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## Does the Adverse Announcement Effect of Climate Policy Matter? - A Dynamic General Equilibrium Analysis

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# Does the Adverse Announcement Effect of Climate Policy Matter? - A Dynamic General Equilibrium Analysis\*

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## Abstract

We consider a carbon emissions tax announced today, but implemented after a known time-lag. Before implementation, the announcement induces higher emissions than without intervention. In welfare terms, this adverse announcement effect could more than outweigh the gain after tax implementation. We quantify a ‘critical lag’ such that a shorter (longer) implementation lag is a welfare gain (loss) over no-intervention. We identify resource scarcity as the main driver for a short critical lag. The model is a global Ramsey Model extended by an exhaustible carbon resource and linked to a climate model.

Keywords: Climate Policy, Announcement Effect, Dynamic General Equilibrium, Non-renewable Resource, Backstop, Green Paradox, Welfare Evaluation

JEL Classification: E21, E32, F44, G21, G28

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# 1 Introduction

Mitigation of climate change is a key issue on any policy-maker's agenda. Most climate policies aim to reduce carbon emissions by restricting fossil fuel demand. The focus on the demand side of fossil fuels may lead to a 'green paradox' (Sinn 2008), i.e. to a situation where well-intentioned climate policies actually trigger an acceleration of climate change. These well-intentioned policies fail because they neglect fossil fuel supplier reactions. A green paradox may occur when (a) there is incomplete coverage of climate policies, (b) climate policies become stricter over time, or (c) because climate policies are implemented with a time-lag (see van der Werf and Di Maria (2012) for an overview).<sup>1</sup>

This paper focuses on climate policies with a time-lag because bureaucratic and political procedures often impede immediate implementation. The Kyoto Protocol is an example for lagged implementation. It was agreed upon in December 1997, became effective in 2005, and was implemented in 2008 (United Nations Framework Convention on Climate Change 2014). The Paris Agreement did not lead to an immediate emissions restrictions, either. An implementation lag leads to an adverse 'announcement effect': the owners of carbon resources anticipate the restriction on resource demand in the future. They are induced to extract more resources before implementation and to sell the extracted resources at a lower price than in a world without climate policy. This pushes up emissions during the implementation lag. The adverse announcement effect is well established in the literature and partly documented in the data. It may have even already been triggered by the political and scientific debate on climate policy (Jensen et al. 2015).

Di Maria et al. (2012) analyze the announcement effect in a model with two resource types differing in carbon content. They compare emissions up to the implementation of an announced emissions cap to laissez-faire development and find that emissions increase during the implementation lag. Smulders et al. (2012) develop a model with capital accumulation and both clean and dirty energy inputs. They show that an increase in emissions may even occur if resources are not scarce or if the implementation date is uncertain. Jensen et al. (2015) discuss theoretically which factors influence the announcement effect itself and its impact on climate policy. Di Maria et al. (2014b) use the announcement of the Acid Rain Program to analyze the announcement effect empirically. The authors document a drop in coal prices and an increase of coal use for plants relying on spot market deliveries. Overall, they suggest that

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<sup>1</sup>Eichner and Pethig (2011), for example, analyze conditions for a green paradox in a two-period / three-country model. Nachtigall and Rübhelke (2016) introduce learning-by-doing in the renewable energy sector in a two-period model and find that, in such a model, the green paradox resulting from a tax in the second period may not hold anymore.

no paradox occurred because the adverse announcement effect was reduced by institutional factors like long-term contracts. This cannot always be taken for granted. Lemoine (2013) provides evidence for announcement effects on the US energy market.

The question that remains unanswered so far concerns the consequences of an adverse announcement effect with regard to climate policy in quantitative terms. In cases where immediate implementation of, say, an emissions tax is impossible, will *laissez-faire*, i.e. no emissions tax, be better for overall welfare than an emissions tax with a time-lag on implementation? Compared to *laissez-faire*, lagged implementation means that emissions are higher before implementation but lower afterwards. It is not clear a priori which effect dominates in welfare terms. As pointed out by Jensen et al. (2015), the cumulative effect of greenhouse gas emissions may be more important than the change in the timing of emissions caused by the adverse announcement effect. Obviously, a policy intervention that is optimal from the instant of implementation onward will always be welfare-improving if the implementation lag is sufficiently short. If, however, the implementation lag is too long, the adverse announcement effect may dominate. Then it is preferable to do nothing, rather than announcing and implementing a policy with too long an implementation lag. If there are good reasons to suspect that the implementation lag may be too long to render the net welfare effect positive, a responsible environmental policy should opt for *laissez-faire*. It is thus vital not just to know that a green paradox may materialize, but also to know whether it is likely and which factors have a quantitatively relevant influence.

In this paper, we define a ‘critical lag’ for an announced emissions tax to analyze the consequences of an adverse announcement effect. This critical lag is defined such that a shorter (longer) implementation lag is a welfare gain (loss) over the *laissez-faire* scenario. To determine the critical lag, we build a global economy-climate model—an integrated assessment model—with endogenous carbon resource supply and a time-lag between emissions and damages. The climate module is borrowed from the DICE-model, Nordhaus (2008b). In the economic model, firms produce output by using effective labor, capital, and a non-renewable resource.<sup>2</sup> The non-renewable resource is introduced in a Hotelling-framework with rising extraction costs. Even though the empirical validation of the Hotelling-framework is mixed (Livernois 2008), Anderson et al. (2014) show that using the drilling instead of the extraction decision matches the data on oil extraction quite well. This provides evidence that the intertemporal arbitrage condition inherent in the Hotelling framework is still applicable. Strand (2007) also uses a Hotelling framework (without climate module) to consider the role of a lag between signing a

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<sup>2</sup>Di Maria et al. (2012) and Di Maria et al. (2014a) do not include capital, while Smulders et al. (2012) do not include a non-renewable resource.

technology treaty and its implementation for carbon emissions. For a lag of 10 years, he finds an adverse announcement effect.

In an extension to our baseline model, we also introduce a backstop technology in a way that allows a complete switch away from natural resources. In global, forward-looking integrated assessment models<sup>3</sup>, fossil fuel supply is usually either not represented at all (DICE) or only represented in terms of rising extraction costs, without considering a scarcity rent or supply-side reactions (e.g. MIND, ENTICE-BR). Such an approach cannot capture a possible reaction from resource owners.<sup>4</sup> The model presented in Golosov et al. (2014) is an exception to the extent that it includes resource owners' behavior, represented by a Hotelling framework. In contrast to our model, Golosov et al. (2014) do not include time-lags between emissions and damages.<sup>5</sup>

We propose to include the backstop in the model in a way that (a) permits the use of fossil fuels only, (b) the simultaneous use of fossil fuels and the backstop, and (c) a complete switch to the backstop. This bridges the gap between the theoretical literature and integrated assessment modeling. The theoretical literature sees a backstop as a perfect substitute for fossil fuels given a certain price (e.g. Hoel and Kverndokk (1996); Tahvonen (1997); van der Ploeg and Withagen (2012)), while most integrated assessment models use a constant elasticity of substitution framework to aggregate fossil fuels and the backstop (Gerlagh and van der Zwaan 2004; Popp 2006). With a CES technology, fossil fuels extracted will be used forever, however high the price becomes. In our model, a sufficiently high price for extracted fossil fuels will trigger a complete switch to the backstop.

We assume the emissions tax to be welfare-maximizing as of the point in time at which it is implemented. Welfare would of course be higher if, at the very beginning, a tax were announced that is optimal as of the point in time at which it is announced, subject to the constraint of lagged implementation. Such a policy would, however, be time-inconsistent and therefore not credible. When it comes to the point in time of implementation, revising the plan in a way that is optimal from the point in time of implementation would raise welfare.

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<sup>3</sup>The selection of the Integrated Assessment Models discussed here is based on the overview of Stanton et al. (2009). We consider global, forward-looking models (classified as 'Welfare Maximization Models') together with the model by Golosov et al. (2014). Table 5 in the Appendix gives an overview.

<sup>4</sup>Blanford et al. (2009) and Bosetti et al. (2009) use integrated assessment models to analyze the developing countries' anticipated participation in the climate coalition. Bosetti et al. (2009) explicitly consider leakage effects, but neither Blanford et al. (2009) nor Bosetti et al. (2009) discuss the resource owners' response to announced participation.

<sup>5</sup>See Bretschger and Karydas (2014) for an analytical model with time-lag between emissions and damages. One could see the paper of Golosov et al. (2014) as part of a movement to advance the analytical understanding of integrated assessment models (see also e.g. Golosov et al. (2014), Traeger (2014) and Gerlagh and Liski (2015)), partly building on workhorse models from analytical macroeconomics (Gerlagh and Liski 2015). Golosov et al. (2014), Traeger (2014) and Gerlagh and Liski (2015) all rely on log-utility and in their calibrations make reference to the DICE model. Traeger (2014) and Gerlagh and Liski (2015) present approaches that consider uncertainty in more detail, but that do not include a resource sector.

We thus consider the announcement of a tax that is optimally designed with regard to the point in time at which it is implemented.

As the economic dynamics are fast compared to climate dynamics, we use numerical simulations to complement the theoretical analysis with first quantitative insights. We show that the critical lag is strongly reduced by resource scarcity, and to a lesser extent by a lower time preference rate and higher productivity growth. In turn, damage parameters and climate sensitivity matter for the welfare effect of climate policy, but they do not have a large impact on the critical lag. These parameters have a scaling effect, but no timing effect. The availability of a backstop technology greatly increases the critical lag.

The remainder of this paper proceeds as follows: In the next section we introduce the model and solve both for the market equilibrium and for the social planner's solution. We also calculate the optimal tax path and introduce the welfare measure. The solution approach and the calibration follow in section 3. In section 4 and 5 we discuss results and robustness. The paper closes with our conclusions.

