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**Working Paper**

## Non-parametric estimation of marginal propensities to consume: The case of regression splines

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**WORKING PAPERS**

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NON-PARAMETRIC ESTIMATION OF  
MARGINAL PROPENSITIES TO CONSUME: THE  
CASE OF REGRESSION SPLINES

Casper Nordal Jørgensen  
University of Copenhagen and  
Danmarks Nationalbank

and

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# DANMARKS NATIONALBANK **WORKING PAPERS**

## NON-PARAMETRIC ESTIMATION OF MARGINAL PROPENSITIES TO CONSUME: THE CASE OF REGRESSION SPLINES

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## RESUME

I papiret undersøges anvendeligheden af en ikke-parametrisk metode, en såkaldt regression spline, til estimation af husholdningers marginale forbrugstilbøjelighed. I forhold til eksisterende metoder kræver denne tilgang færre teoretiske antagelser for identifikation. Mere specifikt vises det i papiret at en regression spline-model for forbrug estimeret på baggrund af indkomst og aktiver giver et forholdsvis præcist estimat af husholdningernes marginale forbrugstilbøjelighed ved brug af tværsnitsdata. Den foreslåede metode er dog relativt datakrævende. Vi anvender metoden til at estimere den marginale forbrugstilbøjelighed for danske husholdninger baseret på detaljerede indkomst- og formuedata. Vi estimerer en gennemsnitlig marginal forbrugstilbøjelighed ud af likvide aktiver på 49 pct., men finder samtidig en betydelig grad af heterogenitet på tværs af formuefordelingen. For eksempel er forskellen på den gennemsnitligt estimerede marginale forbrugstilbøjelighed mellem 2. og 8. indkomstdecil over 30 procentpoint. Resultaterne indikerer, at det kan være vigtigt at tage højde for heterogeniteten i den marginale forbrugstilbøjelighed ved vurdering af effekter af finanspolitiske tiltag eller af ændringer i de lange renter.

## ABSTRACT

We investigate a non-parametric method to estimating marginal propensities to consume (MPC) using regression splines. This approach complements existing methods by relaxing a number of strong requirements on the part of the theory, otherwise necessary to acquire identification. Specifically, we show that a regression spline of consumption on income and assets yields a surprisingly precise estimate of the MPC at the household-level using only cross-sectional data. As a tradeoff on the part of the regression theory, the proposed method is somewhat data intensive. We use our proposed method to estimate the marginal propensity to consume for each household in Denmark, using detailed tax records on income and wealth. We estimate an aggregate MPC out of liquid assets of 49%, but with considerable heterogeneity over the wealth distribution. For example, we find the difference between the estimated MPC for the average household in the 2nd income decile compared to the 8th income decile to be more than 30 percentage points. Results indicate that it may be important to take the heterogeneity of MPC across households into account when assessing the impacts of proposed fiscal policies or changes in long interest rates.

# Non-Parametric Estimation of Marginal Propensities to Consume: The Case of Regression Splines\*

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March 17, 2017

## Abstract

We investigate a non-parametric method to estimating marginal propensities to consume (MPC) using regression splines. This approach complements existing methods by relaxing a number of strong requirements on the part of the theory, otherwise necessary to acquire identification. Specifically, we show that a regression spline of consumption on income and assets yields a surprisingly precise estimate of the MPC at the household-level using only cross-sectional data. As a trade-off on the part of the regression theory, the proposed method is somewhat data intensive. We use our proposed method to estimate the marginal propensity to consume for each household in Denmark, using detailed tax records on income and wealth. We estimate an aggregate MPC out of liquid assets of 49%, but with considerable heterogeneity over the wealth distribution. For example, we find the difference between the estimated MPC for the average household in the 2nd income decile compared to the 8th income decile to be more than 30 percentage points. Results indicate that it may be important to take the heterogeneity of MPC across households into account when assessing the impacts of proposed fiscal policies or changes in long interest rates.

**Keywords** Non-Parametric Modeling, Consumption, Macroeconomic Policy.

**JEL Classification** C51, E21, E6.

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## 1 Introduction

Households' propensity to consume is of high interest to macroeconomists and policy makers. For example, a policy maker seeking to mitigate the negative aspects of an economic downturn by, say, a direct economic handout or through reliance on automatic stabilizers would need to predict the aggregate effects on spending of such policies. The effect is likely to depend to a large extent on the distributionary profile of the policy. A large literature analyzes the effects of observed stimuli, both to evaluate the cost-benefit of government interventions, as well as to assess the validity of the core macroeconomic model of consumption and savings. The primary approach has been to estimate marginal propensities to consume (MPCs) from spending patterns of households using *natural experiments*, in which a large number of households have received an *unexpected* windfall gain. The need for natural experiments, unfortunately, puts a strain on the amount of empirical data available to analyze the *aggregate* propensity to consume.

The present paper adds to this literature by proposing a simple method, requiring only cross-sectional data on household-level consumption and assets, which can be used to estimate household-level MPCs. We identify household-level MPCs from the first derivative of the household-level consumption functions, which we in turn estimate “non-parametrically” using regression splines. Based on household-level MPCs, the researcher can readily estimate the aggregate marginal propensity to consume of an economy.

Knowledge of aggregate consumption responses to fiscal stimuli is crucial for policy makers to evaluate government policies. Recent government interventions in the U.S. have focused on handing out large sums of money *directly* to individual households. These contrasts with “New Deal” type policies of the past, that focused on creating new jobs by undertaking large publicly funded infrastructure projects. In terms of costs, the size of the Works Progress Administration, of the “Second New Deal”, had an initial budget of about 6.7% of GDP in 1935, while the latest 2008 U.S. stimulus package was only about .7% of GDP. Both types of policies aims at increasing demand, the former by encouraging private spending, the latter by increasing employment - and wage income - through public works. The stimulus packages of the past decades most likely had a shorter time horizon, often planned and executed in the same year. In contrast, the New Deal policies had an impact over a long time with a considerable time lag in implementation. While a large body of literature has studied the effects of “New Deal” type policies, there is less consensus about the effects of direct government handouts. As stressed in Kaplan and Violante (2014), while there are many empirical studies of individual consumption responses to specific unexpected transfers, the literature is still missing in the analysis on aggregate effects of fiscal stimuli. This paper demonstrates a simple method to bridge this gap.



Still, the consumption expenditure effects of recent government handouts have been studied in a number of papers. Johnson et al. (2006) analyze the 2001 US fiscal stimulus package, using the Consumer Expenditure Survey (CEX) and find a “short-run” (3 months) propensity to consume of 20% to 40% on non-durables. They find a higher “long-run” propensity to consume of 60%.<sup>1</sup> Kreiner et al. (2012) use a Danish 2004 stimulus finding a spending propensity between 60% to 75% for small windfall gains. Quite interestingly, they find the propensity to consume increasing in a measure for household impatience. Agarwal and Qian (2014) finds a high propensity to consume, of about 80%, using a Singaporean lump-sum 2011 unexpected government transfer. Parker et al. (2011); Misra and Surico (2014) and Parker (2015) analyze the 2008 US stimulus package and find very low short-run propensities to consume, from 3.5% to 30% on non-durables, but a high degree of heterogeneity across households. For example, Misra and Surico (2014) writes that “[a]lmost half of American families did not adjust their consumption following receipt of the 2001 or 2008 tax rebates.” In order to assess the effects on a stimulus package it seems crucial to identify the households responding to the stimulus, and to which extent they do so.

Jappelli and Pistaferri (2014) use Italian survey data finding a marginal propensity to consume (MPC) of 48 percent on average. They also find a substantial heterogeneity. MPC falls with cash-on-hand, income and financial assets. They find an 11% increase in MPC when moving from highest to lowest income households, and a 25 to 30 percentage point increase when moving from the top to the bottom quantiles of the cash-on-hand distribution. Jappelli and Pistaferri (2014) further argue that the strong heterogeneity cannot be replicated in a representative agent model. Their study, however, has to rely on *self-reported* consumption responses to an imaginary windfall gain. Alternative studies using survey data, Paiella (2007); Arrondel et al. (2015), find similar substantial heterogeneity in estimated MPCs across the wealth and income distribution.

These findings suggest that taking current level of income and assets into account might be crucial when estimating marginal propensities to consume. The heterogeneity in estimated MPCs across households further suggests that using a single agent (or a single equation) framework for predicting the aggregate propensity to consume of an economy might not be adequate. We address both concerns by proposing to estimate the *household-level* consumption function as a first stage, instead of directly focusing on the aggregate MPC. This will allow estimation of consumption responses *conditional* on

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<sup>1</sup> Johnson et al. (2006) distinguished between “short-run” and “long-run” to highlight that while, on average, ultimately 60% of the 2001 stimulus was consumed within a year, it was not consumed immediately upon receiving. Probably, “short-run” is a more appropriate term for a 1-year horizon. Most of the studies cited below are concerned with the share spent on an economic gain over a 12 months period. The same is true for our estimates, since we use annual data to identify consumption functions.

income, wealth and expected future income.

We show in a simulation exercise of Deaton (1991)'s canonical model of the household-level consumption and savings choice that a cubic regression Spline yields a surprisingly accurate estimate of the true consumption policy function even for high degrees of serial correlation in the income process. We then show that the 1st derivative of the same regression Spline can be used as an estimate of the true MPC, again with high accuracy. The cubic regression Spline greatly outperforms alternative non-structural regressions.

In the empirical application we use Danish register data to estimate consumption policy functions at the household-level. In spite of the very flexible estimation setup, the estimated policy functions show a remarkable similarity with the theoretical policy function derived in Deaton (1991). The corresponding marginal propensities to consume at the household level also display a high degree of similarity with the theory. Aggregating the household-level MPCs, our preferred method yields an aggregate MPC of approximately 49 percent.

In the next section we discuss the theory behind the consumption-savings function of modern macroeconomics and show how we can use this to estimate *empirical* consumption functions. Section 3 presents our empirical application. In this section we estimate the aggregate MPC of Denmark and discuss how it vary across households. Section 4 concludes.

## 2 Theory

This section discuss a new approach to estimating individual-level marginal propensities to consume using only cross-sectional data. Based on the canonical consumption-savings model of Deaton (1991) we show that a “non-parametric” regression model delivers a good fit of the consumption policy function. We then show that the true marginal propensity to consume can be estimated from the first derivative of the fitted consumption policy function.

### 2.1 The Model

Since we will base our estimation approach on the logic of how one solves the economic model of Deaton (1991), it is useful to provide a short description of the standard solution method. We take as given the canonical model of Deaton (1991).<sup>2</sup> An agent has the following utility function

$$U_t = \sum_t \beta^t \mathbb{E}_t \frac{C_t^{1-\gamma}}{1-\gamma}, \quad (1)$$

where  $\beta^i$  is the subjective discount factor between period  $t$  and period  $t+i$ ,  $C_t$  is consumption expenditures in period  $t$  and  $\gamma$  is the coefficient of relative risk aversion. The agent receives a stochastic income each period centered around a *permanent* income  $\bar{Y}$ , with a transitory component  $e_t$ ,<sup>3</sup>

$$Y_t = \bar{Y} + e_t, \quad (2)$$

$$e_t = \rho e_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2). \quad (3)$$

The parameter  $\rho$  determines the extent of persistency of the transitory income component. The agent has access to the asset  $A_{t+1}$  with an interest rate  $r_t$ . The budget identity is

$$C_t + A_{t+1} = Y_t + (1 + r_t)A_t, \quad (4)$$

which has to be obeyed in every period  $t$ . The agent’s problem is to maximize (1) subject to (2)-(4).  $A_{t+1}$  and  $C_t$  are choice variables,  $Y_t$  and  $r_t$  are state variables, and  $A_t$  is a transition variable.

<sup>2</sup> Deaton builds on Modigliani and Brumberg (1954) and Bewley (1977). For further discussion see e.g. Carroll (1997, 2001); Gourinchas and Parker (2002).

<sup>3</sup> This assumption is obviously rather simplistic, since it is very likely that an agent *jointly* decides on how much or how hard to work and how much to spend on consumption. We make the assumption of an exogenous income stream in order to make the model as simple as possible. One could introduce a great deal of inter-connectedness over time by modeling the permanent income process as an AR or ARMA process, see e.g. Carroll (1997).

