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ORIGINAL RESEARCH

Optimal ordering quantities for substitutable deteriorating items under joint replenishment with cost of substitution

Vinod Kumar Mishra[1]

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Abstract In this paper we develop an inventory model, to determine the optimal ordering quantities, for a set of two substitutable deteriorating items. In this inventory model the inventory level of both items depleted due to demands and deterioration and when an item is out of stock, its demands are partially fulfilled by the other item and all unsatisfied demand is lost. Each substituted item incurs a cost of substitution and the demands and deterioration is considered to be deterministic and constant. Items are order jointly in each ordering cycle, to take the advantages of joint replenishment. The problem is formulated and a solution procedure is developed to determine the optimal ordering quantities that minimize the total inventory cost. We provide an extensive numerical and sensitivity analysis to illustrate the effect of different parameter on the model. The key observation on the basis of numerical analysis, there is substantial improvement in the optimal total cost of the inventory model with substitution over without substitution.

Keywords Inventory control · Substitutable items · Cost of Substitution · Deterioration · Optimal ordering quantity · Joint replenishment

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Introduction

As we know that at any retails or supermarket the occurrence of temporary stock-outs is a very common phenomenon in the categories of frequently purchased items and it is also very common to see at any retails or supermarket, customers who willing to purchase certain items will be willing to purchase the substitute items, if they faced the situation of the stock-outs. A survey report of Anupindi et al. (1998) also observed the same phenomenon, in which he found that 82–88% of buyer would be willing to buy the substitute items if the desired items are out of stock. The substitutable items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the inventory system of substitutable items, i.e. the effect of deterioration plays a vital role in the decision of ordering quantity of substitutable deteriorating items. When substitution will take place an additional cost is incurred, known as substitution cost. Such substitution costs may arise due to a variety of reasons: the cost of the reworking required on an item to make it substitutable for the other, loss of a customer's goodwill due to substitution, etc. Deterioration of physical goods in stock is a very realistic feature and there is a big need to consider it in the inventory modelling of substitutable items. Tang and Yin (2007) categorizes the substitution as stock-out-based substitution, price-based substitution and assortment-based substitution. Recently, Kim and Bell (2011) categorizes the substitution as symmetrical substitution and asymmetrical substitution. As they define, the definition of these categories are: stock-out based substitution corresponds to a situation in which a customer may purchase another product as a substitute, when the preferred product is out of stock, price-based

substitution corresponds to a situation in which a retailer uses different pricing to make certain products substitutable, assortment-based substitution occurs when products with similar attributes are substitutable while symmetrical substitution occurs when all of the unfulfilled demands of one items are completely fulfilled by the demands of the substitute items and asymmetrical substitution occur when partial fraction of unfulfilled demands are added to the demands of the substitutable item. Based on the categories defined as above, this paper lies in the category of asymmetrical stock-out-based substitution. In the recent years, little bit attention has been given in the research for the stock-out-based substitution within the EOQ setting under deterministic demand and to the best of our knowledge no one consider the concept of deterioration for the substitutable items with deterministic demand and joint replenishment. Recently, Salameh et al. (2014) developed the joint replenishment policy for substitution by considering the deterministic demand, closely related to this paper, but they have not considered the concept of deterioration which is more realistic to determine the accurate optimal ordering quantity in the current era of competitive business strategies. Large numbers of the literature are available in inventory modelling of substitutable and deteriorating item separately. Thus, in subsequent paragraph, first, we discuss about recent and previous advancement in the inventory modelling of deteriorating items then inventory modelling of substitutable items.

The journey of inventory modelling was started in second decade of nineteenth century when Harris (1915) developed the first inventory model and this model was generalized by Wilson (1934) by deriving the formula to obtain the economic order quantity (EOQ). The inventory of deteriorating items was first studied by Whitin (1957) in which he considered the fashion goods as deteriorating items. Further there are several researcher who gave different inventory model of deteriorating item under different realistic situations, the reader may refer the review paper on inventory of deteriorating items of Raafat (1991), Goyal and Giri (2003), Li et al. (2010) and Bakker et al. (2012) and Khanlarzade et al. (2014) for detailed review of the literature of inventory of deteriorating items. Recently, Taleizadeh (2014a, b) gave economic order quantity model for deteriorating and evaporating items with consecutive and advanced payment, respectively.

The first inventory model of substitutable item was studied by McGillivray and Silver (1978) by considering that all of the substitutable items have the same unit variable cost and shortage penalty. Parlar and Goyal (1984) developed the similar model for optimal ordering decisions for stochastic demands. Pasternack and Drezner (1991) numerically proved that if the products are not substitutable then the associated optimal order quantities can be larger or smaller. In this sequence, Drezner et al. (1995) developed an EOQ model with substitution for two substitutable products and compare the results with no substitution. Ernst and Kouvelis (1999) suggested an efficient numerical search algorithm for the optimal stocking levels for three partially substitutable products. Gurnani and Drezner (2000) extended the model of Drezner et al. (1995) for multiple products. Mishra and Raghunathan (2004) gave new explanation for why retailers might be interested in vendor-managed inventory and showed that vendor-managed inventory intensifies the competition between two manufacturers of competing brands. Under joint replenishment policy (JRP), Porras and Dekker (2008) provided a complete analysis and presented a new inventory model over JRP when a correction is made for the empty replenishment and Hong and Kim (2009) gave a genetic algorithm for JRP and devised an unbiased estimator to find out the exact cost. In continuation of this, Schulz and Telha (2011) theoretically showed that the JRP with constant demands may have no polynomial-time algorithm. Taleizadeh et al. (2015) gave Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor-managed inventory system with deteriorating items. Recently, Krommyda et al. (2015), Salameh et al. (2014), Rasouli and NakhaiKamalabadi (2014), and Gerchack and Grosfeld (1999) developed inventory model for two substitutable item with deterministic demand, constant holding cost and fixed ordering cost but no one considered the effect of deterioration in inventory decision of substitutable items. Zhao et al. (2014) analyzed the pricing decision for two substitutable product with price-dependent probabilistic demand with fixed ordering cost and constant holding cost while Ye (2014), Huang and Ke (2014), Li et al. (2013), Li and You (2012), Hsieh (2011), and Xue and Song (2007) developed the inventory policy for multiple substitutable item with stochastic demand, fixed ordering cost and constant holding cost.

