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Abstract

We examine an open economy’s strategy to reduce its carbon emissions by replacing its consumption of coal—very carbon intensive—with gas—less so. Unlike the standard analysis of carbon leakage, unilateral carbon-reduction policies with more than one carbon energy source may turn counter-productive, ultimately increasing world emissions. Thus, we establish testable conditions as to whether a governmental emission-reduction commitment warrants the exploitation of gas, and whether such a strategy increases global emissions. We also characterize the extent to which this unilateral policy makes the rest of the world’s emission commitments more difficult to meet. Finally, we apply our results to the case of the US.

JEL-Codes: Q410, Q580, H730, F180.

Keywords: unilateral climate policy, carbon emission reduction, shale gas, gas-coal substitution, coal exports, carbon leakage, US policy, counter-productive policy.

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I. Introduction

Natural gas is the fossil fuel that releases the least CO2 when burned. Now more than ever, it is hoped that a large replacement of very carbon intensive fuels by shale gas can help reduce carbon emissions and, therefore, significantly mitigate a climate problem labeled “the ultimate commons problem of the twenty-first century” (Stavins, 2011). For example, an increasing number of top CO2 emitting countries that are endowed with substantial shale gas deposits plan to meet their emission reduction commitments by promoting this resource; among them, the US, Russia, China, and the UK. This substitution is mostly manifest in the power generation sector in which electricity can be economically produced from both steam coal and natural gas. In a sufficiently long-run perspective, over which the appropriate infrastructure can be built, gas can virtually replace coal and other traditional fuels for all uses.

The hope that shale gas can play a major role in national climate policy strategies has been substantiated by academic experts—e.g., the MIT report of Jacoby, O’Sullivan, and Paltsev (2011). Not surprisingly, this option is also supported by the industry, which is an evident implementation advantage over traditional climate mitigation strategies.¹

However, two important aspects of the rise of shale gas have raised serious questions about its climate impact. The first—and most obvious—one concerns the net relative contribution of gas to global warming, once the leakage of methane at the production level is taken into account. This first aspect has been addressed in the field of natural sciences and raises specific regulatory challenges.² Although our results will connect with the relative climate impact of gas,³ our analysis deals more directly with the second concerning aspect of the rise of gas: international coal leakage. For example, according

¹BP, BG Group, Eni, Statoil and Total recently declared in a joint letter to media (June 1, 2015): “We urge governments to take decisive action at December’s UN summit. We are also united in believing such action should recognize the vital roles of natural gas and carbon pricing in helping to meet the world’s demand for energy more sustainably.”

²For a synthetic review on this aspect and on the perspective of regulating the leakage of methane due to fracking, see, for example, https://www.scientificamerican.com/article/epa-will-regulate-methane-emissions-from-oil-and-gas-wells/.

³Our application will consider the absence of scientific consensus around this parameter.
to Light, Kolstad, and Rutherford (1999), the international competitive market for coal implies a particularly high leakage potential. By contrast, the transport of gas—in particular, its shipment—is highly more challenging, which explains that gas is still virtually all consumed where it is produced. As a consequence, the domestic replacement of coal by shale gas releases amounts of tradable coal, whose supply meets the foreign energy demand, and, therefore, contributes to increase emissions in the rest of the world. For example, the empirical evidence reported in Section II suggests a relationship between the recent boom of shale gas, the reduction in US CO2 emissions, and the peak in US coal exports. See also the recent projections by Chakravorty, Fischer, and Hubert (2015) on the development of shale gas in China.

There are two main reasons why this problem deserves a particular attention in the context of the current energy landscape. First, in the aftermath of the Paris Climate Agreement, governments will have to rely on unilateral initiatives to meet their respective emission reduction commitments. Indeed, in the light of both the agreement and the preceding COP21 talks, the project of penalizing carbon at the global level in a coordinated manner seems unrealistic.

The second reason motivates our research more specifically: It is that the rise of gas as an intermediate (less carbon containing) energy source fundamentally modifies the analysis of unilateral climate policy. Indeed, with more than one carbon energy source, our results highlight that a large country’s unilateral emission reduction may ultimately increase global carbon emissions if it is achieved by promoting an intermediate source of energy like gas. This theoretical possibility sharply differs from the standard analysis with a single source of carbon, which predicts that leakage cannot exceed 100%. The difference may be explained as follows. With a single source of carbon, any carbon penalty—be it unilateral—causes its total supply to contract; leakage, in that case, reallocates the consumption of a smaller total carbon quantity. With multiple carbon-generating energy sources, things are not so simple: A unilateral carbon penalty not only reduces the total production of the most polluting sources, it may also boost—under some condition
that we establish—the domestic production of intermediate sources like gas to replace the domestic consumption of the former. This boost—under another condition that we establish—may be of such an extent that it more than compensate, at the global level, the carbon reduction due to the global contraction in the most polluting sources; in this case, the total quantity of carbon is increased and, therefore, carbon leakage from the carbon reducing economy is augmented to more than 100% by this economy’s exports of the most carbon intensive energy sources.

To our knowledge, however, there exists no analysis of unilateral emission reduction policies with more than one carbon energy source. This is so despite the fact that the large replacement of coal by gas is a relevant option in several top-emitting regions.

To analyze this new situation, we examine a highly stylized open economy, considering the minimal set of ingredients involved. There are two regions: the home country and the rest of the world. The home country relies on two substitutable carbon energy inputs: coal—more carbon intensive—and gas—less so. By contrast, the rest of the world cannot use the home country’s gas, but may trade coal with the latter. In each region, there is a single representative energy consumer and a single firm representative of the sector supplying carbon energies; their demands and supplies depend on prices only. Policy-induced leakage corresponds to the displacement of carbon emissions from one region to another, which results from changes in demand and supply of the representative consumers and producers. In this setup, we address the question whether the domestic rise of gas can help reduce domestic and global CO2 emissions, and how this rise affects foreign regions’ ability to meet their own carbon emission commitments. Our analysis rests on a static representation of the energy market, in the spirit, for example, of Hoel (1994) and Harstad (2012). In Section VI, however, we explain how our results carry over to a dynamic setting in which energy sources are endogenously developed and extracted

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4 The supply and demand functions of representative producers and consumers can be interpreted as reflecting economic decisions by a continuum of individual agents, each deciding whether to consume or produce an infinitesimal quantity in each region. Our model would need to be modified to reflect in a more realistic fashion leakage induced by agents’ decisions to relocate their activity.
This paper lies at the intersection of mainly two strands of literature. On the one hand, it is complementary with recent papers on the leakage effect that limits the effectiveness of unilateral climate policies—see, among other important contributions, Eichner and Pethig (2011), Ritter and Schopf (2014), and Fischer and Salant (2017). In general, leakage only limits, but does not more than compensate, the effect of the unilateral policy. Indeed, the above studies have focused on the simplifying case in which there is a single polluting energy source. We extend this literature to the case of more than one polluting source, which gives rise to the possibility that leakage exceed 100%, so that a unilateral well-intentioned policy may turn counter-productive. Such a possibility can be interpreted as the leakage counterpart of the “green paradox” (Sinn, 2008).

On the other hand, this article is complementary with the resource economics literature that has dealt with the coexistence of several polluting energy sources—see, among other papers, Chakravorty, Moreaux, and Tidball (2008), Henriët and Schubert (2015), and Coulomb and Henriët (2017), which examine closed economy situations, as when policies are implemented at the world level. We extend this literature to the case in which an open economy implements a climate policy unilaterally.

The rest of the article is structured as follows. In Section II, we discuss an important example: the US climate strategy, the rise of shale gas, and the concomitant peak in US coal exports. Section III presents our model. First, Section IV examines the relationship between a unilateral CO2 reduction commitment and the increase in gas production. Second, it assesses the effect of more gas on world CO2 emissions. Third, it draws implications for the adjustment of climate policy in the rest of the world. The analysis yields testable conditions establishing in which contexts the promotion of natural gas is justified from the perspective of an individual country’s emissions objective and from a global perspective, and the extent to which this promotion undermines the rest of the world’s efforts to meet its own CO2 emission commitment. In Section V, we review existing empirical estimates for the relevant parameters, and we apply our previously
obtained formulas to the case of the US. In Section VI, we discuss two aspects that our
analysis purposely omits: first, the fact that energy sources are actually exploited over
time and, second, the fact that gas is starting to get internationally traded.

II. An Important Example: The US Climate Strategy, the Shale Gas
Boom, and the Peak of US Coal Exports

The issue addressed in this paper is particularly well illustrated by recent developments
in the US climate policy project and in the US energy sector: namely, the rise of shale
gas, the US climate policy plan to rely on gas supply, the replacement of coal by gas in the
US power sector, and the recent peak of US coal exports. These developments, because
of their magnitude, are likely to have an impact on the world energy policy landscape.
Indeed, the US is the second top carbon emitting economy. It is also the most important
gas producer, the second top coal producer and consumer, and, last but not least, the
top coal reserve holder.

Since 2011, the US CO2 emissions have been regulated by the EPA under the Clean Air
Act Federal law. As a matter of fact, the ratification of the Paris Climate Agreement by
President Obama commits—at least for the next four years—the US Federal Government
to a 26 – 28% reduction in CO2 emissions by 2025 with respect to their 2005 level. To
meet this commitment, the previous US Administration’s plan has been to rely on the
rapid development of gas production from the shale resource in the aftermath of the
“fracking” revolution of the early 2000s. For example, in his June 25, 2013 Speech on
Climate Change, President Obama put things this way:

My administration pledged to reduce America’s greenhouse gas emissions
. . . And today, we produce more natural gas than anybody else. So we are
producing energy. And these advances have grown our economy, they have
created new jobs, they can’t be shipped overseas—and, by the way, they have
also helped drive our carbon pollution to its lowest levels in nearly twenty
years. Since 2006, no country on Earth has reduced its total carbon pollution
by as much as the US . . . In fact, many power companies have already begun modernizing their plants, and creating new jobs in the process. Others have shifted to burning cleaner natural gas instead of dirtier fuel sources . . . Today, we use more clean energy . . . which is supporting hundreds of thousands of good jobs. We waste less energy, which saves you money at the pump and in your pocketbooks. And guess what—our economy is 60% bigger than it was twenty years ago, while our carbon emissions are roughly back to where they were twenty years ago.