Assuming a constant interest rate, the interior solution to finite the problem yields the well-known Euler equation for consumption

$$\forall t < T : \quad C_t^{-\gamma} = \beta R E_t \left[ C_{t+1}^{-\gamma} \right], \quad (5)$$

where  $R \equiv (1 + r)$ , and the optimal consumption at  $t = T$  is  $C_T = Y_T + RA_{T-1}$ . The optimal consumption expenditure should be such that the marginal value of consumption equals the discounted, both subjective through  $\beta$  and objective through  $R$ , expected marginal value of future consumption expenditure.<sup>4</sup> But how can  $C_t$  be computed if we don't know  $C_{t+1}$ ?

## 2.2 Policy Function

The standard trick to finding the optimal  $C_t$  is to compute the *expected*  $C_{t+1}$  conditional on each choice of  $C_t$  and then find the one  $C_t$  that satisfies (5) and maximizes the objective function (1). If  $\beta R < 1$  equation this recursive structure is a contraction mapping and has a unique solution.

Hence, as is standard, we assume that  $\beta R < 1$ . Next, define  $x_t \equiv RA_t + Y_t$  as “cash on hand”, and the marginal utility of money for a certain level of spending as  $\lambda(C_t)$ . We seek to find a *consumption policy function*  $c_t = f(x_t)$  such that

$$p(x_t) = \max \left\{ \lambda(x_t), \beta R \int p(R(x_t - \lambda^{-1}[p(x_t)]) + Y_{t+1}) dF(Y_{t+1}|Y_t) \right\}, \quad (6)$$

where  $p(x_t) = \lambda[f(x_t)]$ , we have written out the mathematical expectations operator  $E_t$  as an integral over the income  $Y_{t+1}$ . With a solution for  $p(x_t)$  we readily find the consumption policy function  $f(x_t) = \lambda^{-1}[p(x_t)]$  as the inverse of the marginal utility of consumption. Notice that the policy function depends on the level of cash on hand  $x_t$  but not on future levels of cash on hand. The future aspect is solely captured by the income process, specifically by the expected future income *conditional* on the current income draw  $Y_t$ . Hence, for stable preferences and real interest rate, changes in consumption choices arise solely through income and the level of cash on hand. Our problem is to find the functional form of  $p(\cdot)$ , which we do numerically by backwards induction.<sup>5</sup>

<sup>4</sup> Two things are worth noting here: 1) This holds for any time period except from the last period. Here the agent will simply consume all she has, because there is no tomorrow; and 2) equation (5) is a condition for an *interior* solution. If, for some reason, the agent believes that she can consume much more tomorrow than today, then  $E_t C_{t+1}^{-\gamma} << C_t^{-\gamma}$  and she would like to increase consumption today. If she is constrained by doing so, through, for example, a liquidity constraint, we have a corner solution and the Euler condition will not hold. Technically (5) is a *constraint*, and in general it must be that  $C_t^{-\gamma} \geq \beta R E_t C_{t+1}^{-\gamma}$ .

<sup>5</sup> We assume a finite time horizon, discretize the income process and impose that agents will consume all resources in the final period, hence  $c_T = x_T$ . We can then compute the optimal consumption choice for  $t = T - 1$ , knowing the choice at  $t = T$ . With this we obtain an optimal response for  $t = T - 1$ ,

Two such consumption policy functions are shown in figure 1. Panel (a) shows the consumption function for an income process with no persistence in the transitory income component,  $\rho = 0$ .<sup>6</sup> In the, perhaps more realistic, scenario in which transitory income shocks are persistent, the policy function becomes state dependent. Panel (b) shows the resulting consumption policy functions for 5 distinct income draws. The lowest line depicts the resulting policy function for an agent with the lowest income draw, while the highest graph shows the resulting function if the agent had received the highest income draw. The lines in between represents (from the bottom up) the 25th, 50th (median) and 75th percentile of the income draws.

The marginal propensity to consume is the gradient of the consumption policy function. For sufficiently low amounts of cash on hand the optimal consumption response is to consume all available resources, i.e the agent will have a marginal propensity to consume of 1. As the level of cash on hand increases the MPC decreases. We notice that there appears to be a “kink” in the policy function when the level of cash on hand is close to the value of the permanent income. The MPC sharply drops after hitting the kink and appears to rapidly converge to a small number. The pattern of the consumption policy functions depicted in figure 1 is robust to a wide range of specifications of the income processes and assets definitions.

Figure 1 is interesting when considering how to estimate marginal propensities to consume. If we have data on consumption, income and the level of cash on hand for a set of households, we could infer their consumption policy function and use the functional form of this empirical policy function to calculate MPCs. Unfortunately, the kink presents a challenge with the empirical fitting, because it requires a rather flexible function to capture the curvature of the consumption policy function. Below we discuss two different approaches. A global (polynomial regression) approximation, and a local (regression Spline) approximation.

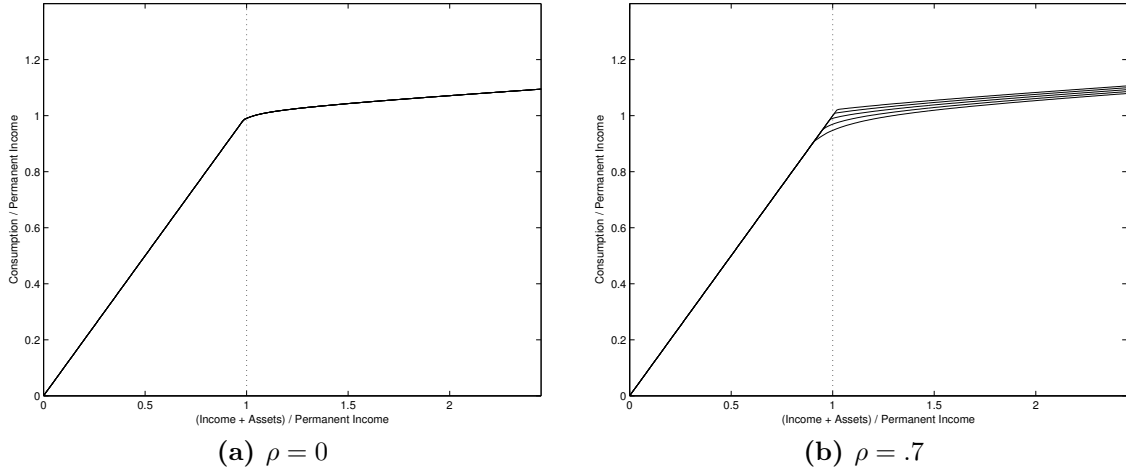
### 2.3 Global Approximation of the Policy Function

A number of studies have use logarithmic transformations of the observed consumption, income and asset variables in an effort to linearize the consumption policy function. An alternative could be to use non-transformed data and regress a higher order polynomial in income and cash on hand on total consumption. We therefore evaluate the following

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which we can use for  $t = T - 2$  and so forth. See Carroll (2006) for a comprehensible description on the solution algorithm.

<sup>6</sup>Figure 1a is a replication of Deaton (1991) figure 1, where his  $\rho$  is our  $1 - \gamma$  and his  $\delta$  is our  $\beta$ .



**Figure 1:** Consumption Policy Functions.

*Notes:* Vertical line represents where cash on hand equals permanent income. Model solved and calibrated on a quarterly interval, with yearly values  $\beta = .95$ ,  $\mu = 2$ ,  $R = 1.05$ ,  $Y_t = \bar{Y} + e_t$  where  $\bar{Y} = 100$ ,  $e_t = \rho e_{t-1} + \epsilon_t$ , and  $\epsilon \sim N(0, 10)$ . The normal distribution is approximated by a 17-point discrete process with a width of  $m = 3$  using the method of Tauchen.

four regression models

$$C_t = \beta_0 + \beta_1 \cdot \log(x_t) + e_t, \quad (7)$$

$$C_t = \sum_{i=0}^I \beta_i \cdot x_t^i + e_t. \quad (8)$$

$$C_t = \beta_0 + Y_t + \alpha_1 \cdot Y_t \log(x_t) + \beta_1 \cdot \log(x_t) + e_t, \quad (9)$$

$$C_t = \sum_{i=0}^I \alpha_i \cdot Y_t x_t^i + \sum_{i=0}^I \beta_i \cdot x_t^i + e_t. \quad (10)$$

where  $\alpha_i$  and  $\beta_i$  are regression parameters to be estimated,  $e_t$  is an error term, and  $I \leq 5$ . Models (9) and (10) represents the insight from figure 1 panel (b), where the consumption policy function depends on the income draw. The parameters of each regression model is readily estimated from OLS. Note that it is not uncommon to find a version of the above model formulated in terms of income rather than cash on hand. In appendix A we present an argument for why we believe this might be an inadequate approach.

**Marginal Propensity to consume** We estimate the marginal propensity to consume by taking the first derivate of each regression model

$$MPC_t^{log} = \beta_1/x_t, \quad (11)$$

$$MPC_t^{poly \otimes Y} = \sum_{i=1}^I i \cdot \beta_i x_t^{i-1}. \quad (12)$$

$$MPC_t^{log \otimes Y} = \alpha_1 \cdot Y_t/x_t + \beta_1/x_t, \quad (13)$$

$$MPC_t^{poly \otimes Y} = \sum_{i=1}^I i \cdot \alpha_i Y_t x_t^{i-1} + \sum_{i=1}^I i \cdot \beta_i x_t^{i-1}. \quad (14)$$

## 2.4 Local Polynomial Approximation - Regression Splines

A standard polynomial regression experiences difficulties in approximating the consumption policy function at certain values of cash on hand, due to the curvature of the policy function. This arises because the polynomial regression is a *global* approximation of the data. In contrast, the regression Spline is a *local* approximation of the data. This allows for the flexibility of a polynomial function, while maintaining a potentially higher degree of stability compared to a global polynomial. Specifically, the regression Spline is smooth around its joints, meaning that we can obtain a very flexible global fit, while maintaining continuity of the resulting regression function. This latter feature is especially relevant considering our goal of estimating marginal propensities to consume.

Following de Boor (1972) we now define a local regression model for consumption,  $C_t$ , against cash on hand,  $x_t$ . The regression model for a Spline of *order*  $k$  (degree  $k - 1$ ) with  $N$  *interior knots*<sup>7</sup> is defined as

$$C_t = \sum_{i=0}^{N+k-1} \beta_i B_{i,k}(x_t) + e_t, \quad (15)$$

where the  $B_{i,k}(x_t)$ 's are the *Spline basic functions* and  $e_t$  is an error term. The Spline basic functions are defined recursively as

$$B_{i,k} = \omega_{i,k}(x_t) B_{i,k-1}(x_t) + (1 - \omega_{i+1,k}(x_t)) B_{i+1,k-1}(x_t), \quad k > 0, \quad (16)$$

---

<sup>7</sup>In our specifications the end knots are simply the minimum as well as the maximum values observed. I.e. we have  $N + 2$  knots, the first being the minimum value, the second knot being the first interior knot and so forth. The final knot is thus the maximum observed value.

with

$$B_{i,0} = \begin{cases} 1, & \text{for } t_i \leq x_t < t_{i+1}, \\ 0, & \text{otherwise,} \end{cases} \quad (17)$$

and  $\omega_{i,k}(x_t)$  is an “interpolation weight” between  $B_{i,k-1}(x_t)$  and  $B_{i+1,k-1}(x_t)$

$$\omega_{i,k}(x_t) = \frac{x_t - t_i}{t_{i+k-1} - t_i}, \quad \text{for } t_{i+k} \neq t_i, \quad (18)$$

and  $t_i \in T = \{t_{-\tau}, \dots, t_0, t_1, \dots, t_{N+\tau}\}$  are knot values. For example, for a cubic Spline regression model (order  $k = 4$ ) with, say, 5 interior knots we calculate 9 basis functions,  $\{B_{0,3}, B_{1,3}, \dots, B_{8,3}\}$ . Note that the number of basis functions increases with the number of interior knots defined *and* with the order of the Spline. A Spline of order 2 ( $k = 2$ ) is the simple linear interpolation.

We use quantile knots in order to guarantee support between each knot value. I.e. for  $N$  interior knots, we define the knot sequence  $\{t_0, t_1, \dots, t_N, t_{N+1}\}$  where for  $i = 1, \dots, N$ ,  $t_i$  is the  $i/N$ 'th percentile of  $x_t$ ,  $t_0 = \min\{x_t\}$  and  $t_{N+1} = \max\{x_t\}$ . This guarantees that we have an equal share of observations between each knot.

