condition, provides a relationship between the elasticity of marginal cost and the market share of firm \( j \)

\[ \frac{\varepsilon c_j' (e)}{c_j(e)} = \frac{e}{I} \frac{\sigma - (\sigma - 1) s_j}{s_j (\sigma - 1) (1 - s_j)}. \]

(12)

We now explore the relationship between the equilibrium quantities and the degree of competition. The number of firms affects both the intensity of price competition and the incentives for public good provision by solving the necessary conditions (11) and (12) to relate the equilibrium level of environmental quality to the equilibrium price and market share. Assume that the relation between cost and environmental quality is given by the isoelastic function \( c_j(e) = \beta e^{-\gamma} \), \( \gamma > 0 \). Then there is a symmetric equilibrium with \( p_j = p^* = P \), \( v_j = e/N \), and \( s_j = 1/N \), where

\[ p^* = \beta \left( \frac{\varepsilon^{YH}}{\varepsilon^{YH} - 1} \right)^{1+\gamma} \left[ \frac{N}{\varepsilon^{YH}} \right]^\gamma. \]

(13)

Solving for the equilibrium environmental expenditure by each firm

\[ E^* = \frac{\gamma I}{N^2} \frac{\varepsilon^{YH} - 1}{\varepsilon^{YH}}, \]

(14)

so that aggregate expenditure is

\[ e^* = \frac{\gamma I}{N} \frac{\varepsilon^{YH} - 1}{\varepsilon^{YH}}. \]

(15)
The first proposition describes the impact of an increase in the number of firms on the environmental variables.\(^8\) The proof of this result, and all the results that follow, is in the Appendix.

**Proposition 1** An increase in the number of firms:
(i) reduces the environmental expenditure of each firm;
(ii) reduces aggregate environmental expenditure;
(iii) increases the production cost of each firm.

The proposition shows that an increase in competition leads to lower environmental contribution by each firm. This result is a consequence of the free-rider effect becoming stronger when there are more firms: an increase in competition reduces the incentive to contribute to the environmental public good. Moreover, in this model, the free-rider effect is so strong that a reduction in competition actually increases total environmental contributions. This result holds for two or more firms so we can conclude that environmental quality is highest when there is monopoly and the free-rider effect is eliminated. This is in sharp contrast to the private provision of a public good by households for which there is no clear effect of numbers on provision (see Myles, 1995). The result shows that competition will not be good for the environment when firms are relied upon to finance environmental improvements.

The second proposition determines the relationship between the number of firms and the price level. In the absence of environmental contributions an increase in the number of firms increases the elasticity of demand which leads to a lower equilibrium price. With environmental contribution an increase in the number of firms, on the one hand, raises production cost as environmental contributions fall, and so increases the equilibrium price, but on the other hand, reduces price through the standard elasticity effect. The latter effect becomes small when the number of firms is large, but, in general, the overall effect depends on cost and demand elasticities. The elasticity of demand affects the marginal incentive to contribute to the environment: a higher elasticity results in larger contributions. Environmental contributions increase as the cost elasticity increases because the greater effect of contribution on cost makes a larger contribution worthwhile despite the free riding. The resolution of these effects is described in the proposition. Denote the initial number of firms by \(N\), and define \(
\gamma_1 = \frac{2}{\sigma - 1}\) and \(\bar{N} = 1 + \frac{1 + \sqrt{1 + 4(1 + \gamma)\gamma}}{2\gamma}\).

**Proposition 2** When there is an increase in the number of firms:
(i) if \(\gamma < \gamma_1\), equilibrium price decreases when \(N < \bar{N}\) and increases when \(N > \bar{N}\);
(ii) if \(\gamma \geq \gamma_1\), equilibrium price increases.

Proposition 2 provides the surprising conclusion that a reduction in competition can actually lead to a fall in the equilibrium price. The mechanism is clear from proposition 1: when the number of firms is lower the contribution of each to environmental improvement is higher and all benefit from a reduction in cost. In some cases the cost reduction is sufficiently great to feed through the pricing decision into a reduction in the equilibrium price. Table 1 provides an example of the non-monotonic behavior of the equilibrium price identified in case (ii) of proposition 2.\(^9\) Assume \(\sigma = 1.5\) and \(\gamma = 0.3\). The threshold value \(\bar{N} = 4.14\), so the equilibrium price falls from \(N = 2\) until \(N = 4\) and rises from \(N = 5\).

<table>
<thead>
<tr>
<th>(N)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<tbody>
<tr>
<td>(p^*)</td>
<td>14.32</td>
<td>12.10</td>
<td>11.78</td>
<td>11.85</td>
<td>12.06</td>
</tr>
</tbody>
</table>

Table 1: Equilibrium price

The next step is to determine how the profit level of each firm is affected by changes in the intensity of competition. If there were no externality then profit would decrease monotonically as the number of active firms increased. This need not be the case with the environmental externality. A change in the number of firms affects equilibrium profits via three channels: the market share of each firm, the equilibrium price, and

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\(^8\)In the analysis the number of firms is treated as a continuous variable so the propositions are formally valid for a marginal increase. The results are valid for a discrete increase provided the increase does not cause the number of firms to cross the various critical values defined in the propositions.  

\(^9\)For all the examples that follow \(\beta = 1\) and \(I = 1\).
the level of cost. To explore the potential outcomes substitute (15-14) into (10) to determine the equilibrium profit of each firm as

$$
\pi^* = \frac{I}{N \varepsilon YH} \left[ 1 - \frac{\gamma}{N} (\varepsilon YH - 1) \right] = \frac{I}{N^2} \frac{N^2 - \gamma (N - 1) (\sigma - 1)}{N \sigma - \sigma + 1}.
$$

(16)

It can be seen directly that profits are non-negative as

$$\gamma \leq \frac{N^2}{(\sigma - 1) (N - 1)}. \quad (17)$$

We assume that (17) is satisfied so that the symmetric equilibrium we have characterized is consistent with non-negative profit.

The relationship between profit and the number of firms is stated in the following proposition. The proposition shows that the effect of competition on profit depends on elasticity of production cost with respect to environmental contributions and the elasticity of demand. When the cost elasticity is relatively high it becomes possible for profit to rise with increased competition over a range of values of the initial number of firms, \( N \). Defining \( \gamma_2 = \frac{27}{(\sigma - 1)(8\sigma + 1)}, \gamma_3 = \frac{36}{(\sigma - 1)(10\sigma + 1)}, \gamma_4 = \frac{4}{\sigma - 1}, \) and \( \gamma_5 = \frac{N^2}{(\sigma - 1)(N - 1)} \) permits the statement of proposition 3.

**Proposition 3** When there is an increase in the number of firms:

(i) For any \( N \geq 2 \):

(a) if \( \gamma \leq \gamma_2 \), the profit of each firm falls;

(b) if \( \gamma_2 < \gamma \leq \gamma_3 \), there exists \( \bar{N} \geq 3 \) such that the profit of each firm rises for \( N < \bar{N} \) and falls for \( N > \bar{N} \);

(c) if \( \gamma_3 < \gamma \leq \gamma_4 \), there exists \( \bar{N} \geq 3 \) such that the profit of each firm rises for \( N < \bar{N} \) and falls for \( N > \bar{N} \);

(ii) For any \( N \geq 3 \) there exists \( \bar{N} \geq 3 \) such that the profit of each firm rises for \( N < \bar{N} \) and falls for all \( N > \bar{N} \) if \( \gamma_4 < \gamma \leq \gamma_5 \).

As an illustration of profit increasing with more competition, consider the case of \( \gamma = 7, \sigma = 1.5 \). This corresponds to part (i)(c), with \( \bar{N} = 4.55 \). For \( 2 \leq N \leq 5 \) profit is increasing, but decreases with more competition starting from \( N = 5 \). This is shown in table 2.

<table>
<thead>
<tr>
<th>( N )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
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<tbody>
<tr>
<td>( \pi^* )</td>
<td>0.0500</td>
<td>0.0555</td>
<td>0.0625</td>
<td>0.0629</td>
<td>0.0604</td>
<td>0.0571</td>
</tr>
</tbody>
</table>

Table 2: Firm profit

To understand how competition affects social welfare it is necessary to consider aggregate profit and consumer utility. The total profit of the monopolistically competitive sector is

$$
\Pi^* = \frac{I}{\varepsilon YH} \left[ 1 - \frac{\gamma}{N} (\varepsilon YH - 1) \right] = \frac{I}{N} \frac{N^2 - \gamma (N - 1) (\sigma - 1)}{\sigma (N - 1) + 1}.
$$

(18)

It can be seen directly that the aggregate profit of the firms can either rise or fall as the number of firms increases, depending on the elasticity of demand and the elasticity of production cost. A characterization of when each possibility arises is given in the following proposition, for which \( \gamma_7 = \frac{1}{2} \). It is surprising to observe that profit can rise or fall as the number of firms increases but, if it ever does fall, then it must eventually start to rise again. This property is counter that of the standard model.

**Proposition 4** When the number of firms increases:

(i) if \( 0 < \gamma < \gamma_6 \), aggregate profit falls;

(ii) if \( \gamma_6 < \gamma < \gamma_4 \), there exists \( N^* > 2 \) such that aggregate profit falls for \( N < N^* \) and rises for \( N > N^* \);

(ii) if \( \gamma_4 < \gamma < \gamma_5 \), aggregate profit rises.
The second component of social welfare is the utility level of the consumer. The consumer cares directly about the number of firms because variety is beneficial and cares indirectly through the determination of equilibrium prices. Substituting from (2) and (13) into (1), indirect utility at a symmetric equilibrium as a function of the number of firms is

\[ V^* = \frac{\gamma}{\beta} I^{1+\gamma} \left[ \frac{(\sigma - 1)}{N\sigma - \sigma + 1} \right]^{1+\gamma} N^{1-\gamma}. \] (19)

The next proposition shows that despite the variety effect it is possible for utility to monotonically decrease as the number of firms increases, or to initially rise and then decrease. This is because of the competing price and variety effects. The proposition implies that when production cost responds strongly to the environmental expenditures (\( \gamma \) is sufficiently high) consumers are better-off with less competition: their gain from a lower price outweighs the loss from reduced variety in goods. On the other hand, when production cost does not fall much when environmental expenditures increase (\( \gamma \) is sufficiently low) more competition benefits the consumer. In the intermediate range of \( \gamma \) consumer utility can change non-monotonically with an increase in competition; in particular, in case (ii) utility is maximized at \( \tilde{N} > 2 \). Defining \( \gamma_7 = \frac{3\sigma - 1}{(\sigma - 1)^2} \) gives the following proposition.