Two years after this statement, the US policy project of replacing the steam coal input by natural gas in the US power generation sector was strengthened by the proposal of the Clean Power Plan to command this transition.\footnote{Officially, the enforcement of the plan has been temporarily halted by the Supreme Court. Meanwhile, in practice, an increasing number of States—including Republican-held ones—are taking initiatives so as to meet the plan’s requirements.}

Figure 1 shows that the replacement of coal by gas has been effective for the past few years, and that this movement has gone hand in hand with the development of gas. It also indicates that the policy promotion of gas has accelerated this transition in recent years.\footnote{Figure 1 shows yearly consumption. Gas use has notoriously overtaken coal in the US power generation sector for some months in 2015 for the first time in history. See, for example, \url{http://www.nytimes.com/2015/07/14/business/natural-gas-overtakes-coal-in-us-electric-generation.html}.}

Given the large CO2 impact of the US power sector, this coal-gas substitution has indeed contributed—of course, among other factors—to the reduction of the US CO2 emissions (Feng, Davis, Sun, and Hubacek, 2015, and Kotchen and Mansur, 2016). For example, Figure 2 shows the fall in CO2 emissions generated by energy consumption in the US.

As President Obama emphasized in his 2015 speech, the economic success surrounding the gas boom has entailed domestic benefits, despite the reduction in CO2 emissions that it has induced. However, the gas boom and, therefore, its policy promotion, caused—again, among other factors\footnote{For example, in 2011, massive flooding in Australia prevented Australian coal to be delivered to}—large amounts of coal to be released that ultimately met
the foreign demand for cheap energy. Figure 3 shows the peak in net US coal exports that has been concomitant with the replacement of coal by gas in the US power sector.

The above developments are likely to persist under the recently elected US Administration. First, despite President Trump’s plan that the US not be party any longer to the Paris agreement after 2020—and irrespective of the ultimate decision of the next US Administration on this matter—the pressure towards a decrease in US CO2 emissions is likely to be continued by the public actions of US states and by the self-regulation of companies—see, for example, The Economist, June 5, 2017. Second, the rise of natural gas will continue to be publicly supported, as confirmed by President Trump on June 29, 2017, thus accompanying the decreasing trend in US CO2 emissions.

Third, and most importantly, US coal exports are likely to continue increasing. On the one hand, according to specialists, the ongoing replacement of coal by gas will generate a potential for US coal exports to keep rising in the future.8 The realization of this potential has been limited, under the Obama Administration, by the successful opposition to the US coal exports by China, which has been compensated by US coal. Besides, the US coal exports in the past few years have served less distant markets, in South America and Europe.

of environmental groups to the building of new coal-export terminals needed to meet
the growing coal demand in Asia. However, as the newly elected US Administration
notoriously supports the coal industry, it is to be anticipated that these projects will
receive a more favorable regulatory treatment. On the other hand, the rise in US coal
exports has become an objective in itself of the Trump Administration. For example, in
his recent speech at the “Unleashing American Energy Event” on June 29, 2017, President
Trump made the following announcement:

The Department of the Treasury will address barriers to the financing of
highly efficient, overseas coal energy plants. Ukraine already tells us they
need millions and millions of metric tons right now. There are many other
places that need it, too. And we want to sell it to them, and to everyone else
all over the globe who need it.

The perspective of rising US coal exports has caused growing concerns both in the
academic sphere—e.g., Meredith Fowlie’s contribution to the blog of the Energy Institute

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9See, for example, https://www.bloomberg.com/news/articles/2014-11-21/
at Haas, Berkeley,\textsuperscript{10} and Knittel, Metaxoglou and Trindade (2016)—and in the NGO sector, and will continue to do so under the current US administration. This paper seeks to address these concerns. Indeed, existing studies about the coal-gas policy-induced substitution have mostly focused on the changes within the US economy—e.g., Burtraw et al. (2014), Knittel, Metaxoglou, and Trindade (2015), and Cullen and Mansur (2017). A recent interesting addition to this literature is due to Wolak (2016) who presents simulations relating the coal-gas substitution in the US and the global coal market, assuming a zero price-elasticity of the foreign demand for coal. According to very recent elasticity estimations for coal exports and imports by Knittel, Metaxoglou, Soderbery, and Trindade (2017), the latter assumption is justified in a short-term perspective. Both studies imply that US coal exports do not significantly contribute to increase the rest of the world’s CO2 emissions in the short run.

There exists no theoretical analysis that integrates the US policy objective of reducing domestic CO2 emissions, the coal-gas substitution that this objective induces, and the resulting change in coal exports. Our findings are complementary with Wolak (2016) and

\textsuperscript{10}Available at \url{https://energyathaas.wordpress.com/2014/07/28/will-coal-exports-abroad-offset-hard-won-carbon-reductions-at-home/}.  

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Knittel et al. (2017) in several respects. First, they bring up theoretical insights as to the logic of policy-induced CO2 leakage in presence of more-than-one carbon energies. Second, they deliver testable conditions that can be applied not only to the US—as we do in Section V—but also to any gas-rich economy that considers the option of producing more gas to achieve its CO2 reduction commitments. Third, the application of our formulas to the case of the US reinforces Wolak’s (2016) and Knittel et al.’s (2017) conclusions, although, taking a longer-run perspective, we depart from the assumption of a perfectly price-inelastic demand for coal.

III. A Simple Model of an Open Economy Using Coal and Gas

A. Basics

Regions. There are two regions. The domestic open economy of interest will be called “Home,” and variables related to this country will accordingly be denoted by the superscript “H.” The rest of the world will be treated as a single open economy which will be called “Foreign,” and variables related to it will be denoted by the superscript “F.”

Coal supply. In each of the two regions, there is a price-taking representative firm supplying coal. Coal being tradable across regions, competitive markets will establish a single international coal price $p_c$. The Home and Foreign coal supplying firms respectively produce amounts $s^H_c$ and $s^F_c$—expressed in energy units—which are determined by the following supply functions of the coal price $p_c$:

$$s^H_c = S^H_c(p_c)$$

and

$$s^F_c = S^F_c(p_c),$$

which are both assumed non negative, differentiable and strictly increasing for all $p_c \geq 0$.

Gas supply. For simplicity, gas is only produced in the Home country by a price-taking representative firm, which does not export it.\footnote{In Section VI, we explain how the analysis extends to the possibility that gas be exported.} Its production $s^H_g$—expressed in energy

$$s^H_g = S^H_g(p_g),$$
units—is given by the supply function of the domestic price of gas $p_g$

$$s_g^H = s_g^H(p_g),$$

which is assumed non negative, differentiable and strictly increasing for all $p_g \geq 0$.

**Energy demand by the Foreign country.** For simplicity, the rest of the world does only rely on the coal energy: There is a price-taking representative consumer of electric energy in the Foreign country, and electricity is solely produced from coal through a linear technology which, in energy units, is “one-for-one.” Therefore, the Foreign country’s coal consumption $x_c^F$ is determined by the energy demand function

$$x_c^F = D_c^F(p_c),$$

which is assumed non negative, differentiable and strictly decreasing for all $p_c \geq 0$.

**Energy demand by the Home country.** The domestic economy relies on both coal and gas: There is a competitive representative consumer of electric energy in the Home country, and electricity can be produced equivalently from coal or gas through a one-for-one energy transformation technology. Since coal and gas are perfectly substitutable, competitive markets will establish a single final energy price, irrespective of the source of energy. We will denote this final price by $p$. Therefore, the Home country’s consumption $x^H$ of coal and gas is determined by the energy demand function

$$x^H = D^H(p),$$

which is assumed non negative, continuous and strictly decreasing for all $p \geq 0$. The domestic consumption $x^H$ corresponds to the consumption of the domestically produced gas $s_g^H$ and a residual consumption of coal $x_c^H$:

$$x^H = x_c^H + s_g^H.$$ 

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12. Our model is compatible with other energies—e.g., gas or alternative energies—being produced and consumed in the Foreign country. Indeed, the demand function $D_c^F$ may be interpreted as the residual demand for coal after other locally produced energies have been consumed, with no implications on our results.

13. This demand function may be interpreted as the residual energy demand after other—e.g., alternative—energies have been used.
B. Laissez-Faire Equilibrium

By assumption, the energy market is competitive. In this subsection, we assume away any policy intervention. Public policy will be introduced in the next section.

In the sequel, as will be clear shortly below, we will focus on the empirically relevant equilibria in which the Home economy produces electricity from coal and gas at the same time. Since the latter are assumed perfectly substitutable, such interior equilibria are characterized by the following no-arbitrage equality,\(^{14}\) relating the equilibrium domestic final energy price to the equilibrium domestic producer prices of coal and gas:

\[
\tilde{p} = \tilde{p}_c = \tilde{p}_g; \tag{1}
\]

a “\(^{\sim}\)” on top of a variable or function will be used to indicate that this variable or function is evaluated at the market equilibrium.