## 2 The Model

The dynamic general equilibrium model combines an economic module with a climate module. Economic dynamics determine resource use and emissions, while the climate module translates emissions into temperature change. Temperature, in turn, affects output. The resource represents all fossil fuels lumped together, measured in terms of their carbon content. Due to the climate externality, the decentralized market equilibrium does not have an equivalent optimization representation. We therefore first derive the equilibrium conditions for the decentralized economy with an exogenous tax imposed on carbon use, subsequently deriving the tax rate leading to the welfare optimum.

With respect to notation, dotted variables (e.g.  $\dot{X}$ ) denote time derivatives and hats denote growth rates ( $\hat{X} := \dot{X}/X = d \log X/dt$ ). Latin letters denote variables that depend on time (with exemptions stated). Row vectors are written as  $[\dots]$ , column vectors as transposed row vectors  $[\dots]^\top$ .

### 2.1 The economic module

Firms and households interact on competitive markets. Firms produce the final output  $Y$  with capital  $K$ , resource flow  $R$ , and effective labor  $E$  as inputs according to a Cobb-Douglas

technology

$$Y = \Omega(T_1)K^\alpha R^\beta E^\lambda, \quad (1)$$

with partial production elasticities  $\alpha > 0$ ,  $\beta > 0$ ,  $\lambda > 0$  and  $\alpha + \beta + \lambda = 1$ . Output is scaled by the damage function  $\Omega(T_1)$  based on Nordhaus and Boyer (2000) and Nordhaus (2008b). The specification assumes the percentage loss of GDP to be a quadratic function of the global mean surface temperature  $T_1$ . Labor is constant, but its productivity grows exogenously such that effective labor  $E$  grows at the exogenous rate  $\hat{E}$ . The resource flow  $R$  represents a mixture of fossil fuels extracted from an exhaustible carbon resource stock  $S$ . As a common denominator, the fossil fuel inputs are measured in terms of their respective carbon content, which is emitted one-to-one into the atmosphere after having been burned in production. Hence,  $R$  stands for both input and emissions measured in GtC/a.<sup>6</sup> The carbon resource stock—‘resource stock’ in the following—also represents a mixture of fossil fuels converted into carbon and measured in GtC.

Due to perfect competition, input prices equal their respective marginal productivities. Taking the output good as numéraire, we obtain

$$\iota + \delta = \alpha Y/K, \quad (2)$$

$$q + r = \beta Y/R, \quad (3)$$

with resource price  $q$ , interest rate  $\iota$ , depreciation rate  $\delta$ , and specific tax rate  $r$ . Imposing the tax either on the resource input or on the emissions makes no difference because the carbon extracted as a component of fossil fuel is emitted one-to-one as a component of CO<sub>2</sub>. The tax is paid by the firms.

Input flow  $R$  is extracted from a continuous set of privately-owned carbon sources exploitable at costs per unit of flow that vary across sources. In their role as resource owners, households are price-takers selling the flow on a competitive market. It is well-known that under these conditions, sources are exploited in the order of ascending unit costs such that—provided the interest rate is positive—unit costs can be expressed as a non-increasing function  $k(S)$  of the total resource stock  $S$  in all as yet-unexploited sources (see e.g. Herfindahl (1967); Laitner (1984); Solow and Wan (1976)).

Furthermore, resource owners collectively act like a representative owner of all sources who, at any time  $t$ , chooses an extraction path  $R(\tau)$ ,  $\tau \geq t$ , so as to maximize the present (as of time  $t$ ) value of future net revenues from selling the flows at prices taken as given. The asset

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<sup>6</sup>Units are printed in **teletype font**. Table 6 in the Appendix lists the units.



value of the remaining stock  $S(t)$  at time  $t$  is thus

$$v(t, S(t)) = \frac{1}{D(t)} \max_{R(\tau), \tau \geq t} \int_t^\infty D(\tau) R(\tau) (q(\tau) - k(S(\tau))) d\tau, \quad (4)$$

subject to

$$\dot{S} = -R. \quad (5)$$

The discount factor is  $D$ . Its law of motion is

$$\dot{D} = -\iota D, \quad (6)$$

with boundary condition  $D(0) = 1$ .

Optimum conditions<sup>7</sup> are

$$q = k(S) + p, \quad (7)$$

with co-state  $p$  of  $S$  displaying the marginal asset value per unit resource stock, and with

$$\dot{p} = \iota p + Rk'(S), \quad (8)$$

the augmented Hotelling rule. The transversality condition<sup>8</sup> is

$$\lim_{t \rightarrow \infty} D(t)p(t)S(t) = 0. \quad (9)$$

The consumption side of the model is standard. An immortal representative household owns all assets  $a$  of the economy, inelastically supplies labor, and receives the tax revenue collected by the state as a lump sum. The state has no other role to play than collecting the tax and channeling it to the household's budget. The asset value is the value of the capital stock plus the value of the resource stock,

$$a = K + v. \quad (10)$$

As output is transformed one-to-one into investment, capital is measured in units of the numéraire.

The household receives utility from consumption  $C$ . As usual, instantaneous utility  $u$

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<sup>7</sup>Derived from the current-value Hamiltonian  $\mathcal{H} = R(q - k(S)) - pR$ .

<sup>8</sup>In addition, the following two conditions have to be fulfilled: (1) There exists a number  $m$  such that  $|D(t)p(t)| \leq m$  for all  $t \geq t_0$ . (2) There exists a number  $t'$  such that  $p(t) \geq 0$  for all  $t \geq t'$  (Sydsæter et al. 2005). These conditions also have to be fulfilled in the case of all other transversality conditions.

has the constant intertemporal elasticity of substitution form with intertemporal elasticity of substitution  $1/\theta$ . Maximizing the discounted utility flow and taking into account the budget constraint displayed by the production balance of the economy<sup>9</sup>

$$\dot{K} = Y - \delta K - Rk(S) - C \quad (11)$$

yields the well-known Keynes-Ramsey rule

$$\hat{C} = (i - \rho)/\theta, \quad (12)$$

with time-preference rate  $\rho$  and the transversality condition

$$\lim_{t \rightarrow \infty} D(t)K(t) = 0. \quad (13)$$

Integrating the Keynes-Ramsey rule, we can write optimal consumption in levels as

$$C = (e^{\rho t} B D)^{-1/\theta}, \quad (14)$$

with an endogenous constant  $B$ . Note that, unlike the other variables written in Latin letters,  $B$  does not depend on time.

## 2.2 The climate module

The climate module is taken from the DICE model by Nordhaus (2008b).<sup>10</sup> It consists of two connected sub-systems. The first sub-system describes the carbon cycle, i.e. the evolution of the carbon masses between the three carbon reservoirs atmosphere  $M_1$ , upper ocean  $M_2$ , and lower ocean  $M_3$ . It reads

$$\dot{M} = \Gamma M + [R + \nu, 0, 0]^\top, \quad (15)$$

with  $M := [M_1, M_2, M_3]^\top$ , the carbon-transition matrix  $\Gamma$  and carbon emissions  $R$  from carbon used in the production process and emissions from land use change  $\nu$ . Carbon stocks are measured in gigatons of carbon **GtC**, and carbon flows in gigatons of carbon per annum **GtC/a**.

The second sub-system describes the impact on temperature of the carbon concentration in the atmosphere. It reads

$$\dot{T} = \Lambda T + [\Pi(M_1, t), 0]^\top, \quad (16)$$

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<sup>9</sup>Appendix B gives a detailed derivation relating the household's budget constraint to the production balance of the economy.

<sup>10</sup>We have adapted his discrete time version to continuous time to match the economic module.

with  $T := [T_1, T_2]^\top$ . The global mean surface temperature  $T_1$  dynamically interacts with the temperature of the lower oceans  $T_2$ , as described by the matrix  $\Lambda$ . Temperatures are measured as differences in  $^\circ\text{C}$  from their level in the year 1900. The function  $\Pi(M_1, t)$  describes radiative forcing. Figure 1 illustrates the interactions.

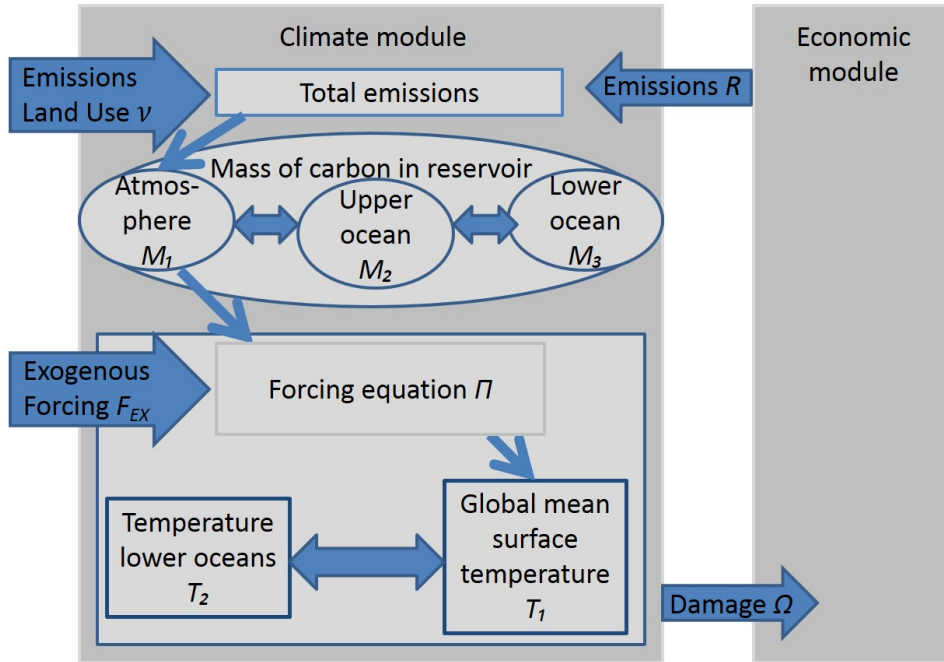


Figure 1: Interactions in the climate module.