**Proposition 5** When there is an increase in the number of firms:

(i) if \( 0 < \gamma < \gamma_1 \), consumer utility rises;

(ii) if \( \gamma_1 < \gamma < \gamma_7 \), there exists \( \tilde{N} > 2 \) such that consumer utility rises for \( N < \tilde{N} \) and falls for \( N > \tilde{N} \);

(ii) if \( \gamma > \gamma_7 \), consumer utility falls.

To illustrate the non-monotonicity consider the example of \( \gamma = 1.3, \sigma = 2 \) which corresponds to case (ii) with \( \tilde{N} = 5.24 \). The table shows that utility is maximized when the number of firms is \( N = 5 \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>0.0913</td>
<td>0.1229</td>
<td>0.1322</td>
<td>0.1344</td>
<td>0.1340</td>
<td>0.1325</td>
</tr>
</tbody>
</table>

Table 3: Consumer utility

In the absence of a link between environment and cost, a reduction in the number of firms decreases social welfare because of the loss of both competitiveness and variety. Propositions 1 to 5 have demonstrated that the cost saving from environmental contribution and the strategic interaction in providing environmental contributions can lead to non-standard comparative statics effects in which greater competition can lead to a higher price and, in some case, lower consumer utility. The question we now address is whether it is possible for the cost-reduction effect to be sufficient to reverse the standard result that welfare falls as the number of firms is reduced.

The social welfare analysis is based on the use of a compensation argument. As the number of firms changes, the aggregate profit of the producers and the utility of the consumers change. Clearly, social welfare falls if both profit and utility fall, and rises if both rise. When profits and utility change in opposite directions, we identify a social welfare increase as occurring when the gainers can compensate the losers. That is, if the increase in profit exceeds the additional income required by consumers as compensation, or if the consumers can compensate firms by transferring income to them to restore lost profitability, then a social welfare increase is obtained. The change in profit and utility caused by a change \( \Delta N \) in the number of firms can be approximated by \( \Delta \Pi \approx \frac{d\Pi}{dN} \Delta N \) and \( \Delta V \approx \frac{dV}{dN} \Delta N \). Suppose that \( \Delta V < 0 \) and \( \Delta \Pi > 0 \). Take \( \Delta I \) from the firms and transfer it to the consumer so that the transfer returns the consumer to the original level of utility, so \( \frac{dV}{dN} \Delta N + \frac{dV}{dI} \Delta I = 0 \). Then social welfare increases if profits do not fall below the original level

\[ \Delta \Pi + [\Delta I] = \left( \frac{d\Pi}{dN} + \frac{dV}{dN} \right) \Delta N \geq 0. \] (20)

It can be seen that if the change \( \Delta N \) causes profit to fall and utility to rise, a transfer \( \Delta I \) from the consumer to the firms gives the same condition for a welfare increase.

Using (18) and (19), and denoting \( \frac{dW}{dN} = \frac{d\Pi}{dN} + \frac{dV}{dN} \), we have
Clearly, \(
\frac{dW}{dN} = O \left( N^{-1} \right) ,
\)
with \(\frac{I \left( \frac{1}{\sigma - 1} - \gamma \right) }{N(1 + \gamma)}\) being the leading term. Therefore, for \(N\) sufficiently large (provided that profit is non-negative) the sign of the change in welfare will be determined by the sign of the term \(\frac{1}{\sigma - 1} - \gamma\).

Figure 1 shows three distinct possibilities for the effect of an increase in the number of firms. The calculations assume \(\sigma = 2.5\). When \(\gamma = 0.5\) utility always increases with \(N\) while aggregate profit falls for smaller \(N\) and rises for larger \(N\). Since \(\frac{dW}{dN}\) is positive for all \(N\), the firms can always be compensated for the loss in profit, and so social welfare increases when there are more active firms. The case of \(\gamma = 1\) sees profit first fall then rise, with the opposite true for utility. For higher \(N\), the extra profit is not enough to compensate the consumer. Hence, the welfare-maximizing number of firms is between 4 and 5. Finally, for \(\gamma = 3\) profit increases with \(N\) (production cost falls rapidly) but utility and the extra profit is not enough to compensate the consumers. In this case, profit is only non-negative for \(N \geq 3\), so welfare is maximized when there are 3 firms. These results show that when the number of firms is high, aggregate profit changes slower with a change in \(N\) than consumer welfare so that greater competition benefits society when production cost is not very sensitive to the environmental expenditures of firms. Conversely, when production cost is highly sensitive to environmental expenditures, less competition is better for society: the loss in aggregate profits with fewer firms is outweighed by the gains to the consumers.

The analysis has explored the incentives of firms to make environmental expenditures and the economic consequences of the level of competition. It has been shown that the public good nature of environmental expenditures combines with the preference for variety to generate some striking non-standard results. These arise because more competition exacerbates the free-rider problem and leads to a decline in environmental quality. This raises the cost of production and puts upward pressure on equilibrium prices. The increases in production cost and prices lie behind the non-standard impacts of competition on profit and utility. The fact that less competition can sometimes increase social welfare seems to offer support for the public interest defence. We will hold back from addressing whether or not that is the correct interpretation until the detailed analysis in section 6.
4 Brand Image

The frequent portrayal in advertising of the positive environmental pedigree of products is abundant evidence that an environmental brand image is very profitable. Our second perspective on environmental expenditures is to model their role in the improvement of brand image: we imagine consumers to care about the environmental quality of the products they consume. The concern can be related to real environmental attributes, such as beauty products not being tested on animals or food products that are grown organically, or equally it can depend only on the cultivated brand image of the product with an absence of any real underlying environmental benefit. The analysis of this section applies to both these interpretations but we distinguish between the two in sections 5 and 6.

We now extend the Yang and Heijdra model to include brand image. To do this, assume that the quantity of consumption of good \( j \), \( q_j \), and the perceived environmental brand image (or quality), \( z_j \), enter multiplicatively into a CES utility function

\[
U(q, z) = \left[ \sum_{j=1}^{N} (q_j z_j)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}.
\]

The demand function that results from the maximization of utility is given by

\[
q_j z_j = \frac{1}{NP_Z} \left( \frac{p_j z_j}{P_Z} \right)^{-\sigma},
\]

where the quality-adjusted price index for the \( N \) products is\(^{12}\)

\[
P_Z = \left( \frac{1}{N} \sum_{j=1}^{n} \left( \frac{p_j}{z_j} \right)^{1-\sigma} \right)^{1/(1-\sigma)}.
\]

The improvement of brand image by a firm is a costly activity. To model this situation we assume that the marginal cost of production of good \( j \) is constant at \( c_j \) and independent of the quality level. To achieve quality level \( z_j \) for its product firm \( j \) incurs a fixed cost of \( C(z_j) \), with \( C' > 0, C'' \geq 0 \). This cost can be interpreted as encompassing expenditure on advertising, brand promotion, and actual environmental expenditures. Firm \( j \) chooses its product price and environmental quality to maximize profit

\[
\pi_j = (p_j - c_j) q_j - C(z_j).
\]

The necessary conditions for the maximization are

\[
\frac{p_j - c_j}{p_j} = \frac{1}{z_j} = \frac{1}{\sigma - (\sigma - 1) s_j},
\]

where \( s_j = \frac{1}{N} \left( \frac{p_j}{z_j} \right)^{1-\sigma} \), and

\[
z_j C'(z_j) = \frac{(\sigma - 1) s_j (1 - s_j)}{\varepsilon_j Y H}.
\]

The first result concerns the effect of an increase in the number of firms on the equilibrium quality level. This result, and those that follow, assume that the initial equilibrium is symmetric.

---

\(^{10}\) Another application is fair trade. A fair-trade label appeals to particular groups of consumers and so raises demand. At the same time, inputs acquired from fair-trade sources typically result in extra costs for producers. Arguably, support of fair trade may contribute to the protection of environment (say, local small-scale farms might be likely to use “greener”, albeit less efficient technologies).

\(^{11}\) The model has some similarities to that of Sengupta (2015). In the Sengupta model consumers do not observe whether firms invest in green technology or not but try to make an inference from price. In our model all consumers observe \( z \) and accept the observation as a sign of environmental quality.

\(^{12}\) The corresponding quantity index adjusted for environmental quality is \( Q_Z = \left[ \frac{1}{N} \sum_{j=1}^{N} (z_j q_j)^{(\theta-1)/\varphi} \right]^{\theta/(\sigma-1)} \).
Proposition 6  An increase in the number of firms reduces equilibrium product quality.

Proposition 6 shows that in equilibrium the environmental quality of each product and, therefore, the level of environmental expenditure of each firm, is higher when the number of firms is lower. Hence, a less competitive market environment encourages greater development of environmental brand image. The result arises because a lower number of active firms increases the marginal value of boosting brand image because of the intensification of non-price competition.

To investigate how firm profit depends on the level of competition we assume the cost function for quality is given by $C(z) = \frac{\mu}{N}z^\theta$, with $\mu \geq 1$ and $\theta > 0$.\footnote{This cost function implies isoelastic marginal cost, $C'(z^*) = 1 - \mu^{-1}$.} Solving the first-order conditions for the symmetric equilibrium determines the profit level of a firm as

$$\pi^* = \frac{I}{N\varepsilon^YH} \left[ 1 - \frac{1}{\mu} (\varepsilon^YH - 1) \right]. \tag{28}$$

It is assumed that $1 + \mu - \varepsilon^YH > 0$ to ensure the level of profit is strictly positive. A comparison of (28) to (16) for the production cost model shows that the expressions are similar, but with $\frac{I}{N}$ replaced by $\frac{1}{\mu}$. This reflects the difference between the nature of environmental expenditure in the two models: it provides only a private to the firm in the brand quality model, whereas in the production cost model environmental improvement has an external effect on all firms in the economy. Brand image changes the Yang and Heijdra model by adding a dimension of non-price competition so the effects of changing the number of firms on profit are standard.