This paper makes the model of Krommyda et al. (2015), Salameh et al. (2014), and Rasouli and NakhaiKamalabadi (2014) more realistic and applicable by taking into account the effect of deterioration and cost of substitution on inventory of the substitutable items. Demand is considered as a constant function for both mutually substitutable items. If one of the items is out of stock then its demand will be fulfilled by the second item and if any demand is not met by substitutable item it will be completely lost. Both the products are order jointly and replenishment cycle is same for both the items.

The rest of the paper is organized as follows: In the next section, we describe the assumptions and notations used in the entire article, "Formulation and solution" Section gives

the detail of mathematical formulation and solution procedure of the model, extensive numerical analysis and convexity shown graphically in "Numerical and sensitivity analysis" Section and article ends with summary and conclusions of the article.

Assumptions and notations

For the mathematical formulation of the inventory model, the following assumptions and notations are used.

Assumptions

(a) The two items are ordered jointly in every ordering cycle.
(b) The demand rates and deterioration rates are known and constant for both items.
(c) The procurement lead time is zero and replenishment rates for both items are infinite.
(d) When an item is completely depleted and it subsequently becomes out of stock and there is on-hand inventory of the second item available, then the second item while supplying its own demands substitutes the demands of the first item during stock-out period. This substitution need not be the full substitution. It can be limited to a fraction (known as the substitution rate) of the total demand of the first item during stock-out period. The remaining un-substituted demand of the first item is lost.

Notations

Notation is grouped into parameters of the model, intermediate variables, and derived functions.

Parameters	
D_1, D_2	Demand rate of item 1 and item 2
θ	Deterioration rate
α_1, α_2	Substitution rate of item 1 by item 2
Q_1^*, Q_2^*	Optimal ordering quantities of item 1 and item 2, respectively
A_1, A_2	Fixed ordering cost per order of item 1 and item 2
i	Rate of holding cost of item 1 and item 2
C_1, C_2	Item cost per unit of item 1 and item 2
C_{S12}	Unit substitution cost for item 1 if substituted by item 2
C_{S21}	Unit substitution cost for item 2 if substituted by item 1
π_1, π_2	Lost sale cost per unit of item 1 and item 2, respectively
Intermediate variables	
p	Portion of time when substitution occur
z	Inventory level of item when other item is out of stock
t_1	Time when level of inventory of substituted item completely depleted
t_2	Time when level of inventory of substitute item completely depleted in case of no substitution
Derived functions	
$I_1^1(t)$	Inventory level of item 1 when item 1 depletes before item 2 at time t, $0 \le t \le t_1$
$I_2^1(t)$	Inventory level of item 2 when item 1 depletes before item 2 at time t, $0 \le t \le t_1$
$I_3^1(t)$	Inventory level of item 2 when item 1 depletes before item 2 and substitution take place at time t, $0 \le t \le t_1 + p$
$I_1^2(t)$	Inventory level of item 1 when item 2 depletes before item 1 at time t, $0 \le t \le t_2$
$I_2^2(t)$	Inventory level of item 2 when item 2 depletes before item 1at time t, $0 \le t \le t_2$
$I_3^2(t)$	Inventory level of item 1 when item 2 depletes before item 1 and substitution take place at time t, $0 \le t \le t_2 + p$
TC11 (Q_1, Q_2)	Total cost with substitution in per ordering cycle of item 1
TC12 (Q_1, Q_2)	Total cost with substitution in per ordering cycle of item 2
TC (Q_1, Q_2)	Total cost with substitution in per ordering cycle of both item
TC1 (Q_1, Q_2)	Total cost per unit time with substitution in per ordering cycle of both item when item 1 deplete before item 2
TC2 (Q_1, Q_2)	Total cost per unit time with substitution in per ordering cycle of both item when item 1 deplete before item 2
TC_{WS} (Q_1, Q_2)	Total cost per unit time without substitution

Formulation and solution

As stated before, we consider an inventory system for two mutually substitutable deteriorating items under the assumptions as mentioned in "Assumptions and notations" Section. At the beginning of the replenishment cycle, the retailer orders Q_1 and Q_2 units of item 1 and item 2, respectively, whose consumption rates are D_1 and D_2. The inventory level of both items gradually depletes due to deterioration and consumption rate. There are three possibilities cases,

Case 1 Item 1 depletes before item 2, i.e. if at time t_1 the inventory of item 1 is out of stock, as depicted in Fig. 1, then item 2 partially substitutes the item 1 with substitution rate α_1 and a portion of unmet demand for item 1 is assumed to be lost with the rate of $(1 - \alpha_1)$.

