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Costs of Change, Political Polarization, and Re-election Hurdles*

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Abstract

We develop and study a two-period model of political competition with office- and policy-motivated candidates, in which *(i)* changes of policies impose costs on all individuals and *(ii)* such costs increase with the magnitude of the policy change. We show that there is an optimal positive level of costs of change that minimizes policy polarization and maximizes welfare. One interpretation of this finding is that societies with intermediate levels of conservatism achieve the highest welfare and the lowest polarization levels. We apply our model to the design of optimal re-election hurdles. In particular, we show that raising the vote-share needed for re-election above 50% weakly reduces policy polarization and tends to increase welfare. Furthermore, we identify circumstances where the optimal re-election hurdle is strictly larger than 50%.

Keywords: elections; democracy; political polarization; costs of change; re-election hurdles; political contracts

JEL Classification: D7, H4

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1 Introduction

1.1 Motivation and model

Motivation

In this paper, we introduce a new dynamic model of two-candidate political competition. The novelty of the model is that changes in policies are costly for candidates and voters and such costs increase with the extent of the policy change. We use the model to address policy polarization both from a positive and normative perspective. Our study is motivated by three empirical facts: *(i)* policy changes are often costly; *(ii)* policy and party polarization have increased in recent times; *(iii)* policies are typically more moderate than the positions of parties. In the following we discuss each of these observations.

When a new policy is implemented, the so-called *costs of change* refer to expenditures or losses that occur only due to the presence of an established status-quo policy. Thus, these costs only materialize because actual policies shift, thereby enabling dynamic links between preferences and policies across periods. Costs of change can mainly occur either because additional investments are required to smooth the transition from the status-quo policy to the new policy or because the policy shift renders original investments in physical and human capital obsolete.[1] Whatever the source of the costs of change, they are ultimately borne by citizens—be it in the form of taxes, of higher prices for products and services, of low returns on assets, or of job destruction.[2] We provide several examples.

The shift with regard to military alliances is an example of a policy reversal that implies large transaction costs: *"The [...] motivation sustaining current alliance relations is the desire to avoid paying the transaction costs of creating alliances that are better aligned with current challenges. These costs are unquestionably significant, politically and militarily"*.[3] In the case of the Iraq War, estimates of 2006 indicate that demobilizing 20,000 out of the total 160,000 troops would have required $6-10bn—roughly one percent of the total war costs up to that date (Bilmes and Stiglitz, 2006). Another example of expenditures aimed at smoothing the transition from the status quo to a new policy is the Patient Protection and Affordable Care Act in the US (see Public Law (2010b)), which required additional efforts on the part of the government to facilitate that insurees, insurers, and health care providers comply with the new law. Quite often, the realization of policy changes also requires substantial government efforts in terms of

[1] Fixed costs brought about by investments related to new policies are considered in Section 9.1.

[2] See e.g. Hall (2013) on how changes in labor regulation affects social welfare through job losses.

[3] Kori Schake in *The American Interest*. `http://www.the-american-interest.com/2011/05/01/the-allies-we-need/` (retrieved 8 May 2015).

political bargaining, policy design, communication to citizens, and overcoming resistance from interest groups—efforts that would not be necessary if there were no established status quo. Fighting political resistance may thus entail opportunity costs for the parliament. One of the most famous examples in this respect is the Dodd-Frank Wall Street Reform and Consumer Protection Act (see Public Law (2010a)), which took up a large amount of Congress time before it was implemented in 2010.

Implementing new regulations on how to calculate risk and capital requirements—e.g. banking regulation (see Basel III[4])—render existing software and human capital to calculate risk and capital requirements obsolete. Analogous phenomena occur for environmental regulations, energy production laws, or health care regulations. A startling example of asset devaluation associated with policy changes is the immediate phase out of nuclear energy in Germany which has caused large write-downs of the value of existing capital as such capital becomes idle. Moreover, the implementation of such decisions will require additional investments to shut down the nuclear plants forever.[5]

In all the above instances the costs of change are not independent from the extent to which policies change and, in fact, the larger the policy change is, the higher are the costs for all citizens.[6] In this paper we focus on the type of policies mentioned above—i.e., policies whose change or reversal brings about significant costs that increase with the extent of the policy change.[7]

The second empirical observation that motivates the analysis set out in the present paper is the substantial increase of political polarization in recent times in the United States—and to a lower extent in Europe. This will be discussed in detail in Section 2. The term *political polarization* refers to various phenomena. While *social polarization* refers to the degree of polarization of the citizens' political preferences, *party polarization* refers to the gap between the ideal policies of

[4]See `http://www.bis.org/bcbs/basel3.htm` (retrieved 16 June 2015).

[5]One should also note that some policies are only effective in the long run. Aborting such policies before they pay off would cause losses. Via the Marshall Plan, for instance, the US granted loans to many European countries after World War II. Changing the US international affair policy toward isolation before the loans had reached their maturity could have incentivized some of the indebted countries not to pay back the loans, thereby imposing costs on the US.

[6]We stress that, as it is the case in our model, citizens may have different preferences over policies. Hence, citizens and politicians are alike only from the point of view of costs of change. Moreover, it will suffice in our model that the perceived—but not the real—costs of change be the same for all politically-active voters.

[7]Of course, there exist policies which can be reversed without causing barely any cost. However, even for policies whose implementation does not cause large costs—such as legalizing gay marriage for example—a reversal would be costly due to the partners' investments into their relationships.

the different political parties.[8,9] Finally, office-holders from different parties implement different policies. The degree of *policy polarization* refers to the difference between these implemented policies.[10] The empirical literature suggests that in the United States political polarization have increased in the last decades.

Finally, our paper is motivated by the evidence that policy polarization tends to be lower than party polarization (Wiesehomeier and Benoit, 2009). That is, office-holders' policy choices tend to be more moderate than their parties' ideal policies. The results shown in our paper replicate this observation and thus they offer an explanation for the partial misalignment between the policies implemented and the positions of parties.

Model

We develop a model in which changes of policies are costly and they increase with the magnitude of the policy change. The model enables us to study how these *costs of change*—as well as social and party polarization—impact on policy polarization and how optimally chosen *re-election hurdles* could reduce the latter and increase welfare. A re-election hurdle denotes the vote-share threshold that an incumbent has to attain in order to be re-elected. It will be sufficient to consider a two-period setting.

At the beginning of each period, two candidates, whose ideal policies coincide with the ideal policies of their parties, compete in an election for an executive office.[11] Once in office, the winning candidate faces issues in two different dimensions. On the one hand, he has to decide with regard to a *policy* on which citizens have dissenting preferences; on the other hand, he carries out a *public project* that benefits every citizen, including himself, in the same way. Hence, office-holders always carry out the public project. Candidates diverge in their *ability* to carry out this public project, and if they belong to different political parties, they also diverge in their preferred policy position. The ability of a candidate is only learned by everyone (the candidate himself, the other candidate, and the electorate) at the end of his first term. A candidate who loses an election is replaced in the next election by a new candidate from the same political party (i.e. with the same policy preference) but possibly with a different degree of ability.

[8]Some authors, e.g. Krasa and Polborn (2014), use the term *ideological polarization* instead of social polarization.

[9]The definition of party polarization given here applies to two-party systems—like the one we consider in this paper—but could be adapted to multi-party systems.

[10]If parties' ideal policies (resp. candidates' policy choices) are located symmetrically with respect to the median voter's ideal policy, party polarization (resp. policy polarization) can be equivalently defined by the distance between parties' ideal policies (resp. candidates' policy choices) and the median voter's preferred position.

[11]Our model can also be applied to two-candidate races for parliamentary seats.

While social and party polarization are given, office-holders endogenously choose policies in both periods. Consequently, the level of policy polarization arises endogenously, and this is the object we primarily focus on. As set out above and in contrast to standard models, we assume that changing the policy from the first period to the second period is costly and, moreover, this cost of change will increase with the extent of the policy change.[12]

1.2 Main Results and broader implications

Positive results

It is intuitive that costs of change will constitute an impediment for candidates setting out to implement their preferred policies once they are in power. The reason is that choosing their ideal policy positions may require large policy changes which might not be supported by a majority of voters when such a policy is crucial for the political debate. With regard to the Iraq War in the US presidential elections of 2004 and 2008, for instance, it is very likely that the status-quo policy—i.e. the decision to go to war in the first place—may have significantly affected the voting decision of many voters that were concerned about which candidate could bring the war to a satisfactory end.

An important result of our paper is that costs of change always moderate the policy choices of policy-makers, thereby increasing their chances of re-election. Furthermore, we show that the degree of the reduction in the level of policy polarization varies with the marginal cost of change, but in a non-monotonic way.[13] We prove that there is a range of moderate marginal cost levels at which policy polarization is minimal but remains positive. The more marginal costs differ from this range of values, the closer policy polarization will come to party polarization. In the limit where there are either no costs of change or the marginal cost of change is infinite, policy polarization matches party polarization.

Our comparative statics analysis regarding variations in the marginal cost of change reveals further insights in relation to welfare, where the latter is measured by utilitarian welfare or, equivalently in our model, by the median voter's utility. Indeed, costs of change weakly increase welfare. Moreover, welfare is single-peaked in the marginal cost of change. In other words, there exists a moderate level of marginal costs of change for which welfare is higher than when there are either no costs of change or marginal costs of change are very large. Therefore, moderate costs associated with changes in policies not only minimize policy polarization but also have a

[12] Although in the basic model we consider costs of change to be linear in the absolute difference between the policies adopted in the two periods, in the last part of the paper we analyze the robustness of our results by investigating other, more general specifications for these costs.

[13] Our main positive results are given in Theorem 2.

socially beneficial effect.[14]

Normative results: optimal re-election hurdles

Because the currently observed high levels of policy polarization may be a concern for the society, we also take a normative approach and ask whether higher re-election hurdles for incumbents might reduce policy polarization. Increased re-election hurdles have been advocated as a way of counteracting socially detrimental incumbency advantages (Gersbach, 2010), and they have actually been introduced in democracy.[15] For instance, in the Canton of Zurich, long-term Social-democrat deputies of the Swiss National Council need two thirds of the votes to be eligible for the next term (see Art. 7 in SP Kanton Zürich (2010)).[16]

When electoral support for the incumbents tends to persist, re-election hurdles for incumbents can also emerge implicitly as a combination of two existing institutional features of the political system: term limits and qualified majorities to amend the rules regulating such limitations. Indeed, to stay in office beyond the limit imposed by law, the incumbent mostly needs a majority of more than 50% to remove such limitations.[17] In the US, for instance, a two-term limit for the presidential election exists at the constitutional level, and numerous states and cities have implemented similar rules both for the executive and the representative branches. An instance of a legislative modification aimed at enabling an extra term for the incumbent occurred in the elections for the mayor of New York in 2009. In this example, Michael Bloomberg succeeded in staying in office for a third term after the City Council of New York agreed to modifiy the two-term limit that was in place.

In this paper we investigate the effect of increased re-election hurdles on policy polarization. We use the term *extra-hurdle* to denote a share of votes that, when added to a 50% share of the votes, constitutes the re-election hurdle, i.e. the vote-share threshold that an incumbent has to attain in order to be re-elected. Note that if an incumbent does not obtain enough votes

[14]We stress that when they materialize, costs of change generate a disutility for both voters and candidates.

[15]There is a large literature on the existence and causes of incumbency advantages of US congressmen (see e.g. Alford and Brady (1989), Gelman and King (1990), and Levitt and Wolfram (1997)). Erikson et al. (1993) find that governors have similar advantages when seeking re-election.

[16]For the 2011 elections of the Swiss National Council, three candidates had to reach this two-third majority in order to be nominated for the elections—one of them did not reach the threshold: see `http://www.nzz.ch/aktuell/startseite/sp-nationalraetin-anita-thanei-quorum-1.10577885` (retrieved 13 May 2015).

