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**Working Paper**

## Endogenous growth and the Taylor principle

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Martin Micheli

## Endogenous Growth and the Taylor Principle

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Martin Micheli<sup>1</sup>

# Endogenous Growth and the Taylor Principle

## Abstract

*This paper analyzes conditions for determinacy in a new Keynesian model with endogenous growth. Endogenous growth shrinks the determinacy region considerably. Complying with the Taylor principle is not sufficient for determinacy, which decreases in the spillovers from actual on potential output. Monetary policy therefore has to be more aggressive than in an exogenous growth setup to ensure determinacy.*

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# 1 Introduction

Prior to the Great Recession, most economists viewed aggregate supply to determine an economy's potential output. Aggregate demand would explain fluctuations around this exogenous trend. Slow recoveries and hysteresis following the Great Recession (Ball, 2014; Fatas and Summers, 2016) however, are indicative of spillovers from aggregate demand on potential output (Summers, 2014; Yellen, 2016). Endogenous growth models allow for such spillovers.

This paper analyzes conditions for stability in a monetary model with endogenous growth. Up till now, the literature has focused on optimal monetary policy in such models (Lai and Chin, 2010; Annicchiarico and Rossi, 2013; Ikeda et al., 2014). We derive sufficient conditions for determinacy in such a model and show that complying with the Taylor principle, implying that nominal interest rates react more than one for one to inflation, is not sufficient prevent the occurrence of sunspot equilibria.

## 2 The model

### 2.1 Representative household

The economy is populated by infinitely many households with unit mass one. The representative household derives utility from consumption  $C$ , disutility from working  $L$ , and maximizes expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \left( \log(C_t) - \chi \frac{L_t^{1+\eta}}{1+\eta} \right). \quad (1)$$

$\beta$  represents the household's discount factor. Utility maximization is subject to the budget constraint

$$P_t C_t + B_t = P_t w_t L_t + (1 + r_{t-1}^B) B_{t-1} + P_t \Pi_t. \quad (2)$$

$P$  is the price level,  $B$  are one period bonds that pay the nominal interest rate  $r^B$ ,  $w$  is the real wage, and  $\Pi$  is the household's share of real firm profits. The household's first order conditions are the Euler (3) and the labor supply equation (4).

$$C_t^{-1} = \beta C_{t+1}^{-1} (1 + r_t^B) \frac{P_t}{P_{t+1}} \quad (3)$$

$$C_t^{-1}w_t = \chi L_t^\eta , \quad (4)$$

## 2.2 Final good firms

Perfectly competitive firms combine intermediate goods  $Y_t(f)$  to final good bundles  $Y_t$ . Firms minimize expenditures for intermediate products  $\int_0^1 P_t(f)Y_t(f)df$ , with the price  $P_t(f)$  of the intermediate good of type  $f$ . They face the production function

$$Y_t = \left( \int_0^1 (Y_t(f))^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}} . \quad (5)$$

The demand for the individual intermediate good is  $Y_t(f) = \left( \frac{P_t(f)}{P_t} \right)^{-\theta} Y_t$ .

## 2.3 Intermediate good firms

Infinitely many intermediate good firms operate under monopolistic competition. The individual firm  $f$  produces output  $Y_t(f)$  using labor  $L_t(f)$  as input factor.

$$Y_t(f) = Z_t L_t(f) . \quad (6)$$

$Z$  is a freely available stock of knowledge that determines labor productivity. It might therefore be thought of as a learning by doing externality, which has been labeled as serendipitous learning (Annicchiarico et al., 2011). This stock of knowledge is exogenous to the individual firm but endogenous on the aggregate level. It determines next period's labor productivity and by this is the root of endogenous growth.

$$Z_t = \varphi_{t-1} (Y_{t-1})^\alpha \quad (7)$$

$\varphi$  and  $\alpha$  determine how aggregate production increases the stock of knowledge. We use  $\varphi$  to pin down the rate of balanced growth,  $\alpha$  determines how strong fluctuations in economic activity affect potential output. Note that for  $\alpha = 0$ , the model collapses to a standard new Keynesian model.