With each  $B_{i,k}(x_t)$  calculated we can simply estimate the  $\beta_i$ 's in (15) with OLS. Using the parameter estimates we can obtain an expression for the consumption policy function

$$\hat{C}_t(x_t) = \sum_{i=0}^{N+k-1} \hat{\beta}_i B_{i,k}(x_t). \quad (19)$$

We plot one such estimated consumption policy function in figure 3.

**Marginal Propensity to Consume** de Boor (1972) shows that the derivative of a Spline function is another Spline function of lower order. Specifically, the first derivative of (15) is

$$MPC^{Spline}(x_t) = \frac{\partial C_t}{\partial x_t} = (k-1) \sum_{i=0}^{N+k-1} A'_i B_{i,k-1}(x_t), \quad (20)$$

where

$$A'_i = \frac{\beta_i - \beta_{i-1}}{t_{i+k-1} - t_i}, \quad \text{for } t_{i+k-1} > t_i. \quad (21)$$



**Interaction with Income** If income shocks are persistent, it proves useful to take the cross-product of cash on hand and income and define the regression model

$$C_t^Y = \sum_{i=0}^{N+k-1} \alpha_i Y_t \cdot B_{i,k}(x_t) + \sum_{i=0}^{N+k-1} \beta_i B_{i,k}(x_t) + e_t, \quad (22)$$

where  $Y_t$  is income. The Spline basis functions,  $B_{i,k}(x_t)$ , are calculated as in (16) and the “interpolation weights”,  $\omega_{i,k}(x_t)$  are calculated as in (17). We estimate (22) with OLS and can easily obtain predicted values by the estimated consumption policy function. Taking the first derivative as above yields an expression for the estimated marginal propensity to consume conditional on cash on hand and income

$$MPC^{Spline \otimes Y}(x_t) = (k-1) \sum_{i=1}^{N+k-1} \tilde{A}'_i Y_t \cdot B_{i,k-1}(x_t) + (k-1) \sum_{i=1}^{N+k-1} A'_i B_{i,k-1}(x_t), \quad (23)$$

where  $A'_i$  is defined above and

$$\tilde{A}'_i = \frac{\alpha_i - \alpha_{i-1}}{t_{i+k} - t_i}, \quad \text{for } t_{i+k} > t_i. \quad (24)$$

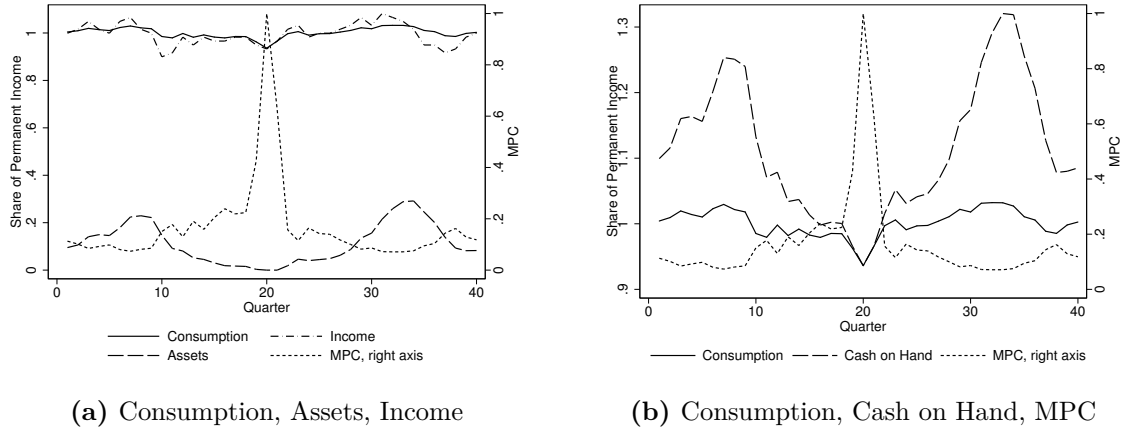
## 2.5 Simulation Exercise

We solve the policy function, (6), for six different income processes, (2), obtaining a dataset of 4,000 agent-quarter observations with their income draws and resulting consumption and asset choices. We calculate the true marginal propensity to consume of each agent conditional on their level of cash on hand and income draws. Using this simulated dataset we explore the goodness-of-fit of the global and local approximations of the consumption policy functions and the resulting estimated marginal propensity to consume.

Following Deaton (1991), we set  $\beta = .95$  and set the permanent income  $\bar{Y} = 100$ . We solve for the policy function,  $f(x_t, Y_t | \Omega)$  from (6), for each of the following set of parameters

$$\Omega = \Omega^\mu \times \Omega^{\sigma_Y^2} \times \Omega^\rho,$$

where  $\Omega^\mu = \{2, 2.5, 3\}$ ,  $\Omega^{\sigma_Y^2} = \{5, 10, 15\}$  and  $\Omega^\rho = \{0, .7\}$ . We simulate the income path for 1,000 agents over 40 quarters taking random draws from the income process (2)-(3) and calculating the corresponding consumption-savings choice of each agent using the policy function  $f(x_t, Y_t | \Omega)$ . We further calculate the *exact* marginal propensity to consume out



**Figure 2:** Simulated Consumption-Savings Path.

*Notes: Policy responses of one agent simulated over 40 quarters, using the converged consumption function, and parameter values, shown in figure 1 for  $\rho = .7$ .*

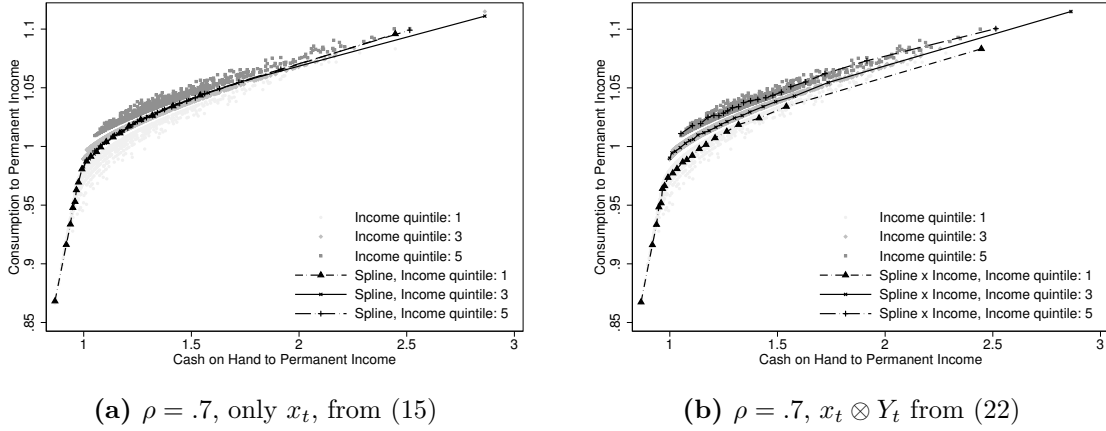
for each agent and each quarter as

$$MPC(x_t, Y_t|\Omega) := \frac{f(x_t + \epsilon, Y_t|\Omega) - f(x_t, Y_t|\Omega)}{\epsilon}, \quad (25)$$

where  $\epsilon$  is “small”. Since for  $\lim_{\epsilon \rightarrow 0+} MPC(x_t, Y_t|\Omega) = \frac{\partial f(x_t, Y_t|\Omega)}{\partial x_t}$ , we interpret  $MPC(x_t, Y_t|\Omega)$  as the “marginal propensity to consume out of liquidity”. I.e. (25) measures the share an agent with income of  $Y_t$ , cash on hand of  $x_t$  and subject to the economic environment  $\Omega$  would consume out of an unexpected windfall gain.

Figure 2 shows the income, assets, consumption expenditures and MPC path of a particular agent with a randomized initial level of assets. Note how consumption closely tracks income over the period, with the agent starting off spending less than she receives in income. This reverses after about 10 quarters after which the agent depreciate all her assets until she is basically living from hand to mouth. We also plot the calculated marginal propensity to consume, from (25), which highlights the consumption-savings tradeoff of the agent. For the first 20 quarters the agent has a relatively low MPC, less than .2. As soon as she has depleted her assets, however, her MPC rapidly increases until hitting the upper limit of 1 in the 25th quarter, at which point she is willing to consume the full amount of any hand-out she might receive. As her income increases a few quarters later, her MPC quickly falls to a level equal to the “pre-crisis” quarters resulting in the build-up of an asset buffer.

Of the 40,000 simulated agent-quarter observations, we take a random sample of 4,000 agent-quarter observations to mimick the case in which the researcher only has access to cross-sectional data. Using this sample we estimate models (7)-(10) and (15)-(22)



**Figure 3:** Comparing Estimated Consumption Policy Functions.

*Notes:* Parameters as in figure 1 simulated for 4,000 agents. Grey shaded dots represents simulated (true) consumption. Each connected line shows the predicted consumption for the 1st, 5th, ..., 95th and 99th percentile of the total resource distribution conditional on income.

using the simulated data to compare their goodness-of-fit. Table 1 shows the root mean standard deviation (RMSD) of the true values of consumption and MPC and the root mean squared errors (RMSE) for a set of the regression models. We use the RMSD to illustrate the noise of the data coming from the income process.<sup>8</sup>

Column 2 (“Poly”) shows the predicted consumption and MPC values for the single regression model (7) or (8) with the lowest RMSE for the consumption prediction. I.e. for each sample we estimate the logarithmic model (7) and the polynomial models in (8) for  $I = 1, \dots, 5$  and calculate the RMSE of the prediction  $\hat{C}_t$  for each model. Each row of column 1 in table 1 shows the lowest obtained RMSE. Likewise column 4 (“Poly”) shows the RMSE of the preferred global polynomial model of cash on hand interacted with income, (9)-(10). We notice that there is almost no difference between the RMSE for the models without persistent income shocks,  $\rho = 0$ . For example, when  $\sigma_y^2 = 10$  the RMSE for the best polynomial is .0037 while it is .0035 for the best polynomial in which income and cash on hand is interacted. In contrast, when income is serially correlated,  $\rho = .7$ , the regression models with interacted income and cash on hand obtains a remarkable better fit (lower RMSE) when predicting the optimal consumption choice. For example, when  $\sigma_y^2 = 10$  the RMSE for the preferred income-interacted polynomial is almost half (.0056) that of the preferred standard polynomial (.011). The second part of the table, however, shows that the interacted model does not appear to improve on the simple regression

<sup>8</sup>  $RMSD(x) \equiv [(N-1)^{-1} \sum_i (\bar{x} - x)^2]^{1/2}$  is an unbiased estimator of the standard deviation of the random variable  $X$  if it is distributed identically and independently. This is obviously not true for  $\rho \neq 0$ , and so the RMSD in table 1 should not be interpreted as the standard deviation of the consumption or MPC observations, but as a simple metric for the amount of variation in the data.

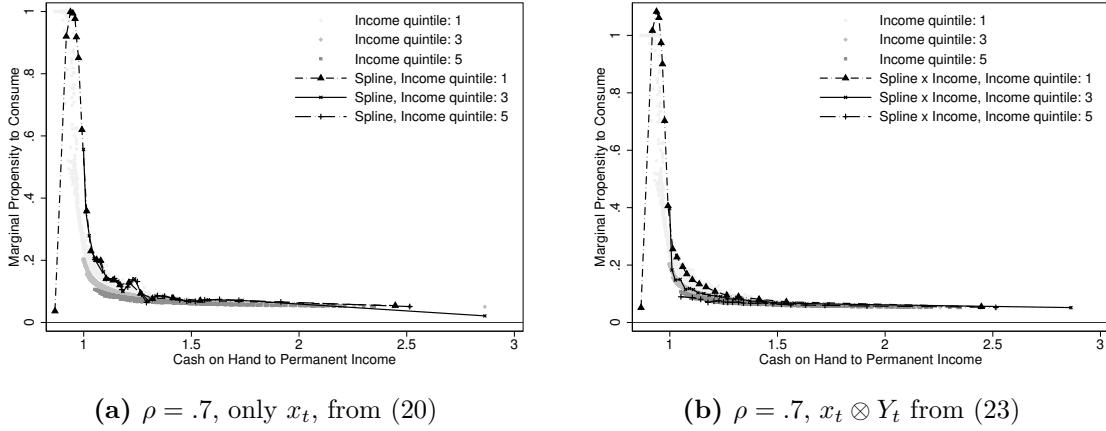
model in fitting the marginal propensity to consume. We see that the RMSE of both the model with and with the income interaction is between .14 to .23, which is rather high considering that the true MPC only takes on values between 0 and 1. This suggests that inferring the MPC from the global approximation models entails a considerable amount of uncertainty.