[17]Note that unlike explicit extra-hurdles, implicit extra-hurdles operate along two elections: if, e.g., in the case of a two-term limit, the vote-share obtained by the incumbent—and his supporting coalition—in the election following his first term is large enough, he can pass a law to remove the term limitation; the incumbent can then stand for a second re-election only if he obtained a large support in the previous election. Under the assumption that incumbents can preserve or increase their vote-share, both implicit and explicit extra-hurdles would, however, bring similar outcomes.

to attain the re-election hurdle, the challenger will be elected.[18,19] In our model with positive costs of policy change, we find the following four results regarding the effects of extra-hurdles on policy polarization and welfare:[20]

The first result shows that higher re-election hurdles induce office-holders to adopt policies in the first period that are closer to the median voter's ideal policy, so policy polarization declines with non-zero extra-hurdles. In our model, the fact that positive extra-hurdles yield more moderate first-period policies than a zero extra-hurdle is triggered by two forces.

On the one hand, when the share of the electorate needed for re-election is higher than half, the critical voter for incumbent support is no longer the median voter but a more partisan voter whose ideal policy is closer to the challenger's than to the incumbent's ideal policy. Accordingly, a positive extra-hurdle raises the office-holder's interest in choosing a policy closer to the median voter's preferred policy. By doing so, the incumbent can become more attractive for voters on the other side of the political spectrum because policy changes in the second period will be more moderate if the incumbent wins the election.[21] This will occur despite the fact that the challenger's ideal policy is closer to the critical voter's ideal policy, as the election of the challenger would cause significant costs of change.

On the other hand, positive extra-hurdles can reduce the chances of an incumbent being re-elected. This might in principle induce him to try steamroller tactics and choose a policy in the first period that is as close as possible to his preferred policy, as this would subsequently keep his own policies and the policies of the challenger closer to his ideal policy position. However, as the latter will undertake a significant change of policy if he is elected, the office-holder will prefer to moderate his policy choice in exchange for reducing the costs associated with the likelier event of the challenger being elected in the second election.

Overall, we show that there exists a critical positive extra-hurdle such that, when the actual extra-hurdle is lower than the critical level, the first effect will moderate the policy chosen in the first period, while when the actual extra-hurdle is above the critical level, the second effect will do so.

[18] The implementation of an extra-hurdle that is strictly larger than 50% but below 100% can be viewed as a weaker and flexible form of term limits. Similarly to the latter, extra-hurdles treat incumbents and challengers asymmetrically.

[19] An alternative would be to set up a second competition involving two new candidates if an incumbent has been deselected for not surpassing the re-election hurdle but has nevertheless attained a vote-share above 50%. This modification would ensure that any office-holder is elected with at least half of the votes. The analysis of this alternative lies beyond the scope of this paper and is left for further research.

[20] Our main normative results are given in Theorem 1.

[21] The utility of office-holders depends on being in office, on the policy chosen, and on the level of public project provision. We assume that the desire to hold office is sufficiently strong to ensure that, once in power in the first period, candidates will choose a policy ensuring that their re-election chances are maximized.

The second result concerns the impact of extra-hurdles on welfare. The welfare of the median voter depends on three components: the policies chosen in both periods, the public project carried out in each period, and the costs associated with policy changes. Raising the extra-hurdle from 0% to some level below the critical level mentioned above neither affects expected benefits from public projects nor the likelihood that an incumbent will be deselected. But it reduces policy polarization and expected costs of change, and thus increases welfare. If an extra-hurdle above the critical level is implemented, policy polarization is still reduced compared to no extra-hurdle being implemented. However, in addition to this, the likelihood of incumbent deselection rises, which leads to higher expected costs of change. Nevertheless, we find that, compared to a usual 50% majority, an extra-hurdle above the critical level increases welfare, as the reduction in policy polarization compensates the increase in expected costs of change.[22]

Our third result is that the critical extra-hurdle mentioned above is *optimal* in the sense that it maximizes welfare and minimizes policy polarization. We also show that below this optimal extra-hurdle, a marginal increase of the extra-hurdle always weakly increases welfare and weakly reduces policy polarization. Because the optimal extra-hurdle depends on the parameters of the model, marginal changes in re-election hurdles are a cautious approach focused on directional welfare improvements. Moreover, such an optimal extra-hurdle is always positive, and it is therefore associated with a re-election hurdle higher than 50%.

Finally, for the sake of our fourth result, we allow candidates to offer binding extra-hurdles themselves instead of complying with the re-election hurdle chosen by the public. We show that in this scenario candidates will endogenously choose a welfare-maximizing extra-hurdle. Binding extra-hurdles might take the form of a *re-election hurdle contract*.[23] In such a contract, a candidate would stipulate an extra-hurdle that, when added to a 50% share of the votes, constitutes the re-election hurdle that is applied when he seeks re-election.

Broader implications

On the one hand, the preceding results may have broader implications on democracy. Although our notion of welfare already accounts for policy polarization in one of its components, there are additional reasons to be concerned about policy polarization per se. Indeed, as we will set out below, the literature suggests that significant policy polarization may have negative mid- and long-term consequences on the functioning of democracy and the quality of its outcomes.

[22] In the US, the success rate of the governors that have sought re-election since World War II is about 72%. Indeed, see `http://governors.rutgers.edu/on-governors/us-governors/when-governors-seek-re-election` (retrieved 13 May 2015). Due to this incumbency advantage, it seems unlikely that moderate extra-hurdles will strongly reduce the incumbents' re-election chances.

[23] This is a special political contract. On the theory and implementation of such contracts, see Gersbach (2012).

Our results suggest that higher re-election hurdles can reduce policy polarization and thus also decrease additional perils associated with the latter.

On the other hand, both the model and the costs of change can be interpreted in a broader sense. The latter, in particular, may capture psychological fear of change or aversion to the perceived ambiguity of the outcomes of policy changes instead of costs in material terms. The higher such psychological costs, the higher the degree of conservatism of the society. Typically, costs of the latter type would be expected to rise with the extent of the policy change. For instance, small adjustments in the competition rules for health-care providers may be associated with negligible psychological costs for the citizens, but such costs may be large if a large-scale reform of the health care system—like the one in the Patient Protection and Affordable Care Act in the US—is undertaken. In the latter case, limited information about a multitude of intended and unintended consequences of these large changes may manifest itself as ambiguity about benefits and costs for citizens and as aversion towards such ambiguity translating into psychological costs.[24]

The rest of the paper is organized as follows: In Section 2 we review the papers related to our article. In Section 3 we outline our baseline model, where costs of change are linear in the absolute difference between the policies chosen in the two periods. In Section 4 we compute equilibrium policy choices and re-election probabilities as a function of the extra-hurdle. In Section 5 we study the implications of extra-hurdles on policy polarization and welfare and set out our main positive and normative results. In Section 6 we modify the model and allow candidates to offer re-election hurdle contracts before the first election. We show that contracts are offered such that optimal extra-hurdles are implemented. In Section 7 we show that our main results remain unchanged if we allow the citizenry to have polarized ideal policy positions. In Section 8 we extend the model so that it accounts for a status quo policy. In Section 9 we study the case where costs of change are not linear. Section 10 concludes.

2 Relation to the Literature

The present paper is related to several strands of the literature.

Electoral competition

Our model of candidate competition for winner-take-all elections shares features with the stan-

[24]There may be instances in which information is deliberately withheld from the public by office-holders, which contributes to the psychological costs as citizens remain unsure whether they have access to the most important information.

dard literature on electoral competition. First, as in Hansson and Stuart (1984), Duggan and Fey (2005), and Krasa and Polborn (2010a,b) for example, each candidate is both office-motivated and policy-motivated and has some exogenous characteristics: in our model, his most preferred policy and his randomly chosen ability.

Second, we consider a policy on which agents have dissenting preferences and a public project regarding which all citizens have common preferences. Voters and candidates have quadratic—and hence single-peaked—preferences over policies chosen in the two periods.[25] Moreover, due to the existence of costs of change, they also care about the difference between the two policies. As a consequence, agents' preferences regarding pairs of policies chosen in different periods are not separable in time. Non-separable preferences not related to costs of change are considered e.g. in Krasa and Polborn (2014).

Costs of change in policies

To the best of our knowledge, hardly any models in the literature have considered costs associated with changes in policies. An exception is Glazer et al. (1998), who show that if the fixed costs of change are large, the incumbent may choose a policy that is more extreme than his own ideal policy and the electorate may decide to vote for him to prevent a policy change.

The main novelty of our paper is not to show that costs associated with policy changes can be exploited strategically, but to analyze how the *extent* of these costs may influence society. We show in particular that when the marginal cost of change is intermediate, the existence of costs of change turns out to be beneficial to the society, as they reduce policy polarization.

Recently, Forand (2014) and Nunnari and Zápal (2014) have examined infinite-horizon models where candidates can fully commit to a policy before the election and where policies are bound not to change as long as the office-holder stays in office. Our paper is complementary to this work because in our model, the costs of change affect all subsequent office-holders, but they only offer a partial commitment device. Moreover, our main focus is how the existence of such a partial commitment tool can reduce polarization and increase welfare.

Policy commitment

In electoral competition models, it is a divisive issue whether candidates can commit to policy positions or not. In the classic formulation of Hotelling (1929) and Downs (1957), politicians can do so. Yet, various other strands of literature, most notably models of political accountability, assume that competitive elections are not enough to guarantee—by means of electoral rewards

[25]The assumption that preferences are quadratic is not crucial, but it considerably simplifies the exposition of the results.

and punishments—that politicians who do not stick to their promises are deselected. Standard theoretical models of electoral accountability—see Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989), Persson et al. (1997) or Ashworth (2012) for a recent review—contain two basic elements: *(i)* an electorate that decides whether to re-elect an incumbent or elect a challenger based on some performance measure, and *(ii)* an incumbent that can anticipate the electorate's behavior and respond to it by choosing a certain action during his tenure before elections are held.

Accordingly, our model can be interpreted as a model of imperfect accountability—with two periods separated by an election—in which the costs of policy change allow the first-period incumbent to partially commit to a particular policy. The present paper tries to bridge the aforementioned divide between full and lack of commitment for politicians. With partial commitment, the first-period policy choice provides an anchor for future behavior, as it influences the voters' decisions and potential second-period candidates' policy choices. One of the main results in our paper points out that the existence of limited commitment—in the form of costs of change—can induce some moderation in the policy choices of office-motivated politicians who care about policies, without reducing welfare.

Political polarization

There is a large body of literature that deals with political polarization, both from a theoretical and an empirical point of view. The literature informs us that, in a democracy, all three manifestations of political polarization (party, social, policy) can potentially influence each other. Our paper adds to this literature by *(i)* investigating how the degree of policy polarization arises endogenously from party polarization, social polarization, and costs of change, and *(ii)* proposing a novel institutional feature of winner-take-all elections that curbs policy polarization without creating welfare losses.

The literature on polarization has focused on the existence of and trends in political polarization, their causes and their consequences. We briefly review this literature. In the case of the United States, the existence and rapid increase of policy polarization is well-established, as demonstrated e.g. by the evolution of the voting patterns of Democratic and Republican legislators of the House of Representatives and Senate (see e.g. Poole and Rosenthal (1984, 2001), McCarty et al. (2006), and Theriault (2008)). Whereas the preferences of an ample fraction of citizens may have been less subject to polarization, the preferences of citizens with high partisan identification have become more polarized. This has led to an increase in social polarization and, in turn, to a rise in party polarization, as the views of partisan citizens help shape party ideology (McCarty et al., 2006).