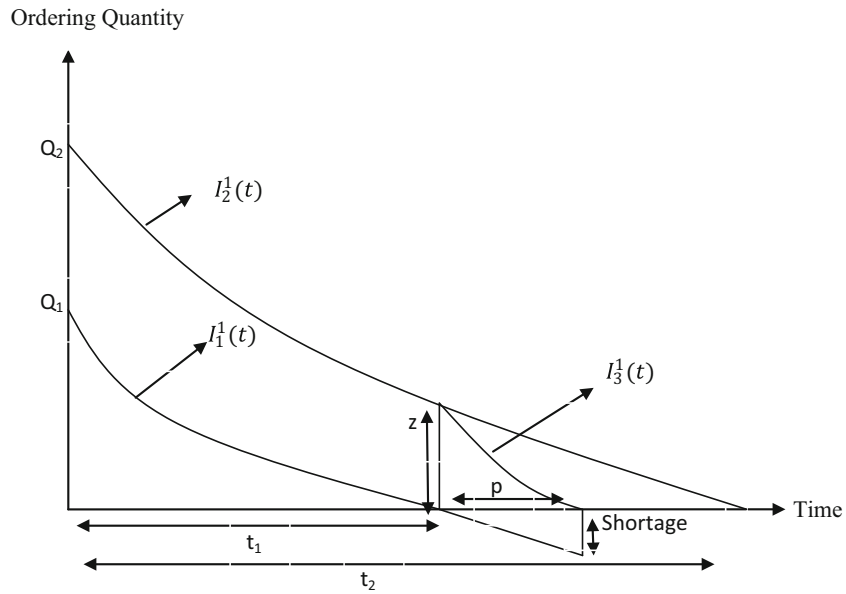

Fig. 1 Graphical representation of first scenario of inventory model when $t_1 < t_2$

Case 2 Item 2 depletes before item 1, i.e. if at time t_2 the inventory of item 2 is out of stock, as depicted in Fig. 2, then item 1 partially substitutes the item 2 with substitution rate α_2. A portion of unmet demand for item 2 is assumed to be lost with the rate of $(1 - \alpha_2)$.

Case 3 The inventory level of both items becomes zero simultaneously and no substitution will take place, as depicted in Fig. 3.

Case 1 (Fig. 1) Item 1 is substituted by item 2 ($t_1 < t_2$).

As depicted in Fig. 1, the inventory level of both item is governed by the following differential equations

$$\frac{\mathrm{d}I_1^1(t)}{\mathrm{d}t} + \theta I_1^1(t) = -D_1; \quad 0 \le t \le t_1 \tag{1}$$

with boundary condition $I_1^1(0) = Q_1$ and $I_1^1(t) = 0$.

$$\frac{\mathrm{d}I_2^1(t)}{\mathrm{d}t} + \theta I_2^1(t) = -D_2; \quad 0 \le t \le t_1$$

with boundary condition $I_2^1(0) = Q_2$ and $I_2^1(t_1) = I_3^1(t_1)$. (2)

$$\frac{\mathrm{d}I_3^1(t)}{\mathrm{d}t} + \theta I_3^1(t) = -(D_2 + \alpha_1 D_1); \quad t_1 \le t \le t_1 + p$$

with boundary condition $I_3^1(t_1) = I_2^1(t_1)$ and $I_3^1(t_1 + p) = 0$. (3)

The solutions of Eqs. (1)–(3) are

$$I_1^1(t) = \frac{D_1}{\theta}\left(\mathrm{e}^{\theta(t_1 - t)} - 1\right); \quad 0 \le t \le t_1 \tag{4}$$

$$I_2^1(t) = \frac{(Q_2\theta + D_2)}{\theta \mathrm{e}^{\theta t}} - \frac{D_2}{\theta}; \quad 0 \le t \le t_1 \tag{5}$$

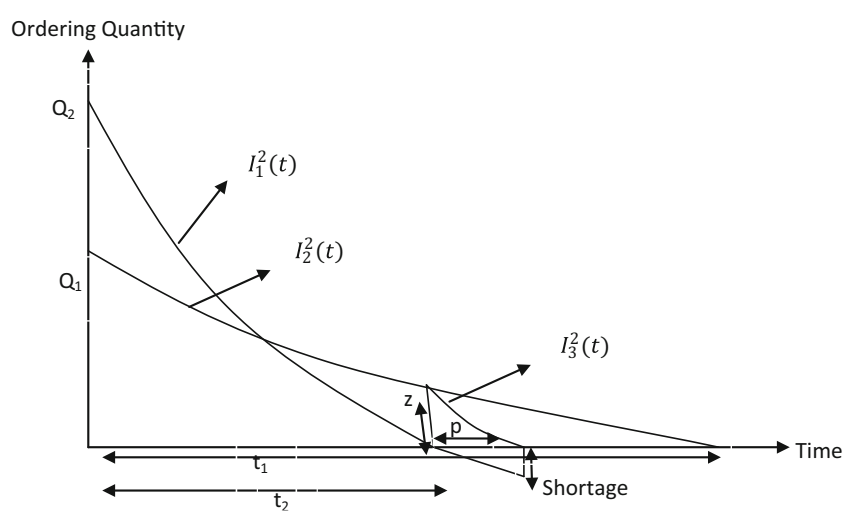

Fig. 2 Graphical representation of second scenario of inventory model when $t_1 > t_2$

