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Optimal extraction paths with electric power generation

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OPTIMAL EXTRACTION PATHS WITH ELECTRIC POWER GENERATION

HEMF Working Paper No. 07/2017

by

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and

Christoph Weber

July 2017

UNIVERSITÄT DUISBURG ESSEN

Open-Minded

Optimal Extraction Paths with Electric Power Generation

by Andreas A. Renz and Christoph Weber

Abstract

It would seem that Hotelling's rule and its related models of resource extraction and electricity production as largest consumer of scarce resources are closely related. However, although fixed costs and a non-storable product are essential in characterizing electricity markets, they can hardly be found in respective literature. We show optimal extraction paths when coal, gas and a renewable with differing fixed and variable costs as well as carbon intensities are considered. The technology with lowest fixed costs will then – though relying on a scarce resource – always be used in perpetuity. The high fixed-cost fossil technology may be exploited at a definite point of time if it is relatively scarce or also used ad infinitum.

Keywords : Scarce resources, Optimal control theory, Hotelling, Valuation, Non-renewable resource, Pollution target, Climate change, Peak-load-pricing
JEL-Classification : C61, Q32, Q41, L94

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1 Introduction

For decades, fossil energy sources have been the backbone of global energy supply. This has not changed despite of international agreements such as the Kyoto Protocol to limit global warming by constraining carbon emissions. Stern (2007) and others show that this lack of action will lead to a fundamental welfare loss in the future. A major reason behind might be the time delay between cause and effect. So differs an optimal consumption of fossil fuels in a near-term view heavily from one in a long-term perspective.

Hotelling (1931) first describes optimal extraction and price paths for exhaustible resources. Herfindahl (1967) proves that if multiple resources compete, the order of extraction is given by increasing order of respective cost. While this conclusion has been widely discussed in literature, most models rely on cost as differentiating argument (c.f. Solow and Wan (1976), Kemp and van Long (1980)). Chakravorty and Krulce (1994) show that if differences in resources quality cannot be characterized independently from demand, Herfindahl's rule may no longer apply. Instead, there exist periods where multiple resources are either extracted jointly or exclusively.

When atmospheric accumulation of carbon dioxide has been identified as cause for global warming, Hotelling literature was given a new perspective while introducing a constraint on global emission in addition to the scarcity of fossil resources. Tahvonen (1997) shows that a backstop technology may then be used simultaneously with a scarce resource despite of higher unit costs if externality costs of pollution are included via a damage function. Chakravorty et al. (2006) consider political targets to limit carbon emissions by a ceiling on the stock of pollution.[1] Depending on an exogenous change of demand, possible extraction paths may then also include a phase where both the expensive backstop and a nonrenewable fossil resource are used simultaneously.

Chakravorty et al. (2008) assess the optimal order of extraction of two nonrenewable and a backstop technology. In contrast to the tradition of Hotelling they completely neglect extraction cost, so polluting characteristics and abundances become the only differentiators. Counter-intuitive from Herfindahl's perspective, the "bad", more polluting resource is then used first until the stock of pollution is filled up to the maximum target. Then, the less polluting resource is used first, followed by the more polluting. This implies a potential switching from the dirty resource to the cleaner one and back what has also not been observed in earlier Hotelling literature. They conclude that the initial endowments of the two resources will determine the order of extraction.

The literature mentioned describes energy demand from a mainly general perspective.[2] Consequently, the differing characteristics of multiple energy sectors (e.g., Electricity, Heating, Transportation) are not considered. Electricity markets, which bear the lion's share of global energy supply, are characterized by the (nearly) non-storability of electric power. Furthermore, it is the energy sector which has a great potential for shifting the production mix. But here, costs have to be considered in a more differentiated way. E.g. gas power plants are economical to use despite of high variable costs when meeting rare peak demands because fixed costs are low. Contrarily, a high utilization of base load coal power plants can compensate for an initially much higher investment. Peak load pricing has shown to be an appropriate means to describe these types of markets (c.f. Steiner (1957), Hirshleifer (1958), Boiteux (1960), Crew and Kleindorfer (1979), Chao (1983) and

[1] A targeted atmospheric concentration of e.g. 550 pm does directly transform into an absolute target amount of atmospheric carbon dioxide and vice versa.

[2] Chakravorty and Krulce (1994) describe optimal paths where demand is heterogeneous.

Weber (2005)).

In this paper we characterize the solution to a two-resource-model where a fluctuating demand is described by a load duration curve. The technologies are characterized by different operating and fixed costs, polluting intensities and efficiencies. For illustrative reasons we use coal which is assumed to show high fixed and low operating costs and is very polluting with low efficiency, and gas as a technology with inverse characteristics. Wind forms a backstop technology with highest fixed costs. We consider goals to limit global warming by a ceiling on the stock of pollution.

We show if initial endowments with resources are low, all three technologies are used from the very beginning simultaneously. The technology with lowest fixed costs will be part of the efficient mix until infinity. So, in contrast to classical Hotelling models we do not always see a definite time of exploitation, though, all resources will be fully exploited.

If endowments with fossil fuels are large, an initial path of non-zero time length, where the renewable is not yet part of the mix, will result. As soon as it has joined the mix its share will continuously increase. If initial endowments are large enough, an intermediate phase, where pollution stays at the maximum allowable level, results. Here, natural dilution and anthropogenic emissions are counterbalancing each other. This ceiling phase could but does not have to be characterized by a constant energy mix. Eventually, the mix will tend towards a clean one.

We further propose a straightforward definition of relative scarcity. This question has often been answered by a comparison of reserves-to-production ratios. Certainly, a energy market will adjust the usage if one resource turns out to exhaust early and therefore a definition on today's basis may be misleading. Our approach distinguishes itself as we consider optimal paths. If coal is relatively scarce, it will be exploited at a definite point of time. If it is gas, both resources will be used upon infinity. This becomes visible when comparing current Hotelling rents under consideration of differing fixed costs. Gas will be used upon infinity in any case.

2 A Hotelling model distinguishing fixed and variable cost components

Consider an economy with a fixed price-inelastic electricity demand which is characterized by a fluctuating load. Given the (nearly) non-storability of electrical energy, demand has to be met instantaneously. Let it be described by a load duration curve (ldc) $D : [0; S] \mapsto \mathbb{R}_+, s \to D(s)$ which is continuously decreasing to zero over the interval $[0; S]$. The ldc shows the load over a representative period S (typically a year, say, operating period) arranged in decreasing order of magnitude (cf. figure 1).

Welfare W is the negative sum of operating and user costs to make capacities available discounted at a constant rate of $\rho>0$. It is defined by $W = -\int_0^\infty (\sum C_{inv}(t) + \sum C_{op}(t))e^{-\rho t}dt$. At each point of time t, the electricity demand characterized by $D(s)$ is met.[3] We consider three technologies indexed by $u = 1, 2, 3$ in decreasing order of fixed costs f_u. Technology 1 (say, wind) is clean and has highest fixed and no variable costs. Technology 2 (coal) has medium fixed and relatively low operating costs c_u. It is very polluting at low efficiency (or high heat rate h_u).

[3] We intentionally model a row of consecutive periods in a continuous way. So, the ldc describes the distribution of demand at each point of time t which is then assumed to be constant over time: $\frac{\partial D(s)}{\partial t} = 0$.

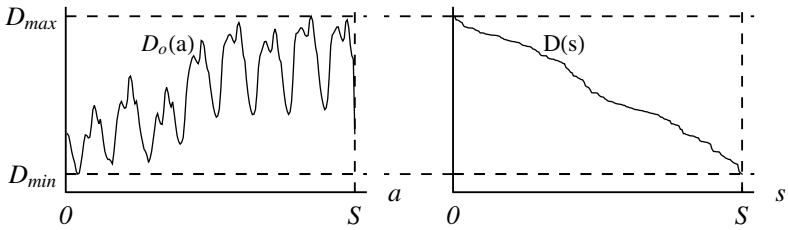

Figure 1: Chronological load demand development $D_o(a)$ over a defined operating period S and respective load duration curve (ldc) $D(s)$ for the same period.

Technology 3 (gas) has lowest fixed, but highest operating costs. It is less polluting than coal and has a relatively high efficiency. f_u incurs per capacity installed K_u and represents the user cost of capital to make capacities available permanently.[4] c_u is assumed to be constant in scale, occurs per energy unit produced Q_u and includes plant operating and extraction costs.[5] The renewable does not depend on depletive input factors while each of the other two technologies rely on finite fossil resources.[6] The extraction rates can then be described as follows:

$$\dot{X}_u(t) = -h_u \cdot Q_u(t) \text{ for } u = 2,3 \text{ with } X_u(t=0) = X_u^0, \tag{1}$$

while the total energy demand must be covered by the three available resources ($\sum_{u=1}^{3} Q_u(t) = \int_0^S D(s)ds$) for $t \geq 0$. Let $Z(t=0)$ be the initial excess over natural steady-state concentration of CO_2 in the atmosphere, $Z(t)$ the current. It is increased by burning fossil fuels and decreased by a natural dilution rate proportional to $Z(t)$. $\alpha > 0$ describes the natural decay rate, ζ_u the technology-specific emissions per electricity unit produced. Consider commitments to limit global warming implemented as a political limit on the stock of pollution $\bar{Z}$.[7] Then,

$$\dot{Z}(t) = \sum_{u=2}^{3} \zeta_u \cdot Q_u(t) - \alpha \cdot Z(t) \text{ with } Z(t) \leq \bar{Z} \text{ and } 0 < \alpha < 1. \tag{2}$$

Let the production portfolio indirectly be defined by minimum s_u^{min} and maximum operating hours s_u^{max} per technology u and the ldc (cf. figure 2). In a portfolio that consists of wind, coal and gas, the installed capacities of each technology $u = 1,2,3$ at each point of time t are then sufficiently defined by

$$K_u(s_u^{min}(t), s_u^{max}(t)) = D(s_u^{min}(t)) - D(s_u^{max}(t)), \tag{3}$$

[4] The definitions for fixed and operating cost are similar to the standard peak load pricing model if we normalize t with the length of the representative period S: $t = a/S$ (cf. Renz and Weber (2017)).