The causes for the increase in political polarization in the case of the US Congress are diverse. On the one hand, an increase in party polarization seems to be a consequence of the geographical changes in partisan alignment (see e.g. Rohde (2010) or Theriault (2004)). For instance, the south has become more Republican and, in turn, the policies proposed by the politicians hoping to win a seat have shifted to the right. Moreover, Layman et al. (2006) point to incumbent-friendly redistricting. These forces have weakened the potential opposition within each district and have made policies more coincident with the preferences of their primary constituency, which is usually dominated by extreme voters. On the other hand, Theriault (2006) suggests that the degree of party and policy polarization has been promoted by the way in which roll-call votes are structured and the issues decided by those roll calls, while other authors have pointed to changes in the legislative agenda and the strategies of party leaders (Roberts and Smith, 2003) or to the leadership selection system (Heberlig et al., 2006), in which extreme candidates tend to excel. More generally, Grosser and Palfrey (2014) suggest that high party polarization may arise in the context of open, take-it-all elections when citizens are poorly informed about candidates' ideal policies.

Whether high levels of political polarization are beneficial or detrimental for a society is not clear a priori. Clearly, there may be strong negative consequences. For instance, increased party polarization may undermine the trust in the policy-making process and lead to legislative gridlock (Jones, 2001; Binder, 1999). Moreover, high party polarization encourages disinterest in politics, party disidentification, and a decline in turnout (Fiorina et al., 2005). The vast literature on political polarization has pointed to a number of other effects—some negative, some positive—triggered by high political polarization levels.

First, increased party polarization has driven the political debate in America to stark confrontation, which discourages open deliberation on policy issues especially in the media (Sinclair, 2002) but also in Congress (Jamieson and Falk, 2000). Second, heterogeneous societies tend to be socially polarized and are more likely to be ruled by governments that choose bad policies (see Alesina and La Ferrara (2005) for a comprehensive overview) and are more prone to conflict (see Montalvo and Reynal-Querol (2005)). Third, Testa (2012) shows that societies where social polarization is large are more likely to be characterized by high party polarization, but this latter feature helps to control the government by the electorate, as it raises electoral stakes. Fourth, increasing party and policy polarization levels have clarified party importance (Hetherington, 2001) and have made it easier for the citizens to vote ideologically as party platforms are significantly different from each other. Lastly, we note that trends in economic inequality and policy polarization have largely moved in tandem over the last 50 years (McCarty et al., 2006).

3 The Model

3.1 General set-up

We examine a two-period model ($t = 1, 2$) in which, before each period begins, a society elects an office-holder for an executive position to whom it delegates policy-making. The society is composed of a continuum of voters of measure 1, with each voter being indexed by $i \in [0, 1]$. Policy-makers differ in their *policy orientation*; there are right-wing and left-wing policy-makers. We denote by R and L the pool of all right-wing and left-wing policy-makers, respectively. We assume that there are at least two candidates in each pool. At both election dates, two candidates—one from R and one from L—compete for office. The defeated candidate from the first election does not run for office in the second election, and he is replaced by another candidate. Hence, while the first election is an open race, i.e. a race between two new candidates, in the second election one candidate is the incumbent and the other candidate is the challenger. Throughout the paper, we denote policy-makers by k, k', or k''. Independently of his policy orientation, each candidate is one of three *types*: Candidate $k \in R \cup L$ is characterized by his *ability* a_k, which is drawn from a discrete symmetric distribution with support $\{-A, 0, A\}$, where $A > 0$. We assume that

$$P\{a_k = 0\} = \rho \quad \text{and} \quad P\{a_k = A\} = P\{a_k = -A\} = \frac{1-\rho}{2}, \tag{1}$$

where $\rho \in (0, 1)$.[26] A candidate with ability A has outstanding ability. If a candidate has ability $-A$, he is considered to have very low skills. We note that $E[a_k] = 0$, so the ability of a candidate with zero ability coincides with expected ability.[27,28] In each period $t \in \{1, 2\}$, the office-holder, denoted by $k \in R \cup L$, faces issues in two different dimensions:

[26] We will see in Section 4.2 that, according to our model, the larger ρ is, the larger will be the advantage that incumbents enjoy in elections against challengers. That is, ρ is correlated with the incumbency advantage. For an illustration of the bias towards incumbents in the US, see `http://governors.rutgers.edu/on-governors/us-governors/when-governors-seek-re-election` (retrieved 13 May 2015). Since World War II, the re-election rates of governors have increased almost every decade up to nearly 90% in the period 2010–2013.

[27] For simplicity and without loss of generality we normalize expected ability to zero. Note that this normalization does not affect our results.

[28] We stress that the ability of the challenger in the second election is again drawn from $\{-A, 0, A\}$, according to the probabilities given in (1).

- *Public Project P:* He undertakes a public project, denoted by g_{kt}. For simplicity, we assume that the output of the project is directly proportional to the ability of the office-holder, i.e.

$$g_{kt} = a_k.^{29}$$

Hence, with regard to the public project, only the ability of the office-holder matters. The public project includes all business-as-usual activities of the government and office-holders always carry out P.

- *Policy I:* He chooses a policy I from a one-dimensional policy space $[0, 1]$, which impacts each voter and each candidate differently. We use $i_{kt} \in [0, 1]$ to denote his policy choice.

Candidates in their first term observe their ability during or after the public project P has been realized. For each period $t \in \{1, 2\}$, voters observe both i_{kt} and g_{kt} during the term. From g_{kt} voters can infer the ability of the incumbent in the first term. Accordingly, at the end of period $t = 1$ and before the second election takes place, the ability of the incumbent is common knowledge. The sequence of events is summarized in Figure 1.

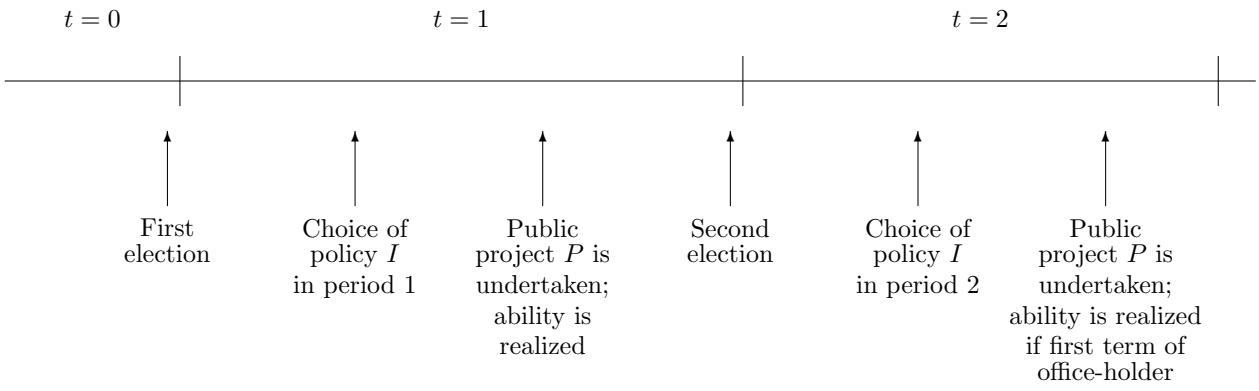


Figure 1: Timeline of the sequence of events.

3.2 Utilities of voters and policy-makers

3.2.1 Instantaneous utilities

Voters and policy-makers derive utility from P and I. Suppose that $k \in R \cup L$ is in office in period t. First, all voters and policy-makers derive the same utility from the public project,

[29] Any linear production function $g_{kt} = \gamma a_k$, with $\gamma > 0$, would yield the same results. To reduce notational complexity, we set $\gamma = 1$. Moreover, by choosing an alternative functional form for g_{kt}, such as $g_{kt} = a_k + d$, with $d > A$, one could ensure that output is always positive.

given by the instantaneous utility function

$$U^P(g_{kt}) = g_{kt}.$$

Second, agents have dissenting preferences on policy I. We order voters according to their most preferred choice of I, thus voter i's ideal policy regarding I is i. Then voter $i \in [0,1]$ derives utility from a choice i_{kt} in period t according to

$$U_i^I(i_{kt}) = -(i_{kt} - i)^2.$$

Note that the assumption of a continuum of voters indexed by $i \in [0,1]$ implies that the voters' ideal policies are uniformly distributed in the electorate. Arbitrary candidates $k' \in R$ and $k'' \in L$ have ideal policies $\mu_{k'} = \mu_R$ and $\mu_{k''} = \mu_L$, respectively. We assume that

$$\frac{1}{2} < \mu_R \leq 1 \quad \text{and} \quad \mu_L = 1 - \mu_R. \tag{2}$$

Hence, the ideal policies of the candidates are distributed symmetrically around the median position. If $i_{kt} \in [0,1]$ has been chosen by office-holder k in period t, a candidate $k' \in R \cup L$ derives utility

$$U_{k'}^I(i_{kt}) = -(i_{kt} - \mu_{k'})^2$$

from policy I. Additionally, an office-holder obtains private benefits $b > 0$ from holding office in a particular period. The benefits b account for all sources of utility that politicians derive from office beyond policy choices. Those benefits include such things as ego rents and satisfaction from holding power and heading a branch of government, plus additional career opportunities after office-holding.

3.2.2 Linear costs of change

A key feature in our model is that policy changes are assumed to be costly for voters and candidates. More precisely, given a policy $i_{k1} \in [0,1]$, the policy choice in the second period, $i_{k'2} \in [0,1]$, imposes additional costs (or utility losses) on voters and policy-makers alike, equal to

$$U^c(i_{k1}, i_{k'2}) = -c \cdot |i_{k1} - i_{k'2}|. \tag{3}$$

The parameter $c > 0$ is the marginal cost of a policy change.[30] Hence, the so-called *costs of change* linearly increase with the absolute difference between the policies adopted in both periods. The existence of such costs has been discussed in detail in the Introduction.

[30] We assume that c is a common parameter for voters and policy-makers. As long as the costs are low, assuming otherwise does not change our results qualitatively but complicates notation unnecessarily.

We point out that the specification of the costs of change given in (3) does not include the possibility that these costs may be non-linear. In Section 9 we analyze convex cost functions and fixed costs of change.

3.2.3 Lifetime utility

Voters and policy-makers discount utility in the second period with a common factor β, where $0 < \beta \leq 1$. Let candidates $k \in R \cup L$ and $k' \in R \cup L$ be in power in $t = 1$ and $t = 2$, respectively. Then the lifetime utility of any voter $i \in [0, 1]$ is given by

$$U^P(g_{k1}) + U_i^I(i_{k1}) + \beta \cdot \left[U^P(g_{k'2}) + U_i^I(i_{k'2}) + U^c(i_{k1}, i_{k'2})\right]. \tag{4}$$

Similarly, the lifetime utility of any policy-maker $k'' \in R \cup L$ is given by

$$U^P(g_{k1}) + U_{k''}^I(i_{k1}) + \mathcal{I}_{k''}(k) \cdot b + \beta \cdot \left[U^P(g_{k'2}) + U_{k''}^I(i_{k'2}) + U^c(i_{k1}, i_{k'2}) + \mathcal{I}_{k''}(k') \cdot b\right],$$

where $\mathcal{I}_{k''}(\cdot)$ is an indicator variable for holding (resp. not holding) power, i.e.

$$\mathcal{I}_{k''}(x) = \begin{cases} 1 & \text{if } k'' = x, \\ 0 & \text{otherwise.} \end{cases}$$

3.3 Majority rule and tie-breaking rules

In the first election, a candidate is elected according to the simple majority rule. We assume that, in the case of a tie, each candidate wins the election with a probability equal to $\frac{1}{2}$.