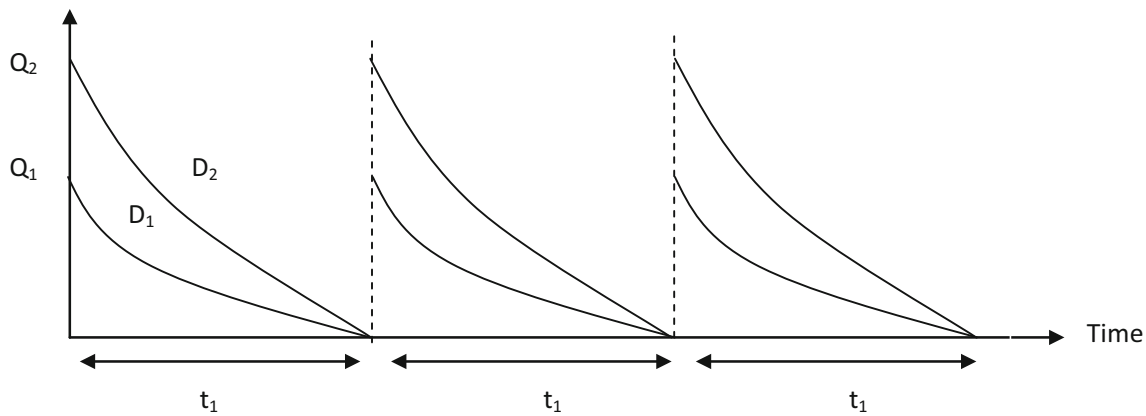

Fig. 3 Graphical representation of third scenario under joint replenishment

$$I_3^1(t) = \frac{D_2 + \alpha_1 D_1}{\theta}\left(e^{\theta(t_1+p-t)} - 1\right); \quad t_1 \le t \le t_1 + p \tag{6}$$

The cost components per cycle consist of (a) costs related to item 1 (b) costs related to item 2 (c) lost sale costs and (d) substitution costs.

(a) The total cost associated with item 1 per ordering cycle consists of fixed ordering cost, purchase cost and holding cost, and can be expressed as

$$\mathrm{TC11}(Q_1, Q_2) = \left(A_1 + C_1 Q_1 + \frac{iC_1}{\theta^2}\left(\theta Q_1 - D_1 \ln\left(\frac{\theta Q_1 + D_1}{D_1}\right)\right)\right). \tag{7}$$

(b) The total cost associated with item 2 per ordering cycle consists of fixed ordering cost, purchase cost and holding cost, and expressed as

$$\mathrm{TC12}(Q_1, Q_2) = \left(\begin{array}{l} A_2 + C_2 Q_2 + \frac{iC_2}{\theta^2}\left(\theta Q_2 - D_2 \ln\left(\frac{\theta Q_1 + D_1}{D_1}\right)\right) \\ -\frac{iC_2(D_1\alpha_1 + D_2)}{\theta^2}\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right) \end{array}\right). \tag{8}$$

(c) The Lost sale cost is incurred due to demand for the item 1, which can be expressed as

$$\text{Lost sale cost} = \frac{\pi_1 D_1}{\theta}\left((1-\alpha_1)\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right)\right). \tag{9}$$

(d) The substitution cost is incurred according to the number of units of item 1 substituted by item 2 at the rate of C_{S12} per unit substituted and can be expressed as

$$\text{Substitution cost} = \frac{C_{S12} D_1 \alpha_1}{\theta}\left(\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right)\right). \tag{10}$$

Thus, the total cost per ordering cycle TC (Q_1, Q_2), from Eqs. (7)–(10) is given as

$$\mathrm{TC}(Q_1, Q_2) = \left[\begin{array}{l} A_1 + A_2 + C_1 Q_1 + C_2 Q_2 + \frac{iC_1}{\theta^2}\left(\theta Q_1 - D_1 \ln\left(\frac{\theta Q_1 + D_1}{D_1}\right)\right) \\ + \frac{iC_2}{\theta^2}\left(\theta Q_2 - D_2 \ln\left(\frac{\theta Q_1 + D_1}{D_1}\right)\right) \\ - \frac{iC_2(D_1\alpha_1 + D_2)}{\theta^2}\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right) \\ + \frac{\pi_1 D_1}{\theta}\left((1-\alpha_1)\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right)\right) \\ + \frac{C_{S12} D_1 \alpha_1}{\theta}\left(\ln\left(\frac{D_1(\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2)}{(D_1\alpha_1 + D_2)(\theta Q_1 + D_1)}\right)\right) \end{array}\right]. \tag{11}$$

Finally, for case 1 (when $t_1 < t_2$), TC1 (Q_1, Q_2), the average total cost per unit time (say a year) is obtained by multiplying the total cost per ordering cycle by the average number of cycles per year $\left(\theta \Big/ \ln\left(\frac{\alpha_1\theta Q_1 + D_1\alpha_1 + \theta Q_2 + D_2}{D_1\alpha_1 + D_2}\right)\right)$ and is given as