[5] Strictly speaking it includes extraction, processing, delivery to plant and plant operation.

[6] The technologies $u = 2,3$ are then also distinctly defined by the respective resource and vice versa.

[7] This limit is assumed to be tight enough to not allow an infinite gas-only mix.

and the respective electricity produced is

$$Q_u(s_u^{min}(t), s_u^{max}(t)) = \int_{s_u^{min}(t)}^{s_u^{max}(t)} D(s)ds + D(s_u^{min}(t)) \cdot s_u^{min}(t) - D(s_u^{max}(t)) \cdot s_u^{max}(t). \tag{4}$$

The upper operating time limit of the highest fixed cost technology in the portfolio is consequently defined by the operating hours per period S, the lower limit of the lowest fixed cost technology equals zero. Furthermore, $\sum K_u(t) = D_{max}$ to ensure that peak demands can be met with capacities installed at all times.

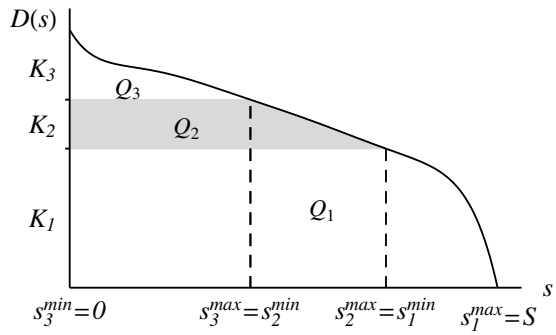

Figure 2: Determination of installed capacities K_u and energy produced per technology Q_u via operating time limits and the ldc.

The social planner maximizes utility (or negative cost) by choosing appropriate operating limits as follows:

$$\max_{s_u^{min}(t), s_u^{max}(t)} \int_0^\infty \left[-\sum_{u=1}^{3} f_u \cdot K_u(s_u^{min}(t), s_u^{max}(t)) - \sum_{u=1}^{3} c_u \cdot Q_u(s_u^{min}(t), s_u^{max}(t)) \right] e^{-\rho t} dt. \tag{5}$$

Proposition 1. *The optimal energy mix at each point of time t is defined by a set of operating limits*

$$s_{u,v}(t) = \frac{f_u - f_v}{c_{var,v}(t) - c_{var,u}(t)} \tag{6}$$

between competing technologies $u < v$. $s_{u,v}(t)$ forms the maximum operating time of technology v and the minimum time of the higher fixed-cost technology u. The variable costs $c_{var,u}(t)$ are the sum of 'pure' operating costs c_u, the scarcity rent $\lambda_u(t)$ and the external cost of pollution $\lambda_p(t)$:

$$c_{var,u}(t) = c_u + h_u \cdot \lambda_u(t) - \zeta_u \cdot \lambda_p(t) \tag{7}$$

Proof. See appendix A.1. □

So, the efficient portfolio can be defined by a set of $s_{u,v}(t)$ in the way that for operating hours $s > s_{u,v}$ with $u < v$, technology u is favorable, for operating hours $s < s_{u,v}$ it is v. (6) reveals that the operating hours for a specific technology in an optimal solution could also be derived using intersections of cost curves $C_u(s,t) = f_u + c_{var,u}(t)s$ of competing technologies (see left-hand side of figure 3 for illustration). So it is in line with solutions for an optimal portfolio in deterministic capacity planning (cf. Steffen and Weber (2013)). It confirms intuition since a technology u is part of the portfolio where its costs are minimum considering both fixed and variable costs as a function of operating time s (see 'efficiency border' in figure 3). This efficient operating time of technology u is then limited by a lower bound $s_{u,v}$, where it competes with the adjacent technology with lower fixed cost (or zero for gas), and an upper bound $s_{w,u}$, where it competes with a technology with higher investment costs (or S for the technology with highest investment cost).

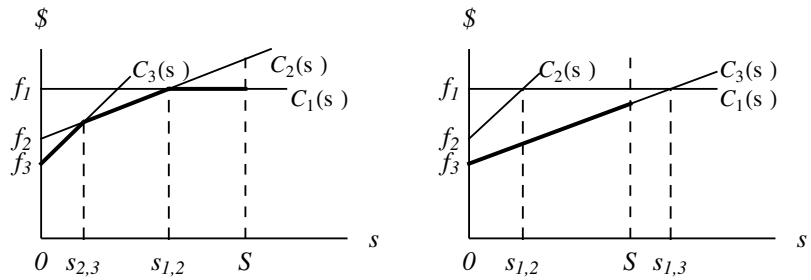

Figure 3: Formation of the efficiency border (fat line). On the left-hand side, all three technologies are part of the efficient portfolio, on the right-hand side, technology 3 is the only one.

Consequently, a technology may not qualify to be part of the efficient portfolio (refer to the right-hand side of figure 3): Then, a technology could be dominated[8] by a combination of other ones (see for example $u = 2$) or its efficient operating time could lie outside the relevant operating range, (see $u = 1$ as another example). Renz and Weber (2017) comprehensively describe which conditions must be fulfilled to make a technology part of the efficient portfolio. However, technologies may leave or enter the efficient portfolio over time when variable costs change.

While operating and investment costs are assumed to be constant over time, scarcity rents and external cost of pollution (say, emission costs) change. Their dynamics are defined by

$$\dot{\lambda}_u(t) = \rho \cdot \lambda_u(t) - \frac{\partial L(t)}{\partial X_u(t)} = \rho \cdot \lambda_u(t) \Rightarrow \lambda_u(t) = \lambda_u^0 \cdot e^{\rho t} \tag{8}$$

and

$$\dot{\lambda}_p(t) = \rho \cdot \lambda_p(t) - \frac{\partial L(t)}{\partial Z(t)} = (\rho + \alpha)\, \lambda_p(t) + \nu_c(t), \tag{9}$$

with the complementary slackness condition as follows

$$\nu_c(t) \geq 0,\ \bar{Z} - Z(t) \geq 0,\ \nu_c(t)\left(\bar{Z} - Z(t)\right) = 0. \tag{10}$$

[8] Here, 'dominate' means lower cost in the respective operating range $s \in [0; S]$.

According to (8) any scarcity rent $\lambda_u(t)$ will rise exponentially at the rate of discount at all times. If the stocks available are low or emission targets loose, $\bar{Z}$ is not binding and $\lambda_p(t) \equiv 0$. Define this Hotelling path as $s^H_{u,v}(t) = s_{u,v}(\lambda_u(t) > 0, \lambda_p(t) \equiv 0)$.

The transversality conditions at infinity are

$$\lim_{t \to +\infty} e^{-\rho t} \cdot \lambda_u(t) \cdot X_u(t) = 0 \tag{11}$$

and

$$\lim_{t \to +\infty} e^{-\rho t} \cdot \lambda_p(t) \cdot Z(t) = 0. \tag{12}$$

Furthermore, we specify the expensiveness of the renewable so that wind would not be part of the efficient portfolio if no fossil scarcity or emission targets existed, i.e.

$$s_{1,3} = \frac{f_1 - f_3}{c_3} \geq S \text{ or } s_{1,2} = \frac{f_1 - f_2}{c_2} \geq S. \tag{13}$$

In this case the mix consisted of coal and gas only, i.e.

$$s_{2,3} = \frac{f_2 - f_3}{c_3 - c_2} < S. \tag{14}$$

Economic sense then yields the necessary condition[9] for an optimal solution as follows:

$$h_u \cdot \int_0^\infty Q_u(t)dt = X_u^0 \text{ for } u = 2, 3. \tag{15}$$

3 Optimal control and corresponding extraction paths

In this section, we begin the analysis by discussing the role of wind as backstop technology presuming a 'pure' Hotelling path ($\lambda_p(t) \equiv 0$). We then accommodate for two competing fossil technologies with differing abundances by introducing a definition of relative scarcity on the basis of optimal extraction paths. Eventually, we describe optimal extraction where highly abundant initial stocks make a carbon ceiling binding.

3.1 Backstop technology entering the production mix

We begin by showing in which way the scarcity rent becomes a function of the initial resource stock and how its increase over time determines the efficient portfolio mix.

Proposition 2. *If initial endowments with fossil resource stocks are scarce, an expensive renewable technology will complement the efficient production portfolio from the very beginning. If the initial resource stocks are abundant, e.i. $X_3^0 > X_3^R(X_2^0)$, - and emission targets are not extremely tight, - the renewable will complement the portfolio at a later point of time.*

Proof. See appendix A.2. □

[9] Equation (8) in (11) gives a similar condition.

Proposition 2 says that if the initial resource stocks are abundant, the bounded demand limits its use and not its scarcity. Then, the scarce resources will be used exclusively until the resource stocks have decreased to a level where the increased scarcity makes the renewable technology economical to use.[10] Such a path, with only one fossil technology available to support intuition, is determined by its maximum upper operating bound $s^H_{1,3}(t)$ and the requirement for complete exploitation formulated in (15).[11] It is depicted in figure 4.