Cost minimization yields an expression for real marginal cost  $RM C$

$$RM C_t = \frac{w_t}{Z_t} . \quad (8)$$

Intermediate good firms set intermediate good prices to maximize profit. We assume staggered price setting a la Calvo. Each period, only the share  $1 - \omega$  of firms is able to



adjust prices. This results in the optimal price  $P_{jt}$  for firms that are able to adjust prices

$$\frac{P_{jt}}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{i=0}^{\infty} (\omega\beta)^i C_{t+i}^{-1} Y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta} RMC_{t+i}}{E_t \sum_{i=0}^{\infty} (\omega\beta)^i C_{t+i}^{-1} Y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}} \quad (9)$$

and the law of motion for the price level

$$P_t^{1-\theta} = (1 - \omega) P_{jt}^{1-\theta} + \omega P_{t-1}^{1-\theta} . \quad (10)$$

## 2.4 Aggregation

In equilibrium, markets clear. Aggregate production equals aggregate consumption

$$Y_t = C_t . \quad (11)$$

The aggregate production function of intermediate good producers reads

$$D_t Y_t = Z_t L_t , \quad (12)$$

with  $L_t = \int_0^1 L_t(f) df$ , price dispersion  $D_t = \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\theta} df$  and the law of motion for price dispersion

$$D_t = (1 - \omega) \left( \frac{P_{jt}}{P_t} \right)^{-\theta} + \omega \left( \frac{P_{t-1}}{P_t} \right)^{-\theta} D_{t-1} . \quad (13)$$

## 2.5 Balanced Growth

The economy is described by the equations (3), (4), (7), (8), (9), (10), (11), (12), (13), the definition of price dispersion  $D_t$ , and a rule for monetary policy. This system however, is instationary due to serendipitous learning. We stationarize the system by expressing all variables in terms of deviations from balanced growth. We define  $x^* = \frac{x_t}{Z_t} \forall x_t \in \{y_t, c_t, w_t\}$ ,  $\gamma_t = \frac{Z_t}{Z_{t-1}}$ , and  $\varphi^* = \frac{\varphi_t}{Z_t^{1-\alpha}}$ . The last equality implies that efficiency of serendipitous learning  $\varphi$  increases in the stock of knowledge. This assumption is necessary to ensure balanced growth for  $\alpha \in (0, 1)$ .

## 2.6 Monetary policy

The central bank adjusts nominal interest rates to deviation of inflation ( $\pi_t$ ) from the inflation target ( $\pi$ ) and to deviations of real output from its balanced growth path ( $y^*$ ).

$$\frac{1 + r_t^b}{1 + r^b} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t^*}{y^*}\right)^{\phi_y} \quad (14)$$

## 2.7 Calibration

We calibrate six parameters<sup>1</sup>  $\alpha, \beta, \eta, \omega, \phi_\pi$ , and  $\phi_y$  for a quarterly frequency and analyze deviations from the zero inflation balanced growth path ( $\pi = 1$ ).  $\beta$  is set to 0.99 representing an annual real interest rate of 4%. The inverse of the Frish elasticity is  $\eta = 1$ .

We mainly use to values for the frequency of price adjustments. We set  $\omega = 4/7$  representing an average price duration of 7 Months as in the US and  $\omega = 10/13$  representing an average price duration of 13 Months similar to what is observed in the Euro area (Dhyne et al., 2006). Ball (2014) finds that the elasticity of potential output with respect to real economic activity  $\alpha$ , is close to one. We set  $\alpha = 0.9$ . We vary the parameter values of  $\omega$  and  $\alpha$  as well as the parameters in the central bank's reaction function to show how these parameters affect determinacy.

## 3 Interest rate policy and aggregate stability

We can rewrite the system, which allows us to derive analytical results for the regions of determinacy. We define  $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ .

$$\begin{pmatrix} \hat{y}_{t+1}^* \\ \hat{\pi}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 - \alpha + \phi_y + \kappa \frac{1+\eta}{\beta} & \phi_\pi - \frac{1}{\beta} \\ -\kappa \frac{1+\eta}{\beta} & \frac{1}{\beta} \end{pmatrix}}_A \begin{pmatrix} \hat{y}_t^* \\ \hat{\pi}_t \end{pmatrix} \quad (15)$$

The rational expectations equilibrium is stable if both eigenvalues of  $A$  lie outside the unit circle. This is true if i)  $\det A > 1$ ,  $\det A - \text{tr } A > -1$  and  $\det A + \text{tr } A > -1$  or ii)  $\det A - \text{tr } A < -1$  and  $\det A + \text{tr } A < -1$  (Woodford, 2003, Appendix C.1). As both, the trace  $(1 - \alpha + \phi_y + \kappa \frac{1+\eta}{\beta} + \frac{1}{\beta})$  and the determinant  $(\frac{1}{\beta} (1 - \alpha + \phi_y + \kappa (1 + \eta) \phi_\pi))$  of  $A$  are strictly positive given our calibration and plausible values for  $\phi_y$ , ii) is clearly violated.