### 2.3 The system of equations

The equations that describe the model are summarized in Table 1, with algebraic and differential equations in the left-hand column and boundary conditions on the right.  $\bar{M}$  and so forth denote initial stocks. If we plug in (7) for the resource price  $q$ , the model has thirteen unknown functions of time, namely three carbon masses stacked in the vector  $M$ , two temperatures stacked in the vector  $T$ , output  $Y$ , consumption  $C$ , resource flow  $R$ , resource stock  $S$ , the marginal asset value per unit of stock  $p$ , capital  $K$ , interest rate  $\iota$ , and discount factor  $D$ . The model also contains the integration constant  $B$ . There are eight differential equations with initial boundary conditions corresponding to the variables  $M$ ,  $T$ ,  $D$ ,  $S$ , and  $K$ . Furthermore, there are four algebraic equations for the variables  $Y$ ,  $C$ ,  $\iota$ , and  $R$ . There is an additional differential equation for  $p$ . Finally, there are two terminal boundary conditions enabling us to determine both  $p$  and  $B$ , provided the differential equation system has two unstable eigenvalues. This turns out to be the case.

Table 1: Overview equilibrium conditions market setting

$\dot{M}$	$= \Gamma M + [\nu + R, 0, 0]^\top$	(15)	$M(0)$	$= \bar{M}$
$\dot{T}$	$= \Lambda T + [\Pi(M_1, t), 0]^\top$	(16)	$T(0)$	$= \bar{T}$
$Y$	$= \Omega(T_1)K^\alpha R^\beta E^\lambda$	(1)		
$C$	$= (BDe^{\rho t})^{-1/\theta}$	(14)	$\lim_{t \rightarrow \infty} D(t)K(t)$	$= 0$ (13)
$\dot{D}$	$= -\iota D$	(6)	$D(0)$	$= 1$
$\iota$	$= \alpha Y/K - \delta$	(2)		
$R$	$= \beta Y/(k(S) + p + r)$	(3)		
$\dot{p}$	$= \iota p + Rk'(S)$	(8)	$\lim_{t \rightarrow \infty} D(t)p(t)S(t)$	$= 0$ (9)
$\dot{S}$	$= -R$	(5)	$S(0)$	$= \bar{S}$
$\dot{K}$	$= Y - \delta K - C - Rk(S)$	(11)	$K(0)$	$= \bar{K}$

## 2.4 The optimal tax

So far, the tax rate has been taken as an exogenous policy instrument. To derive an optimal path for the tax rate, we solve the planner's problem of choosing a time path—starting at  $t = 0$ —for consumption  $C$  and resource extraction  $R$  to maximize the representative household's utility

$$U = \int_0^\infty u(C)e^{-\rho\tau} d\tau,$$

subject to technological, resource and climate constraints

$$\dot{K} = \Omega(T_1)K^\alpha R^\beta E^\lambda - \delta K - C - Rk(S),$$

$$\dot{S} = -R,$$

$$\dot{M} = \Gamma M + [\nu + R, 0, 0]^\top,$$

$$\dot{T} = \Lambda T + [\Pi(M_1, t), 0]^\top,$$

given inherited state variables  $K$ ,  $S$ ,  $M$ , and  $T$  at  $t = 0$ .

The present value Hamiltonian is

$$\begin{aligned} \mathcal{H} = & u(C)e^{-\rho t} + \left( \Omega(T_1)K^\alpha R^\beta E^\lambda - \delta K - C - Rk(S) \right) P_K \\ & - R P_S + \left( M^\top \Gamma^\top + [\nu + R, 0, 0] \right) P_M + \left( T^\top \Lambda^\top + [\Pi(M_1, t), 0] \right) P_T. \end{aligned} \quad (17)$$

$P_K$  and so forth are the costates associated with states  $K$  and so forth.  $P_M$  and  $P_T$  are column vectors. If  $\mathcal{H}_R$  and so forth denote derivatives of the Hamiltonian with respect to  $R$  and so

forth, the static and dynamic efficiency conditions read, respectively,

$$\mathcal{H}_R = 0 \quad \Rightarrow \quad 0 = P_K \frac{\beta Y}{R} - P_S + P_{M_1} - P_K k(S), \quad (18)$$

$$\mathcal{H}_C = 0 \quad \Rightarrow \quad P_K = C^{-\theta} e^{-\rho t}, \quad (19)$$

$$\mathcal{H}_K = -\dot{P}_K \quad \Rightarrow \quad -\dot{P}_K = P_K \left( \frac{\alpha Y}{K} - \delta \right), \quad (20)$$

$$\mathcal{H}_S = -\dot{P}_S \quad \Rightarrow \quad -\dot{P}_S = -P_K k'(S) R, \quad (21)$$

$$\mathcal{H}_M = -\dot{P}_M \quad \Rightarrow \quad -\dot{P}_M = \Gamma^\top P_M + \left[ P_{T_1} \frac{\partial \Pi}{\partial M_1}, 0, 0 \right]^\top, \quad (22)$$

$$\mathcal{H}_T = -\dot{P}_T \quad \Rightarrow \quad -\dot{P}_T = \Lambda^\top P_T + \left[ P_K \frac{\partial Y}{\partial T_1}, 0 \right]^\top. \quad (23)$$

The transversality condition is

$$\lim_{t \rightarrow \infty} Z(t)^\top P(t) = 0, \quad (24)$$

with  $Z(t)$  and  $P(t)$  denoting column vectors of states and costates in the corresponding order.

To relate the optimal solution to the market outcome, one has to relate the costates—the shadow prices—to the market prices. The product of the integration constant  $B$  and the discount factor  $D$  corresponds to the shadow price  $P_K$ , the marginal asset value per unit of resource stock in the decentralized outcome  $p$  corresponds to the shadow price of the resource stock transformed into units of the numéraire  $P_S/P_K$ .

The decentralized market will fulfill the optimality conditions if the tax rate is<sup>11</sup>

$$r = -P_{M_1}/P_K. \quad (25)$$

The tax rule (32) has an obvious interpretation:  $-P_{M_1}$  is the marginal utility loss from an extra unit of carbon in the atmosphere. It is translated into units of the numéraire by dividing it through  $P_K$ , the marginal utility of an extra unit of the numéraire.

## 2.5 Welfare evaluation

For the welfare evaluation of policies, we use two measures, a relative and an absolute intertemporal equivalent variation. The former is the constant percentage  $h$  by which consumption  $\check{C}$  of the benchmark scenario must be changed to attain the utility level  $U$  of the policy scenario. A relative intertemporal equivalent variation  $h > 0$  ( $h < 0$ ) indicates a welfare gain (loss) compared to the benchmark. The relative intertemporal equivalent variation is thus implicitly

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<sup>11</sup>See Appendix C for details.

defined by

$$U = \int_0^\infty \frac{\left((1+h)\check{C}\right)^{1-\theta}}{1-\theta} e^{-\rho\tau} d\tau.$$

Using the definition of  $\check{U}$ ,

$$\check{U} = \int_0^\infty \frac{\check{C}^{1-\theta}}{1-\theta} e^{-\rho\tau} d\tau,$$

we obtain

$$h = \left( \left( \frac{U}{\check{U}} \right)^{1/(1-\theta)} - 1 \right).$$

The absolute intertemporal equivalent variation is the amount of the numéraire  $W$  one would have to give to the household in  $t = 0$  to make it as well off as in the policy case,

$$W = h \int_0^\infty \check{D}(\tau)\check{C}(\tau)d\tau. \quad (26)$$

The utility level  $U$  can be calculated comfortably by adding to the system an extra differential equation with an appropriate boundary condition. Define

$$V(t) := \int_t^\infty u(C)e^{-\rho\tau} d\tau,$$

such that  $U = V(0)$ . Taking the time differential delivers the extra differential equation

$$\dot{V} = -u(C)e^{-\rho t}.$$

The transversality condition

$$\lim_{t \rightarrow \infty} V(t) = 0 \quad (27)$$

is obtained from

$$U = \lim_{t \rightarrow \infty} \left( \int_0^t u(C)e^{-\rho\tau} d\tau + V(t) \right) = U + \lim_{t \rightarrow \infty} V(t).$$

Accordingly, we can treat  $V$  as a forward-looking variable that can be determined like the other variables in the system.

### 3 Solution Approach and Calibration

Two important issues arise when operationalizing the model. First, a solution concept has to be picked. Second, appropriate data are needed for calibration.

### 3.1 Solution concept

In the long run, we assume that the model reaches a steady-state (or Balanced-Growth-Path) with constant growth rates. The following two assumptions lead to constant steady-state growth rates: First, as emissions asymptotically tend towards zero, we consider the steady-state system that would prevail if no more emissions entered the climate module. Second, we assume that in the long term the extraction cost component in the production balance (11) grows more slowly than the capital stock, i.e.

$$\hat{R} - \epsilon \hat{S} < \hat{K}, \quad (28)$$

with  $\epsilon$  denoting the elasticity of extraction costs with respect to the resource stock. With the assumption of constant steady-state growth rates,  $\hat{Y}^* = \hat{K}^* = \hat{C}^* = g$  and  $\hat{S}^* = \hat{R}^* = g - \iota^*$ .

To obtain the model solution, we directly solve the dynamic market equilibrium conditions with  $r = 0$  for the market outcome and  $r = -P_M/P_K$  for the planner's solution. We use the `bvp5` two-point boundary-value problem solver in `matlab`. Due to numerical considerations, we solve the model in terms of *stationary transforms* of the variables (see Bröcker and Korzhenevych (2013) for more details) such that the whole model becomes stationary. A stationary model means growth rates that equal zero. A stationary transform is defined by  $\tilde{X} = X e^{-\hat{X}^* t}$ , with  $\hat{X}^*$  denoting the steady-state growth rate. Then the stationary transforms are solved in log-deviations from their respective values in 2005—which are identical to the original variables—if data are available, or from a proxy steady-state if no data are available for 2005. The latter is the case for the shadow values. The proxy steady state is defined like the steady state, but with climate as in  $t = 0$ . Two variables are not solved in log-deviations:  $V$  is used directly, while  $T$  is solved in absolute deviations from the respective 2005 data.