In comparison, column 3 and 5 shows the RMSE for a cubic Spline regression model, (15) and (22), with 10 interior knots. We see from column 3 of the upper part of the table that using a local approximation drastically improves the fit over a global approximation. With no persistence in the income process,  $\rho = 0$ , the RMSE of the Spline is almost two orders of magnitude lower than that of the best fitting global approximation model. For example, when  $\sigma_y^2 = 10$  the RMSE of the Spline is .00016 which is almost 1/200 of the RMSE for the best fitting global approximation model. With positive income persistency,  $\rho = .7$ , the difference becomes less stark, but both versions of the regression Spline continues to obtain a RMSE lower by at least an order of magnitude compared to their global approximation counterparts. The bottom half of table 1 shows, however, that while the regression Spline clearly dominates the global approximation model, the improvement is less stark when predicting MPCs.

Comparing the two regression Splines to each other we notice that they obtain similar values for the RMSE in the economy without income persistence. The income-augmented Spline even has a slightly higher RMSE in the MPC predictions. With income persistency, however, the income-augmented regression Spline has a RMSE of at least an order of magnitude lower than the standard regression Spline. The improvement for the MPC prediction is less notable, but still apparent, with a RMSE of .11 when  $\sigma_y^2 = 10$  compared to .14 for the standard regression Spline.

For a visual illustration of the improvement of the income-augmented Spline regression, figure 3 shows the standard regression Spline (panel (a)) and the income-augmented regression Spline (panel (b)).<sup>9</sup> The varying shaded grey dots are the scatter of consumption against cash on hand conditional on income draws. One notices that the scatter resembles the shape of the consumption policy function from figure 1. One also notices that the darker shaded dots (representing the highest income draws) lies above the lighter shaded dots. Hence, for the same level of cash on hand, a higher income draw leads to a higher observed consumption choice. In panel 3a we notice that the predicted policy functions overlay one another, and therefore does not take into account that varying income

<sup>9</sup> A comment on the construction of the figure is appropriate here. Since, by design, the income augmented Spline perfectly matches a large share of all observations, a plotting of each prediction would become indistinguishable from the true scatter plot of the consumption choices. We therefore obtain, for each income quintile, the consumption choice corresponding to the following percentiles of cash on hand: 0, 5, 10, ..., 100, where 0 represents the minimum value and 100 the maximum value.



**Figure 4:** Comparing Estimated Marginal Propensity to Consume.

*Notes:* Parameters as in figure 1 simulated for 4,000 agents. Grey shaded dots represents simulated (true) MPC. Each connected line shows the predicted MPC for the 1st, 5th, ..., 95th and 99th percentile of the total resource distribution conditional on income.

levels have different effects on the optimal choice of consumption. Panel 3b, in contrast, shows the predicted consumption policy function for the income-augmented Spline, (22). We notice that the predicted policy functions visibly differ from one another and that they much closer resemble the cloud of observed consumption choices in the data. We also notice that for levels of cash on hand greater than 1, both the collection of observed consumption choices and the predicted policy functions appear to follow parallel paths.

Our main interest is the ability of the regression Spline to predict marginal propensities to consume. Hence, figure 4 shows the calculated MPC from the simulation and predicted MPC from the regression Splines. Again, panel (a) shows the standard regression Spline while panel (b) shows the income-augmented regression Spline. We immediately notice that both Splines erroneously predict a very low MPC for agents with the lowest level of cash on hand and the income draw. This could be due to the low density of data in this area. Remember that we force a knot at the minimum and maximum values of cash on hand, and distribute the interior knots so that the number of observations between each knot is identical. This means that at the end points, we might not have a sufficient number of observations to accurately predict the MPCs. Apart from the inconsistency for the lowest levels of cash on hand, we notice that the graphs in panel 4b fits the true MPC surprisingly well. The regression Splines follows the curvature of the MPC plotting to a rather high degree, predicting (correctly) very high MPCs for low levels of cash on hand, and very low MPCs for high levels of cash on hand. For comparison a global polynomial approximation (not shown), cannot capture the extreme curvature of the MPC plotting for low levels of cash hand.

The regression Spline MPC model (20), shown in panel 4a, predicts the same MPC for all agents with identical cash on hand, irrespective of their income draw. This significantly biases the MPC prediction upwards. The income-augmented regression Spline model (23), panel 4b, shows a much better fit to the observed MPCs, correctly predicting *lower* MPCs for agents with identical levels of cash on hand but with a *higher* income draw.

**Table 1:** Model Fitting Statistics, RMSD and RMSE

Income Variance, $\sigma_y^2$	True Consumption	Predicted Consumption, RMSE			
		Total Resources		Total Resources $\times$ Income	
	RMSD (6)	Poly (7)/(8)	Spline (15)	Poly (9)/(10)	Spline (22)
Transitory Income Serial Correlation, $\rho = 0$					
5	0.0097	0.0030	0.000087	0.0033	0.000080
10	0.013	0.0037	0.00016	0.0035	0.00011
15	0.015	0.0045	0.00022	0.0041	0.00016
Transitory Income Serial Correlation, $\rho = .7$					
5	0.023	0.0095	0.0049	0.0046	0.00032
10	0.031	0.011	0.0069	0.0056	0.00046
15	0.039	0.013	0.0078	0.0051	0.00051
Income Variance, $\sigma_y^2$	True MPC	Predicted MPC, RMSE			
		Total Resources		Total Resources $\times$ Income	
	RMSD (25)	Poly (11)/(12)	Spline (20)	Poly (13)/(14)	Spline (23)
Transitory Income Serial Correlation, $\rho = 0$					
5	0.19	0.15	0.11	0.15	0.12
10	0.18	0.14	0.100	0.14	0.11
15	0.18	0.14	0.10	0.14	0.11
Transitory Income Serial Correlation, $\rho = .7$					
5	0.23	0.20	0.16	0.20	0.10
10	0.21	0.16	0.14	0.17	0.11
15	0.20	0.16	0.14	0.17	0.097

*Notes:* Each row shows the root mean squared deviation from the mean (RMSD) for the true consumption and MPC values and the root mean squared error (RMSE) for the regression models. The true Consumption and MPC values are generated using the converged policy function (6) for the utility preference parameters:  $\beta = .95$ ,  $\mu = \{2, 2.5, 3\}$ ,  $R = 1.05$ ,  $\bar{Y} = 100$  and the stated income parameters  $\sigma_y^2 = \{5, 10, 15\}$  and  $\rho = \{0, .7\}$ . Each cell is an average of the resulting RMSD/RMSE for each value of  $\mu$ .

### 3 Application - Marginal Propensity to Consume in Denmark

In this section, we illustrate the usefulness of our new method for estimating marginal propensities to consume (MPCs) in an empirical application. Specifically, we use a detailed data set containing annual income, asset holdings and demographic information on the full population of Danish residents from 2005 to 2013. In the following we describe the data and our measures of income, assets and consumption in detail. We then discuss the sample selection criteria and present summary statistics, before we finish by presenting and discussing the results.

#### 3.1 The Data

Our dataset consists of detailed tax and employment data on Danish households from 2005 to 2013. We combine individual-level, 3rd party reported, tax data, demographic characteristics and employment information to obtain a complete economic picture of more than 98% of Danish households.<sup>10</sup> We use the official price deflator from Statistics Denmark to deflate all monetary variables to 2010 values. We further divide all financial variables with household size to obtain per capita values.

**Income** Annual income,  $Y_t$ , is measured as the sum of all after-tax personal income during the year  $t$ . Data on income is primarily based on third-party reportings to the tax authorities. Our measure thus represents total *disposable* income for the household during year  $t$ , including government transfers.

**Assets and Debt** For each household in the data we observe the stock of a specific set of assets on December 31, year  $t - 1$ . These assets include total bank holdings, the market value of stocks, the market value of bonds, and the value of all houses and apartments owned. We define *liquid* assets for year  $t$ ,  $\tilde{A}_t \equiv RA_{t-1}$ , as the sum of bank holdings, the market value of stocks and the market value of bonds on December 31st, year  $t - 1$ .

We further observe the stock of debt on December 31st, the sum of which we denote as *total debt*,  $D_t$ . With both assets and debt defined, we compute total *net wealth* for year  $t$  as

$$W_t \equiv \tilde{A}_t - D_t.$$

For reasons which will be discussed later, this measure does not include housing wealth.

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<sup>10</sup> The data is provided by Statistics Denmark for research purposes. While all individuals residing in Denmark is contained in the data by default, a small share of individuals has chosen to opt out of providing their information for research purposes.

**Cash on Hand** We define cash on hand,  $X_t$ , as total liquid resources available to the household for consumption in year  $t$ .

$$X_t = Y_t + \tilde{A}_t,$$

Hence,  $X_t$  equals the *flow* of income during year  $t$ , and the sum of the *stock* of liquid interest-accrued resources ( $\tilde{A}_t$ ).<sup>11</sup>

**Imputing Consumption Expenditures** Following Browning and Leth-Petersen (2003); Leth-Petersen (2010); Andersen et al. (2014) we impute consumption expenditures from changes in net wealth from one year to another. In particular, we use the accounting identity

$$C_t \equiv Y_t - S_t = Y_t - (W_t - W_{t-1}).$$

One challenge with this approach is that the change in the value of a household's holding of a particular asset (or liability) does not necessarily reflect a change in the physical stock of that asset, i.e. saving. Changes in the asset's price, i.e. capital gains or losses, are also included, and it is generally not possible to separate the two sources of variation. This means that the imputed measure of consumption can contain substantial measurement error. However, we are able to improve the measure in a number of respects. First, housing assets is by far the largest asset category among households. Families involved in a real estate trade clearly change the physical stock of assets. We exclude families engaged in housing trade from our dataset in both the year in which the real estate sale took place, the previous year and the subsequent year. The remaining variation in housing stock is therefore due to capital gains and losses, and therefore, we disregard housing wealth in our measure of net wealth. Still, however, housing investments cannot be separately identified and is therefore counted as consumption in our measure.<sup>12</sup>

Fluctuations in stock prices is another important source of capital gains or losses for stock-owning families. Unfortunately, our data does not allow us to separate the effect of

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<sup>11</sup> Technically, we run into a challenge when using the definition  $\tilde{A} = RA_{t-1}$  because it *might* include the January paycheck in year  $t$ . Hence, the measured  $\tilde{A}_t$  is upwards biased. We expect this bias to be of lesser importance in imputing the consumption values since it is common for Danish households during our sample period to receive their January paycheck on one of the last banking days of December. The bias could, however, negatively influence our estimated consumption functions and therefore result in a *downward bias* of our MPC estimates.

<sup>12</sup> One could argue that housing investments *should* count as a consumption expenditure if the investment increase the utility of the housing (consumption). On the other hand, how should we define larger housing investments, such as repairs, primarily used for upkeep of the home? If such investments are financed in large part through debt accumulations, and not reflected immediately (or at all) in the home resale price, then such housing investments could create large spikes and sudden drops in our consumption measure.



changing stock prices from the effects of actual buying and selling. Instead, we proceed in two steps. First, we exclude households owning stocks with a value of more than 10,000 DKK ( $\sim \$1,500$ ) in a given year. And second, for the remaining families, we use a crude adjustment based on the overall development in stock markets: For each family, we multiply the value of stock portfolio at the beginning of the year with the over-the-year growth rate of the C20 index, the top-tier index of the Copenhagen Stock Exchange. The result of this calculation can be viewed as an approximation of the capital gain earned on the family's stock portfolio during the year, so we subtract it from the change in the value of the family's stock portfolio. Furthermore, instead of using the change in pension assets, we use data on pension savings in the form of annual pension contributions. This measure is much more precise than data on pension assets during the sample period.