$$\mathrm{TC1}(Q_1,Q_2)=\frac{\theta}{\ln\left(\frac{\alpha_1\theta Q_1+D_1\alpha_1+\theta Q_2+D_2}{D_1\alpha_1+D_2}\right)}\left(\begin{array}{l}A_1+A_2+C_1Q_1+C_2Q_2+\frac{iC_1}{\theta^2}\left(\theta Q_1-D_1\ln\left(\frac{\theta Q_1+D_1}{D_1}\right)\right)\\+\frac{iC_2}{\theta^2}\left(\theta Q_2-D_2\ln\left(\frac{\theta Q_1+D_1}{D_1}\right)\right)\\-\frac{iC_2(D_1\alpha_1+D_2)}{\theta^2}\ln\left(\frac{D_1(\alpha_1\theta Q_1+D_1\alpha_1+\theta Q_2+D_2)}{(D_1\alpha_1+D_2)(\theta Q_1+D_1)}\right)\\+\frac{\pi_1D_1}{\theta}\left((1-\alpha_1)\ln\left(\frac{D_1(\alpha_1\theta Q_1+D_1\alpha_1+\theta Q_2+D_2)}{(D_1\alpha_1+D_2)(\theta Q_1+D_1)}\right)\right)\\+\frac{C_{S12}D_1\alpha_1}{\theta}\left(\ln\left(\frac{D_1(\alpha_1\theta Q_1+D_1\alpha_1+\theta Q_2+D_2)}{(D_1\alpha_1+D_2)(\theta Q_1+D_1)}\right)\right)\end{array}\right) \tag{12}$$

Case 2 (Fig. 2) Item 2 is substituted by item 1 ($t_2 < t_1$).

Following an approach analogous to case 1, for case 2 (when $t_1 > t_2$), TC2 (Q_1, Q_2), the average total cost per unit time (say a year) is

$$\mathrm{TC2}(Q_1,Q_2)=\frac{\theta}{\ln\left(\frac{\alpha_2\theta Q_2+D_2\alpha_2+\theta Q_1+D_1}{D_2\alpha_2+D_1}\right)}\left(\begin{array}{l}A_1+A_2+C_1Q_1+C_2Q_2+\frac{iC_2}{\theta^2}\left(\theta Q_2-D_2\ln\left(\frac{\theta Q_2+D_2}{D_2}\right)\right)\\+\frac{iC_1}{\theta^2}\left(\theta Q_1-D_1\ln\left(\frac{\theta Q_2+D_2}{D_2}\right)\right)\\-\frac{iC_1(D_2\alpha_2+D_1)}{\theta^2}\ln\left(\frac{D_2(\alpha_2\theta Q_2+D_2\alpha_2+\theta Q_1+D_1)}{(D_2\alpha_2+D_1)(\theta Q_2+D_2)}\right)\\+\frac{\pi_2D_2}{\theta}\left((1-\alpha_2)\ln\left(\frac{D_2(\alpha_2\theta Q_2+D_2\alpha_2+\theta Q_1+D_1)}{(D_2\alpha_2+D_1)(\theta Q_2+D_2)}\right)\right)\\+\frac{C_{S21}D_2\alpha_2}{\theta}\left(\ln\left(\frac{D_2(\alpha_2\theta Q_2+D_2\alpha_2+\theta Q_1+D_1)}{(D_2\alpha_2+D_1)(\theta Q_2+D_2)}\right)\right)\end{array}\right) \tag{13}$$

Case 3 (Fig. 3) No Substitution.

Figure 3 illustrates the inventory levels for the case of no substitution. Under a joint replenishment policy, the inventories of both items deplete to zero simultaneously, i.e. $Q_1/D_1 = Q_2/D_2$. The average total cost per unit time for an inventory system without substitution under joint replenishment, $\mathrm{TC_{WS}}$ (Q_1, Q_2), consists only of setup costs, purchase costs and holding costs and is given as

$$\mathrm{TC_{WS}}(Q_1,Q_2)=\frac{\theta}{\ln\left(\frac{\theta Q_1+D_1}{D_1}\right)}\left(\begin{array}{l}A_1+A_2+C_1Q_1+C_2Q_2+\frac{iC_1}{\theta^2}\left(\theta Q_1-D_1\ln\left(\frac{\theta Q_1+D_1}{D_1}\right)\right)\\+\frac{iC_2}{\theta^2}\left(\theta Q_2-D_2\ln\left(\frac{\theta Q_1+D_1}{D_1}\right)\right)\end{array}\right). \tag{14}$$

Solution procedure

To determine the optimal ordering quantities from total cost function. In the next section we will graphically show that the total cost function is a strictly convex function and using this property of total cost function, next we provide an algorithm to obtain the optimal ordering quantities.

Algorithm to obtain the optimal ordering quantities.

Step 1 Initialize all the parameters of the model.
Step 2 Solve the constraint optimization problem

$$P_1:(Q_1,Q_2)\text{ that }\min_{Q_1,Q_2}\mathrm{TC1}(Q_1,Q_2)\text{ subject to }\frac{Q_1}{D_1}\le\frac{Q_2}{D_2}$$

and

$$P_2:(Q_1,Q_2)\text{ that }\min_{Q_1,Q_2}\mathrm{TC2}(Q_1,Q_2)\text{ subject to }\frac{Q_1}{D_1}\ge\frac{Q_2}{D_2}$$

Step 3 Obtain the optimum solution by (Q_1^*,Q_2^*) = Min (P_1,P_2).
Step 4 Stop.

Numerical and sensitivity analysis

In this section, we provide a numerical example to illustrate the proposed model using Maple mathematical modelling package. In Numerical analysis, we use the parameter as defined in Table 1 unless otherwise mentioned.