As boundary conditions, we require a subset of the variables to reach constant growth rates at a final point in time. This is similar to requiring some variables to reach their steady-state values at a finite horizon, a method known from the literature (see Bröcker and Korzhenevych (2013) for a discussion). As there is some arbitrariness in fixing the finite horizon, we vary the horizon to ensure that the choice of the finite horizon does not affect results. We also run a model version using the stable manifold approach proposed by Bröcker and Korzhenevych (2013). The results do not change. The procedure with final boundary conditions is justified as discounting makes the present value welfare for one scenario with a finite and another scenario with an infinite horizon come arbitrarily close to each other.

### 3.2 Calibration

The reference year for model calibration and the starting point of the model ( $t = 0$ ) is 2005. The calibration of the climate module is based on the 2010 version of the RICE/DICE model (Nordhaus 2010), including initial values for carbon masses  $M(0)$  and temperatures  $T(0)$ . Appendix D gives details. Calibration of consumption and of production, including the initial capital stock and the calibration of the resource sector, rely on several data sources explained in the following. Again, Appendix D gives details. We calibrate the model in the laissez-faire set-up. To obtain correct welfare results, we keep the calibration in the alternative runs that include a tax.

The growth rate of effective labor is  $\hat{E} = 0.0219 \%$  1/a to obtain a steady-state growth rate of 1.8 % 1/a (similar to Barro and Sala-i Martin (2004)). Furthermore, the depreciation rate is  $\delta = 0.05$  1/a (Barro and Sala-i Martin 2004). The resource's cost share is  $\gamma = 0.058$ , calculated from from IEA (2007) and EIA (2010). Remaining income is divided between capital (1/3) and labor (2/3) (Barro and Sala-i Martin 2004). We use  $Y(0) = 45.23 \times 10^{12}$  \$/a (World Bank 2009).

For the consumption side of the model, the elasticity of intertemporal substitution  $1/\theta = 2/3$ , and the time-preference parameter  $\rho = 1.5 \%$  1/a (Nordhaus 2010). This yields a real interest rate in steady state of  $\iota^* = 4.2 \%$  1/a.

There are no reliable data available for the initial capital stock  $K(0)$ . As capital dynamics are fast compared to the climate system, so that the steady-state relation is quickly reached, we use the steady-state capital-output ratio  $K/Y = \alpha/(i^* + \delta)$  to calibrate  $K(0)$ . This gives an initial capital stock of  $K(0) = 154.6 \times 10^{12}$  \$.

To calibrate the resource sector, we assume  $\epsilon$  to be constant, i.e. we specify extraction costs as

$$k(S) = \gamma S^{-\epsilon}, \quad (29)$$

similar to Laitner (1984). For condition (28) to hold in steady state,

$$\epsilon < \frac{\iota^*}{\iota^* - g} \quad (30)$$

is required, with steady-state interest rate  $\iota^*$  and steady-state growth rate of the economy  $g$ . To see why, use the fact that the assumption of constant steady-state growth rates leads to  $\hat{Y}^* = \hat{K}^* = \hat{C}^* = g$  and  $\hat{S}^* = \hat{R}^* = g - \iota^*$ . Condition (28) thus reads  $(1 - \epsilon)(g - \iota^*) < g$ , which one can rearrange to get (30). Extraction costs are zero for  $\gamma = 0$  and constant for  $\epsilon = 0$ . For  $\epsilon = 1$ , unit extraction costs double if the stock is halved.



For the initial resource stock  $S(0)$ —the sum of oil, gas, and coal stocks converted into carbon—, we use data on the carbon resource base (the sum of reserves and resources)<sup>12</sup> and set  $S(0) = 3000 \text{ GtC}$  (McGlade and Ekins 2015; UNDP 2000). As ‘Resources’ in particular are only recoverable with technological developments, we assume rising extraction costs and use  $\epsilon = 1$ . Condition (30) holds. The parameter  $\gamma$  is chosen to ensure an initial resource flow of  $R(0) = 7.76 \text{ GtC}$ , which results in an initial resource price of  $q(0) = 0.34 \text{ \$/kgC}$ .<sup>13</sup> Note that the resource price has a cost component,  $k(S) = \gamma S^{-\epsilon}$ , and a rent component,  $p$ . The rent component is endogenous and driven by the abundance of the resource. It turns out to be  $0.03 \text{ \$/kgC}$  in the base calibration.

Given  $q(0)$ , we can calculate rent-to-price ratios—the rent share in the price—to compare the model result with available data. Based on the model results, the rent-to-price ratio is 11% for 2005. Bauer et al. (2013) present data for 2010, with rents close to zero for natural gas and coal in Russia, but rent-to-price ratios around 50% for crude oil in the Middle East and North Africa, natural gas in EU27, and coal in China, and around 30% for crude oil in the USA. The average rent-to-price ratio for some European countries—namely the Netherlands, Denmark, the United Kingdom, and Norway—in 1999 for oil and gas was 34% (calculation based on information from European Commission (2002)). If coal were included, the rent-to-price ratio might decrease. On the one hand, coal with a reserve-to-production ratio<sup>14</sup> of 224 a is relatively more abundant than oil and natural gas with reserve-to-production ratios of 40 and 62 a, respectively (Feygin and Satkin 2004).<sup>15</sup> On the other, deposits vary, e.g. in quality and extraction costs, so that high rents also occur for coal. Summarizing available studies, an average rent-to-price ratio is difficult to determine. It seems that our result is in the lower range. One could try to increase the ratio by assuming the cost component to be smaller. But this would imply a lower initial equilibrium flow price and, accordingly, a larger initial flow, which is at odds with the data.

Overall, the set-up and the calibration of our model leads to results comparable to those found in other studies. An important measure is the predicted global mean surface temperature in the laissez-faire setting. Our model calculates a global mean surface temperature for the year 2100 that is in the range of the IPCC predictions (see Table 2).

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<sup>12</sup>UNDP (2000, p. 481) defines ‘Reserves’ and ‘Resources’ as follows: ‘Reserves: those occurrences of energy sources or mineral that are identified and measured as economically and technically recoverable with current technologies and prices’ and ‘Resources: those occurrences of energy sources or minerals with less certain geological and/or economic/technical recoverability characteristics, but that are considered to become potentially recoverable with foreseeable technological and economic development’.

<sup>13</sup>Note that  $\text{\$/kgC} = 10^{12} \text{\$/GtC}$ .

<sup>14</sup>The reserve-to-production ratio gives the period the resource stock will last at the current extraction rate.

<sup>15</sup>The reserve-to-production ratio in the model is 387 a.

Table 2: Comparison of the predicted global mean temperature increases for the year 2100.

Scenario	our model	B1*	A1B*	A2*	RCP2.6**	RCP8.5**
$\Delta^{\circ}\text{C}$ from 1900	3.1	2.4	3.4	4.2	1	4.1

\*based on IPCC (2007):

A1B assumes a converging world with very rapid economic growth and balanced technological change;

B1 also assumes a converging world, but with a rapid change towards a service and information technology;

A2 considers a fragmented world with increasing population and slower technological change;

\*\*based on Collins et al. (2013):

RCP2.6 is the low-emission scenario, RCP8.5 is the high-emission scenario.

## 4 Results

To determine the critical lag for an announced emissions tax, we calculate the welfare differences between scenarios with an announced emissions tax and a do-nothing policy. The scenario with announcement is a scenario where, at the start of the time horizon, we have the announcement of the implementation of a tax in  $t > 0$  that is optimal from the instant of implementation onward. In  $t = 0$ , agents correctly anticipate the introduction of the tax in  $t > 0$ . As set out before, the time path of the announced intervention starting in  $t > 0$  cannot be chosen optimally from the present point of view, i.e. taking anticipation by the market participants into consideration, because such a policy would be time-inconsistent and thus not credible.

Prior to the implementation of the respective tax the announcement of the emissions tax causes a behavior change in comparison with the do-nothing scenario. This is the announcement effect. Figure 2 illustrates the announcement effect. It shows the percent of excess carbon emissions in the scenario with an implementation lag of 10 years compared to a do-nothing policy over time. The positive numbers indicate that in the case with announcement emissions are initially higher than emissions in the case without intervention. After the tax has been implemented, emissions are lower compared to the do-nothing policy. The difference in emissions initially increases over time for the following reason: the initial capital stock is identical in both cases, but as resource use is higher in the case with announcement, output is higher as well. Thus the capital stock adapts and grows faster in the case with announcement. Higher capital stock increases marginal productivity of the resource, so that resource use also increases relative to the do-nothing policy. Di Maria et al. (2012) also discuss higher resource use during the announcement period ('abundance effect'), while Smulders et al. (2012) discuss the consumption-saving trade-off faced by the households.