**Sample Selection** We limit our focus to a sample of households with a head aged 30 - 60 years, with a minimum annual income of 25,000 DKK, and where neither partner is self-employed nor a full-time student. As mentioned above, we also drop any household in year  $t - 1, t, t + 1$  if the household engaged in a real estate transaction or moved during year  $t$ . We also exclude any household in year  $t - 1, t, t + 1$  which was formed from two individual households in year  $t$ , dissolved in year  $t$ , lost a head of family in year  $t$ , or expanded the family in year  $t$ .<sup>13</sup> Out of the approximately 2.4 million Danish households aged 30-60 in a given year, we are left with about .66 million households per year after enforcing the requirements above as well as discarding outliers with respect to income, liquid assets and consumption. Our final sample used for estimation consists of 6.0 million household-year observations.

**Grouping by Socio-Economic Status** Following Gourinchas and Parker (2002) we construct household socio-economic (SE) profiles in order to obtain an estimate of “permanent income” for each profile. We use the definition provided by Statistics Denmark on individual-level socio-economic status, educational level, employment status as well as information on home ownership. We construct an individual-level “life-time” employment status as the highest obtained employment status for the individual during the sample period. In terms of education, we use the educational level of the highest educated individual in the household within 12 major categories. For example, we distinguish between whether the highest educated individual in a household holds a Trade School degree (e.g. Carpenter) or a Non-Academic Bachelor degree (e.g. Nurse). We finally augment this category with an indicator for whether the household owns at least one home (*owner*), and an

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<sup>13</sup> For example, two households  $A$  and  $B$  are counted as two separate households in any year  $\tau < t - 1$ , and a single household in any year  $\tau > t + 1$ , if they combined their households in year  $\tau = t$ .

indicator for whether this home is a co-op run home (*co-op*).<sup>14</sup> Hence, our socio-economic grouping is

$$SE_t = \max_t \{empl_t^1\} \times \max_t \{empl_t^2\} \times \max_{1,2,t} \{educ_t^i\} \times I(\text{owner}) \times I(\text{co-op}), \quad (26)$$

where  $empl_t^i$  is the position of employment (e.g. middle-management) of partner  $i$  during year  $t$  and  $educ_t^i$  is the highest household level education obtained until and including year  $t$ , and  $I(\cdot)$  is an indicator function. Time-variation in  $SE_t$  comes from any change in housing and educational status.

**Permanent Income** If Danish households can be characterized by the above-mentioned set of non-overlapping groups, within which each household only differ in income realization, we can run the regression

$$Y_t = \alpha + \sum_i \beta_i \cdot I(SE_t = SE_i) + age + age^2 + \epsilon_t, \quad (27)$$

and use the predicted values as a measure for “permanent” income,  $\bar{Y}(SE_t)$ , for each SE group.<sup>15</sup> The “permanent income” measure is constant over time for each SE group. We scale all household-level monetary variables,  $C_t, Y_t, X_t$ , with  $\hat{Y}(SE_t)$  to obtain “normalized” values for consumption expenditures,  $c_t$ , disposable income,  $y_t$ , and cash on hand,  $x_t$ . Note that we write “permanent income” in quotes because of a number of obvious challenges with claiming that the predicted values from regression (27) represents the *true* permanent income level. For example, it might be reasonable to assume that the permanent income level is *household-specific*. Since we do not use the panel dimension of the data in the regression, it would be an overstretch to claim  $\hat{Y}(SE_t)$  as the true permanent income level for each household.<sup>16</sup>

### Summary Statistics

Table 2 shows summary statistics for the total sample broken down by deciles of the ratio of current to permanent income, i.e.  $y_t = Y_t/\hat{Y}(SE_t)$ . The ratio of current to permanent income gives an indication for whether the household is earning above-average

<sup>14</sup> With make a distinction between co-op and “regular” homes, because the prices of co-op homes and taxes are significantly different from those of “regular” homes in Denmark. Co-op ownership account for a significant fraction of homeownership in Denmark, especially in the larger cities.

<sup>15</sup> We estimate the parameters in (27) using OLS and cluster standard errors on  $se = \max_t \{se_t^1\} \times \max_t \{se_t^2\} \times \max_{1,2,t} \{educ_t^i\}$ , i.e. the time-invariant component of the household SE group.

<sup>16</sup> As a robustness to the following analysis, we have estimated (27) with the fixed-effects estimator and used the predicted values  $\hat{Y}^{FE}(SE_t)$  to normalize the variables  $C_t, Y_t, X_t$ . Our results are qualitatively unchanged.

wage-income, which is critical for consumption-savings decisions.

We note that the average estimated “permanent income” level is increasing in the income to “permanent income” ratio (from hereon “income-ratio”), except for the top two deciles. This suggests that the 9th and 10th income-ratio deciles represents households with a, perhaps, large temporary spike in their current income, that is not represented by a (predictable) increase in their future income level.

Home ownership is very common for the households in our sample, with 61% of all households owning the home in which they reside. Home ownership is increasing in the income-ratio, except for the 10th decile. In terms of demographics, the average household in our sample consists of almost 2.4 individuals with household head(s) approximately 44 years of age. Overall, 9% of households have experienced a significant unemployment spell during the past 24 months. Consistent with intuition, we observe the unemployment is rate decreasing in the income-ratio. The last four columns shows that average cash on hand,  $X$ , income,  $Y$ , liquid assets,  $A$ , and imputed consumption,  $C$ , are all increasing in the income-ratio deciles.

Table 2: Summary Statistics by Deciles of Income to Permanent Income

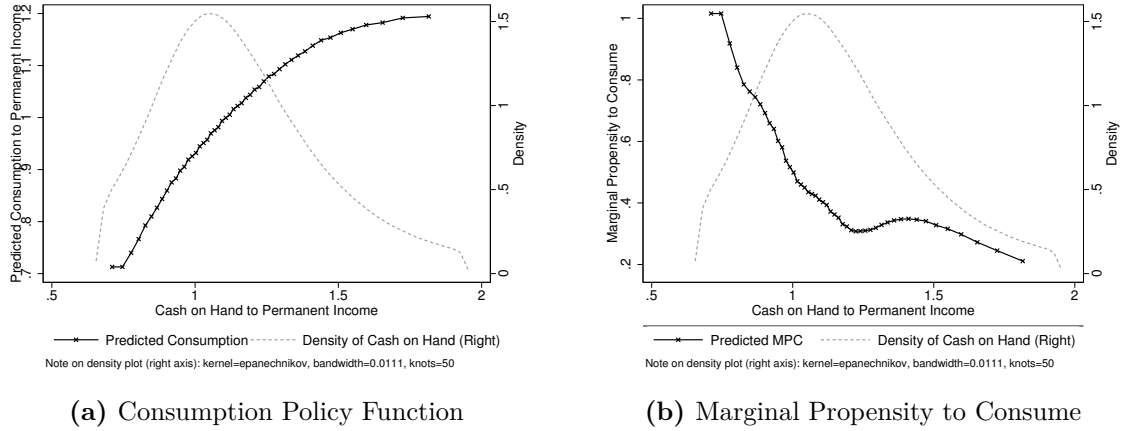
Income Decile♣	Obs.	Perm. Inc.* , (27)	Shares			Averages					
			Home	Co-op	Unempl.†	Size	Age	Cash on Hand*	Income*	Liq. Assets*	Cons.*
1st	599,529	184.4 31.6	0.4 0.49	0.1 0.29	0.21 0.41	1.6 1.04	43.5 8.5	127.7 56.4	107.7 28.5	20.1 44.7	123.3 57
2nd	599,529	190.4 29.7	0.54 0.5	0.07 0.25	0.19 0.39	2.07 1.27	43.9 8.5	173 58.1	146.6 23.9	26.4 48.7	157.8 56.8
3rd	599,528	195.1 27.9	0.61 0.49	0.06 0.24	0.13 0.33	2.32 1.33	43.9 8.5	197.6 61.3	166.5 24.2	31.1 52.1	175 57.3
4th	599,528	197.6 26.7	0.64 0.48	0.06 0.24	0.1 0.29	2.45 1.36	43.9 8.5	215.2 63.7	180.6 24.7	34.5 54.5	186.5 57.4
5th	599,529	198.8 26.4	0.64 0.48	0.07 0.25	0.08 0.26	2.52 1.38	43.9 8.5	229.9 66.9	192.3 25.7	37.6 57.4	195.4 57.6
6th	599,530	199.2 26.4	0.62 0.48	0.07 0.26	0.06 0.24	2.54 1.39	43.9 8.6	243.6 70.7	203.2 27.1	40.4 60.5	203.5 58.1
7th	599,526	199.2 26.9	0.6 0.49	0.08 0.27	0.06 0.23	2.56 1.4	43.9 8.5	257.2 74.7	214.5 29.1	42.7 63.4	212 58.8
8th	599,530	198.8 27.6	0.58 0.49	0.08 0.28	0.05 0.22	2.57 1.41	43.8 8.5	272.6 79.7	227.7 31.9	44.9 66.6	221.7 60.1
9th	599,527	197.3 28.9	0.55 0.5	0.09 0.28	0.04 0.21	2.57 1.4	43.7 8.4	291.5 85.6	244.8 36.4	46.7 69.7	234.6 62.1
10th	599,528	184.5 33.1	0.45 0.5	0.07 0.26	0.05 0.22	2.65 1.43	43.1 8.5	312.4 91.9	268.5 44.6	43.9 70.1	252.6 65.1
Total	5,995,284	194.5 29.2	0.56 0.5	0.08 0.26	0.1 0.3	2.38 1.38	43.7 8.5	232.1 89.3	195.3 54.4	36.8 60	196.2 69.2

Note: Standard deviation in italics.

♣ : Deciles of income to permanent income.

‡ : Share of household where one or both heads has experienced at least six months unemployment during past 24 months.

\* : In 1,000 DKK.



**Figure 5:** Estimated Consumption Policy Function and MPC.

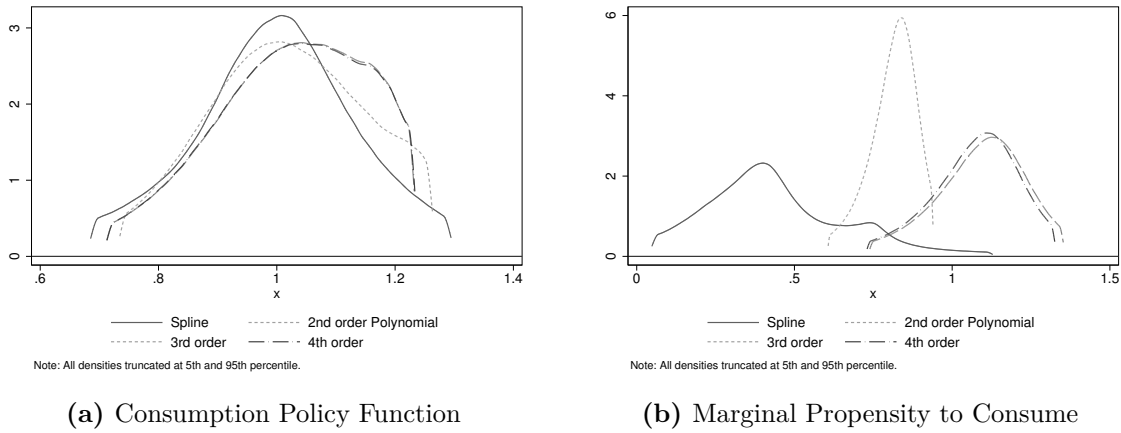
*Notes:* The consumption policy function is the predicted values from the regression Spline, (22). The marginal propensity to consume is the first derivative of the regression spline, (22), calculated in (23). Each connected line shows the predicted consumption to permanent income or MPC for the 3th, 5th, ..., 97th and 99th percentile of cash on hand.

### 3.2 Regression Results

As discussed in the previous section, we can estimate the consumption policy function from a regression Spline on cross-sectional data. We assume households have identical consumption preferences and identical income processes for each socio-economic (SE) classification. Hence, the only difference between households –within each SE group– is their income realization.

