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**Working Paper**

## Destroying Surplus and Buying Time in Unanimity Bargaining

Economics Working Paper Series, No. 16/248

**Provided in Cooperation with:**

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*Suggested Citation:* Britz, Volker (2016) : Destroying Surplus and Buying Time in Unanimity Bargaining, Economics Working Paper Series, No. 16/248, ETH Zurich, CER-ETH - Center of Economic Research, Zurich,  
<https://doi.org/10.3929/ethz-a-010646708>

This Version is available at:

<https://hdl.handle.net/10419/171691>

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Working Paper 16/248  
May 2016

Economics Working Paper Series



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# Destroying Surplus and Buying Time in Unanimity Bargaining

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This version, May 2016

## Abstract

In non-cooperative bargaining games in the tradition of Rubinstein, the proposer derives bargaining power from the prospect of a costly delay which would follow the rejection of a proposal. We consider a unanimity bargaining game in which the proposer can strategically choose to prolong this delay. Prolonging the delay increases the proposer's bargaining power, but is assumed to come at a cost and thus cause an inefficiency. We use an appropriate refinement of stationary subgame-perfect equilibrium as the solution concept. We characterize equilibrium strategies and payoffs. We establish conditions on model parameters under which equilibrium is or is not efficient. For inefficient equilibria, we quantify the extent of the inefficiency. Moreover, we study the relation between the number of players and the degree of inefficiency. We find that inefficient equilibria become more inefficient the more players there are. Moreover, the parameter region in which an efficient equilibrium is possible shrinks when the number of players increases.

**Keywords:** Bargaining, Surplus Destruction, Discount Factor, Timing

**JEL Codes:** C72, C78

# 1 Introduction

We study a non-cooperative bargaining game for  $n$  players in which the proposer can choose to prolong the time lapse which occurs in the negotiation if her proposal is rejected. In order to impose a longer time lapse, a proposer must make some costly effort and thus destroy surplus. Hence, the proposer faces a trade-off between an increase in her bargaining power and a decrease in the size of the surplus.

Unanimity bargaining games in the tradition of Rubinstein (1982) take the following form: In each round, one player makes a proposal. Unless the proposal is unanimously accepted, a new round begins after a costly delay. Typically, one assumes that a payoff which occurs with a delay of one round is discounted by a fixed factor. The proposer derives bargaining power from the cost of delay expressed in this discount factor. Hence, in a unanimity bargaining game, a decision to reject a proposal effectively implies a decision to destroy some share of the surplus. Consequently, the proposer can appropriate the amount by which the surplus would shrink if her proposal was rejected. Indeed, Shaked and Sutton (1984) describe a player's equilibrium payoff as corresponding to the "sum of the shrinkages" of the surplus which occur in those rounds where the player is the proposer. That sum depends on the rate at which the surplus shrinks and on the frequency with which the bargaining protocol calls on a particular player to be the proposer. While the original Rubinstein game has been extended to an arbitrary number of players and various different bargaining protocols, these basic considerations have been confirmed as a standard result. Using stationary subgame-perfect equilibrium as the appropriate solution concept, variations of this result have been shown for many different bargaining protocols, see Britz, Herings, and Predtetchinski (2010, 2014), Kultti and Vartiainen (2010), Laruelle and Valenciano (2008), Miyakawa (2008).

In the present paper, we consider the following model: There are  $n$  players bargaining over a given surplus. In each bargaining round, one player is the proposer. She chooses which shares of the current surplus she offers to each player, and which share of the current surplus she destroys. If all players accept the shares offered to them, then the game ends. If no agreement is reached in the current bargaining round, then the next bargaining round takes place after a time lapse. The length of this time lapse increases in the share of the surplus which the proposer has destroyed. If the proposer has not destroyed any surplus, then the next bargaining round occurs after an exogenously fixed minimal time lapse, which may or may not be equal to zero. We assume that players are impatient and have a common rate of time preference. These modeling choices reflect the idea that the proposer can "commit" herself to her proposal and forestall any counter-proposal for some time, but entering into such a commitment is costly.

The bargaining protocol is as follows: Without loss of generality, we assume that the

first proposal is made by Player 1. Any subsequent proposal is made by the player who first rejected the previous proposal. Once a proposal has been made, players respond to the proposal sequentially in a fixed order. It is well-known that this responder order itself has no bearing on the analysis of equilibrium payoffs, see for example Britz et al. (2014). In our model, the proposer faces the following trade-off: She can “buy time” during which no alternative to her proposal can be put forward. This comes at the cost of destroying surplus which is to the detriment of all players. If she does “destroy surplus and buy time,” then due to impatience, this makes responding players more reluctant to reject a proposal, and thus enhances the proposer’s bargaining power.

Stationary subgame-perfect Nash equilibrium is the standard solution concept for  $n$ -player unanimity bargaining games. We adopt a slight technical refinement of this equilibrium concept, which we call the “bargaining equilibrium.”

Our main results are as follows: We provide a complete characterization of bargaining equilibria for any possible configuration of the basic model parameters. These parameters are the rate of time preference, the share of the surplus which has to be destroyed in order to gain one unit of extra time lapse, the exogenously fixed minimal time lapse, and the number of players. On the path of play of any bargaining equilibrium, agreement is reached immediately on the first proposal. For some values of basic model parameters, this agreement is efficient in the sense that the proposer finds it optimal not to destroy any surplus. For other parameter values, however, the equilibrium allocation is inefficient since the proposer does find it optimal to destroy surplus. A bargaining equilibrium always exists. Moreover, the bargaining equilibrium is unique except in the knife-edge case where the exogenously fixed minimal time lapse is equal to zero. In that case, there can be a multitude of efficient bargaining equilibria which support a range of allocations of the surplus. We show that the equilibrium level of surplus destruction vanishes in the limit as the rate of time preference becomes either sufficiently small or sufficiently large. For any given number of players, we establish a tight upper bound on the equilibrium level of surplus destruction. No such bound exists, however, if the number of players is taken arbitrarily large. In principle, the entire surplus could be destroyed in a bargaining equilibrium in the limit as the number of players grows without bound.

Many authors have considered bargaining models in which surplus destruction is not (only) an immediate consequence of a rejection of a proposal, but is an independent strategic decision by a player. For instance, Fernandez and Glazer (1991) and Haller and Holden (1990) model wage bargaining between a firm and a union where the union can decide to go on strike after an unsuccessful bargaining round. Haller and Holden (1990) point out the difference between surplus destruction, in this case a strike, as an immediate consequence of the rejection of a proposal, and surplus destruction as an independent strategic deci-

sion. Manzini (1999) considers a bargaining model with two players where one player can destroy some surplus when his proposal is rejected. The Rubinstein equilibrium persists, but also a new equilibrium arises which is favorable to that player. In Manzini (1996), partial surplus destruction occurs automatically (not as a strategic choice) after every other rejection. Houba (1997) interleaves the rounds of the bargaining process with the rounds of some infinitely repeated game so that every time a proposal is rejected, another round of the underlying game is played.

Avery and Zemsky (1994) and Busch et al. (1998) analyze a class of bargaining games in which, following the rejection of a proposal, a player can decide to destroy surplus. Due to a multiplicity of equilibria in these games, the proposer can make a credible threat to destroy surplus unless her proposal is accepted. In this way, she effectively decreases the other player's discount factor, and thus enhances her own bargaining power. Our model shares with Avery and Zemsky (1994) and Busch et al. (1998) the idea that the proposer uses surplus destruction in order to effectively reduce the responders' discount factor. One crucial difference is, however, that we allow the proposer to destroy surplus only in the beginning of a bargaining round without conditioning on a rejection. Thus, the multiplicity of equilibria which drives the results in Avery and Zemsky (1994) and Busch et al. (1998) does not play a role in our analysis. Another related paper is Yildirim (2007) who has players in a bargaining game exert costly effort in order to increase their probability of becoming a proposer, and thus their bargaining power.

The rest of the paper is organized as follows: Section 2 contains the formal description of the model and the equilibrium concept. A characterization of bargaining equilibria is derived in Section 3. An analysis of efficient bargaining equilibria is given in Section 4, while some results pertaining to bargaining equilibria with surplus destruction are displayed in Section 5. We quantify the equilibrium level of surplus destruction in Section 6, and we offer some concluding remarks in Section 7.

## 2 Model description

We consider a unanimity bargaining game with players  $N = \{1, \dots, n\}$  who are negotiating on the division of an exogenously given surplus. Bargaining proceeds in *rounds*; the first bargaining round takes place at some time  $\tau = 0$ , while the timing of any further bargaining rounds is endogenously determined. The size of the surplus at any time  $\tau \geq 0$  is denoted by  $\Pi^\tau$ . The initial surplus size  $\Pi^0$  is normalized to one, while the size  $\Pi^\tau$  of the surplus at any time  $\tau > 0$  is endogenously determined.