Despite the negative announcement effect, an optimal emissions tax implemented with a lag is still desirable if the overall welfare effect of the policy is positive. Figure 3 plots the

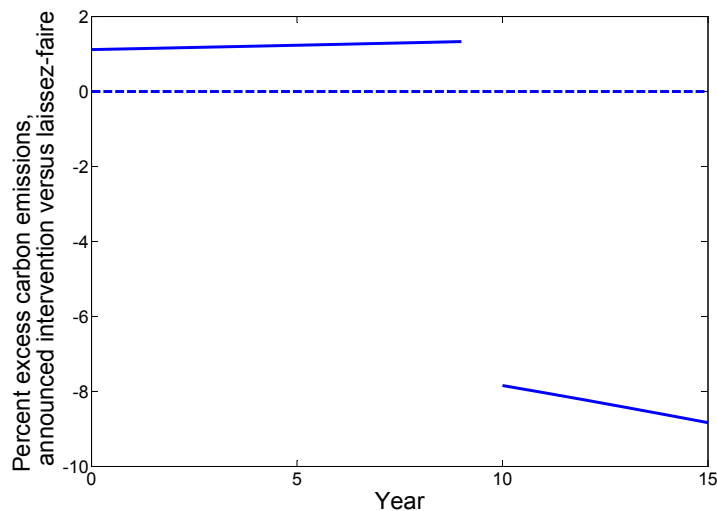


Figure 2: Emissions: Laissez-faire vs. announcement period of 10 years.

welfare gain of the (delayed) intervention compared to no-intervention over the length of the implementation lag. A positive number indicates a welfare gain, a negative number indicates a welfare loss, which implies that, in welfare terms, no intervention would be better. For an announcement period of zero, the figure displays the welfare gain of the optimal policy. We find that the overall welfare effect depends on the length of the announcement period. It is positive for small periods, negative for intermediate periods, and approaches zero in the very long term. A ‘critical lag’ for favorable, i.e. welfare-enhancing, climate policy arises. It is defined such that a shorter implementation lag is welfare-increasing compared to no-intervention, but a longer implementation lag is welfare-reducing. The critical lag is 69 years. If one looks at the Kyoto-Protocol with an implementation lag of 11 years, the critical lag is long enough not to make policy-makers worry about the adverse announcement effect impeding lagged policy implementation. Still, as Figure 3 shows, earlier implementation is a welfare gain compared to later implementation.

With an instantaneously implemented optimal emissions tax, one gains a relative intertemporal equivalent variation of  $h = 0.0353 \%$  over laissez-faire corresponding to an absolute intertemporal equivalent variation of  $W = 463.4 \times 10^9 \text{ \$}$ , a share of 1.02 % of 2005 GDP. The gain from optimal climate policy is small, a typical result especially in globally integrated assessment models, as positive and negative regional effects cancel each other out.<sup>16</sup> The in-

<sup>16</sup>The relative equivalent variation of optimal policy as compared to laissez-faire in the DICE model 2007 is larger than our welfare result due to differences in resource scarcity. The DICE model does not model resource extraction but places an upper limit on carbon emissions. The carbon limit in the DICE model 2007 is 6000 GtC, higher than in our model. Also, in our model extraction costs go up rapidly with ongoing exploitation. If carbon is scarce, markets tend to economize its use, which turns out to be good for the climate. Simulations with a reduced carbon limit in the DICE model 2007 GAMS version (see Nordhaus (2008a)) support our argument

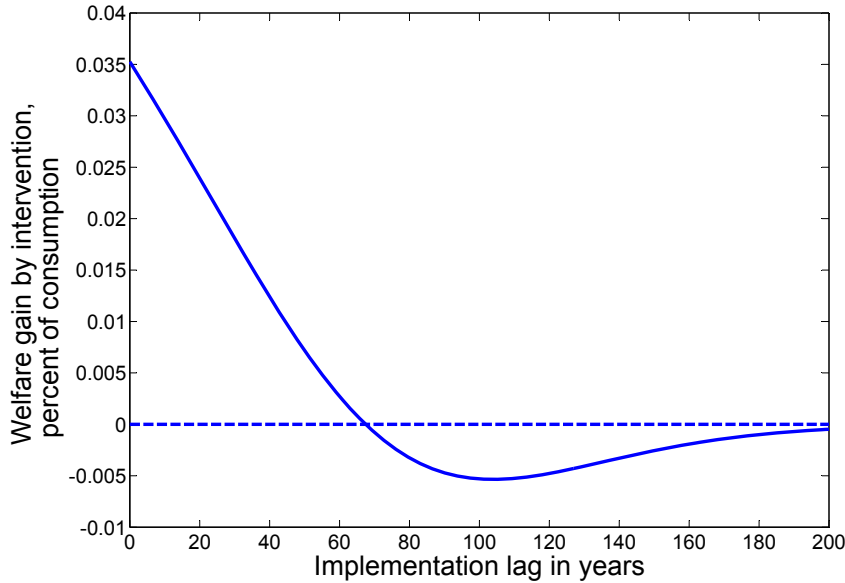


Figure 3: Critical lag: the welfare effect of lagged policy implementation depending on the length of the implementation lag.

clusion of threshold effects would almost certainly increase the welfare effect of optimal climate policy.

Figure 4 shows the optimal specific tax in  $\$/tC$  for the period 2005-2100 and the period 2005-2015 on the left and the right panel, respectively. The optimal specific tax increases over time. Written as value added tax, it initially increases and eventually decreases. The right panel also shows the path of the tax implemented with a lag of 15 years. It shows that the tax implemented with a time lag lies slightly above the tax that was immediately implemented.

## 5 Robustness

The aim of this paper is to evaluate the adverse announcement effect and to determine a critical lag for climate policy. Accordingly, the sensitivity analysis focuses on model and parameter choices that may affect the timing of emissions and the effect of that timing on welfare. Before considering changes in parameter and initial stock values, we begin by considering the effect of an available backstop technology. Then we discuss the effect of the model's characteristics on the critical lag.

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that resource scarcity explains the differences in the welfare effects of optimal climate policy. Thus, resource scarcity means less room for climate policy to enhance welfare. This result is in line with Golosov et al. (2014), who compare the effect of an optimal tax on scarce oil and abundant coal, relative to the laissez-faire allocation. They find that the effect of the tax is large on coal use, but small on oil use.

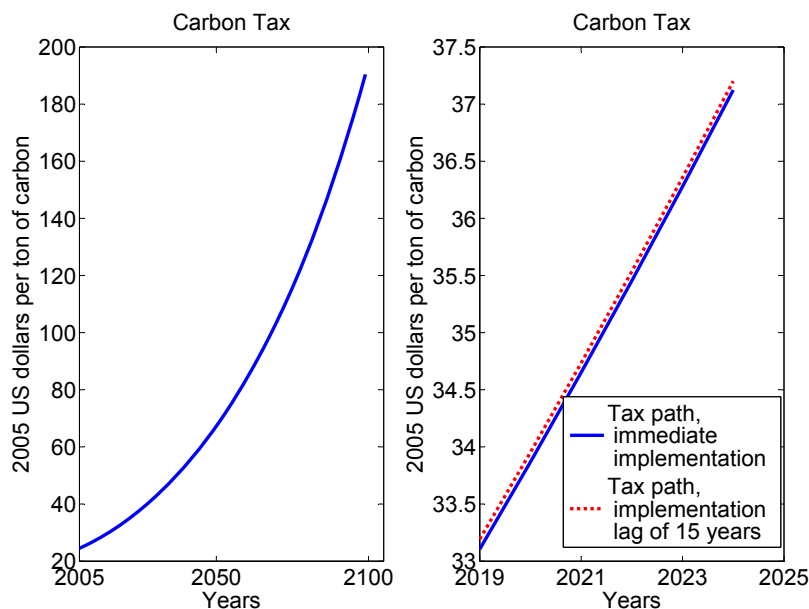


Figure 4: Optimal specific tax.

## 5.1 A Backstop Technology

So far, the model does not allow for a backstop technology that would make it possible to switch to carbon-free production if the resource price becomes too high. A backstop technology enlarges the intervention possibilities, as climate policy can now affect not only the timing of emissions but also the total amount emitted by inducing a switch to the backstop. The introduction of a backstop technology would thus lead to a larger welfare gain from an instantaneously implemented emissions tax and could also make a difference to the critical lag.

Contrary to the model presented earlier—the ‘baseline’ model—, firms now produce output by using ‘Energy’  $N$  instead of using the resource directly. Energy can be produced in three ways, (1) by transforming the resource to energy one-to-one, (2) by combining the resource and the output good as inputs to energy production, or (3) by exclusively transforming the output good into energy. Using output to produce energy inputs—i.e. using a non-polluting, non-exhaustible input to produce energy—represents the ‘backstop technology’ (as e.g. in van der Ploeg (2013), Hoel and Kverndokk (1996), and Tahvonen (1997)). Unit costs of producing the energy input from output are constant in terms of output, but high. Marginal extraction costs of the resource rise with the decreasing resource stock, so that a switch to the backstop occurs before the resource stock is physically depleted. Alternatively, one could model the backstop technology with decreasing marginal costs. The point in time when both ways to produce energy have equal marginal costs is what matters most.

Firms choose the minimal-cost technology. If the resource price, including tax, is low

enough, they will choose the first option. In a medium range, they will choose the second. If the price is high enough, they will opt for the third.

The respective energy production function is constant returns to scale with unit cost function  $g(\tilde{q})$ , where  $\tilde{q} := q + r$  denotes the resource price including tax, if there is any. The price of the other input (the output good) is suppressed, as it equals one (the output good is the numéraire). The inputs per unit of energy are thus  $d_1 = g'$  and  $d_2 = g - g'\tilde{q}$  for the resource and the output good, respectively.

Inputs are non-perfect substitutes. But differently from common specifications like CES, Inada is supposed not to hold. If  $\tilde{q}$  is below a certain lower bound  $\underline{q}$ , only the resource is used. To make this case comparable to the baseline specification in Section 2, we assume this to be the case in the initial situation, i.e.  $\underline{q} > \tilde{q}(0)$ . If, on the other hand,  $\tilde{q}$  is higher than a certain upper bound  $\bar{q}$ , the output good alone is used; this is the pure backstop case. Finally, in the intermediate case, both inputs are combined.

Formally, let  $g(\tilde{q})$  be continuously differentiable with

$$\begin{aligned} d_1(\tilde{q}) = 1 \text{ and } d_2(\tilde{q}) = 0 & \quad \text{for } \tilde{q} \leq \underline{q}, \\ g(\tilde{q}) \text{ quadratic in } \tilde{q} & \quad \text{for } \underline{q} < \tilde{q} < \bar{q}, \\ d_1(\tilde{q}) = 0 & \quad \text{for } \bar{q} \leq \tilde{q}. \end{aligned}$$

This uniquely specifies the unit cost function. Figure 5 shows the resulting unit costs ( $g(\tilde{q})$ ) as well as the inputs per unit of energy ( $d_1(\tilde{q})$  and  $d_2(\tilde{q})$ ).