In the second election, we allow a potentially different majority rule to be used in deciding who will hold office. Specifically, the incumbent is re-elected if his vote-share in the second election is equal to or larger than $\frac{1}{2} + \delta$, where $\delta \in \left[0, \frac{1}{2}\right]$. The value of δ measures the increase of the re-election hurdle for the incumbent; we therefore call it *extra-hurdle*. For instance, $\delta = 0$ corresponds to the simple majority rule, while $\delta = \frac{1}{2}$ implies that an incumbent is re-elected only if he obtains unanimous support. If the incumbent's vote-share is strictly less than $\frac{1}{2}+\delta$, he will be deselected and the challenger wins the election. For ease of presentation, the incumbent is assumed to win the election if he receives a vote-share of exactly $\frac{1}{2} + \delta$.[31]

3.4 Assumptions on the parameters

For the sake of analytical tractability, we assume that policy-makers' benefits from holding office, b, are sufficiently large to ensure that any policy-maker will prefer being in office to not

[31] Without this assumption we would need to discretize the set of possible policy choices in order to guarantee the existence of a best response of the office-holder in $t = 1$.

being in office under any constellation of parameters that we analyze in the model.[32] This assumption guarantees that, in equilibrium, the office-holder's policy choice always maximizes his re-election probability. Moreover, A is assumed to be large enough to ensure that, in equilibrium, an incumbent with ability A is always re-elected, while an incumbent with ability $-A$ never is.[33] To simplify notation, we further suppose that $\beta = 1$, as results can easily be extended to any value of $\beta \in (0, 1]$. Finally, all voters are assumed to vote sincerely, i.e., each of them votes for the policy-maker from whom he expects the highest utility.[34,35]

3.5 Notion of equilibrium and informational assumptions

Voters observe the policy choice and the output of the public project during the term. From the latter they infer the ability of the incumbent. An equilibrium in our model is a perfect Bayesian Nash equilibrium of the game displayed in Figure 1. We use $\mathcal{G}$ to denote this game.

4 Analysis of the Game

In this section, we determine the equilibria of game $\mathcal{G}$ in order to assess how key parameters and variables determine policy choices. Because left- and right-wing candidates' ideal policy positions are symmetrically distributed around the median (see (2)) and are common knowledge, and since the distribution of a_k is independent of the policy orientation of candidate k, the median voter is indifferent between the left-wing and the right-wing candidate when the first election takes place and each candidate receives a vote-share of $\frac{1}{2}$ in the first election.[36] According to the election rule, both candidates have a 50% chance of winning the first election. We assume that, without loss of generality, a right-wing politician is in office in the first period. Let $\mathcal{G}^R$ (resp. $\mathcal{G}^L$) be the game that starts after a candidate from pool R (resp. L) has been chosen as first office-holder. We note that once the equilibria of $\mathcal{G}^R$ have been determined, the equilibria of $\mathcal{G}^L$, and therefore of $\mathcal{G}$, immediately follow by symmetry. In the following, we solve $\mathcal{G}^R$ by looking for sequentially rational equilibria.

[32] It will suffice to assume that $b > 2 + c + 4A$.

[33] It will suffice to assume that $A > 1 + c$.

[34] As there are binary decisions in each period, there is nothing to be gained from strategic voting.

[35] This assumption mirrors the voting behavior of individuals in a large but finite society when each individual might be pivotal.

[36] Whether indifferent voters vote for the left- or the right-wing candidate does not influence the outcome of the election. Indeed, since the median voter has measure zero, the election is tied, independently of his behavior.

4.1 The second period

We start with the analysis of the policy-makers' behavior in the second period. As there is no further election, the office-holder will choose his policy to maximize his expected utility at the beginning of $t = 2$.

Proposition 1
Let $k \in R$ and $k' \in R \cup L$ be the office-holders in $t = 1$ and $t = 2$, respectively. In $t = 2$, the best response of k' to a policy i_{k1} chosen in the first period is given by

$$i_{k'2}(i_{k1}) = min\left\{max\left\{\mu_{k'} - \frac{c}{2}, i_{k1}\right\}, \mu_{k'} + \frac{c}{2}\right\} =: \begin{cases} i^*_{R2}(i_{k1}) & \text{if } k' \in R, \\ i^*_{L2}(i_{k1}) & \text{if } k' \in L, \end{cases} \tag{5}$$

where $\mu_{k'}$ is the ideal policy of policy-maker k'.

Proof: See Appendix A.

The expression in (5) comprises several cases in one compact formula. It is thus useful to provide intuition and some special cases. We first observe from Proposition 1 that for any given first-period policy choice $i_{k1} \in [0, 1]$, the second-period responses of right- and left-wing policy-makers satisfy

$$i^*_{R2}(i_{k1}) \geq i^*_{L2}(i_{k1}),$$

as right-wing candidates gain more (or lose less) from shifting policies to the right than left-wing candidates. Second, in order to assess the impact of costs of change on policy choices in $t = 2$, the following corollary, which immediately follows from Proposition 1, is useful.

Corollary 1
Suppose that $k \in R$ is the office-holder in both periods and that he has selected $i_{k1} \leq \mu_R$. Then,

(i) *If $c = 0$, then $i^*_{R2}(i_{k1}) = \mu_R$.*

(ii) *$i^*_{R2}(i_{k1}) = max\{\mu_R - \frac{c}{2}, i_{k1}\}$.*

(iii) *Suppose that $i_{k1} = \frac{1}{2}$, $c = \mu_R - \frac{1}{2}$. Then $i^*_{R2}(i_{k1}) = \frac{1}{2} \cdot \left(\mu_R + \frac{1}{2}\right)$.*

Corollary 1 illustrates that the second-period office-holder does not select his ideal policy, as costs of change prevent him from completely indulging in his own preferences. If, as in case *(iii)*, the previous office-holder has adopted the median position, the costs of change will induce the office-holder to select a policy somewhere between his ideal policy and the median position.

4.2 The first period

We next study the decisions taken in the first period. We distinguish two cases. In one case, we will consider circumstances where costs of change are not too large and allow significant policy changes by office-holders with opposing preferences. Specifically, we assume that

$$0 < \frac{c}{2} < \mu_R - \frac{1}{2}. \tag{6}$$

We note that the upper bound in the above inequality,

$$\Pi := \mu_R - \frac{1}{2} \in \left(0, \frac{1}{2}\right],$$

measures the degree of *party polarization*. Indeed, if Π is close to a half, parties' ideal policies are very far away, so the interests of both parties are opposed. By contrast, if Π is close to zero, parties' ideal policies are very close, so both parties will primarily focus on the interests of those voters located close to the median voter. Assumption (6) ensures that costs of change are small compared to the level of party polarization. In the main body of the text, we will focus on this case, while in Appendix B we consider the case $c \geq 2\Pi$. Our results regarding polarization are qualitatively similar in both cases—and the technical steps for proving them are substantially the same. Nevertheless, some properties of the solutions do not carry over from low costs of change to higher costs.[37]

Next we analyze the second election, in which all citizens select either the incumbent, $k \in R$, or the new left-wing candidate.

Proposition 2

Let $\delta \in \left[0, \frac{1}{2}\right]$ and $k \in R$ be in office in $t = 1$. Then, in equilibrium of $\mathcal{G}^R$, k will be re-elected in the second election if and only if his policy choice in $t = 1$, denoted by i_{k1}, and his ability level, a_k, satisfy

$$a_k \geq a_\delta(i_{k1}), \tag{7}$$

where

$$\begin{aligned} a_\delta(i_{k1}) = &\left(i^*_{R2}(i_{k1}) - \left(\frac{1}{2} - \delta\right)\right)^2 + c\,|i^*_{R2}(i_{k1}) - i_{k1}| \\ &- \left(i^*_{L2}(i_{k1}) - \left(\frac{1}{2} - \delta\right)\right)^2 - c\,|i^*_{L2}(i_{k1}) - i_{k1}| \end{aligned} \tag{8}$$

and $i^*_{R2}(\cdot)$ and $i^*_{L2}(\cdot)$ are given in (5).

[37] For instance, we will see later that the minimization of ex-ante policy polarization and the maximization of social welfare have unique solutions only if (6) holds.

Proof: See Appendix A.

Proposition 2 indicates that for any fixed first-period policy choice i_{k1}, the larger the extra-hurdle δ, the higher the ability level needed to achieve re-election. Indeed, under (6), $i^*_{R2}(i_{k1})$ is strictly larger than $i^*_{L2}(i_{k1})$ for all $i_{k1} \in [0,1]$ and thus, for fixed i_{k1},

$$\frac{da_\delta(i_{k1})}{d\delta} = 2(i^*_{R2}(i_{k1}) - i^*_{L2}(i_{k1})) > 0.$$

Next we analyze how the office-holder in $t = 1$, $k \in R$, chooses his policy i_{k1} depending on $\delta \in \left[0, \frac{1}{2}\right]$.

Proposition 3
Let $\delta \in \left[0, \frac{1}{2}\right]$, and let $k \in R$ be in office in $t = 1$. Then k's policy choice in $t = 1$, in equilibrium of $\mathcal{G}^R$, will be

$$i_{k1}(\delta) = i^*_{R1}(\delta) := \begin{cases} \mu_R - \frac{c}{2} \cdot \frac{1-\rho}{3+\rho} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \mu_R + \frac{c}{2} - 2\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right], \\ \mu_R - \frac{c}{2} \cdot \frac{1+\rho}{3-\rho} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \end{cases} \tag{9}$$

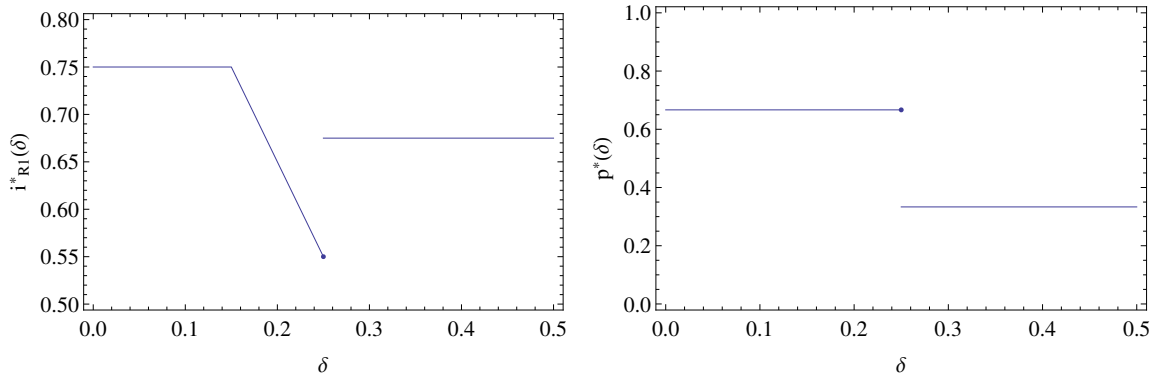
Proof: See Appendix A.

In Figure 2(a), we illustrate Proposition 3 with a plot of $i^*_{R1}(\delta)$ for $\mu_R = 0.8$, $c = 0.5$, and $\rho = \frac{1}{3}$. We see that any non-zero extra-hurdle δ weakly moderates the first-period policy choice compared to $\delta = 0$. Moreover, $i^*_{R1}(\delta)$ is closest to the median voter at $\delta = \frac{c}{2}$. The following expression for k's re-election probability is computed in the proof of Proposition 3 and is helpful for a more detailed interpretation of the plot in Figure 2(a):[38]

$$p^*(\delta) = \begin{cases} \frac{1+\rho}{2} & \text{if } \delta \in \left[0, \frac{c}{2}\right], \\ \frac{1-\rho}{2} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \end{cases} \tag{10}$$

Function $p^*(\delta)$ is illustrated in Figure 2(b). As (10) shows, the re-election probability of an incumbent is larger than $\frac{1}{2}$ when extra-hurdles are low. This indicates an incumbency advantage. Let us briefly outline the intuitive reasons for the shape of function $i^*_{R1}(\delta)$ and the incumbency advantage. There are two different mechanisms that induce the first-period office-holder to choose a weakly more moderate policy when a non-zero extra-hurdle is in force. First, if δ is non-zero, the critical voter is closer to the challenger's ideal policy. Therefore the office-holder chooses a more moderate policy in order to be more attractive to the critical voter. This behavior may help to ensure re-election of the office-holder if he turns out to have an ability of zero,

[38] More precisely, (10) follows from the combination of (48), (49), (50), and (51) from Appendix A.



