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## A quantitative theory of time-consistent unemployment insurance

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# A Quantitative Theory of Time-Consistent Unemployment Insurance

Yun Pei and Zoe Xie

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JEL classification: E61, J64, J65, H21

Key words: time-consistent policy, unemployment insurance, labor market, business cycle

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# A Quantitative Theory of Time-Consistent Unemployment Insurance \*

Yun Pei and Zoe Xie<sup>†</sup>

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## ABSTRACT

During recessions, the U.S. government substantially increases the duration of unemployment insurance (UI) benefits through multiple extensions. This paper seeks to understand the incentives driving these increases. Because of the trade-off between insurance and job search incentives, the classic time-inconsistency problem arises. This paper endogenizes a time-consistent UI policy in a stochastic equilibrium search model, where a government without commitment to future policies chooses the UI benefit level and expected duration each period. A longer duration increases unemployed workers' consumption but reduces job search, leading to higher future unemployment. Quantitatively, the model rationalizes most of the variations in benefit duration during the Great Recession. We use the framework to evaluate the effects of the 2009-2013 benefit extensions on unemployment and welfare.

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# 1 INTRODUCTION

This paper examines the U.S. government’s decision to extend unemployment insurance (UI) durations during recessions. The U.S. government has extended UI benefit durations in response to high unemployment during every recession since the 1970s. In particular, during and following the Great Recession, the maximum UI duration more than tripled compared to normal times, and the whole extension program lasted for more than four years. Studies in the optimal policy literature find it hard to reconcile the optimal UI policy over the business cycle with the direction and scale of the extensions policy.<sup>1</sup> Departing from the optimal policy literature that assumes government commitment to future policies, this paper studies the decisions of a government who cannot or would not make promises about future policies.

Government commitment is an important margin. In the presence of the trade-off between insurance and job search incentive, the classic time-inconsistency problem arises. Intuitively, before workers decide job search effort, the government optimally wants to use less generous benefits to incentivize search; once search and unemployment have realized, the government instead wants to use more generous benefits to help smooth out consumption.<sup>2</sup> A government without commitment cannot make promises about future benefit policies, and so it lacks the proper tools to incentivize job search when the workers’ search intensity depends on the expected future value of unemployment. Evidence suggests that benefit duration extensions during recessions are discretionary decisions based on current economic conditions (e.g. productivity, unemployment) and are not pre-committed policies.<sup>3</sup> Therefore, to understand the government’s behavior, it is natural to consider a government who cannot make promises about future policies.

We find that a government without commitment optimally chooses benefit extensions that quantitatively resemble those in the U.S. between 2009 and 2013. Lack of commitment is the key to generating this quantitative result: It is *ex-ante* optimal to promise less generous benefits when the unemployment is high, but a government without commitment cannot make such promises. As a result, our quantitative results are more in line with the reality than do models with government commitment. Because of the quantitative relevance of the model, we also use it to evaluate the impacts of benefit extensions on the aggregate unemployment rate and welfare during the Great Recession. Our approach addresses the common endogeneity problem of policy evaluations; it also allows the workers and firms to form expectations about future extension policies that are chosen endogenously by the government.

Underlying these results is an equilibrium business cycle model with a government and private sector consisting of workers and firms. We use the concept of Markov-perfect equilibrium to characterize the decisions of a government without commitment. Because the equilibrium restricts the government’s policy rules to depend only on current payoff-relevant states, the policies are time-

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<sup>1</sup> See, for example, [Jung and Kuester \(2015\)](#) and [Mitman and Rabinovich \(2015\)](#)

<sup>2</sup> Section [7.1](#) provides a numerical illustration of this time-inconsistency problem.

<sup>3</sup> We discuss this in Section [6.1](#).

consistent. Specifically, a welfare-maximizing central government chooses the UI benefit level and the probability that the benefit expires (“benefit exhaustion probability”) each period depending on the current levels of unemployment and aggregate productivity.<sup>4</sup> Modeling the benefit exhaustion probability rather than a fixed length of benefits keeps the government’s decision tractable. At the same time, the inverse of benefit exhaustion probability gives the expected duration, which allows for comparison with empirical evidence on benefit extensions. A key assumption here is that once benefits expire, the unemployed worker does not regain eligibility until he finds a job. Such a UI system is consistent with the actual UI policy in the U.S. Under this assumption, the unemployed workers with benefits search less than those without benefits. As a result, benefit duration policy today, through changing the proportion of insured unemployed workers, directly impacts the states of the economy (unemployment and the measure of benefit-eligible unemployed workers) inherited by the future government and thus the future policies. In equilibrium, the government follows the same policy rules each period.

The private sector’s decisions are modeled using a search-matching model with risk-averse workers, endogenous search intensity by the unemployed, and business cycles driven by shocks to aggregate labor productivity. Unemployed workers search for jobs, while firms post vacancies. Both parties make decisions given the government’s policy choices. Because future government policies affect their expected future value, their decisions also depend on their expectations about future government policies. Generous future benefit policies reduce worker’s incentives to search, which in turn lowers firm’s incentives to create vacancies. Since the government’s duration policy directly affects the future states of the economy and in turn affecting the private sector’s expectations about future policies, the government’s policy decision has to take into account the effect of expectations on private choices.<sup>5</sup>

The main trade-off associated with the government’s duration policy is between insurance and incentives.<sup>6</sup> A longer duration increases the UI coverage today and thus raising the average insurance for the unemployed workers. It also reduces the average job search through an increase in the share of unemployed workers receiving benefits, which raises the future unemployment and alters the private sector’s expectations about future policies. Over the business cycle, the UI duration is strongly countercyclical. In response to a drop in productivity, the expected future productivity is low,

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<sup>4</sup> Because we want to use the model to study observed UI variations, we keep the policy parameters as close to reality as possible. For example, we abstract from the type of employment history-dependent UI policy analyzed in [Hopenhayn and Nicolini \(1997\)](#).

<sup>5</sup> The government’s choice of the benefit level, in contrast to the duration, is static in our model. In particular, benefit level does not affect the future unemployment, and so there is no intertemporal dimension to the choice of benefit level. The government sets benefit level to achieve the best income redistribution within the period.

<sup>6</sup> UI can also affect economic activities through liquidity (e.g. [Shimer and Werning 2007](#); [Chetty 2008](#)) and aggregate demand (e.g. [Kekre 2015](#); [Di Maggio and Kermani 2016](#)) channels. The workers in our setup do not borrow and save, and so we essentially shut down the liquidity channel. Also, literature emphasizing liquidity effect has focused on benefit level, while we examine changes of the UI duration, which plausibly have little liquidity effect on the unemployed. In theory, UI, as a redistribution of income from rich to poor, can raise the aggregate demand because of differential marginal propensities to consume. The empirical support for this channel has studied the effect of different benefit levels. The effect of duration changes is less clear.

which implies a low marginal return to production tomorrow and a low marginal gain from job creation today. As a result, the cost of a higher expected duration is low, and the government raises UI duration. As the unemployment rate rises, the marginal gain from increasing UI duration is higher as more unemployed workers receive benefits, and as a result, the expected duration increases further.<sup>7</sup>

Because our model generates cyclical movements of UI policy that are consistent with empirical observations, the model can potentially be used to answer quantitative questions about UI benefit extensions. Before that, we validate our model along three dimensions. First, we check if a calibrated version of the model correctly predicts the generosity of UI policy in steady state. Calibrated to the steady-state U.S. labor market and the average replacement ratio, the model generates a steady-state UI duration of 26.3 weeks, compared to 26 weeks in the U.S. Second, without directly targeting the business cycle properties in calibration, the calibrated model generates labor market volatilities that are very close to the data. Lastly, we test to see how well our model can account for variations in UI duration in recessions. We apply the model to the U.S. economy during the Great Recession by feeding in the exogenous job separation rates from the data and calibrating labor productivity to match the observed path of unemployment rates from 2008 to the end of 2013. Overall, our model matches the variations in duration very well, generating the correct timing of duration changes as well as 80% of the overall increase in UI duration.

An implication of our theory is that the Markov policy, by increasing UI duration in recessions, contributes to higher unemployment. Using the calibrated model, we ask how much of the increase in the unemployment rate can be accounted for by UI benefit extensions as opposed to general economic conditions. In particular, we compare the Markov-perfect equilibrium to an economy where the government does not change UI policy in recessions. The same paths of job separation rates and productivity shocks, which generate a peak of 10% unemployment rate in the Markov equilibrium, lead to a peak of only 6.7% under the no-extensions policy. In other words, about 3.3 percentage point increase in the unemployment rate during this period can be accounted for by rising UI benefit extensions.

An important mechanism of our model works through expectations: when unemployment is high, the unemployed workers expect longer UI durations in the future, and in response reduces job search. To investigate how much workers' expectation contributes to the 3.3-percentage point unemployment gap, we turn off the expectations effect and obtain an unemployment gap of less than 1 percentage point, which implies that more than two-thirds of the unemployment gap attributable to UI benefit extensions is driven by expectations. We also perform welfare evaluations of the extension policy.

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<sup>7</sup> The idea that the welfare gains and costs of UI vary over the business cycle is not new. For example, [Krueger and Meyer \(2002\)](#) argue that the efficiency loss from reduced search effort may be smaller during a recession than during a boom. More recently, [Kroft and Notowidiglo \(2015\)](#) empirically estimate the moral hazard cost and consumption smoothing benefit of UI benefits, and they find that the marginal welfare cost from generous benefits is procyclical and the marginal welfare gain is modest and varies positively with unemployment rate. While they focus on the changing moral hazard effect of UI benefits on individual workers, we investigate the optimal government's response to the changing efficiency loss.

This paper makes two main contributions. First, we characterize the time-consistent UI policy over the business cycle of a government without commitment to future policies. While a long tradition of literature have studied the optimal UI policy (for example, [Hopenhayn and Nicolini 1997](#); [Wang and Williamson 2002](#); [Shimer and Werning 2008](#); [Chetty 2008](#); [Golosov, Maziero, and Menzio 2013](#)), the cyclical response of optimal UI is a relatively new topic. Even less is known about the time-consistent UI policy.<sup>8</sup> The present paper fills the gap by characterizing time-consistent UI policy using the Markov-perfect equilibrium concept, focusing on its cyclical responses to changes in the underlying economic conditions. In addition to generating realistic cyclicality, our theory delivers quantitatively relevant variations in the UI durations, thus giving us a framework to address quantitative questions.

Second, this paper quantifies the impact of benefit extensions on unemployment and welfare. We are not the first to evaluate the extensions policy (see, for example, [Fujita 2010](#); [Rothstein 2011](#); [Nakajima 2012](#); [Hagedorn, Manovskii, and Mitman 2015](#); [Mitman and Rabinovich 2016](#)). Our approach addresses the potential policy endogeneity of most empirical works. Among the structural studies, [Nakajima \(2012\)](#) computes a deterministic transition economy where the extension policy path is exogenous and slowly revealed in stages. [Mitman and Rabinovich \(2016\)](#) estimate a government’s response function using historical extensions and unemployment data. Our approach is more similar to the latter in that the private sector forms expectations about future policies based on the policy rules in a stochastic environment, and our evaluation of the extensions policy is also more in line with their results.<sup>9</sup>

In addition to the two main contributions, this paper also makes empirical contributions. Empirically, we document that UI duration increased during all recessions since the 1970s. We find consistent patterns across these recessions. First, UI durations reach their highest levels around the time that unemployment rates peak. Second, recessions with higher peak unemployment rates are also associated with larger UI duration increases.

The rest of the paper proceeds as follows. Section 2 describes the model environment and defines the private-sector competitive equilibrium. Section 3 defines the Markov-perfect equilibrium. We characterize the solution to the government’s problem using the GEE and solve the equilibrium. Section 4 describes the parametrization strategy. We conduct equilibrium analysis in Section 5 by presenting the Markov government’s policy rules and discussing their implications for the labor market. Section 6 provides quantitative analysis of UI benefit extensions during recessions. Section 7 compares the long-run properties of Markov and Ramsey policies. Section 8 concludes. We relegate

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<sup>8</sup> See, for instance, [Alesina and Tabellini \(1990\)](#); [Klein and Ríos-Rull \(2003\)](#); [Chari and Kehoe \(2007\)](#); [Battaglini and Coate \(2008\)](#); [Yared \(2010\)](#) for studies of other time-consistent policies.

<sup>9</sup> Admittedly, we don’t need a model without government commitment to perform this policy evaluation. In fact, computing the endogenous government policy limits the amount of details in our model; for example, we don’t have private borrowing/saving decision and the tier structure of benefit extensions as present in [Nakajima \(2012\)](#). Nevertheless, we find by using an endogenous government policy we capture the uncertain expectation about future policies. In [Nakajima \(2012\)](#), the private sector expects that the extensions are temporary and will eventually revert back to steady state. In our case, the private sector forms expectations about future policies based on the anticipated future economic conditions.

all derivations, sensitivity analysis, additional figures, and a detailed review of the related literature to the Appendix.

## 2 MODEL

In this section, we describe the model environment and characterize the competitive equilibrium. The model is based on a search-matching model with aggregate productivity shocks.

### 2.1 Model environment

Time is discrete and infinite. The model is inhabited by a mass of infinitely lived workers and firms. The measure of workers is normalized to one. In any given period, a worker can be either employed or unemployed. Some unemployed workers receive UI benefits. Workers are risk averse and maximize expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) - v(s_t)]$$

where  $\mathbb{E}_0$  is the period 0 expectation factor, and  $\beta$  is the time discount factor. Period utility comprises of utility from consumption of goods  $U(c)$  and disutility from job search activity  $v(s)$ . Utility is increasing in  $c$  and decreasing in  $s$ . To study the insurance incentive of the government we assume that  $U(\cdot)$  is a concave function. Only unemployed workers choose positive search intensity; that is, there is no on-the-job search. Each period, an employed worker gets paid wages from production. Wage determination technology is specified later in this section. An unemployed worker, if on unemployment benefits, receives  $b$  from the government. In addition, an unemployed worker also produces  $h$ , which we take as the combined value of leisure, home production, and welfare. There are no private insurance markets and workers cannot save or borrow.

Firms are risk neutral and maximize the expected discounted sum of profits, with the same discount factor  $\beta$ . A firm can be either matched to a worker (and producing) or vacant. A vacant firm posting a vacancy incurs a flow cost  $\kappa$ .

Unemployed workers and vacancies form new matches. Let  $I$  and  $V$  denote the aggregate search by unemployed worker and the aggregate vacancy posting by firms, respectively. Then the number of new matches formed in a period is given by the matching function  $M(I, V)$ . The matching function exhibits constant returns to scale, is strictly increasing and strictly concave in both arguments, and is bounded above by the number of expected matches:  $M(I, V) \leq \min\{I, V\}$ . The job-finding probability per efficiency unit of search intensity,  $f$ , and the job-filling probability per vacancy,  $q$ , are functions of labor market tightness,  $\theta = V/I$ . More specifically,

$$\begin{aligned} f(\theta) &= \frac{M(I, V)}{I} = M(1, \theta) \\ q(\theta) &= \frac{M(I, V)}{V} = M\left(\frac{1}{\theta}, 1\right). \end{aligned}$$

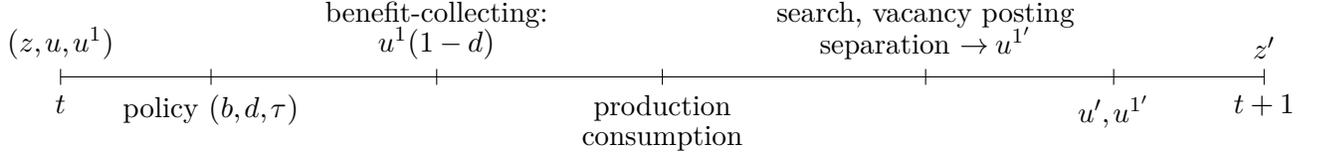


Figure 1: Timing of events.

Following the assumptions made on  $M$ ,  $f(\theta)$  is increasing in  $\theta$  and  $q(\theta)$  is decreasing in  $\theta$ . The job-finding probability for an unemployed searching with intensity  $s$  is  $sf(\theta)$ . Existing matches are destroyed exogenously with constant job separation probability  $\delta$ .

Each period, a matched pair of a worker and a firm produces  $z$ , where  $z$  is the aggregate labor productivity.  $z$  is equal to  $\bar{z}$  in steady state.

## 2.2 Government policy

The government cannot borrow or lend; instead, it balances the budget each period. The government finances unemployment benefits  $b$  through a lump sum tax,  $\tau$ , on all workers, both employed and unemployed.<sup>10</sup> The government budget constraint is

$$\tau = u_{benefit}b. \quad (1)$$

The government decides the generosity of the UI program by varying (1) benefit level,  $b \geq 0$ , and (2) the benefit exhaustion probability  $d$  ( $1/d$  is the expected duration). Once a benefit level and exhaustion probability are determined, benefit-eligible unemployed workers receive benefits  $b$  with probability  $d$ .

## 2.3 Timing

The timing of events within a period is illustrated in Figure 1 and is as follows. The economy enters period  $t$  with a measure of the total unemployed workers  $u$  and a measure of the benefit-eligible unemployed workers  $u^1$ . The aggregate shock  $z$  is then realized.  $(z, u, u^1)$  are the aggregate states of the economy.

Once government policies  $(b, d, \tau)$  for the period are announced,  $u_{benefit} = u^1(1 - d)$  workers collect benefit. In other words, with probability  $d$ , benefit-eligible unemployed workers lose benefit status in this period.

Employed workers produce  $z$  and receive wages  $w$ . Unemployed workers produce  $h$  and, if collecting benefits, receive  $b$ . All workers pay a lump sum tax  $\tau$ .

<sup>10</sup> We experiment with alternative tax structures where either only employed workers pay tax, or employed and benefit-eligible unemployed workers pay tax. Results are not presented in the paper but are available upon request.

Given aggregate states and government policies for the period, unemployed workers with and without benefits choose search intensity  $s^1$  and  $s^0$ , respectively. At the same time, firms decide how many vacancies to post, at cost  $\kappa$  per vacancy. The aggregate search is then  $I = u^1(1-d)s^1 + (u - u^1(1-d))s^0$ , aggregate vacancy posting is  $V$ , and market tightness is equal to  $\theta = V/I$ . The fraction of unemployed workers with and without benefits who find jobs is  $f(\theta)s^1$  and  $f(\theta)s^0$ , respectively. At the same time, a fraction  $\delta$  of the existing  $1 - u$  matches are exogenously destroyed. Newly unemployed workers and unemployed workers with benefits constitute next period's state  $u^1$ .<sup>11</sup>

The laws of motion of unemployed workers are

$$\begin{aligned}
\text{total unemployed:} \quad u' &= \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^0)(u-u^1(1-d)) + (1-f(\theta)s^1)u^1(1-d)}_{\text{previously unemployed who didn't find job}} \quad (2) \\
\text{benefit-eligible unemployed:} \quad u^1 &= \underbrace{\delta(1-u)}_{\text{newly unemployed}} + \underbrace{(1-f(\theta)s^1)u^1(1-d)}_{\text{unemployed with benefits who didn't find job}} \quad (3)
\end{aligned}$$

## 2.4 Workers

Denote by  $g$  the government policy  $(b, d, \tau)$ . In what follows we suppress the functional arguments in  $\theta$ , which is an object determined in equilibrium. Wage  $w$  also depends on states of the economy, and may be an equilibrium object. The wage determination process is specified later. A worker unemployed and with benefits consumes  $h + b - \tau$  and chooses search intensity  $s^1$ ; an unemployed worker without benefits consumes  $h - \tau$  and chooses search intensity  $s^0$ . With probability  $f(\theta)s$ ,  $s = \{s^0, s^1\}$ , he finds a job and starts working the following period. Let  $V^e(z, u, u^1; g)$  and  $V^u(z, u, u^1; g)$  be the values of an employed and an unemployed worker, respectively, with the beginning-of-period measures of unemployed workers with and without benefits  $(u, u^1)$  and realized aggregate shock  $z$ , given government policy for that period  $g = (b, d, \tau)$ .