To introduce the backstop into the baseline model and to solve it, one has to modify some equations, calibrate the parameters describing the backstop technology, and take into account the fact that steady-state behavior differs from the baseline model.

With the new technology, the production balance of the economy changes to

$$\dot{K} = Y + rN - \delta K - Ng(\tilde{q}) - C.$$

In addition, (3) changes to

$$g(\tilde{q}) = \beta Y/N. \tag{31}$$

The decentralized market is will fulfill the optimality conditions if the tax rate is

$$r = -P_{M_1}/P_K, \tag{32}$$

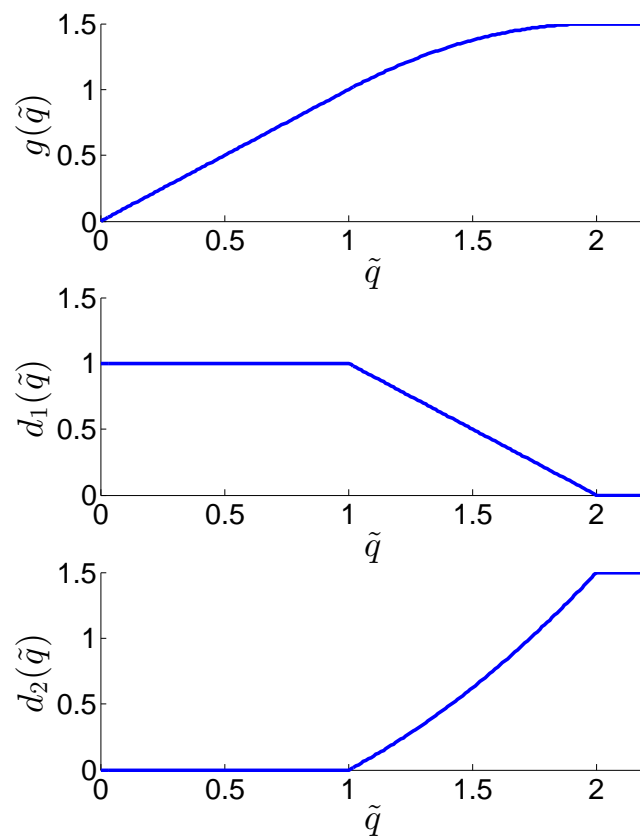


Figure 5: Illustration of the unit costs ( $g(\tilde{q})$ ) as well as the inputs per unit of energy ( $d_1(\tilde{q})$  and  $d_2(\tilde{q})$ ), with  $\underline{q} = 1$  and  $\bar{q} = 2$ .

as before.

The calibration of the model version with a backstop technology follows the baseline set-up. In addition, based on Popp (2006) and Nordhaus (2008b), the carbon prices limiting the transition phase to the backstop are chosen to be  $\underline{q} = 2.5q(0)$  and  $\bar{q} = 3.5q(0)$ , respectively. The resulting backstop cost per unit of energy in the pure backstop case (i.e. for  $\tilde{q} > \bar{q}$ ) is  $(\bar{q} + \underline{q})/2 = 3\bar{q}(0)$ .

The long-term behavior of the economy in a model with backstop technology differs from the baseline version, as the switch to the backstop technology leads to  $R = -\dot{S} = 0$  in the steady state. The resource loses its economic value,  $p = 0$ . The economic module reduces to the well-known Ramsey model. The Ramsey model becomes stationary when written in terms of ‘per effective labor’. As before, the climate module is stationary in nature.

How does allowing for a backstop technology change the previous results? The column ‘Backstop’ in Table 3 displays welfare results for a lagged implemented policy measure compared to no-intervention for different time-lags. As expected, the welfare effect of an instantaneous implementation is larger in the case with a backstop technology than in the model without backstop ( $h=0.0499$  % compared to  $0.0353$  % in the baseline case). Also, the introduction of a backstop technology leads to a larger critical lag. Unlike the baseline case, for an implementation lag of 70 years, the welfare effect is still positive compared to no-intervention.

Table 3: Overview welfare results ( $h$  in %) for different implementation lags and two backstop specifications

Lag in years	$h$ in %	
	Backstop	Later backstop
0	0.0499	0.0445
50	0.0240	0.0190
70	0.0153	0.0111

The backstop is defined with  $\underline{q} = 2.5q(0)$  and  $\bar{q} = 3.5q(0)$ . The later backstop is defined with  $\underline{q} = 3q(0)$  and  $\bar{q} = 4q(0)$ .

The column ‘Later Backstop’ in Table 3 presents results for a ‘later’ backstop in the sense that the backstop is more expensive und therefore only becomes profitable at a later point in time. We use  $\underline{q} = 3q(0)$  and  $\bar{q} = 4q(0)$  (compared to  $\underline{q} = 2.5q(0)$  and  $\bar{q} = 3.5q(0)$ ). We see that welfare results are lower in the second case. This indicates that the more expensive the backstop technology and the later it becomes profitable, the closer the results are to the baseline case without backstop technology.



## 5.2 Sensitivity analysis of parameter values

The main concern is that the critical lag is too short. To better identify factors that reduce its length, we introduce parameter variation within plausible ranges in a direction that shortens the critical lag. If the critical lag is increasing (decreasing) in a parameter, we reduce (raise) it to an acceptable lower (upper) bound. First, we modify one parameter at a time, leaving the other parameter values as in the benchmark. Second, we change several parameters to construct a case that is most unfavorable for our claim that the lag is long enough. Third, we also discuss the effect of different initial resource and capital stocks.

Table 4 shows the parameter variations and results in terms of  $h$  and the critical lag. Column 3 indicates whether the critical lag is increasing ('+') or decreasing ('-') in the respective parameter. We do not vary the depreciation rate, as its impact on the critical lag is negligible.<sup>17</sup> The same holds for the forcing parameters that represent climate sensitivity. Even though climate sensitivity is uncertain (Meinshausen et al. 2009), it seems to have little impact on the critical lag.

Table 4: Sensitivity analysis w.r.t. parameter values

Parameter	Baseline Value	Effect on Critical Lag	Value for Robustness	$h$ (in %)	Critical Lag
$\theta$	1.5	+	1.1	0.0720	64
$\rho$	0.015	+	0.001	0.1274	58
$g$	0.018	-	0.037	0.0233	48
$\epsilon$	1	+	0	0.0512	64
$\omega$	1	+	0.8	0.0228	69
$\eta_1$	0.1206	+	0.0600	0.0033	67

$\rho, g, \delta$  are measured in  $1/\mathbf{a}$ ,  $\omega$  scales damages  $\Omega$ ,  $\theta, \epsilon, \omega$  are unitless.

The sensitivity analysis encompasses parameters that influence discounting: time-preference parameter  $\rho$ , elasticity of intertemporal substitution  $1/\theta$ , steady-state growth rate  $g$  via the growth rate of effective labor  $\hat{E}$  and extraction costs (via  $\epsilon$ ). Furthermore, we include parameters of the damage function. The current calibration gives 0.024% output loss if the surface temperature increases by 2°C from now, which seems tiny. Moreover, we discuss the impact of climate sensitivity. This parameter translates accumulated emissions in the atmosphere into temperature increase.

The effect of the inverse of the elasticity of intertemporal substitution  $\theta$  on the critical lag is positive, so we take its lower bound for the sensitivity analysis. As consumers usually cannot defer consumption through time without incurring costs, we take the elasticity of intertemporal

<sup>17</sup>The depreciation rate  $\delta$  shows no effect on the critical lag for a range of 0.04  $1/\mathbf{a}$  and 0.1  $1/\mathbf{a}$ , values considered in Gerlagh and van der Zwaan (2004) and Nordhaus (2010).

substitution to be below one (as e.g. found by Attanasio and Weber (1995)) and set  $\theta = 1.1$  as the lower bound value. For the time preference rate, we take the ‘Stern’ Value (Gerlagh and Liski 2012) as lower bound,  $\rho = 0.001$  1/a. For the growth rate of the economy, which can be calibrated by adapting the growth rate of effective labor  $\hat{E}$ , we use the upper bound value reported in Nakicenovic and Swart (2000),  $g = 0.037$  1/a. In most cases, these value choices are within the ranges reported in Drupp et al. (2015).

For the extraction cost curve, we decrease  $\epsilon$  to  $\epsilon = 0$ . This represents constant extraction costs. The rent component becomes high and there may be more to gain from re-allocating extraction over time. Still, we find that the impact on the critical lag is moderate.

To examine the effect of the damage function on the critical lag, we scale the damage term with  $\omega$  according to  $(1 + \omega(\xi_1 T_1 + \xi_2 T_1^{\xi_3}))^{-1}$ . The effect on the critical lag is small. For  $\omega = 0.8$ , there is no effect. For  $\omega = 2$ , the critical lag slightly increases to 70 years. Making our model reproduce—in 2005—estimates on the social cost of carbon such as 90 \$/tC requires inflating the damage term much more. To obtain 90\$/tC (Watkiss 2011), one would have to choose a scaler as high as  $\omega \approx 5$ .

If the parameter  $\eta_1$  that represents climate sensitivity is halved, the gain of optimal climate policy is greatly reduced, but the critical lag is barely affected. A larger climate sensitivity is more detrimental for welfare.

The results in Table 4 indicate that, *ceteris paribus*, none of the changes in parameter values shorten the critical lag to any serious extent. A combination of  $\theta = 1.1$ ,  $\rho = 0.001$ , and  $g = 0.037$  gives  $h = 0.224\%$  and a critical lag of 38 years.

Besides parameter values, different values for the initial stocks may affect results, especially as, for the initial capital stock, there are no reliable data available, and for the initial resource stock, several definitions—and accordingly, different values—are available.

The calibration based on the steady-state capital-output ratio leads to  $K(0) = 154.6 \times 10^{12}$  \$. Reducing this value by roughly one third, to  $K(0) = 100 \times 10^{12}$  \$, gives  $h = 0.0375\%$  and a critical lag of 63 years.