(a) Equilibrium policy choice of right-wing policy-maker in $t = 1$.

(b) Re-election probability in equilibrium.

Figure 2: Illustration of the equilibrium analysis in $t = 1$ using the parameter values $\mu_R = 0.8$, $c = 0.5$, $A = 2$, and $\rho = \frac{1}{3}$.

because the challenger would create significant costs of change.[39] Second, if δ is sufficiently large, re-election cannot be reached when ability is zero. In this case, the office-holder will still choose a moderate policy in order to reduce the expected costs of change, which are increasing in the probability of deselection. Note that for $\delta \in \left[0, \frac{c}{2}\right]$, the first mechanism takes effect and $p^*(\delta) = \frac{1+\rho}{2}$. That is, the incumbent is re-elected when he has at least zero ability, and he has an incumbency advantage. However, for extra-hurdles above $\frac{c}{2}$, re-election with zero ability cannot be ensured. In this case, the second mechanism kicks in, and $p^*(\delta) = \frac{1-\rho}{2}$. Since ability is drawn from a discrete distribution, $p^*(\delta)$ and $i^*_{R1}(\delta)$ are discontinuous at $\delta = \frac{c}{2}$.

4.3 Equilibrium outcomes

Sections 4.1 and 4.2 enable us to describe the equilibrium outcomes of game $\mathcal{G}$. From Propositions 1 and 3, it immediately follows that the equilibrium policy choices in $\mathcal{G}^R$ are given by

$$i^*_{R1}(\delta) = i^*_{R2}(\delta) = \begin{cases} \mu_R - \frac{c}{2} \cdot \frac{1-\rho}{3+\rho} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \mu_R + \frac{c}{2} - 2\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right], \\ \mu_R - \frac{c}{2} \cdot \frac{1+\rho}{3-\rho} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right] \end{cases} \quad \text{and} \quad i^*_{L2}(\delta) = \mu_L + \frac{c}{2}, \tag{11}$$

for each given $\delta \in \left[0, \frac{1}{2}\right]$. Note that for all δ, $i^*_{L2}(\delta)$ is strictly smaller than $i^*_{R1}(\delta)$. Due to (2), and because the distribution of a_k is independent of the policy orientation of candidate k, the equilibrium policy choices of $\mathcal{G}^R$ and $\mathcal{G}^L$ are symmetrically distributed around $\frac{1}{2}$. More

[39] Note that benefits from holding office are large, so the office-holder will choose a policy that maximizes his re-election probability.

precisely, the equilibrium policy choices in $\mathcal{G}^L$ are given by

$$i_{L1}^{**}(\delta) = i_{L2}^{**}(\delta) = \begin{cases} \mu_L + \frac{c}{2} \cdot \frac{1-\rho}{3+\rho} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \mu_L - \frac{c}{2} + 2\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right], \\ \mu_L + \frac{c}{2} \cdot \frac{1+\rho}{3-\rho} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right] \end{cases} \quad \text{and} \quad i_{R2}^{**}(\delta) = \mu_R - \frac{c}{2}. \tag{12}$$

In $\mathcal{G}$, there are four possible voting outcomes: (R,R), (R,L), (L,L), and (L,R), depending on the types of first- and second-period office-holders. For each $\delta \in \left[0, \frac{1}{2}\right]$, the equilibrium policy choices for each of these voting outcomes are entirely characterized once the policy orientations of both office-holders are known. In particular, for each $\delta \in \left[0, \frac{1}{2}\right]$, there are four different outcomes that can appear in an equilibrium of game $\mathcal{G}$. Those outcomes can be denoted by (R,R), (R,L), (L,L), and (L,R). In (R,R) and (R,L), the equilibrium policy choices, as functions of δ, are given by $i_{R1}^{*}(\delta)$, $i_{R2}^{*}(\delta)$, and $i_{L2}^{*}(\delta)$ from (11). Similarly, in (L,L) and (L,R), the equilibrium policy choices are given by $i_{L1}^{**}(\delta)$, $i_{L2}^{**}(\delta)$, and $i_{R2}^{**}(\delta)$ from (12). We note that the realization of ability in the first period only influences whether the office-holder gets re-elected, but it has no impact on the policy choice for a given office-holder in the second period. We mentioned earlier that the probability of winning the first election is equal to $\frac{1}{2}$ for both initial candidates. Furthermore, in equilibrium, the re-election probability, conditional on being in office in $t = 1$, is given by (10) for both candidates. Hence, the equilibrium outcomes of $\mathcal{G}$ occur with probabilities

$$p_{(R,R)}(\delta) = p_{(L,L)}(\delta) = \frac{p^*(\delta)}{2} \quad \text{and} \quad p_{(R,L)}(\delta) = p_{(L,R)}(\delta) = \frac{1 - p^*(\delta)}{2}, \tag{13}$$

where $p^*(\delta)$ is given by (10).

We conclude this section by focusing for a moment on the specific situation where $c = 0$. In this case, both candidates will choose their ideal policies whenever they are in office, since there are no costs to prevent them from indulging in their own preferences. Considering (11) and (12) at the limit where c tends to zero shows that, in equilibrium, we indeed approximate this solution.[40]

5 Policy Polarization and Welfare

In this section we explore how the levels of *policy polarization* and *welfare* depend on the model parameters and the institutional variable δ. Specifically, we assess how c, μ_R, and ρ impact policy polarization when $\delta = 0$ and how policy polarization and welfare vary with $\delta \in \left[0, \frac{1}{2}\right]$.

[40] Another extreme case corresponds to the situation where c is very large. In Appendix B, we show that for $c \geq 4\Pi$, the office-holder of the first period chooses his ideal policy and, due to large costs of change, the office-holder in $t = 2$ is forced to choose the same policy.

We show that choosing certain strictly positive values for δ reduces ex-ante policy polarization. Moreover, we prove that policy polarization is minimized for the same value of δ that maximizes welfare or, equivalently, the expected lifetime utility of the median voter. If $\delta = 0$, we find that moderate levels of c reduce policy polarization and increase welfare, relative to the case where there are no costs of change.

5.1 Polarization and welfare concepts

First we define the concept of *ex-post policy polarization*.[41]

Definition 1

Let $k \in R \cup L$ and $k' \in R \cup L$ be the office-holders in $t = 1$ and $t = 2$, respectively, and let $(i_{k1}, i_{k'2}) \in [0,1] \times [0,1]$ be their respective policy choices. Then, ex-post policy polarization is defined as

$$EPP(i_{k1}, i_{k'2}) = \frac{\left|i_{k1} - \frac{1}{2}\right| + \left|i_{k'2} - \frac{1}{2}\right|}{2}.$$

Because candidates may or may not be deselected after the first term, what matters for voters is *ex-ante policy polarization*, i.e. the expected value of ex-post policy polarization over all possible equilibrium outcomes. Accordingly, we define ex-ante policy polarization as follows:

Definition 2

Ex-ante policy polarization is the expected value of ex-post policy polarization.

From the analysis of Section 4.3, it follows that ex-ante policy polarization can be written as

$$\begin{aligned} EAP(\delta) =& p_{(R,R)}(\delta) \cdot EPP(i^*_{R1}(\delta), i^*_{R2}(\delta)) + p_{(R,L)}(\delta) \cdot EPP(i^*_{R1}(\delta), i^*_{L2}(\delta)) \\ &+ p_{(L,L)}(\delta) \cdot EPP(i^{**}_{L1}(\delta), i^{**}_{L2}(\delta)) + p_{(L,R)}(\delta) \cdot EPP(i^{**}_{L1}(\delta), i^{**}_{R2}(\delta)), \end{aligned}$$

where the equilibrium policy choices, $i^*_{R1}(\delta)$, $i^*_{R2}(\delta)$, $i^*_{L2}(\delta)$, $i^{**}_{L1}(\delta)$, $i^{**}_{L2}(\delta)$, and $i^{**}_{R2}(\delta)$ are given by (11) and (12), and the equilibrium outcome probabilities, $p_{(R,R)}(\delta)$, $p_{(R,L)}(\delta)$, $p_{(L,L)}(\delta)$, and $p_{(L,R)}(\delta)$ are given by (13). From Section 4.3, we know that $i^*_{R1}(\delta)$ and $i^{**}_{L1}(\delta)$ are symmetric around $\frac{1}{2}$. The same holds for $i^*_{R2}(\delta)$ and $i^{**}_{L2}(\delta)$, and $i^*_{L2}(\delta)$ and $i^{**}_{R2}(\delta)$, respectively. Moreover, we know that $p_{(R,R)}(\delta) = p_{(L,L)}(\delta)$ and $p_{(R,L)}(\delta) = p_{(L,R)}(\delta)$. Therefore, $EAP(\delta)$ coincides with the ex-ante policy polarization of $\mathcal{G}^R$, that is,

$$EAP(\delta) = p^*(\delta) \cdot EPP(i^*_{R1}(\delta), i^*_{R2}(\delta)) + (1 - p^*(\delta)) \cdot EPP(i^*_{R1}(\delta), i^*_{L2}(\delta)), \tag{14}$$

[41] Ex-post policy polarization could be defined in different ways. Note that, qualitatively, the results of this paper would be the same if, in Definition 1, we were to define ex-post policy polarization as, say, $EPP(i_{k1}, i_{k'2}) := \frac{1}{2}\left(i_{k1} - \frac{1}{2}\right)^2 + \frac{1}{2}\left(i_{k'2} - \frac{1}{2}\right)^2$.

where $p^*(\delta)$ is given by (10).

To measure welfare, we take the integral over all voters' expected lifetime utilities. As voters' ideal policy positions are symmetrically distributed around the median voter's ideal policy, $i = \frac{1}{2}$, and since all voters are affected in the same way by the public project and by costs of change, maximizing the utilitarian welfare function is equivalent to maximizing the median voter's expected lifetime utility. According to (4), expected lifetime utility of the median voter consists of three terms, the expected lifetime utility from public projects (EU^P), the expected lifetime utility from policies ($EU^I_{\frac{1}{2}}$), and the expected lifetime utility from costs of change (EU^c). As in the case of policy polarization, it suffices to consider expected lifetime utility over all equilibrium outcomes of $\mathcal{G}^R$. This is due to the symmetries of the equilibrium policy choices and the fact that the distribution of a_k is independent of the policy orientation of k. Thus, welfare as a function of $\delta \in \left[0, \frac{1}{2}\right]$ is given by

$$W(\delta) = EU^P(\delta) + EU^I_{\frac{1}{2}}(\delta) + EU^c(\delta). \tag{15}$$

Here we use δ as the argument of the utility functions because δ uniquely determines both the expected output of the public project and the expected policy choice in both periods. We note that if $k \in R$ is the first-period office-holder and $k' \in L$ is the challenger in the second election, the three components of welfare are given by

$$\begin{aligned}
EU^P(\delta) &= p^*(\delta) \cdot 2E\left[a_k | k \text{ re-elected and } p_\delta(i_{k1}) = p^*(\delta)\right] \\
&\quad + (1 - p^*(\delta)) \cdot \left\{E\left[a_k | k \text{ not re-elected and } p_\delta(i_{k1}) = p^*(\delta)\right] + E[a_{k'}]\right\}, \qquad (16) \\
EU^I_{\frac{1}{2}}(\delta) &= p^*(\delta) \cdot \left[-\left(i^*_{R1}(\delta) - \frac{1}{2}\right)^2 - \left(i^*_{R2}(\delta) - \frac{1}{2}\right)^2\right] \\
&\quad + (1 - p^*(\delta)) \cdot \left[-\left(i^*_{R1}(\delta) - \frac{1}{2}\right)^2 - \left(i^*_{L2}(\delta) - \frac{1}{2}\right)^2\right], \qquad (17) \\
EU^c(\delta) &= p^*(\delta) \cdot \left[-c|i^*_{R1}(\delta) - i^*_{R2}(\delta)|\right] \\
&\quad + (1 - p^*(\delta)) \cdot \left[-c|i^*_{R1}(\delta) - i^*_{L2}(\delta)|\right], \qquad (18)
\end{aligned}$$

with $p^*(\delta)$ from (10) and $i^*_{R1}(\delta)$, $i^*_{R2}(\delta)$, and $i^*_{L2}(\delta)$ from (11).