In this context, the equilibrium price \(\tilde{p}\) is characterized by the balance between energy demand and supply at the world level:

\[
D_H(\tilde{p}) + D_F(\tilde{p}) = S_c^H(\tilde{p}) + S_c^F(\tilde{p}) + S_g^H(\tilde{p}), \tag{2}
\]

where we assume that \(D_H(0) + D_F(0) > S_c^H(0) + S_c^F(0) + S_g^H(0)\), so as to eliminate the uninteresting situation in which there exists no equilibrium with non-zero energy consumption. Since the left-hand side and right-hand side of (2) are respectively strictly decreasing and increasing, \(\tilde{p} > 0\) is uniquely defined.

The equilibrium condition (2) may be written in the following way, highlighting the equality between the (residual) demand for coal by the Home country, on the left-hand

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\(^{14}\)In practice, coal and gas inputs are not perfectly substitutable at the plant level because power plants are typically fuel specific. Instead, they are substitutable at the industry level in a sufficiently-long-term perspective that allows the building of new coal- and gas-fired power plants. That is why, despite the fact that the US power generation sector is currently investing in new coal- and gas-fired plants simultaneously, the equality between the price of coal and the price of gas for an equivalent amount of power is not exactly observed. It is important to note, nevertheless, that the respective costs of using these two fuels have been rapidly converging since 2005, whether or not other operating expenses are integrated—see, e.g., [http://www.eia.gov/electricity/annual/html/epa_08_04.html](http://www.eia.gov/electricity/annual/html/epa_08_04.html). This convergence reflects that the short-run arbitrage between the two substitutable energies tends to vanish in the long run.
side, and the world coal supply net of the rest of the world’s demand, on the right-hand
side:
\[ D^H(\tilde{p}) - S^H_g(\tilde{p}) = S^H_c(\tilde{p}) + S^F_c(\tilde{p}) - D^F(\tilde{p}). \]  

(3)

We make the following assumption:
\[ D^H(\tilde{p}) > S^H_g(\tilde{p}) > 0, \]  

(4)

which implies that there exists a non-zero residual demand \( D^H(\tilde{p}) - S^H_g(\tilde{p}) > 0 \) for coal in the Home country in the equilibrium. Assumption (4) formally validates our earlier-
mentioned focus on situations in which coal and gas are used simultaneously in the Home
country.

The equilibrium price of energy \( \tilde{p} \) defined by (2) determines all other variables: domestic gas production \( \tilde{s}^H_g = S^H_g(\tilde{p}) \); domestic and foreign coal production \( \tilde{s}^H_c = S^H_c(\tilde{p}) \) and \( \tilde{s}^F_c = S^F_c(\tilde{p}) \); domestic electricity consumption from coal and gas \( \tilde{x}^H = D^H(\tilde{p}) \), and, therefore, domestic coal consumption \( \tilde{x}^H_c = \tilde{x}^H - \tilde{s}^H_g \); rest-of-the-world coal consumption \( \tilde{x}^F_c = D^F(\tilde{p}) \).

It follows that the equilibrium also determines the Home country’s net exports of coal, on the left-hand side, which meet the Foreign country’s net imports, on the right-hand
side:
\[ \tilde{s}^H_c - (\tilde{x}^H - \tilde{s}^H_g) = \tilde{s}^F_c - \tilde{x}^F_c, \]  

(5)

net exports or imports may be positive or negative, with no consequence on our qualitative
results.

IV. Domestic CO2 Reduction and Gas Promotion

In this section, we examine a policy aiming to reduce the CO2 emissions that are generated by the use of coal and gas in the Home country.

A. CO2 Emissions

We assume that, per unit of energy, coal consumption and gas consumption generate respectively \( \theta_c \) and \( \theta_g \) units of CO2. We further assume that coal is more CO2 intensive
than gas:

\[ \theta_c \geq \theta_g > 0. \]

Therefore, domestic CO2 emissions amount to

\[ e^H = \theta_c x_c^H + \theta_g s_g^H. \]  

(6)

B. Domestic CO2 Commitment and Implementation

Assume now that the Home country is committed to limit its CO2 emissions \( e^H \), so that it remains below the exogenous cap \( \bar{e}^H \):

\[ \theta_c x_c^H + \theta_g s_g^H \leq \bar{e}^H. \]  

(7)

However this commitment is implemented, it necessarily translates into a penalty for using CO2 that augments the market price of coal and gas, whether this penalty is explicit or implicit.\(^\text{15}\) Consider, for simplicity, that this penalty is explicit: For example, (7) is implemented by a carbon tax or by a competitive market for emission rights, giving rise, in either case, to an explicit CO2 price, which we denote by the variable \( \tau^H \geq 0 \). In equilibrium, this variable is endogenously determined in such a way that the emission commitment is met. If the implementation system is a carbon tax, the domestic government establishes the tax level \( \tilde{\tau}^H \) so that (7) is satisfied. If there is a tradable permit system, \( \tilde{\tau}^H \) is the equilibrium price of a right to emit one CO2 unit out of the quota \( \bar{e}^H \). Obviously, if the cap \( \bar{e}^H \) is soft in the sense that it falls short of equilibrium emissions \( \theta_c \tilde{x}_c^H + \theta_g \tilde{s}_g^H \) realized in absence of policy, as described in the previous section, then \( \tilde{\tau}^H = 0 \), and the analysis of the previous section applies; otherwise, the cap is strong, constraint (7) will be active and \( \tilde{\tau}^H > 0 \).

In turn, the CO2 price \( \tau^H \) amounts to varying taxes on coal and gas, proportional to their CO2 intensity. That is, under the commitment (7), the additional cost of using coal is \( \theta_c \tau^H \) and the additional cost of using gas is \( \theta_g \tau^H \). Therefore, the user prices of

\(^{15}\)In practice, for example in the US, regulatory standards are often used, imposing tighter constraints to the biggest sources of CO2 emissions.
coal and gas in the Home country become respectively $p_c + \theta_c \tau^H$ and $p_g + \theta_g \tau^H$ and the no-arbitrage condition (1) which must prevail in equilibrium should be adjusted as follows:

$$\tilde{p} = \tilde{p}_c + \theta_c \tilde{\tau}^H = \tilde{p}_g + \theta_g \tilde{\tau}^H.$$  \hspace{1cm} (8)

This condition relates the equilibrium producer prices for coal $\tilde{p}_c$ and gas $\tilde{p}_g$ to the equilibrium domestic price for energy $\tilde{p}$ and the equilibrium domestic price of CO2, $\tilde{\tau}^H$.

Therefore, the former producer prices are given by

$$\tilde{p}_c = \tilde{p} - \theta_c \tilde{\tau}^H$$  \hspace{1cm} (9)

and

$$\tilde{p}_g = \tilde{p} - \theta_g \tilde{\tau}^H.$$  \hspace{1cm} (10)

Like in Section III, equilibrium prices must balance supply and demand on the world energy market; using expressions (9) and (10), the equilibrium condition (2) becomes

$$D^H(\tilde{p}) + D^F(\tilde{p} - \theta_c \tilde{\tau}^H) = S^H_c(\tilde{p} - \theta_c \tilde{\tau}^H) + S^E_c(\tilde{p} - \theta_c \tilde{\tau}^H) + S^H_g(\tilde{p} - \theta_g \tilde{\tau}^H).$$ \hspace{1cm} (11)

Equilibrium prices must further satisfy the binding commitment (7):

$$\theta_c \left[ D^H(\tilde{p}) - S^H_g(\tilde{p} - \theta_g \tilde{\tau}^H) \right] + \theta_g S^H_g(\tilde{p} - \theta_g \tilde{\tau}^H) = \bar{e}^H.$$ \hspace{1cm} (12)

It can be verified that the unique solution $(\tilde{p}, \tilde{\tau}^H)$ of the system (11)-(12) determines producer prices $\tilde{p}_c$ and $\tilde{p}_g$ by (9) and (10), and, therefore, all equilibrium quantities. In particular, the equilibrium domestic gas production

$$\tilde{s}_g^H = S^H_g(\tilde{p}_g) = S^H_g(\tilde{p} - \theta_g \tilde{\tau}^H)$$

will be examined in the next subsection.

C. Effect of Domestic CO2 Reduction on Domestic Gas Production

We now turn to the effects of a reduction in the domestic CO2 cap $\bar{e}^H$. In this subsection, we focus on the reaction of the domestic production of gas; the effect on world CO2 emissions will be examined thereafter.
For simplicity, we consider an infinitesimal change \( d\bar{e}^H < 0 \), starting from the equilibrium level of emissions \( \theta_c \tilde{x}^H_c + \theta_g \tilde{s}^H_g \) in absence of emission commitment, as in the previous section. That means that, by assumption, prior to the change in \( \bar{e}^H \), the constraint (7) is not active so that \( \tilde{\tau}^H = 0 \) and the equilibrium is the one characterized in Section III.

Intuition suggests—and Appendix A confirms—that the change \( d\bar{e}^H < 0 \) causes \( \tilde{\tau}^H \) to become strictly positive, thus introducing penalties on both coal and gas used in the Home country. Accordingly, the domestic final price of energy \( \tilde{p} \) increases, so that domestic energy consumption \( \tilde{x}^H \) decreases, as expected.

According to (9) and (10), it follows that the producer prices \( \tilde{p}_c \) and \( \tilde{p}_g \) for coal and gas are each affected in two opposite directions: On the one hand, they are pushed upwards by the rise in the final energy price \( \tilde{p} \), and, on the other hand, they are impacted negatively by the increase in the carbon penalty \( \tilde{\tau}^H \), to the extent of their respective CO2 intensities \( \theta_c \geq \theta_g \). As far as coal is concerned, Appendix A verifies that the reduction in the domestic emission cap systematically induces the producer price \( \tilde{p}_c \) to decrease, thus reducing coal production \( \tilde{s}^H_c \); this intuitive reaction is the same as in standard models with a single polluting energy.