The optimization problem of an unemployed worker *without* benefits (superscript 0 denotes no benefits) is

$$\begin{aligned}
V^0(z, u, u^1; g) &= \max_{s^0} U(h - \tau) - v(s^0) + f(\theta)s^0\beta\mathbb{E}V^e(z', u', u^1; g') \\
&\quad + (1 - f(\theta)s^0)\beta\mathbb{E}V^0(z', u', u^1; g'), \quad (4)
\end{aligned}$$

and the problem of an unemployed worker *with* benefit is

$$\begin{aligned}
V^1(z, u, u^1; g) &= \max_{s^1} U(h + b - \tau) - v(s^1) + f(\theta)s^1\beta\mathbb{E}V^e(z', u', u^1; g') \\
&\quad + (1 - f(\theta)s^1)\beta\mathbb{E} \left[ d'V^0(z', u', u^1; g') + (1 - d')V^1(z', u', u^1; g') \right]. \quad (5)
\end{aligned}$$

<sup>11</sup> Effectively, newly unemployed workers qualify for benefits with probability  $1 - d$ . In reality, newly unemployed workers qualify for benefits with at least two quarters of earnings and must pass an “earnings test” that depends on individual state policies. We model it as a probability for simplicity here.

A worker entering a period employed produces and consumes his wage  $w$  minus tax  $\tau$ . With probability  $\delta$ , he loses his job and becomes unemployed the following period. There is no intra-temporal search, so a newly separated worker remains unemployed for at least one period. The Bellman equation of an employed worker is then

$$V^e(z, u, u^1; g) = U(w - \tau) + (1 - \delta)\beta\mathbb{E}V^e(z', u', u^1; g') + \delta\beta\mathbb{E}\left[d'V^0(z', u', u^1; g') + (1 - d)'V^1(z', u', u^1; g')\right]. \quad (6)$$

## 2.5 Firms

To be matched with a worker and start production, a firm posts a vacancy.<sup>12</sup> A firm that posts a vacancy incurs a flow cost  $\kappa$ . With probability  $q(\theta)$ , a vacancy is filled and ready for production the following period. Let  $J^u(z, u, u^1; g)$  be the value of an unmatched firm posting a vacancy. The Bellman equation of an unmatched firm is

$$J^u(z, u, u^1; g) = -\kappa + q(\theta)\beta\mathbb{E}J^e(z', u', u^1; g') + (1 - q(\theta))\beta\mathbb{E}J^u(z', u', u^1; g'), \quad (7)$$

where  $J^e(z, u, u^1; g)$  is the value of a matched firm. In equilibrium, under free-entry condition, the firm will post vacancies  $v(z, u, u^1; g)$  until  $J^u(z, u, u^1; g) = 0$ .

A matched firm receives output net of wages  $z - w$ . With constant probability  $\delta$ , a match is destroyed at the end of period. The Bellman equation of a matched firm is

$$J^e(z, u, u^1; g) = z - w + (1 - \delta)\beta\mathbb{E}J^e(z', u', u^1; g') + \delta\beta\mathbb{E}J^u(z', u', u^1; g'). \quad (8)$$

## 2.6 Wage determination

When a match is formed, the economic rent is shared between the firm and the worker. To introduce wage rigidity, we set wages to be a function of productivity. In particular, wages increase in labor productivity  $z$ , but less than one to one. As such, workers and firms share the risk of fluctuating aggregate labor productivity.

While Nash bargaining is widely used in the search and matching framework, people have used other wage determination specifications. The main challenge of Nash bargaining is its difficulty in generating the amount of observed fluctuations in unemployment. Various works in the literature have used alternate specifications to introduce wage rigidity to achieve the desired fluctuations: [Hall and Milgrom \(2008\)](#) adopt alternating-offer wage bargain; [Landais, Michailat, and Saez \(2010\)](#) and [Nakajima \(2012\)](#) use a specification similar to the one we use; [Jung and Kuester \(2015\)](#) introduce countercyclical bargaining power of workers. The main advantage of the specification we use is it allows us to calibrate the degree of wage rigidity directly from data. The main drawback is because

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<sup>12</sup> The firms can be viewed as a representative firm with a collection of jobs and several vacancies.

wages do not depend on workers' outside options, benefit policies have no effect on wages, which is the macro channel emphasized in [Hagedorn, Karahan, Manovskii, and Mitman \(2013\)](#).

## 2.7 Competitive equilibrium

DEFINITION 1. (Competitive equilibrium) Given a policy  $g = (b, d, \tau)$  and initial conditions  $(z^-, u^-, u^{1-})$ , a competitive equilibrium consists of  $(z, u, u^1)$ -measurable functions for worker's search intensity  $s^0(z, u, u^1; g)$  and  $s^1(z, u, u^1; g)$ , market tightness  $\theta(z, u, u^1; g)$ , total unemployment  $u'(z, u, u^1; g)$ , and the measure of unemployed workers with benefit  $u^{1'}(z, u, u^1; g)$ , and value functions  $V^e(z, u, u^1; g)$ ,  $V^0(z, u, u^1; g)$ ,  $V^1(z, u, u^1; g)$ ,  $J^e(z, u, u^1; g)$ , and  $J^u(z, u, u^1; g)$  such that for all  $(z, u, u^1; g)$ :

- the value functions satisfy the worker and firm Bellman equations (4)-(8);
- the search intensities  $s^0$  and  $s^1$  solve the unemployed worker's maximization problem of (4) and (5), respectively;
- the market tightness  $\theta$  is consistent with the free-entry condition,  $V^u(z, u, u^1; g) = 0$ ;
- measures of unemployment satisfy the laws of motion (2)-(3).

## 2.8 Characterization of private-sector optimality

The competitive equilibrium can be characterized by three optimality conditions.<sup>13</sup> Appendix D contains a derivation of the optimality conditions. In what follows, primes denote variables of the following period, and subscripts denote derivatives.

The optimal choice of search intensity  $s^0$  and  $s^1$  for the unemployed worker is characterized by

$$\text{no benefit: } \frac{v_s(s^0)}{f(\theta)} = \beta \mathbb{E} \left[ U(w' - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \quad (9)$$

$$\begin{aligned} \text{with benefit: } \frac{v_s(s^1)}{f(\theta)} &= \beta \mathbb{E} d' \left[ U(w' - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \\ &+ \beta \mathbb{E} (1 - d') \left[ U(w' - \tau') - U(h + b' - \tau') + v(s^{1'}) + (1 - f(\theta')s^{1'} - \delta) \frac{v_s(s^{1'})}{f(\theta')} \right]. \quad (10) \end{aligned}$$

The worker's optimality conditions state that the marginal cost (left-hand side) of increasing the job-finding probability equals the marginal benefit (right-hand side). The marginal cost is the marginal disutility of search of the unemployed worker weighted by the aggregate job-finding rate per efficiency unit of search. The marginal benefit is the sum of utility gain from being employed in the next period and the benefit of economizing on future search cost. A higher future benefit  $b'$  or a higher future duration  $1/d'$  reduces the utility gain from being employed in the next period and thus lowers the marginal benefit of search today.

<sup>13</sup> To economize on notation, we suppress the dependence on  $(z, u, u^1; g)$ . It should be understood throughout that the optimal decisions are functions with arguments  $(z, u, u^1; g)$ .

From firm’s free-entry condition

$$\frac{\kappa}{q(\theta)} = \beta \mathbb{E} \left[ z' - w' + (1 - \delta) \frac{\kappa}{q(\theta')} \right], \quad (11)$$

where the marginal cost (left-hand side) equals the marginal benefit (right-hand side) of a filled vacancy. The marginal cost is the flow cost of posting a vacancy weighted by the probability of filling that vacancy. The marginal benefit is the profits from employing a worker. Because a newly formed match does not become operational until the next period, the benefit from production has components only from the next period.

### 3 MARKOV EQUILIBRIUM

In this section, we define the Markov-perfect equilibrium in our economy. We assume the government is a utilitarian planner, who maximizes the expected value of a worker’s utility. The government policy instruments include benefit level  $b$ , expected duration  $1/d$ , and tax  $\tau$ . We consider government policies that are time consistent using the Markov-perfect equilibrium, à la [Klein, Krusell, and Ríos-Rull \(2008\)](#).

Intuitively, one can think of the economy as having a different government each period. Each successive government chooses only current policy, taking future governments’ policies as given. In other words, today’s government cannot directly choose future policies. Instead, both today’s government and private sector take form an expectation about future government policy rules when making decisions. Like [Klein, Krusell, and Ríos-Rull \(2008\)](#), we focus on equilibria where government policy depends *differentiably* on the aggregate states of the economy.<sup>14</sup>

The timing of events is illustrated in [Figure 1](#). Because the economy consists of a mass of workers and firms, the private sector take future government policies as given.<sup>15</sup> The equilibrium described above can be equivalently stated as an equilibrium where the government chooses policy and private-sector allocations together given the state of the economy. To reduce the number of policy instruments in the government’s problem, we use the following function derived from the government’s budget constraint to express tax

$$\mathcal{T}(u^1, b, d) := u^1(1 - d)b.$$

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<sup>14</sup> While there is not a proof for the existence and uniqueness of Markov-perfect equilibrium, [Chatterjee and Eyigungor \(2014\)](#) provide argument for the existence of Markov-perfect equilibrium with continuous decision rules. Our companion paper ([Pei and Xie 2015](#)) uses numerical method to provide evidence for unique *differentiable* equilibrium in a setup similar to the present one.

<sup>15</sup> The current government policies are decided before the private sector moves. The future government policies will depend on future states, which are affected by how the private sector moves today. Each worker or firm does not consider how their action will affect future policies through changing future states.

The *government period return function* is equal to the average welfare of all workers, and is given by

$$\begin{aligned}
R(z, u, u^1, b, d, s^0, s^1) = & \quad (1 - u)U(w(z) - \mathcal{T}(u^1, b, d)) && \text{worker} \\
& + (u - u^1(1 - d)) [U(h - \mathcal{T}(u^1, b, d)) - v(s^0)] && \text{unemployed without benefit} \\
& + u^1(1 - d) [U(h + b - \mathcal{T}(u^1, b, d)) - v(s^1)] && \text{unemployed with benefit}
\end{aligned}$$

DEFINITION 2. (Markov-perfect equilibrium) A Markov-perfect equilibrium consists of a value function  $G$ , government policy rules  $\Psi^b$  and  $\Psi^d$ , and private decision rules  $\{S^0, S^1, \Theta, \Gamma, u^{1-}\}$  such that for all aggregate productivity  $z$  and unemployment states  $(u, u^1)$ ,  $b = \Psi^b(z, u, u^1)$ ,  $d = \Psi^d(z, u, u^1)$ ,  $s^0 = S^0(z, u, u^1)$ ,  $s^1 = S^1(z, u, u^1)$ ,  $\theta = \Theta(z, u, u^1)$ ,  $u' = \Gamma(z, u, u^1)$ , and  $u^{1'} = \Gamma^1(z, u, u^1)$  solve

$$\max_{b, d, s^0, s^1, \theta, u', u^{1'}} R(z, u, u^1, b, d, s^0, s^1) + \beta \mathbb{E}G(z', u', u^{1'})$$

subject to

- The worker's laws of motion

$$f_1(u, u^1, d, s^0, s^1, \theta, u') := u' - \delta(1 - u) - f(\theta)(s^0 - s^1)u^1(1 - d) - (1 - f(\theta)s^0)u = 0 \quad (12)$$

$$f_2(u, u^1, d, s^1, \theta, u^{1'}) := u^{1'} - \delta(1 - u) - (1 - f(\theta)s^1)u^1(1 - d) = 0; \quad (13)$$

- The private-sector optimality conditions below, writing  $\mathcal{O} = (z, u, u^1)$  to economize on notation

$$\begin{aligned}
& \eta_1(s^0, \theta, \mathcal{O}'; \Psi^b, \Psi^d, S^0, S^1, \Theta) \\
:= & \frac{v_s(s^0)}{f(\theta)} - \beta \mathbb{E} \left[ U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^0(\mathcal{O}')) \right] \\
& - \beta \mathbb{E} \left[ (1 - f(\Theta(\mathcal{O}'))S^0(\mathcal{O}')) \frac{v_s(S^0(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} - \delta \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} \right] = 0 \quad (14)
\end{aligned}$$

$$\begin{aligned}
& \eta_2(s^1, \theta, \mathcal{O}'; \Psi^b, \Psi^d, S^0, S^1, \Theta) \\
:= & \frac{v_s(s^1)}{f(\theta)} - \beta \mathbb{E} \Psi^d(\mathcal{O}') \left[ U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^0(\mathcal{O}')) \right] \\
& - \beta \mathbb{E} \Psi^d(\mathcal{O}') \left[ (1 - f(\Theta(\mathcal{O}'))S^0(\mathcal{O}')) \frac{v_s(S^0(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} - \delta \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} \right] \\
& - \beta \mathbb{E} (1 - \Psi^d(\mathcal{O}')) \left[ U(w(z') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) - U(h + \Psi^b(\mathcal{O}') - \mathcal{T}(u^{1'}, \Psi^b(\mathcal{O}'), \Psi^d(\mathcal{O}'))) + v(S^1(\mathcal{O}')) \right] \\
& - \beta \mathbb{E} (1 - \Psi^d(\mathcal{O}')) [1 - f(\Theta(\mathcal{O}'))S^1(\mathcal{O}') - \delta] \frac{v_s(S^1(\mathcal{O}'))}{f(\Theta(\mathcal{O}'))} = 0 \quad (15)
\end{aligned}$$

$$\eta_3(\theta, \mathcal{O}'; \Theta) := \frac{\kappa}{q(\theta)} - \beta \mathbb{E} \left[ z' - w(z') + (1 - \delta) \frac{\kappa}{q(\Theta(\mathcal{O}'))} \right] = 0; \quad (16)$$

- The government value function satisfies the functional equation

$$G(\mathcal{O}) \equiv R(z, u, u^1, \Psi^b(\mathcal{O}), \Psi^d(\mathcal{O}), S^0(\mathcal{O}), S^1(\mathcal{O})) + \beta \mathbb{E}G(z', \Gamma(\mathcal{O}), \Gamma^1(\mathcal{O})).$$

The Markov government chooses current policy, knowing how the private sector will behave given the policy.<sup>16</sup> More specifically, the current Markov government weighs the trade-off between current and future welfare. By choosing a longer expected duration  $1/d$ , the current government increases the share of unemployed workers receiving benefits today, thus raising the current welfare. At the same time, because of moral hazard problem, unemployed workers on benefits choose lower search intensity, and as a result higher duration reduces the average search intensity, leading to higher future unemployment and lower future welfare.<sup>17</sup>

In addition, because all successive governments follow the same set of policy rules, the current government, by choosing current policy, affects the policies of future governments through changing future states of the economy. This *disciplining effect*, through private-sector expectations of future policies, affects job search of unemployed workers on benefits today, and through general equilibrium effects, affects job search of benefit-ineligible unemployed and firms' vacancy posting. The current government correctly anticipates this effect when choosing today's policy. Proposition 1 provides the conditions that characterize government decisions. The proof involves deriving the GEE and is included in Appendix D.

PROPOSITION 1. Given the aggregate states of the economy and the private-sector optimality conditions, the unemployment benefit policy  $b$  in the Markov-perfect equilibrium is characterized by

$$R_b = 0, \quad (17)$$

and policy  $d$  associated with the expected benefit duration can be characterized by the GEE

$$\begin{aligned} 0 &= R_d - f_{1d}\lambda \\ &+ \frac{f_{2d}}{f_{2u^{1'}}} \left\{ \frac{\eta_{1u^{1'}}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u^{1'}}}{\eta_{2s^1}} \left[ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] + \dots \right. \\ &\left. \dots + \frac{\eta_{3u^{1'}}}{\eta_{3\theta}} \left[ -\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left( R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \right\} \\ &+ \beta \mathbf{E} \left( -\frac{f_{2d}}{f_{2u^{1'}}} \right) [R'_{u^1} - \lambda' f'_{1u^1}] \\ &+ \beta \mathbf{E} \left( -\frac{f_{2d}}{f_{2u^{1'}}} \right) \left( -\frac{f'_{2u^1}}{f'_{2d}} \right) [R'_d - \lambda' f'_{1d}], \end{aligned} \quad (18)$$

<sup>16</sup> Equivalently, in the setup here, the government chooses both policy and private-sector allocations, taking into consideration private-sector optimality conditions.

<sup>17</sup> A secondary effect exists through taxation. With longer duration, more unemployed workers receive benefits and the lump-sum tax is higher. The scale of this marginal cost of longer duration is small relative to the other two effects.

where  $\lambda$  is the shadow price of unemployment characterized by

$$\begin{aligned}
0 &= \lambda \\
&+ \left\{ \frac{\eta_{1u'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u'}}{\eta_{2s^1}} \left[ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] + \dots \right. \\
&\dots + \left. \frac{\eta_{3u'}}{\eta_{3\theta}} \left[ -\lambda f_{1\theta} - \frac{f_{2\theta}}{f_{2d}} (R_d - \lambda f_{1d}) - \frac{\eta_{1\theta}}{\eta_{1s^0}} (R_{s^0} - \lambda f_{1s^0}) - \frac{\eta_{2\theta}}{\eta_{2s^1}} \left( R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right) \right] \right\} \\
&- \beta \mathbb{E} [R'_e - \lambda' f'_{1e}] \\
&- \beta \mathbb{E} \left( -\frac{f'_{2u}}{f'_{2d}} \right) [R'_d - \lambda' f'_{1d}]. \tag{19}
\end{aligned}$$

The benefit level  $b$  affects only current welfare and does not have an effect on the states of future economy.<sup>18</sup> As a result,  $b$  is set at a level that equates the marginal benefit (higher consumption for unemployed workers with benefits) and marginal cost (higher lump-sum tax) on current welfare. The equation  $R_b = 0$  reflects such incentives.

In contrast, the choice of  $d$  is more complex. The GEE (Equation 18) provides a way to interpret the incentives of the government when choosing  $d$ . From (18), any change in  $d$  has four effects. First, it directly affects the trade-off between current consumption and future unemployment (first line). In particular, a lower  $d$  (higher expected duration) increases current welfare by increasing the share of unemployed workers receiving benefits. This is the *insurance* effect. At the same time, a lower  $d$  also reduces average search, thus increasing future unemployment.<sup>19</sup> Second, through changing the expectation of future duration it affects current job search of benefit-collecting unemployed workers. This is the *moral hazard* effect. Changes in search of benefit-collecting unemployed workers in turn affect average job search and vacancy posting through general equilibrium effects (second and third lines). Third, any change in  $d$  affects future consumption through changing future unemployment (fourth line). This and the second effect together represent the “search/leisure” trade-off—lower  $d$  increases future unemployment, which in equilibrium reduces search today through a higher expected  $d'$ , thus increasing current welfare. Lastly, through changing  $d'$ , any change in  $d$  changes the future trade-off between consumption and unemployment (last line). The weight on the last line can be thought of as  $dd'/dd$  holding the two flow equations at zero and unemployment after the next period unchanged. The government determines current  $d$  by setting the net marginal value of  $d$  to zero.

Note that the GEE does not contain explicitly the derivative of  $\Psi^d$ ; it appears indirectly in the

<sup>18</sup> While the current benefit level does not affect search behavior, higher expected future benefit levels reduce current search incentive of an unemployed worker with benefits.