In order to compare different extra-hurdles, it is useful to introduce the following optimality concepts:

Definition 3

An extra-hurdle $\delta \in \left[0, \frac{1}{2}\right]$ is called

- *W-optimal if it maximizes welfare,*
- *P-optimal if it minimizes ex-ante policy polarization.*

Furthermore, when comparing a setting where non-zero extra-hurdles have been implemented to a setting without extra-hurdles, it is useful to introduce the following definitions:

Definition 4

An extra-hurdle $\delta \in \left(0, \frac{1}{2}\right]$ *is called*

- *W-increasing if* $W(\delta) > W(0)$,
- *weakly W-increasing if* $W(\delta) \geq W(0)$,
- *P-reducing if* $EAP(\delta) < EAP(0)$,
- *weakly P-reducing if* $EAP(\delta) \leq EAP(0)$.

5.2 Optimal extra-hurdles (our normative results)

We can now state our main normative results.

Theorem 1

In equilibrium of $\mathcal{G}$*, the following holds:*

(i) Any extra-hurdle $\delta \in \left(0, \frac{1}{2}\right]$ *is both weakly W-increasing and weakly P-reducing.*

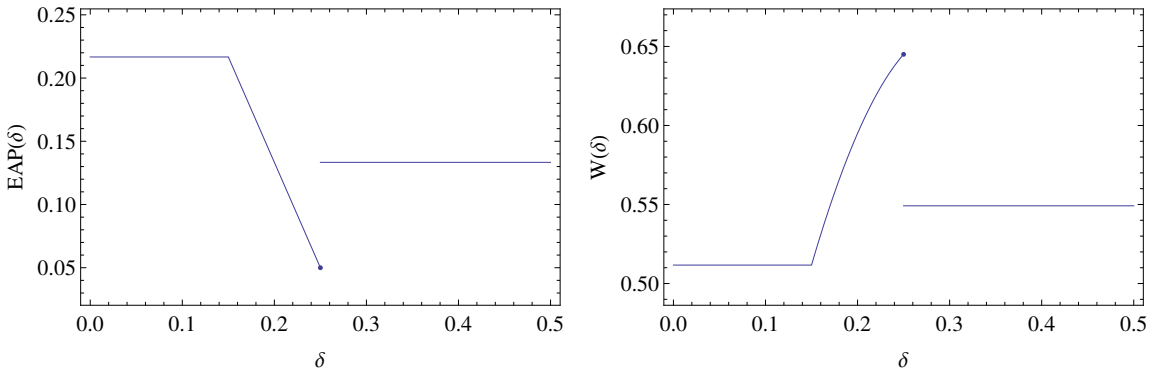
(ii) There exists some $\delta^* \in \left(0, \Pi\right)$ *that is both W- and P-optimal. Moreover,*

$$\{\delta^*\} = \underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) = \underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmax}} W(\delta).$$

Proof: See Appendix A.

According to *(ii)* of Theorem 1, it is optimal, in terms of both welfare and ex-ante policy polarization, to introduce δ^* as the extra-hurdle for incumbents.[42] In the proof of Theorem 1 we show that $\delta^* = \frac{c}{2}$. Statement *(i)* in Theorem 1 shows that the introduction of any non-zero extra-hurdle weakly increases welfare and weakly reduces ex-ante policy polarization with respect to the simple majority rule. That is, $\delta = 0$ is never optimal. The intuition and understanding of Theorem 1 is developed with the illustrations in Figures 3(a) and 3(b), where $EAP(\delta)$ and $W(\delta)$ are plotted for $\mu_R = 0.8$, $c = 0.5$, $A = 2$ and $\rho = \frac{1}{3}$. The behavior of ex-ante policy polarization as a function of δ immediately follows from the shape of $i^*_{R1}(\delta)$, which is depicted in Figure 2(a) and driven by the two mechanisms described at the end of Section 4.2. The behavior of $W(\delta)$ is more subtle. Note that $EU^P(\delta)$ is constant and equal to $\frac{A(1-\rho)}{2}$,

[42] Note that not only $EAP(\delta)$ but also ex-post policy polarization of each equilibrium outcome of $\mathcal{G}$ is minimized at δ^*. Moreover, besides total welfare, $W(\delta)$, two of its components, $EU^I_{\frac{1}{2}}(\delta)$ and $EU^c(\delta)$, are uniquely maximized at δ^*.



(a) Ex-ante policy polarization.

(b) Welfare.

Figure 3: Illustration of the equilibrium values of ex-ante policy polarization and welfare, using the parameter values $\mu_R = 0.8$, $c = 0.5$, $A = 2$ and $\rho = \frac{1}{3}$.

independently of δ. Thus, it suffices to analyze $EU^I_{\frac{1}{2}}(\delta)$ and $EU^c(\delta)$. $EU^I_{\frac{1}{2}}(\delta)$ is increasing (decreasing) when $EAP(\delta)$ is decreasing (increasing). For $\delta \in [0, \delta^*]$, expected costs of change are weakly decreasing because the re-election probability, $p^*(\delta)$, is constant, the first-period policy choice approaches the median when δ increases, and $i^*_{R1}(\delta) = i^*_{R2}(\delta)$. Hence, welfare is weakly increasing for $\delta \in [0, \delta^*]$. For $\delta > \delta^*$, expected costs of change are larger than for $\delta = 0$, because the deselection probability of the incumbent is larger. Nevertheless, since the increase in expected costs of change is outweighed by the reduction in ex-ante policy polarization, any extra-hurdle above δ^* is W-increasing. Moreover, $W(\delta)$ is uniquely maximized at δ^*, since $EU^P(\delta)$ is constant and both $EU^I_{\frac{1}{2}}(\delta)$ and $EU^c(\delta)$ are maximized at δ^*.

The plots in Figures 3(a) and 3(b) also reveal that below the critical value δ^*, a marginal increase in the extra-hurdle weakly increases welfare and weakly reduces ex-ante policy polarization. This is an important result. Indeed, because the exact value of c—and thus of $\delta^* = \frac{c}{2}$—may be uncertain, a cautious approach to improving welfare might consist in marginally increasing the extra-hurdle.

5.3 Comparative statics (our positive results)

It is also instructive to consider comparative statics effects for the case of $\delta = 0$. The results stated in this section immediately follow from the expressions of $EAP(\delta)$ and $W(\delta)$ computed in the proof of Theorem 1. First we examine what happens when the cost parameter c varies and all other quantities are held constant. Our main positive results are given in the following theorem:[43]

[43] We stress that Theorem 2 holds for all $c > 0$, i.e. (6) need not be satisfied for Theorem 2 to hold.

Theorem 2

Let $\delta = 0$. Then, the following holds:

(i) $EAP(c|\delta = 0) \leq EAP(0|\delta = 0)$ *and* $W(c|\delta = 0) \geq W(0|\delta = 0)$, *for any* $c > 0$.

(ii) There exists a unique $c^* > 0$ *such that, for all* $0 \leq c_1 < c_2 < c^* < c_3 < c_4$,

$$\begin{cases} EAP(c_1|\delta = 0) \geq EAP(c_2|\delta = 0) \geq EAP(c^*|\delta = 0) \\ EAP(c^*|\delta = 0) < EAP(c_3|\delta = 0) \leq EAP(c_4|\delta = 0) \\ W(c_1|\delta = 0) < W(c_2|\delta = 0) < W(c^*|\delta = 0) \\ W(c^*|\delta = 0) > W(c_3|\delta = 0) \geq W(c_4|\delta = 0) \end{cases}$$

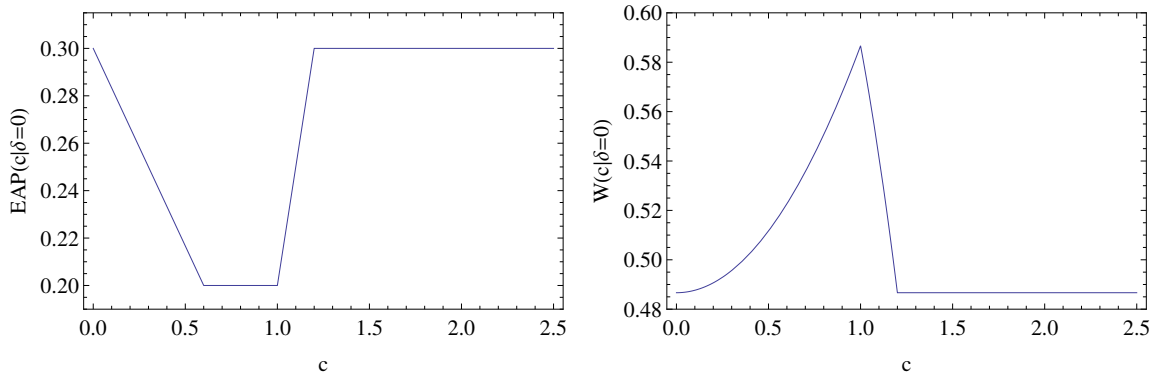
hold.

Proof: See Appendix A.

The behavior of ex-ante policy polarization and welfare as a function of the cost parameter—in the absence of extra-hurdles, i.e. for $\delta = 0$—is depicted in Figure 4(a) and Figure 4(b). These plots illustrate Theorem 2. Figure 4(b) shows that *(i)* welfare is weakly higher when policy changes are costly than when there are no costs of change and *(ii)* $W(c|\delta = 0)$ is single-peaked as a function of c. In particular, the fact that moderate overhead costs of policy changes could be welfare-improving is a very interesting insight. Figure 4(a) shows that *(i)* $EAP(c|\delta = 0) \leq EAP(0|\delta = 0)$ for all non-zero values of c, and *(ii)* the larger the difference is between c and c^*, the (weakly) larger ex-ante policy polarization will be.[44] Statement *(i)* of Theorem 2 follows from the fact that positive values of the cost parameter c yield weakly more moderate policy choices than $c = 0$. For small c, the behavior of $EAP(c|\delta = 0)$ and $W(c|\delta = 0)$ in response to marginal changes in the cost parameter can be explained in the following way: For $c < 2\Pi$, any marginal increase in c yields strictly more moderate policy choices in both periods. Thus $EAP(c|\delta = 0)$ is strictly decreasing for $c < 2\Pi$. Moreover, welfare is strictly increasing for small c, since the effect of moderated policies outweighs the effect of increasing the marginal cost of change.