As far as gas is concerned, things are not so simple. Gas is polluting, but less so than coal. For such an intermediate energy, the rise in the final energy price \( \tilde{p} \) may more than compensate the increase in the carbon penalty \( \tilde{\tau}^H \), so that the producer price of gas \( \tilde{p}_g = \tilde{p} - \theta_g \tilde{\tau}^H \) and, therefore, domestic gas production \( \tilde{s}^H_g = S_g(\tilde{p}_g) \), may increase as a result of the emission cap reduction \( d\bar{e}^H < 0 \).

In Appendix A, the analysis of the total differentiation of (11)-(12) with respect to \( \bar{e}^H, \tilde{\tau}^H \) and \( \tilde{p} \) shows the following result.

**Proposition 1 (Effect of domestic CO2 reduction on gas production)** A reduction of CO2 emissions in the Home country warrants a higher production of gas if and only if

\[
\frac{\theta_c - \theta_g}{\theta_g} > \tilde{r}_0 \equiv \frac{\tilde{\xi}^H_c}{\tilde{x}^H_F \tilde{\xi}^H_\xi + \tilde{x}^H_c \tilde{\xi}^H_s}.
\]  

(13)
We have used the following notations: $\xi_{D} \equiv -D^{H}(p)/x^{H} > 0$ is the price elasticity of the domestic energy demand, $\xi_{D} \equiv -D^{F}(p_{c})/x^{F} > 0$, the price elasticity of coal demand in the rest of the world, and $\xi_{S} \equiv S^{H}(p_{c})/s_{c} > 0$ the price elasticity of the world coal supply, where $s_{c} = S_{c}(p_{c}) \equiv S_{c}^{H}(p_{c}) + S_{c}^{F}(p_{c})$ is the world supply of coal.

Proposition 1 provides a testable condition according to which the reduction of domestic emissions in a gas-producing country justifies that more gas be produced. This condition relates, on the one hand, the rate of increase in pollution $(\theta_{c} - \theta_{g})/\theta_{g} \geq 0$ from gas to coal with, on the other hand, demand and supply price elasticities and market shares evaluated in equilibrium.

For any observed elasticities and market shares, the proposition tells that more gas should be produced when coal is sufficiently more CO2 intensive than gas. For example, in the limit case in which gas would tend to be CO2 free $(\theta_{g} \mapsto 0)$, the left-hand side of (13) would tend to be infinitely high, so that the condition would be systematically satisfied. Indeed, in the standard model in which only one of two perfectly substitutable energy sources is polluting, the reduction of pollution commands to increase the production of the non-carbon substitute. Also for example, if coal and gas were equally polluting $(\theta_{c} - \theta_{g} = 0)$, the fact that the right-hand side of (13) is non negative implies that the condition would never be satisfied. Indeed, in this limit case, there would be a single homogeneously polluting energy source with no substitute, requiring that its production be reduced to decrease pollution.

However, for sensible values of CO2 intensities $\theta_{c}$ and $\theta_{g}$, whether condition (13) is satisfied and, therefore, gas production should be increased depends on the properties of the emission-reducing open economy, which are reflected in the right-hand side $\tilde{r}_{0}$ of the condition. The analysis of this term indicates that relying on gas to reduce CO2 emissions is most likely to be justified if $\tilde{x}^{H}$ is low, if $\tilde{x}_{c}^{F}$ and $\tilde{s}_{c}$ are high, and if $\tilde{\xi}_{S} s_{c}$ is low, that is, for small energy-consuming open economies, in a world in which coal has a large market share, especially in the rest of the world, and in which the price of coal will have little impact on coal supply. Therefore, this formula may be satisfied for some
gas-producing countries and not for others, implying different policy recommendations about the promotion of gas.

For example, in Section V, we will examine how Proposition 1 applies to the case of the US.

D. Effect of Domestic CO2 Reduction on Coal Exports and World Emissions

We now examine the impact of reducing the domestic CO2 cap \( \bar{e}^H \) on world CO2 emissions. In our model, total CO2 emissions \( e^W \) not only consist of the domestic emissions \( e^H \) defined in (6), but also of emissions \( e^F \) released by the rest of the world:

\[
e^W = e^H + e^F. \tag{14}
\]

By assumption, the former are set to the binding limit \( e^H = \bar{e}^H \) as per (7) and are, therefore, reduced accordingly. At the same time, the Foreign country’s use of coal releases equilibrium CO2 emissions

\[
\tilde{e}^F = \theta_c \tilde{x}_c^F, \tag{15}
\]

where \( \tilde{x}_c^F = D^F(\tilde{p}_c) \).

Were the foreign coal demand \( D^F \) perfectly price inelastic, as it might be in the short run, the rest of the world’s CO2 emissions would never increase.\(^\text{16}\) In a medium- to long-term perspective over which coal demand becomes elastic, however, emissions \( \tilde{e}^F \) are systematically increased as a result of the domestic CO2 reduction.\(^\text{17}\) In this context, as mentioned above—and shown in Appendix A—the domestic CO2 reduction policy necessarily reduces the producer price \( \tilde{p}_c \) of the most carbon intensive coal energy, inducing a rise in the equilibrium use of coal \( \tilde{x}_c^F \) in the rest of the world. This is so despite the fact that the decreased coal producer price \( \tilde{p}_c \) induces a reduction in coal production

\(^{16}\)For example, this is the assumption that Wolak (2016) makes and that Knittel et al.’s (2017) short-run estimation supports.

\(^{17}\)Although coal demand in the Foreign country can be interpreted as the residual demand for coal after some other local energies have been used, our simplifying formulation implies that CO2 emissions from these other energies are omitted, as if they were all non-carbon energies. Taking into account the CO2 emissions generated by other carbon energies in the Foreign country would slightly modify the model, with no implications on our qualitative results.
\( \tilde{s}_c^H \) and \( \tilde{s}_c^F \) in both regions. This is the effect highlighted by the standard leakage analysis focusing on a single carbon energy source. It follows that the net coal imports \( \tilde{s}_c^F - \tilde{x}_c^F \) of the Foreign country, and, by (5), the net coal exports of the Home country in direction of the rest of the world, increase systematically as a result of the domestic CO2 emission reduction. This stresses the central role of the latter, identified in Section II in the case of the US.

Therefore, the policy-induced reduction in domestic CO2 emissions is, at least partly, compensated by the increase in emissions in the rest of the world due to increased domestic coal exports. In fact, our next result indicates that this compensation may more than offset the domestic CO2 reduction, ultimately causing world CO2 emissions to rise. In other words, unlike the standard leakage analysis with a single carbon energy source, the rate of CO2 leakage associated with the domestic CO2 reduction—see Appendix B for details—

\[
\frac{d\tilde{e}^F}{d\tilde{e}^H} = \frac{\tilde{\xi}_{DH} \tilde{s}_c^H \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) + \tilde{\xi}_{DF} \tilde{s}_c^F \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right)}{\tilde{\xi}_{DH} \tilde{s}_c^H \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) + \tilde{\xi}_{DF} \tilde{s}_c^F \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right)}
\]

may exceed 100%. Accordingly, in this subsection, we address the question of the effectiveness of the domestic unilateral CO2 reduction policy: Under which circumstances does this policy remain less than compensated by the concomitant increase in emissions in the rest of the world? In other words, under which circumstances does the emission leakage rate remain less than 100%?

In Appendix B, the analysis of the total differentiation of world emissions \( \tilde{e}^W = \tilde{e}^H + \tilde{e}^F \) and of the leakage rate (16) with respect to \( \tilde{e}^H \) shows the following result.

**Proposition 2 (Leakage from domestic CO2 reduction and effect on world CO2)**

A reduction of CO2 emissions in the Home country effectively contributes to reduce world CO2 emissions—i.e., the leakage rate is less than 100%—if and only if

\[
\frac{\theta_c - \theta_g}{\theta_g} < \frac{\tilde{\xi}_{DH} \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right)}{\tilde{\xi}_{DF} \tilde{s}_c^F + \tilde{\xi}_{DH} \tilde{s}_c^H + \tilde{\xi}_{DF} \tilde{s}_c^H + \tilde{\xi}_{DH} \tilde{s}_c^F} \left[ 1 + \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \tilde{\xi}_{Sc} \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \tilde{\xi}_{Sc} \left( \frac{\theta_c - \theta_g}{\theta_g} + 1 \right) \tilde{\xi}_{DH} \right].
\]
Proposition 2 provides a testable condition according to which the reduction of domestic emissions in a gas-producing country effectively contributes to reduce world emissions. Like (13), condition (17) relates, on the one hand, the rate of increase in pollution \((\theta_c - \theta_g)/\theta_g \geq 0\) from gas to coal with, on the other hand, demand and supply price elasticities and market shares.

Comparing (17) with (13), one can immediately see that their left-hand sides are identical, equal to the rate of increase in pollution from gas to coal. The comparison further reveals that the first fraction on their right-hand sides are similar, equal to \(\tilde{r}_0\) as defined in (13). Moreover, by definition of the elasticity variables, the term between brackets on the right-hand side of (17) happens to be more than one—rather than equal to one in (13).

Condition (17), therefore, can only be violated if condition (13) is satisfied: It means that the domestic CO2 reduction policy may only be counter-productive—and the leakage rate be more than 100%—if it is accompanied by a development of gas as per Proposition 1. In particular, in the extreme situation in which coal supply is perfectly inelastic, as when \(\tilde{\xi}_{sc} = 0\), the right-hand sides of (17) and (13) become identical to each other, so that the two conditions are complementary. In this example, obviously, a domestic CO2 reduction policy accompanied by a development of gas does not induce a reduction of coal use at all, thus systematically leading to a more-than-100% leakage rate. Otherwise, when the domestic CO2 reduction justifies to limit both coal and gas use in the Home country, world emissions will never increase.