Proposition 7  An increase in the number of firms reduces the equilibrium profit of each firm and aggregate profit.

The indirect utility function at a symmetric equilibrium is

$$V = \frac{I^{\frac{1}{\mu}+1}}{c} \left[ \frac{\theta}{\mu} \right]^{\frac{1}{\mu}} \left[ \frac{\varepsilon^YH - 1}{\varepsilon^YH} \right]^{1+\frac{1}{\mu}} N^{\frac{1}{\mu}-\frac{1}{\mu}}. \tag{29}$$

This allows the relationship between utility and $N$ to be derived.

Proposition 8  When there is an increase in the number of firms:

(i) if $\varepsilon^YH < 1 < \mu < \sigma - 1$, utility increases for $N < \hat{N}$ and decreases for $N > \hat{N}$ where $\hat{N} > 2$;

(ii) if $\sigma - 1 < \mu$, utility increases.

Note that the condition that guarantees non-negativity of profit for the brand image model implies a tighter restriction on the admissible range of parameters than in the production cost model. This rules out the case when consumer utility is monotonically decreasing in $N$ (compare to case (iii) in proposition 5), and, at the same time, case (i) occurs for a narrower range of the model parameters, and this shrinks further for higher $N$.

The compensation argument is now used to assess the social welfare effect of changing the number of firms. The change in social welfare is given by

$$\frac{dW}{dN} = \frac{I}{\mu N} \left[ \frac{\mu}{\sigma - 1} - 1 + \frac{1 + \mu}{(N - 1) (\varepsilon^YH)^2} \right].$$

It can be seen that $\frac{dW}{dN} = O \left(N^{-1}\right)$, with the leading term being $\left[ \frac{1}{\sigma - 1} - \frac{1}{\mu} \right] N^{-1}$. The interesting situation is when welfare decreases as the number of firms increases. A necessary (but not sufficient) condition for this to occur is $\mu < \sigma - 1$. The possibility for a smaller number of firms to deliver higher welfare is demonstrated in figure 2. For $\mu = 1.7$ the change, $\frac{dW}{dN}$, is always positive so more firms give higher welfare. When $\mu = 1.4$ the optimal number of firms is between 3 and 4. Finally, for $\mu = 1.1$ a smaller number of firms always gives higher welfare (but note profits are non-negative only for $N < 4$).
Environmental expenditures that enhance brand image create a private benefit for the firm making the expenditure. From this perspective, the expenditures are just a form of non-price competition so it is not surprising that these expenditures are highest when the number of firms is small. Other than this effect, increased competition benefits the consumer to the detriment of the firms. The welfare impact of increasing the number of firms is less clear-cut. The example illustrated three different possibilities for the welfare effect including the surprising outcome that welfare can decrease when there is greater competition.

5 Costs and Brand Image

As a necessary development before discussion of the public interest defence we now describe the equilibrium that arises when the production cost model and the brand image model are combined. To do this we assume that there are three different forms of environmental spending. The first two forms determine the environmental brand image of the product, while the third form affects the level of environment but not brand image.

Brand image is determined by a combination of expenditure on nominal environmental quality improvement and expenditure on real quality improvement. By nominal quality improvement we have in mind expenditure on promotion of environmental brand image without any beneficial environmental activity taking place. In contrast, real quality improvement is brought about by actual environmental activity so it enhances environment quality and reduces the production cost of firms. Nominal and real quality are aggregated into the environmental brand image as perceived by consumers. The third form of environmental expenditure causes a direct improvement in the quality of the environment but is not included in the perception of product quality by consumers.

The profit of firm \( j \) with total environmental expenditure \( E_j \) is

\[
\pi_j = [p_j - c(e)] q_j - E_j.
\] (30)

Amount \( E^1_j \) is allocated to the nominal quality improvement, \( E^2_j \) to real quality improvement, and \( E^3_j \) to direct expenditure on environmental improvement. By definition, \( E^1_j + E^2_j + E^3_j = E_j \). The level of environmental quality is given by aggregating real and direct expenditures

\[
e = (1 - \eta) \sum_{i=1}^{N} E^2_i + \eta \sum_{i=1}^{N} E^3_i,
\] (31)

Figure 2: Welfare impact \((\sigma = 2.5)\)

13
so that the effect of expenditure on quality is proportional to spending, but the rate of proportionality is different for the two types of expenditure. Eliminating $E^3_i$:

$$e = \eta \sum_{i=1}^{N} E_i - (2\eta - 1) \sum_{i=1}^{N} E^1_i - \eta \sum_{i=1}^{N} E^2_i.$$  \hfill (32)

We are interested in an interior solution, where a strictly positive amount is allocated to each type of expenditure. This implies that we must assume $\frac{1}{2} < \eta < 1$ otherwise expenditure on real quality improvement will strictly dominate direct expenditure, so no direct expenditure would occur.

Consumer preferences are determined by the quantity and quality of the $N$ goods as described in (22). The perceived brand image of good $j$ is determined by a combination of the nominal and the real environmental qualities. The aggregation of the two expenditures into an overall level of quality, $z_j$, is determined by the CES function

$$z_j = \left[ (1 - \xi) (z^1_j)^\alpha + \xi (z^2_j)^\alpha \right]^{1/\alpha}, \quad \xi \in [0, 1].$$  \hfill (33)

The cost of attaining quality level $z^k_j$ is given by the cost function $C_k (z^k_j)$. The quality levels and the allocation of environmental expenditure for firm $j$ must satisfy the two equalities

$$E^1_j = C_1 (z^1_j), \quad E^2_j = C_2 (z^2_j).$$  \hfill (34)

The next result determines the necessary conditions of firm $j$ for the choice of price and the allocation of environmental expenditure.

**Lemma 1** The profit-maximizing choices of firm $j$ are described by the necessary conditions:

$$p_j = c(e) \frac{e^{\frac{\beta}{\gamma}}}{e^{\frac{\beta}{\gamma}} - 1},$$  \hfill (35)

$$z^1_j C'_1 (z^1_j) = I s_j \left( 1 - s_j \right) \left( \sigma - 1 \right) (1 - \xi) z^1_j z^1_j \frac{1}{z^1_j},$$  \hfill (36)

$$C'_1 (z^1_j) \frac{z^1_j}{z^1_j} \frac{C'_2 (z^2_j)}{C^2_j (z^2_j)} = 2\eta - 1 - \xi \frac{z^1_j}{z^2_j},$$  \hfill (37)

$$-\eta c'(e) q_j - 1 = 0.$$  \hfill (38)

The first condition (35) captures the mark-up of price over marginal cost. Condition (36) relates the marginal cost of nominal quality to the marginal benefit improvement. The third condition, (37), balances the allocation of expenditure between the two forms of brand image on the basis of effectiveness and the ratio of marginal costs. The fourth, (38), equates the marginal benefit of additional environmental expenditure to the marginal cost.

For analytical tractability we now assume that the cost functions are isoelastic

$$c(e) = \beta e^{-\gamma}, \quad \beta > 0, \quad \gamma > 0;$$  \hfill (39)

$$C_k (z^k_j) = \left( \frac{z^k_j}{\theta_k} \right)^\mu, \quad \theta_k > 0, \quad \mu \geq 1; \quad k = 1, 2.$$  \hfill (40)

Using these functional forms it is possible to determine the allocation of environmental expenditure between nominal and real quality improvement and the equilibrium quality of the environment.

The first result determines the choices of each firm, $j$, as a function of equilibrium market share.

**Lemma 2** The allocation of expenditure on nominal and real quality improvement is determined by:

$$E^1_j = \frac{1 - \xi s_j I}{\omega} \left[ 1 - \frac{1}{e^{\frac{\beta}{\gamma}}} \right],$$  \hfill (41)

$$E^2_j = \left[ \frac{1 - \xi}{\xi} \frac{2\eta - 1}{\eta} \right]^{\mu/(\alpha - \mu)} \left[ \frac{\theta_1}{\theta_2} \right]^{\alpha/(\alpha - \mu)} E^1_j.$$  \hfill (42)
where \( \omega = 1 - \xi + \xi \left[ \frac{1 - \xi \theta_1}{\xi \theta_2} 2^{\eta - 1} \right]^{\alpha/(\alpha - \mu)} \).

When the equilibrium is symmetric between firms the results in lemma 2 can be employed to find an explicit solution for the equilibrium environmental contribution of each firm and the equilibrium environmental quality.

**Lemma 3** The total environmental contribution by each firm is

\[
E^* = \frac{I}{N} \left( \frac{\gamma}{N} + \frac{1}{\mu} \right) \varepsilon^{yH} - 1, \tag{43}
\]

and aggregate environmental expenditure is

\[
e^* = \frac{\gamma \eta I}{N} \varepsilon^{yH} - 1. \tag{44}
\]

Observe that the solution (43) for equilibrium expenditure is similar to (14) for the production cost model. The additional term, \( \frac{1}{\mu} \), reflects the indirect contribution to environmental quality of brand image. Aggregate environmental expenditure in equilibrium differs from (15) by a constant factor. The additional factor, \( \frac{\gamma \eta}{N} \), captures the marginal productivity of the direct environmental expenditures on the quality of the environment. Hence, by proposition 1, environmental expenditures in this combined model will decrease if the number of firms increases.

The individual and aggregate profits in a symmetric equilibrium are given by the following expressions:

\[
\pi^* = \frac{I}{N \varepsilon^{yH}} \left[ 1 - \left( \frac{\gamma}{N} + \frac{1}{\mu} \right) \left( \varepsilon^{yH} - 1 \right) \right], \tag{45}
\]

\[
\Pi^* = \frac{I}{\varepsilon^{yH}} \left[ 1 - \left( \frac{\gamma}{N} + \frac{1}{\mu} \right) \left( \varepsilon^{yH} - 1 \right) \right]. \tag{46}
\]

Profit will be non-negative in equilibrium if \( \frac{\gamma}{N} + \frac{1}{\mu} \leq \frac{1}{\varepsilon^{yH} - 1} \). The equilibrium level of firm profit and the level of aggregate profit differ from the results for the production cost model, (16) and (18), only by the inclusion of the additional term \( \frac{1}{\mu} \). This does not significantly affect the comparative statics, so the impact of an increase in the number of firms on individual and aggregate profit is described by propositions 3 and 4.