<sup>19</sup>  $f_{1d} = f(\theta)(s^0 - s^1)u^1$  is the marginal change in unemployment when  $d$  changes.

private-sector auxiliary function derivatives. In particular, the derivatives with respect to  $u'$  are

$$\begin{aligned}\eta_{1u'} &\equiv \frac{\partial \eta_1}{\partial u'} + \underbrace{\frac{\partial \eta_1}{\partial b'} \Psi_u^{b'} + \frac{\partial \eta_1}{\partial d'} \Psi_u^{d'} + \frac{\partial \eta_1}{\partial s^{0'}} S_u^{0'} + \frac{\partial \eta_1}{\partial s^{1'}} S_u^{1'} + \frac{\partial \eta_1}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}} \\ \eta_{2u'} &\equiv \frac{\partial \eta_2}{\partial u'} + \underbrace{\frac{\partial \eta_2}{\partial b'} \Psi_u^{b'} + \frac{\partial \eta_2}{\partial d'} \Psi_u^{d'} + \frac{\partial \eta_2}{\partial s^{0'}} S_u^{0'} + \frac{\partial \eta_2}{\partial s^{1'}} S_u^{1'} + \frac{\partial \eta_2}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}} \\ \eta_{3u'} &\equiv \frac{\partial \eta_3}{\partial u'} + \underbrace{\frac{\partial \eta_3}{\partial \theta'} \Theta_u'}_{\text{disciplining effect}},\end{aligned}$$

and the derivatives with respect to  $u^{1'}$  are

$$\begin{aligned}\eta_{1u^{1'}} &\equiv \frac{\partial \eta_1}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_1}{\partial b'} \Psi_{u^{1'}}^{b'} + \frac{\partial \eta_1}{\partial d'} \Psi_{u^{1'}}^{d'} + \frac{\partial \eta_1}{\partial s^{0'}} S_{u^{1'}}^{0'} + \frac{\partial \eta_1}{\partial s^{1'}} S_{u^{1'}}^{1'} + \frac{\partial \eta_1}{\partial \theta'} \Theta_{u^{1'}}'}_{\text{disciplining effect}} \\ \eta_{2u^{1'}} &\equiv \frac{\partial \eta_2}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_2}{\partial b'} \Psi_{u^{1'}}^{b'} + \frac{\partial \eta_2}{\partial d'} \Psi_{u^{1'}}^{d'} + \frac{\partial \eta_2}{\partial s^{0'}} S_{u^{1'}}^{0'} + \frac{\partial \eta_2}{\partial s^{1'}} S_{u^{1'}}^{1'} + \frac{\partial \eta_2}{\partial \theta'} \Theta_{u^{1'}}'}_{\text{disciplining effect}} \\ \eta_{3u^{1'}} &\equiv \frac{\partial \eta_3}{\partial u^{1'}} + \underbrace{\frac{\partial \eta_3}{\partial \theta'} \Theta_{u^{1'}}'}_{\text{disciplining effect}}.\end{aligned}$$

This reflects an important point made earlier—that successive governments agree on a policy rule  $\Psi^d$ . The Markov government does not try to manipulate its successor through changing current  $d$ , hence the absence of directives of  $\Psi^d$  directly from the GEE. The fact that  $\Psi^d$  affects private-sector auxiliary functions captures the fact that how much a higher  $d$  (lower expected duration) increases private-sector job search and vacancy posting depends on how the extra search will reduce next-period unemployment.

The Markov-perfect equilibrium is then characterized by a system of *functional* equations (1), (12)–(16), and (17)–(19). An analytical characterization of the Markov-perfect equilibrium is not possible; instead, we solve for the equilibrium numerically by approximating the government policy rules and private-sector decision rules using the Chebyshev collocation method.

## 4 PARAMETRIZATION

We describe our calibration strategy in this section. The model period is one month. We calibrate the parameters of the Markov equilibrium to match important features of the U.S. labor market between 2003.I and 2007.IV.

The utility function is

$$U(c, s) = \frac{c^{1-\sigma}}{1-\sigma} - v(s),$$

where  $v(\cdot)$  is the search cost function. We assume  $v(\cdot)$  is a non-negative, strictly increasing, and convex function, with the property that  $v(0)$  is bounded and  $v(0) \geq 0$ . We specify the search cost

function to be consistent with the literature:

$$v(s) = \gamma \frac{s^{1+\phi}}{1+\phi}.$$

For any  $\gamma > 0$ ,  $v$  exhibits positive and increasing marginal cost,  $v_s(s) > 0$  and  $v_{ss}(s) > 0$ , and  $v(0) = v_s(0) = 0$ .

We adopt the matching function from [Den Haan, Ramey, and Watson \(2000\)](#), which is also used in [Hagedorn and Manovskii \(2008\)](#) and [Krusell, Mukoyama, and Sahin \(2010\)](#) among others,

$$M(I, V) = \frac{V}{[1 + (V/I)^\chi]^{1/\chi}},$$

where  $I$  is the aggregate job search and  $V$  is the aggregate vacancy posting in the economy. This matching function guarantees that both the job-finding rate,

$$f(\theta) = \frac{\theta}{[1 + \theta^\chi]^{1/\chi}},$$

and the job-filling rate,

$$q(\theta) = \frac{1}{[1 + \theta^\chi]^{1/\chi}},$$

are always strictly less than 1.

We pick three parameters related to preferences. The discount factor  $\beta$  is  $0.99^{1/3}$ , giving a quarterly discount factor of 0.99. The coefficient of relative risk aversion  $\sigma$  is set to 1 (log utility). Finally, the search cost curvature parameter  $\phi$  is set to 1 following the average estimate in the literature.<sup>20</sup>

The externally calibrated parameters are summarized in [Table 1](#). Following the methodology outlined in [Shimer \(2005\)](#), we calculate the average monthly job separation rate from aggregate-level CPS data.<sup>21</sup> This gives an average job-finding rate during 2003.I-2007.IV of 0.40, and an average separation rate  $\delta = 0.02$ .<sup>22</sup> We set the costs of vacancy creation  $\kappa$  to be 58% of monthly labor productivity following [Hagedorn and Manovskii \(2008\)](#).

As in [Shimer \(2005\)](#), labor productivity  $z$  is taken to be the average real output per employed person in the non-farm business sector. This measure is taken from the seasonally adjusted quarterly

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<sup>20</sup> Imposing  $\phi$  equal to 1 gives a quadratic search cost function. This restriction is consistent with estimates by [Yashiv \(2000\)](#), [Christensen et al. \(2005\)](#), and [Lise \(2013\)](#), and calibration work of [Nakajima \(2012\)](#).

<sup>21</sup> To be consistent with our model, we do not adjust for time aggregation error when computing the job separation rate. Therefore, the job separation rate from the data is  $\delta_t = u_{t+1}^s / e_t$ , where  $u^s$  is short-term (one to four weeks) unemployment, and  $e$  is total employment.

<sup>22</sup> Although some may argue that the U.S. economy during 2003.I-2007.IV is *above* the long-run trend, we believe it is an appropriate period to target for the labor market, especially because of the secular downward trend in job separation rate documented by, for example, [Fujita \(2012\)](#). [Appendix B](#) also documents a declining trend in job-finding rate since 1951. Given these trends, using the long-run average job-finding and separation rates would overestimate the recent steady-state numbers.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
$\delta$	U.S. job separation rate	0.02
$\kappa$	Vacancy posting cost	0.58
$\rho$	Persistence of productivity	0.968
$\sigma_\epsilon$	Standard deviation of innovation to productivity	0.0060
$\epsilon_w$	Elasticity of wage with respect to productivity	0.446

Note: Calibration targets are monthly statistics of the U.S. economy.

data constructed by the Bureau of Labor Statistics. We normalize the mean productivity to be  $\bar{z} = 1$ , and assume an AR(1) process for the shock to  $z$ :

$$\log z' = \rho \log z + \sigma_\epsilon \epsilon,$$

where  $\rho \in [0,1)$ ,  $\sigma_\epsilon > 0$ , and  $\epsilon$  are i.i.d. standard normal random variables. We target a quarterly autocorrelation of 0.762 and an unconditional standard deviation of 0.013 for the HP-filtered productivity process. At a monthly frequency this means setting  $\rho = 0.9680$  and  $\sigma_\epsilon = 0.006$ .

Wages are function of productivity with the following functional form,

$$w(z) = \exp(\log \bar{w} + \epsilon_w \log z),$$

where  $\bar{w}$  represents the steady-state share of output for the worker, and  $\epsilon_w$  is the elasticity of the average wage with respect to aggregate productivity. We use data on labor productivity and real wages (constructed using labor shares data) between 1951.I and 2014.IV to estimate  $\epsilon_w = 0.446$ . This means a 1 percentage point increase in labor productivity is associated with a 0.446 percentage point increase in real wages. Our estimate is close to the estimate of 0.449 for 1951.I-2004.IV obtained by [Hagedorn and Manovskii \(2008\)](#).

We jointly calibrate four parameters using steady-state moments. The four parameters are (1) the value of home production (and leisure)  $h$ , (2) the matching function parameter  $\chi$ , (3) the level parameter of search cost  $\gamma$ , and (4) the steady-state wage level  $\bar{w}$ . We use four steady-state moments as targets: (1) the expected UI replacement ratio, (2) the average job-finding rate, (3) the average job-filling rate, and (4) the proportion of unemployed workers with benefits.<sup>23</sup> We follow [Shimer \(2005\)](#) and set the replacement ratio at 40%. The average job-finding rate is the monthly rate at which unemployed workers become employed, and it is 0.40 for 2003.I-2007.IV. Over the same period, the job-filling rate is 0.66.<sup>24</sup> Table 2 reports these internally calibrated parameter and the matching

<sup>23</sup> We use a derivative-free algorithm for least-squares minimization to perform joint calibration. See [Zhang, Conn, and Scheinberg \(2010\)](#) for details.

<sup>24</sup> The job-filling rate is calculated as the job-finding rate divided by the vacancy-unemployment ratio, where the latter is computed using the national unemployment rate reported by the BLS and the nonfarm job openings from the

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
$h$	Value of home production	0.595
$\gamma$	Disutility of search	1.706
$\chi$	Matching parameter	3.462
$\bar{w}$	Steady-state wage	0.979
Target	Data	Model
Average replacement ratio	40%	38.1%
Average job-finding rate	0.40	0.416
% unemployed with benefits	45	45.8
Average job-filling rate	0.66	0.661

Note: Calibration targets are monthly statistics of the U.S. economy 2005.I-2007.IV.

of calibration targets. The calibrated model delivers a benefit duration of 26.3 weeks, very close to the benefit duration of 26 weeks in the U.S. during normal times, thus delivering the first model validation.

Table 3 compares labor market statistics in the pre-2008 U.S. economy and the calibrated Markov economy.<sup>25</sup> The calibrated model does a good job of generating the relevant cyclical properties, which provides the second model validation for our theory. The model also produces negative correlation between unemployment and vacancy, thus preserving the shape of the Beveridge-curve (inverse relation between unemployment and vacancy). Two parameters allow our model to match cyclical properties well. First, we calibrate the elasticity of wage with respect to productivity to match data counterparts. The relatively low wage elasticity means firm's profit and hence vacancy posting are volatile over the business cycle. Second, by targeting the average replacement ratio, we obtain a high value of home production, which contributes to high volatility in job search. Unlike [Shimer \(2005\)](#) and [Hagedorn and Manovskii \(2008\)](#), the benefit level in our model is endogenously chosen by the government and is a function of home production in steady state. As such, we can use the home production value to target replacement ratio.

## 5 EQUILIBRIUM ANALYSIS

In this section, we present the Markov government policy rules and discuss their effects on the equilibrium labor market.

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Job Openings and Labor Turnover Survey. The estimate for the 2003.I-2007.IV period is close to [Den Haan, Ramey, and Watson \(2000\)](#) who use plant-level data during 1972.II-1988.IV and get a job-filling rate of 0.71.

<sup>25</sup> We use statistics from pre-Great Recession as the empirical counterpart to evaluate whether our model with average job separation rate can generate the long-run volatilities. In Section 6 we introduce shocks to the job separation rate to look at the labor market during the Great Recession.

Table 3: Summary Statistics: Cyclicality

Statistic		Productivity $z$	Unemployment $u$	Vacancy $v$	v-u ratio $v/u$
<i>Quarterly U.S. data 1951.I-2007.IV</i>					
Standard deviation		0.013	0.123	0.142	0.257
Correlation matrix	$z$	1	-0.271	0.392	0.339
	$u$	-	1	-0.889	-0.951
	$v$	-	-	1	0.980
	$v/u$	-	-	-	1
<i>Calibrated Markov economy</i>					
Standard deviation		0.013	0.147	0.167	0.273
Correlation matrix	$z$	1	-0.908	0.919	0.982
	$u$	-	1	-0.698	-0.909
	$v$	-	-	1	0.933
	$v/u$	-	-	-	1

Note: Seasonally adjusted unemployment,  $u$ , is constructed by the BLS from the CPS. Vacancy-posting,  $v$ , is [Barnichon \(2010\)](#)'s spliced series of seasonally adjusted help-wanted advertising index constructed by the Conference Board and the job-posting data from the JOLTS. Both  $u$  and  $v$  are quarterly averages of monthly series. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1,600.

## 5.1 Markov equilibrium policy rules

Figure 2 plots the Markov equilibrium policy rules for UI policies holding productivity at the steady state level.<sup>26</sup> In each plot, the solid line represents policy rule, and the dashed line represents steady-state unemployment.

The expected UI duration  $1/d$  increases in total unemployment. The decision on UI duration involves a trade-off between insurance (for higher current consumption) and job-creation (for higher future welfare). When unemployment is high, both the insurance and job-creation incentives are high—the former because more people are unemployed, and the latter because shorter duration would increase job-search incentives for more people through their expectations of future UI policies. In equilibrium, the increase in insurance incentive outweighs the higher job creation incentive, and expected duration is longer at higher unemployment.

In contrast, the UI benefit level  $b$  is lower at higher unemployment, but the scale of change is minuscule, falling by less than 1% from steady state to 10% unemployment. Intuitively, when unemployment is high, the cost of taxation rises slightly more than the gain from insurance. The almost flat policy rule of  $b$  reflects that when unemployment increases the rise in cost of taxation is almost entirely offset by the higher gain from insurance.

<sup>26</sup> We also hold the proportion of unemployed workers with benefits at the steady-state level.

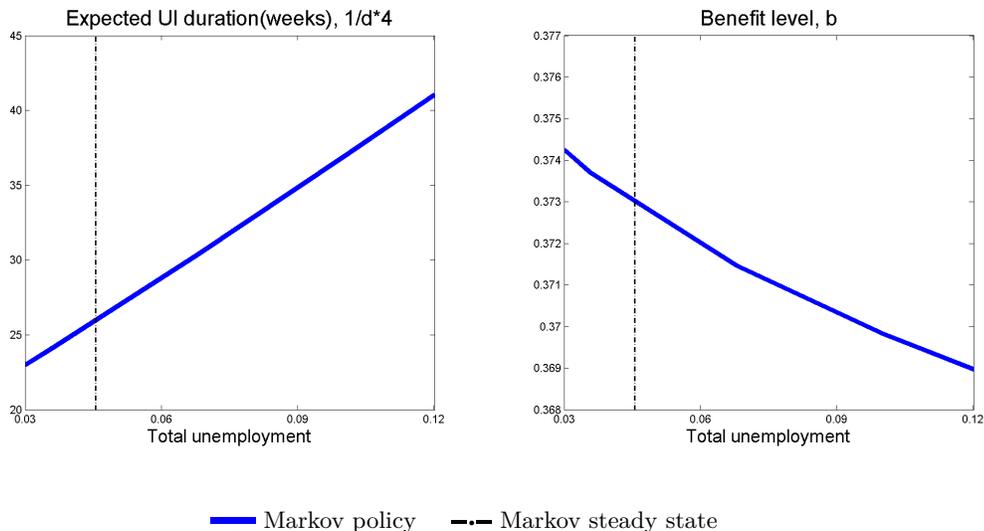


Figure 2: Markov equilibrium government policy rules holding productivity and proportion of benefit-eligible unemployed workers at steady state.

Figure 3 plots the Markov equilibrium government policy rules, holding unemployment (both total unemployment and benefit-eligible unemployment) at the steady-state levels. The expected duration increases dramatically with lower labor productivity, especially when productivity is below its steady state. This is because when productivity is low, the expected productivity next period is also low, assuming a persistent productivity process. As such, the marginal return from production tomorrow (for both workers and firm) is low, as is the cost of low job creation today (high unemployment tomorrow). As a result, the marginal cost of longer duration (lower job creation) is low, and the government chooses long duration. The unemployment benefit level  $b$ , in contrast, increases with higher labor productivity, but the slope is fairly small, indicating very small changes with respect to labor productivity.

## 5.2 Comparative static analysis of government incentives

Because the government's choice of  $d$  (which directly translates into expected duration) involves the trade-off between insurance and moral hazard, we conduct comparative static analysis to understand changes in these two incentives that drive the movements in expected duration. Figure 4 shows responses of these two incentives to changes in unemployment (left panel) and productivity (right panel).<sup>27</sup>

As total unemployment rises, both insurance gains and moral hazard cost are higher. More specifically, when unemployment is higher, longer UI duration has a larger insurance effect because it extends benefit coverage to more unemployed workers. In terms of the GEE (Equation 18), this

<sup>27</sup> The responses over unemployment hold constant the proportion of benefit-eligible unemployed workers and productivity at steady state. The responses over productivity holding total and benefit-eligible unemployment at steady states. Both subplots hold UI policies at steady state.

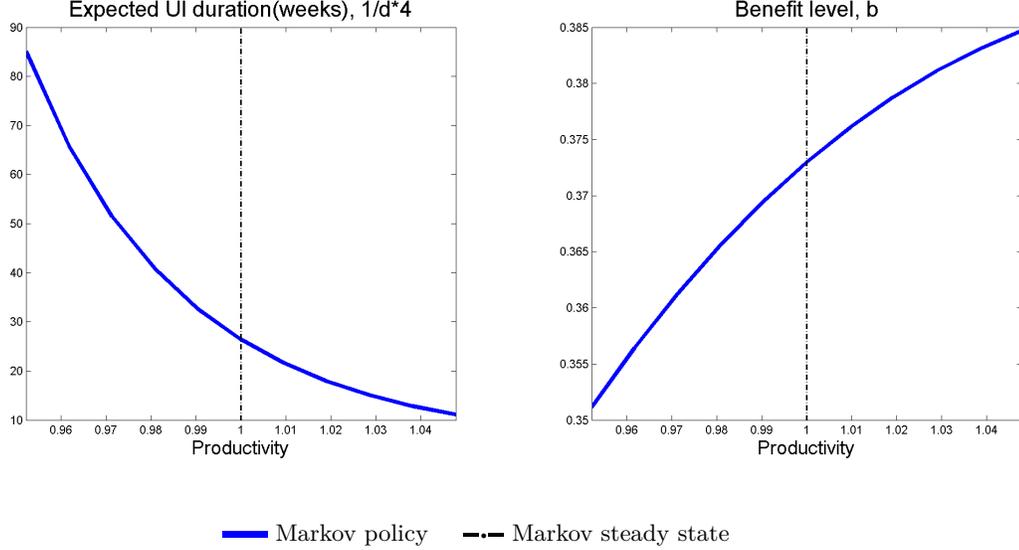


Figure 3: Markov equilibrium government policy rules holding unemployment at steady state.

effect affects the marginal effect of government choice  $d$  (and hence expected duration) on current welfare (part of  $R_d$ ):

$$- u^1 \times \underbrace{[U(h + b - \tau) - U(h - \tau)]}_{\text{welfare gain from giving benefits to an additional worker}} \quad (20)$$

With higher unemployment ( $u$ ) and fixing the proportion of benefit-eligible unemployed workers ( $u^1/u$ ),  $u^1$  in expression (20) is larger, which increases the marginal welfare gain from a smaller  $d$  (longer expected duration). At the same time, higher unemployment makes future unemployment more sensitive to changes in search induced by changes in the current UI policy, and as a result, longer UI duration has a larger moral hazard cost. The left panel of Figure 4 shows that a 1% increase in total unemployment raises the insurance incentive by 16% and moral hazard cost by 9%. The higher increment in insurance incentives means that at a higher unemployment level, the government has a stronger incentive to increase expected UI duration.