Unlike condition (13), the rate of pollution increase from gas to coal \((\theta_c - \theta_g)/\theta_g\) is in general not only involved in the left-hand side of (13) but also in its right-hand side. For any observed equilibrium elasticities and market shares, for which values \((\theta_c - \theta_g)/\theta_g\) is inequality (17) satisfied, the leakage rate (16) less than 100%, and, therefore, the domestic CO2 reduction policy effective? To start with, we examine the two limit cases. First, one can easily verify that if gas tended to be CO2 free \((\theta_g \rightarrow 0)\), the right-hand side of (17) would tend to infinity more rapidly than its left-hand side. In this case, therefore, the
condition would always be satisfied so that the domestic CO2 reduction would always lead to less CO2 at the world level, like in the standard model in which only one energy source is polluting. Second, if gas and coal were equally CO2 intensive \((\theta_c - \theta_g = 0)\), the left-hand side would be zero and would always be strictly less than the right-hand non-negative side. Therefore, in this case, the condition would also be systematically satisfied so that the domestic reduction policy would be effective, like in the standard model in which there is a homogenous carbon energy source.

For intermediate values of the rate of pollution increase from gas to coal, nevertheless, the domestic CO2 reduction may be more than compensated by a more-than-100% leakage rate. More precisely, Appendix B shows that condition (17) is satisfied if and only if the function of \(r = (\theta_c - \theta_g)/\theta_g\)

\[
P(r) = S_c'(\hat{p}_c) \left(1 - \frac{D_{gH}^{H'}(\hat{p})}{S_{gH}^{H'}(\hat{p}_g)}\right)r^2 + \left(D_{g}^{F'}(\hat{p}_c) - \frac{S_c'(\hat{p}_c)D_{gH}^{H'}(\hat{p})}{S_{gH}^{H'}(\hat{p}_g)}\right) r - D_{gH}^{H'}(\hat{p}) \left(1 + \frac{S_c'(\hat{p}_c)}{S_{gH}^{H'}(\hat{p}_g)}\right)
\]

is strictly positive. This function is a polynomial of degree two, which is represented by the U-shaped curve of Figure 4. Its analysis in Appendix B yields the following corollary of Proposition 2.

**Corollary 1 (Counter-productive domestic CO2 reduction)** A reduction of CO2 emissions in the Home country induces a rise in world CO2 emissions—i.e., the leakage rate is more than 100%—

1. Only if

\[
\left(\frac{\bar{x}_F}{\bar{x}_H}\right)^2 - 4\left(\frac{s_c}{\bar{x}_H}\xi_{S_c} + \frac{s_g}{\bar{x}_H}\xi_{S_g} + \frac{\bar{x}_F}{\bar{x}_H}\xi_{DF} + \xi_{DH}\right)\frac{\xi_{DH}\xi_{S_c} s_c}{\xi_{S_g} s_g} > 0,
\]

which guarantees that function \(P(r)\) in (18) admits two real roots \(\underline{\bar{r}} < \bar{r}\);

2. And, provided that (19) is satisfied, if and only if,

\[
\bar{\bar{r}} < \frac{\theta_c - \theta_g}{\theta_g} < \bar{r}.
\]
Corollary 1 helps summarize the conditions under which the domestic CO2 reduction policy happens to be counter-productive.

E. Summary

Assuming that condition (19) is satisfied, Corollary 1 tells that the domestic CO2 reduction policy turns out to be counter-productive when the rate of pollution increase from gas to coal \((\frac{\theta_c - \theta_g}{\theta_g})\) takes intermediate values, in between the two thresholds \(\tilde{r}_0\) and \(\bar{r}\) represented in Figure 4. At the same time, the analysis of Proposition 2 already revealed that the domestic policy may only be counter-productive when the policy warrants the production of more gas, implying that \(\tilde{r}_0 \leq \bar{r}\).

![Figure 4: Domestic CO2 reduction policy, occurrence of gas boom and increase in world CO2 emissions](image)

To sum up, for low values \((\frac{\theta_c - \theta_g}{\theta_g}) \leq \tilde{r}_0\), as when coal and gas are not so different as far their CO2 intensity is concerned, the domestic CO2 reduction objective does not
call for a rise in gas production. For sufficiently high values $\bar{r}_0 < (\theta_c - \theta_g)/\theta_g$, as when gas is significantly less CO2 intensive than coal, the domestic CO2 reduction policy does warrant that more gas be produced. Despite the fact that the promotion of gas induces more coal to be exported from the Home country to the rest of the world, this does not necessarily mean that world emissions are increased. In fact, only for intermediate values $\bar{r}_0 \leq \bar{\bar{r}} < (\theta_c - \theta_g)/\theta_g$, if any, the domestic policy is counter-productive, inducing ultimately more CO2 emissions at the world level. For $\bar{\bar{r}} \leq (\theta_c - \theta_g)/\theta_g$, as when gas has a sufficiently low carbon intensity, the policy does command more gas to be produced and more coal to be exported, yet ultimately contributing to reduce world CO2 emissions.

Last but not least, these various possibilities do not only depend on the rate of pollution increase $(\theta_c - \theta_g)/\theta_g$, but also on the values of the thresholds $\bar{r}_0$, $\bar{\bar{r}}$ and $\bar{\bar{r}}$, which all reflect the observed equilibrium characteristics of the gas-rich Home country committed to reduce its CO2 emissions. This motivates, for example, the application of Section V to the case of the US.

F. CO2 Commitment and Implementation in the Rest of the World

We have hitherto considered that the rest of the world was not committed to any CO2 limitation when examining the domestic CO2 reduction policy. In that case, we have established the conditions under which this policy increases excessively the emissions of the Foreign country so that it may become counter-productive at the world level. In fact, in the aftermath of the Paris agreement, it is interesting to examine the case in which the rest of the world is also committed to a CO2 emission cap. That is what we do in this subsection: We assume that the Foreign country’s CO2 emissions are limited to the exogenous level $\bar{e}^F$. With our simplifying assumption that the Foreign country only relies on the coal energy, that means

$$\theta_c \bar{x}_c^F = \bar{e}^F. \tag{21}$$

In a way similar to the Home country, consider that this limitation is implemented by means of an explicit carbon price $\bar{\tau}^F > 0$, whether it is a carbon tax or the price
of carbon permits. It implies a carbon penalty \( \theta_c \tau^F \) on the Foreign country’s use of coal. Accordingly, the coal consumer price in the rest of the world should be adjusted to become, instead of \( \tilde{p}_c = \tilde{p} - \theta_c \tilde{\tau}^H \) as per equation (9),

\[
\tilde{p}_c = \tilde{p} - \theta_c \tilde{\tau}^H + \theta_c \tilde{\tau}^F,
\] (22)

where, following our previous formulation, \( \tilde{p} - \theta_c \tilde{\tau}^H \) remains the international price of the coal energy. Consequently, the world energy balance condition (11) should be adjusted to become

\[
D^H(\tilde{p}) + D^F(\tilde{p} - \theta_c \tilde{\tau}^H + \theta_c \tilde{\tau}^F) = S^H_c(\tilde{p} - \theta_c \tilde{\tau}^H) + S^F_c(\tilde{p} - \theta_c \tilde{\tau}^H) + S^H_g(\tilde{p} - \theta_g \tilde{\tau}^H). \] (23)

In the context of this subsection, compared with the previous setting in absence of CO2 emission cap in the rest of the world, equilibrium prices \( \tilde{p}, \tilde{\tau}^H \) and \( \tilde{\tau}^F \) are determined so as to satisfy the new world energy market equilibrium condition (23), the Home country’s commitment (12), as well as the new Foreign country’s commitment

\[
\theta_c D^F(\tilde{p} - \theta_c \tilde{\tau}^H + \theta_c \tilde{\tau}^F) = \bar{e}^F. \] (24)

In this new setting, Appendix C shows that the domestic CO2 reduction policy still induces a lower coal price \( \tilde{p}_c \) and more coal exports \( \tilde{x}^H_c - \tilde{x}^H_c \) from the Home country to the rest of the world. Although, by assumption, CO2 emissions in the latter are not increased, the carbon equilibrium penalty \( \tilde{\tau}^F \) should be raised to ensure that the cap (21) is satisfied: It means that the domestic CO2 reduction policy makes it more difficult for the rest of the world to meet its own commitment.

The following proposition—proved in Appendix C—establishes the extent to which the rest of the world should increase its carbon price in response to the domestic CO2 reduction policy, so as to meet its emission limit \( \bar{e}^F \).
Proposition 3 (Domestic CO2 reduction and policy in the rest of the world)

In the face of a reduction of CO2 emissions in the Home country, satisfying the Foreign country’s CO2 commitment requires that the latter raises its carbon penalty relatively to the Home country’s one to an extent given by

\[
\frac{d\tau^F}{d\tau^H} = \frac{\xi_{D^H} + \left(\frac{\theta_c - \theta_g}{\theta_c}\right) \frac{\tau^H}{\tau^H} \xi_g}{\xi_{D^H} + \frac{\tau^H}{\tau^H} \xi_c + \frac{\tau^H}{\tau^H} \xi_g}.
\] (25)

Interestingly, expression (25) of the relative carbon-price rise in the rest of the world is increasing with the rate \(\frac{\theta_c - \theta_g}{\theta_c}\), all other things being equal. This suggests that, in reaction to the domestic CO2 reduction policy, the rest of the world should increase its carbon penalty even more when gas is less CO2 intensive relative to coal.