Consider a bargaining round which takes place at time  $\tau \geq 0$ , when the current surplus is of size  $\Pi^\tau$ . Any such bargaining round has the following structure: One player is the

proposer, let us say for the moment that it is Player  $i$ . She announces a proposal  $\theta^i = (\theta_1^i, \dots, \theta_n^i)$ , where  $\theta_j^i$  is the share of the current surplus  $\Pi^\tau$  which Player  $i$  offers to Player  $j$ . Along with the proposal  $\theta^i$ , the proposer also chooses the share  $\sigma t^i$  of the surplus which she destroys. The time lapse which would occur after a rejection of the proposal  $\theta^i$  is prolonged by  $t^i$  as a result of destroying the share  $\sigma t^i$  of the current surplus. The parameter  $\sigma > 0$  measures how costly it is to “buy time.” The pair  $(\theta^i, t^i)$  is restricted to be *feasible*, that is, it must satisfy inequalities  $\theta^i \geq 0$ ,<sup>1</sup> and  $0 \leq t^i \leq 1/\sigma$ , and, moreover,  $\sum_{j \in N} \theta_j^i \leq 1 - \sigma t^i$ . If Player  $i$  chooses  $(\theta^i, t^i)$  such that  $\theta^i = (0, \dots, 0)$  and  $t^i = 1/\sigma$ , then the game ends and all players receive zero payoffs. If Player  $i$  chooses  $(\theta^i, t^i)$  such that  $t_i < 1/\sigma$ , then Players  $1, \dots, n$  respond sequentially (and in this order) to the proposal  $\theta^i$  by *acceptance* or *rejection*. If all players accept  $\theta^i$ , then the game ends, and each Player  $j \in N$  receives the share  $\theta_j^i$  of the current surplus  $\Pi^\tau$ . As soon as some Player  $j \in N$  rejects  $\theta^i$ , the current bargaining round ends, and Player  $j$  becomes the proposer of the next bargaining round, which takes place at time  $\tau + \Delta + t^i$ , where  $\Delta \geq 0$  denotes the exogenously given minimal time lapse between two bargaining rounds. The remaining surplus in the next bargaining round is  $\Pi^{\tau + \Delta + t^i} = (1 - \sigma t^i)\Pi^\tau$ . We have not yet specified which player makes the proposal in the first bargaining round. Without loss of generality, we assume that it is Player 1.

All players are risk-neutral and impatient, and they share a common discount rate  $r > 0$ . Thus, if Player  $j$  receives a share  $\theta_j^i$  of the surplus  $\Pi^\tau$  at time  $\tau$ , this corresponds to a payoff of  $e^{-r\tau}\theta_j^i\Pi^\tau$  for Player  $j$ . If no proposal is ever accepted (“perpetual disagreement”), then all players receive zero payoffs. We have now completed the description of the bargaining game  $G(\Delta, n, r, \sigma)$ . In the sequel, we will use the notation  $G^i(\Delta, n, r, \sigma)$  to denote a subgame which starts at a history where Player  $i$  needs to choose a pair  $(\theta^i, t^i)$ . We note that a subgame  $G^i(\Delta, n, r, \sigma)$  is equivalent to the entire game  $G(\Delta, n, r, \sigma)$  except for the identity of the initial proposer and the size of the initial surplus.

A *stationary strategy* for a Player  $i \in N$  consists of a pair  $(\theta^i, t^i)$  which Player  $i$  chooses whenever she is the proposer, and of a correspondence  $A^i(t)$  such that Player  $i$  responds to proposer  $j$ ’s choice of  $(\theta^j, t^j)$  by acceptance if  $\theta^j \in A^i(t^j)$ , and by rejection otherwise. Henceforth, we denote a stationary strategy concisely by  $(\theta^i, t^i, A^i(t))$ .<sup>2</sup> A profile of stationary strategies  $(\theta^i, t^i, A^i(t))_{i \in N}$  induces a set  $A(t) = \bigcap_{i \in N} A^i(t)$  of unanimously acceptable proposals for each time lapse  $t$ . Observe that a stationary strategy specifies the proposal

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<sup>1</sup>The vector notation is as follows: We write  $v' \geq v''$  if each component of  $v'$  is equal to or greater than the corresponding component of  $v''$ , with at least one strict inequality. If each component of  $v'$  is strictly greater than the corresponding component of  $v''$ , then we write  $v' \gg v''$ .

<sup>2</sup>The continuation game after Player  $i$ ’s rejection of a proposal depends on the time lapse chosen by the current proposer, but is independent of the current proposer’s identity. Thus, it is appropriate to define a stationary strategy such that the player cannot condition his acceptance or rejection on the identity of the current proposer.

as well as the surplus destruction as shares of the surplus, independently of the current size of the surplus.

A *stationary subgame-perfect Nash equilibrium (SSPE)* is a profile of stationary strategies which is a subgame-perfect Nash equilibrium. For the purpose of the present paper, it is useful to work with an equilibrium concept which slightly refines SSPE in two ways. We now discuss these two refinements in turn:

Take the profile  $(\theta^i, t^i, A^i(t))_{i \in N}$  of stationary strategies, and consider a subgame  $G^k(\Delta, n, r, \sigma)$  which starts at a node where Player  $k$  needs to choose a pair  $(\theta^k, t^k)$ . Suppose that the appropriate restriction of  $(\theta^i, t^i, A^i(t))_{i \in N}$  is played in that subgame. Let  $\gamma_k((\theta^i, t^i, A^i(t))_{i \in N})$  denote Player  $k$ 's share of the initial surplus of that subgame. In particular, if playing the appropriate restriction of  $(\theta^i, t^i, A^i(t))_{i \in N}$  in the subgame  $G^k(\Delta, n, r, \sigma)$  leads to an agreement on some proposal  $\tilde{\theta}$  after a finite delay of length  $\tilde{\tau}$ , then we have  $\gamma_k((\theta^i, t^i, A^i(t))_{i \in N}) = e^{-r\tilde{\tau}}\tilde{\theta}_k$ . If it leads to an immediate agreement on  $\tilde{\theta}$  in the subgame under consideration, then  $\gamma_k((\theta^i, t^i, A^i(t))_{i \in N}) = \tilde{\theta}_k$ . If the subgame  $G^k(\Delta, n, r, \sigma)$  ends with the destruction of the whole surplus or in perpetual disagreement when the appropriate restriction of  $(\theta^i, t^i, A^i(t))_{i \in N}$  is played, then  $\gamma_k((\theta^i, t^i, A^i(t))_{i \in N}) = 0$ . The quantity  $\gamma_k((\theta^i, t^i, A^i(t))_{i \in N})$  is important to define the cut-off above which it is optimal for Player  $k$  to accept a proposal. Indeed, we say that the profile  $(\theta^i, t^i, A^i(t))_{i \in N}$  of stationary strategies satisfies *sincere voting* if

$$A^k(\hat{t}) = \left\{ v \in \mathbb{R}_+^n \mid v_k \geq (1 - \sigma\hat{t})e^{-r(\Delta+\hat{t})}\gamma_k((\theta^i, t^i, A^i(t))_{i \in N}) \right\}, \quad \forall \hat{t} \in [0, 1/\sigma], \quad \forall k \in N.$$

Verbally, sincere voting means that a player accepts a proposal  $v$  if and only if the implementation of the proposal  $v$  would make her weakly better off than becoming the proposer in the next bargaining round. The factor  $e^{-r(\Delta+\hat{t})}$  is applied because of the delay occurring after a rejection of the current proposal. It reflects actual discounting of future payoffs by impatient players. In addition, the factor  $(1 - \sigma\hat{t})$  is applied because any payoffs in the next bargaining round are expressed as shares of a surplus which has shrunk by the share  $\sigma\hat{t}$  relative to the current surplus. We can interpret the entire term  $(1 - \sigma\hat{t})e^{-r(\Delta+\hat{t})}$  as the “implicit discount factor,” arising endogenously from the proposer’s choice of the time lapse.

Restricting attention to sincere voting is a simplification which does not affect the results of our analysis. To be more precise, if we considered subgame-perfect equilibria in stationary strategies (SSPE) without restricting attention to sincere voting, we would be able to show that sincere voting must hold on the equilibrium path of play. However, there could arise SSPE in which sincere voting is violated off the equilibrium path. Such “inessential multiplicity” is of no interest for our analysis of equilibrium payoffs. These issues are well-known in the literature on bargaining games. One example of a more detailed discussion can be found in Britz et al. (2010).



The second refinement which we impose on SSPE is only relevant for bargaining games in which  $\Delta = 0$ . Loosely speaking, we want to ensure that two “subsequent” bargaining rounds cannot take place “at the same time” on an equilibrium path of play. While this is trivially true for games with  $\Delta > 0$ , it requires the following refinement in case  $\Delta = 0$ : We say that a profile of stationary strategies  $(\theta^i, t^i, A^i(t))_{i \in N}$  satisfies *no passing* if  $t^i > 0$  for every  $i \in N$  such that  $\theta^i \notin A(t^i)$ .

A *bargaining equilibrium* is a profile of stationary strategies which satisfies sincere voting and no passing, and which is a subgame-perfect Nash equilibrium.

### 3 A characterization of bargaining equilibria

The purpose of this section is to derive Theorem 3.7 which provides a complete characterization of bargaining equilibria. It is the basis for the analysis in the sequel of the paper. In order to arrive at this characterization of bargaining equilibria, we are first going to establish a sequence of auxiliary results. In particular, we are going to show that on the path of play induced by a bargaining equilibrium, agreement is reached immediately in every subgame, and that every player chooses a proposal which makes all other players indifferent between acceptance and rejection. These equilibrium properties are well-established for “standard” unanimity bargaining games in which the time lapse between bargaining rounds is exogenously given. In the present section, the arguments building towards Theorem 3.7 follow similar lines as in the previous work by Banks and Duggan (2000) and Britz et al. (2010, 2014), but are complicated by the endogenous determination of the time lapse.

One important step towards the characterization of bargaining equilibria is to show that in such an equilibrium every subgame ends in an agreement. In particular, this means that no proposer decides to end the game by destroying the entire surplus, and no perpetual disagreement can occur on the equilibrium path of play. As a first step towards this result, Lemma 3.1 below claims that in a bargaining equilibrium, there is at least one player whose proposal is unanimously accepted.