We now investigate how a change in the number of firms affects consumer welfare. In a symmetric equilibrium indirect utility is given by

\[
V(N) = V_0 N^{1/(\sigma - 1) - 1/\mu} \left[ \frac{\varepsilon^{yH} - 1}{\varepsilon^{yH}} \right]^{1 + 1/\mu}, \tag{47}
\]

where \( V_0 = I^{1 + 1/\mu} \left( \frac{\gamma \eta}{\beta} \right) \left[ \frac{1 - \xi \theta_1}{\xi \theta_2} \right]^{1/\mu} \). The effect upon utility is described in the following proposition.

**Proposition 9** When the number of firms increases:

(i) if \( \sigma < \bar{\sigma} \), utility increases;

(ii) if \( \bar{\sigma} (\gamma, \mu) < \sigma < \sigma_0 \), there exists \( \bar{N} > 2 \) such that utility increases for \( N < \bar{N} \) but decreases for \( N > \bar{N} \);

(iii) if \( \sigma \geq \sigma_0 \) utility falls for all \( N \geq 2 \);

where \( \bar{\sigma} = 1 + \frac{1}{2(\gamma + 1)} \) and the expression for \( \sigma_0 \) is given in the Appendix.

The behavior of utility is monotone for sufficiently large \( N \), but can be non-monotone for small \( N \), depending on the values of \( \gamma, \mu, \) and \( \sigma \). When the cost effect is weak (\( \gamma < \frac{1}{2(\sigma - 1)} - \frac{1}{\mu} \)), the production-cost effect is dominated by the market share effect, so that with the number of firms rising the equilibrium price falls and utility increases. On the other hand, larger values of \( \gamma \) imply a steeper rise in the marginal cost as environmental quality deteriorates. Free-riding leads to high production cost and, hence, higher prices, and when the cost effect is very strong utility falls as the number of firms increases.

Table 5 illustrates the non-monotonic pattern in consumer welfare when \( \gamma = 1, \sigma = 1.6, \) and \( \mu = 1.5 \). In this case, \( \bar{\sigma} = 1.3 < \sigma = 1.6 < \sigma_0 \approx 2.11 \), so this corresponds to case (ii), with \( \bar{N} = 4.61 \).
Finally, we investigate the effect upon welfare of increasing the number of firms. The results have shown that for a large range of the parameters consumer utility and producer profit move in opposite directions as the number of firms changes. Therefore, the effect of competition on social welfare will depend on which of these two components dominates. The change in the social welfare is given by

$$\frac{dW}{dN} = \frac{I}{N} \left[ \frac{1}{1+\gamma+\frac{1}{\mu}} + \frac{\gamma}{N} \left( \frac{N}{(N-1)} - \frac{1}{N} \right) \right].$$

(48)

In this case, again, $\frac{dW}{dN} = O(N^{-1})$, and the leading term is $\frac{1}{N} \left( \frac{1}{1+\gamma+\frac{1}{\mu}} + \frac{\gamma}{N} \left( \frac{N}{(N-1)} - \frac{1}{N} \right) \right)$. Thus, for large $N$ the sign of the change in welfare is determined by the sign of $\left( \frac{1}{1+\gamma+\frac{1}{\mu}} + \frac{\gamma}{N} \left( \frac{N}{(N-1)} - \frac{1}{N} \right) \right)$. For small $N$, depending on the configuration of parameters and on the starting value, an increase in $N$ may lead to an increase or a decrease in welfare. Importantly, there are cases for which an increase in the number of firms decreases profit sufficiently that the consumers are unable to compensate the firms.

These points are illustrated by the three cases shown in figure 3. In the first case, with $\gamma = 0.1$, the production cost externality is low, so there is little incentive for free-riding and more competition is always beneficial. The opposite is true in the third case with $\gamma = 1$ so a substantial externality. In the intermediate case, social welfare is maximal around $N = 3$; a further increase in the number of firms leads to a loss in aggregate profit which exceeds the gains accruing to the consumers.

### 6 Cartel Defence

The public interest defence for allowing a cartel to operate is based on the argument that the additional profitability induces cartel members to make greater environmental contributions that more than offset the welfare loss due to non-competitive pricing. To address whether this defence can be sustained it is necessary to contrast the outcomes with and without a cartel. We know that a cartel will succeed in raising price relative to the Nash equilibrium, but the effect upon environmental expenditures is far less clear.
The results in the previous sections have identified cases in a reduction in competition (meaning a reduced number of active firms) raises welfare because environmental contributions increase. It might be argued that such results demonstrate a basis for the public interest defence but this is not a valid inference. Instead, the defence requires a demonstration that welfare is increased when a fixed number of firms cease competing and begin operating as a cartel. In a model with homogenous products and no environmental expenditures the formation of a cartel will reduce welfare. For example, a collusive duopoly will result in the monopolistic outcome, and monopoly generates lower welfare than duopoly. This reasoning cannot be applied so straightforwardly to the model with environmental contributions since collusion also affects the incentive to make the contributions. It is the consequences of this feature that we have to assess.

The public interest defence requires that the creation of a cartel generates a welfare-enhancing increase in non-market benefits. To address this the Nash equilibrium with no cartel is taken as the starting point and the introduction of a cartel power is modelled as causing an increase in price. Formally we adopt a reform perspective: the market price becomes the control variable (a cartel will always set a higher price than competing firms) and the other choice variables adjust optimally to the price. It is then possible to determine whether the welfare loss due to the price increase is more that offset by a gain from increased environmental contributions. If this is the case, the analysis provides support for the public interest defence. If the converse is true, then the public interest defence fails.

The solution process is to suppress the first-order condition for price and solve the remaining conditions treating price as a parameter of the system. This determines the environmental expenditures as functions of the price. With price equal to the Nash equilibrium value the environmental expenditures will be equal to our existing solutions. To make this analysis feasible it is assumed that the equilibrium is symmetric so that there is a common equilibrium price. Given this background, it is possible to state the key result.

**Lemma 4** An increase in cartel power:

(i) Decreases the environmental contributions of individual firms;
(ii) Decreases the aggregate environmental contribution.

The crucial part of lemma 4 is that total environmental expenditure is decreasing in \( p \). In fact, we have

\[
e = \left( \frac{\eta I \gamma \beta}{N p} \right)^{1/(1+\gamma)},
\]

from which it can be seen directly that \( e \) is decreasing in \( p \) and \( N \). Therefore, a large cartel leads to lower environmental expenditure than a small cartel for a given level of price. Which of a large or small cartel leads to the lowest level of environmental expenditures depends on the interplay between cartel size and the ability to sustain a higher price.

The final result describes the welfare effect of a cartel and the resolution of the initial question about the public interest defence.

**Theorem 1** Social welfare is lower with a cartel than it is with competing firms.

The theorem is established by showing that an increase in \( p \) from the Nash equilibrium level increases profit but decreases utility. The consumer loses directly from the price increase (in a standard model this alone more than offsets the increase in profit) and suffers further loss from the reduction in perceived environmental quality of the products. Because of the optimization of the firms belonging to the cartel, the envelope condition removes the effect of environmental contributions on profit. The intuition behind this result is that environmental expenditures are made for strategic reasons. This is the case whether they create have public benefits for the firms (the “real” expenditures) or private benefits (“nominal” brand image expenditures). The formation of a cartel reduces the need for these strategic expenditures so the level falls as a consequence. No other outcome is possible within this framework.

From this analysis follows the conclusion that there are no grounds for the cartel defence based on public interest. If the cartel is permitted to raise price above the Nash equilibrium level with monopolistic competition then environmental expenditures fall. The competing firms have strategic reasons to make individual environmental contributions but these are reduced as the cartel becomes more successful.
7 Conclusions

Corporations make significant environmental contributions and must expect some commercial advantage from doing so. We have modelled two forms of benefit: an improved environment reducing production cost, and increased demand through cultivation of an environmental brand image. These benefits have very different effects upon the strategic interaction between firms.

An improved environment is a public good that benefits all firms; as a result, strategic interaction in contribution to this public good is added to the strategic interaction in the market place. The consequence is that reduced competition benefits the environment since it reduces the free-rider effect. There are cases in which this can lead to social welfare increasing as the number of competing firms is reduced despite the consumer preference for variety. Brand image is a form of non-price competition and has only private benefits for firms. It is therefore not surprising that increased competition is preferable when there is no direct environmental benefit from brand image.

Combining these two interpretations of environmental contribution allows us to address the public interest defence in a rich analytical framework. Even though there is a wide range of effects at work we are able to demonstrate that cartelization will be harmful to the public interest. The change in the strategic environment that results from the formation of a cartel leads to a reduction in environmental contributions and lower environmental quality. The public interest defence is therefore not sustained even though the firms provide voluntary environmental contributions.