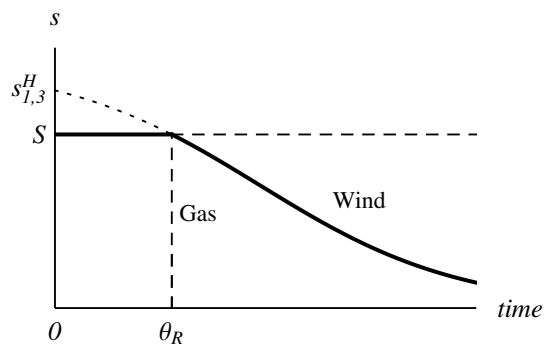

Figure 4: Production portfolio time-path (fat line) with gas ($u = 3$) and wind ($u = 1$) if the ceiling is never binding.

Initially, $s^H_{1,3}(t = 0) = s^{0,H}_{1,3} > S$ and gas is exclusively used (cf. the situation described on the right-hand side of figure 3). The Hotelling rent $\lambda_3(t) = \lambda^0_3 \cdot e^{\rho t}$ will increase until $s^H_{1,3}(t)$ equals S and wind enters the portfolio.[12] Let this point of time be defined as θ_R. Further define X^R_3 as the initial reserve which leads to an $s^{0,H}_{1,3} = S$. Then, for all $X^0_3 > X^R_3$, $s^{0,H}_{1,3} > S$ and gas is used exclusively in the beginning. If the initial stock is less, the initial portfolio already consists of gas and wind.

If both resources are available, the maximum upper operating bound of the combined fossil energy use is defined by $\max\left\{s^H_{1,2}, s^H_{1,3}\right\}$. Consequently, a similar case as the one with a single resource applies.[13]

But in the case of two available resources, if $s^H_{1,3} \geq s^H_{1,2}$, a cost-minimum production mix would suspend coal from the efficient portfolio (cf. again the right-hand side of figure 3). The optimal sequence and development of production portfolios over time and the corresponding extraction rates become a function of the relative and absolute initial endowments with the fossil stocks. This is shown in figure 5. There, scarce combined initial endowments are situated in region I and result in a mix that includes wind for the complete time horizon. The further elements of figure 5 will be discussed in the following.

3.2 Definition of relative scarcity

While the upper operating bound of fossil use must decrease over time, the path of $s^H_{2,3}$ is determined by differing scarcities of coal and gas. Detailing this behavior in the coming section, we still stick to the assumption that the emission ceiling is never binding.

[10] Not 'extremely tight' emission targets mean that such a path is still admissible ($\lambda_p(t) \equiv 0$ for $t \geq 0$).

[11] Equation (4), (6), (7) and (15) give $\theta_R \cdot \int_0^S D(s)ds + \int_{\theta_R}^{\infty} \int_0^{s^H_{1,3}(t)} D(s) - D(s^H_{1,3}(t))dsdt = X^0_3/h_3$.

[12] For the ease of representation, we omit the time argument whenever the context is clear in the following .

[13] Cf. the proof of Proposition 2 in appendix A.2.

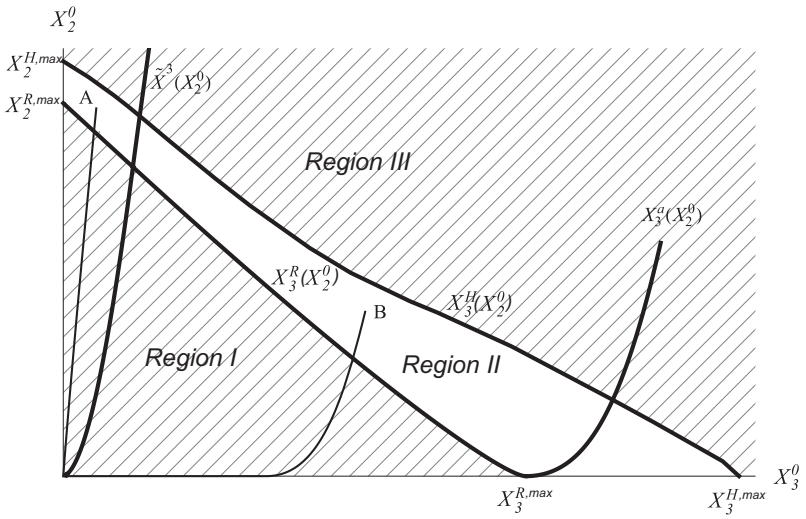

Figure 5: Overview of potential extraction paths per initial endowment. The fat fat lines frame different regions, the thin lines possible extraction paths. (Note: The figure above assumes a linear decreasing ldc.)

Proposition 3. *Coal is relatively scarce compared to gas if the optimal control paths leads to its complete exploitation before infinity. This is the case if the initial ratio of scarcity rents of coal to gas is strictly above their ratio of relative fixed costs advantage towards the renewable:*

$$\frac{h_2 \cdot \lambda_2^0}{h_3 \cdot \lambda_3^0} > \frac{f_1 - f_2}{f_1 - f_3}. \tag{16}$$

If gas is relatively scarce, both resources will be consumed simultaneously until infinity.

Proof. See appendix A.3. □

Depending on the initial resource allocation, coal may be exhausted at a distinct point of time $< \infty$ or it is used jointly with gas and wind up to infinity. The technology with lowest fixed costs (which is gas) must always complement the portfolio. It becomes obvious when looking at the cost curves $C_u(s) = f_u + c_{var,u} \cdot s$: For low operating hours s gas shows always (for $c_{var,3} < \infty$) lowest production costs. We explain respective Hotelling paths of both types of scarcities sequentially in the following:

Gas is relatively scarce, coal is abundant If gas is relatively scarce and coal is abundant, both resources are used until infinity. Consider a case where all three technologies are part of the portfolio. The actual use of coal $Q_2(s_{1,2}^H, s_{2,3}^H)$ is then defined by its lower operating bound, specified in

$$s_{2,3}^H = \frac{f_2 - f_3}{(c_3 - c_2) + (h_3 \cdot \lambda_3 - h_2 \cdot \lambda_2)}, \tag{17}$$

and an upper bound as follows:

$$s^H_{1,2} = \frac{f_1 - f_2}{c_2 + h_2 \cdot \lambda_2}. \tag{18}$$

If (16) is reversed it ensures that $s^H_{1,2} > s^H_{2,3}$ and consequently $Q_2 > 0 \ \forall t \geq 0$.[14] The integral use of gas is defined through $Q_3(0, s^H_{2,3})$ and (15). So, the difference of initial scarcity rents $(h_3 \cdot \lambda^0_3 - h_2 \cdot \lambda^0_2)$ becomes a function of X^0_3 only (cf. appendix A.5.1). It follows that the path of $s^H_{2,3}$, which sufficiently determines the share of gas for the entire time horizon, is independent from X^0_2 as long as the ceiling is not binding.

At each point of time coal and wind compensate the remaining demand.[15] If the combined initial stocks are abundant, it exists a non-zero time period at the beginning where wind is not yet part of the efficient portfolio (and the consumption of coal is defined through $Q_2(s^H_{2,3}, S)$). Corresponding extraction paths would start in the left part of Region I if the combined initial stocks are scarce, or in Region II if the combined stocks are abundant, but the ceiling still not binding. Path A shows an exemplary extraction path.

Refer to figure 6 for the respective control path. In the beginning, only coal and gas are used. The share of gas will continuously decrease compensated by an increase of coal. From θ_R onwards, the renewable complements the portfolio. All three technologies are then used until infinity while the share of wind is continuously increasing.

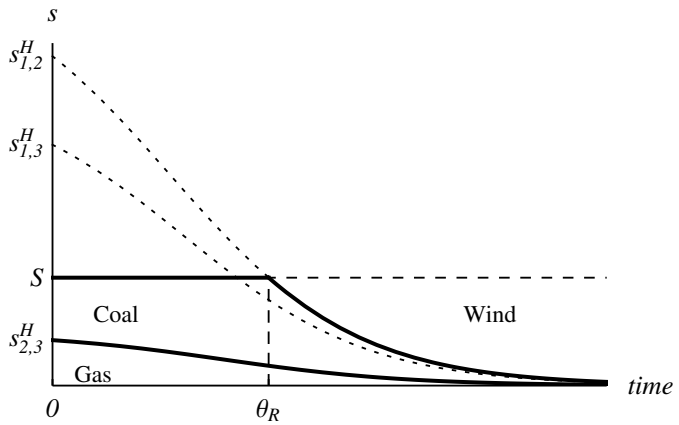

Figure 6: A case with two available initial stocks without CO_2 restrictions and scarce gas resources.

Gas is relatively abundant, coal is scarce The higher the initial endowment with gas (at a given X^0_2), the lower its initial scarcity rent, the higher $s^{0,H}_{2,3}$ and the slighter its decrease. Hence, there exists an $\tilde{X^3}(X^0_2)$ which leads for all $X^0_3 > \tilde{X^3}(X^0_2)$ to a path of $s^H_{2,3}$ slighter decreasing than that of $s^H_{1,2}$ which has to be eventually continuously decreasing in any case (recall equation (18)). Hence, if $X^0_3 \leq \tilde{X^3}(X^0_2)$, we see the development as described in the previous case. Otherwise there exists a time $\theta_{2,E}$ when $s^H_{1,2}$ equals $s^H_{2,3}$ and coal must be terminally exhausted.[16] Such initial endowments are found in the middle part of Region I in figure 5. Again, if the combined

[14] The Hotelling paths are considered here. In appendix A.3, this coherence is described in detail.
[15] If (16) does not hold, $s^H_{2,3} < S$ for all $t \geq 0$ is a consequence of (14).
[16] This is a direct consequence of (15).

initial endowment with coal and gas is abundant, an initial phase where the renewable energy is not part of the efficient portfolio results (see middle part of Region II in figure 5 and Path B as a exemplary extraction path).