**Lemma 3.1.** *If  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, then there is  $i \in N$  such that  $\theta^i \in A(t^i)$ , and  $t^i < 1/\sigma$ .*

**Proof.** In the appendix. □

We have shown that at least one player makes an acceptable proposal in a bargaining equilibrium. Now we are going to consider a proposal which is unanimously accepted, and show that such a proposal makes each player other than the proposer indifferent between acceptance and rejection. Moreover, such a proposal distributes the entire share of the surplus which is not destroyed. It is a standard result in unanimity bargaining that the

proposer “extracts all surplus” from the other players by making them exactly indifferent between acceptance and rejection. Lemma 3.2 below replicates this finding for the game under consideration here.

**Lemma 3.2.** *Suppose that  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, and  $\theta^i \in A(t^i)$ , as well as  $t^i < 1/\sigma$ . Then, we have*

$$\begin{aligned}\theta_j^i &= (1 - \sigma t^i) e^{-r(\Delta + t^i)} \gamma_j((\theta^k, t^k, A^k(t))_{k \in N}), \quad \forall i \in N, \quad \forall j \in N \setminus \{i\}, \\ \theta_i^i &= 1 - \sigma t^i - \sum_{j \in N \setminus \{i\}} \theta_j^i, \quad \forall i \in N.\end{aligned}$$

**Proof.** In the appendix. □

Recall that we have defined  $G^i(\Delta, n, r, \sigma)$  as the subgame in which Player  $i$  makes the initial proposal. Given a profile of stationary strategies  $(\theta^k, t^k, A^k(t))_{k \in N}$ , we have defined  $\gamma_i((\theta^k, t^k, A^k(t))_{k \in N})$  as the share of the surplus which Player  $i$  receives if the appropriate restriction of  $(\theta^k, t^k, A^k(t))_{k \in N}$ , is played in the subgame  $G^i(\Delta, n, r, \sigma)$ . Our next step is to show that  $\gamma_i((\theta^k, t^k, A^k(t))_{k \in N})$  is strictly positive for every Player  $i$ .

**Lemma 3.3.** *If  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, then it holds that  $\gamma_j((\theta^k, t^k, A^k(t))_{k \in N}) > 0$  for all  $j \in N$ .*

**Proof.** In the appendix. □

In the game under consideration, bargaining may continue indefinitely without agreement. Moreover, a proposer could unilaterally end the bargaining process by destroying the whole surplus. One important implication of Lemma 3.3 above is that neither of these two outcomes can occur in a bargaining equilibrium. To see this, suppose that either a perpetual disagreement or the destruction of the entire surplus occurs in some subgame  $G^i(\Delta, n, r, \sigma)$ . This results in zero payoffs for all players, and in particular for the initial proposer in the subgame, thus contradicting Lemma 3.3.

**Corollary 3.4.** *In a bargaining equilibrium  $(\theta^k, t^k, A^k(t))_{k \in N}$ , agreement is reached in finite time in every subgame  $G^i((\theta^k, t^k, A^k(t))_{k \in N})$ .*

Since no player destroys the entire surplus, it follows that the “implicit discount factor” given by  $(1 - \sigma t^i) e^{-r(\Delta + t^i)}$  is strictly positive for every  $i \in N$ . Hence, Lemma 3.2 and Lemma 3.3 readily imply that any proposal  $\theta^i$  which is accepted in a bargaining equilibrium is strictly positive in all components. This property is crucial for the proof of Lemma 3.5 below, which claims that, in a bargaining equilibrium, agreement is reached immediately in every subgame.

**Lemma 3.5.** *In a bargaining equilibrium  $(\theta^k, t^k, A^k(t))_{k \in N}$ , agreement is reached immediately in every subgame  $G^i((\theta^k, t^k, A^k(t))_{k \in N})$ .*

**Proof.** In the appendix. □

We have now shown that, in a bargaining equilibrium, no delay can occur before an agreement in any subgame. In the case of a bargaining game with  $\Delta > 0$ , this readily implies that every player's proposal is unanimously accepted. This result could also be shown for SSPE in general. However, in the case where  $\Delta = 0$ , the immediate agreement property need not always imply that every player's proposal is unanimously accepted: Indeed, the immediate agreement property does not rule out the possibility that a player might choose a zero time lapse along with a proposal which is rejected. If the following proposal were accepted, then agreement would still have occurred "immediately." Under our definition of a bargaining equilibrium, however, this complication is ruled out by the no passing property, so that we have the following corollary.

**Corollary 3.6.** *In a bargaining equilibrium, every player's proposal is unanimously accepted.*

Since every player's proposal is accepted in a bargaining equilibrium, we have

$$\theta_i^i = \gamma_i((\theta^k, t^k, A^k(t))_{k \in N}) > 0$$

for every  $i \in N$ . Now Lemma 3.2 implies that in every bargaining equilibrium,

$$\theta_j^i = (1 - \sigma t^i) e^{-r(\Delta + t^i)} \theta_j^j > 0,$$

for every  $i \in N$  and  $j \in N \setminus \{i\}$ . Therefore, if Player  $i$  is the proposer, she faces the following trade-off: If she chooses  $t^i$ , then she must offer the other players the share  $(1 - \sigma t^i) e^{-r(\Delta + t^i)} \sum_{j \in N \setminus \{i\}} \theta_j^j$  in order to obtain their agreement. This term is clearly decreasing in  $t^i$ . Choosing a greater time lapse improves Player  $i$ 's bargaining position relative to the other players. On the downside, however, choosing  $t^i$  implies that a share  $\sigma t^i$  of the surplus is destroyed. More formally, given proposals  $\theta^j$  of all other players  $j \in N \setminus \{i\}$ , and given that each of these proposals is unanimously accepted, Player  $i$ 's payoff can be thought of as the following function of  $t^i$ :

$$\xi^i(t^i) = (1 - \sigma t^i) \left( 1 - e^{-r(\Delta + t^i)} \sum_{j \in N \setminus \{i\}} \theta_j^j \right).$$

In a bargaining equilibrium, Player  $i$  chooses  $t^i$  so as to maximize  $\xi^i(t^i)$ .

If the profile of stationary strategies  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, then the above analysis implies that

$$A^j(t^i) = \{v \in \mathbb{R}_+^n | v_j \geq \theta_j^i\}, \quad \forall i \in N, \quad \forall j \in N \setminus \{i\}.$$

From now on, it is therefore appropriate to ease the notation and describe a bargaining equilibrium only as a profile  $(\theta^i, t^i)_{i \in N}$ . Theorem 3.7 below unifies the findings derived in this section into a characterization of bargaining equilibria.

**Theorem 3.7.** *The profile  $(\theta^i, t^i)_{i \in N}$  is a bargaining equilibrium if and only if it satisfies the following conditions:*

$$\theta_i^i = 1 - \sigma t^i - \sum_{j \in N \setminus \{i\}} \theta_j^i, \quad (1)$$

$$\theta_j^i = (1 - \sigma t^i) e^{-r(\Delta + t^i)} \theta_j^j, \quad j \in N \setminus \{i\}, \quad (2)$$

$$t^i \in \arg \max_{t \in [0, 1/\sigma]} (1 - \sigma t) \left( 1 - e^{-r(\Delta + t)} \sum_{j \in N \setminus \{i\}} \theta_j^j \right). \quad (3)$$

Verbally, Eqn. (1) says that the part of the surplus which is not destroyed is completely distributed to the players. Eqn. (2) says that each player other than the proposer is offered the share which makes him exactly indifferent between acceptance and rejection. In case  $\Delta > 0$ , setting all the time lapses  $t^1, \dots, t^n$  in Eqns. (1)–(2) equal to zero would yield the equations which are familiar from the equilibrium analysis of standard unanimity bargaining games with an exogenously fixed time lapse  $\Delta > 0$  and concomitant discount factor  $e^{-r\Delta} < 1$ . The novel element of the above equilibrium characterization is the optimization problem (3) which effectively endogenizes the discount factor. We will see that two kinds of solution to this optimization problem are relevant for the analysis: First, a corner solution of the optimization problem corresponds to the case where  $t^i = 0$  is chosen. Second, an interior solution corresponds to the case where some  $t^i > 0$  is chosen.

More formally, consider the optimization problem (3). Take the derivatives

$$\begin{aligned} \partial \xi^i(t) / \partial t &= -\sigma + (\sigma + r(1 - \sigma t)) e^{-r(\Delta + t)} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right) \\ \partial^2 \xi^i(t) / \partial^2 t &= -r\sigma e^{-r(\Delta + t)} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right) - r(\sigma + r(1 - \sigma t)) e^{-r(\Delta + t)} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right). \end{aligned}$$

We see that

$$\partial^2 \xi^i(t) / \partial^2 t < 0 \text{ on } [0, 1/\sigma].$$

Moreover, evaluating the first-order derivative at the points  $t = 0$  and  $t = 1/\sigma$ , we see that

$$\begin{aligned}\partial\xi^i(t)/\partial t|_{t=0} &= -\sigma + (\sigma + r)e^{-r\Delta} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right), \\ \partial\xi^i(t)/\partial t|_{t=1/\sigma} &= -\sigma + \sigma e^{-r(\Delta+1/\sigma)} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right).\end{aligned}$$

We can conclude that there are two possible cases:

1. If  $e^{-r\Delta} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right) \leq \sigma/(\sigma + r)$ , then the first-order derivative  $\partial\xi^i(t)/\partial t$  is non-positive for  $t \in [0, 1/\sigma]$  and strictly negative for  $t \in (0, 1/\sigma)$ . Hence, the optimization problem (3) has only a corner solution at  $t = 0$ .
2. If  $e^{-r\Delta} \left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right) > \sigma/(\sigma + r)$ , then the first-order derivative  $\partial\xi^i(t)/\partial t$  is strictly positive at  $t = 0$ , and strictly monotonically decreasing on the interval  $[0, 1/\sigma]$ . If  $\partial\xi^i(t)/\partial t|_{t=1/\sigma} \geq 0$ , then we would find a corner solution at  $t = 1/\sigma$ . However, we have seen in Corollary 3.4 that this does not occur in a bargaining equilibrium. So we need to consider only the case where  $\partial\xi^i(t)/\partial t|_{t=1/\sigma} < 0$ , in which we find a unique interior solution at some  $t \in (0, 1/\sigma)$ . This solution is given by the first-order condition  $\partial\xi^i/\partial t = 0$ , which can be written as  $\sum_{j \in N \setminus \{i\}} \theta_j^j = \frac{\sigma e^{r(\Delta+t)}}{\sigma + r(1-\sigma t)}$ .