In response to a drop in productivity, both insurance gains and moral hazard cost are lower. In particular, lower productivity leads to lower wages, which increases the employed workers' marginal utility of consumption and reduces the marginal gain from insurance. This effect is small, and disappears if workers are risk neutral. In contrast, the drop in moral hazard cost is larger. In response to a lower productivity, the expected future productivity and wages are also lower, which means that future unemployment leads to a smaller reduction in average consumption. In other words, there is lower moral hazard cost as a result of lower average search induced, and the government can “afford” to choose longer duration. The drop in moral hazard cost is amplified by a drop in job posting—result of lower productivity and hence lower expected future profit—which lowers the response of future unemployment to changes in the duration policy. This amplification accounts for the nonlinear shape

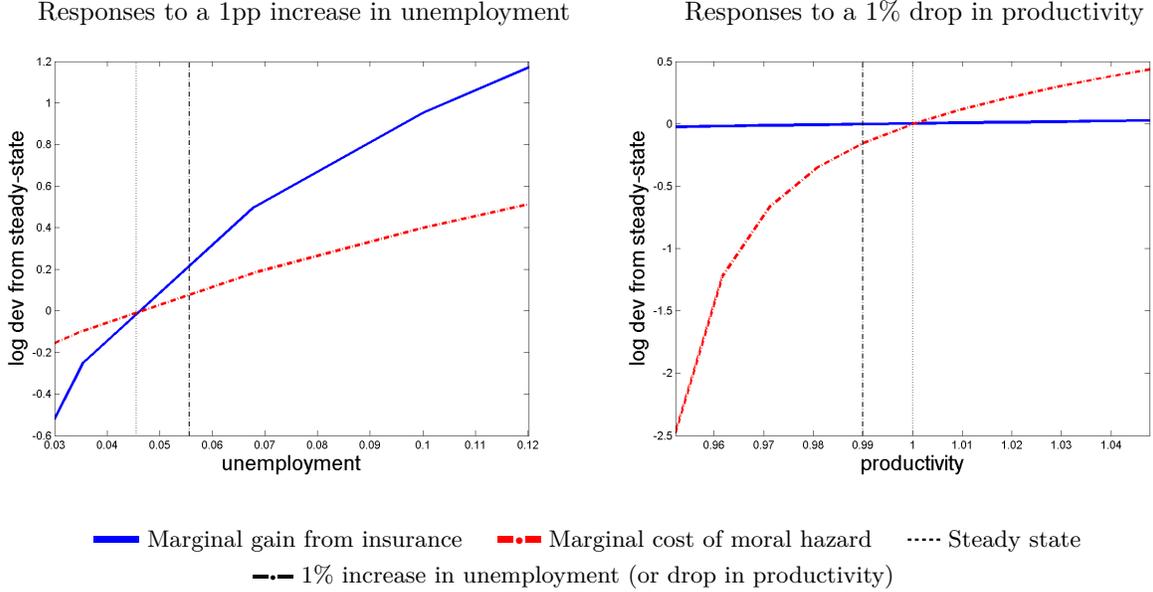


Figure 4: Responses of marginal gain from insurance and marginal moral hazard cost to a 1 percentage-point increase in unemployment or 1% drop in productivity, holding government policies at the steady state.

of duration policy in response to productivity changes.

The variations of marginal welfare gain and cost here are consistent with recent empirical findings by [Kroft and Notowidiglo \(2015\)](#). First, they find that the moral hazard cost is procyclical. The marginal cost of moral hazard here varies positively in both unemployment and productivity and is overall procyclical.<sup>28</sup> Second, they find that the marginal welfare gain from consumption smoothing varies positive with the unemployment but variations are small. The marginal gain from insurance in our mechanism also varies positively with unemployment, but the scale of variation is large. This is because the gain from consumption smoothing that they document is only part of the gain from insurance in our mechanism. Most of the variations in the gain from insurance in the left panel of Figure 4 come from an *extensive* margin: when unemployment is high, the gain from extending benefits to more unemployed workers is high because it increases the average consumption of all unemployed workers.

### 5.3 Impulse response in policy and labor market

We now consider the economy’s response to a one-time, unanticipated drop in productivity. Figure 5 shows the response of the economy to a 1% drop in productivity  $z$  at time 0. We first focus on the responses with Markov policy (solid blue lines). We then compare the responses of the economy with and without (dotted red lines) government policy changes to understand the driving forces behind

<sup>28</sup> While we distinguish between a drop in productivity and an increases in unemployment, the empirical work of [Kroft and Notowidiglo \(2015\)](#) does not. So their result that moral hazard cost is higher when the unemployment rate is lower should not be directly compared to the left panel of Figure 4.

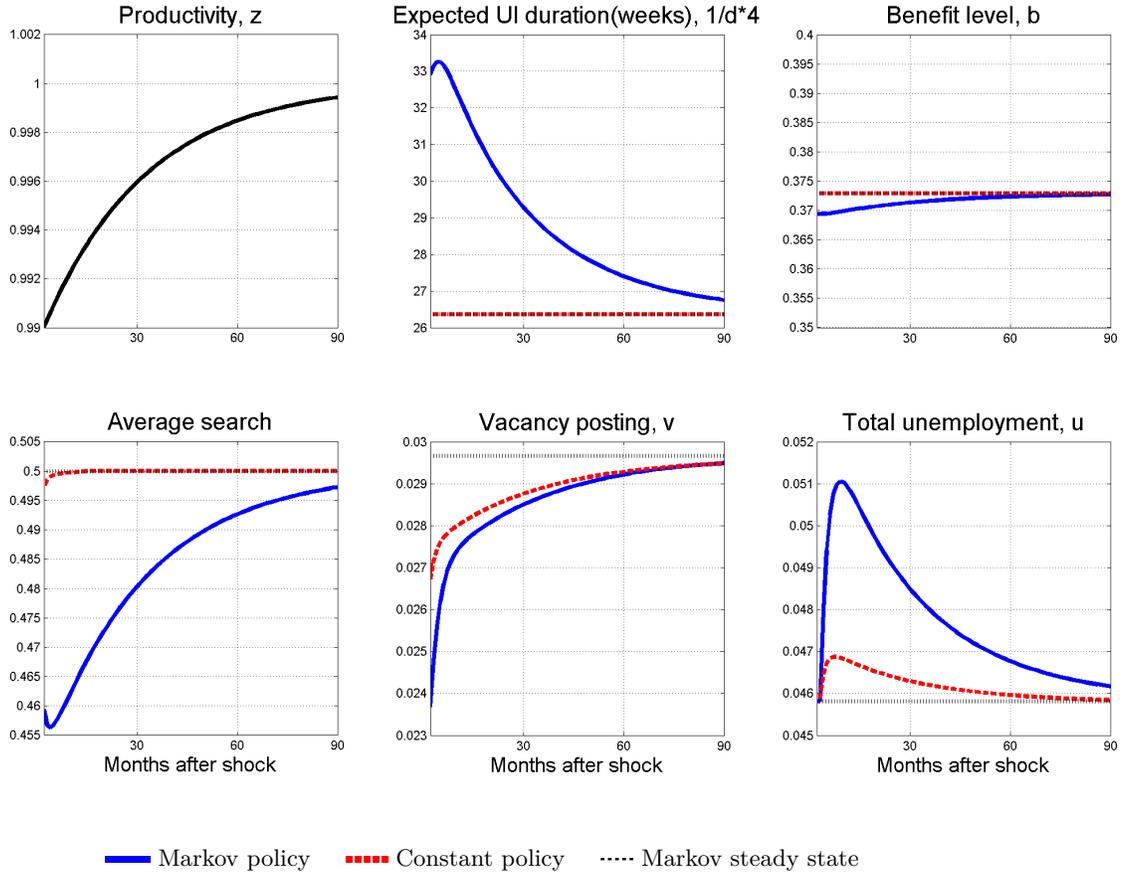


Figure 5: Impulse response to a 1% drop in productivity.

movements in the labor market.<sup>29</sup> Because the transition dynamics are relatively slow, it takes a long time for the economy to return to a steady state. In Figure 5, the time horizon is 90 months or approximately seven and half years.

Upon shock, expected duration rises immediately from 26.3 weeks to 33 weeks and then falls slowly as productivity recovers. By month 30, expected duration has fallen to 29 weeks. Since unemployment is a slow-moving process, it peaks at around month 7, when productivity has already recovered one-fifth of the 1% drop. Because expected duration rises with higher unemployment, the drop in expected duration after the initial rise is slowed by the rising unemployment. Benefit level, in contrast, falls to below steady state on impact, with less than 1% total change, and slowly recovers to the pre-shock steady state as both productivity and unemployment recover.<sup>30</sup>

Search by both insured (benefit-collecting) and uninsured unemployed workers fall upon impact,

<sup>29</sup> Appendix E provides impulse responses of other labor market statistics.

<sup>30</sup> The benefit level in our model is slightly procyclical. This is because during a recession, lower wages and higher total unemployment raises the marginal cost (in utility terms) of providing benefits. Even though the scale of changes is small, the procyclical benefits go against what happens in a typical recession. To be more realistic, it is reasonable to think that during a recession, the government has a more lax budget and does not have to impose lower benefit levels.

which drives average search down by about 10%. While insured unemployed workers search less in response to much longer expected UI duration, the uninsured unemployed workers respond mainly to lower expected future output and wages. Vacancy posting also falls initially but the recovery is much quicker than the job search recovery. By month 6, vacancy posting is more than half-way back to the pre-shock steady-state level. This is because vacancy posting depends on expected future productivity and aggregate search. As search by individual unemployed workers recovers, and with high unemployment during the first few months after shock, *aggregate* search is high.<sup>31</sup> Because higher aggregate search increases the marginal return from vacancy posting, vacancy posting responds to aggregate (and not average) search. Total unemployment increases rapidly to peak in month 7, before gradually falling back to its steady-state level.

To understand to what extent the rise in unemployment is driven by changes in policy versus productivity, we shut down changes in government policy and only allow the labor market to respond to changes in productivity. Compared to unemployment increases with policy changes (solid blue lines), the increases without policy change (dotted red lines) are much more muted (1% versus 5%). Behind the difference in unemployment are smaller drops in both search and vacancy posting without policy change. In particular, the average search drops by less than 1%, compared to a 9% decrease with policy changes. The drop in vacancy posting without policy changes is about half of the drop with policy change. The larger difference in search reflects that job search incentives are distorted by policy changes whereas vacancy posting incentives respond mostly to productivity changes.

## 6 UI DURATION EXTENSIONS IN RECESSION

Because the cyclical properties of Markov equilibrium policy rules are consistent with those of the U.S. policy, in this section we use the theoretical framework to study recessions. We first validate the model by using the model to account for expected duration variations during and after the Great Recession (December 2007 to December 2013). We then compare the Markov policy to alternative UI policies to study the impacts on unemployment and welfare.

### 6.1 Empirical evidence of UI benefit extensions in recessions

We first document variations in UI duration during each recession since the 1970s. Figure 6 plots variations in unemployment and UI duration during all five recession episodes.<sup>32</sup> The shaded areas represent National Bureau of Economic Research (NBER) official recession dates. For each recession episode, the dotted red line (right axis) plots the unemployment rate, and the solid blue line (left axis) plots the maximum expected UI duration in weeks. The timing and extent of changes in UI

<sup>31</sup> Additional impulse response in Appendix E shows above steady-state aggregate search during the first few months after shock.

<sup>32</sup> The recession from January to July 1980 was both shorter and milder than the other recessions. In addition, it was followed immediately by the much longer recession from July 1981 to November 1982. We therefore left out the former recession period.

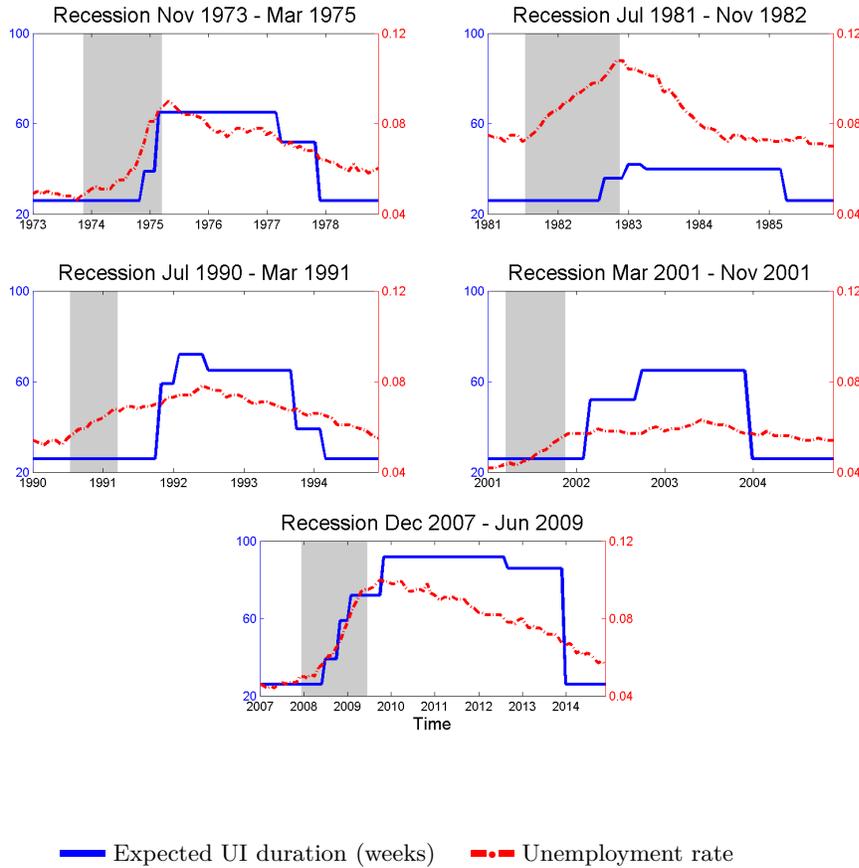
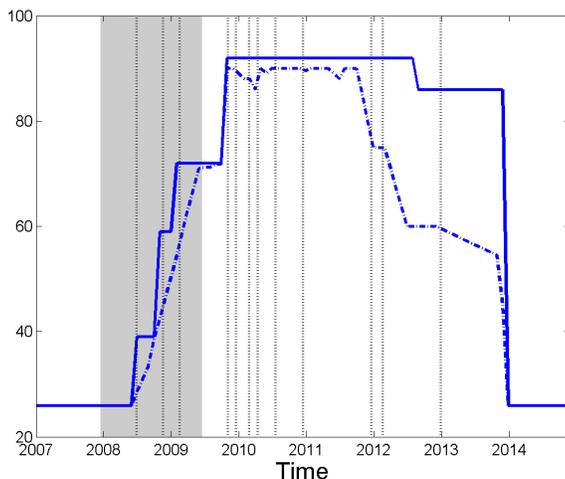


Figure 6: Empirical changes in unemployment (right axis) and UI duration (left axis) during recessions since the 1970s.

duration follow the specifics of the federal unemployment compensation laws, which are available from the U.S. Department of Labor Employment and Training Administration (DOLETA) website. Two things are worth noting. First, during all recession episodes, UI duration reached its highest level around the time unemployment peaked. Second, comparing across recessions, recession with higher unemployment is associated with in general higher expected UI duration. This, however, was not true for the 1980s recession. The fact that Markov benefit duration rises with total unemployment is consistent with the above historical evidence.

Because more detailed data are available for the Great Recession, Figure 7 documents the frequency of legislation on UI policy during and following the recession. The vertical dotted lines indicate the timings of legislation. The frequency of legislation increased substantially from the mid-2008, especially from late 2009, to 2011. This provides evidence that during the recessions the federal government does not follow a prescribed policy rule and instead makes policy choices depending on the contemporary states of the economy.<sup>33</sup> This observation motivates our choice to use the Markov

<sup>33</sup> There is an automatic benefit extensions program called Extended Benefits (EB), whereby the benefit duration is automatically extended when a state's unemployment rate exceeds 6.5% or 8%. The EB extensions are triggered in



— Expected UI duration (weeks)    - - - Weighted UI duration (weeks)    ····· UI legislation

Figure 7: Empirical changes in UI duration and timing of UI-related legislation during the Great Recession.

policy, which is time consistent and without commitment, to describe the policy changes during the recession.

Because state-level implementations of UI benefit extension are conditional on state economic conditions, especially on the state’s insured unemployment rate (IUR) and total unemployment rate (TUR), we use the two statistics to compute whether the state was eligible for longer durations in the month a UI-related legislation was passed during the Great Recession. We then create a weighted measure of expected UI duration across states using the number of total insured unemployed workers in each state as the weight. Appendix A contains more details. Figure 7 plots weighted expected UI duration with a dashed blue line. For the quantitative analyses, we use this weighted average series as the empirical counterpart because it is a more accurate description of the UI duration policy implemented.

## 6.2 The Great Recession

To further our theory, we put the model in an environment similar to the U.S. economy during and following the Great Recession from December 2007 to December 2013. Because our theory focuses on UI, we specify a path for productivity to match the observed unemployment path during the period.<sup>34</sup> We use a piecewise linear productivity process consisting of the decline, the trough and,

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a state regardless of the national economic conditions or what the Congress decides. So in a sense this is a committed extensions program, in contrast to the discretionary extensions implemented in recessions. During the Great Recession, the EB extensions represent about one-third of the total overall maximum extensions. We thank an anonymous reviewer for pointing this out.

<sup>34</sup> Appendix E includes two alternative calibrations where we use productivity  $z$  to either match the path of UI duration or get a best fit for both unemployment and duration. The benchmark calibration of productivity process

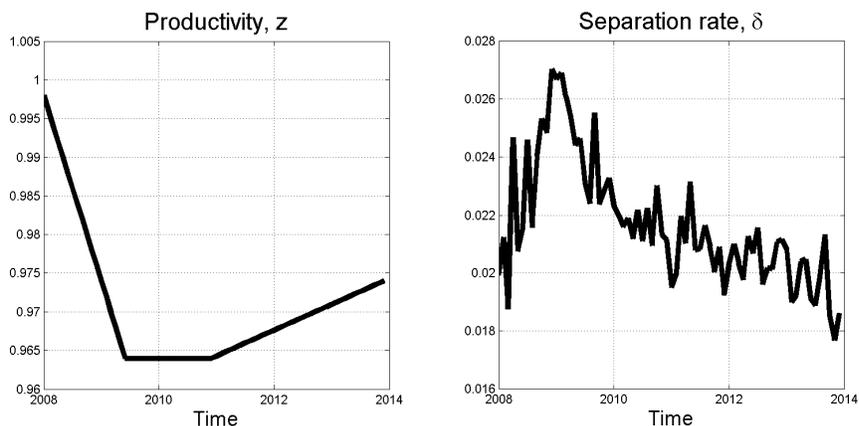


Figure 8: Exogenous shock processes during the Great Recession.

the recovery. It turns out that this simplified way of calibrating the productivity path generates good fit for unemployment. The job separation rates over this period are exogenous and calculated from aggregate-level CPS data. This path is then fed into the model assuming they are unexpected shocks and the agents expect future separation rate to return to its steady state. In Section 6.2.5 we look at the case where the agents have an expectation about how future job separation rate evolves. Figure 8 plots these shock processes.<sup>35</sup>

**Model Fit** Given the shock processes, Figure 9 plots the variations in UI duration, labor market variables, and average welfare generated by the Markov equilibrium (solid blue line). Compared to the U.S. economy (dashed black line), the Markov equilibrium matches well the variations in UI duration and the vacancy-unemployment ratio. In particular, the Markov expected duration policy rises from 26 weeks to slightly below 80 weeks, compared to the maximum of 90 weeks in the U.S. economy. The Markov policy also generates a decline in expected duration, but the decline starts earlier than in the U.S. economy. The Markov policy captures the sudden drop and slow rise in the vacancy-unemployment ratio as observed in the U.S. economy, but underestimates the scale of the drop. One reason for the smaller drop in the vacancy-unemployment ratio is that the model does not have job-to-job transition. During recessions, job-to-job transition, in addition to unemployment-to-job transition, declines, and as a result, vacancy posting should decline more when job-to-job

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here is the preferred one because while unemployment is a smooth process (and reported on a weekly basis), UI duration is not because Congress meets on a relatively fixed schedule that does not respond to changes in economic conditions. Therefore, matching to UI duration is subject to underlying assumptions about meeting schedules.