As for the initial resource stock, several definitions related to different stock estimates are available. In the baseline case, the initial resource stock matches the carbon resource base, i.e. the sum of reserves and resources. To analyze the effect of a lower initial resource stock on the critical lag—as a lower stock reduces the lag—, we match the initial resource stock to the carbon reserves,  $S(0) = 750$  CtC.<sup>18</sup> We find  $h=0.0061\%$  and a critical lag of 18 years.

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<sup>18</sup>The German *Federal Institute for Geosciences and Natural Resources* (Bundesanstalt für Geowissenschaften und Rohstoffe (BGR) 2013) reports estimates of 718 GtC, McGlade and Ekins (2015) report estimates of 790 GtC.

Since estimates of shale oil and shale gas are still not very good (World Energy Council 2010), one could argue in favor not of a smaller, but of an even larger initial resource stock than in our baseline calibration. A larger resource stock would be in line with the DICE model's upper bound of 6000 GtC on emissions. Including unconventional sources, the German *Federal Institute for Geosciences and Natural Resources* (Bundesanstalt für Geowissenschaften und Rohstoffe (BGR) 2013) estimates a resource base even larger than this.

So far, we found only one case suggesting an implementation lag so short that doing-nothing instead of running the risk of a welfare loss due to the announcement effect may be the better choice, namely the case of a small initial resource stock of only 750 GtC. The impact of any policy intervention is, however, small in this scenario, because firms tend to economize on the resource use and the climate is not threatened severely anyway. But there is little evidence supporting this scenario. The climate problem is caused not by too little but by too much in the way of carbon resources.

### 5.3 Possible model extensions

In the following, we discuss the major simplifications of the model design and how they impact the critical lag.

One might ask whether CCS, if allowed, would be chosen as an alternative to the backstop technology at some period in time. It would only be chosen if the tax exceeds CCS costs. Hardisty et al. (2011) find CCS not to be economically or socially viable for marginal external costs of CO<sub>2</sub> emission below 100 \$ per ton CO<sub>2</sub>, corresponding to 0.367 \$/kgC, excluding internal and external costs of transport and storage. If we take this as a cost estimate, we see that our optimal tax rates in the model with backstop technology come close to it shortly before the economy completely switches to the backstop. But one must also take into account the fact that with a viable CCS option rents would be larger, making the backstop more attractive. It is thus unlikely that introducing a CCS option would make a big difference.

The model's resource sector is a simplified representation of natural resource extraction. It only considers extraction costs and abstracts from upfront investments. Cairns (2014) argues that taking account of technological and geological features of oil production—like upfront investment in productive capacity—makes a green paradox less likely. Including these features in our model would delay the critical lag even further.

Furthermore, we use a global model for the analysis that does not include trade between countries and thus does not consider carbon leakage. Compared to a model with several identical countries, where only some of them announce an emissions tax, a global model over-

estimates the role of the adverse announcement effect. The reason is that if not all emitting countries announce an emissions tax the adverse announcement effect is less severe for several countries, as only a part of the overall emissions will be restricted. Thus with a global model, we are on the conservative side, with a tendency to overestimate the adverse announcement effect and to underestimate the critical lag.

One could argue that the possibility to implement the *optimal* climate tax at the point in time when implementation becomes possible is overly optimistic. Still, there is no reason why a tax will necessary be too low. It could also be too high, especially, as it indirectly depends on the time preference rate.

What may reduce the critical lag is the ‘ordering effect’ described by Di Maria et al. (2012). Considering several resources that differ in their carbon content, the announcement of, say, an emissions tax may induce resource owners to use the resource with the higher carbon content first, which further pushes up emissions during the implementation lag. This effect may be weakened by the investments necessary to switch fuels. Di Maria et al. (2014b), for example, provide evidence that many energy generators did not switch to dirtier coal. We leave the detailed analysis of this case to future research.

An important topic in climate change modelling is uncertainty. We partly address this topic in the section on robustness. Still, the model remains deterministic in nature and risk preferences are not modelled in detail. We also leave this to future analysis.

## 6 Conclusion

This paper focuses on the evaluation of the overall welfare effect of announced climate policy measures. The Kyoto Protocol is a prominent example. Between agreement in 1997 and implementation in 2008, there was an announcement period of 11 years. If a climate policy measure—say, an emissions tax—cannot be implemented immediately due to political and bureaucratic procedures, resource owners will in the meantime anticipate future emissions restrictions and increase extraction. This negative effect is well established in the literature. Should the policy still be implemented or not? It is not clear a priori whether the reduction in emissions occurring with tax implementation is able to overcompensate the emissions that have increased during the announcement period.

In our model set-up, we confirm previous findings indicating that an announcement effect—a behavior change prior to tax implementation—does exist and that this effect is quantitatively relevant for the overall welfare effect in integrated assessment models. Assuming that imme-

diate implementation is impossible, we take the analysis one step further and ask how far one could postpone the implementation of the tax without turning welfare effects into the negative. Such an assessment should help guide policy-makers. Our simulations show that, in most scenarios, the critical lag is around 60 years. This is a conservative estimate in the sense that—whenever possible—we made choices that would lead to a large announcement effect and a short critical lag. We identify resource scarcity as the critical factor for a very short critical lag. Still, a large resource stock is more likely. A lower time preference rate and higher productivity growth also reduce the critical lag. The impact of changing parameters related to damages and climate sensitivity on the critical lag is low. With a backstop technology, the critical lag is larger. At all events, earlier implementation leads to a higher welfare effect than later implementation.

For the analysis we include forward-looking resource owners and climate dynamics based on the carbon cycle in a global, forward-looking model. This distinguishes our model from many other global, forward-looking integrated assessment models. Furthermore, we propose a new way of including a backstop technology in integrated assessment models, which allows the simultaneous use of carbon and of the backstop to produce the energy input (as in integrated assessment models) and a complete switch to the backstop for energy production (as in most theoretical models).

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## A Tables

Table 5: Overview integrated assessment models.

Model	Backstop	Resource Scarcity	Climate Dynamics	References
MIND	‘additive aggregation’ of fossil, renewable and trad. non-fossil energy*	based on costs	model, see Kriegler and Bruckner (2004)	Edenhofer (2006)
DEMETER	CES aggregation of fossil fuels and backstop	no constraint	1-box representation as in the early DICE model	Gerlagh and van der Zwaan (2004)
DICE	yes**	constraint on emissions	carbon cycle	Nordhaus (2008b)
ENTICE-BR	CES aggregation of fossil fuels, backstop and energy efficiency knowledge	based on costs	carbon cycle based on DICE	Popp (2006)
Golosov/ Hassler/ Krusell/ Tsyvinski	no	Hotelling approach	linear representation based on RICE	Golosov et al. (2014)
This model	three phases, pure backstop use in last phase	Hotelling approach	carbon cycle based on DICE	

Based on the overview of Stanton et al. (2009) in *Climate and Development*. We have included global, forward-looking (classified as ‘welfare maximization’) models, plus the model presented in Golosov et al. (2014).

\* A CES aggregation of specific capital and fossil fuels is used.

\*\* DICE: ‘The backstop technology is introduced into the model by setting the time path of the parameters in the abatement-cost equation, so that the marginal cost of abatement at a control rate of 100 percent is equal to the backstop price for each year’ (Nordhaus 2008b, p. 24). As resource input is not modelled explicitly, it is not so easy to interpret the switch away from resource use in terms of a backstop technology.

Table 6: Overview units.

Abbreviation	Explanation
\$	2005 US dollars
GtC	gigatons carbon
kgC	kilograms carbon
Gtoe	gigatons oil equivalent
kgoe	kilograms oil equivalent
a	annum
°C	degrees Celsius
$\Delta^\circ\text{C from 1900}$	degrees Celsius difference from temperature level in the year 1900

Note that  $\$/\text{kgC} = 10^{12}\$/\text{GtC}$ .

## B The Household's Optimization Problem

The household chooses consumption  $C$  to maximize utility

$$U = \int_0^{\infty} u(C)e^{-\rho\tau} d\tau$$

subject to the budget constraint, with time-preference rate  $\rho$ . As usual,  $u$  has the constant intertemporal elasticity of substitution form

$$u = \frac{C^{1-\theta}}{1-\theta},$$

with intertemporal elasticity of substitution  $1/\theta$ . The flow budget constraint is

$$\dot{a} = \iota a + (1 - \alpha - \beta)Y + rR - C, \quad (33)$$

stating that saving  $\dot{a}$  equals income minus consumption. Income has three components: interest on the asset, labor income, and tax income collected by the state and paid to the household. The market prevents chain-letter credit financing so that for  $t \rightarrow \infty$ , the present value of the asset must be non-negative (Barro and Sala-i Martin 2004, p. 92),

$$\lim_{t \rightarrow \infty} D(t)a(t) \geq 0.$$

The optimality conditions are the Keynes-Ramsey Rule  $\hat{C} = (\iota - \rho)/\theta$  and the transversality condition

$$\lim_{t \rightarrow \infty} D(t)a(t) = 0. \quad (34)$$

It is convenient to write the budget constraint (33) in a different equivalent way by inserting (2), (3), and (10),

$$\dot{K} + \dot{v} = \iota(K + v) + Y - (\iota + \delta)K - qR - C. \quad (35)$$

Taking the time derivative of (4), using (6) yields

$$\dot{v} = \iota v - R(q - k(S)). \quad (36)$$

Substituting this for  $\dot{v}$  in (35), we obtain the production balance of the economy (11),

$$\dot{K} = Y - \delta K - Rk(S) - C.$$

As both components of  $D(t)a(t) = D(t)K(t) + D(t)v(t)$  are non-negative, for (34) to hold, both components have to approach zero as  $t \rightarrow \infty$ . The transversality condition (34) thus implies (13)

$$\lim_{t \rightarrow \infty} D(t)K(t) = 0.$$

## C The Optimal Tax

To see that

$$r = -\frac{P_{M_1}}{P_K}$$

in the decentralized market leads to the optimal allocation, do the following. Insert  $BD$  for  $P_K$  and  $\hat{D}$  for  $\hat{P}_K$ ,  $-r$  for  $P_{M_1}/P_K$  and  $P_K p$  for  $P_S$ . Then (20) becomes (6) with  $\iota$  from (2), (19) becomes (14), and (18) becomes (3) with  $q = P_S/P_K + k(S)P_K$ . Inserting  $p = P_S/P_K$  into (8) leads to (21). Equations (13) and (9) are the transversality conditions in (24) for  $K$  and  $S$ , respectively.