Turning to i), as the determinant and the trace of  $A$  are strictly positive,  $\det A + \text{tr } A > -1$  is always satisfied. The remaining two conditions for a stable equilibrium can be summarized by

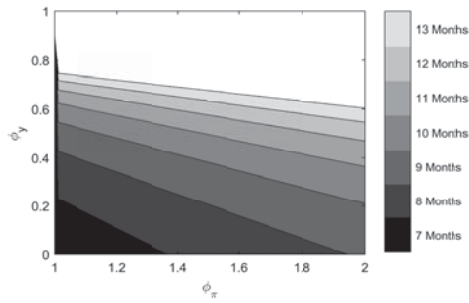
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<sup>1</sup>We analyze the first order approximation around balanced growth, all other parameters vanish from the linearized system of equations.

$$\phi_\pi > \begin{cases} \frac{\beta-1+\alpha}{\kappa(1+\eta)} - \frac{1}{\kappa(1+\eta)}\phi_y & \text{for } \phi_y \leq \frac{\kappa(1+\eta)+1-\beta-\alpha\beta}{-\beta} \\ 1 + \alpha\frac{1-\beta}{\kappa(1+\eta)} - \frac{1-\beta}{\kappa(1+\eta)}\phi_y & \text{for } \phi_y > \frac{\kappa(1+\eta)+1-\beta-\alpha\beta}{-\beta} \end{cases} \quad (16)$$

Figure 1 shows determinacy for different combinations of the parameters in the central bank's reaction function  $\phi_\pi$  and  $\phi_y$  in the baseline calibration. Shaded areas indicate indeterminacy. To show the effect of price stickiness, we allow for average price durations between 7 and 13 months. The space of parameter combinations resulting in indeterminacy increases in the average price durations.

**Figure 1:** Indeterminacy and average price duration



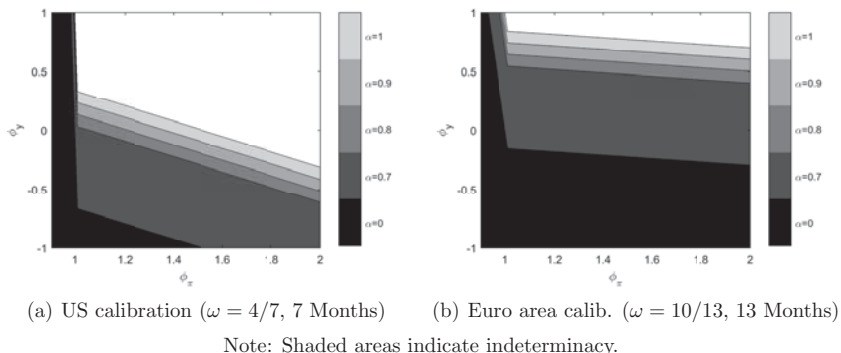
Note: Shaded areas indicate indeterminacy.

Figure 2 illustrates indeterminacy for different values of  $\alpha$ , the elasticity of potential to actual output. As reference, we show indeterminacy for  $\alpha = 0$ , the new Keynesian model with exogenous growth. Panel (a) uses an average price duration of 7 months, panel (b) shows determinacy for an average duration of 13 months. Spillovers from actual on potential output increase indeterminacy.

The intuition for our results is straightforward. In a standard new Keynesian model, adjusting nominal interest rates more than one for one to inflation is sufficient to ensure determinacy if  $\phi_y = 0$ . If the central bank takes real economic activity into account ( $\phi_y > 0$ ), values slightly less than one for  $\phi_\pi$  are possible as long as nominal interest rates rise more than one for one in the long run (Ascari and Ropele, 2009).

Under endogenous growth, the path of potential output depends on economic activity. Inflationary shocks not only lower economic activity but also potential output. The c.p. negative effect of subdued economic activity on inflation is therefore lower in an endogenous growth model than in a standard new Keynesian model. The central bank c.p. has to react stronger to inflationary pressures to ensure determinacy. The threshold

**Figure 2:** Indeterminacy under different potential output elasticities



values for the parameters in the central bank's reaction function to ensure determinacy increase in  $\alpha$  and in the Calvo-parameter  $\omega$  due to increasing variability of potential output in these two parameters.

## 4 Concluding remarks

This paper analyzed determinacy in a monetary model with endogenous growth. We show that endogenous growth increases the space of indeterminate parameter constellations considerably. The Taylor principle is not sufficient to ensure determinacy. Aggregate instability increases in the elasticity of potential output with respect to real economic activity and in the degree of price stickiness. Especially when price stickiness and spillovers from actual on potential output are substantial, central banks should react strong to deviations from their targets for inflation and economic activity to ensure determinacy.

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