V. Numerical Application to the US

Section II stressed the relevance of the US example by documenting the following developments: The boom of gas production in the US has supported the recent decrease in US CO2 emissions, and has been concomitant with a peak in coal exports. The predictions of our model are in line with these developments.

In this section, we apply the theoretical results of Section IV to the case of the US: Do our results justify the promotion of gas in the US as a means to reduce domestic CO2 emissions? Does this strategy effectively contribute to reduce CO2 emissions at the world level? To what extent does this strategy make the rest of world’s CO2 emission commitment difficult to achieve?

For this application, we use sensible approximations of market shares, as well as empirical estimates collected from the existing literature on energy demand and supply elasticities.

A. Empirical Estimates of Parameters and Equilibrium Values

Coal and gas relative CO2 intensity. Following the Intergovernmental Panel on Climate Change (IPCC, 2014, Annex 3, Table A.3.2), the relative CO2 pollution intensity
of coal is approximately \( \theta_c/\theta_g = 2 \), implying that the rate in pollution increase from gas to coal is \((\theta_c - \theta_g)/\theta_g = 1 \). Although the use of this ratio is standard, it is also controversial for mainly two reasons. One is the heterogeneity of the coal resource as far as its carbon content is concerned. Another one, already mentioned in the Introduction, is that gas does not only contribute to climate change by releasing CO2 when burnt but also by potentially releasing methane when extracted—see, e.g., Howarth et al. (2011). To take this controversy into account, we will examine how our application is sensitive to changes in \((\theta_c - \theta_g)/\theta_g \).

**Market shares.** Data from the US Energy Information Administration (EIA)\(^\text{18}\) suggest the following approximation: Were the current world production/consumption of coal and the US gas production/consumption normalized to \( \tilde{s}_c + \tilde{s}_g^H = 8 \) units of energy, it would be decomposed as \( \tilde{s}_c = 7 \) units of coal production, \( \tilde{s}_g^H = 1 \) unit of US gas production, and \( \tilde{x}_g^H = 2 \) units of US energy consumption. It follows that \( \tilde{x}_c^F = 6 \) units of the \( \tilde{s}_c = 7 \) units of world coal production would be consumed in the rest of the world, while the US coal consumption would be of \( \tilde{x}_c^H = 1 \) unit. Also, the US energy consumption \( \tilde{x}_g^H = 2 \) would consist of about \( \tilde{x}_c^H = 1 \) unit of coal consumption and \( \tilde{x}_g^H = 1 \) unit of gas consumption.

**US electricity demand price elasticity.** Various studies estimate the price elasticity of the demand for electricity in the US. Maddala et al. (1997) focus on the residential demand; their average estimates across 49 US States are 0.16 and 0.24 for the short and long run respectively. These orders of magnitude are confirmed by Garcia-Cerrutti (2000), and Bernstein and Griffin (2006). The former studies the residential sector in Californian counties and finds mean elasticity estimates of 0.17 for the short run and 0.19 for the long run. The latter find 0.24 and 0.32 for the short and long run respectively. For the US commercial sector, Paul et al. (2009) find average price elasticities of the electricity demand of 0.11 and 0.29 in the short and long run. In the industrial sector, their estimates are 0.16 in the short run and 0.4 in the long run. Recently, Deryugina et

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\(^\text{18}\)The data used here are available at [https://www.eia.gov/electricity/data.php#consumption](https://www.eia.gov/electricity/data.php#consumption) and [https://www.eia.gov/outlooks/ieo/coal.php](https://www.eia.gov/outlooks/ieo/coal.php).
al. (2017) find the one-year average price elasticity to be 0.14 and the three-year price elasticity to be 0.29 in the residential and small commercial sector.

For their recent simulation, Chakravorty et al. (2015) assume a 0.3 price elasticity of the US final demand for energy, which is in line with the above long-run estimates.

In our model, price elasticities are medium-run responses, i.e., evaluated over periods of time that allow the replacement of coal-fired power stations by gas-fired ones. In reality, the elasticity of the demand for coal and gas induced by the demand for the electricity produced from these energies may differ from the elasticity of the final electricity demand because there are other, alternative ways of producing electricity. However, alternative sources play a minor role in electricity generation.

Therefore, our numerical application will assume the intermediate value of 0.2 for the price elasticity $\tilde{\xi}_{DH}$ of the US demand of coal and gas for electricity generation purposes. **Non-US coal demand price elasticity.** The non-US demand for coal—especially in the top coal-consuming Chinese economy—is often considered to be very inelastic in the short run. This assumption has been recently questioned by Burke and Liao (2015). They estimate the price elasticity of the demand for coal in China using a panel of province-level data over the 1998-2012 period. They find a range 0.3 to 0.7 when responses are considered over a two years period of time.

For their simulation, Chakravorty et al. (2015) assume a price elasticity of the energy demand of 0.4 for the industrial sector and of 0.5 for the commercial and residential sectors.

Accordingly, our numerical application will assume the intermediate value of 0.5 for the price elasticity $\tilde{\xi}_{DF}$ of the demand for coal in the rest of the world. **Coal and gas supply price elasticity.** The price elasticity of fossil fuels’ supply is usually low, even in the long run; it reflects the scarcity of economically exploitable resources. As far as natural gas is concerned, Brown and Krupnick’s (2010) estimates of the long-run price elasticity of supply range from 0.9 to 1.4. The empirical literature on the price elasticity of coal supply is characterized by a large variety of estimates, ranging
from 0.1 to 7.9—see, e.g., Labys et al. (1979), Beck et al. (1991), Dahl and Duggan (1996), Light (1999), Light et al. (1999), and Truby and Paulus (2012).

In our numerical application, we proceed in two basic steps. First, we assume the sensible, but arbitrary, value of 1 for both the price elasticity $\tilde{\xi}_S$ of coal supply and the price elasticity $\tilde{\xi}_S$ of gas supply. Second, we examine how our results are sensitive to changes in these two elasticities.

B. Application Results and Sensitivity Analyses

**CO2 reduction policy in the US and domestic gas production.** According to Proposition 1, condition (13) tells whether the Home country CO2 reduction commitment justifies a rise in gas production. In the case of the US, the values given by the previous subsection yields $\tilde{\eta}_0 = 0.04$ for left-hand-side threshold of condition (13), which largely falls short of the value of 1 for the rate of pollution increase $(\theta_c - \theta_g)/\theta_g$. This application of Proposition 1, therefore, suggests that a reduction of CO2 emissions in the US does warrant that this reduction be met by increasing the US production of gas.

**CO2 reduction policy in the US and world CO2 emissions.** Proposition 2 and its analysis point out that a rise in US gas induced by a reduction of CO2 emissions in the US may be accompanied by a more-than-100% leakage rate causing an ultimate increase in world CO2 emissions.

With the above chosen values, however, we find that the value in (19) is negative, implying by Corollary 1 that a US CO2 reduction reached by means of a domestic rise in gas cannot induce world CO2 emissions to increase, irrespective of the rate $(\theta_c - \theta_g)/\theta_c$ of pollution increase from gas to coal. That means that, in the case of the US with our chosen values, function (18) has no real roots, so that, in Figure 4, the U-shaped curve lies above the horizontal axis. Accordingly, the associated leakage rate as expressed in (16) takes the value of

$$\frac{d\tilde{e}_F}{d\tilde{e}_H} = 41\%.$$  

This number, although relatively high, significantly falls short of the 100% counter-
effectiveness threshold. This result, nevertheless, happens to change dramatically when other values of supply elasticities are considered.

**Variations in supply price elasticities.** The leakage rate of 41% just obtained is highly sensitive to small changes in the price elasticities $\tilde{\xi}_S^c$ and $\tilde{\xi}_S^g$ of coal and gas supply. This is illustrated by the iso-leakage-rate curves of Figure 5 in the coal- and gas-supply elasticities’ space. In particular, Figure 5 shows that the leakage rate may exceed 100% for elasticity values that fall into the range of values that are admitted by the empirical literature, as, for example, with $\tilde{\xi}_S^c = 0.1$ and $\tilde{\xi}_S^g = 1$.

![Figure 5: Curves of iso-leakage-rate and supply elasticities](image)

**Variations in relative CO2 intensities.** By contrast, Figure 6 shows that sensible changes in the rate of pollution increase $(\theta_c - \theta_g)/\theta_g$ around the standard—but controversial—value of 1 does not modify significantly the rate of leakage. For example,
the leakage rate is maximum at 42% when coal is 62% more polluting than gas.

\[ \frac{\theta_c - \theta_g}{\theta_g} \]

Figure 6: Leakage rate and the rate of increase of pollution from gas to coal

At the same time, Figure 6 shows that the often-made simplifying assumption of a single polluting energy may lead to importantly underestimate the leakage rate. For example, if gas were as polluting as coal—i.e., \( \frac{\theta_c - \theta_g}{\theta_g} = 0 \)—as if there were no intermediate energy, the obtained leakage rate would be of 27%, rather than 41%.