We have found that Player  $i$  chooses the level of surplus destruction and the concomitant time lapse depending on the basic model parameters and on the value of  $\left( \sum_{j \in N \setminus \{i\}} \theta_j^j \right)$ . Since every responder becomes the next proposer after he rejects the current proposal, this quantity is the (un-discounted) sum of the reservation payoffs of all players. It can be interpreted as the “price of agreement.”

**Theorem 3.8.** *Let  $(\theta^k, t^k)_{k \in N}$  be a bargaining equilibrium. For every Player  $i \in N$ , it holds that either  $\sum_{j \in N \setminus \{i\}} \theta_j^j \leq \frac{\sigma e^{r\Delta}}{\sigma + r}$  and  $t^i = 0$ , or  $\sum_{j \in N \setminus \{i\}} \theta_j^j > \frac{\sigma e^{r\Delta}}{\sigma + r}$  and  $t^i > 0$ . In the latter case,  $t^i > 0$  is the solution to  $\sum_{j \in N \setminus \{i\}} \theta_j^j = \frac{\sigma e^{r(\Delta+t^i)}}{\sigma + r(1-\sigma t^i)}$ .*

In the next section, we are going to discuss bargaining equilibria in which all players choose zero surplus destruction. We call such equilibria *efficient bargaining equilibria*. In Section 5, we then turn to bargaining equilibria in which at least one player chooses to destroy some surplus. We call such equilibria *bargaining equilibria with surplus destruction*. In the sequel of the paper, we will show that all bargaining equilibria are “symmetric,” that is, all players choose the same surplus destruction. Moreover, we will show that for any given choice of the model parameters, there exists either an efficient bargaining equilibrium, or a bargaining equilibrium with surplus destruction, but not both.

## 4 Efficient bargaining equilibria

In this section, we study bargaining equilibria in which all players choose not to destroy any surplus. The following theorem restates the characterization of a bargaining equilibrium for this case.

**Theorem 4.1.** *The proposals  $(\theta^i)_{i \in N}$  are part of an efficient bargaining equilibrium if the following conditions hold for all  $i \in N$  and all  $j \in N \setminus \{i\}$ :*

$$\theta_i^i = 1 - \sum_{j \in N \setminus \{i\}} \theta_j^i, \quad (4)$$

$$\theta_j^i = e^{-r\Delta} \theta_j^j, \quad (5)$$

$$\theta_i^i \geq r/(\sigma + r). \quad (6)$$

**Proof.** If  $t^i = 0$ , then Eqns. (1)–(2) specialize to Eqns. (4)–(5). Moreover, due to Theorem 3.8, the inequality  $e^{-r\Delta}(\sum_{j \in N \setminus \{i\}} \theta_j^j) \leq \sigma/(\sigma + r)$  is satisfied in an efficient bargaining equilibrium. Combined with Eqns. (4)–(5), this implies Ineq. (6) above.  $\square$

In unanimity bargaining games in the tradition of Rubinstein, one standard result is the uniqueness of SSPE payoffs in the limit as the discount factor goes to one. If the discount factor were exactly equal to one, however, this uniqueness result would break down and every allocation of the surplus could be supported by an SSPE. We will see that efficient bargaining equilibria in our model exhibit a similar discontinuity around the point where the exogenously given minimal time lapse  $\Delta$  is zero. Therefore, in what follows, we will discuss efficient bargaining equilibria for the two cases  $\Delta > 0$  and  $\Delta = 0$  in turn:

Indeed, let us first consider a bargaining game  $G(\Delta, n, r, \sigma)$  with  $\Delta > 0$ . In this case, Eqns. (4)–(5) above amount to a system of  $n^2$  independent equations which allow us to solve for a unique array of proposals  $(\theta^1, \dots, \theta^n)$ . This solution is

$$\begin{aligned} \theta_i^i &= \frac{1}{1 + (n-1)e^{-r\Delta}}, \quad \forall i \in N, \\ \theta_j^i &= \frac{e^{-r\Delta}}{1 + (n-1)e^{-r\Delta}}, \quad \forall i \in N, \quad \forall j \in N \setminus \{i\}. \end{aligned}$$

Of course, these proposals are the same proposals which would be made in an SSPE of an  $n$ -player bargaining game in which  $\Delta > 0$  and  $t^1 = \dots = t^n = 0$  are exogenously fixed. Henceforth, we refer to the allocation which gives the proposer a share  $\frac{1}{1+(n-1)e^{-r\Delta}}$  of the surplus, and which gives each player other than the proposer a share  $\frac{e^{-r\Delta}}{1+(n-1)e^{-r\Delta}}$  of the surplus as the *fixed schedule allocation*. This allocation will be an important benchmark in our later analysis of bargaining equilibria with surplus destruction. If  $\Delta > 0$ , then any

efficient bargaining equilibrium leads to the fixed schedule allocation. However, depending on the basic model parameters, the fixed schedule allocation may or may not be supported by a bargaining equilibrium. More specifically, for the fixed schedule allocation to be a bargaining equilibrium, an additional restriction given by Ineq. (6) must be satisfied. The intuition is as follows: If the fixed schedule allocation does not give the proposer a sufficiently large share of the surplus, then the proposer has an incentive to choose a strictly positive time lapse in order to enhance her bargaining power, and thus depart from the fixed schedule allocation.

Now let us turn to the case of a bargaining game  $G(\Delta, n, r, \sigma)$  with  $\Delta = 0$ . From Eqn. (5), it is immediate that in an efficient bargaining equilibrium, all players make the same proposal, say  $\theta^*$ . As before, Eqn. (4) simply says that the proposal  $\theta^*$  made by all players distributes the entire surplus to the players. Eqns. (4)–(5) do not say anything about the allocation of the surplus to the different players in an efficient bargaining equilibrium. Indeed, this allocation is only restricted by Ineq. (6), which becomes  $\theta_i^* \geq r/(\sigma + r)$  for each  $i \in N$ . Thus, there is a set of surplus allocations that can be supported by efficient bargaining equilibria. The only restrictions are that every player receives at least the share  $r/(\sigma + r)$ , and that the whole surplus is allocated to the players. Recall that in unanimity bargaining games in which the discount factor is exogenously fixed to exactly one, every efficient allocation can be supported by an SSPE. Here, the analogue of the discount factor is the term  $e^{-r\Delta}$  which is exactly equal to one if  $\Delta = 0$ . However, it is not generally true that every allocation can be supported by an efficient bargaining equilibrium if  $\Delta = 0$ . The intuition is as follows: In an efficient bargaining equilibrium, every player's payoff must be sufficiently large so that this player does not find it in his interest to deviate from efficient bargaining equilibrium by choosing a strictly positive time lapse. This condition is satisfied exactly when the player's payoff is at least  $r/(r + \sigma)$ . In the limit as  $r$  goes to zero, this condition becomes ever weaker, and thus any efficient allocation of the surplus is consistent with an efficient bargaining equilibrium. On the other hand, if  $r = \sigma/(n - 1)$ , then each player requires a share  $1/n$  in order to be willing to refrain from surplus destruction. Thus, as  $r$  grows towards  $\sigma/(n - 1)$ , the range of efficient bargaining equilibrium allocations shrinks and eventually collapses into the equal division. If  $r > \sigma/(n - 1)$ , each player requires more than a share  $1/n$  of the surplus in order to be willing to refrain from surplus destruction. Hence, no agreement can be reached without surplus destruction, and an efficient bargaining equilibrium does not exist. We conclude that, in a bargaining game with  $\Delta = 0$ , surplus destruction can only be avoided if  $r$  is sufficiently small.

Next, we state a necessary and sufficient condition for the existence of an efficient bargaining equilibrium, which covers both cases  $\Delta > 0$  and  $\Delta = 0$ .

**Lemma 4.2.** *An efficient bargaining equilibrium exists if and only if  $e^{r\Delta} \geq (n-1)(r/\sigma)$ .*

**Proof.** *If.* Consider an array of proposals  $(\bar{\theta}^i)_{i \in N}$  defined for every  $i \in N$  by  $\bar{\theta}_i^i = \frac{1}{1+(n-1)e^{-r\Delta}}$  and  $\bar{\theta}_j^i = \frac{e^{-r\Delta}}{1+(n-1)e^{-r\Delta}}$  for  $j \in N \setminus \{i\}$ . These proposals satisfy Eqns. (4)–(5). If  $e^{r\Delta} \geq (n-1)(r/\sigma)$ , then  $e^{-r\Delta}(n-1) \leq \sigma/r$ , and so  $\bar{\theta}_i^i \geq \frac{1}{1+\sigma/r} = \frac{r}{r+\sigma}$ , and so Ineq. (6) is satisfied. Indeed, we have shown that proposals  $(\bar{\theta}^i)_{i \in N}$  are part of an efficient bargaining equilibrium.