A corresponding control path is depicted in figure 7. Coal is relatively scarce, the combined initial stocks are abundant enough to result in a non-renewable phase, but not highly so that a ceiling is never binding. First, coal and gas are used simultaneously until at $t = \theta_R$, the renewable complements the mix. All technologies are then used until coal is fully exploited at $t = \theta_{2,E}$. Now, wind and gas are used until infinity.

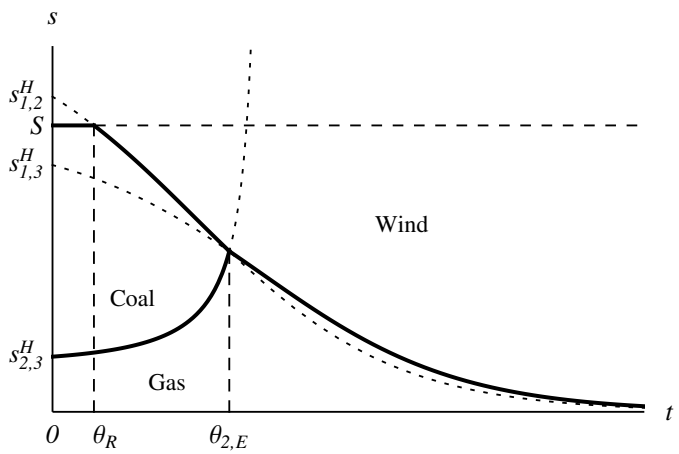

Figure 7: A case with two available initial stocks without CO_2 restrictions and scarce coal resources

Starting from a situation as described in figure 7, the lower the initial stock of coal (at a given X_3^0), the quicker $s^H_{2,3}$ will increase. So, there exists a path $s^H_{2,3}$ that cuts S from below before $s^H_{1,2}$ does from above. Consequently, coal would leave the efficient portfolio and must be exploited before wind complements the portfolio. Initial endowments that result in such paths are situated in the right part of Region II in figure 5. We define the borderline initial stocks $X_3^a(X_2^0)$ where the superscript stands for abundance.[17] For all $X_3^0 \geq X_3^a(X_2^0)$, the renewable does not enter the portfolio before coal ist exploited.[18]

3.3 Production paths when a ceiling is binding

Thus far, the analysis has focused on 'pure' Hotelling paths in the absence of a binding emission ceiling. To bring out this effect we assume high abundance of the combined fossil resource stocks in the following. Such initial endowments would be situated in Region III in figure 5.

Proposition 4. *If combined initial endowments with fossil resource stocks are highly abundant, a ceiling on the stock of pollution becomes binding and $\lambda_p(t)$ is initially strictly less zero. A subsequent ceiling phase, where the stock of pollution stays at the maximum allowable level, is characterized by external cost of pollution increasing to zero. This decrease is a function of the*

[17] Cf. appendix A.5.1.

[18] This initial stock combinations $X_3^a(X_2^0)$ are defined via $\frac{h_2 \cdot \lambda_2^0}{h_3 \cdot \lambda_3^0} = \frac{f_1 - f_2 + S \cdot (c_1 - c_2)}{f_1 - f_3 + S \cdot (c_1 - c_3)}$, the initial endowments derive respectively. Here, wind enters the mix exactly at the point of time when coal is exploited.

specific shape of the ldc if all three available technologies are part of the efficient mix at that time.

Proof. See appendix A.4. □

Proposition 4 says that there exists a threshold of initial stock combinations $(X_3^H(X_2^0))$ where for any $X_3^0 > X_3^H(X_2^0)$, a 'pure' Hotelling path from the very beginning would lead to breaking the ceiling. In this case the carbon ceiling will become binding in the future. Hence, since we assume that $Z(t=0) < \bar{Z}$, the external cost of pollution develop with $\lambda_p^0 \cdot e^{(\rho+\alpha)t}$ in the beginning ($Z(t) < \bar{Z}$ yields $\nu_c = 0$ in (10) what simplifies the solution of (9)). Let $s_{u,v}^c$ describe the paths during this initial phase. During this phase $\dot{Z} > 0$ until the carbon stock reaches the ceiling. Let $t = \theta_C$ specify this point of time. Then, Z stays at the ceiling for a non-zero time period until further increasing scarcity rents shift the mix to a cleaner one with $\dot{Z} < 0$ (Renz and Weber, 2017). This point of time should be specified by θ_H. Here, a Hotelling path begins until infinity.

If only one resource was initially available or coal already exploited, the characterization of the ceiling phase would be fairly simple. $Z(t) = \bar{Z}$ would allow for only a single, consequently stable, mix defined by $Q_u(0, s_{1,u}) = \alpha \cdot \bar{Z}/\zeta_u$ for either $u = 2$ or $u = 3$. But if all three technologies are part of the mix at θ_C, multiple portfolios are possible. To keep the emissions at a constant rate, a higher share of wind (at the expense of coal) could be compensated by a lower share of gas (at the advance of coal) and vice versa. So, the specific shape of the ldc becomes immediately relevant.

Until now, the shape of the ldc has had no direct influence on the composition of the energy portfolio at a certain point of time t.[19] The reason lies in the characteristics of the peak load pricing problem where we are dealing with a combination of fixed cost plus variable costs proportional to output. The upper and lower boundaries of the efficient operating time are defined as the intersection of the relevant cost curves with constant slope. At a single point of time, when both the fixed and variable cost components are well defined and so is the intersection, the position within the ldc does not matter.[20] Now, the positions of the control variables also depend on the share of each technology Q_u since total emission have to be balanced. And because the scarcity rents are exponentially increasing for the entire time horizon, emission cost $\bar{\lambda}_p(t)$, which describes the external cost of pollution during the ceiling phase,[21] become a function of λ_2^0, λ_3^0 and $B(s_{1,2})$ which should characterize the ldc-specific compensation factor:

$$B(s_{1,2}) = \frac{\partial \frac{D(s_{2,3}(s_{1,2}))}{D(s_{1,2})}}{\partial s_{1,2}}.$$

You find a detailed derivation in appendix A.4.

For exemplary clarification, relatively increasing variable costs of gas are pushing the portfolio towards more coal. But without increasing the share of renewables we would break the ceiling. So, the effect of a change in $s_{1,2}$ on $s_{2,3}$ depends on the actual position within the ldc.

Still, the actual control paths differ according to the initial relative endowments with fossil stocks as explained in the following.

[19] Please note that the actual shape of the ldc influences the duration of the different phases in any case since the effect of increasing cost does have a non-linear effect on the resource use. But $\partial s_{u,v}/\partial t$ at a certain time t is independent from the actual position within the ldc.

[20] Cf. figure 3.

[21] Do avoid the wrong implication of a constant factor from a flat ceiling phase we do not omit the time argument here.

Gas is relatively scarce, coal is abundant First, consider gas to be relatively scarce and use the same definition for relative scarcity as stated earlier. As illustrated in figure 8,[22] in the first phase, after wind has joined the mix its share continuously increases. Depending on the initial endowment there are multiple paths possible for the share of gas: It could increase or decrease depending on whether its relative scarcity or the fact that it is cleaner prevails. We may also encounter an initial situation where the mix first shifts away from gas due to a very high initial scarcity rent of gas followed by a trend back towards more gas. The reason would lie in an initially high scarcity rent that would be overcompensated by emission costs which rise exponentially with a higher rate of $(\alpha+\rho)$ (compared to ρ at which the scarcity rents rises). In the second phase $\bar{s}_{1,2}$ and $\bar{s}_{2,3}$ changes simultaneously while $Z = \bar{Z}$. The path implies continuously decreasing emission costs reaching zero at θ_H. Due to the relative scarcity of gas in the third phase, both resources are used until infinity.

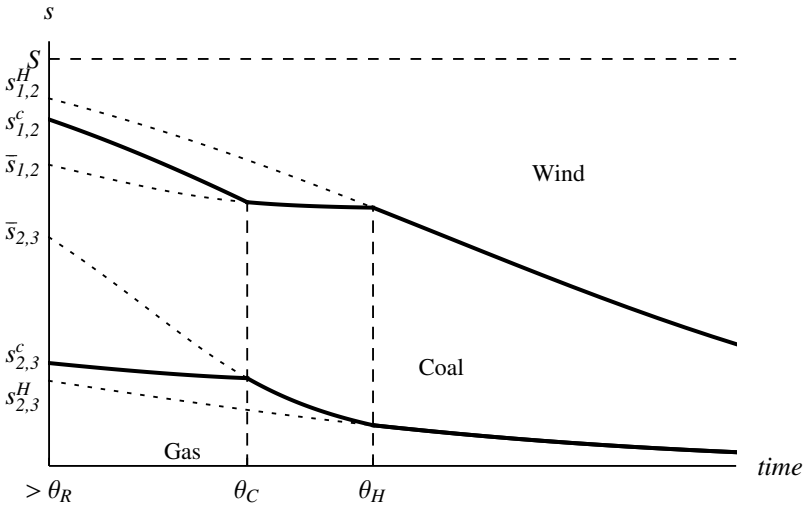

Figure 8: A case with two available initial stocks with CO_2 restrictions and scarce gas resources

Gas is relatively abundant, coal is scarce If we further increase the initial endowment with gas and keep coal fixed, coal becomes relatively scarce and the control path shows a different characteristic. In the beginning, gas and coal are also used simultaneously. The increasing scarcity rents and emission costs lead the renewable to enter the portfolio at θ_R. Its share continuously increases until Z equals $\bar{Z}$. Now, the emission costs decrease while the emission stock stays strictly at the ceiling. When the further increased scarcity rents push the mix to one that is less polluting than the natural dilution, a pure Hotelling path begins. From then on all three technologies are used simulatenously until coal runs out and gas and wind are used until infinity. The related control path is described in figure 9.[23]

Again, if gas is particularly abundant, coal might be exploited before the renewewable has entered the mix.[24] This case and all previous ones are further detailed in appendix A.5.