According to the results in Theorem 2, it would be socially desirable to increase the costs of policy changes if such costs stay sufficiently low. One possibility to make changes more costly could be to require qualified majorities—proportional to the desired policy change—in the law-making process of parliament. These obstacles would most likely impose additional opportunity costs on citizens, as parliamentary members would have to devote more time to reaching an agreement. A second insight of Theorem 2 that relies on the interpretation of costs of change as

[44] In the proof of Theorem 2 we show that c^*, the unique value of c that maximizes welfare and minimizes ex-ante policy polarization, is given by $c^* = (3 + \rho)\Pi$.



(a) $EAP(0)$ as a function of c.

(b) $W(0)$ as a function of c.

Figure 4: Comparative statics for $\delta = 0$: Ex-ante policy polarization and welfare as a function of $c > 0$, for the parameter values $\mu_R = 0.8$, $A = 2$ and $\rho = \frac{1}{3}$.

psychological costs is the following: Welfare is higher in societies with an intermediate degree of conservatism compared to both "wavering societies" (i.e. societies where policy changes do not impose utility losses on citizens) and "highly-conservative societies" (i.e. societies where policy changes impose huge utility losses on citizens).

Second, we consider changes in party polarization Π. We know from (11) that if Π increases, the equilibrium policy choices become more extreme, and thus $EAP(0)$ will increase. In other words, if party polarization increases, so does ex-ante policy polarization, since policy-makers aspire to move towards their ideal policy positions when choosing their policies. Because more extreme policy choices additionally yield higher costs of change in those cases where the first-period office-holder is deselected, increasing Π (or, equivalently, increasing μ_R) will lower welfare. These two effects are confirmed by the proof of Theorem 1, from which we know that, for $\delta = 0$, ex-ante policy polarization and welfare are given by

$$EAP(0) = \Pi - \frac{c(1-\rho)}{4} \tag{19}$$

and

$$W(0) = \frac{A(1-\rho)}{2} - 2\Pi^2 + \frac{c^2(1-\rho)}{2(3+\rho)}, \tag{20}$$

respectively.

Third, consider changes in ρ. Expression (19) reveals that ex-ante policy polarization increases linearly in ρ. The reason for this is twofold. On the one hand, the equilibrium value of re-election probability $p^*(\delta)$, given in (10), increases with ρ, which induces the first-period office-holder to choose a policy that is closer to his ideal policy. On the other hand, this more extreme policy enters into the expression of $EAP(0)$ with a weight that is increasing in $p^*(\delta)$.[45] Finally, as

[45]This can be seen from (73) in Appendix A.

can be seen from (20), $W(0)$ is decreasing in ρ. The effect of more extreme policies and lower expected lifetime utility from the public project is stronger than the influence of the lower probability of deselecting the first-period office-holder.

6 Re-election Hurdle Contracts

In this section we extend our model in such a way that the choice of the extra-hurdle δ is now made by the candidates themselves. More specifically, before the first election, candidates $k \in R$ and $k' \in L$ choose $\delta_k \in \left[0, \frac{1}{2}\right]$ and $\delta_{k'} \in \left[0, \frac{1}{2}\right]$, respectively. Then they both announce their choices simultaneously and publicly and commit to them. Such commitments can occur through *re-election hurdle contracts*—special political contracts outlined in Gersbach (2012). In the first election, the simple majority rule applies; in the case of a tie, each candidate wins the election with probability $\frac{1}{2}$. All voters know the announced values of δ_k and $\delta_{k'}$ when they vote in the first election. In the second election, if k (resp. k') is the office-holder, he will be re-elected if his vote-share is equal to or larger than $\frac{1}{2} + \delta_k$ (resp. $\frac{1}{2} + \delta_{k'}$). We denote this new game by $\mathcal{G}'$. The sequence of events in the extended model is shown in Figure 5.

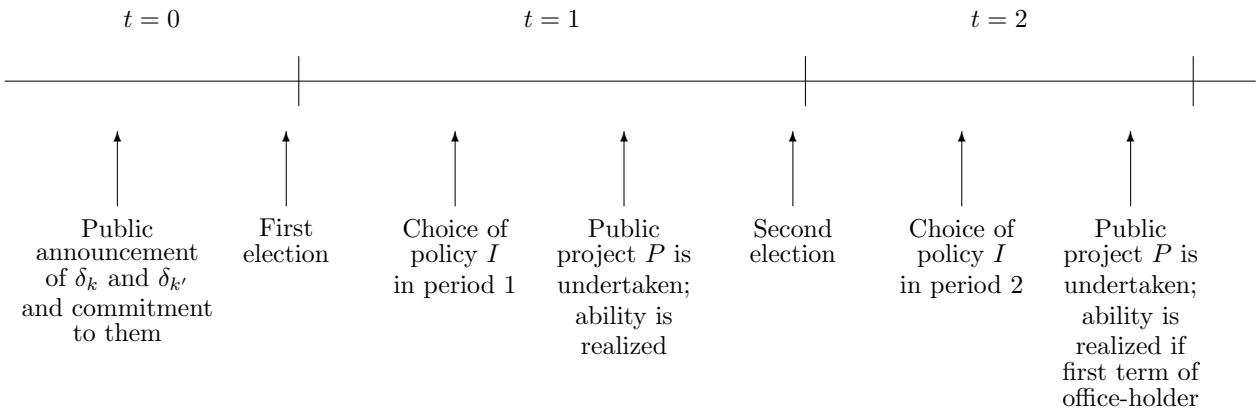


Figure 5: Timeline of the sequence of events in the extended model.

Under the mechanism described above, the following result holds:

Theorem 3

In equilibrium of the game $\mathcal{G}'$, $k \in R$ and $k' \in L$ will commit to $\delta^* = \frac{c}{2}$ and both will win the first election with probability equal to $\frac{1}{2}$.

Proof: See Appendix A.

Theorem 3 shows that in equilibrium the two candidates will voluntarily commit to $\delta^* = \frac{c}{2}$, which is the unique extra-hurdle that is both W- and P-optimal. As a consequence, $\delta^* = \frac{c}{2}$ will be implemented in equilibrium, which makes re-election hurdle contracts a legitimate method for implementing the concept of extra-hurdles in practice. The intuition for the result in Theorem 3 is straightforward. If one candidate commits to a different value, say $\delta_k \neq \delta^*$, the other candidate can secure election (with probability 1) by announcing δ^*, because δ^* is the unique maximizer of the expected lifetime utility of the median voter. Hence, the choice $\delta_k \neq \delta^*$ cannot be a best response, because a candidate is elected with (at least) probability $\frac{1}{2}$ if he chooses δ^*.

7 Social Polarization and Policy Polarization

Up to now, we have assumed that the electorate's ideal policies are distributed uniformly across the interval $[0, 1]$. In the present section, we relax this assumption and allow for non-zero *social polarization*. We examine the implications of extra-hurdles on welfare and ex-ante policy polarization under different degrees of (exogenously given) social polarization. To that purpose, we define the following family of density functions, where $\alpha \in [0, 1]$:

$$f_\alpha(i) = \begin{cases} -4\alpha i + (1+\alpha) & \text{if } i \in \left[0, \frac{1}{2}\right], \\ 4\alpha i + (1-3\alpha) & \text{if } i \in \left(\frac{1}{2}, 1\right]. \end{cases} \tag{21}$$

Any distribution in (21) with $\alpha > 0$ is bimodal, with kinks in $i = 0$ and $i = 1$. Accordingly, this is the description of an electorate with voters centered around two extreme positions.[46] Parameter α reflects the degree of social polarization. In particular, if $\alpha = 0$, the distribution defined by $f_\alpha(\cdot)$ will reduce to a uniform distribution on $[0, 1]$, whereas $\alpha = 1$ captures the situation where there are few centrist voters. The density functions defined in (21) are illustrated in Figure 6.

Since the degree of social polarization can be measured by α, we denote welfare and ex-ante policy polarization by $W_\alpha(\cdot)$ and $EAP_\alpha(\cdot)$, respectively. The following theorem contains the main results on how extra-hurdles impact $EAP_\alpha(\cdot)$ and $W_\alpha(\cdot)$:

[46] Different distributions capturing social polarization could be used instead of the family of distributions given in (21). The results regarding the consequences of extra-hurdles on policy polarization and welfare are likely to be qualitatively the same, but the analysis would become significantly more complex.

Theorem 4

Let $\alpha \in [0,1]$, and let the distribution of the voters' ideal policies have a density $f_\alpha(\cdot)$. Then,

(i) Theorem 1(i) holds.

(ii) Theorem 1(ii) holds. That is, there exists some $\delta^*_\alpha \in (0, \Pi)$ such that

$$\{\delta^*_\alpha\} = \underset{\delta \in [0,\frac{1}{2}]}{\operatorname{argmin}} EAP_\alpha(\delta) = \underset{\delta \in [0,\frac{1}{2}]}{\operatorname{argmax}} W_\alpha(\delta).$$

(iii) δ^*_α is continuous and decreasing in $\alpha \in [0,1]$.

(iv) If some fixed $\delta \in [0, \delta^*_1]$ is chosen, $EAP_\alpha(\delta)$ is weakly decreasing in α and $W_\alpha(\delta)$ is weakly increasing in α.[47]

Proof: See Appendix A.

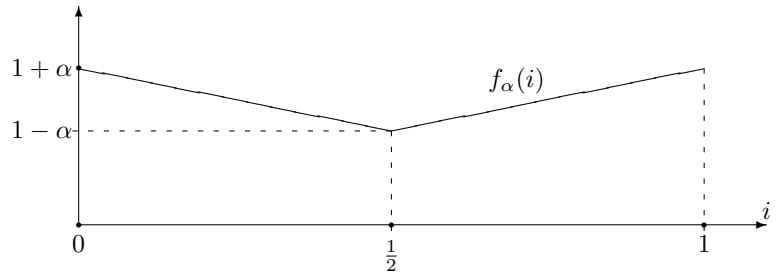


Figure 6: The density functions $f_\alpha(i)$ describe a socially polarized electorate. The parameter $\alpha \in [0,1]$ measures the degree of social polarization.

Statements *(i)* and *(ii)* of Theorem 4 establish that the results of Theorem 1 hold even when the electorate is socially polarized. However, the uniquely determined W- and P-optimal δ is not equal to $\frac{c}{2}$ for each value of $\alpha \in [0,1]$. Indeed, the effect of increasing the extra-hurdle is leveraged by the degree of social polarization. That is, the more polarized a society is, the lower the optimal extra-hurdle will be, so δ^*_α is decreasing in α, as stated in *(iii)*. From the proof of Theorem 4, we immediately see that for $\delta = 0$, i.e. in the absence of extra-hurdles, welfare and ex-ante policy polarization do not depend on the level of social polarization.[48] Hence, policy polarization is not driven by the level of social polarization when the simple majority rule is applied at all stages. As a consequence, the positive results stated in Section 5.3 are robust to the electorate being socially polarized. In particular, Theorem 2 holds independently of the level of social polarization.

[47] Note that $\delta^*_1 = \frac{c^2}{2}$, which is lower than $\frac{c}{2}$.

[48] More precisely, this follows from (87) and (88).

8 Initial Costs of Change

An implicit assumption in the baseline model is that there are no costs of change in $t = 1$, or, equivalently, that there is no status quo policy. That is, in the basic version of our model we have focused on a situation where the significance of costs of change occurs only after the policy in $t = 1$ is chosen. It is thus worth analyzing how our results change if we assume the existence of a status quo policy in $t = 0$, denoted by $i_0 \in [0, 1]$. Suppose that such a status quo policy imposes additional costs of change in $t = 1$, given by

$$U^{\tilde{c}}(i_0, i_{k1}) = -\tilde{c} \cdot |i_0 - i_{k1}|,$$

where i_{k1} is the policy chosen by office-holder k in $t = 1$ and $\tilde{c} \in (0, c]$. These costs are added to the lifetime utility of all agents.