Using the normalized consumption and resource variables, we estimate the consumption policy function from the regression Spline (15), and the marginal propensity to consume using (20). Figure 5 panel (a) shows the resulting *aggregate* consumption policy function while figure 5 panel (b) shows the resulting *aggregate* marginal propensity to consume, both as functions of cash on hand to permanent income.<sup>17</sup> We note the strong similarity with the theoretical consumption policy function and MPC displayed in figures 1 and 4. The consumption policy function in panel (a) is increasing and concave. The estimated marginal propensity to consume in panel (b) is highest for the lowest values of cash on hand and notably decreasing as the level of cash on hand increase. We note that the estimated MPC is not as smooth as the estimated consumption function. We should stress that the estimated MPC shown in panel (b) comes from an aggregation of the

<sup>17</sup> By “aggregate” we mean the average consumption response (over all households) to a certain level of cash on hand. I.e. using the full sample we calculate the 5th, 7th, ..., 93th and 95th percentile of cash on hand. For each of these values we calculate the mean predicted consumption response or MPC of all households with cash on hand in a small interval around this value. Each cross in the figures represents one such “aggregate” consumption or MPC response.



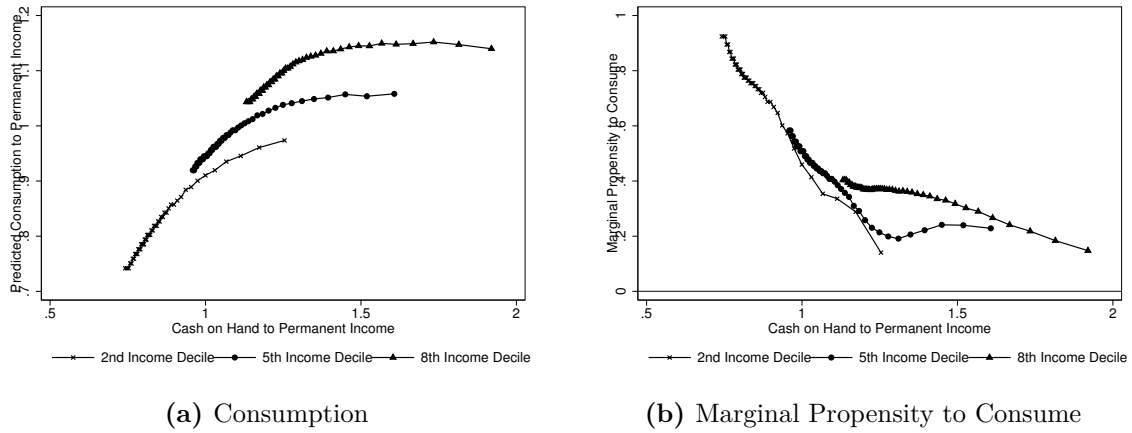
**Figure 6:** Comparing Regression Spline to Polynomials

*Notes: The figure plots estimated kernel densities of predicted consumption values to permanent income (panel 6a) and predicted marginal propensity to consume (panel 6b). The consumption policy function is the predicted values from the regression Spline, (22), and polynomials, equation (10). The marginal propensity to consume is the first derivative of the regression spline, (22), calculated using equation (23), and polynomials calculated using equation (14). See table 6 for a summary table comparing the estimated consumption function and MPC of the regression spline to the polynomial model.*

*individually* estimated MPCs for each household in the sample. The observed “wiggles” are therefore a direct consequence of the flexibility of the regression Spline. However, this does not alone explain the hump beginning at 1.25 of cash on hand to permanent income. Graphing the marginal propensity to consume separately for each income decile shows that the hump arise from households in the higher income deciles with low levels of cash on hand having a larger MPC compared to lower income households with high levels of cash on hand. This can be seen from figure 7b and is discussed in details below.

Figure 6 compares the estimated consumption function and MPC of the regression spline, (22), to 3 different polynomials, (10). We notice from panel 6a that the regression spline does a much better job at capturing the distribution of consumption. This is furthermore clear from the bottom row of table 6 showing that the regression spline has a root mean squared error (RMSE) of .26 and an adjusted  $R^2$  of .938, while the polynomials have RMSEs around .3 and adjusted  $R^2$ 's less than .25. Table 6 in the appendix shows a detailed comparison between the regression spline and the polynomial models.

As discussed in the theoretical section, especially regarding figure 4, it might be important to condition on current income as a proxy for expected future income. Figure 7 graphs aggregate estimated consumption policy functions and MPC for three different income deciles. Similar to figure 3, we notice that the estimated consumption policy functions in panel (a) appear as a shifting of the same underlying function. The consumption functions are clearly concave with a slope close to one for lowest levels of cash on hand



**Figure 7:** Estimated Consumption Policy Function and MPC for 3 Income Deciles.  
*Notes:* The consumption policy function and MPC is the predicted values from the income augmented regression Spline, (22). Each connected line shows the predicted consumption/MPC value for the 1st, 3rd, ..., 97th and 99th percentile of cash on hand.

within each income decile. This appearance is perhaps more noticable in panel (b) showing that the estimated MPC for a given level of cash on hand has a similar shape across income deciles.

In order to compare the overall MPC across income deciles we focus on table 3. The first columns, as in table 2, show the number of household-year observations and the mean estimated permanent income by income deciles. The next four columns show average levels of the consumption share of, respectively, cash on hand, income and permanent income. The average consumption share of income is larger than one, suggesting that a considerable number of households consumed more than their disposable income during our sample period. In line with expectations, we note that the consumption share of cash on hand and income is decreasing with increasing income, reinforcing the picture from figure 5. The consumption share of permanent income is strongly increasing, which is as expected since the consumption to income ratio does not drop much as income to permanent income increases.

The last four columns shows average estimated MPC without conditioning on future expected income, equation (15), and when conditioning on expected future income, equation (22). Average MPC is somewhat higher, at 51% versus 49%, when we do not take future expected income into account. For both methods, the estimated MPC rapidly decrease as the ratio of current to permanent income increases. A very similar picture is obtained when considering income deciles instead of deciles of current to permanent income, cf. appendix B.

The bottom of table 3 shows model fitting statistics of the model without and with

**Table 3:** Consumption Shares and Estimated MPC by Household Income deciles

Income Decile <sup>♣</sup>	Obs. <sup>†</sup>	Permanent Inc. <sup>*</sup> , (27)	Income to		Consumption Share of			Conditioning on Future Income?			
			Cash on Hand	Inc.	Perm. Inc.	Inc.	Perm. Inc.	Cons., (15)	MPC, (20)	Cons., (22)	MPC, (23)
1st	599,529	184.4	0.89	1.04	1.18	0.67	0.67	0.71	0.77	0.68	1.31
		<i>31.6</i>	<i>0.15</i>	<i>0.56</i>	<i>0.63</i>	<i>0.29</i>	<i>0.29</i>	<i>0.15</i>	<i>0.18</i>	<i>0.13</i>	<i>0.84</i>
2nd	599,529	190.4	0.89	0.95	1.08	0.83	0.83	0.87	0.74	0.84	0.65
		<i>29.7</i>	<i>0.14</i>	<i>0.33</i>	<i>0.34</i>	<i>0.26</i>	<i>0.26</i>	<i>0.11</i>	<i>0.17</i>	<i>0.08</i>	<i>0.25</i>
3rd	599,528	195.1	0.88	0.92	1.05	0.9	0.9	0.94	0.65	0.91	0.52
		<i>27.9</i>	<i>0.14</i>	<i>0.31</i>	<i>0.31</i>	<i>0.26</i>	<i>0.26</i>	<i>0.1</i>	<i>0.17</i>	<i>0.07</i>	<i>0.23</i>
4th	599,528	197.6	0.88	0.91	1.03	0.94	0.94	0.99	0.58	0.95	0.43
		<i>26.7</i>	<i>0.14</i>	<i>0.29</i>	<i>0.29</i>	<i>0.26</i>	<i>0.26</i>	<i>0.09</i>	<i>0.17</i>	<i>0.06</i>	<i>0.19</i>
5th	599,529	198.8	0.87	0.89	1.02	0.98	0.98	1.02	0.53	0.99	0.37
		<i>26.4</i>	<i>0.14</i>	<i>0.28</i>	<i>0.27</i>	<i>0.26</i>	<i>0.26</i>	<i>0.08</i>	<i>0.17</i>	<i>0.06</i>	<i>0.17</i>
6th	599,530	199.2	0.87	0.87	1	1.02	1.02	1.05	0.48	1.02	0.34
		<i>26.4</i>	<i>0.14</i>	<i>0.27</i>	<i>0.26</i>	<i>0.27</i>	<i>0.27</i>	<i>0.08</i>	<i>0.17</i>	<i>0.06</i>	<i>0.15</i>
7th	599,526	199.2	0.87	0.86	0.99	1.07	1.07	1.08	0.44	1.06	0.32
		<i>26.9</i>	<i>0.14</i>	<i>0.26</i>	<i>0.25</i>	<i>0.27</i>	<i>0.27</i>	<i>0.07</i>	<i>0.17</i>	<i>0.07</i>	<i>0.14</i>
8th	599,530	198.8	0.87	0.85	0.98	1.12	1.12	1.11	0.42	1.1	0.32
		<i>27.6</i>	<i>0.14</i>	<i>0.26</i>	<i>0.24</i>	<i>0.28</i>	<i>0.28</i>	<i>0.06</i>	<i>0.19</i>	<i>0.07</i>	<i>0.16</i>
9th	599,527	197.3	0.88	0.85	0.96	1.19	1.19	1.15	0.36	1.16	0.34
		<i>28.9</i>	<i>0.14</i>	<i>0.25</i>	<i>0.23</i>	<i>0.29</i>	<i>0.29</i>	<i>0.04</i>	<i>0.2</i>	<i>0.09</i>	<i>0.18</i>
10th	599,528	184.5	0.89	0.85	0.95	1.39	1.39	1.19	0.16	1.3	0.3
		<i>33.1</i>	<i>0.13</i>	<i>0.24</i>	<i>0.21</i>	<i>0.35</i>	<i>0.35</i>	<i>0.03</i>	<i>0.11</i>	<i>0.17</i>	<i>0.17</i>
Total	5,995,284	194.5	0.88	0.9	1.02	1.01	1.01	1.01	0.51	1	0.49
		<i>29.2</i>	<i>0.14</i>	<i>0.32</i>	<i>0.33</i>	<i>0.34</i>	<i>0.34</i>	<i>0.16</i>	<i>0.24</i>	<i>0.18</i>	<i>0.43</i>
Model Fitting Statistics											
								Spline, (15)			
								RMSE	$R_a^2$	RMSE	$R_a^2$
								0.2976	0.9219	0.2611	0.938
								Spline $\otimes Y$ , (22)			

Notes: Standard deviations in italics.

<sup>♣</sup> : Deciles of income to permanent income.<sup>\*</sup>: In 1,000 DKK.



**Table 4:** Aggregate MPC for Different Groups of Households

(a) All (homogeneous MPC)	0.448
(b) Bottom income decile	0.852
(c) Top income decile	0.284
(d) Unemployed	0.569
(e) Homeowners	0.395

*Note:* The table reports the aggregate short-run MPC out of transitory income for selected groups of households. Estimated MPCs larger than 1 or smaller than 0 have been recoded to 1 and 0, respectively. 'Unemployed' refers to households in which at least one adult has experienced at least six months unemployment during past 24 months.

a conditioning on expected future income. Both models display similar values for the adjusted R-square. However, the model with a conditioning on expected future income has a notable lower root mean squared error (RMSE) compared to the model without. Model (23) is therefore our preferred model.

Based on model (23), table 4 reports the aggregate MPC for different groups of households. These results further highlight the large heterogeneity in MPCs across households, thereby also indicating that a single-agent framework may not in all cases be sufficient to predict the aggregate propensity to consume of an economy. Rather, it may be important to take the heterogeneity into account when assessing for example the impacts of fiscal policies or changes in (long) interest rates.

## 4 Conclusion

In this paper we describe a new and simple way of estimating marginal propensities to consume (MPC) using only cross-sectional data. We show theoretically that a local polynomial approximation, a regression Spline, yields accurate results and is robust to both high levels of income variance and income persistency. In fact, the regression Spline is superior to a variety of global polynomial and logarithmic regression models, which are commonly used in the literature.

We then investigate the applicability of the regression Spline on real data, using a sample of Danish individual-level tax data. The regression Spline does a remarkable job at estimating the consumption policy function and marginal propensity to consume, with only a few counterintuitive developments in the tails of the consumption and income distribution.

Aggregating the household-level estimates of marginal propensities to consume, the income augmented regression Spline yields an aggregate (“1-year”) MPC of about 49%. This figure is larger than studies analyzing short-run propensities to consume, but slightly less than studies reporting long-run propensities to consume.