<sup>35</sup> The process we specify for labor productivity is in fact not far-fetched. Labor productivity as measured by average production per person in the nonfarm business sector by 3% from the end of 2007 to the beginning of 2009, which is more than 4% in detrended terms. The difference between our process and the U.S. process is the recovery path. U.S. labor productivity recovered swiftly to pre-2008 levels by the end of 2010, whereas our process follows a slower recovery process. For theories on slow and/or jobless recovery following the Great Recession, see, for example, [Stock and Watson \(2012\)](#), [Shimer \(2012\)](#), and [Heathcote and Perri \(2015\)](#). In addition, [McGrattan and Prescott \(2010, 2014\)](#) suggest that labor productivity calculated from the data—especially during the 1990s and the Great Recession—are mismeasured. This paper does not take a stand on what the true labor productivity is and instead use the observed unemployment path to discipline productivity.

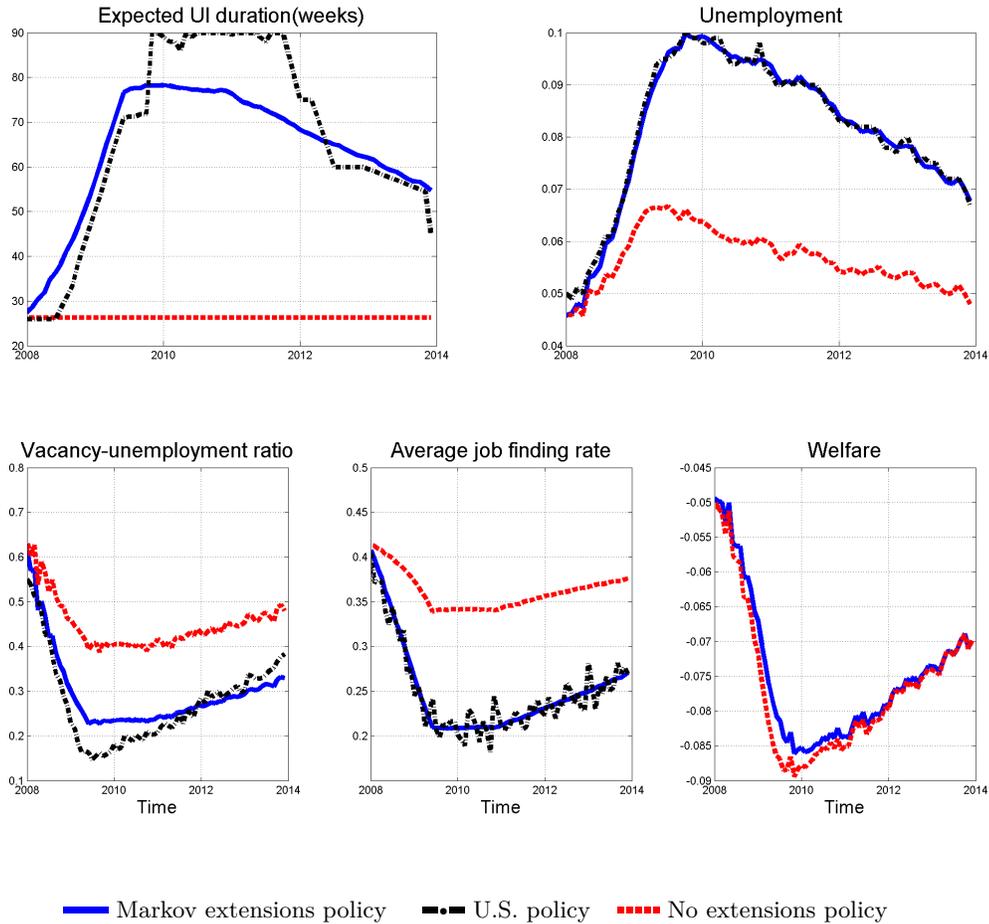


Figure 9: UI duration, unemployment, and welfare during the Great Recession: Model, data, and counterfactual policies.

transition is taken into account.

### 6.2.1 Policy Evaluation

One interesting question we study using the model is whether and how much do benefit extensions contribute to higher the unemployment rates. We use the counterfactual where the government does not do benefit extensions but instead keeps benefit duration at 26 weeks throughout the recession (and the private sector fully understands it). Figure 9 shows that, in contrast to the no-extension policy (dotted red line), the Markov benefit extensions policy leads to higher unemployment rates. At the peak of unemployment, unemployment is lower by more than 3 percentage points in the economy without extensions.

A key prediction of our theory is that search effort is procyclical, that it falls during recessions. This feature is present in the standard search model with endogenous search effort. The empirical

findings on the cyclicity of search effort are mixed.<sup>36</sup> More recently, [Gomme and Lkhagvasuren \(2015\)](#), after controlling for heterogeneity in the unemployed worker’s past wages and hours, find evidence that search is procyclical, consistent with the prediction of structural search literature.

**Welfare Evaluations** We perform the welfare evaluation of extensions at two points in time. First, consistent with how the effect on unemployment is evaluated, we look at the *ex-post* welfare. This is computed at each point in time over the transition path as the average welfare of all workers in the economy, after the realization of shocks. The last panel of [Figure 9](#) plots the welfare during this period. Even though the Markov extensions substantially raised the unemployment, the average welfare is higher with the Markov extensions. The difference in the average welfare over the transition path roughly translates into 0.16% of average monthly consumption.<sup>37</sup>

Second, we compare the welfare at the start of the recession. This is an ex-ante comparison. The difference here is the ex-ante evaluation is done without the knowledge of how the recession would pan out, and agents in the economy expect future shocks to follow an AR(1) process. We assume that an unexpected productivity shock occurs in January 2008, and perform the welfare comparison after the realization of this shock. The question addressed by this exercise is, “*should* the government follow the Markov rule or the no-extensions policy given this shock?” Interestingly, the no-extensions policy gives slightly higher (less than 0.1% in consumption equivalent) average welfare than the Markov policy rule ex ante. The reason for this welfare reversal is that the recession during this period turns out to be both long and severe, which gives more justification for the Markov policy.

### 6.2.2 The effect of expectation

To isolate the effect of expectation in our result, we shut down the channel of private sector’s expectations. In particular, in this exercise we assume that the private sector expects future UI duration (and benefit level) to stay at the steady-state level. At the same time, we assume that the government implements the same policy as before ex-post. In other words, all benefit extensions above the regular 26 weeks are “unanticipated.” [Figure 10](#) illustrates the experiment by comparing the economy under (1) Markov benefit extensions policy (solid blue line), (2) no benefit extensions policy (dotted red line), and (3) unexpected extensions policy (dashed green line).

The unemployment gap between Markov extensions and unexpected extensions policies is large, accounting for about 70% of the unemployment gap between the Markov extensions and the no-extensions economics. This reflects the importance of expectations. When insured unemployed workers rationally expect the government to follow Markov policy, they reduce their job search

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<sup>36</sup> [Shimer \(2004\)](#) and [Mukoyama, Patterson, and Şahin \(2006\)](#), for example, find countercyclical search effort, while [DeLoach and Kurt \(2013\)](#) find evidence of procyclical search effort. See [Gomme and Lkhagvasuren \(2015\)](#) for a brief review of the empirical literature on search effort.

<sup>37</sup> Even though shocks to both productivity and job separation rate contribute to cyclical fluctuations in the model, the shock to productivity actually drives most of the movement. [Appendix E](#) provides an analysis where productivity is kept at its steady-state level to isolate the effect of the shock to job separation rate.

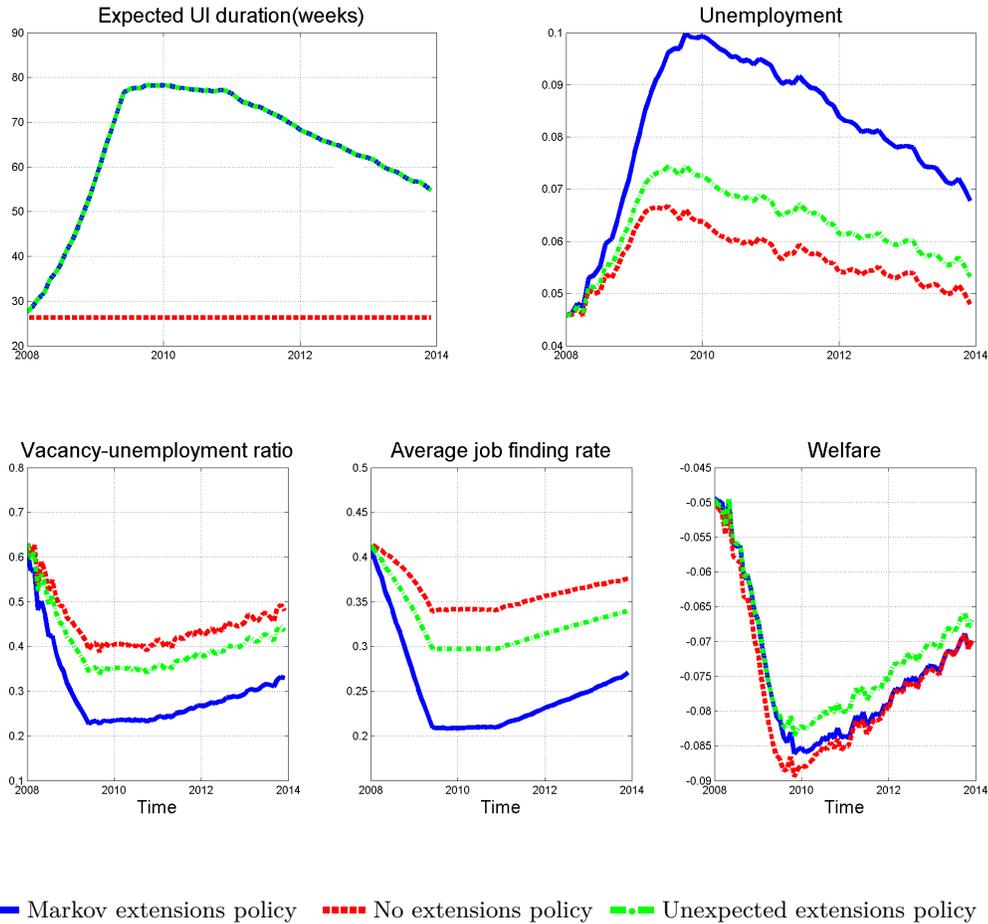


Figure 10: UI duration, unemployment, and welfare during the Great Recession: Effect of expectation.

activities when productivity is low or when unemployment is high, because the expected long UI duration next period distorts their search incentives. In contrast, when unemployed workers expect the government to maintain a no-extensions policy, they do not reduce search as much.

Because the distortion on search incentives works only through unemployed worker’s expectation of future UI policies, this exercise is also a decomposition of the unemployment gap into the “search” wedge and the “composition” wedge. More specifically, the unemployment difference between Markov extensions and the unexpected extensions represents the effect of policy distortion on search behavior; the unemployment difference between the unexpected extensions and no-extensions economics represents the effect of policy in changing the composition of the unemployed population—longer duration increases the proportion of unemployed workers with benefits, thus reducing average search.

Interestingly, the average welfare of under unexpected extensions is higher than in the Markov equilibrium. This is not surprising, because the former economy has both high current consumption—from the ex-post Markov policy—and low future unemployment—due to the expectation of no-extensions policy. But such an economy is in a sense “unsustainable” as it requires that the govern-

ment always be able to “fool” the private sector.

### 6.2.3 Drivers of the *ex-post* welfare gap

The *ex-post* welfare gap between the Markov and no-extensions policies in Figure 9 are driven by two opposing forces: the higher unemployment under the Markov policy reduces the average welfare in the Markov economy relative to the no-extensions policy, whereas the higher proportion of unemployed workers on benefits increases the average welfare of unemployed workers in the Markov economy. Figure 11 illustrates these two forces.

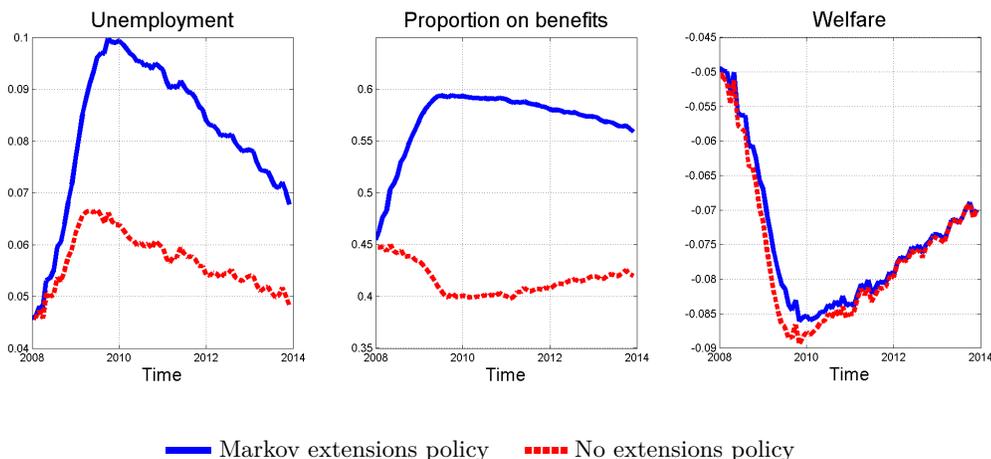


Figure 11: Drivers of welfare gap between Markov extensions and no extensions policies: Unemployment and proportion unemployed on benefits.

The proportion of unemployed workers on benefits (middle panel) is calculated as  $u^1(1-d)/u$ . Under the no-extensions policy, the proportion falls early in the recession. While both the measure of benefit-eligible unemployed worker,  $u^1$ , and the total unemployment,  $u$ , increase in response to rising job separation rates and falling productivity, the rise in total unemployment is larger because the job-finding rates of both insured (benefit-collecting) and uninsured unemployed workers fall. In contrast, under the Markov policy, the initial rise in the proportion is mainly driven by the longer duration policy (lower  $d$ ).

The gap in benefit coverage between the two policies increases over time. At the time when unemployment peaks, about 60% of unemployed workers are covered by UI benefits under the Markov policy, whereas only 40% have benefits under the no-extensions policy. The higher benefit coverage under the Markov policy leads to higher average welfare among unemployed workers relative to the no-extensions policy. The welfare gap (right panel) between the Markov and no-extensions policies is then the result of the welfare cost of higher unemployment being outweighed by the welfare gain from higher benefit coverage ratio. An important reason for this result is that wages are low during

the recession, which lower the marginal cost of unemployment.<sup>38</sup>

#### 6.2.4 Relation to Hagedorn, Manovskii, and Mitman (2015)

The comparison between Markov equilibrium and the economy under no-extensions policy is in line with Hagedorn, Manovskii, and Mitman (2015), who exploit discontinuity at state borders to identify the effect of unemployment benefit extensions. In particular, one way to interpret their empirical result is follows. In states where firms and workers expect good exogenous shocks to the economy, e.g. an oil boom, they also expect lower or no benefit extensions. This increases the expected value of employment to a worker and in turn increases job creation compared to states with bad economic outlooks. This interpretation is very similar to our theory. With the Markov policy, firms and workers *expect* longer benefit durations in recessions—analogueous to states with bad economic outlooks—whereas under the no-extensions policy, the private sector *expects* no-extensions policy—an extreme case of states with good exogenous shocks.

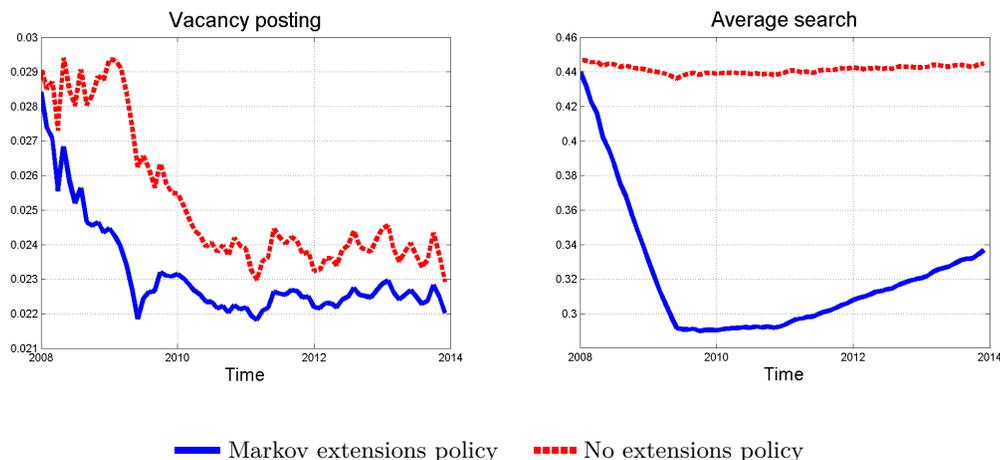


Figure 12: Markov extensions versus no extensions policy: Vacancy posting and average search.

One difference between our theory and Hagedorn, Manovskii, and Mitman (2015) lies in the mechanism underlying our results. In their model, longer benefit duration increases unemployed worker’s outside option, thus increasing their reservation wage. Higher reservation wage then reduces firm’s vacancy posting by reducing profit margin. In our model, longer expected duration makes unemployment less painful and thus reduces job search activity. Reduced search activity then reduces the marginal return to vacancy posting, lowering overall vacancy posting. To see how much of the unemployment gap in Figure 9 comes from differences in vacancy posting as opposed to job search, Figure 12 compares the Markov equilibrium and the economy under no-extensions policy along these two dimensions. Both vacancy posting and average search are higher in the economy with no-

<sup>38</sup> Another reason is that the long duration policy discourages search by the benefit-collecting unemployed workers, which means even if benefit coverage were the same under the two policies, the average welfare of unemployed workers would still be slightly higher under the Markov policy.

extensions UI policy, but the gap is much larger for average search. This comparison reflects that in our model both vacancy posting and search contribute to the unemployment gap, but average search contributes more.

### 6.2.5 Cyclical job separation risk

So far the cyclical job separation rate is both exogenous and unexpected. As an alternative specification, we make the job separation shock contingent on productivity in the model, so that the private sector takes into consideration the cyclical job separation rate—and form expectations accordingly—when making decisions. We specify the job separation rate process as

$$\delta(z) = \bar{\delta} + I_{\delta}(z - \bar{z}),$$

where  $\bar{\delta}$  is the steady-state job separation rate, and  $I_{\delta} < 0$  is the rate of change of the separation rate with respect to aggregate productivity. This formulation has the natural interpretation that the job separation rate increases when profits are low. When labor productivity is low, wages are low as wages are also a function of productivity. Because the elasticity of wages with respect to productivity is less than 1, lower productivity means lower profit, or  $z - w$  in the model. To estimate this process we use job separation and labor productivity data over 1951.I-2014.IV.