According the algorithm as defined above, we solve the constraint optimization problem as defined in step-2 by maple mathematical software. The outputs of step-2 are P_1: $Q_1 = 116.08$, $Q_2 = 91.34$, $\mathrm{TC1}(Q_1,Q_2) = 2000.79$ and P2: $Q_1 = 199.45$, $Q_2 = 19.34$, $\mathrm{TC2}(Q_1,Q_2)$ = 2069.98. When comparing the total cost of both constraint optimization problems in step-3 of the algorithm then first optimization problems (P_1) lead to the optimal solution. Thus, the optimal ordering quantities in this case is $(Q_1^*,Q_2^*) = (116.8,\ 91.34)$ and the optimal total cost is 2000.79. The optimal total cost of inventory model with no substitution under same assumptions using Eq. (14) with traditional calculus method the outputs are $Q_1 = 178.70$, $Q_2 = 44.67$, $\mathrm{TC2}(Q_1,Q_2)$ = 2096.98. Comparing the output (optimal total cost) of both inventory model then difference in optimal total cost and percentage improvement are 96.19 and 4.59, respectively.

To examine the nature of average total cost function, we plot the total cost function with varying order quantity of item 1 and item 2. The results are shown in Figs. 4, 5 and 6.

As Figs. 4, 5 and 6 show that the average total cost function is strictly convex function. Thus, average total cost function (Eq. 12) always leads to the unique optimal solution.

Next, we carry out a sensitivity analysis of the optimal total cost and optimal ordering quantities according to given values of different parameter of the model. The percentage improvements in the optimal total cost according to the values of various parameters are also presented. The numerical results are given in Table 2.

Now, we investigate the decrease in total cost when substitution is possible and compared to the case without substitution with respect to various parameters of the system. The results are shown in Figs. 7, 8 and 9.

While the extent and relative rate may vary, general nature of the percentage improvement in TC1 over TC_{WS} follows the intuitive reasoning. The findings are presented in Table 3.

Table 1 Initial parameters used for numerical analysis

Parameter	Item-1	Item-2
Consumption rate (D_1, D_2)	200	50
Deterioration rates (θ)	0.01	0.01
Substitution rates (α_1, α_2)	0.2	0.4
Setup costs (A_1, A_2)	300	300
Rate of holding cost (i)	2	2
Cost per unit (C_1, C_2)	3	3
Lost sale costs (π_1, π_2)	6	4
Substitution costs (C_{S12}, C_{S21})	2	2

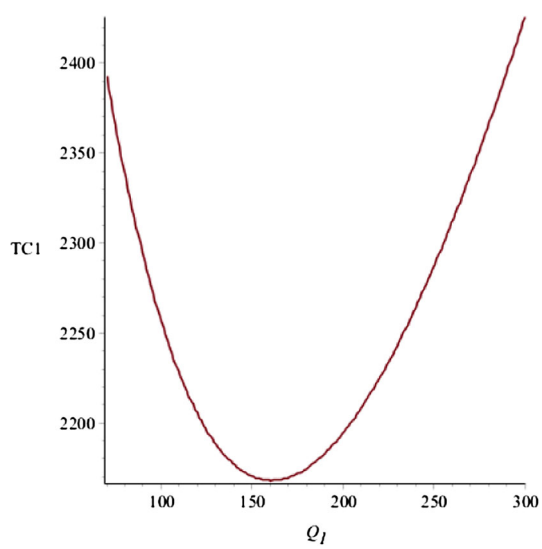

Fig. 4 Total cost (TC1) versus Q_1 at fixed Q_2

As we know that the main objective of an organization, dealing the inventory control, is to provide the right products to the right place, at the optimal price and time, and in the good quality with optimal quantity. With the help of this inventory model, which provides a detailed analysis of substitutable deteriorating items with numerical examples, manager can increase the firm's capability and their performance to deal the inventory of their organizations.

Summary and conclusions

In this paper, we presented an inventory model for two substitutable deteriorating items under joint replenishment in each replenishment cycle. If at any moment of time, the inventory level of one item is out of stock then second item will partially substitute the first item and vice versa. A