**CO2 reduction policy in the US and policy in the rest of the world.** According to Proposition 3, formula (25) indicates the relative carbon penalty increase that the rest of the world must implement to ensure that its CO2 commitment remains satisfied in the face of the domestic CO2 reduction. With the values chosen in the previous subsection, the application of this formula tells us that, were the US policy raising the US price of carbon by $10, the rest of the world should react by raising its carbon price by $1.7. With a single resource \( \theta_g = \theta_c \)—as when the presence of gas as an intermediate energy is ignored—the same increase in the US price of carbon would be offset by a rise in rest-of-the-world carbon price by only $0.48.
VI. Concluding Remarks

Our analysis stresses that, with an intermediate carbon energy source like gas, a well-intentioned unilateral CO2 reduction policy may be more than offset by a more-than-100% leakage rate, making the policy counter-productive at the world level. This sharply contrasts with the standard analysis of unilateral policies with a single carbon energy source, in which the leakage rate is always less than 100%. In this new context, an examination was needed of the circumstances under which a unilateral policy relying on gas turns counter-productive. We have established simple and testable conditions (i) under which a domestic CO2 reduction warrants that gas production be increased and (ii) under which such an increase effectively helps reduce CO2 emissions at the world level.

Our results look simple. However, they are new and they shed light on a currently important policy option. Indeed, in the aftermath of the Paris Climate Agreement, countries will rely on unilateral initiatives to meet their CO2 reduction targets, and, in this context, a number of large gas-rich economies hope to do so by increasing their gas production.

Our formulas can be applied to any such gas-rich region to help evaluate whether the option of relying on gas effectively contributes to reducing CO2 emissions at the world level. For example, our application to the most important US case suggests that the rise of gas in the US is not only warranted from the perspective of the national current CO2 commitment, but also from the perspective of a need to reduce CO2 emissions at the world level. This finding confirms and consolidates the conclusion of Wolak (2016) and Knittel et al. (2017), obtained under the extreme assumption that the foreign demand for coal is perfectly inelastic. At the same time, our application identifies the central role of coal- and gas-supply elasticities, which highly affect the impact of the US policy; therefore, it calls for further empirical research to estimate more precisely these elasticities.

For this analysis, we have purposely used a highly stylized setting, focusing on the
aspects that seemed to be the most fundamental ones: an open-economy relying on carbon-generating coal and gas, using its gas domestically and trading coal with the rest of the world. Consequently, our results have been obtained under simplifying conditions and one may question whether they survive more complex settings. As already explained in the main text, the extension of the model to the case in which other energies can be used to produce electricity—including foreign gas in the rest of the world—is straightforward once energy demand functions are reinterpreted as residual demands after other energies have been used. Two other aspects are omitted in our analysis, however, which deserve further discussion.

A. Dynamic Coal and Gas Supplies

First, coal and gas energy sources are fossil fuels that are produced over time. One extension of our analysis to the case in which coal and gas supply is dynamic is straightforward. Assume that both energy sources are costlessly produced over some time horizon by Hotelling-style (Hotelling, 1931) competitive sectors seeking to maximize long-term profits. Consider that these sectors develop exploitable reserves prior to extracting them at some convex exploration and development costs, in the manner first proposed by Gaudet and Lasserre (1988). In this case, it can easily be verified that the formulation of the model in terms of cumulative quantities over the time horizon is isomorphic to the static model of Sections III and IV; the analysis of a reduction in long-term total emissions in the Home country yields the same results as Propositions 1, 2 and 3, and Corollary 1. The formulas would only differ by the notion of supply elasticities involved, which would emerge as elasticities of the long-run production of reserves, rather than static supply elasticities; this difference highlights that the elasticity notion that is relevant for our analysis should reflect sufficiently long-run supply responses.

B. International Gas Trade

The second aspect that needs to be discussed is the possibility that natural gas be traded. As mentioned in the Introduction, the international trade of gas is highly challenging
in comparison with coal. However, the former is progressively becoming reality. For example, following a wave of investment in Liquefied Natural Gas export terminals, the US has shipped natural gas since February last year.

In fact, our analysis extends in a relatively straightforward way to the possibility that the rest of the world may import some gas produced in the Home country. There are two basic cases.

First, assume that coal and gas are perfectly substitutable not only in the Home country but also in the Foreign region. There are two possibilities. If coal and gas are used simultaneously in the Home country, as in our main analysis, the no-arbitrage condition \( \tilde{p}_c + \theta_c \tilde{\tau}^H = \tilde{p}_g + \theta_g \tilde{\tau}^H \) prevails as per (8), and implies that \( \tilde{p}_c < \tilde{p}_g \). In this case, no gas will ultimately be used in the rest of the world and the analysis of the main text applies. If instead coal and gas are used simultaneously in the rest of the world, the counterpart of (8) for the Foreign country \( \tilde{p}_c = \tilde{p}_g \) must hold, contradicting (8): \( \tilde{p}_c + \theta_c \tilde{\tau}^H > \tilde{p}_g + \theta_g \tilde{\tau}^H \). In this case, no coal is used in the Home country, and the analysis reduces to the standard leakage model in which a unilateral CO2 reduction never increases world CO2 emissions.

Second, assume that coal and gas are imperfect substitutes in the rest of the world. For simplicity, consider that there is an independent demand for gas \( D^F_g(p_g) \) in the Foreign country. This allows gas to be used at the same time as coal not only in the Home country but also in the rest of world. In this case, our analysis survives, provided that the domestic supply of gas is reinterpreted as a residual supply, after the rest-of-the-world demand has been served.

To conclude, whether it is internationally traded or not, a carbon intermediate energy source like gas, used at the same time as another more carbon intensive energy source like coal, retains its central role which modifies both the qualitative and quantitative analysis of unilateral climate policies and international leakage.
REFERENCES


APPENDICES

A Proof of Proposition 1

In order to alleviate notations, variables' functions will be omitted throughout the following appendices, as long as it does not cause ambiguity.

Final energy price and domestic CO2 price

Totally differentiating the system (11)-(12) with respect to $\tilde{e}^H$, $\tilde{p}$ and $\tilde{\tau}^H$ yields two linear equations which can be written as the following matrix equation:

$$
\begin{pmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{pmatrix}
\begin{pmatrix}
d\tilde{p} \\
d\tilde{\tau}^H
\end{pmatrix} =
\begin{pmatrix}
0 \\
1
\end{pmatrix} d\tilde{e}^H,
$$

where, using the simplifying notation $S_c \equiv S_c^H + S_c^F$,

$$
\begin{align*}
A_1 &= D^{Ht} + D^{Ft} - (S_c' + S_g^{Ht}) < 0, \\
A_2 &= -\theta_c D^{Ft} + \theta_c S_c' + \theta_g S_g^{Ht} > 0, \\
A_3 &= \theta_c D^{Ht} - (\theta_c - \theta_g) S_g^{Ht} < 0, \\
A_4 &= \theta_g (\theta_c - \theta_g) S_g^{Ht} > 0.
\end{align*}
$$

(A.1)

The signs of terms $A_1$, $A_2$, $A_3$ and $A_4$ follow from our assumptions.

Inverting the matrix, one obtains

$$
\begin{pmatrix}
d\tilde{p} \\
d\tilde{\tau}^H
\end{pmatrix} =
\frac{1}{A_1 A_4 - A_2 A_3}
\begin{pmatrix}
A_4 & -A_2 \\
-A_3 & A_1
\end{pmatrix}
\begin{pmatrix}
0 \\
1
\end{pmatrix} d\tilde{e}^H
$$

or, equivalently,

$$
\begin{pmatrix}
d\tilde{p} \\
d\tilde{\tau}^H
\end{pmatrix} =
\frac{1}{A_1 A_4 - A_2 A_3}
\begin{pmatrix}
-A_2 \\
A_1
\end{pmatrix} d\tilde{e}^H.
$$

(A.2)

By (A.1), $A_1 < 0$, $-A_2 < 0$, while $A_1 A_4 - A_2 A_3$ can easily be reduced as follows and, therefore, shown to be positive:

$$
A_1 A_4 - A_2 A_3 = -\theta_g^2 D^{Ht} S_g^{Ht} + (\theta_c - \theta_g)^2 S_g^{Ht} (S_c' - D^{Ft}) - \theta_c^2 D^{Ht} (S_c' - D^{Ft}) > 0. 
$$

(A.3)

One can conclude

$$
\frac{d\tilde{p}}{d\tilde{e}^H} < 0 \quad \text{and} \quad \frac{d\tilde{\tau}^H}{d\tilde{e}^H} < 0.
$$

(A.4)

Coal price on the international market

By (9), the equilibrium coal price is $\tilde{p}_c = \tilde{p} - \theta_c \tilde{\tau}^H$. Differentiating this equation with respect to $\tilde{e}^H$ and using (A.2), one obtains:

$$
\frac{d\tilde{p}_c}{d\tilde{e}^H} = -\frac{1}{A_1 A_4 - A_2 A_3} (A_2 + \theta_c A_1),
$$

where $A_1 A_4 - A_2 A_3 > 0$ by (A.3), and where, by (A.1), $A_2 + \theta_c A_1$ can easily be shown to be negative:

$$
A_2 + \theta_c A_1 = \theta_c D^{Ht} - (\theta_c - \theta_g) S_g^{Ht} < 0.
$$
One can conclude
\[ \frac{d\tilde{p}_c}{d\bar{e}^H} > 0. \] \hspace{1cm} (A.5)

**Home country energy consumption and coal and gas supplies**

The equilibrium energy consumption in the Home country is \( \tilde{x}_c^H = D^H(\tilde{p}) \). Differentiating with respect to \( \bar{e}^H \) and using (A.4), one obtains
\[ \frac{d\tilde{x}_c^H}{d\bar{e}^H} = D^H(\tilde{p}) \frac{d\tilde{p}}{d\bar{e}^H} > 0. \]

The domestic supply of coal is \( \tilde{s}_c^H = S_c^H(\tilde{p}_c) \). Differentiating with respect to \( \bar{e}^H \) and using (A.5), one obtains
\[ \frac{d\tilde{s}_c^H}{d\bar{e}^H} = S_c^H \frac{d\tilde{p}_c}{d\bar{e}^H} > 0. \]