As before, productivity shock  $z$  is exogenously specified to match the unemployment process. The resulting labor productivity process requires a smaller drop than before—3.2% as opposed to 3.6% without changing separation risk. This is because the presence of countercyclical separation rate reinforces the effect of productivity shock. Figure 13 shows the transitions for this alternate specification.

## 6.3 Other recessions

As noted in the empirical analysis, longer UI duration is not just a phenomenon during the Great Recession. Comparing across recession episodes since the 1970s, recessions with higher unemployment were associated with, in general, higher UI durations. In this section, we test whether our model delivers this characteristic. Because of the declining secular trend in job-finding and separation rates, we need to recalibrate the model parameter to the pre-recession period for each recession episode. Table 4 summarizes the labor market statistics for the pre-recession window for each recession.

As with the Great Recession, we use the path of job separation rate from data, and target observed unemployment path to recover the path of productivity for each recession. Figure 14 displays model-generated expected UI duration (solid blue line), unweighted UI duration from data (dashed blue line), and model-generated unemployment (broken red line, right axis) for each recession documented in the empirical analysis. Three observations are worth noting. First, the model matches increases in UI duration reasonably well, producing more than 60% of the increases (solid blue line vs. dashed

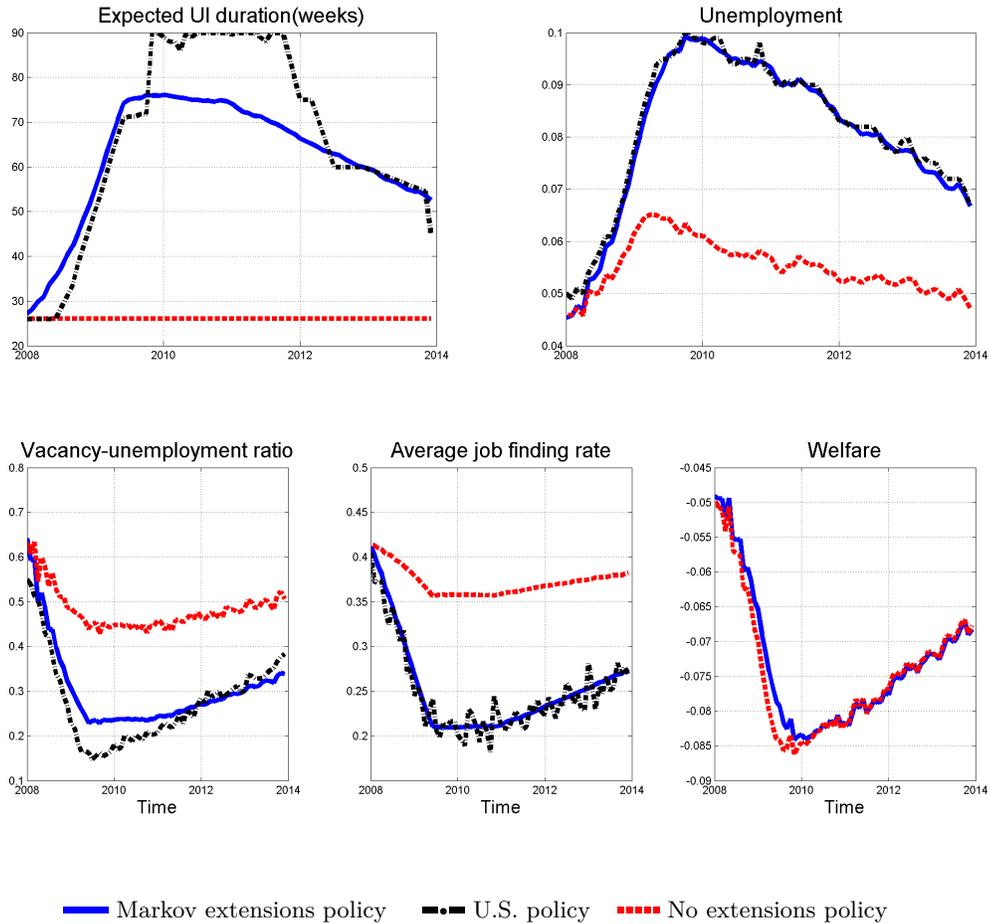


Figure 13: With cyclical separation risk: UI duration, unemployment, and welfare during the Great Recession.

blue line) in each recession. Second, consistent with patterns documented from the data, during all four recessions, model-generated UI duration reached its highest level around the time unemployment peaked. Lastly, recessions where unemployment was higher (broken red line) also had, in general, higher model-generated UI duration (solid blue line), except the 1980s recession. This evidence shows that, as an additional model validation, our theory is able to generate not only quantitatively significant UI duration increases in recessions, but also cross-time patterns consistent with the data.

While our model can rationalize benefit extensions during recessions, our theory is not successful in generating the lack of actions (shortening of benefit durations) during normal times. However, it is worth noting that the response of the Markov policy to productivity is not entirely symmetric. In fact, the duration policy is nonlinear, increasing faster over regions of low productivity than it decreases over regions of high productivity.<sup>39</sup> This indicates that during times of high productivity the Markov government would like to reduce benefit duration but the magnitude is much less than the increases during a recession.

<sup>39</sup> Shown in Figure 3 and discussed in Section 5.2.

Table 4: Calibration Targets for Other Recessions

Recession	Pre-recession period	Pre-recession labor market statistics		
		Separation	Job finding	Job filling
Nov 1973-Mar 1975	1973.I-1973.III	0.026	0.51	0.71
Jul 1981-Nov 1982	1980.II-1981.I	0.033	0.41	0.71
Jul 1990-Mar 1991	1988.I-1990.II	0.027	0.47	0.71
Mar 2001-Nov 2001	1999.I-2000.IV	0.020	0.49	0.66

Note: Job-filling rate pre-2000 are from [Den Haan, Ramey, and Watson \(2000\)](#).

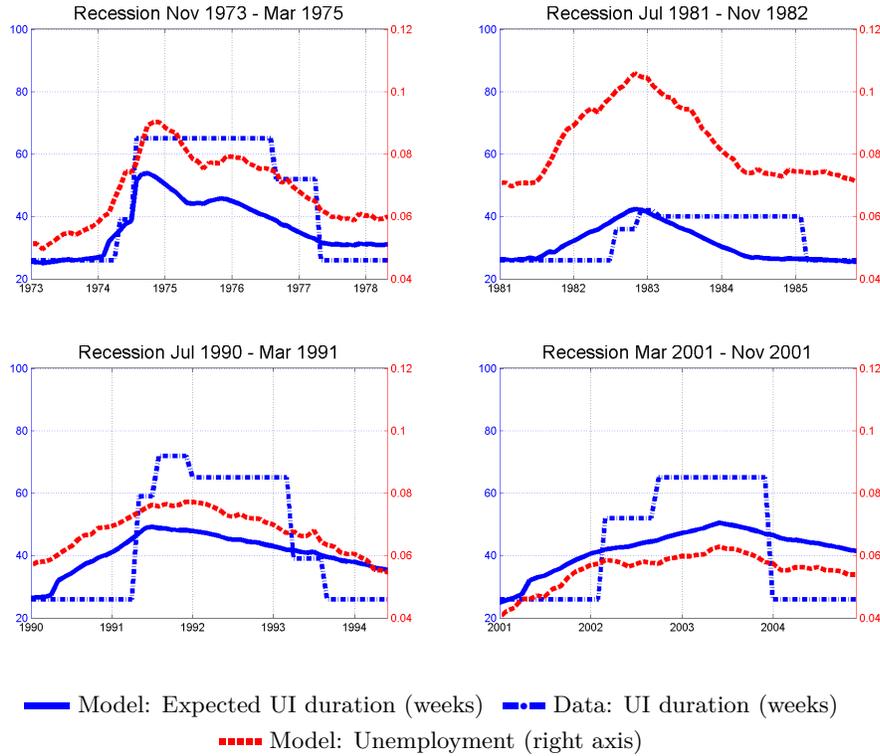


Figure 14: UI duration and unemployment in other recessions: Model versus data

## 7 THE ROLE OF COMMITMENT

Our theory assumes no commitment by the government. In this section, we compare the Markov policy with the policy chosen by a Ramsey government to illustrate the role of commitment.

The Ramsey government has commitment to all its future policies at the beginning of time. The government's decision problem is therefore to choose a sequence of unemployment benefit and duration and tax policies  $\{b_t, d_t\}_{t=0}^{\infty}$  to maximize the worker's utility, taking into account how the private sector will respond to these policies. At time 0, the government decides on its policies for all future periods and for all possible realizations of shocks. The private sector takes government

policies as given and follows the timing described in Section 2.

Equivalently, the government's problem can be written as one of choosing policies  $\{b_t, d_t, \tau_t\}_{t=0}^{\infty}$ , and allocation and prices  $\{s_t^0, s_t^1, \theta_t, u_{t+1}, u_{t+1}^1\}_{t=0}^{\infty}$  to maximize utility subject to the government budget constraint and competitive equilibrium conditions. Formally,

**DEFINITION 3.** (Ramsey policy) Given initial measures of unemployed population  $(u_0, u_{b,0})$  and aggregate labor productivity  $z_0$ , the optimal government policy with commitment consists of a sequence of benefit level and duration and taxes  $\{b_t, d_t\}_{t=0}^{\infty}$  that solves

$$\max_{\{b_t, d_t, s_t^0, s_t^1, \theta_t, u_{t+1}, u_{t+1}^1\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t R(u_t, u_t^1, b_t, d_t, s_t^0, s_t^1)$$

over the set of all policies that satisfy the worker's flow equations (2)-(3), and the private-sector optimality conditions (9)-(11), for all time  $t$  and aggregate shock  $\{z_t\}_{t=0}^{\infty}$ .

For easy exposition, we use auxiliary functions  $\tilde{f}_1, \tilde{f}_2, \tilde{\eta}_1, \tilde{\eta}_2$ , and  $\tilde{\eta}_3$  to denote the flow equations and the three private-sector optimality conditions (9)-(11), respectively. Note that the three private-sector optimality conditions play the role of incentive constraints in the optimal policy problem, similar to the incentive constraints in a principal-agent setup such as [Hopenhayn and Nicolini \(1997\)](#).

To derive a set of conditions that characterize the Ramsey policy, we let  $\beta^t \pi^t \lambda_t, \beta^t \pi^t \lambda_{b,t}, \beta^t \pi^t \mu_t, \beta^t \pi^t \mu_{b,t}$ , and  $\beta^t \pi^t \gamma_t$  be the Lagrange multipliers on  $\tilde{f}_1, \tilde{f}_2, \tilde{\eta}_1, \tilde{\eta}_2$ , and  $\tilde{\eta}_3$ , where  $\pi^t$  is the probability of a history realization  $\{z_0, z_1, \dots, z_t\}$  given an initial condition  $z_0$ .

**PROPOSITION 2.** Given initial conditions and the private-sector optimality conditions, the optimal government policy can be characterized by the following government's first-order conditions with respect to  $b_t, d_t, s_t^0, s_t^1, \theta_t, u_{t+1}$ , and  $u_{t+1}^1$  for all time  $t > 0$  (highlights represent differences with the Markov government optimality conditions):

$$\begin{aligned} b: & \quad \mu_{t-1} \frac{\tilde{\eta}_{1b',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2b',t-1}}{\beta} - R_{b,t} = 0 \\ d: & \quad \mu_{t-1} \frac{\tilde{\eta}_{1d',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2d',t-1}}{\beta} + \lambda_t f_{1d,t} + \lambda_{b,t} f_{2d,t} - R_{d,t} = 0 \\ s^0: & \quad \mu_{t-1} \frac{\tilde{\eta}_{1s^0',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2s^0',t-1}}{\beta} + \lambda_t f_{1s^0,t} + \mu_t \tilde{\eta}_{1s^0,t} - R_{s^0,t} = 0 \\ s^1: & \quad \mu_{t-1} \frac{\tilde{\eta}_{1s^1',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2s^1',t-1}}{\beta} + \lambda_t f_{1s^1,t} + \lambda_{b,t} f_{2s^1,t} + \mu_{b,t} \tilde{\eta}_{2s^1,t} - R_{s^1,t} = 0 \\ \theta: & \quad \mu_{t-1} \frac{\tilde{\eta}_{1\theta',t-1}}{\beta} + \mu_{b,t-1} \frac{\tilde{\eta}_{2\theta',t-1}}{\beta} + \gamma_{t-1} \frac{\tilde{\eta}_{3\theta',t-1}}{\beta} + \lambda_t f_{1\theta,t} + \lambda_{b,t} f_{2\theta,t} + \mu_t \tilde{\eta}_{1\theta,t} + \mu_{b,t} \tilde{\eta}_{2\theta,t} + \gamma_t \tilde{\eta}_{3\theta,t} = 0 \\ u: & \quad \lambda_t f_{1u',t} - \beta \mathbb{E} \{ R_{u,t+1} - \lambda_{t+1} f_{1u,t+1} - \lambda_{b,t+1} f_{2u,t+1} \} = 0 \\ u^1: & \quad \lambda_{b,t} f_{1u^1',t} + \mu_t \tilde{\eta}_{1u^1,t} + \mu_{b,t} \tilde{\eta}_{2u^1',t} - \beta \mathbb{E} \{ R_{u^1,t+1} - \lambda_{t+1} f_{1u^1,t+1} - \lambda_{b,t+1} f_{2u^1,t+1} \} = 0 \quad (\text{RAM}) \end{aligned}$$

where primes denote next period, and subscripts are derivatives.

The period- $t$  solution is state dependent. It depends on the current productivity  $z_t$  and the beginning-of-period unemployment level  $u_t$ , as well as multipliers  $(\mu_{t-1}, \mu_{b,t-1}, \gamma_{t-1})$ .  $\mu_t$  and  $\mu_{b,t}$

are the marginal values of relaxing the optimal search condition for the unemployed worker without and with benefit, respectively, and  $\gamma_t$  is the marginal value of relaxing the firm's equilibrium free-entry condition. The presence of  $\mu_{t-1}$ ,  $\mu_{b,t-1}$ , and  $\gamma_{t-1}$  as stated in the optimal policy captures commitment—the Ramsey government in period  $t$  has to deliver these marginal values, which it promised workers and firms in period  $t - 1$ .

The key difference between the conditions characterizing the Ramsey and Markov policies is that the Markov optimality conditions do not contain promised marginal values from the previous period (terms containing  $\mu_{t-1}$ ,  $\mu_{b,t-1}$ , and  $\gamma_{t-1}$  as highlighted in red in RAM), because the Markov government lacks commitment to future policies.  $\mu_{t-1}$ ,  $\mu_{b,t-1}$ , and  $\gamma_{t-1}$  represent the *marginal private values* (shadow price) of optimal job-search and vacancy-posting behaviors in period  $t - 1$ . These marginal values are affected by expected policy and allocations of period  $t$ . For example, more generous UI in period  $t$  reduces expected gains from search and vacancy posting, thus reducing job creation in period  $t - 1$ . Because the Markov government cannot commit, it does not internalize how current policy affects incentives in the previous period. As a result, its policy does not depend on the values of  $\mu_{t-1}$ ,  $\mu_{b,t-1}$ , and  $\gamma_{t-1}$ . In contrast, the Ramsey government chooses policies that can deliver these promises, thus their presence in the Ramsey optimality conditions.

Note that commitment is assumed in the Ramsey case. If given the choice to break a promise, the government will deviate from the sequence of policies prescribed by the government at time 0. The government at period  $t$  has an incentive to promise low future unemployment benefits to encourage search and vacancy posting, because as explained in Section 2, current search (mainly search of the benefit-collecting unemployed workers) is higher when expected future UI duration is shorter. However, after the employment outcome in period  $t$  is realized, the government has an incentive to provide insurance to more unemployed workers by choosing longer duration. This incentive to deviate from the original plan is what constitutes time inconsistency in the Ramsey problem.

## 7.1 A simple example to illustrate time inconsistency

We consider a simple example to illustrate the presence of time inconsistency in the Ramsey problem. To simplify the illustration, we restrict the government to one policy instrument, the benefit level, which the government sets for a future date. Higher future benefit level increases consumption of ex-post unemployed workers but reduces ex-ante job search incentives.

There are two periods and one unit measure of workers. Workers search in the first period and consume in the second period. Assume no time discounting and firms. In the first period,  $1 - \bar{u}$  of workers are guaranteed a job in the second period. The remaining  $\bar{u} = 0.05$  workers choose how much to search,  $s \in (0, 1)$ , for a job starting in the second period. Search incurs utility costs governed by the convex function  $v(s)$ . Worker's utility of consumption is given by  $U(c)$ .

With probability  $s$ , the worker finds a job and receives wage  $\bar{w} = 1$  in the second period; otherwise, he receives unemployment benefit  $b \in (0, 1)$ . Optimal choice of search is thus characterized by

$v_s(s) = U(\bar{w}) - U(b)$ . The number of unemployed workers in the second period is  $u = (1 - s)\bar{u}$ . The government in this economy chooses  $b$  at the beginning of period 1 to maximize average utility

$$\begin{aligned} W &= (1 - u)U(\bar{w}) + u[U(b) - v(s)] \\ \text{subject to } u &= (1 - s)\bar{u} \\ v_s(s) &= U(\bar{w}) - U(b). \end{aligned}$$

Essentially, the government is solving

$$\max_{s \in (0,1)} [1 - (1 - s)\bar{u}]U(\bar{w}) + (1 - s)\bar{u}[U(\bar{w}) - v_s(s) - v(s)]$$

with first-order condition given by

$$\bar{u}[v_s(s) + v(s)] - (1 - s)\bar{u}[v_{ss}(s) + v_s(s)] = 0.$$

Let  $U(c) = \log(c)$  and  $v(s) = \frac{s^2}{2}$ . The government optimally chooses  $s^* = 0.549$ ,  $b^* = 0.578$ , and  $u^* = 0.0226$ , with average utility  $W^* = -0.0158$ .

Now suppose the government can revise benefits after workers have chosen  $s$ . Then the ex-post optimal policy is  $\hat{b} = 1$ , with ex-post average utility given by  $\hat{W} = (1 - u^*)\log \bar{w} + u^* [\log \hat{b} - (s^*)^2/2] = -0.0034 > W^*$ . In fact, any  $\hat{b} > b^*$  will result in higher ex-post average utility. The fact that there exists a better policy ex-post illustrates the time inconsistency in this setup; time inconsistency, in turn, means lack of commitment leads to different policy outcomes than an economy with government commitment.

## 7.2 The long-run effect of commitment

We compare the steady-state Markov and Ramsey policies. The difference here is that the Markov government lacks commitment over future policies, and hence does not consider the effect of current policy on past allocations. For a fair comparison, we re-calibrate the Ramsey economy to match the same set of steady-state statistics used to calibrate the Markov economy. Table 5 shows these calibrated parameters and the target moments. The exogenously calibrated parameters are the same as before.

Table 6 compares the steady-state benefit duration policy in the Markov economy and in the Ramsey economy. Because both economies are calibrated to match the same steady-state replacement ratio, benefit duration is the only source of policy difference in this comparison. Consistent with the difference highlighted before, the Ramsey policy is less generous than the Markov policy because the Ramsey government internalizes the effect of current policy on previous job creation.

Table 5: Internally Calibrated Parameters: Ramsey

Parameter	Description	Value
$h$	Value of home production	0.397
$\gamma$	Disutility of search	4.432
$\chi$	Matching parameter	2.263
$\bar{w}$	Steady-state wage	0.979
Target	Data	Model
Average replacement ratio	40%	37.9%
Average job-finding rate	0.40	0.400
% unemployed with benefits	45	44.6
Average job-filling rate	0.66	0.660

Note: Calibration targets are monthly statistics of the U.S. economy 2005.I-2007.IV.

Table 6: Steady States: Markov versus Ramsey Policy

Statistic	Markov	Ramsey
Duration(weeks), $1/d$	26.3	16.8

Note: Steady states are computed using parameters calibrated to the same set of steady-state moments.