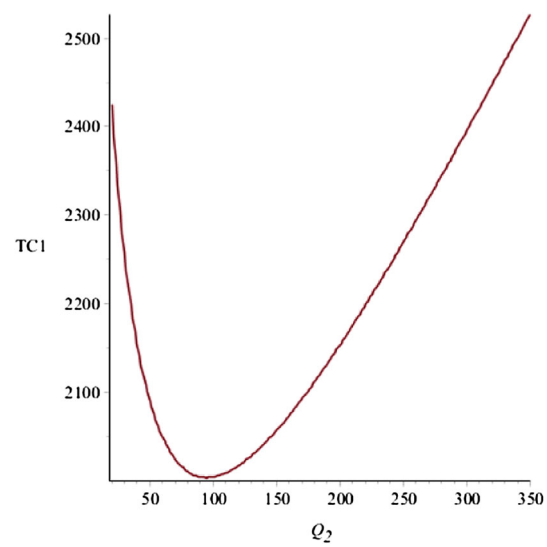

Fig. 5 Total cost (TC1) versus Q_2 at fixed Q_1

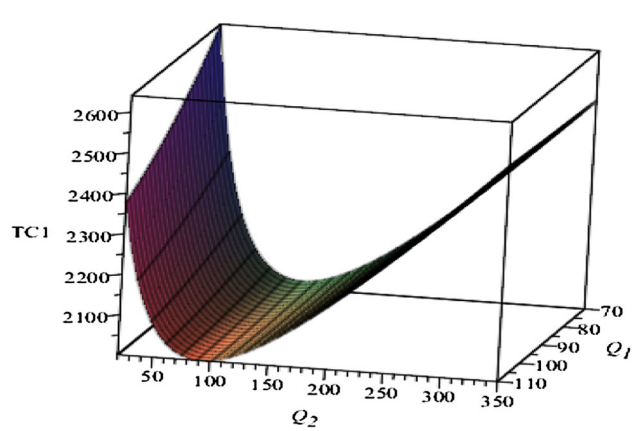

Fig. 6 Total cost (TC1) versus ordering quantity of item 1 and item 2

Table 2 Sensitivity analysis optimal total cost and optimal ordering quantities

Parameter	Value of parameter	Optimal total cost and optimal ordering quantity with substitution			Optimal total cost and optimal ordering quantity without substitution			% Improvement in optimal total cost
		Q_1^*	Q_2^*	TC1	Q_{1WO}^*	Q_{2WO}^*	TC_{WS}	
C_1/C_2	2/3	251.02	11.30	1627.30	208.73	52.18	1703.77	4.49
	3/3	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	4/3	52.67	118.20	2086.32	158.75	39.68	2465.73	15.39
	5/3	18.09	129.72	2114.08	144.27	36.06	2817.47	24.97
	6/3	0	134.16	2118.99	133.14	33.28	3156.49	32.87
i	2	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	3	77.51	83.67	2205.54	145.99	36.49	2397.90	8.02
	4	58.18	76.59	2371.44	126.46	31.61	2651.73	10.57
	5	46.57	70.83	2514.68	113.13	28.28	2875.43	12.55
	6	38.82	66.14	2642.58	103.28	25.82	3077.72	14.14
π_1	3	16.58	130.47	1636.74	178.70	44.67	2096.98	21.95
	4	49.75	120.82	1778.57	178.70	44.67	2096.98	15.18
	5	82.91	107.95	1900.96	178.70	44.67	2096.98	9.35
	6	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	7	199.45	19.34	2069.36	178.70	44.67	2096.98	1.32
$A_1 = A_2$	300	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	400	116.08	115.11	2144.14	206.39	51.59	2305.72	7.01
	500	116.08	135.38	2266.38	230.80	57.70	2489.70	8.97
	600	116.08	153.36	2374.78	252.88	63.22	2656.09	10.59
	700	116.08	169.68	2473.21	273.18	68.29	2809.16	11.96
C_{S12}	1	107.79	95.89	1978.22	178.70	44.67	2096.98	5.66
	2	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	3	124.37	86.49	2021.57	178.70	44.67	2096.98	3.60
	4	132.66	81.32	2040.40	178.70	44.67	2096.98	2.70
	5	140.96	75.80	2057.10	178.70	44.67	2096.98	1.90
α_1	0.2	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	0.4	199.45	19.34	2069.36	178.70	44.67	2096.98	1.32
	0.6	199.45	19.34	2069.36	178.70	44.67	2096.98	1.32
	0.8	199.45	19.34	2069.36	178.70	44.67	2096.98	1.32
	1.0	199.45	19.34	2069.36	178.70	44.67	2096.98	1.32
θ	0.01	116.08	91.34	2000.79	178.70	44.67	2096.98	4.59
	0.05	113.82	92.18	2016.93	177.98	44.49	2118.27	4.78
	0.10	111.11	93.16	2036.92	177.09	44.27	2144.63	5.02
	0.15	108.52	94.06	2056.69	176.21	44.05	2170.72	5.25
	0.20	106.06	94.88	2076.27	175.34	43.83	2196.58	5.48

portion of unmet demand for both items is assumed to be lost and each substituted item incurs a cost of substitution. For the situation as mentioned above, we mathematically formulated an inventory model and developed a solution procedure to obtain the optimal ordering quantities.

The numerical analysis showed that as rate of holding cost, setup cost and deterioration rate increases, the percentage improvement in optimal total cost with substitution over without substitution also increases and if we increase the shortage cost or cost of substitution, the percentage improvement decreases in total optimal cost with substitution over without substitution while percentage improvement becomes constant when substitution rate of item 1 becomes equal or more from substitution rate of item 2.

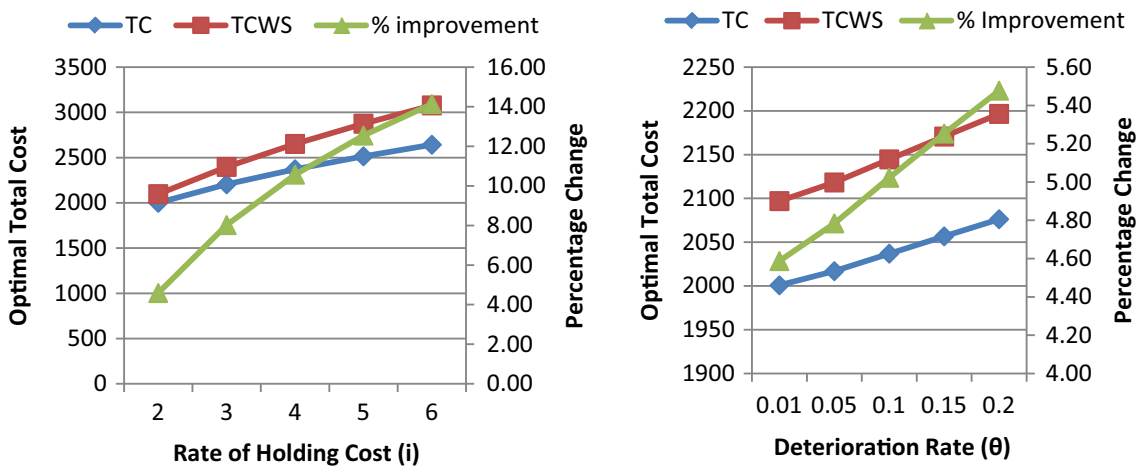

Fig. 7 % improvement in TC over TCWS with ratios of item cost (C_1/C_2) and rate of holding Cost (i)