[22] During the ceiling phase $s^c_{1,2}$ decreases slower than $s^c_{2,3}$ as at the former border coal is replaced by clean wind and at the latter border the fossil technology gas is replaced by the other fossil technology coal.

[23] During the ceiling phase $s^c_{1,2}$ decreases slower than $s^c_{2,3}$ as at the former border coal is replaced by clean wind and at the latter border the fossil technology gas is replaced by the other fossil technology coal.

[24] In this specific case during the ceiling phase, the portfolio would only consist of gas and the renewable.

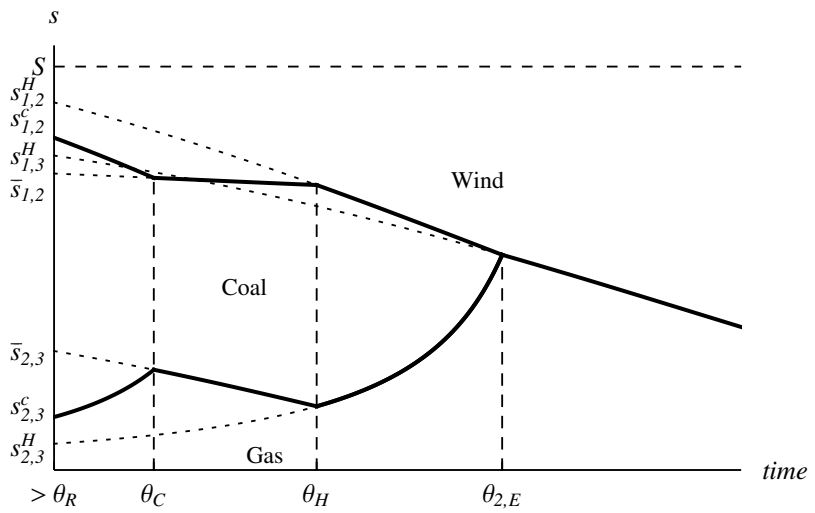

Figure 9: A case with two available initial stocks with CO_2 restrictions and scarce coal resources

4 Conclusion

This paper combines Hotelling's theory, that was extended to an emission constrained environment by Chakravorty et al. (2006), with a peak load pricing approach to sufficiently describe electricity production markets in a similar context. The maximum amount of CO_2 in the atmosphere as well as the resource stocks are constrained. The demand is fixed and characterized by a load duration curve which is used to illustrate the relationship between generating capacity requirements and capacity utilization in electricity production. Our approach allows to accommodate differing characteristics of electricity production technologies in terms of pollution, efficiency, variable and fixed costs. For the illustration of possible extraction paths, we use a three technology case with two polluting technologies which are coal, a high-carbon, high-fixed-low-variable cost, low efficiency technology, and gas, a technology with inverse characteristics. A green, supply-independent, technology with high-fixed and no variable costs supplements the two fossil ones.

We show that the extractions paths substantially depend on the initial endowments with coal and gas. A relatively low initial endowment with the high-fixed cost fossil technology leads to its complete exploitation, the low-fixed cost technology is used until infinity in either case. This difference in behavior does not occur in existing Hotelling literature.

If emission targets are tight (or the combined initial stocks are highly abundant), a phase where the CO_2-amount stays at the maximum allowable level results. This phase persists until the available stocks have decreased to a level and the increased scarcity pushed the production mix to a cleaner one.

We could also show that reduction targets may be attained via a shift towards the less polluting fossil fuel in the beginning. Its share should then decrease again when a renewable technology complements the portfolio. The more competitive the renewable, the earlier it will complement the portfolio and the longer the fossil fuels will suffice. Hence, equity between generation will increase if costs of renewable technologies can be reduced quickly.

A Proof and details of control paths

A.1 Proof of Proposition 1

Building on (5), subject to (1), (2), (3) and (4) the Lagrangian of the current-value Hamiltonian writes

$$\begin{aligned}\mathcal{L}(t) = &- \sum_{u=1}^{3} f_u \cdot K_u(s_u^{min}(t), s_u^{max}(t)) \\ &- \sum_{u=1}^{3} c_u \cdot Q_u(s_u^{min}(t), s_u^{max}(t)) \\ &- \sum_{u=2}^{3} \lambda_u(t) \cdot h_u \cdot Q_u(s_u^{min}(t), s_u^{max}(t)) \\ &+ \lambda_p(t) \left[\sum_{u=2}^{3} \zeta_u \cdot Q_u(s_u^{min}(t), s_u^{max}(t)) - \alpha \cdot Z(t) \right] \\ &+ \nu_c(t) \cdot \left(\bar{Z} - Z(t) \right),\end{aligned} \tag{19}$$

where $\lambda_u(t)$ is the Hotelling rent of resource u, $\lambda_p(t)$ is the externality cost of pollution and $\nu_c(t)$ the Karush-Kuhn-Tucker multiplier factoring in the ceiling on the stock of pollution. Since $\sum_{u=1}^{3} Q_u(t) = \int_0^S D(s)ds$ and $\sum_{u=1}^{3} K_u(t) = D(0)$ for all $t \geq 0$, the maximum operating bound of one technology must be the minimum of another one.[25] So we define:

$$s_{u,v}(t) = s_u^{max}(t) = s_v^{min}(t) \text{ with } v > u.$$

Then, the first order conditions for an interior solution are as follows:

$$\begin{aligned}\frac{\partial L(t)}{\partial s_{u,v}(t)} =& D_{s_{u,v}(t)} \cdot (-f_v + f_u - c_u \cdot s_{u,v}(t) \\ &+ c_v \cdot s_{u,v}(t) - \lambda_u(t) \cdot h_u \cdot s_{u,v}(t) + \lambda_v(t) \cdot h_v \cdot s_{u,v}(t) \\ &+ \lambda_p(t) \cdot (\zeta_u - \zeta_v) \cdot s_{u,v}(t)) = 0, \text{ with } u = 1, 2, 3, \ v > u.\end{aligned}$$

Consequently, the internal solution of a cost minimum portfolio is defined as follows:

$$s_{u,v}(t) = \frac{f_u - f_v}{(c_v - c_u) + (h_v \cdot \lambda_v(t) - h_u \cdot \lambda_u(t)) - \lambda_p(t)\,(\zeta_v - \zeta_u)},$$

or, when cost components per technology are grouped together, by

$$s_{u,v}(t) = \frac{f_u - f_v}{(c_v + h_v \cdot \lambda_v(t) - \lambda_p(t) \cdot \zeta_v) - (c_u + h_u \cdot \lambda_u(t) - \lambda_p(t) \cdot \zeta_u)},$$

[25] The maximum operating bound of the technology with highest fixed costs in the portfolio equals S, the minimum operating bound of the technology with lowest fixed costs equals 0.

which allows variable technology-specific cost components to be aggregated to yield

$$s_{u,v} = \frac{f_u - f_v}{c_{var,v}(t) - c_{var,u}(t)}.$$

Remark that if and only if u and v are part of the efficient portfolio (and are not dominated by other technologies in the relevant operating range $s \in [0; S]$), $s_{u,v}$ describes an internal solution to the problem described.

A.2 Proof of Proposition 2

Renz and Weber (2017) describe which conditions must be fulfilled to make a technology u part of the efficient portfolio on the basis of the relative position of control variables $s_{u,v}$ towards each other. The costs of the technologies $u = 1, 2, 3$ as a function of operating time may be written by

$$C_u(s) = f_u + c_{var,u} \cdot s$$

with $c_{var,u}$ defined in (7). Then, $s_{u,v}$ defined in (6), has the following characteristics:

$$C_u(s) > C_v(s) \ \forall s < s_{u,v} \ \wedge \ C_u(s) \leq C_v(s) \ \forall s \geq s_{u,v}. \tag{20}$$

First, assume there is no gas available. If $S = \infty$,[26] the actual use of coal is then defined via the respective energy produced $Q_2(0, s^H_{1,2})$ at each point of time $t \geq 0$. Further assume that emission targets are not binding ($\lambda_p \equiv 0$). The integral consumption of coal is so a function of λ^0_2 and according to (15) the following equation must hold for an optimal solution:

$$X^0_2 = h_2 \cdot \int_0^\infty \int_0^{s^H_{1,2}} D(s) - D(s^H_{1,2}) ds dt$$

with

$$s^H_{1,2} = \frac{f_1 - f_2}{c_2 + h_2 \cdot \lambda^0_2 \cdot e^{\rho t}}.$$

$s^{0,H}_{1,2} = s^H_{1,2}(t = 0) > 0$ is then continuously increasing in X^0_2 on $[0; \infty]$ while $s^H_{1,2}$ is continuously decreasing over time with $\lim_{t \to \infty} s^H_{1,2} = 0$. Consequently there must also exist a $X^0_2 = X^R_2$ with $s^{0,H}_{1,2} = S < \infty$. Then, for all $X^0_2 > X^R_2$ $s^{0,H}_{1,2} > S$ and for all $X^0_2 \leq X^R_2$ $s^{0,H}_{1,2} \leq S$. Since $s^H_{1,2} \geq S$ suspends wind from the efficient portfolio, any $X^0_2 > X^R_2$ must lead to an initial no-wind phase. A proof for a gas-only case is derived similarly.