Our results further confirm the large heterogeneity in MPCs across households, which is also found in previous studies. This may have important implications for macroeconomic modelling as well as for assessing the impacts of proposed fiscal policies or changes in long interest rates.

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## A Normalizing consumption by income

An often used alternative to the regression models of section 2 is

$$C_{i,t}/Y_{i,t} = \alpha + \eta X_{i,t} + \beta x_{i,t}/Y_{i,t} + \varepsilon_t, \quad (28)$$

where  $Y_{i,t}$  is *labour* (or “normal”) income in year  $t$  for household  $i$ .<sup>18</sup> Note that (28) is identical to (10) for all  $\alpha_1, \dots, \alpha_I$  and  $\beta_0, \beta_1, \dots, \beta_I$  equal to zero. Normalizing consumption expenditures by labour income yields a regression on the fraction of consumption out of (current) income, from which multiple authors<sup>19</sup> interpret the estimated coefficient  $\beta$  as a *marginal propensity to consume* out of wealth. We see three problems with estimating regression equation (28) and with relating  $\hat{\beta}$  to MPC out of wealth; 1) it does not allow for a constant term, 2) only resource and expenditure variables are normalized by labour income which does not allow a one to one relationship with equation (8) or (10), and perhaps most importantly, 3) normalizing with current income could yield inconclusive results. We discuss each point briefly below.

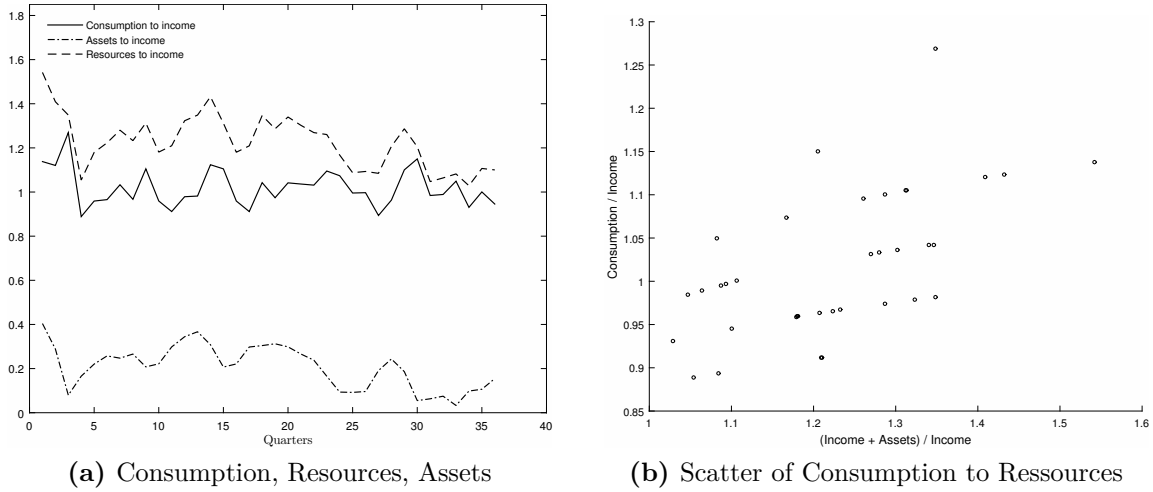
The first problem, that (28) does not allow for a constant term, is seen directly by multiplying the equation by  $Y_{i,t}$ . While the theoretical consumption function, figure 1, seems to suggest that there should be no constant term in the functional form for the consumption function, we would a) need to test for this empirically and therefore still include a constant term, and b) realize that if the household is allowed to borrow, i.e. allowing assets be negative, it would shift the curve to the left, and therefore introduce a non-zero constant term in the consumption function. In order to allow for such a constant term we would need to introduce a variable  $\tilde{Y} = 1/Y$  in regression equation (28).

The second problem, that only the resource and expenditure variables are normalized by (current) income, breaks the clear connection with the theoretical model. In order to align the theory with the regression equation, all controls should also be interacted with (or “normalized” by) current income. While this may be a reasonable approach it is absent from equation (28). A more clear approach, if one were to believe the normalization in (28), would be to estimate the regression on data subset by the controls  $X_{i,t}$ , i.e. simply treating households with different  $X_{i,t}$ ’s as so different, that they cannot have identical  $\beta$ ’s and therefore cannot be pooled into the same estimation.

The third and perhaps the most important problem, the normalization with current income, is that current income is *not* the (theoretically) most important variable when determining consumption. Indeed, as is discussed extensively by Carroll (1997, 2001,

<sup>18</sup> Regressions of this form are often estimated on cross-sectional data, thus suppressing the  $i$  subscript and assuming away household-specific effects.

<sup>19</sup> See e.g. Paiella (2007) and Arrondel et al. (2015).



**Figure 8:** Normalizing by Income.

*Note:* Variables normalized by actual labour income. Plotting for one agent over 36 quarters, using the converged consumption function shown in 1. Parameters as in figure 1.

2006) and Blundell et al. (2008), one should normalize by *permanent income* if one were to normalize at all. As seen in figure 2 consumption is much less volatile than current income, so if one divides consumption by current income, one *increases the volatility* of the “normalized” consumption! This is clear from panel (a) of figure 8. Hence a scatter plot of consumption versus resources, in which both have been normalized by current income, yields rather uninformative results, as seen in panel (b) of figure 8.

One should therefore either normalize the data by *permanent income* or not normalize at all.

## **B Supplementary Tables**

**Table 5:** Consumption Shares and Estimated MPC by Household Income Deciles

Income Decile♣	Obs.†	Permanent Inc.* (27)	Income to		Consumption Share of			Conditioning on Future Income?			
								No		Yes	
			Cash on Hand	Cash on Hand	Cash on Hand	Inc.	Perm. Inc.	Cons., (15)	MPC, (20)	Cons., (22)	MPC, (23)
2nd	711369	<i>34</i> 181.9	<i>0.16</i> 0.87	<i>0.53</i> 0.93	<i>0.61</i> 1.07	<i>0.31</i> 0.87	<i>0.31</i> 0.87	<i>0.22</i> 0.93	<i>0.2</i> 0.65	<i>0.18</i> 0.88	<i>0.58</i> 0.57
3rd	711369	<i>27.5</i> 190	<i>0.16</i> 0.86	<i>0.93</i> 0.91	<i>0.34</i> 1.05	<i>0.3</i> 0.93	<i>0.3</i> 0.93	<i>0.19</i> 0.99	<i>0.23</i> 0.55	<i>0.15</i> 0.94	<i>0.34</i> 0.42
4th	711369	<i>23.7</i> 195	<i>0.16</i> 0.85	<i>0.31</i> 0.88	<i>0.31</i> 1.04	<i>0.3</i> 0.97	<i>0.3</i> 0.97	<i>0.16</i> 1.02	<i>0.23</i> 0.49	<i>0.13</i> 0.98	<i>0.29</i> 0.34
5th	711369	<i>22.7</i> 199.3	<i>0.16</i> 0.84	<i>0.3</i> 0.86	<i>0.29</i> 1.02	<i>0.3</i> 1	<i>0.3</i> 1	<i>0.15</i> 1.04	<i>0.24</i> 0.43	<i>0.13</i> 1	<i>0.26</i> 0.28
6th	711369	<i>22.4</i> 203.5	<i>0.16</i> 0.84	<i>0.29</i> 0.84	<i>0.28</i> 1	<i>0.3</i> 1.03	<i>0.3</i> 1.03	<i>0.14</i> 1.06	<i>0.24</i> 0.38	<i>0.13</i> 1.02	<i>0.23</i> 0.24
7th	711369	<i>22.5</i> 207.7	<i>0.16</i> 0.83	<i>0.28</i> 0.82	<i>0.27</i> 0.99	<i>0.3</i> 1.06	<i>0.3</i> 1.06	<i>0.14</i> 1.06	<i>0.23</i> 0.34	<i>0.13</i> 1.05	<i>0.21</i> 0.2
8th	711369	<i>22.7</i> 212	<i>0.17</i> 0.82	<i>0.27</i> 0.8	<i>0.26</i> 0.97	<i>0.3</i> 1.09	<i>0.3</i> 1.09	<i>0.13</i> 1.07	<i>0.23</i> 0.28	<i>0.13</i> 1.07	<i>0.2</i> 0.16
9th	711369	<i>23.1</i> 216.9	<i>0.17</i> 0.82	<i>0.26</i> 0.78	<i>0.25</i> 0.95	<i>0.31</i> 1.14	<i>0.31</i> 1.14	<i>0.13</i> 1.07	<i>0.23</i> 0.21	<i>0.14</i> 1.1	<i>0.18</i> 0.13
10th	711369	<i>23.3</i> 222.8	<i>0.17</i> 0.81	<i>0.26</i> 0.74	<i>0.23</i> 0.91	<i>0.32</i> 1.23	<i>0.32</i> 1.23	<i>0.13</i> 1.06	<i>0.22</i> 0.09	<i>0.14</i> 1.15	<i>0.16</i> 0.07
		<i>23.8</i>	<i>0.17</i>	<i>0.24</i>	<i>0.21</i>	<i>0.33</i>	<i>0.33</i>	<i>0.13</i>	<i>0.19</i>	<i>0.15</i>	<i>0.13</i>
Total	7113691	199.2	0.84	0.86	1.02	1	1	1.01	0.41	0.99	0.33
	<i>30</i>		<i>0.17</i>	<i>0.93</i>	<i>0.33</i>	<i>0.33</i>	<i>0.33</i>	<i>0.18</i>	<i>0.29</i>	<i>0.18</i>	<i>0.38</i>
Model Fitting Statistics											
								Spline, (15)			
								RMSE	$R_a^2$	Spline $\otimes Y$ , (22)	$R_a^2$
								0.302	0.922	0.267	0.936

Notes: Standard deviations in italics.

♣ : Deciles of income.

\*: In 1,000 DKK.



**Table 6:** Predicted Consumption and Estimated MPC by Household Income deciles

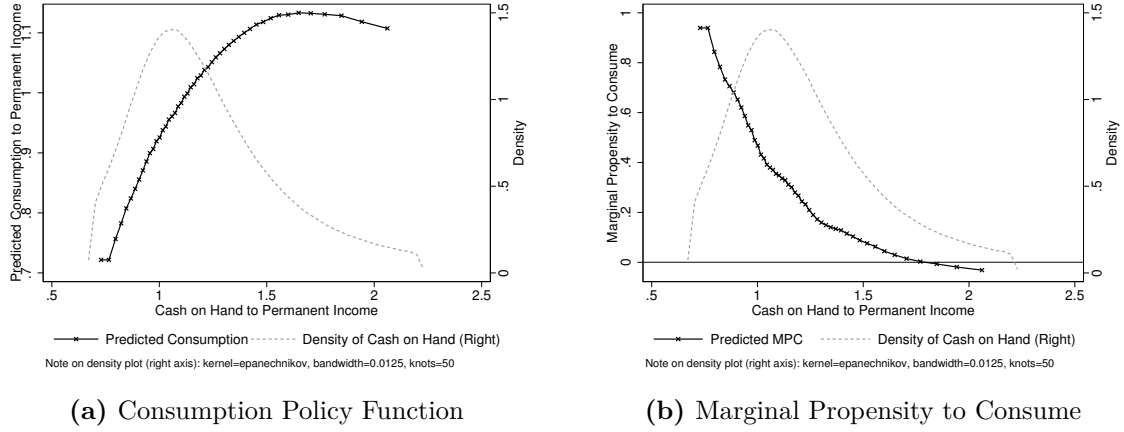
Income	Permanent Decile♣Obs. <sup>†</sup>	Regression Spline / Conditioning on Future Income?			Polynomial / Conditioning on Future Income?		
		No		Yes	Yes, Linear		Yes, 4th order
		Cons., (15)	MPC, (20)	Cons., (22)	MPC, (23)	Cons., (10)	MPC, (14)
1st	599,529	184.4 <i>31.6</i>	0.71 <i>0.15</i>	0.68 <i>0.13</i>	1.31 <i>0.84</i>	0.86 <i>0.09</i>	0.33 <i>0</i>
2nd	599,529	190.4 <i>29.7</i>	0.87 <i>0.11</i>	0.84 <i>0.08</i>	0.65 <i>0.25</i>	0.92 <i>0.09</i>	0.33 <i>0</i>
3rd	599,528	195.1 <i>27.9</i>	0.94 <i>0.1</i>	0.91 <i>0.07</i>	0.52 <i>0.23</i>	0.95 <i>0.09</i>	0.33 <i>0</i>
4th	599,528	197.6 <i>26.7</i>	0.99 <i>0.09</i>	0.95 <i>0.06</i>	0.43 <i>0.19</i>	0.97 <i>0.09</i>	0.33 <i>0</i>
5th	599,529	198.8 <i>26.4</i>	1.02 <i>0.08</i>	0.99 <i>0.06</i>	0.37 <i>0.17</i>	1 <i>0.1</i>	0.33 <i>0</i>
6th	599,530	199.2 <i>26.4</i>	1.05 <i>0.08</i>	1.02 <i>0.06</i>	0.34 <i>0.15</i>	1.02 <i>0.1</i>	0.33 <i>0</i>
7th	599,526	199.2 <i>26.9</i>	1.08 <i>0.07</i>	1.06 <i>0.07</i>	0.32 <i>0.14</i>	1.04 <i>0.1</i>	0.33 <i>0</i>
8th	599,530	198.8 <i>27.6</i>	1.11 <i>0.06</i>	1.1 <i>0.07</i>	0.32 <i>0.16</i>	1.07 <i>0.11</i>	0.33 <i>0</i>
9th	599,527	197.3 <i>28.9</i>	1.15 <i>0.04</i>	1.16 <i>0.09</i>	0.34 <i>0.18</i>	1.1 <i>0.11</i>	0.33 <i>0</i>
10th	599,528	184.5 <i>33.1</i>	1.19 <i>0.03</i>	1.3 <i>0.17</i>	0.3 <i>0.17</i>	1.19 <i>0.14</i>	0.33 <i>0</i>
Total	5,995,284	194.5 <i>29.2</i>	1.01 <i>0.16</i>	1 <i>0.18</i>	0.49 <i>0.43</i>	1.01 <i>0.14</i>	0.33 <i>0</i>
Model Fitting Statistics		Spline, (15)		Spline $\otimes Y$ , (22)		Poly., 1st order, (10)	
		RMSE	$R_a^2$	RMSE	$R_a^2$	RMSE	$R_a^2$
		0.2976	0.9219	0.2611	0.938	0.1639	0.3098
						0.2412	0.2951