## 8 CONCLUSION

This paper develops a quantitative theory of how a welfare-maximizing government uses UI policy to balance the incentives of insurance against the cost of moral hazard arising from distortion in unemployed workers' job search incentives. We use the concept of Markov-perfect equilibrium to study a time-consistent UI policy, where the government makes decisions each period contingent on payoff-relevant aggregate states of the economy. In the steady state, our theory delivers an expected UI duration close to the U.S. policy. Over the business cycle, the UI benefit level stays roughly constant and the expected duration rises during recessions. Both the steady state and cyclical properties of our theory are consistent with policies in the U.S. since the 1970s.

We then use the theoretical framework to study benefit extensions in the U.S. from 2008 to 2013. We find that compared to a UI policy fixed at the 2007 level, the Markov policy, which matches most of the variations in benefit duration observed in the data, leads to an increase of 3 percentage points in the unemployment at its peak. Of this unemployment gap, we find that more than two-thirds is driven by private-sector expectations: unemployed workers expect longer future UI durations in recessions and as a result reduce job search. More importantly, we find that a longer UI duration during recessions can welfare improving. Compared to the scenario where the government does not

change UI policies, the benefit extensions lead to ex-post higher welfare during the 2008-13 period, although it is not necessarily welfare improving for a shorter and less severe recession. This finding provides a new perspective to the debate over whether UI benefit extensions are good policy.

Because our theory assumes no commitment by the government, we compare the Markov policy to the Ramsey policy to see how much non-commitment matters in the long run. When calibrated separately to the same set of targets, the steady-state Ramsey UI duration is much lower, reflecting the Ramsey government's ex-ante incentives to stimulate job search.

Several simplifying assumptions are made for tractability. First, neither workers nor government can save or borrow. Because savings provide self-insurance to workers, allowing workers to save will reduce the need for government-provided insurance policy. At the same time, credit access may reduce search by the unemployed (see, for example, [Herkenhoff \(2015\)](#)), thus exacerbating moral hazard associated with search. The reduced need for insurance and increased moral hazard problem will likely reduce the cyclical response of benefit duration. Allowing government access to the credit market will increase the cyclical responses of the benefit duration and likely make the benefit level less procyclical (or more acyclical). This is because the government can borrow to finance a generous benefit policy in bad times and pay back the debt with tax revenue in good times.

The second assumption is that government policy takes effect right away. In reality, there often is a time lag between legislation and implementation. Allowing the government to announce policy changes before implementation gives workers and firms time to react to the potential changes, which may mitigate the effect of policy changes ("announcement effect"). However, by looking at UI legislations during the Great Recession, we find that most extensions came into effect shortly after announcement. Furthermore, the announcement effect, if any, is likely small quantitatively. [Nakajima \(2012\)](#), for example, incorporates announcement effect in his evaluation of benefit extensions and finds minor quantitative effect associated with announcement.

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## A UNEMPLOYMENT INSURANCE BENEFIT EXTENSIONS IN THE GREAT RECESSION

States unemployment insurance and the federal government have adjusted unemployment benefits through the duration margin. In normal circumstances and under the regular program: Unemployment Compensation (UC), an eligible unemployed worker may receive unemployment benefits up to 26 weeks in most states. During economic downturn, automatic benefits extensions are triggered under the Extended Benefits (EB) program. The duration is 13 or 20 weeks depending on the state's insured unemployment rate (IUR) or the total unemployment rate (TUR). In addition, the Emergency Unemployment Compensation (EUC08) has been launched in 2008 and has been redefined in the ARRA context in 2009. It also increases the maximum benefits duration. Four waves called "Tiers" have been implemented. The first one (Tiers I) is effective without any conditions on states' experience with unemployment. Tiers II, III and IV require a condition on the IUR and/or TUR to be effective.

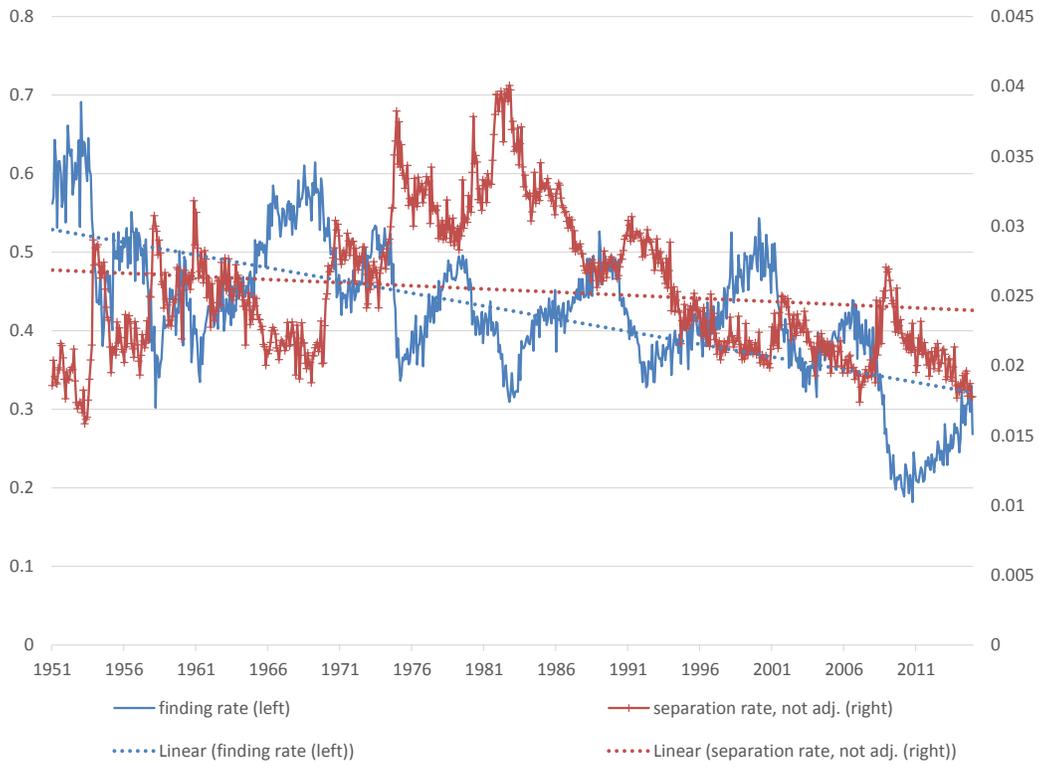
For these purposes, we extract the series of the IUR and TUR for 51 states of the US and compute if the state is eligible for the EB and the EUC08 programs. The sum of these three programs gives the maximum duration of unemployment benefits for each state. It is weighted in order to build an aggregate indicator. We assume the weights are equal to the number of total insured unemployed workers in the state divided by the total insured unemployed workers in the US. Statistics on insured unemployment population comes from the U.S. Department of Labor Employment and Training Administration. Table 7 reports a timeline for policy changes and unweighted expected maximum duration under the EUC08 and EB programs.

Table 7: Federally-Funded Unemployment Insurance Extensions 2008-2013

Start date	Program extension of EUC08	End date	Additional Weeks
Jul 2008	13 weeks for all states	Nov 2008	13
Nov 2008	Tier I: 20 weeks for all states Tier II: 13 weeks for states with $TUR \geq 6\%$	Mar 2009	33
Mar 2009	keep existing structure	Nov 2009	33
Nov 2009	Tier I - 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states $TUR \geq 6\%$ Tier IV: 6 weeks if states $TUR \geq 8.5\%$	Dec 2009	53
Dec 2009	keep existing structure	Aug 2010	53
Mar 2010	keep existing structure	Sep 2010	53
Apr 2010	keep existing structure	Nov 2010	53
Jul 2010	keep existing structure	May 2011	53
Dec 2010	keep existing structure	Jun 2012	53
Dec 2011	keep existing structure	Aug 2012	53
Feb 2012	Tier I: 20 weeks for all states Tier II: 14 weeks for all states Tier III: 13 weeks if states $TUR \geq 6\%$ Tier IV: 6 weeks if states $TUR \geq 8.5\%$ (16 weeks if no active EB and $TUR \geq 8.5\%$ )	May 2012	53
Jun 2012	Tier I: 20 weeks for all states Tier II: 14 weeks if states $TUR \geq 6\%$ Tier III: 13 weeks if states $TUR \geq 7\%$ Tier IV: 6 weeks if states $TUR \geq 9\%$	Sep 2012	53
Sep 2012	Tier I: 14 weeks for all states Tier II: 14 weeks if states $TUR \geq 6\%$ Tier III: 9 weeks if states $TUR \geq 7\%$ Tier IV: 10 weeks if states $TUR \geq 9\%$	Dec 2012	47
Jan 2013	keep existing structure	Dec 2013	47
Start date	Program extension of EB	End date	Additional Weeks
Feb 2009	6.5% 13 week IUR and $IUR \geq 110\%$ of prior 3 years 8% 13 week IUR and $IUR \geq 110\%$ of prior 3 years	Dec 2013	13 26

Source: DOLETA, [Whittaker and Isaacs \(2013\)](#), [Albertini and Poirier \(2015\)](#)

## B SECULAR DECLINE IN JOB FINDING AND SEPARATION RATES



## C REVIEW OF RELATED LITERATURE

This paper is closely related to two strands of literature: the literature on UI and the literature on time-consistent public policy.

The literature on UI dates back to [Mortensen \(1977\)](#), who argues that unemployment insurance reduces job search effort by the unemployed. Since Mortensen, the majority of this literature takes one of two approaches: either studying the effects of actual UI policy or looking for an optimal policy. The present paper aims to bridge the two approaches by endogenizing government choice of UI policy. Here we take the stance that the government, when making UI policies, is unable to commit to future policies.

One of the classic empirical results in public finance is that social insurance programs such as UI reduce labor supply. Earlier works include [Moffitt \(1985\)](#) and [Meyer \(1990\)](#), who show that a 10% increase in unemployment benefits raises average unemployment durations by 4–8% in the U.S. [Krueger and Meyer \(2002\)](#) and [Gruber \(2007\)](#), for example, interpret this finding as evidence that UI has significant moral hazard costs. Our framework relies on a similar mechanism. When the *expected* payoff from unemployment is high relative to future wages, unemployed workers search less actively. [Chodorow-Reich and Karabarbounis \(2015\)](#) construct a time series of the opportunity cost of employment and find that the cost is procyclical and volatile over the business cycle.

More recently, [Chetty \(2008\)](#) explores an alternate explanation for the link between unemployment benefits and duration. He argues that unemployment benefits increase cash on hand for the unemployed, and thus reduces search intensity. This effect is stronger for unemployed workers with tighter liquidity constraints. Because this “liquidity effect” has the socially beneficial effect of correcting credit market failure, the truly optimal benefit level, as [Chetty \(2008\)](#) argues, should be higher than if such an effect were ignored. For tractability, the present paper abstracts from the credit market and therefore cannot directly control for the liquidity effect.

Since the Great Recession of 2007-2009, there has been renewed interest in evaluating the quantitative effect of UI on unemployment rate. The reason behind this renewed interest is the extension of expected maximum unemployment benefit duration from 26 to over 90 weeks at the peak of the recession and afterward. The literature has a wide range of estimates. On the one end, [Rothstein \(2011\)](#) estimates a small effect of benefit extensions on unemployment (about 0.1-0.5 percentage-point in the unemployment rate). On the other end, [Hagedorn, Manovskii, and Mitman \(2015\)](#), using the natural experiment of neighboring counties in two states having similar labor market conditions but different unemployment benefit durations, find a large effect of benefit extensions on employment (0.0161 log-point increase in employment as a result of a 1% drop in benefit duration). In between the two extremes, [Nakajima \(2012\)](#) estimates that benefit extensions led to 1.4 percentage-point increase in unemployment rate during 2007–2012 (or approximately 30% of the overall increase). [Fujita \(2010\)](#), using data from the Current Population Survey, obtains an estimate between 0.8 and 1.8 percentage points. Our paper answers a related but more relevant question—what are the welfare implications of benefit extensions? And the answer is that despite contributing to higher unemployment rate, benefit extensions are actually welfare-improving.

The literature on optimal UI has traditionally adopted a principal-agent framework (e.g., [Hopenhayn and Nicolini \(1997\)](#), [Wang and Williamson \(2002\)](#), [Shimer and Werning \(2007, 2008\)](#), and [Golosov, Maziero, and Menzio \(2013\)](#)). This framework allows moral hazard frictions to be characterized in a steady state, but it becomes intractable when extended to a business cycle environment. The literature typically shows that the optimal benefit should decline with the unemployment duration of an individual worker. For tractability,

our paper abstracts from duration-dependent benefits. More recently, [Mitman and Rabinovich \(2015\)](#) study optimal benefits over the business cycle in a search-matching framework with endogenous unobservable search intensity. [Jung and Kuester \(2015\)](#) take a more general approach by studying the optimal mix of unemployment benefits, hiring subsidies, and layoff taxes in a recession. These papers assume that the government is able to commit to future policies. Although this assumption is innocuous and standard for normative analysis, such policies are time inconsistent and thus hardly implementable without government commitment. The empirical section of the present paper documents frequent voting on UI-related issues during recession, which indicates that the government does not follow a prescribed policy rule when making UI policies. Our paper complements the literature by characterizing a time-consistent UI policy. Our companion paper ([Pei and Xie 2015](#)) compares the cyclical responses of the Markov and Ramsey governments to highlight the effect of commitment. Intuitively, the Ramsey government wants people to search hard in normal times when unemployment is low and the marginal return to search is high. To this end, the Ramsey government promises less generous benefits during recessions.

The current paper is also related to the literature on time-consistent public policy (see, e.g., [Alesina and Tabellini \(1990\)](#), [Chari and Kehoe \(2007\)](#), [Battaglini and Coate \(2008\)](#), and [Yared \(2010\)](#)).<sup>40</sup> Methodologically, our paper follows [Klein, Krusell, and Ríos-Rull \(2008\)](#) to characterize the Markov-perfect equilibrium of a dynamic game in terms of a generalized Euler equation (GEE). Whereas [Klein, Krusell, and Ríos-Rull \(2008\)](#) focus on a deterministic economy, we are interested in how government policy responds to business cycle fluctuations. Recent applications of Markov-perfect equilibrium include [Song, Storesletten, and Zilibotti \(2012\)](#), who study intergenerational conflict over debt in a politico-economic environment.

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<sup>40</sup> See [Klein and Ríos-Rull \(2003\)](#) for a detailed review of the earlier literature.

## D DERIVATIONS AND PROOFS

### D.1 Derivation of private sector optimality conditions

Throughout this section, we drop the dependence of functions on productivity shock  $z$  to economize on notation.

- Solving problem of unemployed person without benefit by taking derivative with respect to  $s^0$

$$\frac{v_s(s^0)}{f(\theta)} = \beta[V^{e'} - V^{0'}] \quad (21)$$

Solving problem of unemployed person with benefit by taking derivative with respect to  $s^1$

$$\frac{v_s(s^1)}{f(\theta)} = \beta[V^{e'} - d'V^{0'} - (1-d')V^{1'}] \quad (22)$$

Using worker's bellman equations

$$\begin{aligned} V^e - V^0 &= U(\bar{w} - \tau) - [U(h - \tau) - v(s^0)] + \beta(1 - f(\theta)s^0)[V^{e'} - V^{0'}] - \beta\delta[V^{e'} - (1-d')V^{1'} - d'V^{0'}] \quad (23) \\ V^e - dV^0 - (1-d)V^1 &= d[U(\bar{w} - \tau) - U(h - \tau) + v(s^0) + \beta(1 - f(\theta)s^0)(V^{e'} - V^{0'}) - \beta\delta(V^{e'} - (1-d')V^{1'} - d'V^{0'})] \\ &\quad + (1-d)[U(\bar{w} - \tau) - U(h + b - \tau) + v(s^1) + \beta(1 - f(\theta)s^1 - \delta)(V^{e'} - d'V^{0'} - (1-d')V^{1'})] \quad (24) \end{aligned}$$

Combine (23) with (21) and (22)

$$V^e - V^0 = U(\bar{w} - \tau) - U(h - \tau) + v(s^0) + (1 - f(\theta)s^0) \frac{v_s(s^0)}{f(\theta)} - \delta \frac{v_s(s^1)}{f(\theta)}$$

Update one period, and substitute into (21)

$$\frac{v_s(s^0)}{f(\theta)} = \beta \left[ U(\bar{w} - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right]$$

Combine (24) with (21) and (22)

$$\begin{aligned} &V^e - dV^0 - (1-d)V^1 \\ &= d \left[ U(\bar{w} - \tau) - U(h - \tau) + v(s^0) + (1 - f(\theta)s^0) \frac{v_s(s^0)}{f(\theta)} - \delta \frac{v_s(s^1)}{f(\theta)} \right] \\ &\quad + (1-d) \left[ U(\bar{w} - \tau) - U(h + b - \tau) + v(s^1) + (1 - f(\theta)s^1 - \delta) \frac{v_s(s^1)}{f(\theta)} \right] \end{aligned}$$

Update one period, and substitute into (22)

$$\begin{aligned} \frac{v_s(s^1)}{f(\theta)} &= \beta d' \left[ U(\bar{w} - \tau') - U(h - \tau') + v(s^{0'}) + (1 - f(\theta')s^{0'}) \frac{v_s(s^{0'})}{f(\theta')} - \delta \frac{v_s(s^{1'})}{f(\theta')} \right] \\ &\quad + \beta(1-d') \left[ U(\bar{w} - \tau') - U(h + b' - \tau') + v(s^{1'}) + (1 - f(\theta')s^{1'} - \delta) \frac{v_s(s^{1'})}{f(\theta')} \right] \end{aligned}$$

- From unmatched firm's value function, assuming free entry, i.e.  $J^0(u, u^1) = 0$

$$\frac{\kappa}{q(\theta)} = \beta J^1(u', u^{1'})$$

Then firm's value function can be rewritten as

$$J^1(u, u^1) = z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta)}$$

Update one period

$$J^1(u', u^{1'}) = z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta')}$$

Substitute into the first equation

$$\frac{\kappa}{q(\theta)} = \beta \left[ z - \bar{w} + (1 - \delta) \frac{\kappa}{q(\theta')} \right]$$

## D.2 Proof of proposition 1: derivation of Markov GEE

Throughout this section, we drop the dependence of functions on productivity shock  $z$  to economize on notation. Let  $\lambda, \lambda_b, \mu, \mu_b, \gamma$  be the Lagrange multipliers on (12)-(16), respectively.

1. Take derivatives of government's problem with respect to  $b, d, s^0, s^1, \theta, u'$  and  $u^{1'}$

$$\begin{aligned} b: & \quad R_b = 0 \\ d: & \quad \lambda f_{1d} + \lambda_b f_{2d} - R_d = 0 \\ s^0: & \quad \lambda f_{1s^0} + \mu \eta_{1s^0} - R_{s^0} = 0 \\ s^1: & \quad \lambda f_{1s^1} + \lambda_b f_{2s^1} + \mu_b \eta_{2s^1} - R_{s^1} = 0 \\ \theta: & \quad \lambda f_{1\theta} + \lambda_b f_{2\theta} + \mu \eta_{1\theta} + \mu_b \eta_{2\theta} + \gamma \eta_{3\theta} = 0 \\ u: & \quad \lambda f_{1u'} + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} = \beta G'_u \\ u^1: & \quad \lambda_b f_{2u^{1'}} + \mu \eta_{1u^{1'}} + \mu_b \eta_{2u^{1'}} + \gamma \eta_{3u^{1'}} = \beta G'_{u^1} \quad (\text{FOC}) \end{aligned}$$

where primes denote next period, and subscripts are derivatives. The first equation above  $R_b = 0$  characterize the government's decision on benefit level.