If both fossil resources are available and $S = \infty$, for $s^H_{1,2} > s^H_{1,3}$, $Q_2(s^H_{1,2}, s^H_{2,3}) > 0$ and $Q_3(0, s^H_{2,3}) > 0$, and for $s^H_{1,2} \leq s^H_{1,3}$, $Q_2 = 0$ and $Q_3(0, s^H_{1,3}) > 0$. In both cases, the combined fossil energy produced can be defined by

$$Q_f(t) = \max_{u \ u=2,3} \int_0^{s^H_{1,u}} D(s) - D(s^H_{1,u}) ds. \tag{21}$$

[26] Whenever we make this assumption, we assume a finite total demand ($\int_0^S D(s) ds < \infty$).

So, $\max\left\{s_{1,2}^H, s_{1,3}^H\right\}$ marks the upward limit of efficient operating time of the combined fossil resources. Consequently, a proof for a case with both resources available is obtained again along the same lines. We conclude

$$\exists X_3^R(X_2^0)\ \forall X_2^0 \in [0; X_2^R] \tag{22}$$

with an initial no-wind phase for all $X_3^0 > X_3^R(X_2^0)$ and an initial portfolio already including wind for all $X_3^0 < X_3^R(X_2^0)$. If $X_3^0 = X_3^R(X_2^0)$, wind is part of the portfolio for $t > 0$, but not in $t = 0$.

This proof holds as long as in a path with $s_{1,2}^{0,H} = S$ and $X_3^0 = 0$, the ceiling would not be binding. So, the ceiling must not be extremely tight. If it was, a path starting in $s_{1,2}^{0,H} = S$ with $\lambda_p \equiv 0$ would not be admissible, so wind must complement the portfolio from the very beginning.

A.3 Proof of Proposition 3

Assume there exists an initial stock $\tilde{X}^2(X_3^0)$ for all $X_3^0 \in [0, \infty)$ which is characterized as follows: For all $X_2^0 < \tilde{X}^2(X_3^0)$ coal is exploited at $t = \theta_{2,E} < \infty$ (say, Case b) at the latest, otherwise it may be used until infinity (say, Case a). For the time being, we assume that there is no upper limit for the demand ($S = \infty$) and no emission target ($\lambda_p \equiv 0$) for all $t \geq 0$.

With (1) and (4), the share, respectively the consumption of coal, is then defined by

$$\dot{X}_2 = -h_2 \cdot Q_2(s_{2,3}^H, s_{1,2}^H). \tag{23}$$

$\dot{X}_2$ is strictly decreasing in $s_{1,2}^H$ and increasing in $s_{2,3}^H$, and the consumption of coal equals zero if $s_{2,3}^H \geq s_{1,2}^H$.

Case a: If $s_{2,3} < s_{1,2}$ for $t \geq 0$, $\dot{X}_2 > 0$ for $t \geq 0$. After applying this inequality condition to (6)-(8), we conclude

$$\frac{c_2 + h_2 \cdot \lambda_2^0 \cdot e^{\rho t}}{c_3 + h_3 \cdot \lambda_3^0 \cdot e^{\rho t}} < \frac{f_1 - f_2}{f_1 - f_3}\ \forall t \geq 0.$$

At infinity, the inequality condition is fulfilled if and only if

$$\frac{h_2 \cdot \lambda_2^0}{h_3 \cdot \lambda_3^0} < \frac{f_1 - f_2}{f_1 - f_3}. \tag{24}$$

The definition of the time paths of $s_{2,3}^H$ and $s_{1,2}^H$ does allow for only one intersection in the positive time range. So, since $X_2^0 > 0$, $s_{2,3}^H < s_{1,2}^H$ for t=0, $\dot{X}_2 > 0$ for all $t \geq 0$ and the total consumption is derived by integrating (23) from zero to ∞. Furthermore the total consumption of gas equals $\int_0^\infty h_3 \cdot Q_3(0, s_{2,3})dt$. Via (15) the system with the two variables λ_2^0 and λ_3^0 is well-defined (see also A.5.1).

Case b: Just as in the previous case is $\dot{X}_2 = 0$ if $s_{2,3}^H \geq s_{1,2}^H$. Also, the definition of the time paths of $s_{2,3}^H$ and $s_{1,2}^H$ does allow for only one intersection in the positive time range. So, since $X_2^0 > 0$, $s_{2,3}^H < s_{1,2}^H$ for t=0. If (24) is reversed, $s_{2,3}^H$ and $s_{1,2}^H$ intersect at a point of time $t > 0$ which we specify by $\theta_{E,2} < \infty$. Consequently, $\dot{X}_2 = 0$ for $t \geq \theta_{E,2}$. Since an optimal solution

requires $\int_0^\infty h_2 \cdot Q_2 dt = X_2^0$ and the Hamiltonian has to be piecewise continuous, the point of time when both curves intersect must equal the time when coal is exploited. So, coal is exploited at a point of time $\theta_{E,2} < \infty$ if and only if $\frac{h_2 \cdot \lambda_2^0}{h_3 \cdot \lambda_3^0} > \frac{f_1 - f_2}{f_1 - f_3}$. The respective initial endowments $X_2^0(X_3^0)$ derive respectively through the requirements for a complete consumption.

At the beginning of this proof we made the assumption that $S = \infty$ and that no emission targets apply. If we relax these conditions, the initial control paths $s_{u,v}$ become also a function of λ_p and $S < \infty$. But because of the limited initial endowments with fossil resources and natural dilution rate (as formulated in (2)), there must exist a time θ_H when the ceiling is no more binding ($\lambda_p \equiv 0$) for all $t \geq \theta_H$ (cf. Chakravorty et al. (2006)). For $t > \theta_H$ (and $t > \theta_R$), either coal is still available and the proof above applies or coal is already exploited. Consequently, whenever $\frac{h_2 \cdot \lambda_2^0}{h_3 \cdot \lambda_3^0} > \frac{f_1 - f_2}{f_1 - f_3}$, coal will be exploited at the point of time when $s_{2,3}^H = s_{1,2}^H$ at the latest.

A.4 Proof of Proposition 4

We follow Chakravorty et al. (2008) to show that there exist critical stocks $X_2^H(X_3^0 \equiv 0)$ and $X_3^H(X_2^0 \equiv 0)$ with the following characteristics: If $X_u^0 \leq X_u^H$, $\lambda_p \equiv 0$ for $t \geq 0$. And if $X_u^0 > X_u^H$, $\lambda_p < 0$ for an initial time period $t \in [0; \theta_H) < \infty$. The same holds for $X_2^H(X_3^0)$ with $X_3^0 \in \big[0; X_3^H(X_2^0 \equiv 0)\big]$. If initially $\lambda_p < 0$, three distinct time phases result (Renz and Weber, 2017): An initial phase where $Z < 0$, $\dot{Z} > 0$ and $\lambda_p < 0$, a second phase of non-zero length where $Z = \bar{Z}$ and $\lambda_p < 0$ (say, ceiling phase) and a third phase where $Z < 0$, $\dot{Z} < 0$ and $\lambda_p \equiv 0$ (say, Hotelling phase). Furthermore, we show how the ceiling phase is characterized.

The critical stocks From equation (2) follows that if $X_2^0 = 0$, it exists a $\lambda_3^0 < \infty$ such that $\zeta_3 \cdot Q_3(0, s_{1,3}^0) < \alpha \cdot Z_0$.[27] The respective initial stock of gas is defined by (15). In this case λ_3^0 is a function of X_3^0 only. Consider a case where $s_{1,3}^0 = S$. Since the ceiling should not allow an infinite gas-only mix, the consequent paths implies an initial phase where $\zeta_3 \cdot Q_3 > \alpha \cdot Z$ and Z is increasing. Because Z is continuous per definition and $\lim_{t \to \infty} s_{1,3} = 0$ (respectively $\lim_{t \to \infty} Z_t = 0$), the path of Z then implies a maximum $Z_{max}(s_{1,3}^{0,H} = S)$.[28] If this maximum was greater than $\bar{Z}$, $\theta_R = 0$ and an initial no-wind path was not admissible. So, we assume that $Z_{max}(s_{1,3}^0 = S) < \bar{Z}$. Then, the larger X_3^0, the larger $s_{1,3}^{0,H} > 0$, the longer the time interval $t \in [0; \theta_R]$ where $Q_3(0, S)$, the higher $Z_{max}(X_3^0)$. So, there exists a $X_3^0(X_2^0 \equiv 0)$, where $Z_{max} = \bar{Z}$ which we define by $X_3^H(X_2^0 \equiv 0)$ with the characteristics described.

If we further assume that a path with $s_{1,2}^{0,H} = S$ and $X_3^0 \equiv 0$ is also admissible,[29] a proof for $X_2^H(X_3^0 \equiv 0)$ is obtained similarly. The consequence is that any path with $X_2^0, X_3^0 > 0$, $\lambda_p \equiv 0$ and $max\{s_{1,3}, s_{1,2}\} = S$ does also not break the ceiling. Furthermore, $\bar{Z} > 0$ and $\lim_{t \to \infty} Z(t) = 0$. Recalling the continuity of Z there must also exist a Z_{max}, and, for similar reasons as in the one-resource case, a $X_2^R(X_3^0)$ with $X_3^0 \in \big[0; X_3^R(X_2^0 \equiv 0)\big]$. It is easy to see that $X_3^R(X_2^0)$ equals $X_3^{R,max}$ with $X_3^{R,max} = \max\big\{X_3^R(X_2^0 \geq 0)\big\}$.[30]

[27] The respective $s_{1,3}^0$ is strictly less S since otherwise an infinite gas-only would not break the ceiling what we have excluded in the initial assumptions.