A Equilibrium in the production cost model

The necessary conditions for choosing price and environmental contribution are

\[
\frac{\partial \pi_j}{\partial p_j} = q_j + \left[ p_j - c_j \left( \sum_{i=1}^{n} E_i \right) \right] \frac{\partial q_j}{\partial p_j} = 0,
\]

\[
\frac{\partial \pi_j}{\partial v_j} = -q_j \frac{\partial c_j \left( \sum_{i=1}^{n} E_i \right)}{\partial v_j} - 1 = 0.
\]

The first-order condition with respect to price gives the standard markup condition:

\[
\frac{p_j - c_j(e)}{p_j} = \frac{1}{\epsilon_j^{YH}}.
\] (50)

The second profit-maximization equation for choice of environmental improvement gives

\[-c_j'(e) = \frac{1}{q_j},
\]

or

\[-c_j'(e) = \frac{p_j}{I s_j} \] (51)

These two equations can be used to express the equilibrium profit as

\[
\pi_i = (p_i - c_i) q_i - v_i = \frac{p_i q_i}{\epsilon_i^{YH}} - E_i
\]

\[
= \frac{1}{\epsilon_i^{YH}} c_i \frac{\epsilon_i^{YH}}{\epsilon_i^{YH}} - 1 \left[ -\frac{1}{c_i} \right] - E_i
\]

\[
= \left[ - \frac{e c_j'(e) \left( \epsilon_i^{YH} - 1 \right)}{c_j} \right]^{-1} e - E_i.
\] (52)

We can use (7) and (50) in (51) to rewrite the latter as

\[- \frac{e c_j'(e)}{c_j(e)} = \frac{e}{I s_j (\sigma - 1)} \frac{\sigma - (\sigma - 1) s_j}{s_j (\sigma - 1) (1 - s_j)}.
\] (53)
Assuming constant elasticity of marginal cost,

\[ c_j(e) = \beta_j e^{-\gamma}, \quad \gamma > 0, \]

the equilibrium level of environmental quality is given by

\[ e = I \gamma s_j (\sigma - 1) (1 - s_j) / (\sigma - (\sigma - 1) s_j), \]

and prices are given by

\[ p_j = c_j(e) \frac{\varepsilon_j^{YH}}{\varepsilon_j^{YH} - 1} \]

\[ = \beta_j \left[ I \gamma s_j (\sigma - 1) (1 - s_j) / (\sigma - (\sigma - 1) s_j) \right]^{-\gamma} \frac{\sigma - (\sigma - 1) s_j}{(\sigma - (\sigma - 1) s_j)} \]

\[ = \frac{\beta_j}{[I \gamma s_j]^{\varepsilon_j^{YH}}} \left[ \frac{\sigma - (\sigma - 1) s_j}{(\sigma - (\sigma - 1) s_j)} \right]^{1+\gamma} = \frac{\beta_j}{[I \gamma s_j]^{\varepsilon_j^{YH}}} \left[ 1 + \frac{1}{(\sigma - 1) (1 - s_j)} \right]^{1+\gamma}. \]

In a symmetric interior equilibrium \( p_i = p_j = p = P, \) \( s_i = 1/N, \) and \( E_i = E_j = E = e/N \) for all \( i \) and \( j, \) so

\[ e^* = \frac{I \gamma \varepsilon_j^{YH} - 1}{N} = \frac{(N - 1) (\sigma - 1) \gamma I}{(N - 1)(\sigma - 1) + N N}, \]

(54)

\[ p^* = \beta \left( \frac{N}{\gamma I} \right) \left[ \frac{\varepsilon_j^{YH}}{\varepsilon_j^{YH} - 1} \right]^{1+\gamma} \]

\[ = \beta \left( 1 + \frac{1}{N \sigma - 1 (N - 1)} \right)^{1+\gamma} \left[ \frac{N}{\gamma I} \right], \]

(55)

and the equilibrium environmental expenditure by each firm is, therefore,

\[ E^* = \frac{I \gamma \varepsilon_j^{YH} - 1}{N^2} = \frac{(N - 1) (\sigma - 1) \gamma I}{(N - 1)(\sigma - 1) + N N^2}. \]

(57)

**Proof of Proposition 1.** Rewrite (14) as

\[ E^* = \frac{\gamma I}{N^2 + \frac{N^3}{(N-1)(\sigma-1)}}, \]

so that

\[ \frac{\partial E^*}{\partial N} = - \frac{\gamma I}{\left[ N^2 + \frac{N^3}{(N-1)(\sigma-1)} \right]^2} \left[ \frac{2N + 3N^2 (N - 1) - N^3}{(N - 1)^2 (\sigma - 1)} \right] \]

\[ = - \frac{2N^2 \gamma I}{\left[ N^2 + \frac{N^3}{(N-1)(\sigma-1)} \right]^2} \left[ 1 + \frac{N (N - 3/2)}{(N - 1)^2 (\sigma - 1)} \right] < 0. \]

(58)

(i) The overall level of environmental quality in a symmetric equilibrium is given by

\[ e^* = NE^* = \frac{\frac{\gamma I}{N^2}}{\frac{N^3}{(N-1)(\sigma-1)}}, \]

and so

\[ \frac{\partial e^*}{\partial N} = - \frac{\gamma I}{\left[ N + \frac{N^2}{(N-1)(\sigma-1)} \right]^2} \left[ 1 + \frac{2N (N - 1) - N^2}{(N - 1)^2 (\sigma - 1)} \right] \]

\[ = - \frac{\gamma I}{\left[ N + \frac{N^2}{(N-1)(\sigma-1)} \right]^2} \left[ 1 + \frac{N (N - 2)}{(N - 1)^2 (\sigma - 1)} \right] < 0. \]

(59)
Table 1: Cost reduction, competition and price

<table>
<thead>
<tr>
<th>(γ, σ)</th>
<th>N</th>
<th>p*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; γ &lt; \frac{2}{\sigma - 1}; σ &gt; 1</td>
<td>2 &lt; N \leq N \text{ or } N &gt; \bar{N}</td>
<td>p^<em>(N + 1) &lt; p^</em>(N)</td>
</tr>
<tr>
<td>\frac{2}{\sigma - 1} \leq γ \leq \frac{N^*}{(\sigma - 1)(N - 1)}; σ &gt; 1</td>
<td>N \geq 2</td>
<td>p^<em>(N + 1) &gt; p^</em>(N)</td>
</tr>
</tbody>
</table>

(ii) This follows trivially from (i) and the assumption c(\epsilon) < 0.

**Proof of Proposition 2.** For arbitrary \( N \geq 2 \) we have

\[
\begin{align*}
p^* &= \beta \left( 1 + \frac{1}{\sigma - 1} \frac{N}{N - 1} \right)^{\frac{1 + \gamma}{\gamma}} \left[ \frac{N}{\gamma I} \right]^{\gamma}, \\
\frac{1}{p^*} \frac{dp^*}{dN} &= -\frac{1 + \gamma}{1 + \frac{1}{\sigma - 1} \frac{N}{N - 1} (\sigma - 1) (N - 1)^2} + \frac{\gamma}{N} \\
&= -\frac{1 + \gamma}{(\sigma - 1) (N - 1)^2 + N (N - 1)} + \frac{\gamma}{N} \\
&= \frac{\gamma \sigma (N - 1)^2 - (N - 1) - (1 + \gamma)}{N \left[ (\sigma - 1) (N - 1)^2 + N (N - 1) \right]} = \frac{\Phi(N - 1)}{N \left[ (\sigma - 1) (N - 1)^2 + N (N - 1) \right]}.
\end{align*}
\]

The denominator is positive, and the numerator is a quadratic polynomial,

\[
\Phi(N - 1) = \gamma \sigma (N - 1)^2 - (N - 1) - (1 + \gamma)
\]

with the discriminant

\[
D = 1 + 4 \gamma \sigma (1 + \gamma) > 0.
\]

Thus, \( \Phi(N - 1) \) has two real roots,

\[
N_{\pm} = 1 \pm \sqrt{1 + 4(1 + \gamma)\gamma \sigma / 2\gamma \sigma}.
\]

Clearly, \( N^+ < 0 \). Therefore, \( \Phi(N - 1) > 0 \) for all \( N \geq \max\{2, N^+\} \), and \( \Phi(N - 1) < 0 \) for \( 2 < N < N^+ \) provided \( N^+ > 2 \), or \( N^+ - 1 > 1 \). To establish the condition for the latter we note that the root \( x^+ = -\frac{-b + \sqrt{b^2 - 4ac}}{2a} \) of a quadratic polynomial of the form \( ax^2 + bx + c \) with \( a > 0 \) satisfies \( x^+ > 1 \) if \( a + b + c < 0 \). Applying this to \( \Phi(N - 1) \) we find

\[
\gamma \sigma - 1 - (1 + \gamma) = -2 + \gamma (\sigma - 1) < 0 \iff \gamma < \frac{2}{\sigma - 1}.
\]

Full characterization that takes into account the profit non-negativity condition (17) is summarized in Table 1.

**Proof of Proposition 3**

For arbitrary \( N \geq 2 \) differentiation with respect to \( N \) gives

\[
\frac{\partial p^*}{\partial N} = \frac{I}{N^2 (\varepsilon^y H)^2 v(N)},
\]

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where
\[ v(N) \equiv -\sigma + \frac{2\gamma \sigma (\sigma - 1)}{N} - \frac{\gamma (\sigma - 1)(4\sigma - 1)}{N^2} + \frac{2\gamma (\sigma - 1)^2}{N^3}. \]

In principle, one can calculate the roots of a cubic polynomial exactly, but the expressions are cumbersome and give little insight. Instead, note that
\[ v'(N) = -\frac{2\gamma \varepsilon^{YH}(\sigma - 1)}{N^3} (N - 3) \leq 0 \text{ for } N \geq 3 \]

and \( \lim_{N \to \infty} v(N) = -\sigma < 0. \) Therefore, the sign of \( v(N) \) (and, hence, the sign of \( \frac{\partial v}{\partial N} \)) will be determined by the pattern in \( v(2) \) and \( v(3). \) It is easy to show that \( v(2) = \sigma \left[ \frac{\gamma}{2} (\sigma - 1) - 1 \right] \) and \( v(3) = \frac{\gamma}{27} (\sigma - 1)(8\sigma + 1) - \sigma. \) Thus, both \( v(2) \) and \( v(3) \) are both positive for \( \gamma > \frac{4}{\sigma - 1} \) (note that in this case profits are negative for \( N = 2, \) and so at least three firms can exist in the equilibrium). In this case, \( v(N) \) is positive and increasing up to some \( N \geq 3, \) decreasing for all \( N \geq 3, \) and, since \( \lim_{N \to \infty} v(N) = -\sigma < 0, \) by continuity, \( \exists \bar{N} > 3 \) such that \( v(N) < 0 \) for all \( N > \bar{N}. \) This proves part (d). Next, \( v(2) \) and \( v(3) \) are both negative for \( 0 < \gamma < \frac{4}{(\sigma - 1)(6\sigma - 1)} \). In this case \( v(N) < 0 \) for all \( N \geq 2. \) This proves part (a). Finally, \( v(2) < 0 < v(3) \) for \( \frac{2\gamma}{(\sigma - 1)(6\sigma - 1)} < \gamma < \frac{4}{\sigma - 1}. \) In this case, by the same argument as for part (d), \( \exists \bar{N} > 3 \) such that \( v(N) < 0 \) for all \( N > \bar{N}. \) The profits of a firm at \( N = 2 \) and \( N = 3 \) must be compared directly, because \( v(N) \) (and, hence, \( \frac{\partial v}{\partial N} \)) switches sign between these two points. Straightforward calculation gives \( \pi^*(2) - \pi^*(3) = \frac{36\gamma - \gamma(\sigma - 1)(10\sigma + 1)}{(3\sigma + 1)(2\sigma + 1)}. \) Therefore, \( \pi^*(2) \leq \pi^*(3) \) for \( \gamma \geq \frac{36\gamma}{(\sigma - 1)(10\sigma + 1)}. \) This proves parts (b)-(c).