2. Take derivative of Bellman equation with respect to  $u$  and  $u^1$ , respectively

$$G_u = R_u + R_b \Psi_u^b + R_d \Psi_u^d + R_{s^0} S_u^0 + R_{s^1} S_u^1 + \beta G'_u \Gamma_u + \beta G'_{u^1} \Gamma_u^1 \quad (\text{ENV1})$$

$$G_{u^1} = R_{u^1} + R_b \Psi_{u^1}^b + R_d \Psi_{u^1}^d + R_{s^0} S_{u^1}^0 + R_{s^1} S_{u^1}^1 + \beta G'_u \Gamma_{u^1} + \beta G'_{u^1} \Gamma_{u^1}^1 \quad (\text{ENV2})$$

substitute the last two FOCs into ENV1 and ENV2 to eliminate  $\beta G'_u$  and  $\beta G'_{u^1}$

$$\begin{aligned} G_u = & \quad R_u + R_b \Psi_u^b + R_d \Psi_u^d + R_{s^0} S_u^0 + R_{s^1} S_u^1 \\ & \quad + \Gamma_u \{ \lambda f_{1u'} + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} \} + \Gamma_u^1 \{ \lambda_b f_{2u^{1'}} + \mu \eta_{1u^{1'}} + \mu_b \eta_{2u^{1'}} + \gamma \eta_{3u^{1'}} \} \quad (25) \end{aligned}$$

$$\begin{aligned} G_{u^1} = & \quad R_{u^1} + R_b \Psi_{u^1}^b + R_d \Psi_{u^1}^d + R_{s^0} S_{u^1}^0 + R_{s^1} S_{u^1}^1 \\ & \quad + \Gamma_{u^1} \{ \lambda f_{1u'} + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} \} + \Gamma_{u^1}^1 \{ \lambda_b f_{2u^{1'}} + \mu \eta_{1u^{1'}} + \mu_b \eta_{2u^{1'}} + \gamma \eta_{3u^{1'}} \} \quad (26) \end{aligned}$$

3. Differentiate private-sector optimality conditions ( $\eta_1, \eta_2, \eta_3$ ) and worker's flow equations ( $f_1$  and  $f_2$ ) with respect to  $u$

$$\eta_{1u'}\Gamma_u + \eta_{1u^{1'}}\Gamma_u^1 + \eta_{1s^0}S_u^0 + \eta_{1\theta}\Theta_u = 0 \quad (27)$$

$$\eta_{2u'}\Gamma_u + \eta_{2u^{1'}}\Gamma_u^1 + \eta_{2s^1}S_u^1 + \eta_{2\theta}\Theta_u = 0 \quad (28)$$

$$\eta_{3u'}\Gamma_u + \eta_{3u^{1'}}\Gamma_u^1 + \eta_{3\theta}\Theta_u = 0 \quad (29)$$

$$f_{1u} + f_{1d}\Psi_u^d + f_{1s^0}S_u^0 + f_{1s^1}S_u^1 + f_{1\theta}\Theta_u + f_{1u'}\Gamma_u = 0 \quad (30)$$

$$f_{1u} + f_{1d}\Psi_u^d + f_{1s^1}S_u^1 + f_{1\theta}\Theta_u + f_{1u'}\Gamma_u^1 = 0 \quad (31)$$

4. Substitute (27)-(31) and the FOCs into (25)

$$G_u = R_u - \lambda f_{1u} - \lambda_b f_{2u} \quad (32)$$

Similarly, differentiate private-sector optimality conditions and the worker's flow equations with respect to  $u^1$ , and substitute into (26)

$$G_{u^1} = R_{u^1} - \lambda f_{1u^1} - \lambda_b f_{2u^1} \quad (33)$$

5. Update (32)-(33) and substitute into the last two FOCs, respectively

$$\lambda f_{1u'} + \mu \eta_{1u'} + \mu_b \eta_{2u'} + \gamma \eta_{3u'} = \beta [R'_u - \lambda' f'_{1u} - \lambda'_b f'_{2u}] \quad (34)$$

$$\lambda_b f_{2u^{1'}} + \mu \eta_{1u^{1'}} + \mu_b \eta_{2u^{1'}} + \gamma \eta_{3u^{1'}} = \beta [R'_{u^1} - \lambda' f'_{1u^1} - \lambda'_b f'_{2u^1}] \quad (35)$$

6. Combine the FOCs to get rid of Lagrange multipliers (leaving only  $\lambda$ )

$$\lambda_b = \frac{1}{f_{2d}} [R_d - \lambda f_{1d}] \quad (36)$$

$$\mu = \frac{1}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] \quad (37)$$

$$\mu_b = \frac{1}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \quad (38)$$

$$\gamma = -\frac{1}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \quad (39)$$

7. Rewrite (34)-(35) explicitly by substituting (36)-(39)

$$\begin{aligned} & \lambda f_{1u'} + \frac{\eta_{1u'}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u'}}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \\ & - \frac{\eta_{3u'}}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[ R_{s^1} - \lambda f_{1s^1} - \frac{f_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \\ = & \beta \left\{ R'_u - \lambda' f'_{1u} - \frac{f'_{2u}}{f'_{2d}} [R'_d - \lambda' f'_{1d}] \right\} \quad (\text{GEE1}) \end{aligned}$$

$$\begin{aligned}
& \frac{f'_{2u^1}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1u^1}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2u^1}}{\eta_{2s^1}} \left\{ R_{s^1} - \lambda f_{1s^1} - \frac{f'_{2s^1}}{f_{2d}} [R_d - \lambda f_{1d}] \right\} \\
& - \frac{\eta_{3u^1}}{\eta_{3\theta}} \left\{ \lambda f_{1\theta} + \frac{f_{2\theta}}{f_{2d}} [R_d - \lambda f_{1d}] + \frac{\eta_{1\theta}}{\eta_{1s^0}} [R_{s^0} - \lambda f_{1s^0}] + \frac{\eta_{2\theta}}{\eta_{2s^1}} \left[ R_{s^1} - \lambda f_{1s^1} - \frac{f'_{2s^1}}{f_{2d}} (R_d - \lambda f_{1d}) \right] \right\} \\
= & \beta \left\{ R'_{u^1} - \lambda' f'_{1u^1} - \frac{f'_{2u^1}}{f'_{2d}} [R'_d - \lambda' f'_{1d}] \right\} \quad (\text{GEE2})
\end{aligned}$$

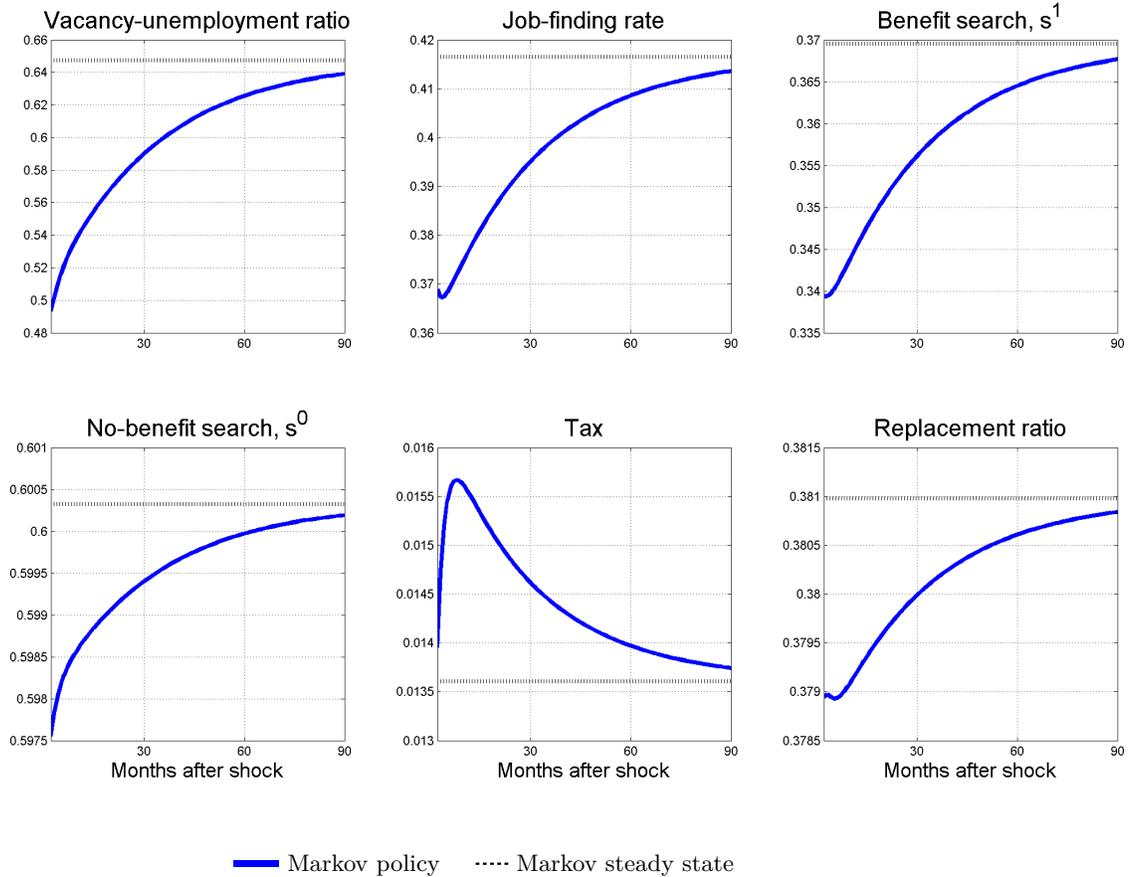
Equation (GEE2) characterizes the government's decision on  $d$ , where  $\lambda$  has the interpretation of the shadow price of unemployment and is characterized by equation (GEE1). Re-arrange to get the equations in Proposition 1.

## E ADDITIONAL QUANTITATIVE ANALYSES

### E.1 Additional impulse responses

This section contains impulse response of some labor market statistics to a one-time 1% negative shock to productivity. This figure complements Figure 5.

Figure 15: Additional plots: Impulse response to 1% negative shock to productivity.

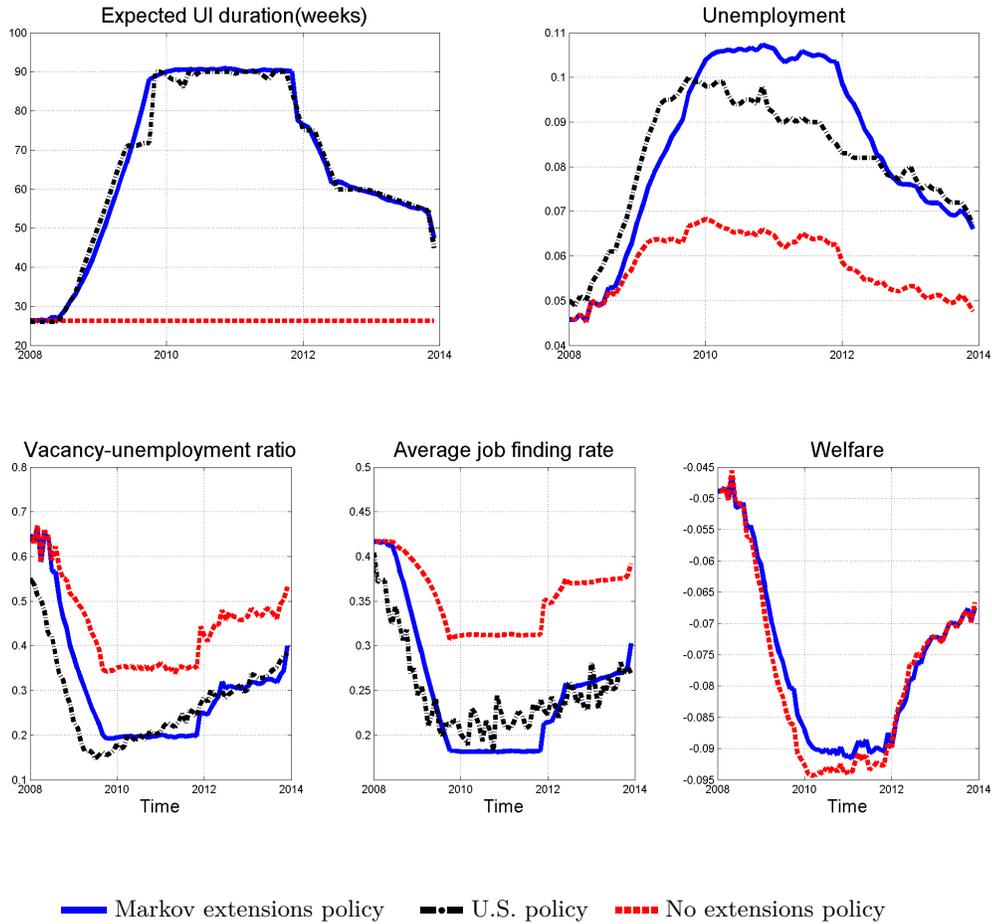


## E.2 Alternative calibration of productivity path during the Great Recession

This section presents alternative calibration of the productivity path  $z_t$ . Compare to Figure 9.

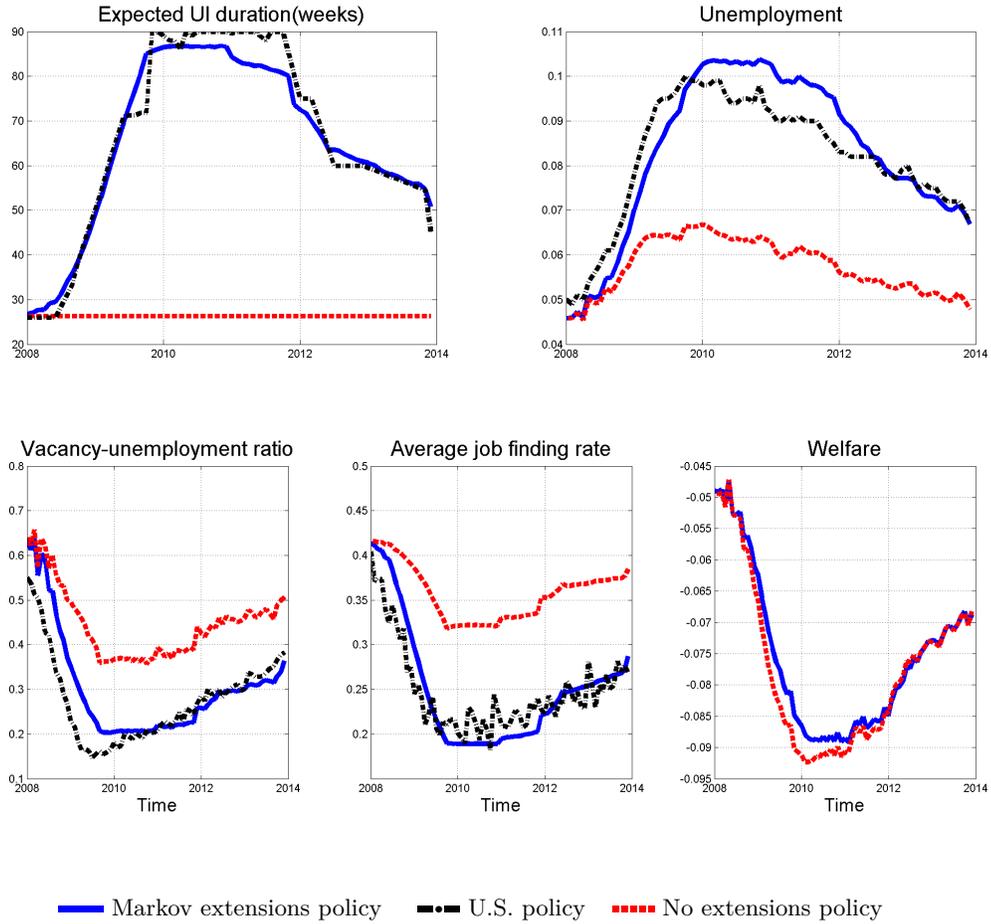
### E.2.1 Calibrating productivity path to match benefit duration

Figure 16: Calibrating productivity to match benefit duration: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).



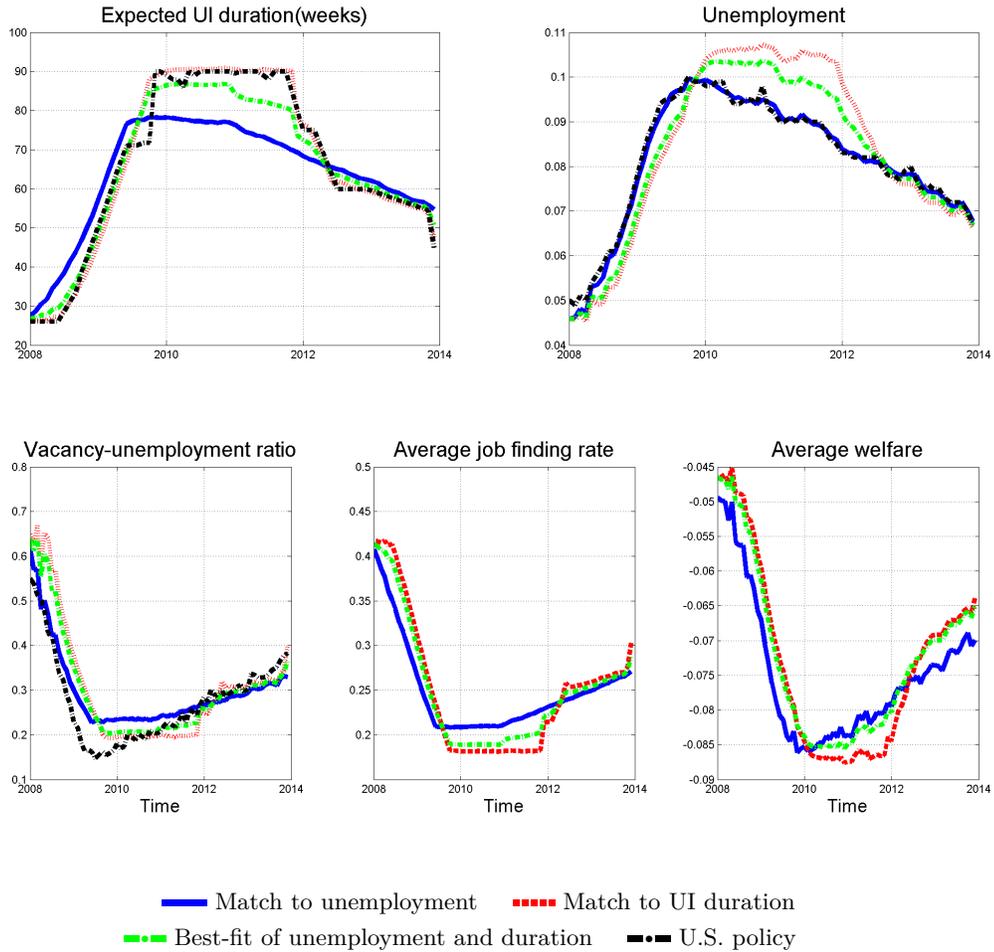
E.2.2 Calibrating productivity path for the best fit of unemployment and benefit duration

Figure 17: Calibrating productivity path for best fit: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).



### E.2.3 Comparison of different calibration strategies of productivity path

Figure 18: Different calibrations of productivity path: Markov policy to match unemployment (solid blue line) versus Markov policy to match duration (dotted red line) versus Markov policy for best-fit (dashed green line) versus U.S. data (dashed black line).



### E.3 Isolating quantitative effects from job separation shock

This section restricts productivity shock  $z$  to be constant at its steady-state level. The only exogenous shock here is the shock to job separation rate  $\delta$ . Compared to Figure 9, both unemployment and expected UI duration are much lower. Thus, productivity shock (and not shock to job separation rate) drives most of the cyclical variations in the model.

Figure 19: Expected UI duration and unemployment: Markov policy (solid blue line) versus constant policy (dotted red line) versus U.S. data (dashed black line).