*Only If.* Suppose that  $(\bar{\theta}^i)_{i \in N}$  is an array of proposals of an efficient bargaining equilibrium. Then, for any  $i \in N$ , we have  $1 = \theta_i^i + \sum_{j \in N \setminus \{i\}} \theta_j^i = \theta_i^i + e^{-r\Delta} \sum_{j \in N \setminus \{i\}} \theta_j^j$ . Due to Ineq. (6), it holds that  $\theta_k^k \geq r/(\sigma + r)$  for every  $k \in N$ , so we have the inequality  $(\frac{r}{\sigma+r}) + (n-1)e^{-r\Delta}(\frac{r}{\sigma+r}) \leq 1$ . Equivalent transformation yields  $e^{r\Delta} \geq (\frac{r}{\sigma})(n-1)$ , as desired. □

For the case where  $\Delta = 0$ , we have argued before that an efficient bargaining equilibrium exists if  $r$  is sufficiently small. This is confirmed by the above theorem. If  $\Delta > 0$ , however, notice that the term  $e^{r\Delta}/r$  grows without bound both as  $r$  goes to zero and as  $r$  goes to infinity. Thus, in bargaining games with  $\Delta > 0$ , an efficient bargaining equilibrium exists for sufficiently small as well as for sufficiently large  $r$ . The intuition is as follows: If  $r$  is very large, the exogenously given minimal time lapse  $\Delta > 0$  already confers so much bargaining power to the proposer that she does not find it worthwhile to destroy surplus in order to gain extra bargaining power. In Section 6, we will give a condition for the existence of an intermediate range of  $r$  so that a bargaining equilibrium with surplus destruction does exist.

## 5 Bargaining equilibria with surplus destruction

As a next step, we are going to complement the previous section with an analysis of bargaining equilibria with surplus destruction. By definition, a bargaining equilibrium with surplus destruction is a bargaining equilibrium in which at least one player destroys surplus. One of the results in this section is that all bargaining equilibria are “symmetric,” that is, in a bargaining equilibrium, all players choose the same surplus destruction and thus the same time lapse. In particular, this will imply that in a bargaining equilibrium with surplus destruction, *all* players choose a strictly positive amount of surplus destruction.

As in the previous section, we begin the analysis with a statement which specializes Theorem 3.7.



**Lemma 5.1.** *Suppose that  $(\theta^k, t^k)_{k \in N}$  is a bargaining equilibrium with surplus destruction. Then, the following equalities hold for every  $i \in N$  such that  $t^i > 0$ :*

$$\theta_i^i = \frac{r(1 - \sigma t^i)^2}{\sigma + r(1 - \sigma t^i)}, \quad (7)$$

$$\sum_{j \in N \setminus \{i\}} \theta_j^i = \frac{\sigma(1 - \sigma t^i)}{\sigma + r(1 - \sigma t^i)}, \quad (8)$$

$$\sum_{j \in N \setminus \{i\}} \theta_j^j = \left( \frac{\sigma}{\sigma + r(1 - \sigma t^i)} \right) e^{r(\Delta + t^i)}. \quad (9)$$

**Proof.** If  $t^i > 0$  in a bargaining equilibrium, then  $t^i > 0$  must be an interior solution to the optimization problem (3). The corresponding first-order condition is Eqn. (9). From Eqns. (1)–(2), we find  $\theta_i^i = 1 - \sigma t^i - (1 - \sigma t^i)e^{-r(\Delta + t^i)} \sum_{j \in N \setminus \{i\}} \theta_j^j$ . By substitution from Eqn. (9) above, we find

$$\theta_i^i = 1 - \sigma t^i - \frac{\sigma(1 - \sigma t^i)}{\sigma + r(1 - \sigma t^i)} = \frac{r(1 - \sigma t^i)^2}{\sigma + r(1 - \sigma t^i)},$$

thus establishing Eqn. (7). Combining the above equality with Eqn. (1), we find Eqn. (8), as desired.  $\square$

We show next that there cannot be a bargaining equilibrium in which one player destroys surplus and another player does not.

**Lemma 5.2.** *If  $(\theta^k, t^k)_{k \in N}$  is a bargaining equilibrium and  $t^i > 0$  for some  $i \in N$ , then  $(t^1, \dots, t^n) \gg 0$ .*

**Proof.** Suppose by way of contradiction that  $(\theta^k, t^k)_{k \in N}$  is a bargaining equilibrium, and  $t^i > 0$  but  $t^j = 0$  for some  $j \in N \setminus \{i\}$ . Using Eqns. (1)–(2), it follows that  $\theta_j^j + e^{-r\Delta} \sum_{k \in N \setminus \{j\}} \theta_k^k = 1$ , which can be rewritten as

$$(1 - e^{-r\Delta})\theta_j^j + e^{-r\Delta} \sum_{k \in N} \theta_k^k = 1.$$

We consider a unilateral deviation by Player  $i$  from the supposed bargaining equilibrium. Under this deviation, Player  $i$  chooses a zero time lapse instead of  $t^i > 0$ , and instead of the proposal  $\theta^i$ , she makes a proposal  $\tilde{\theta}^i$  defined as follows:

$$\begin{aligned} \tilde{\theta}_k^i &= e^{-r\Delta} \theta_k^k, \quad \forall k \in N \setminus \{i\}, \\ \tilde{\theta}_i^i &= e^{-r\Delta} \theta_i^i + (1 - e^{-r\Delta}) \theta_j^j. \end{aligned}$$

We see that  $\sum_{k \in N} \tilde{\theta}_k^i = 1$ , so that the pair  $(\tilde{\theta}^i, 0)$  is feasible. Due to sincere voting, we have  $\tilde{\theta}^i \in A(0)$ , so the proposal  $\tilde{\theta}^i$  is unanimously accepted. We see that the deviation is

profitable for Player  $i$  if the inequality  $e^{-r\Delta}\theta_i^i + (1 - e^{-r\Delta})\theta_j^j > \theta_i^i$  is satisfied, or, equivalently, if  $\theta_j^j > \theta_i^i$ . Indeed, due to Theorem 3.8, we know that  $e^{-r\Delta} \sum_{k \in N \setminus \{j\}} \theta_k^k \leq \frac{\sigma}{\sigma + r}$ . By Eqns. (1)–(2), it follows that  $\theta_j^j \geq \frac{r}{\sigma + r}$ . But from Eqn. (13), we have that  $\theta_i^i = \frac{r(1 - \sigma t^i)^2}{\sigma + r(1 - \sigma t^i)} < \frac{r}{\sigma + r}$ , and thus  $\theta_i^i < \theta_j^j$ . Indeed, we have constructed a profitable deviation for Player  $i$  and thus obtained a contradiction.  $\square$

The next lemma claims that in a bargaining equilibrium with surplus destruction, all players choose the same level of surplus destruction, and thus the same time lapse.

**Lemma 5.3.** *If  $(\theta^k, t^k)_{k \in N}$  is a bargaining equilibrium with surplus destruction, then  $t^1 = t^2 = \dots = t^n$ .*

**Proof.** Define a function

$$\psi(t) = \frac{r(1 - \sigma t)^2 + \sigma e^{r(\Delta + t)}}{\sigma + r(1 - \sigma t)}. \quad (10)$$

Suppose that  $(\theta^k, t^k)_{k \in N}$  is a bargaining equilibrium with surplus destruction. We have already shown that this implies  $(t^1, \dots, t^n) \gg 0$ . Thus, Eqns. (7) and (9) holds for all  $i \in N$ . Summing up these two equations we find that  $\psi(t^i) = \sum_{k \in N} \theta_k^k$  for every  $i \in N$ . We have left to show that the function  $\psi(t)$  is strictly monotonic on the relevant interval  $t \in (0, 1/\sigma]$ . Indeed, consider the first-order derivative

$$\begin{aligned} \psi'(t) &= \left( \frac{r\sigma e^{r(\Delta + t)} - 2r\sigma(1 - \sigma t)}{\sigma + r(1 - \sigma t)} \right) + r\sigma \left( \frac{\sigma e^{r(\Delta + t)} + r(1 - \sigma t)^2}{(\sigma + r(1 - \sigma t))^2} \right) \\ &= \left( \frac{r\sigma}{\sigma + r(1 - \sigma t)} \right) (e^{r(\Delta + t)} - 2(1 - \sigma t) + \psi(t)). \end{aligned}$$

Observing that the denominator  $\sigma + r(1 - \sigma t)$  is strictly positive for any  $t \in (0, 1/\sigma]$ , we only have to show that

$$e^{r(\Delta + t)} + \psi(t) > 2(1 - \sigma t).$$

For any  $t > 0$ , we have

$$e^{r(\Delta + t)} > e^{r\Delta} \geq 1 > 1 - \sigma t.$$

Moreover, invoking once more the fact that  $\sigma + r(1 - \sigma t) > 0$  for  $t \in (0, 1/\sigma]$ , it is easily verified that  $\psi(t) > 1 - \sigma t$ , and the proof is complete.  $\square$

Consider a bargaining equilibrium  $(\theta^k, t^k)_{k \in N}$  with surplus destruction. We have shown that all players choose the same strictly positive time lapse. Our next step is to determine

this time lapse. Using Eqns. (2) and (7), we can conclude that in a bargaining equilibrium with surplus destruction, there is a “proposer share”  $x > 0$  such that  $x = \theta_i^i$  for every  $i \in N$ , and there is a “responder share”  $y > 0$  such that  $y = \theta_j^i$  for every  $i \in N$  and  $j \in N \setminus \{i\}$ . From Eqns. (7)–(9) above, we can infer that in any bargaining equilibrium with surplus destruction, the proposer share  $x$ , the responder share  $y$ , and the time lapse  $t$  satisfy the following equalities:

$$\begin{aligned} x/y &= e^{r(\Delta+t)}(1-\sigma t)^{-1}, \\ x/y &= \left(\frac{r}{\sigma}\right)(1-\sigma t)(n-1), \end{aligned}$$

and, consequently, the equilibrium choice of the time lapse satisfies

$$e^{r(\Delta+t)} = \left(\frac{r}{\sigma}\right)(1-\sigma t)^2(n-1).$$

Now we can state our results concisely as the following characterization of bargaining equilibria with surplus destruction:

**Theorem 5.4.** *In a bargaining equilibrium with surplus destruction, every player chooses the time lapse  $t$  which solves*

$$e^{r(\Delta+t)} = \left(\frac{r}{\sigma}\right)(1-\sigma t)^2(n-1). \quad (11)$$

Moreover, every Player  $i \in N$  chooses the proposal  $\theta^i$  given by:

$$\theta_i^i = \frac{r(1-\sigma t)^2}{\sigma + r(1-\sigma t)}, \quad (12)$$

$$\theta_j^i = \left(\frac{\sigma(1-\sigma t)}{\sigma + r(1-\sigma t)}\right) / (n-1), \quad \forall j \in N \setminus \{i\}. \quad (13)$$

Consider Eqn. (11) which determines the time lapse in a bargaining equilibrium with surplus destruction. The left-hand side of the equality is increasing on  $t \in [0, 1/\sigma]$ , while the right-hand side is decreasing on  $t \in [0, 1/\sigma]$ . Therefore, Eqn. (11) can be solved for  $t > 0$ , and thus a bargaining equilibrium with surplus destruction exists, if and only if the intercept at  $t = 0$  of the left-hand side lies below the intercept at  $t = 0$  of the right-hand side. Indeed, we are now going to give a necessary and sufficient condition for the existence of a bargaining equilibrium with surplus destruction.

**Lemma 5.5.** *A bargaining equilibrium with surplus destruction exists if and only if  $e^{r\Delta} < \left(\frac{r}{\sigma}\right)(n-1)$ .*

**Proof.** *If.* Suppose that  $e^{r\Delta} < \left(\frac{r}{\sigma}\right)(n-1)$ . Then, there is  $\bar{t} > 0$  such that  $e^{r(\Delta+\bar{t})} = \left(\frac{r}{\sigma}\right)(1-\sigma\bar{t})^2(n-1)$ . For every  $i \in N$ , define  $\bar{\theta}_i^i = \frac{r(1-\sigma\bar{t})^2}{\sigma+r(1-\sigma\bar{t})}$ , and

$\bar{\theta}_j^i = \left( \frac{\sigma(1-\sigma\bar{t})}{\sigma+r(1-\sigma\bar{t})} \right) \left( \frac{1}{n-1} \right)$  for  $j \in N \setminus \{i\}$ . Moreover, let  $\bar{t}^k = \bar{t}$  for every  $k \in N$ . It is now easily verified that  $(\bar{\theta}^k, \bar{t}^k)_{k \in N}$  is a bargaining equilibrium.

*Only If.* Suppose that  $(\bar{\theta}^k, \bar{t}^k)_{k \in N}$  is a bargaining equilibrium. Then, for every  $k \in N$ , the equality  $e^{r(\Delta+\bar{t}^k)} = \left( \frac{r}{\sigma} \right) (1-\sigma\bar{t}^k)^2(n-1)$  is satisfied. Since  $\bar{t}^k \in (0, 1/\sigma]$ , it holds that  $\left( \frac{r}{\sigma} \right) (1-\sigma\bar{t}^k)^2(n-1) < \left( \frac{r}{\sigma} \right) (n-1)$  and  $e^{r(\Delta+\bar{t}^k)} > e^{r\Delta}$ , and hence  $e^{r\Delta} < \left( \frac{r}{\sigma} \right) (n-1)$ , as desired. □

Lemma 4.2 and Lemma 5.5 readily imply results on the existence and uniqueness of bargaining equilibria which we summarize in the following theorem.

**Theorem 5.6.** *1. In a bargaining game  $G(\Delta, n, r, \sigma)$  with  $\Delta > 0$ , there exists a unique bargaining equilibrium. It is efficient if the inequality  $e^{r\Delta} \geq \left( \frac{r}{\sigma} \right) (n-1)$  holds, and it involves surplus destruction otherwise.*

*2. In any bargaining game  $G(0, n, r, \sigma)$  where  $\Delta = 0$ , there exists a bargaining equilibrium. If  $1/r \leq \sigma/(n-1)$ , then there is a range of efficient bargaining equilibria. Otherwise the bargaining equilibrium is unique, and it involves surplus destruction.*

In the previous section, we have introduced the notion of the “fixed schedule allocation.” This is the equilibrium allocation which would result in a unanimity bargaining game in which players cannot destroy surplus and thus cannot affect the time lapse  $\Delta > 0$  which occurs between bargaining rounds. Intuitively, the bargaining game which we consider gives the proposer an additional source of power by allowing her to destroy surplus and thereby prolong the time lapse between rounds. It is therefore intuitive that the advantage of the proposer over the other players is greater in a bargaining equilibrium with surplus destruction than under the fixed schedule allocation. On the other hand, the fixed schedule allocation is efficient. Hence, the question arises whether the proposer is better off in the bargaining game under consideration here than in a more standard unanimity bargaining setting which leads to the fixed schedule allocation. We show next that this is indeed the case.

**Lemma 5.7.** *In a bargaining equilibrium with surplus destruction, the proposer receives more than his fixed schedule allocation.*

**Proof.** In the appendix. □

Together with the immediate agreement property, this result says that introducing the possibility of surplus destruction into a unanimity bargaining game is beneficial for the initial proposer only, but harmful for all other players.

## 6 Quantifying equilibrium surplus destruction

In this section, we address in more detail the question how much of the surplus is destroyed in a bargaining equilibrium. In particular, we are interested in the effect that the basic model parameters  $(\Delta, n, r, \sigma)$  have on the equilibrium level of surplus destruction. We have found in the previous section that, in a bargaining equilibrium with surplus destruction, the time lapse  $t$  satisfies the equality

$$e^{r(\Delta+t)} - \left(\frac{r}{\sigma}\right) (1 - \sigma t)^2 (n - 1) = 0.$$

We can easily verify that the partial derivatives relevant for a comparative statics analysis can be signed as follows:

$$\begin{aligned} \partial \left( e^{r(\Delta+t)} - \left(\frac{r}{\sigma}\right) (1 - \sigma t)^2 (n - 1) \right) / \partial t &> 0, \\ \partial \left( e^{r(\Delta+t)} - \left(\frac{r}{\sigma}\right) (1 - \sigma t)^2 (n - 1) \right) / \partial \Delta &> 0, \\ \partial \left( e^{r(\Delta+t)} - \left(\frac{r}{\sigma}\right) (1 - \sigma t)^2 (n - 1) \right) / \partial \sigma &> 0. \end{aligned}$$

The above derivatives with respect to  $t$  and with respect to  $\Delta$  have the same sign: Indeed, if the exogenously fixed minimal time lapse  $\Delta$  increases slightly, then the equilibrium level of surplus destruction decreases. Loosely speaking, the exogenously fixed time lapse and the additional time lapse chosen by the proposer are substitutes.

Likewise, the above derivatives with respect to  $t$  and with respect to  $\sigma$  have the same sign. Indeed, the parameter  $\sigma$  can be interpreted as the cost of gaining more bargaining power. If this cost slightly decreases, then the proposer finds it attractive to enhance her bargaining power by choosing a higher time lapse.

An analysis of the effect of the model parameters  $r$  and  $n$  on the equilibrium time lapse is less straight-forward: First, the number of players being an integer, it would not be meaningful to consider the effect of an infinitesimal change in  $n$ . Second, with regard to the parameter  $r$ , examining the derivative

$$\partial \left( e^{r(\Delta+t)} - \left(\frac{r}{\sigma}\right) (1 - \sigma t)^2 (n - 1) \right) / \partial r = (\Delta + t) e^{r(\Delta+t)} - \frac{(1 - \sigma t)^2 (n - 1)}{\sigma},$$

we see that the effect of a small change of  $r$  on the equilibrium time lapse cannot be signed unambiguously.

In the remainder of this section, we will study in more detail the role of parameters  $r$  and  $n$  in determining the equilibrium time lapse as well as the concomitant amount of surplus destruction. In particular, we are going to show the following results: Given any number of players  $n$ , we can compute an upper bound  $\eta(n)$  on the level of surplus destruction which can occur in a bargaining equilibrium. We demonstrate that this bound is increasing in the number of players. It is approximately equal to 0.22 when there are two players, and it approaches one in the limit as the number of players grows without bound. For any number of players  $n$ , the concomitant bound on equilibrium surplus destruction  $\eta(n)$  is “tight” in the sense that it can actually be attained in the bargaining equilibrium if  $\Delta$  is sufficiently small and if  $r$  is chosen appropriately. Equilibrium surplus destruction is increasing in  $r$  for small  $r$ , and decreasing in  $r$  for large  $r$ . Consequently, for any level of surplus destruction strictly below  $\eta(n)$ , and given some sufficiently small  $\Delta$ , there are always two values of  $r$  so that this level of surplus destruction is supported by a bargaining equilibrium.

For a more formal analysis, it is convenient to rewrite Eqn. (11) as follows:

$$\frac{e^{r(\Delta+t)}}{r} = \left( \frac{(1-\sigma t)^2}{\sigma} \right) (n-1).$$

For  $r \in (0, \infty)$ , and  $\Delta + t > 0$ , define the function  $\mu(r) = e^{r(\Delta+t)}/r$ , and consider its derivatives

$$\begin{aligned} \mu'(r) &= e^{r(\Delta+t)}(r(\Delta+t) - 1)r^{-2}, \\ \mu''(r) &= e^{r(\Delta+t)}((r(\Delta+t) - 1)^2 + 1)r^{-3}. \end{aligned}$$

Since  $\mu''(r) > 0$  for  $r > 0$ , we can use the first-order condition  $\mu'(r) = 0$  to show that  $\mu(r)$  attains the minimum at  $r = 1/(\Delta + t)$ , and, at that minimum, it evaluates to  $\mu(1/(\Delta + t)) = (\Delta + t)e$ . Moreover, given that  $\Delta + t > 0$ , we can verify that  $\mu(r)$  grows without bound in the limit as  $r \downarrow 0$  and in the limit as  $r \rightarrow \infty$ . Hence, we have the following lemma.