## D Calibration

Table 7: Overview variables

Symbol	Explanation	Value	Unit	Source
$C$	consumption		$10^{12}$ \$/a	endogenous
$\nu$ in 2005	emissions from land use change	1.1	GtC/a	Nordhaus (2008b), see Note 1
$R$ in 2005	extracted carbon resource flow = emissions	7.7578	GtC/a	EIA (2010), IEA (2007), see Note 2
$M_1$ in 1750	carbon mass in atmosphere	596.4	-GtC	Nordhaus (2008b), Nordhaus (2010)
$M_1$ in 2005	carbon mass in atmosphere	808	GtC	Nordhaus (2010)
$M_2$ in 2005	carbon mass in upper ocean	1600	GtC	Nordhaus (2010)
$M_3$ in 2005	carbon mass in lower ocean	10010	GtC	Nordhaus (2010)
$P_M$	shadow prices of carbon stocks		utils/GtC	endogenous
$P_T$	shadow prices of temperatures		utils/ $\Delta^\circ\text{C}$ from 1900	endogenous
$P_K$	shadow price of capital stock		utils/ $10^{12}$ \$	endogenous
$P_S$	shadow price of resource stock		utils/GtC	endogenous
$q$ in 2005	resource flow price	0.3415	\$/kgC	EIA (2010), see Note 3

$i^*$	steady-state interest rate	0.042	1/a	see Note 4
$S$ in 2005	resource stock	3000	GtC	McGlade and Ekins (2015); UNDP (2000)
$T_1$ in 2005	global mean surface temperature	0.83	$\Delta^\circ\text{C}$ from 1900	Nordhaus (2010)
$T_2$ in 2005	temperature lower ocean	0.0068	$\Delta^\circ\text{C}$ from 1900	Nordhaus (2010)
$U$	overall utility		present value utils	endogenous
$u$	instantaneous utility		utils/a	endogenous
$Y$ in 2005	GDP	45.23	$10^{12}\$/\text{a}$	World Bank (2009)
$r$	tax rate		$\$/\text{kgC}$	endogenous
$h$	relative intertemporal equivalent variation		1	endogenous
$W$	absolute intertemp. equiv. variation		$10^{12}\$$	endogenous

Table 8: Parameters taken from literature

Symbol	Explanation	Value	Unit	Source
$\Gamma$	carbon transition matrix	$\begin{bmatrix} -0.0120 & 0.0047 & 0 \\ 0.0120 & -0.0052 & 0.0001 \\ 0 & 0.0005 & -0.0001 \end{bmatrix}$	1/a	Nordhaus (2010), see Note 5
$\delta$	depreciation rate	0.05	1/a	Barro and Sala-i Martin (2004)
$\alpha$	income fraction capital	0.314	1	$(1 - \beta)/3$ , Barro and Sala-i Martin (2004, p. 58)

$\theta$	inverse of the intertemporal elasticity of substitution	1.5	1	Nordhaus (2010)
$\Lambda$	temperature interaction matrix	$\begin{bmatrix} -0.0345 & 0.0066 \\ 0.0050 & -0.0050 \end{bmatrix}$	1/a	Nordhaus (2010), see Note 6
$\rho$	time preference rate	0.015	1/a	Nordhaus (2010)
$\underline{q}, \bar{q}$	prices limiting the transition phase to the backstop	$2.5q(0), 3.5q(0)$	1	based on Popp (2006) and Nordhaus (2008b)

Table 9: Functions

Symbol	Explanation	Value	Unit	Source
$\Pi(M_1, t)$	radiative forcing	$\eta_1 \log\left(\frac{M_1(t)}{M_1 \text{ in } 1750}\right) + \eta_2 F_{EX}$	$^{\circ}\text{C/a}$	Nordhaus (2010), see Note 8
$\Omega(T_1)$	damage function	$\frac{b}{1 + \omega\xi_1 T_1 + \omega\xi_2 T_1^{\xi_3}}$		Nordhaus (2008b), see Note 9
$k(S)$	extraction cost function	$\gamma S^{-\epsilon}$	$\$/\text{kgC}$	see Section 3

Table 10: Calibrated parameters

Symbol	Explanation	Value	Unit	Source
$K$ in 2005	capital stock		$10^{12}\$$	$K(2005) = \alpha Y(2005)/(\iota^* + \delta)$
$\beta$	income fraction resource	0.058	1	Calibration: $\frac{q_{2005} R_{2005}}{Y_{2005}}$ , see Note 2
$\hat{E}$	growth rate of effective labor	0.029	1/a	Barro and Sala-i Martin (2004) (page 13), see Note 7
$\gamma$	scaler extraction costs			see Section 3
$\epsilon$	exponent extraction costs	1	1	see Section 3

## Notes

1. Total emissions in GtC/a are  $R + \nu$ , with  $\nu = \nu(0)e^{-.0101t}$ . The value of carbon emissions from land-use change in  $t = 0$ , i.e. in 2005 is 1.1 GtC/a (Nordhaus 2008b). Division by 10 adjusts the original value 11 GtC from **per decade** to **per year**. The yearly shrinking rate of the emissions from land-use change is also taken from Nordhaus (2008b) and transformed into its continuous time yearly counterpart, i.e. from  $(1 - 0.1)^{t-1}$  to  $\log(1 - 0.01) = -.0101$ .
2. Data on resource flows and stocks as well as on emissions are based on IEA (2007). Because the model only uses one carbon resource, we merge the data on oil, coal, and gas and convert units into gigatons of carbon, GtC. We use emissions in 2005 over  $R$  in 2005 to calculate a conversion factor.
3. The resource price  $q$  in 2005 is a weighted average of the average 2005 prices for oil, steam coal, and natural gas in \$/kgoe, 0.3562, 0.0964 and 0.3109, respectively. The coal and gas prices are roughly estimated average prices 2005 from IEA (2007), while the oil price is the average of the ‘Weekly All Countries Spot Price FOB Weighted by Estimated Export Volume’ (EIA 2010). We take the shares of each resource in total resource consumption 2005 as weights and convert the units into \$/kgC. The calculation yields  $q = 0.3415$  in \$/kgC.
4. The steady-state interest rate calculated in the model is  $\iota^* = \rho + \theta g = 4.2\% \text{ 1/a}$ . This is close to the average interest rate of the countries listed in Inter-Agency Group on Economic and Financial Statistics (IAG) (2010). Barro (1987) finds an average interest rate of 3.45 % 1/a for the UK for 1729-1918, Eurostat (2011) states an average interest rate of 4.33 % 1/a for 2001-2010, Mendoza (1991) and Oviedo (2005) use 4 % 1/a as a world interest rate, but Oviedo (2005) cites studies that use an interest rate of up to 14.9 % 1/a. The interest rate in the DICE model is 5-6 % 1/a. Thus our model yields a sensible steady-state interest rate.
5. We calculate the carbon transition matrix according to  $\Gamma = \frac{B_1 - I}{10}$ , using its discrete time counterpart, say  $B_1$ , in Nordhaus (2010), with

$$B_1 = \begin{bmatrix} -0.0120 & 0.0047 & 0 \\ 0.0120 & -0.0052 & 0.0001 \\ 0 & 0.0005 & -0.0001 \end{bmatrix},$$

and the identity matrix  $I$ .  $B_1 - I$  is divided by 10 because one period in the DICE model corresponds to a decade.

6. We calculate the temperature interaction matrix according to  $\Lambda = \frac{B_2 - I}{10}$ , using its discrete time counterpart, say  $B_2$ , in Nordhaus (2010), with

$$B_2 = \begin{bmatrix} .6553 & .0660 \\ .05 & 0.95 \end{bmatrix}.$$

As before, we divide by 10 because one period in the DICE model corresponds to a decade.



7. The rate of technological progress is calibrated to match a steady-state growth rate of 1.8 % 1/a. Barro and Sala-i Martin (2004) (page 13) find an average per-capita growth rate of 1.9% 1/a for major developed countries over a century. For  $g = 0.018 = \frac{(1-\alpha-\beta)\hat{E}-\rho\beta}{1-\alpha-\beta+\beta\theta}$  based on  $\hat{q}^* = \iota^*$ , meaning that  $k(S)$  grows more slowly than  $p$  in the long term, the rate of technological progress is  $\lambda = 0.018/(1-\alpha-\beta)(1-\alpha-\beta+\beta\theta) + \rho\beta = 2.19\% \text{ 1/a}$ .
8.  $F_{EX}$  is the exogenous radiative forcing in **watt/m<sup>2</sup>**. It increases linearly from -0.06 to 0.3 **watt/m<sup>2</sup>** within 100 years and stays constant afterwards. The parameter values  $\eta_1 = 0.1206 \text{ °C/a}$  and  $\eta_2 = 0.022 \text{ (°C/a)(m}^2\text{/watts)}$  are based on Nordhaus (2008b, 2010) and adjusted to the time scale, i.e. the value given by Nordhaus divided by 10.
9. We take the parameters  $\xi_1 = 0.0018$ ,  $\xi_2 = 0.0023$  and  $\xi_3 = 2$  from Nordhaus (2010) and use the scaler  $b$  to ensure  $Y(0) = \bar{Y}$ . The parameter  $\omega$  serves for sensitivity analysis and equals one if not specified otherwise.

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