By (10), the domestic supply of gas is \( \tilde{s}_g^H = S_g^H(\tilde{p} - \theta_c \tilde{x}_c^H) \). Differentiating with respect to \( \bar{e}^H \), one obtains
\[ \frac{d\tilde{s}_g^H}{d\bar{e}^H} = S_g^H(\tilde{p}) \frac{d\tilde{p}}{d\bar{e}^H} - \theta_c d\tilde{x}_c^H \frac{d\bar{e}^H}{d\bar{e}^H} > 0, \]
which, using the expressions in (A.2), becomes
\[ \frac{d\tilde{s}_g^H}{d\bar{e}^H} = \frac{-S_g^H A_2 + \theta_c A_1}{A_1 A_4 - A_2 A_3}. \]

In this expression, (A.1) allows to rewrite \( A_2 + \theta_c A_1 \) as follows:
\[ A_2 + \theta_c A_1 = (\theta_c - \theta_g) (S_c^H - D^{F^I}) + \theta_g D^{H^I}. \]

Since, by (A.3), \( A_1 A_4 - A_2 A_3 > 0 \), and, by assumption, \( S_g^H > 0 \), it follows that \( d\tilde{s}_g^H / d\bar{e}^H \) and \( (\theta_c - \theta_g) (S_c^H - D^{F^I}) + \theta_g D^{H^I} \) have opposite signs. Therefore, \( d\tilde{s}_g^H / d\bar{e}^H < 0 \)—as when a reduction in \( \bar{e}^H \) causes an increase in gas production \( \tilde{s}_g^H \)—is equivalent to
\[ (\theta_c - \theta_g) \left( S_c^H - D^{F^I} \right) + \theta_g D^{H^I} > 0, \]
which is also
\[ \frac{\theta_c - \theta_g}{\theta_g} > \frac{-D^{H^I}}{S_c^H - D^{F^I}}, \]
from which condition (13) is obtained after using the elasticity notations presented in the main text immediately after Proposition 1. This proves the proposition.

**B Proof of Proposition 2 and Corollary 1**

(14) and (15), together with the expression of \( \tilde{p}_c \) in (9), imply the following expression of the world CO2 emissions:
\[ \tilde{e}_c^W = \bar{e}^H + \theta_c D^F(\tilde{p} - \theta_c \tilde{x}_c^H). \]

Differentiating with respect to \( \bar{e}^H \), one obtains
\[ \frac{d\tilde{e}_c^W}{d\bar{e}^H} = 1 + \theta_c D^F(\frac{d\tilde{p}}{d\bar{e}^H} - \theta_c \frac{d\tilde{x}_c^H}{d\bar{e}^H}), \]
which, after using the expressions in (A.2), becomes
\[ \frac{d\tilde{e}_c^W}{d\bar{e}^H} = 1 - \frac{\theta_c D^F \left( A_2 + \theta_c A_1 \right)}{A_1 A_4 - A_2 A_3}. \]
After replacing the terms given in (A.1), and using the simplifying notation \( r \equiv (\theta_c - \theta_g)/\theta_g \), simple manipulations allow to obtain

\[
\frac{d\tilde{e}^W}{d\tilde{e}^H} = \frac{\theta^2_c S_{g}^{H'}}{A_1 A_4 - A_2 A_3 (1 + r)^2} P(r) \tag{B.1}
\]

with

\[
P(r) = S'_c \left( 1 - \frac{D^{H'}}{S_{g}^{H'}} \right) r^2 + \left( D^{F'} - 2 \frac{S'_c D^{H'}}{S_{g}^{H'}} \right) r - D^{H'} \left( 1 + \frac{S'_c}{S_{g}^{H'}} \right), \tag{B.2}
\]

which has been reported in (18). In (B.1), \( A_1 A_4 - A_2 A_3 > 0 \) by (A.3), and, by assumption, \( S_{g}^{H'} > 0 \). Therefore, \( d\tilde{e}^W/d\tilde{e}^H \) and \( P(r) \) as expressed in (B.2) have the same sign.

After simple manipulations using the elasticity notations of the main text, it follows that \( d\tilde{e}^W/d\tilde{e}^H > 0 \) is equivalent to condition (17). This proves Proposition 2.

The leakage rate, as expressed in (16), can be obtained in a similar way since, by (14),

\[
\frac{d\tilde{e}^F}{d\tilde{e}^H} = \frac{d\tilde{e}^W}{d\tilde{e}^H} - 1;
\]

clearly, this rate is less than 100% when \( d\tilde{e}^W/d\tilde{e}^H > 0 \) and more than 100% otherwise.

\( P(r) \) in (B.2) is a polynomial of degree two. Since its second degree coefficient \( S'_c \left( 1 - D^{H'}/S_{g}^{H'} \right) \) is positive, it satisfies \( \lim_{r \to +\infty} P(r) = +\infty \). Moreover, it satisfies \( P(0) = -D^{H'} \left( 1 + S'_c/S_{g}^{H'} \right) > 0 \).

It follows that \( P(r) \)—and, equivalently \( d\tilde{e}^W/d\tilde{e}^H \)—can only be negative if it admits two real roots; in this case, it will be negative for values of \( r \) in between these roots. It is the case if and only if the polynomial’s determinant

\[
\Delta = D^{F'2} + 4 \left( S'_c + S_{g}^{H'} - D^{F'} - D^{H'} \right) \frac{S'_c D^{H'}}{S_{g}^{H'}} \tag{B.3}
\]

is strictly positive. This positivity condition is expressed in (19). It is clearly a necessary condition for the possibility that \( d\tilde{e}^W/d\tilde{e}^H \) be negative, as when the domestic CO2 reduction is counter-productive at the world level, proving the first part of Corollary 1.

If this condition \( \Delta > 0 \) is satisfied, the two roots \( \tilde{\epsilon} < \tilde{\bar{r}} \) of \( P(r) \) as labelled in the main text are

\[
\tilde{\epsilon} \equiv \frac{2 S'_c D^{H'} - D^{F'} - \sqrt{\Delta}}{2 S'_c \left( 1 - D^{H'}/S_{g}^{H'} \right)} \tag{B.4}
\]

and

\[
\tilde{\bar{r}} \equiv \frac{2 S'_c D^{H'} - D^{F'} + \sqrt{\Delta}}{2 S'_c \left( 1 - D^{H'}/S_{g}^{H'} \right)}, \tag{B.5}
\]

where \( \Delta > 0 \) is given by (B.3).
These two roots, assuming that they exist so that $\Delta > 0$ in (B.3), can be shown to be positive as follows. First, following a famous property of second degree polynomials, the roots’ product is

$$\tilde{r}\bar{r} = \frac{-D'H\left(1 + \frac{S'}{Sg}\right)}{S_c'\left(1 - \frac{D'H}{Sg}\right)},$$

which is positive, implying that the two roots have the same sign. Second, following another famous property of second degree polynomials, the roots’ sum is

$$\tilde{r} + \bar{r} = \frac{2S'D'H - D^{FI}}{S_c'(1 - \frac{D'H}{Sg})}.$$  \hspace{1cm} (B.6)

In the latter fraction, the denominator is positive by our assumptions. At the same time, the positivity of $\Delta$ in (B.3) can easily be shown to imply the inequality $D^{FI} < 4\frac{S'D'H}{Sg} \left(1 + \frac{D'H}{D''} - \frac{S_c'S'}{Sg}\right)$, where the fact that the term between parentheses is less than one implies, in turn, $D^{FI} < 4\frac{S'D'H}{Sg} < 2\frac{S'D'H}{Sg}$. It follows that the fraction’s numerator in (B.6) and, therefore, the roots’ sum are positive. Having already established that the roots have the same sign, one can conclude that this sign is positive.

In fact, the analysis and comparison of Propositions 1 and 2 in the main text revealed that $0 < \tilde{r}_0 < \tilde{\tilde{r}}$.

To sum up, provided $\Delta > 0$—and, therefore, condition (19) in Corollary 1—$P'(r)$ is strictly negative—and, therefore, so is $d\tilde{e}^W/d\tilde{e}^H$—for and only for all rates of pollution increase $r = (\theta_c - \theta_g)/\theta_g$ within the non-empty positive interval $(\tilde{r}, \tilde{\tilde{r}})$. This proves the second point of Corollary 1.

C Proof of Proposition 3

Totally differentiating the equilibrium condition (23) with respect to $\tilde{p}$, $\tilde{\tau}^H$ and $\tilde{\tau}^F$, and rearranging terms, one obtains

$$A_1d\tilde{p} + A_2d\tilde{\tau}^H + \theta_cD^{FI}d\tilde{\tau}^F = 0,$$ \hspace{1cm} (C.1)

where the notations defined in (A.1) have been used.

Proposition 3 assumes that the Foreign country’s emissions are limited as per (24), which implies that the coal price $\tilde{p}_c = \tilde{p} - \theta_c\tilde{\tau}^H + \theta_c\tilde{\tau}^F$ therein, as given in (22), is held unchanged. Its total derivative with respect to $\tilde{p}$, $\tilde{\tau}^H$ and $\tilde{\tau}^F$ is, therefore, zero:

$$d\tilde{p} - \theta_c d\tilde{\tau}^H + \theta_c d\tilde{\tau}^F = 0.$$ \hspace{1cm} (C.2)

Combining equations (C.1) and (C.2) by substituting $d\tilde{p}$, one obtains

$$\frac{d\tilde{\tau}^F}{d\tilde{\tau}^H} = -\frac{D'H + \frac{\theta_c - \theta_g}{\theta_c}S'H'}{-D'H + S_c' + S_g'H'},$$

from which equation (25) is derived after using the elasticity notations of the main text. This proves Proposition 3.