[28] Compare to the assumption that the emission targets are not extremely tight in Proposition 2.

[29] The ceiling is not broken.

[30] $X_2^{R,max}$ is obtained similarly.

Characterization of the ceiling phase Renz and Weber (2017, Proposition 3) show that if a ceiling becomes binding, a ceiling phase of non-zero time length where $Z = \bar{Z}$ must follow. In contrast to Renz and Weber (2017), in this program the two available differently polluting resources allow for multiple shares of polluting technologies during this ceiling phase. To account for this fact, we introduce two new variables and assume a net CO_2-emission equal to zero:

$$\zeta_2 \cdot Q_2 + \zeta_3 \cdot Q_3 = \alpha \cdot \bar{Z} \ \forall t \in [\theta_C, \theta_H] \tag{25}$$

If all technologies are part of the efficient portfolio, $Q_{\bar{f}}(t)$ (see also (21)) should describe the total amount of energy produced via fossil fuels as follows:

$$Q_{\bar{f}}(s_{1,2}) \equiv Q_2 + Q_3 = \int_0^{s_{1,2}} D(s) - D(s_{1,2}) ds. \tag{26}$$

It is monotonically increasing in $s_{1,2}$. Q_2 is monotonically decreasing and Q_3 increasing in $s_{1,2}$. They can be expressed as follows:

$$Q_2(s_{1,2}) = \frac{\alpha \cdot \bar{Z} - \zeta_3 \cdot Q_{\bar{f}}(s_{1,2})}{\zeta_2 - \zeta_2},$$

and

$$Q_3(s_{1,2}) = \frac{\zeta_2 \cdot Q_{\bar{f}}(s_{1,2}) - \alpha \cdot \bar{Z}}{\zeta_2 - \zeta_3}.$$

We further define $R_{\bar{f}}(t)$, the share of gas within the fossil mix, with

$$R_f(s_{1,2}) \equiv \frac{Q_3}{Q_{\bar{f}}(s_{1,2})},$$

which is then also monotonically increasing in $s_{1,2}$.

During the time interval, where $Z = \bar{Z}$, the Lagrangian in (19) can then be reformulated by

$$\begin{aligned}
\mathcal{L}(s_{1,2}) = &- (f_1 - f_2) \cdot D(s_{1,2}) - (f_2 - f_3) \cdot D(s_{2,3}(s_{1,2})) \\
&- f_3 \cdot D(S) - \lambda_2 \cdot h_2 \cdot (1 - R_{\bar{f}}(s_{1,2})) \cdot Q_{\bar{f}}(s_{1,2}) - \lambda_3 \cdot h_3 \cdot R_{\bar{f}}(s_{1,2}) \cdot Q_{\bar{f}}(s_{1,2}) \\
&+ \lambda_p \left(\zeta_2 \cdot (1 - R_{\bar{f}}(s_{1,2})) \cdot Q_{\bar{f}}(s_{1,2}) + \zeta_3 \cdot R_{\bar{f}}(s_{1,2}) \cdot Q_{\bar{f}}(s_{1,2}) - \alpha \cdot \bar{Z}\right) \\
&+ \nu_c \cdot \left(\bar{Z} - Z\right),
\end{aligned}$$

which gives us an optimality condition for an inner solution as follows:

$$\bar{s}_{1,2}(t) = \frac{f_1 - (f_2 - (f_2 - f_3) \cdot \frac{\partial \frac{D(s_{2,3}(s_{1,2}))}{D(s_{1,2})}}{\partial s_{1,2}})}{\frac{1}{\zeta_2 - \zeta_3} \cdot [(\zeta_2 \cdot c_3 - \zeta_3 \cdot c_2) + (\zeta_2 \cdot h_3 \cdot \lambda_3 - \zeta_3 \cdot h_2 \cdot \lambda_2)]}. \tag{27}$$

To ease representation, we write

$$B(s_{1,2}) = \frac{\partial \frac{D(s_{2,3}(s_{1,2}))}{D(s_{1,2})}}{\partial s_{1,2}}.$$

We can make the development of the externality costs explicit, by equating $\bar{s}_{1,2}(t)$ with the 'standard' optimality term formulated in Proposition 1:

$$s_{1,2} = \bar{s}_{1,2}(t) = \frac{f_1 - f_2}{c_2 + h_2 \cdot \lambda_2 - \zeta_2 \cdot \bar{\lambda}_p(t)}.$$

Then, the development of the externality costs is defined as follows:

$$\begin{aligned}\bar{\lambda}_p(\lambda_2^0, \lambda_3^0, s_{1,2}) =& \frac{(f_1 - f_2)(c_2 - c_3 + h_2\lambda_2 - h_3\lambda_3)}{(f_1 + f_2(-1 + B(.)) - f_3 B(.))(-\zeta_2 + \zeta_3)} \\ &- \frac{(f_2 - f_3)B(.)(c_2 + h_2\lambda_2)}{(f_1 + f_2(-1 + B(.)) - f_3 B(.))\zeta_2}\end{aligned}$$

λ_p will increase from $\lambda_p(t = \theta_C) < 0$ to zero at θ_H. During this time interval, $\bar{s}_{2,3}(t)$ is implicitly defined via the requirement for balanced CO_2-emissions formulated in equation (25).

A.5 Details of selected control paths

In the following, we describe how the actual control paths can be derived for initial endowments displayed in figure 5 if the technology characteristics and emission targets are known. Whenever possible we draw on (3) and (4).

A.5.1 Efficient control paths if the ceiling is never binding

Gas is relatively scarce, coal is abundant

If the combined initial resource stocks are not highly abundant so that a CO_2 ceiling is never binding, and gas is relatively scarce and coal abundant there are three unknowns λ_2^0, λ_3^0 and θ_R, with three equotations as follows:

- Cumulative use of coal:

$$\int_0^{\theta_R} Q_2(s_{2,3}^H, S)dt + \int_{\theta_R}^{\infty} Q_2(s_{2,3}^H, s_{1,2}^H)dt = X_2^0/h_2$$

- Cumulative use of gas:

$$\int_0^{\infty} Q_3(0, s_{2,3}^H)dt = X_3^0/h_3$$

- Wind entering efficient portfolio:

$$s_{1,2}^H(\lambda_2^0, t = \theta_R) = S$$

If the combined initial resource stocks are low $X_3^0 < X_3^R(X_2^0)$, $\theta_R \equiv 0$.

This case hold also if $X_3^0 = 0$. Then, we have to replace $s_{2,3}^H$ by zero and the 'Cumulative use of coal' becomes irrelevant. Replace coal by gas and $u = 2$ by $u = 3$ to get the description of the case where $X_3^0 = 0$.

Gas is relatively abundant, coal is scarce

If the combined initial resource stocks are not highly abundant so that a CO_2 ceiling is never binding, and gas is abundant and coal relatively scarce there are four unknowns λ_2^0, λ_3^0, θ_R and $\theta_{2,E}$, with four equotations as follows:

- Cumulative use of coal:

$$\int_0^{\theta_R} Q_2(s_{2,3}^H, S)dt + \int_{\theta_R}^{\theta_{2,E}} Q_2(s_{2,3}^H, s_{1,2}^H)dt = X_2^0/h_2$$

- Cumulative use of gas:

$$\int_0^{\theta_{2,E}} Q_3(0, s_{2,3}^H)dt + \int_{\theta_{2,E}}^{\infty} Q_3(0, s_{1,3}^H)dt = X_3^0/h_3$$

- Wind entering efficient portfolio:

$$s_{1,2}^H(\lambda_2^0, t = \theta_R) = S$$

- Point of time when coal is exhausted:

$$s_{2,3}^H(\lambda_2^0, \lambda_3^0, t = \theta_{2,E}) = s_{1,2}^H(\lambda_2^0, t = \theta_{2,E})$$

If the combined initial resource stocks are scarce (e.i. $X_3^0 < X_3^R(X_2^0)$), $\theta_R \equiv 0$.

Gas is relatively abundant, coal is particularly scarce

If the combined initial resource stocks are not highly abundant so that a CO_2 ceiling is never binding, and gas is abundant and coal particularly relatively scarce there are four unknowns λ_2^0, λ_3^0, t_1 and t_2, with four equotations as follows:

- Cumulative use of coal:

$$\int_0^{t_1} Q_2(s_{2,3}^H, S)dt = X_2^0/h_2$$

- Cumulative use of gas:

$$\int_0^{t_1} Q_3(0, s_{2,3}^H)dt + (t_2 - t_1) \cdot Q_3(0, S) + \int_{t_2}^{\infty} Q_3(0, s_{1,3}^H)dt = X_3^0/h_3$$

- Point of time when coal is exhausted:

$$s_{2,3}^H(\lambda_2^0, \lambda_3^0, t = t_1) = S$$

- Wind entering efficient portfolio:

$$s^H_{1,3}(\lambda^0_3, t = t_2) = S$$

[Figure 10 here]

A.5.2 Efficient control paths if a will become binding in the future

Gas is relatively scarce, coal is abundant

If two available resources are highly abundant so that a CO_2 ceiling will become binding, and gas is relatively scarce and coal relatively abundant there are six unknowns λ^0_2, λ^0_3, λ^0_p, θ_R, θ_C and θ_H, with six equotations as follows:

- Cumulative use of coal:

$$\int_0^{\theta_R} Q_2(s^c_{2,3}, S)dt + \int_{\theta_R}^{\theta_C} Q_2(s^c_{2,3}, s^c_{1,2})dt$$
$$+ \int_{\theta_C}^{\theta_H} Q_2(\bar{s}_{2,3}(t), \bar{s}_{1,2}(t))dt + \int_{\theta_H}^{\infty} Q_2(s^H_{2,3}, s^H_{1,2})dt = X^0_2/h_2$$

  with $\bar{s}_{1,2}(t)$ defined in (27) and $\bar{s}_{2,3}(t)$ defined by $\bar{s}_{1,2}(t)$ and (26).