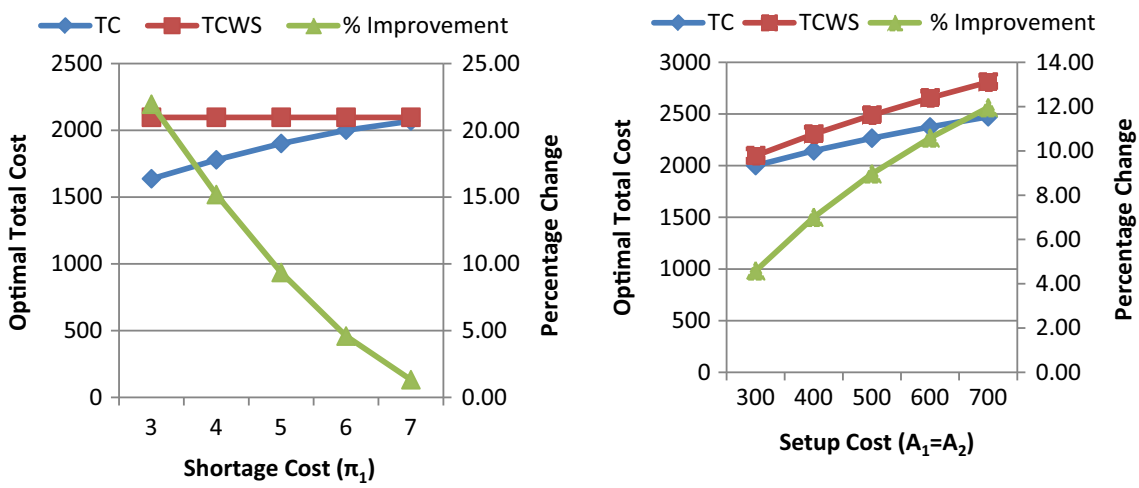

Fig. 8 % improvement in TC over TC_{WS} with variation in shortage cost (π_1) and setup cost ($A_1 = A_2$)

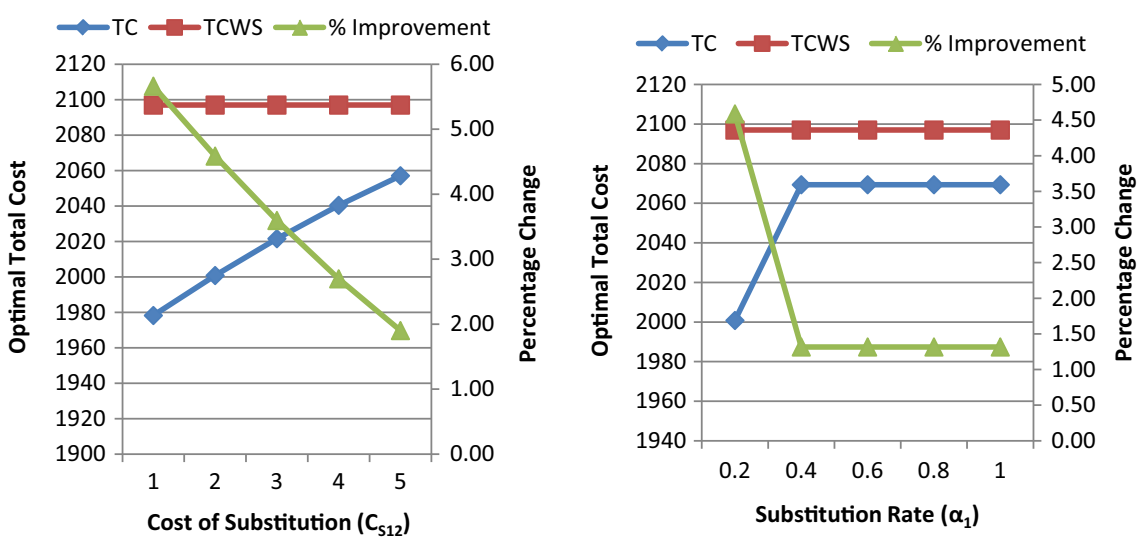

Fig. 9 % improvement in TC over TC_{WS} with cost of substitution (C_{S12}) and substitution rate (α_1)

Table 3 Improvement in total optimal cost with substitution over total cost without substitution

Parameters	Variation	Optimal total cost without substitution TC_{WS}	Optimal total cost TC_1	Percentage improvement
C_1/C_2	Increases	Increases	Increases	Increases
I	Increases	Increases	Increases	Increases
π_1	Increases	Constant	Increases	Decreases
A_1	Increases	Increases	Increases	Increases
C_{S12}	Increases	Constant	Increases	Decreases
α_1	Increases	Constant	Increases	Decreases
θ	Increases	Increases	Increases	Increases

In addition, as this paper only considered two deteriorated items, joint replenishment and same replenishment cycle, while future researches can focus on more than two items, different replenishment cycle for different items, multiple supplier and retailers, trade credit mechanism, supplier retailer cooperation, etc.

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