**Proof of Proposition 4** For \( N \geq 2 \) the aggregate profit of \( N \) firms is given by
\[
\Pi^* = \frac{I}{\varepsilon^{YH}} \left[ 1 - \gamma \varepsilon^{YH} \frac{1}{N} \right] = \frac{I}{\varepsilon^{YH}} \left[ 1 - \gamma \varepsilon^{YH} N + \frac{\gamma}{\varepsilon^{YH} N} \right].
\]

First we solve the non-negativity condition, \( \Pi^* \geq 0. \) This is equivalent to
\[
N^2 - \gamma (\sigma - 1) N + \gamma (\sigma - 1) \geq 0.
\]
Hence, \( \Pi^* \geq 0 \) for all \( N \) if \( \gamma < \frac{4}{\sigma - 1}; \) otherwise, \( \Pi^* \geq 0 \) for all \( N \geq N^+ \equiv \frac{1}{2} \left[ \gamma (\sigma - 1) + \sqrt{\gamma (\sigma - 1) [\gamma (\sigma - 1) - 4]} \right] > 2.
\]

Differentiation gives
\[
\frac{d\Pi^*}{dN} = \frac{I}{N^2 (\varepsilon^{YH})^2} \left[ -(\sigma - 1) \left( 1 + \frac{\gamma}{N} \right) - \gamma \varepsilon^{YH} + \gamma (\varepsilon^{YH})^2 \right]
\]
\[
= \frac{I}{N^2 (\varepsilon^{YH})^2} v(N).
\]

Note that
\[
v'(N) = 2\gamma \varepsilon^{YH} \frac{\sigma - 1}{N^2} > 0,
\]
\[
\lim_{N \to \infty} v(N) = (\sigma - 1)(\gamma - 1) \text{ and } v(2) = (\sigma - 1)(\frac{4}{3})(\sigma - 1) - 1 \text{. Therefore, the following cases are possible: (a) } v(N) < 0 \text{ for all } N \geq 2 \text{ when } 0 < \gamma < \frac{1}{\sigma} < \frac{1}{\sigma - 1}; \text{ in this case the aggregate profits fall with } N \text{ for all } N \geq 2; (b) } v(2) < 0, \lim_{N \to \infty} v(N) = (\sigma - 1)(\gamma - 1) > 0, \text{ and, by the intermediate value theorem } \exists N^* > 2 \text{ such that } v(N^*) = 0, \text{ when } \frac{1}{\sigma} < \gamma < \frac{1}{\sigma - 1}; \text{ in this case the aggregate profits fall with } N \text{ increasing up to } N^* \text{ and rise for } N > N^*, \text{ and (c) } v(N) > 0 \text{ for all } N \geq 2 \text{ when } \gamma > \frac{4}{\sigma - 1}; \text{ in this case the aggregate profits always rise with } N \text{ whenever it is positive, } N \geq N^+ \text{ defined above.}
\]

To calculate \( N^* \) we solve
\[
-(\sigma - 1) \left( 1 + \frac{\gamma}{N} \right) - \gamma \varepsilon^{YH} + \gamma (\varepsilon^{YH})^2 = 0.
\]
This can be rewritten as
\[
-(\sigma - 1) \left( 1 + \frac{\gamma}{N} \right) - \gamma \left( \sigma - \frac{\sigma - 1}{N} \right) \frac{(\sigma - 1)(N - 1)}{N} = 0,
\]
or, after obvious simplification,
\[ \gamma (\sigma - 1) x^2 - 2 \gamma x - 1 + \gamma \sigma = 0, \]
where \( x = \frac{1}{\bar{N}} \). This gives \( N^* = \frac{\sigma - 1}{\sigma - \sqrt{\sigma + (\sigma - 1)/\gamma}} \).

**Proof of Proposition 5.** For arbitrary \( N \geq 2 \) differentiation gives
\[
\frac{N}{V} \frac{dV}{dN} = \frac{1 + \gamma}{\varepsilon Y H (N - 1)} + \frac{1}{\sigma - 1} - \gamma
= \frac{1 + \gamma}{\varepsilon Y H n} + \frac{1}{\sigma - 1} - \gamma \equiv v(n),
\]
where \( n = N - 1 \). Thus, \( \lim_{n \to -\infty} v(n) = \frac{1}{\sigma - 1} - \gamma \). Furthermore,
\[
v'(n) = -\frac{1 + \gamma}{(\varepsilon Y H n)^2} \left( \varepsilon Y H + n \left( \frac{\sigma - 1}{n + 1} \right) \right) < 0.
\]
Thus, three cases are possible: (1) \( v(1) > 0 \), \( \lim_{n \to -\infty} v(n) > 0 \), and, thus, \( v(n) > 0 \forall n \geq 1 \); (2) \( v(1) > 0 \), \( \lim_{n \to -\infty} v(n) < 0 \), and, thus, by the intermediate value theorem, \( \exists \bar{n} > 1 : v(n) = 0 \); (3) \( v(1) < 0 \), \( \lim_{n \to -\infty} v(n) < 0 \), and, thus, \( v(n) < 0 \forall n \geq 1 \). Since \( v(1) = \frac{3\sigma - 1}{\sigma - 1} - \gamma \frac{\sigma - 1}{\gamma + 1} \), these three cases correspond to
(1) \( 0 < \gamma < \frac{1}{\sigma - 1} \); (2) \( \frac{1}{\sigma - 1} < \gamma < \frac{3\sigma - 1}{\sigma - 1} \); and (3) \( \gamma > \frac{3\sigma - 1}{\sigma - 1} \). To calculate \( \bar{n} \) note that \( v(n) = 0 \) reduces to a quadratic equation, \( n^2 - \frac{n}{\gamma (\sigma - 1) n - 1} - \frac{\gamma + 1}{\gamma (\sigma - 1) n - 1} = 0 \). Let \( a = 1 \), \( b = -\frac{n}{\gamma (\sigma - 1) n - 1} \), \( c = -\frac{1}{\gamma (\sigma - 1) n - 1} \). Note that \( a > 0 \) and, for case (2), \( a + b + c > 0 \). Therefore, the roots of \( an^2 + bn + c \) satisfy \( n^* < 1 < n^+ \), where \( n^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \), and, hence, \( \bar{n} = n^+ \). This completes the proof.

**Proof of Proposition 6** At a symmetric equilibrium \( \frac{p^j}{p^z} = 1 \) so \( s_j = \frac{1}{\bar{N}} \) and (26) simplifies to
\[
p^* - c = \frac{1}{\varepsilon Y H} \left\{ \begin{array}{l} \frac{\sigma - 1}{\bar{N}}, \\ \frac{\sigma}{\bar{N} - \sigma}, \end{array} \right\} N = \frac{1}{N}, \quad N \geq 2,
\]
and (27) becomes
\[
z^* C' (z^*) = \left\{ \begin{array}{l} \frac{I Y H}{N}, \\ \frac{I}{N^{1.5}}, \end{array} \right\} N = \frac{1}{N}, \quad N \geq 2.
\]
Logarithmic differentiation of (61) gives
\[
\frac{[C'' (z^*) - 1]}{C' (z^*)} \frac{dz^*}{dN} = -\left[ \frac{1}{N} + \frac{1}{\varepsilon Y H} \frac{d\varepsilon Y H}{dN} \right],
\]
which, upon rearrangement, shows that
\[
\frac{dz^*}{dN} = -\frac{z^*}{1 + \varepsilon C (z^*)} \times \left\{ \begin{array}{l} 1, \\ \frac{1}{\gamma (N - 1)^{1.5}}, \end{array} \right\} N = \frac{1}{N}, \quad N \geq 2,
\]
where \( \varepsilon C (z^*) = \frac{1}{z^*} \frac{C'' (z^*)}{C' (z^*)} > 0 \) is the elasticity of marginal cost.

**Proof of Proposition 7** The profit level of an individual firm is given by,
\[
\pi^* = (p^* - c) q^* - \frac{z^* \mu}{\theta} = \frac{I}{N \varepsilon Y H} \left[ 1 + \mu - \varepsilon Y H \frac{\mu}{\mu} \right].
\]
The equilibrium quality level is given by
\[
z^* = \left[ \frac{\theta I \varepsilon Y H - 1}{\mu \varepsilon Y H} \right]^{1/\mu},
\]
so
\[
\pi^* = \frac{I}{N \varepsilon Y H} \left( 1 + \mu - \varepsilon Y H \frac{\mu}{\mu} \right).
\]

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Profit is only non-negative if $1 + \mu > \varepsilon Y H$ and we impose this constraint on the model parameters. The aggregate profit, $\Pi^*$, is

$$\Pi^* = \frac{I}{\varepsilon Y H} \left( 1 + \frac{\mu - \varepsilon Y H}{\mu} \right).$$

Clearly, since $\frac{d\Pi^*}{dN} = \frac{\sigma - 1}{N^2} > 0$, both $\pi^*$ and $\Pi^*$ are decreasing in $N$. More specifically,

$$\frac{d\pi^*}{dN} = -\frac{I}{N^2 \varepsilon Y H} \left[ 1 + \frac{\sigma - 1}{N} \left( \frac{1}{\varepsilon Y H} + \frac{N - 1}{\mu} \right) \right] < 0,$$

$$\frac{d\Pi^*}{dN} = -\frac{I}{N \varepsilon Y H} \left( \frac{\sigma - 1}{N} + \frac{1 + \mu}{\mu} \right) < 0.$$

**Proof of Proposition 8.** The proof is the same as for Proposition 5. The profit non-negativity condition requires $\frac{1}{\mu} \leq \frac{1}{\varepsilon Y H} - 1$. To see when this is compatible with $\frac{1}{\mu} \leq \frac{3\sigma - 1}{(\sigma - 1)^2}$, note that

$$\frac{3\sigma - 1}{(\sigma - 1)^2} - \frac{1}{\varepsilon Y H - 1} = \frac{1}{\sigma - 1} \left( 3\sigma - 1 - \frac{1}{\varepsilon Y H - 1} \right) \geq 0$$

for $N \geq 3 \frac{1}{2} - \frac{1}{2\sigma} > 1$.