Notes: Standard deviations in italics.

♣ : Deciles of income to permanent income.

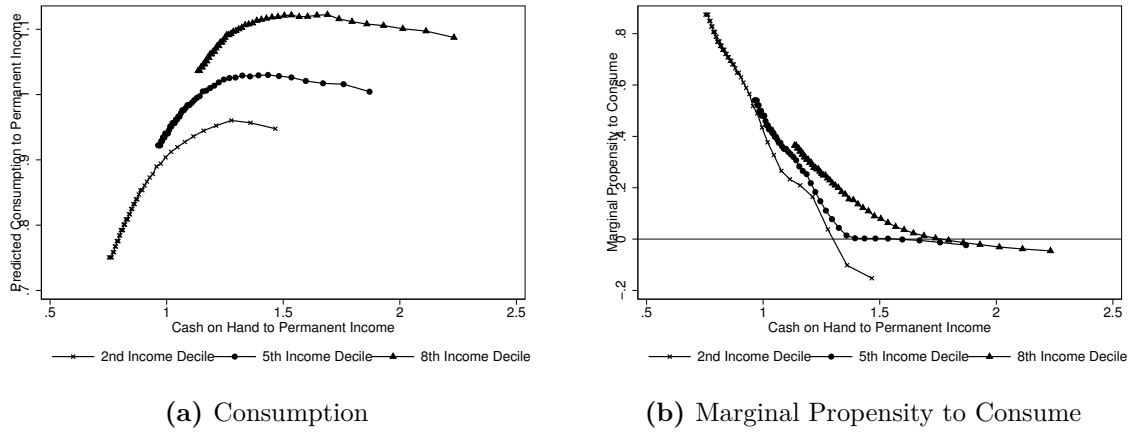
\*: In 1,000 DKK.

## C Figures and Tables Including Large Stockowners



**Figure 9:** Estimated Consumption Policy Function and MPC.

*Notes:* The consumption policy function is the predicted values from the regression Spline, (22). The marginal propensity to consume is the first derivative of the regression spline, (22), calculated in (23). Each connected line shows the predicted consumption to permanent income or MPC for the 3th, 5th, ..., 97th and 99th percentile of cash on hand.



**Figure 10:** Estimated Consumption Policy Function and MPC for 3 Income Deciles.

*Notes:* The consumption policy function and MPC is the predicted values from the income augmented regression Spline, (22). Each connected line shows the predicted consumption/MPC value for the 1st, 3rd, ..., 97th and 99th percentile of cash on hand.

**Table 7:** Aggregate MPC for Different Groups of Households

(a) All (homogenous MPC)	0.267
(b) Bottom income decile	0.713
(c) Top income decile	0.055
(d) Unemployed	0.403
(e) Homeowners	0.211

*Notes:* The table reports the aggregate short-run MPC out of transitory income for selected groups of households. Estimated MPCs larger than 1 or smaller than 0 have been recoded to 1 and 0, respectively. 'Unemployed' refers to households where at least one adult have experienced at least six months unemployment during past 24 months.

**Table 8:** Summary Statistics by Deciles of Income to Permanent Income, Stocks Included

Income Decile♣	Obs.	Perm. Inc.* , (27)	Shares			Averages						
			Home	Co-op	Unempl.†	Size	Age	Cash on Hand*	Income*	Liq. Assets*	Cons.*	
1st	711,370	188.3 <i>32.4</i>	0.44 <i>0.5</i>	0.09 <i>0.29</i>	0.21 <i>0.41</i>	1.67 <i>1.08</i>	43.8 <i>8.5</i>	139.5 <i>68.9</i>	111.7 <i>29.6</i>	27.8 <i>57.5</i>	128 <i>58.9</i>	
2nd	711,369	195.6 <i>30.2</i>	0.59 <i>0.49</i>	0.07 <i>0.25</i>	0.18 <i>0.38</i>	2.16 <i>1.28</i>	44.3 <i>8.5</i>	188.8 <i>71.7</i>	152 <i>24.5</i>	36.8 <i>62.5</i>	163.2 <i>58.4</i>	
3rd	711,369	200.1 <i>28.5</i>	0.66 <i>0.47</i>	0.06 <i>0.24</i>	0.12 <i>0.32</i>	2.39 <i>1.33</i>	44.3 <i>8.5</i>	214.9 <i>75.6</i>	171.6 <i>24.9</i>	43.3 <i>66.5</i>	179.4 <i>58.7</i>	
4th	711,369	202.4 <i>27.4</i>	0.67 <i>0.47</i>	0.06 <i>0.24</i>	0.09 <i>0.28</i>	2.51 <i>1.36</i>	44.4 <i>8.5</i>	234.2 <i>79.7</i>	185.5 <i>25.4</i>	48.7 <i>70.6</i>	190.2 <i>58.7</i>	
5th	711,370	203.4 <i>27.2</i>	0.67 <i>0.47</i>	0.07 <i>0.25</i>	0.07 <i>0.25</i>	2.56 <i>1.37</i>	44.4 <i>8.6</i>	250.5 <i>84.1</i>	197.1 <i>26.5</i>	53.4 <i>74.5</i>	198.7 <i>59.1</i>	
6th	711,368	203.8 <i>27.4</i>	0.66 <i>0.47</i>	0.07 <i>0.26</i>	0.06 <i>0.23</i>	2.59 <i>1.38</i>	44.4 <i>8.6</i>	265.8 <i>88.8</i>	208 <i>28.1</i>	57.8 <i>78.3</i>	206.5 <i>59.5</i>	
7th	711,369	203.9 <i>27.8</i>	0.65 <i>0.48</i>	0.08 <i>0.27</i>	0.05 <i>0.22</i>	2.61 <i>1.39</i>	44.4 <i>8.6</i>	281.3 <i>94.1</i>	219.4 <i>30.1</i>	61.9 <i>82.3</i>	214.7 <i>60.4</i>	
8th	711,369	203.6 <i>28.6</i>	0.62 <i>0.48</i>	0.08 <i>0.27</i>	0.04 <i>0.21</i>	2.62 <i>1.4</i>	44.3 <i>8.5</i>	298.4 <i>99.7</i>	232.7 <i>33</i>	65.6 <i>86</i>	224.1 <i>61.7</i>	
9th	711,369	202.3 <i>29.8</i>	0.59 <i>0.49</i>	0.08 <i>0.28</i>	0.04 <i>0.2</i>	2.63 <i>1.39</i>	44.2 <i>8.4</i>	319 <i>106.8</i>	249.9 <i>37.3</i>	69.1 <i>90.1</i>	236.6 <i>63.5</i>	
10th	711,369	188.6 <i>34</i>	0.49 <i>0.5</i>	0.07 <i>0.26</i>	0.05 <i>0.21</i>	2.66 <i>1.42</i>	43.5 <i>8.6</i>	336 <i>112.1</i>	271.8 <i>44.8</i>	64.2 <i>90.4</i>	253.1 <i>66</i>	
Total	7,113,691	199.2 <i>30</i>	0.61 <i>0.49</i>	0.07 <i>0.26</i>	0.09 <i>0.29</i>	2.44 <i>1.37</i>	44.2 <i>8.5</i>	252.8 <i>106.2</i>	200 <i>54.7</i>	52.9 <i>77.7</i>	199.4 <i>69.8</i>	

Note: Standard deviation in italics.

♣ : Deciles of income to permanent income.

† : Share of household where one or both heads has experienced at least six months unemployment during past 24 months.

\* : In 1,000 DKK.

Table 9: Summary Statistics by Deciles of Income, Stocks Included

Income Decile♣	Obs.	Perm. Inc.* , (27)	Shares			Averages					
			Home	Co-op	Unempl.†	Size	Age	Cash on Hand*	Income*	Liq. Assets*	Cons.*
1st	711370	163 34	0.31 0.46	0.09 0.28	0.19 0.39	1.57 1.04	42.6 9	120.5 52.4	101.8 21.5	18.6 46.8	115.2 49.3
2nd	711369	181.9 27.5	0.48 0.5	0.07 0.26	0.23 0.42	2.07 1.31	43.4 9.2	174.7 59.2	143.8 7.6	30.9 58.4	154.4 49.4
3rd	711369	190 23.7	0.59 0.49	0.06 0.24	0.14 0.35	2.3 1.32	43.5 9	202.8 62.9	165 5	37.9 62.5	173.9 51.8
4th	711369	195 22.7	0.63 0.48	0.06 0.24	0.1 0.3	2.42 1.33	43.7 8.9	223.8 66.9	180.2 4	43.6 66.7	186.6 52.8
5th	711369	199.3 22.4	0.65 0.48	0.07 0.25	0.07 0.26	2.51 1.35	43.9 8.7	242.7 71.5	193.4 3.7	49.3 71.3	197.3 54.1
6th	711369	203.5 22.5	0.66 0.47	0.07 0.26	0.05 0.23	2.57 1.36	44.2 8.5	260.5 75	206.2 3.7	54.3 74.8	207.1 55.2
7th	711369	207.7 22.7	0.67 0.47	0.08 0.27	0.04 0.2	2.63 1.37	44.5 8.3	280.2 79.7	219.8 4.2	60.4 79.5	217.3 56.4
8th	711369	212 23.1	0.68 0.47	0.08 0.27	0.03 0.18	2.68 1.38	44.9 8.1	302.8 84.6	235.7 5.1	67.1 84.3	229 58.1
9th	711369	216.9 23.3	0.68 0.46	0.08 0.27	0.03 0.16	2.77 1.38	45.4 7.7	332.8 90.9	257 7.5	75.9 90.3	244.1 60.3
10th	711369	222.8 23.8	0.71 0.45	0.08 0.27	0.02 0.13	2.88 1.37	46.1 7.3	387.5 103.4	297 20	90.5 100	269.5 64
Total	7113691	199.2 30	0.61 0.49	0.07 0.26	0.09 0.29	2.44 1.37	44.2 8.5	252.8 106.2	200 54.7	52.9 77.7	199.4 69.8

Note: Standard deviation in italics.

♣ : Deciles of income.

† : Share of household where one or both heads has experienced at least six months unemployment during past 24 months.

\* : In 1,000 DKK.