- Lemma 6.1.** *1. Let the parameters  $\Delta, n, \sigma$  and a time lapse  $t$  be such that  $(\Delta + t)e > \left( \frac{(1-\sigma t)^2}{\sigma} \right) (n-1)$ . Then, whatever the value of  $r$ , there is no bargaining equilibrium with the time lapse  $t$ .*
- 2. Let the parameters  $\Delta, n, \sigma$  and a time lapse  $t$  be such that  $(\Delta + t)e = \left( \frac{(1-\sigma t)^2}{\sigma} \right) (n-1)$ . Then, there is exactly one  $r$  such that the bargaining game  $G(\Delta, n, r, \sigma)$  has a bargaining equilibrium with time lapse  $t$ .*
- 3. Let the parameters  $\Delta, n, \sigma$  and a time lapse  $t$  be such that  $(\Delta + t)e < \left( \frac{(1-\sigma t)^2}{\sigma} \right) (n-1)$ . Then, there exist values  $\bar{r} > 1/(\Delta + t)$  and  $\underline{r} < 1/(\Delta + t)$  such that the bargaining*

games  $G(\Delta, n, \bar{r}, \sigma)$  and  $G(\Delta, n, \underline{r}, \sigma)$  each have a bargaining equilibrium with time lapse  $t$ .

The above lemma implies that a bargaining equilibrium with the time lapse  $t$  is only possible when the inequality  $(\Delta + t)e \leq (n - 1)(1 - \sigma t)^2/\sigma$  is satisfied. Since  $\Delta \geq 0$ , this implies the inequality  $te \leq (n - 1)(1 - \sigma t)^2/\sigma$ , which can be written in the quadratic form as

$$(\sigma t)^2 - \left(2 + \frac{e}{n - 1}\right)(\sigma t) + 1 \geq 0.$$

Taking into account the restriction that  $t \in [0, 1/\sigma]$ , this yields the solution

$$\sigma t \leq 1 + \frac{e}{2(n - 1)} - \sqrt{\frac{1}{4} \left(2 + \left(\frac{e}{n - 1}\right)\right)^2 - 1} =: \eta(n).$$

We have now derived an upper bound on the amount of surplus destruction which can occur in a bargaining equilibrium, whence the following theorem.

**Theorem 6.2.** *In a bargaining equilibrium with surplus destruction, we have  $\sigma t \leq \eta(n)$ .*

Notice that the upper bound  $\eta(n)$  on the equilibrium amount of surplus destruction does not depend on  $\sigma$  at all. Instead of saying that the level of surplus destruction in a bargaining equilibrium is bounded above by  $\eta(n)$ , we could equivalently say that the time lapse in a bargaining equilibrium is bounded above by  $\eta(n)/\sigma$ .

From the above argument, we can conclude that if  $\Delta = 0$ , then for any arbitrary value of  $\sigma$ , we can find  $r$  such that the bargaining equilibrium does involve the level of surplus destruction  $\eta(n)$ . In this sense, the upper bound on surplus destruction is “tight.” If  $\Delta > 0$ , however, then not all levels of surplus destruction below  $\eta(n)$  are possible in a bargaining equilibrium.

Observe that  $\eta(n + 1) > \eta(n)$  for any  $n \geq 2$ : More surplus destruction becomes possible as the number of players increases. If there are two players, then the surplus destruction is bounded above by  $\eta(2) \approx 0.22$ . For any  $n \geq 3$ , it holds that  $\eta(n) \geq \frac{1 - \eta(n)}{n}$ . That is, if there are at least three players, then for suitable choices of  $\Delta$  and  $r$ , there can be a bargaining equilibrium in which the amount of surplus which is destroyed exceeds the amount of surplus allocated to the “average” player. Destroying half of the surplus in equilibrium is possible if there are seven or more players. Notice that  $\eta(n)$  approaches one in the limit as  $n$  grows without bound. In principle, with a very large number of players, almost the entire surplus could be destroyed in equilibrium. This does not, however, contradict our earlier finding that a bargaining equilibrium with surplus destruction gives the proposer a higher payoff than the fixed schedule allocation. The reason is that the proposer’s fixed schedule allocation tends to zero as the number of players becomes sufficiently large.

We conclude that in the bargaining game under consideration, a higher number of players harms the efficiency of the bargaining process in two different ways: First, we have shown in Section 4 that an efficient bargaining equilibrium exists if and only if  $e^{r\Delta}/r \geq (n-1)/\sigma$ . For any fixed values of the other model parameters, this inequality fails to hold when  $n$  is too large. Indeed, the scope for an efficient bargaining equilibrium vanishes as the number of players becomes large. Second, we have shown in the present section that a bargaining equilibrium with surplus destruction can involve a higher level of surplus destruction the more players there are.

One more implication of Lemma 6.1 and of Theorem 5.6 is that a bargaining game  $G(\Delta, n, r, \sigma)$  does not have a bargaining equilibrium with surplus destruction if  $\Delta \geq \frac{n-1}{\sigma e}$ . If, however,  $0 < \Delta < \frac{n-1}{\sigma e}$ , then one can find an intermediate range of values for  $r$  such that the game  $G(\Delta, n, r, \sigma)$  admits a bargaining equilibrium with surplus destruction, while values of  $r$  which are lower or higher than this range lead to an efficient bargaining equilibrium. Using the continuity of  $\mu(r)$ , we can show that the equilibrium time lapse and the concomitant level of surplus destruction vanish in the limit as  $r$  approaches either end point of this range.

Finally, we note that a similar continuity property holds in the case where  $\Delta = 0$ . In that case,  $t$  is given by the equality

$$e^{rt}/r = \frac{(1 - \sigma t)^2(n-1)}{\sigma}.$$

We know from Theorem 5.6 that a bargaining equilibrium with surplus destruction exists if  $r > \sigma/(n-1)$ . Since  $t \in (0, 1/\sigma]$  in a bargaining equilibrium with surplus destruction, the right-hand side belongs to the interval  $[0, (n-1)/\sigma)$ , and is therefore bounded. For any fixed  $t$ , however, the expression  $e^{rt}/r$  grows without bound as  $r$  goes to infinity. Hence, in the limit as  $r$  goes to infinity, the equilibrium time lapse  $t$  must vanish. Now consider the limit as  $r$  approaches  $\sigma/(n-1)$  from above. In the limit, we find  $e^{rt}/r = (1 - \sigma t)^2$ , and hence  $t = 0$ . We conclude that the amount of surplus destruction in a bargaining equilibrium peaks for some intermediate value of the discount rate, but vanishes if players are either sufficiently patient or sufficiently impatient.

## 7 Conclusion

We have revisited a unanimity bargaining game in the tradition of Rubinstein (1982). In this kind of non-cooperative bargaining game, a proposer derives bargaining power from the loss of surplus which would occur if her proposal were rejected. The longer is the delay which occurs after such a rejection, and the more impatient the players are, the greater is the proposer's bargaining power. There is a vast literature on such unanimity



bargaining games in which both the time lapse and the players' impatience are exogenously given model parameters. We have augmented this type of unanimity bargaining game by modeling the time lapse following the rejection of a proposal as a strategic choice by the proposing player. The proposer can engage in an activity which costs a share of the surplus but prolongs the time lapse which would follow the rejection of her proposal. We have identified parameter conditions under which the proposer does not find it optimal to prolong the time lapse, and so the "standard" equilibrium results are replicated. Moreover, we have characterized parameter scenarios under which the proposer does destroy some amount of surplus in order to prolong the time lapse. In equilibrium, the proposer is better off than in a bargaining game in which the time lapse is exogenously fixed. For any given number of players, we have established a tight upper bound on the amount of surplus destruction. This bound depends only on the number of players. In the limit as the number of players becomes very large, the entire surplus can be dissipated. The relation between the number of players and surplus destruction is twofold: First, in a bargaining equilibrium with surplus destruction, the maximal amount of surplus destruction increases with the number of players. Second, the parameter region which admits an efficient bargaining equilibrium shrinks as the number of players increases.

# Appendix

## Proof of Lemma 3.1.

Suppose by way of contradiction that  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, and that for every  $k \in N$ , it holds either that  $\theta^k \notin A(t^k)$ , or that  $t^k = 1/\sigma$ . Then, on the path of equilibrium play, either there is perpetual disagreement, or the entire surplus is destroyed. In either case, all players receive zero payoffs. Hence, in the supposed bargaining equilibrium, we have  $\gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) = 0$  for all  $k \in N$ . Due to sincere voting, this implies that  $A(0) = \mathbb{R}_+^n$ . Take a Player  $i$ , and consider a history at which Player  $i$  choses a proposal and a time lapse. Suppose that he deviated from  $(\theta^k, t^k, A^k(t))_{k \in N}$  by choosing a zero time lapse along with the proposal  $(1/n, \dots, 1/n) \in A(0)$ . This proposal is unanimously accepted, and so the deviation is profitable for Player  $i$ .