Thus, the case of $\frac{1}{\mu} > \frac{3\sigma - 1}{(\sigma - 1)^2}$ (consumer utility decreasing with $N$) is ruled out, and the range for parameters for the intermediate case, $\frac{1}{\mu} < \frac{1}{\sigma - 1} < \frac{3\sigma - 1}{(\sigma - 1)^2}$, becomes $\frac{1}{\mu} < \frac{1}{\sigma - 1} < \frac{1}{\varepsilon Y H - 1}$, which shrinks as $N$ gets larger and $\varepsilon Y H$ gets closer to $\sigma$.

**Proof of Lemma 1** Firm $j$ maximizes profit:

$$\max_{\{p_j, \delta_j^1, \delta_j^2, E_j\}} \pi_j = [p_j - c(e)] q_j - E_j.$$

The first-order condition for choice of $p_j$ is

$$\frac{\partial \pi_j}{\partial p_j} = q_j + [p_j - c(e)] \left[ \frac{\partial q_j}{\partial p_j} + \frac{\partial q_j}{\partial P_z} \frac{\partial P_z}{\partial p_j} \right] = 0.$$

Using (8)

$$\frac{p_j - c(e)}{p_j} = [\sigma - (\sigma - 1) s_j(N)]^{-1}, \quad (63)$$

where $s_j(N) = \frac{1}{N} \left[ \frac{p_j}{P_z} \right]^{1-\sigma}$. Solving for $p_j$ gives (35) in the text.

The first-order condition for $\delta_j^1$ is

$$\frac{\partial \pi_j}{\partial \delta_j^1} = -c'(e) \frac{\partial e}{\partial \delta_j^1} q_j + [p_j - c(e)] \left[ \frac{\partial q_j}{\partial z_j} + \frac{\partial q_j}{\partial P_z} \frac{\partial P_z}{\partial z_j} \right] \frac{\partial z_j}{\partial \delta_j^1} = 0, \quad (64)$$

From (34) we have

$$\frac{\partial z_j^k}{\partial \delta_j^1} = \frac{E_j}{C_k'(z_j^1)}, \quad k = 1, 2. \quad (65)$$

From (32) we have

$$\frac{\partial e}{\partial \delta_j^1} = -(2\eta - 1).$$

Using these two conditions (64) can be written as

$$-(2\eta - 1) E_j c'(e) q_j = [p_j - c(e)] \left[ \frac{\partial q_j}{\partial z_j} + \frac{\partial q_j}{\partial P_z} \frac{\partial P_z}{\partial z_j} \right] \frac{\partial z_j}{\partial \delta_j^1} \frac{E_j}{C_k'(z_j^1)}.$$
This can be rearranged as

\[
-(2\eta - 1) C'_1 \left( z_j^1 \right) c' (e) = \frac{p_j - c(e)}{q_j} \left[ \frac{\partial q_j}{\partial z_j} + \frac{\partial q_j}{\partial P_z} \frac{\partial P_z}{\partial z_j} \right] \frac{\partial z_j}{\partial z_j^1} = \left[ p_j - c(e) \right] \left[ z_j \frac{\partial q_j}{\partial z_j} + \frac{P_Z}{q_j} \frac{\partial q_j}{\partial P_z} \frac{z_j}{z_j} \frac{\partial P_z}{\partial z_j} \right] \frac{1}{z_j} \frac{\partial z_j}{\partial z_j^1}. \tag{66}
\]

For the quality-adjusted Dixit-Stiglitz preferences

\[
\frac{z_j \partial q_j}{q_j \partial z_j} = \frac{P_Z}{q_j} \frac{\partial q_j}{\partial P_z} = 1 - \sigma, \quad \frac{z_j \partial P_z}{P_z \partial z_j} = -\frac{1}{N} \left[ \frac{p_j}{z_j} \right]^{1-\sigma} = -s_j (N),
\]

and from the first-order condition for price

\[
p_j - c(e) = \frac{c(e)}{(\sigma - 1) \left[ 1 - s_j (N) \right]} \tag{67}
\]

Therefore, (66) becomes

\[
-(2\eta - 1) C'_1 \left( z_j^1 \right) c' (e) = \frac{c(e)}{z_j} \frac{1}{z_j} \frac{\partial z_j}{\partial z_j^1}. \tag{68}
\]

From (33) we have

\[
\frac{\partial z_j}{\partial z_j} = (1 - \xi) \frac{z_j}{z_j^1} \left( z_j^1 \right)^{\alpha-1}
\]

Substituting into (68) gives

\[
-(2\eta - 1) z_j^1 C'_1 \left( z_j^1 \right) c' (e) = c(e) (1 - \xi) \left( \frac{z_j^1}{z_j} \right)^{\alpha}.
\]

Rearranging this condition gives (36).

The first-order condition for \( \delta_j^1 \) is

\[
\frac{\partial \pi_j}{\partial \delta_j^1} = -c'(e) \frac{\partial e}{\partial \delta_j^1} q_j + \left[ p_j - c(e) \right] \left[ \frac{\partial q_j}{\partial z_j} + \frac{\partial q_j}{\partial P_z} \frac{\partial P_z}{\partial z_j} \right] \frac{\partial z_j}{\partial z_j} \frac{\partial z_j}{\partial \delta_j^1} = 0. \tag{69}
\]

Rearranging and dividing the (64) by (69), gives

\[
\frac{\partial e/\partial z_j^1}{\partial e/\partial \delta_j^1} = \frac{\partial z_j/\partial z_j^1}{\partial z_j/\partial \delta_j^1} \frac{\partial z_j^1/\partial \delta_j^1}{\partial z_j^2/\partial \delta_j^2}. \tag{70}
\]

Using (32) and (33) this condition becomes

\[
\frac{2\eta - 1}{\eta} = \frac{1 - \xi}{\xi} \left( \frac{z_j^1}{z_j^2} \right)^{\alpha-1} \frac{\partial z_j^1/\partial \delta_j^1}{\partial z_j^2/\partial \delta_j^2}.
\]

Using (65) gives (37).

(38) follows directly from differentiation of profit.

**Proof of Lemma 2** Using (40) to substitute into (37) gives

\[
z_j^2 = \left( \frac{1 - \xi}{\xi} \frac{2\eta - 1}{\eta} \right)^{1/(\alpha - \mu)} z_j^1, \tag{71}
\]

and, therefore, using (33)

\[
z_j = z_j^1 \left[ 1 - \xi + \xi \left( \frac{1 - \xi}{\xi} \frac{2\eta - 1}{\eta} \right)^{\alpha/(\alpha - \mu)} \right]^{1/\alpha}. \tag{72}
\]
Use (33) and substitute (72) into (36) to obtain
\[
\frac{\mu}{\theta_1} (z_j^1)^\alpha = \frac{s_j(1 - s_j)}{\varepsilon_j^H} \frac{1 - \xi}{\omega} (\sigma - 1) I,
\] (73)
where
\[
\omega = 1 - \xi + \xi \left( \frac{1 - \xi}{\xi} \theta_2 \frac{2\eta - 1}{\eta} \right)^{\alpha/(\alpha - \mu)}
\] (74)
Using (34), (40) and (7), this gives (41).
From (34) and (40)
\[
E_j^2 = \frac{(z_j^1)^\mu}{\theta_2}.
\]
Using (71)
\[
E_j^2 = \frac{\left( \frac{1 - \xi}{\xi} \theta_2 \frac{2\eta - 1}{\eta} \right)^{\mu/(\alpha - \mu)}}{\theta_2} \left( z_j^1 \right)^\mu
\]
\[
= \left( \frac{1 - \xi}{\xi} \frac{2\eta - 1}{\eta} \right)^{\mu/(\alpha - \mu)} \left( \frac{\theta_1}{\theta_2} \right)^{\alpha/(\alpha - \mu)} E_1^j,
\]
which is (42).

**Proof of Lemma 3**
The proof determines the equilibrium value of \( e \) first and then deduces the expenditure, \( E \), of each firm.
Rearrange (38) as
\[
-\eta c'(e) \frac{d^j q_j}{p_j} = 1.
\] (75)
At a symmetric equilibrium \( p_j q_j = \frac{I}{N} \). Use this fact and (35) to write (75) as
\[
-\eta c'(e) \frac{I}{N} = c(e) \frac{\varepsilon_j^H}{\varepsilon_j^H - 1}.
\] (76)
Since \( c(e) = \beta_j e^{-\gamma} \), substitution into (76) and rearrangement gives \( e \) as determined by (44).
At a symmetric equilibrium (31) gives
\[
e = N \left[ \eta E - \eta E_1 - (2\eta - 1) E_2 \right]
\]
\[
= N \eta \left[ E - E_1 \left( 1 + \frac{2\eta - 1}{\eta} \frac{E_2}{E_1} \right) \right].
\] (77)
Setting \( s_j = \frac{1}{N} \) for the symmetric case in (41) and (42), and substituting these into (77) gives
\[
e = N \eta \left[ E - \frac{I}{\mu N} \frac{\varepsilon_j^H}{\varepsilon_j^H - 1} \right].
\] (78)
Finally, substituting (44) into (78) and rearranging gives (43).