- Cumulative use of gas:

$$\int_0^{\theta_C} Q_2(0, s^c_{2,3})dt + \int_{\theta_C}^{\theta_H} Q_2(0, \bar{s}^c_{2,3,t})dt$$
$$+ \int_{\theta_H}^{\infty} Q_2(0, s^H_{1,2})dt = X^0_2/h_2$$

- Wind entering efficient portfolio:

$$s^H_{1,2}(\lambda^0_2, t = \theta_R) = S$$

- Reaching the ceiling:

$$s^c_{1,2}(\lambda^0_2, \lambda^0_p, t = \theta_C) = \bar{s}_{1,2}(\lambda^0_2, t = \theta_C)$$

- Begin of Hotelling path:

$$\bar{s}_{1,2}(\lambda^0_2, t = \theta_H) = s^H_{1,2}(\lambda^0_2, t = \theta_C)$$

- Cumulative emissions

$$\zeta_3 \cdot \left(\int_0^{\theta_R} Q_2(s^c_{2,3}, S)dt + \int_{\theta_R}^{\theta_C} Q_2(s^c_{2,3}, s^c_{1,2})dt \right)$$
$$\zeta_2 \cdot \left(\int_0^{\theta_C} Q_2(0, s^c_{2,3})dt \right) - \int_0^{\theta_C} \alpha \cdot Z = \bar{Z} - Z_0$$

Gas is relatively abundant, coal is scarce

If two available resources are highly abundant so that a CO_2 ceiling will become binding, and gas is relatively abundant and coal scarce there are seven unknowns λ_2^0, λ_3^0, λ_p^0, θ_R, θ_C, θ_H and $\theta_{2,E}$, with seven equotations as follows:

- Cumulative use of coal:

$$\int_0^{\theta_R} Q_2(s_{2,3}^c, S)dt + \int_{\theta_R}^{\theta_C} Q_2(s_{2,3}^c, s_{1,2}^c)dt$$
$$+\int_{\theta_C}^{\theta_H} Q_2(\bar{s}_{2,3}(t), \bar{s}_{1,2}(t))dt + \int_{\theta_H}^{\theta_{2,E}} Q_2(s_{2,3}^H, s_{1,2}^H)dt = X_2^0/h_2$$

- Cumulative use of gas:

$$\int_0^{\theta_C} Q_2(0, s_{2,3}^c)dt + \int_{\theta_C}^{\theta_H} Q_2(0, \bar{s}_{2,3}(t))dt$$
$$+\int_{\theta_H}^{\theta_{2,E}} Q_2(0, s_{1,2}^H)dt + \int_{\theta_{2,E}}^{\infty} Q_2(0, s_{1,3}^H)dt = X_2^0/h_2$$

- Wind entering efficient portfolio:

$$s_{1,2}^H(\lambda_2^0, t = \theta_R) = S$$

- Reaching the ceiling:

$$s_{1,2}^c(\lambda_2^0, \lambda_p^0, t = \theta_C) = \bar{s}_{1,2}(\lambda_2^0, t = \theta_C)$$

- Begin of Hotelling path:

$$\bar{s}_{1,2}(\lambda_2^0, t = \theta_H) = s_{1,2}^H(\lambda_2^0, t = \theta_C)$$

- Point of time when coal is exhausted:

$$s_{1,2}^H(\lambda_2^0, t = \theta_{2,E}) = s_{1,3}^H(\lambda_3^0, t = \theta_{2,E})$$

- Cumulative emissions:

$$\zeta_2 \cdot \left(\int_0^{\theta_R} Q_2(s_{2,3}^c, S)dt + \int_{\theta_R}^{\theta_C} Q_2(s_{2,3}^c, s_{1,2}^c)dt \right)$$
$$+\zeta_3 \cdot \left(\int_0^{\theta_C} Q_2(0, s_{2,3}^c)dt \right) - \int_0^{\theta_C} \alpha \cdot Z = \bar{Z} - Z_0$$

Gas is relatively abundant, coal is particularly scarce

If two available resources are highly abundant so that a CO_2 ceiling will become binding, and gas is relatively abundant and coal particularly scarce there are eight unknowns λ_2^0, λ_3^0, λ_p^0, θ_R, θ_C, θ_H, $\bar{s}_{1,3}$ and $\theta_{2,E}$, with seven equotations as follows:

- Cumulative use of coal:

$$\int_0^{\theta_R} Q_2(s^c_{2,3}, S)dt + \int_{\theta_R}^{\theta_{2,E}} Q_2(s^c_{2,3}, s^c_{1,2})dt = X^0_2/h_2$$

- Cumulative use of gas:

$$\int_0^{\theta_{2,E}} Q_2(0, s^c_{2,3})dt + \int_{\theta_{2,E}}^{\theta_C} Q_2(0, s^c_{1,3})dt$$
$$+ \int_{\theta_C}^{\theta_H} Q_2(0, \bar{s}_{1,3}(t))dt + \int_{\theta_H}^{\infty} Q_2(0, s_{1,3})dt = X^0_2/h_2$$

- Wind entering efficient portfolio:

$$s^c_{1,2}(\lambda^0_2, t = \theta_R) = S$$

- Reaching the ceiling:

$$s^c_{1,3}(\lambda^0_3, \lambda^0_p, t = \theta_C) = \bar{s}_{1,2}(\lambda^0_2, t = \theta_C)$$

- Definition of the ceiling phase:

$$Q_3(0, \bar{s}_{1,3}) = \alpha \cdot \bar{Z}/h_3$$

- Begin of Hotelling path:

$$\bar{s}_{1,3}(t = \theta_H) = s^H_{1,3}(\lambda^0_3, t = \theta_H)$$

- Point of time when coal is exhausted:

$$s^c_{1,2}(\lambda^0_2, t = \theta_{2,E}) = s^c_{1,3}(\lambda^0_3, t = \theta_{2,E})$$

- Cumulative emissions

$$\zeta_2 \cdot \left(\int_0^{\theta_R} Q_2(s^c_{2,3}, S)dt + \int_{\theta_R}^{\theta_{2,E}} Q_2(s^c_{2,3}, s^c_{1,2})dt \right)$$
$$+\zeta_3 \cdot \left(\int_0^{\theta_{2,E}} Q_2(0, s^c_{2,3})dt + \int_{\theta_{2,E}}^{\theta_C} Q_2(0, s^c_{1,3})dt \right)$$
$$- \int_0^{\theta_C} \alpha \cdot Z = \bar{Z} - Z_0$$

[Figure 11 here]

A.6 Figures for the Appendix

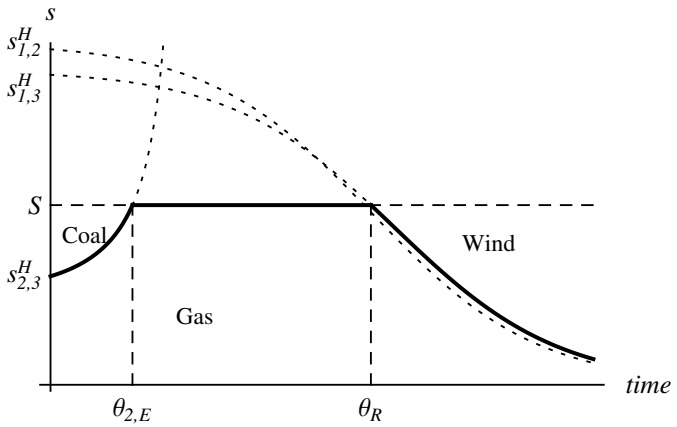

Figure 10: A case with two available initial stocks without CO_2 restrictions and particularly scarce coal resources

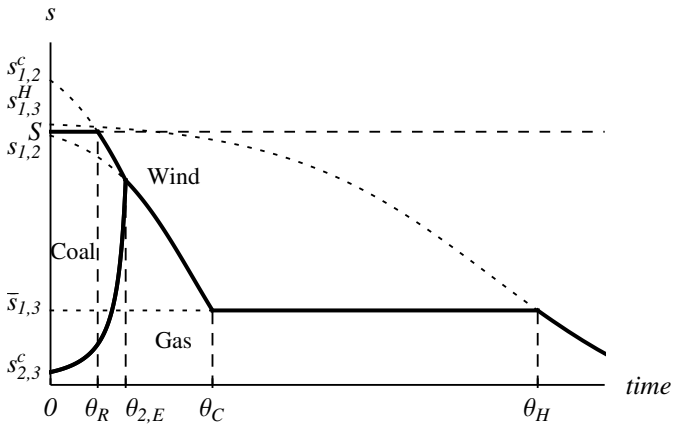

Figure 11: A case with two available initial stocks with CO_2 restrictions and particularly scarce coal resources

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