□

## Proof of Lemma 3.2.

*Part 1.* Due to sincere voting,  $\theta^i \in A(t^i)$  implies  $\theta_j^i \geq (1 - \sigma t^i) e^{-r(\Delta + t^i)} \gamma_j((\theta^k, t^k, A^k(t))_{k \in N})$  for all  $i, j \in N$ . Suppose by way of contradiction that  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, but there are Players  $i \in N$  and  $j \in N \setminus \{i\}$  such that

$$\theta_j^i > (1 - \sigma t^i) e^{-r(\Delta + t^i)} \gamma_j((\theta^i, t^i, A^i(t))_{i \in N}).$$

Let  $\tilde{\theta}_j^i = \theta_j^i - \varepsilon$  and  $\tilde{\theta}_i^i = \theta_i^i + \varepsilon$  while  $\tilde{\theta}_k^i = \theta_k^i$  for all  $k \in N \setminus \{i, j\}$ . For  $\varepsilon > 0$  sufficiently small, it holds that  $\tilde{\theta}^i \in A(t^i)$ . Hence, making the proposal  $\tilde{\theta}^i$  instead of  $\theta^i$  is a profitable deviation for Player  $i$ .

*Part 2.* Since equilibrium proposals are required to be feasible, the inequality  $\theta_i^i \leq 1 - \sigma t^i - \sum_{j \in N \setminus \{i\}} \theta_j^i$  holds in a bargaining equilibrium. We have to show that it must hold with equality. Suppose not. Then, a proposal  $\tilde{\theta}^i$  defined by  $\tilde{\theta}_i^i = \theta_i^i + \varepsilon$  and  $\tilde{\theta}_j^i = \theta_j^i$  for  $j \in N \setminus \{i\}$  would be acceptable and still satisfy the inequality  $1 - \sigma t^i \geq \sum_{k \in N} \tilde{\theta}_k^i$  for  $\varepsilon > 0$  sufficiently small.

□

## Proof of Lemma 3.3.

**Step 1.** Suppose by way of contradiction that  $(\theta^k, t^k, A^k(t))_{k \in N}$  is a bargaining equilibrium, but there is  $j \in N$  such that  $\gamma_j((\theta^k, t^k, A^k(t))_{k \in N}) = 0$ . Suppose that there is some  $t' \in [0, 1/\sigma]$  such that

$$(1 - \sigma t') e^{-r(\Delta + t')} \sum_{k \in N} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) < 1 - \sigma t'.$$

Then, let  $\theta'$  be a proposal given by  $\theta'_k = (1 - \sigma t') e^{-r(\Delta + t')} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N})$  for every  $k \in N \setminus \{j\}$ , and  $\theta'_j = \varepsilon > 0$ . Due to the above inequality, the proposal  $\theta'$  is feasible for  $\varepsilon > 0$  sufficiently small. Moreover, due to sincere voting, it holds that  $\theta' \in A(t')$ . So, Player  $j$  can

profitably deviate from the supposed bargaining equilibrium by choosing  $(\theta', t')$ , thus receiving the share  $\theta'_j = \varepsilon > 0$  instead of the share  $\gamma_j((\theta^k, t^k, A^k(t))_{k \in N}) = 0$ . We have now shown by contradiction that

$$(1 - \sigma t)e^{-r(\Delta+t)} \sum_{k \in N} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) \geq 1 - \sigma t, \quad \forall t \in [0, 1/\sigma]. \quad (14)$$

**Step 2.** It follows from Lemma 3.1 that there is  $i \in N$  such that  $t^i < 1/\sigma$  and  $\theta^i \in A(t^i)$ . Due to sincere voting, this implies the inequality

$$(1 - \sigma t^i)e^{-r(\Delta+t^i)} \sum_{k \in N} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) \leq 1 - \sigma t^i.$$

But in Step 1 above, we have shown that the reverse inequality (14) holds for all  $t \in [0, 1/\sigma]$ , and in particular for  $t^i$ . Therefore,

$$(1 - \sigma t^i)e^{-r(\Delta+t^i)} \sum_{k \in N} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) = 1 - \sigma t^i. \quad (15)$$

**Step 3.** Due to Lemma 3.2, we have that  $\theta_k^i = (1 - \sigma t^i)e^{-r(\Delta+t^i)} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N})$  for every  $k \in N \setminus \{i\}$ . Moreover, since the proposal  $\theta^i$  is accepted, it holds by definition of  $\gamma_i(\cdot)$  that  $\theta_i^i = \gamma_i((\theta^k, t^k, A^k(t))_{k \in N})$ . Now the equality (15) derived in Step 2 above can be written as

$$(1 - \sigma t^i)e^{-r(\Delta+t^i)} \theta_i^i + \sum_{k \in N \setminus \{i\}} \theta_k^i = 1 - \sigma t^i.$$

But due to Lemma 3.2, it holds that  $\theta_i^i + \sum_{k \in N \setminus \{i\}} \theta_k^i = 1 - \sigma t^i$ . Hence, we conclude that  $(1 - \sigma t^i)e^{-r(\Delta+t^i)} = 1$ , and thus  $t^i = 0$  and  $\Delta = 0$ . If  $\Delta > 0$  in the bargaining game under consideration, then we have obtained a contradiction, and the proof of the lemma is complete. We have left to show the lemma for the case  $\Delta = 0$ .

**Step 4.** We are considering the case with  $\Delta = 0$ , and we have shown in Step 3 above that  $t^i = 0$ . Hence, the Eqn. (15) derived in Step 2 above simplifies to

$$\sum_{k \in N} \gamma_k((\theta^k, t^k, A^k(t))_{k \in N}) = 1.$$

Substituting into the Ineq. (14) derived in Step 1 above, and taking into account  $\Delta = 0$ , we find that

$$(1 - \sigma t)e^{-rt} \geq 1 - \sigma t, \quad \forall t \in [0, 1/\sigma].$$

The interval  $[0, 1/\sigma]$  is non-degenerate, hence there is  $t^* \in (0, 1/\sigma)$  which satisfies the inequality

$$(1 - \sigma t^*)e^{-rt^*} \geq 1 - \sigma t^*.$$

Since  $t^* < 1/\sigma$ , we can divide by  $1 - \sigma t^*$  to show that there is  $t^* \in (0, 1/\sigma)$  such that  $e^{-rt^*} \geq 1$ , the desired contradiction.  $\square$

**Proof of Lemma 3.5.** Let  $(\theta^k, t^k, A^k(t))_{k \in N}$  be a bargaining equilibrium. Now suppose that there is a Player  $j \in N$  such that  $\theta^j \notin A(t^j)$ . We have shown before that there is no perpetual disagreement in a bargaining equilibrium, and no player destroys the entire surplus. Thus, the subgame following rejection of  $\theta^j$  ends in agreement on the proposal  $\theta^i$  of some Player  $i$ . Denote the delay between the rejection of  $\theta^j$  and the acceptance of  $\theta^i$  by  $\hat{\tau}$ . Consider a deviation by Player  $j$ , who chooses  $(\tilde{\theta}^j, \tilde{t}^j) = (\theta^i, t^i)$  instead of  $(\theta^j, t^j)$ . Since  $\theta^i \in A(t^i)$ , this deviation leads to immediate acceptance of the proposal  $\tilde{\theta}^j$ , and Player  $j$  receives the share  $\tilde{\theta}_j^j = \theta_j^i$ . But in the supposed bargaining equilibrium, Player  $j$  receives that same share  $\theta_j^i$  without the delay  $\hat{\tau}$ . Moreover, combining Lemma 3.3 with Lemma 3.2, we see that  $\theta_j^i > 0$ . Therefore, due to impatience, the deviation is profitable unless  $\hat{\tau} = 0$ , as claimed by the lemma.  $\square$

**Proof of Lemma 5.7.**

Suppose that  $\bar{t} > 0$  is a time lapse chosen by all players in a bargaining equilibrium with surplus destruction, and let  $\bar{x} = \frac{r(1-\sigma\bar{t})^2}{\sigma+r(1-\sigma\bar{t})}$  be the “proposer share” in that bargaining equilibrium. Suppose by way of contradiction that  $\bar{x} \leq \left(\frac{1}{1+(n-1)e^{-r\Delta}}\right)$ . In this bargaining equilibrium, a Player  $j \in N$  accepts any proposal which gives him at least the payoff  $(1 - \sigma\bar{t})e^{-r(\Delta+\bar{t})}\bar{x}$ . In particular, due to the suppositions that  $\bar{t} > 0$  and  $\bar{x} \leq \left(\frac{1}{1+(n-1)e^{-r\Delta}}\right)$ , this implies that Player  $j$  accepts the proposal  $\tilde{\theta}$  defined as follows:

$$\begin{aligned}\tilde{\theta}_j &= (1 - \sigma\bar{t})e^{-r\Delta} \left( \frac{1}{1 + (n-1)e^{-r\Delta}} \right), \quad j \in N \setminus \{i\}, \\ \tilde{\theta}_i &= 1 - (n-1)(1 - \sigma\bar{t})e^{-r\Delta} \left( \frac{1}{1 + (n-1)e^{-r\Delta}} \right).\end{aligned}$$

In a bargaining equilibrium, there is no profitable deviation for Player  $i$ , so in particular, it is not profitable for this player to propose  $\tilde{\theta}$  and obtain the concomitant payoff  $\tilde{\theta}_i$ . Hence, we have the inequality  $\tilde{\theta}_i \leq \bar{x} \leq \left(\frac{1}{1+(n-1)e^{-r\Delta}}\right)$ . Substituting for  $\tilde{\theta}_i$  in this inequality yields

$$1 - \left( \frac{(n-1)(1 - \sigma\bar{t})e^{-r\Delta}}{1 + (n-1)e^{-r\Delta}} \right) \leq \left( \frac{1}{1 + (n-1)e^{-r\Delta}} \right).$$

Equivalent transformation yields  $(n-1)e^{-r\Delta}\sigma\bar{t} \leq 0$ , contradicting the supposition that  $\bar{t} > 0$ .  $\square$

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