**Proof of Proposition 9** For arbitrary \( N \geq 2 \) the log-derivative of \( V(N) \) gives
\[
\frac{N}{V} \frac{dV}{dN} = \frac{1}{\sigma - 1} - 2 \left( \gamma + \frac{1}{\mu} \right) + \frac{\gamma (\gamma + \frac{1}{\mu})}{n + 1 + \gamma (\mu)} + \frac{1 + \gamma}{n \sigma} - \frac{\sigma - 1 + \gamma + \frac{1}{\sigma}}{n + \frac{1}{\sigma}}
\]
\[
\equiv v(n), \ n \equiv N - 1.
\]
Function \( v(n) \) is continuous and continuously differentiable for all \( n \geq 1 \), \( \lim_{n \to \infty} v(n) = \frac{1}{\sigma - 1} - 2 \left( \gamma + \frac{1}{\mu} \right) \geq 0 \) for \( \sigma \leq 1 + \frac{1}{2(\gamma + \frac{1}{\mu})} = \delta (\gamma, \mu) \), and \( \lim_{n \to \infty} v'(n) = 0 \). Furthermore, observe that \( v(n) \) is strictly decreasing in \( n \):
\[
v'(n) = \frac{\gamma (\gamma + \frac{1}{\mu})}{(n + 1 + \gamma (\mu))^2} - \frac{1 + \gamma}{n^2 (n + \frac{1}{\sigma})^2} \left[ \frac{n^2}{\sigma} + 2 \frac{n}{\sigma} + \frac{1}{\sigma^2} \right] < 0.
\]
It is straightforward to show that
\[ v(1) = -\frac{1}{\sigma^2 - 1} (a\sigma^2 + b\sigma + c), \]
where \( a = (\gamma + \frac{1}{\mu})\frac{4+\gamma\nu}{2+\gamma\nu}, \)
\[ b = -\left(3 + 2 \left(\gamma + \frac{1}{\mu}\right)\right), \]
\[ c = 1 + \left(\gamma + \frac{1}{\mu}\right)\frac{\gamma\nu}{2+\gamma\nu}. \]
Let \( \sigma^+ \) and \( \sigma^- \) be the solutions of \( v(1) = 0; \sigma^\pm = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \).
Since \( a > 0 \) and \( a + b + c = -2 < 0 \), we have \( \sigma^- < 1 < \sigma^+ \).
Let \( \sigma_0(\gamma, \mu) = \sigma^+ \).
Then, \( v(1) < 0 \) for \( \sigma > \sigma_0 \) and \( v(1) > 0 \) for \( 1 < \sigma < \sigma_0 \).
Furthermore, one can show that \( \sigma_0(\gamma, \mu) > \sigma^\ast(\gamma, \mu) \) for all \( \{\gamma, \mu\} \).
Therefore, \( v(n) < 0 \forall n \geq 1 \) for \( \sigma > \sigma_0 \) and \( v(n) > 0 \forall n \geq 1 \) for \( 1 < \sigma < \sigma^\ast \).
In the intermediate case, for \( \sigma^\ast < \sigma < \sigma_0 \) by the intermediate value theorem \( v(n) = 0 \) for some \( n_0 > 1 \).
Let \( N = n + 1 \). This completes the proof.

**Proof of Lemma 4**

The first-order condition for \( E_j^1 \) is
\[ \frac{\partial \pi_j}{\partial E_j^1} = -q_j c' (e) \frac{\partial e}{\partial E_j^1} + [p_j - c(e)] \left[ \frac{\partial q_j}{\partial z_j} \frac{\partial P_z}{\partial z_j} \right] \frac{\partial z_j}{\partial z_j} \frac{\partial z_j^1}{\partial E_j^1} = 0, \] (79)
From (34)
\[ \frac{\partial z_j}{\partial E_j^1} = \frac{1}{C_k' (z_j^1)}, \quad k = 1, 2, \] (80)
and from (32)
\[ \frac{\partial e}{\partial E_j^1} = 1 - 2\eta. \]
Using these two conditions (79) can be written as
\[ (1 - 2\eta) C_1' (z_j^1) c' (e) = [p_j - c(e)] (\sigma - 1) (1 - s_j) \frac{1}{z_j} \frac{\partial z_j}{\partial z_j^1}. \] (81)
Since
\[ \frac{\partial z_j}{\partial z_j^1} = (1 - \xi) \frac{z_j}{z_j^\alpha} (z_j^1)^{\alpha-1} \]
substituting into (81) gives
\[ (1 - 2\eta) z_j^1 C_1' (z_j^1) c' (e) = [p_j - c(e)] (\sigma - 1) (1 - s_j) (1 - \xi) \left( \frac{z_j^1}{z_j^1} \right)^\alpha. \] (82)
From (72)
\[ z_j = z_j^1 \left[ 1 - \xi + \xi \left( \frac{1 - \xi \theta_0 2\eta}{\xi \theta_2 / \eta} \right)^{\alpha/(\alpha-\mu)} \right]^{1/\alpha} \]
\[ = z_j^1 \omega^{1/\alpha}, \] (83)
and the cost function
\[ C_1 (z_j^1) = \frac{(z_j^1)^\mu}{\theta_1}. \] (84)
Substitute (83) and (84) into (81)
\[ \frac{\mu}{\theta_1} (z_j^1)^\mu = \left[ \frac{p_j - c(e)}{c'(e)} \right] (\sigma - 1) (1 - s_j) \frac{1}{1 - 2\eta} \frac{1 - \xi}{\omega}. \] (85)
But, by definition,
\[ E_j^1 = C_1 (z_j^1) = \frac{(z_j^1)^\mu}{\theta_1}, \]
which gives

\[ E_1^j = \frac{1}{\mu} \left[ \frac{p_j - c(e)}{c'(e)} \right] (\sigma - 1) (1 - s_j) \frac{1}{1 - 2\eta} \frac{1 - \xi}{\omega}. \]  

(86)

The remaining necessary conditions for the optimization of firm \( j \) are

\[ \frac{C_i'(z_j)}{C_i'(z_j)} = \frac{2\eta - 1}{\eta} \frac{1 - \xi}{\xi} \left[ \frac{z_j}{z_j^*} \right]^{x-1}, \]  

(87)

and

\[ -\eta c'(e) q_j - 1 = 0. \]  

(88)

From (88)

\[ c'(e) q_j \frac{p_j}{p_j} = -\frac{1}{\eta}, \]

and at a symmetric equilibrium

\[ pq = \frac{I}{N}, \]

so

\[ c'(e) = -\frac{1}{\eta} \frac{N}{I} p, \]

Using the definition of \( c(e) \)

\[ \gamma \beta e^{-(\gamma + 1)} = \frac{1}{\eta} \frac{N}{I} p \]

which can be solved to give total environmental contribution as a function of \( p \)

\[ e = \left( \frac{\eta \gamma \beta}{N p} \right)^{1/(1+\gamma)}. \]  

(89)

Next, write (86) at the symmetric equilibrium as

\[ E^1 = \frac{1}{\mu} \left[ \frac{p}{c'(e)} - \frac{c(e)}{c'(e)} \right] (\sigma - 1) (1 - s_j) \frac{1}{1 - 2\eta} \frac{1 - \xi}{\omega} \]

\[ = \frac{1}{\mu} \left[ -\frac{\eta I}{N} + \frac{1}{\gamma} \right] (\sigma - 1) (1 - s_j) \frac{1}{1 - 2\eta} \frac{1 - \xi}{\omega}, \]

and use (89) to give the solution

\[ E^1 = \left[ \frac{\eta I}{N} - \frac{1}{\gamma} \left( \frac{\eta \gamma \beta}{N p_j} \right)^{1/(1+\gamma)} \right] \frac{N - 1}{N} \frac{\sigma - 1}{\gamma} \frac{1 - \xi}{\mu \omega}. \]  

(90)

To obtain \( E_j^2 \) use (87) and \( C_i(z_j^*) = \left( \frac{z_j^*}{\theta_1} \right)^{x} \) to give

\[ z_j^2 = z_j \left( \frac{\theta_1}{\theta_2} \frac{2\eta - 1}{\eta} \frac{1 - \xi}{\xi} \right)^{1/x}. \]

Since

\[ E_1^j = C_1(z_j^1), \quad E_2^j = C_2(z_j^2), \]

it follows that

\[ E^2 = \left( \frac{z_j^1}{\theta_1} \frac{\theta_1}{\theta_2} \frac{\theta_1}{\theta_2} \frac{2\eta - 1}{\eta} \frac{1 - \xi}{\xi} \right)^{1/x} \]

\[ = E_1^{1/2} \left( \frac{\theta_1}{\theta_2} \frac{\theta_1}{\theta_2} \frac{2\eta - 1}{\eta} \frac{1 - \xi}{\xi} \right)^{1/x} \]

\[ = \left[ \frac{\eta I}{N} - \frac{1}{\gamma} \left( \frac{\eta \gamma \beta}{N p_j} \right)^{1/(1+\gamma)} \right] \frac{N - 1}{N} \frac{\sigma - 1}{\gamma} \frac{1 - \xi}{\mu \omega} \frac{\theta_1}{\theta_2} \frac{2\eta - 1}{\eta} \frac{1 - \xi}{\xi} \right)^{1/x}. \]  

(91)
Finally, $E_j$ can be obtained from noting that

$$
e = (1 - \eta) \sum_{i=1}^{N} E_i^1 + \eta \sum_{i=1}^{N} (E_i - E_i^1 - E_i^2)$$

$$= (1 - 2\eta) NE^1 + \eta NE - \eta NE^2.$$

Solving

$$E = \frac{e}{\eta N} + E^2 - \frac{1 - 2\eta}{\eta} E^1$$

$$= \frac{1}{\eta N} \left( \frac{\eta I\gamma^2}{Np_j} \right)^{1/(1+\gamma)} + \left[ \frac{\theta_1}{\theta_2} \left( \frac{\theta_1 2\eta - 1 1 - \xi}{\eta - \xi} \right) \pi^p - \frac{1 - 2\eta}{\eta} \right] E^1$$

$$= \frac{1}{\eta N} \left( \frac{\eta I\gamma \beta}{Np_j} \right)^{1/(1+\gamma)}$$

$$+ \left[ \frac{\theta_1}{\theta_2} \left( \frac{\theta_1 2\eta - 1 1 - \xi}{\eta - \xi} \right) \pi^p - \frac{1 - 2\eta}{\eta} \right] \left[ \frac{\eta I}{N} - \frac{1}{\gamma} \left( \frac{\eta I\gamma \beta}{Np_j} \right)^{1/(1+\gamma)} \right] \frac{N - 1 \sigma - 1 1 - \xi}{N - 2\eta - 1 \mu \omega}.$$

(92)

References