Using methods similar to those used in Sections 4.2 and 5.2, the following theorem can be shown:[49]

Theorem 5

Let the status quo policy be $i_0 = \frac{1}{2}$, and let $\tilde{c} \in (0, (1 - \rho) \cdot c)$. Then, Theorem 1 holds.

Theorem 5 reveals that our normative results on welfare and ex-ante policy polarization are robust against introducing a status quo policy $i_0 = \frac{1}{2}$, under the condition that $\tilde{c}$, the unitary cost of change in the first period, is smaller than $(1 - \rho) \cdot c$. The intuition for this result is the following: The first-period policy choice is still driven by the two effects mentioned in Section 4.2. That is, up to some critical value of δ, the office-holder will move closer to the median voter when δ grows larger, in order to ensure re-election with ability zero. For extra-hurdles above the critical level, he cannot ensure re-election with ability zero and will thus choose a more partisan policy. Due to costs of change in the first period, there is now an additional third effect that induces the first-period office-holder to choose a moderate policy. Indeed, for given $\delta \in \left[0, \frac{1}{2}\right]$, the larger $\tilde{c}$ grows, the closer the first-period policy choice will be to the median voter, because the office-holder minimizes the cost of change that he incurs. Thus, the initial costs of change reinforce the policy-moderating effect of extra-hurdles and, if $\tilde{c}$ is small, leave the maximizer of welfare and the minimizer of ex-ante policy polarization unchanged. Much as in Theorem 5, it can be shown that, for $(1 - \rho) \cdot c < \tilde{c} \leq c$, statement *(i)* of Theorem 1 still holds but *(ii)* does not, because the third effect is overly strong when $\tilde{c}$ is close to c and when δ is above the critical level mentioned above. In this case, the W-optimal and the P-optimal extra-hurdles may differ. However, both are still larger than zero,

[49] The detailed proof is available upon request. Note that the most challenging part of the proof is finding an expression for $i^*_{R1}(\delta)$, but this can be done similarly to the proof of Proposition 3.

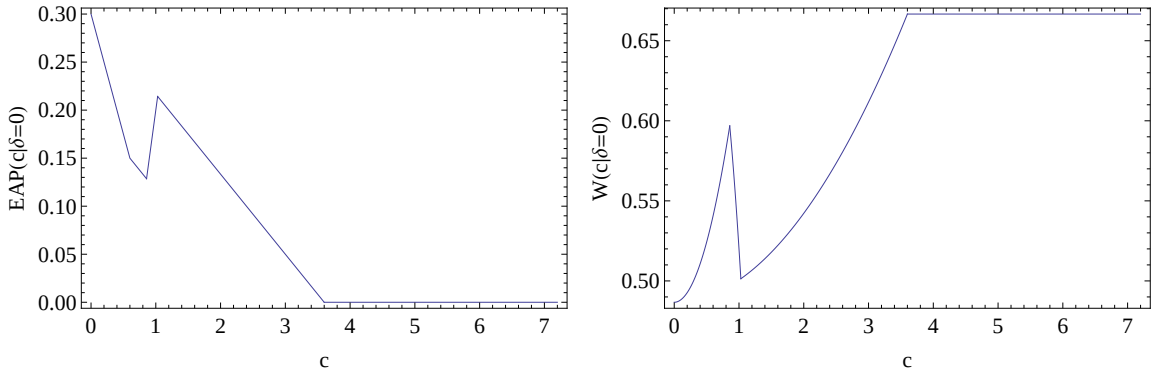
and any W-optimal extra-hurdle is P-reducing. Similarly, any P-optimal δ is W-increasing. If, for instance, $\tilde{c} = c$, then

$$\underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) = \left(\frac{c}{2}, \frac{1}{2}\right]$$

and, conditional on c being smaller than $\frac{2(2\mu_R - 1)}{4+\rho}$,

$$\underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmax}} W(\delta) = \left\{\frac{c}{2}\right\}.$$

That is, the sets of W-optimal and P-optimal extra-hurdles are disjoint. However, all $\delta \in \left(\frac{c}{2}, \frac{1}{2}\right]$ are W-increasing, and $\delta = \frac{c}{2}$ is P-reducing.



(a) $EAP(0)$ as a function of c.

(b) $W(0)$ as a function of c.

Figure 7: Comparative statics for $\delta = 0$, $i_0 = \frac{1}{2}$, and $\tilde{c} = \theta \cdot c$: Ex-ante policy polarization and welfare as a function of $c > 0$, for the parameter values $\mu_R = 0.8$, $A = 2$, $\rho = \frac{1}{3}$, and $\theta = \frac{1-\rho}{2} = \frac{1}{3}$.

Next, we analyze the robustness of Theorem 2 with respect to the introduction of a status quo policy and initial costs of change. For this purpose, let $i_0 = \frac{1}{2}$ and $\delta = 0$. Moreover, we assume that $\tilde{c} = \theta \cdot c$, for some $\theta \in [0, 1]$. Then we can show that any non-zero cost level $c > 0$ still yields more moderate policy choices than $c = 0$.[50] This is why Theorem 2*(i)* still holds in the present setting. However, statement *(ii)* of Theorem 2 is not satisfied anymore, because first-period policy choices are no longer single-dipped in c. The behavior of ex-ante policy polarization and welfare as a function of c (and implicitly, of $\tilde{c}$) is illustrated in Figures 7(a) and 7(b). These plots illustrate that with $i_0 = \frac{1}{2}$ and $\tilde{c} = \theta \cdot c$, large marginal costs of change (in contrast to moderate levels of c) will minimize ex-ante policy polarization and maximize welfare. The intuition for this result is clear: If c and $\tilde{c}$ are very large, candidates choose their policies to be equal to i_0 in both periods, which minimizes costs of change and maximizes the median voter's utility from policies.

[50] The detailed computations are available upon request.

9 Non-linear Costs of Change

Throughout the paper, we have assumed that costs of change are linear in the difference between the policies adopted in the two periods. Under linear costs, we have fully characterized the equilibrium of the game $\mathcal{G}$ depending on Π and c, and we have shown that *(i)* setting a threshold for re-election higher than the usual majority rule weakly reduces ex-ante policy polarization and weakly increases the median voter's welfare, and *(ii)* there exists a unique non-zero extra-hurdle that is simultaneously polarization-minimizing and welfare-maximizing. In this section, we explore the robustness of our results for other and more general specifications: additional presence of fixed costs and convex cost functions.

9.1 Fixed costs of change

In the Introduction, we mentioned the possibility that changing policy I will entail fixed costs. In this section, we assume that in addition to the linear costs of change, there are also fixed costs of change. More specifically, when policies $i_{k1} \in [0,1]$ and $i_{k'2} \in [0,1]$ are implemented in $t=1$ and $t=2$, respectively, all voters and policy-makers incur costs in $t=2$ equal to

$$U^c(i_{k1}, i_{k'2}) = -c \cdot |i_{k1} - i_{k'2}| - K \cdot \mathbb{1}(i_{k1}, i_{k'2}),$$

with $K > 0$ and

$$\mathbb{1}(i_{k1}, i_{k'2}) = \begin{cases} 1 & \text{if } i_{k1} \neq i_{k'2}, \\ 0 & \text{otherwise.} \end{cases}$$

As in the baseline model, we focus on comparatively small values of c. More precisely, we assume that

$$0 < \frac{c}{2} < \Pi - \sqrt{K}. \tag{22}$$

We obtain the following theorem:

Theorem 6

Theorem 1 holds if

$$0 < K < min\left\{\overline{K}_1, \overline{K}_2\right\}, \tag{23}$$

where

$$\overline{K}_1 = \frac{(2\Pi - c)^4}{36} \quad \textit{and} \quad \overline{K}_2 = \frac{2c^2(1-\rho)}{(3-\rho)^2}.$$

The logical steps for proving Theorem 6 are the same as for Theorem 1.[51] The intuition for the result in Theorem 6 is as follows: Fixed costs that are not too large $\left(\text{i.e. below } min\left\{\overline{K}_1, \overline{K}_2\right\}\right)$

[51] The detailed and involved calculations, covering all the different cases that need to be distinguished, are available upon request. As in the baseline model, the most challenging part of the proof is finding an expression for the first-period policy choice in equilibrium, similar to the proof of Proposition 3.

increase the incumbency advantage of the first-period office-holder. Indeed, the critical extra-hurdle up to which he can reach re-election with ability zero is increasing as a function of K and for any given δ below this critical value he can reach re-election with ability zero by choosing a weakly more polar position compared to the case without fixed costs. This effect does not fundamentally change the equilibrium dynamics, it merely increases the unique W- and P-optimal extra-hurdle. As a consequence, the results of Theorem 1 hold with

$$\delta^* = \frac{c}{2} + \frac{\sqrt{K}(2\Pi - c)}{2(2\Pi - c - \sqrt{K})} > \frac{c}{2},$$

given that (22) and (23) are satisfied. Figure 8 illustrates the constraints on c and K given in (22) and (23). The area below the solid curve contains all pairs of values (K, c) that satisfy

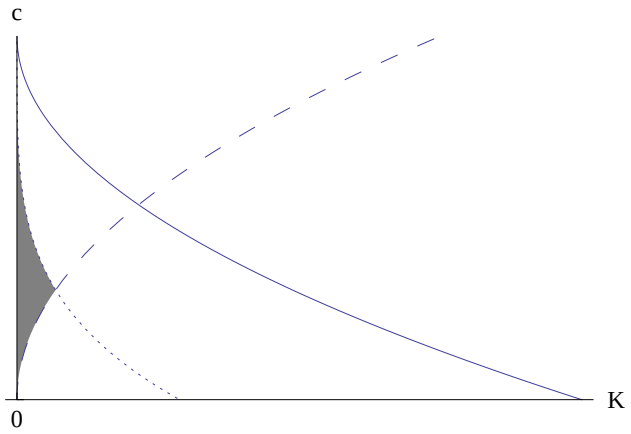


Figure 8: Sketch of the conditions (22) and (23): All pairs (K, c) below the solid curve satisfy (22). If (K, c) lies to the left of the dotted (resp. dashed) curve, then it additionally satisfies $K < \overline{K}_1(c)$ (resp. $K < \overline{K}_2(c)$). The shaded area represents all (K, c) that fulfill (22) and (23).

condition (22). The shaded area represents all such pairs that additionally satisfy condition (23). More precisely, if (K, c) lies to the left of the dotted curve, then $K < \overline{K}_1(c)$ is satisfied. Similarly, any pair (K, c) to the left of the dashed curve satisfies $K < \overline{K}_2(c)$.

Next, we analyze the behavior of $EAP(c|\delta = 0)$ and $W(c|\delta = 0)$ as functions of c. It can be shown that for all K and c that satisfy (22) and $K < \overline{K}_1(c)$ (i.e. for all pairs (K, c) to the left of the dotted curve in Figure 8), any marginal increase in c will reduce ex-ante policy polarization and increase welfare.[52] That is, for small values of c, the behavior of $EAP(c|\delta = 0)$ and $W(c|\delta = 0)$ is in line with Theorem 2. However, there exists no $K > 0$ such that our results would enable us to compute $EAP(c|\delta = 0)$ and $W(c|\delta = 0)$ for all $c < 2\Pi - 2\sqrt{K}$, because the dotted curve in Figure 8 is strictly below the solid curve for all $K > 0$. This is why our results

[52]The detailed proof is available upon request.

do not enable us to confirm that Theorem 2 holds for all $c > 0$ in a setting with fixed costs of change.

9.2 Convex costs of change

In this section, we investigate the robustness of our results when the cost function in (3) is replaced by

$$U^c(i_{k1}, i_{k'2}) = -c \cdot |i_{k1} - i_{k'2}|^\eta,$$

with $\eta \in (1, 2]$.[53] On the one hand, $\eta > 1$ implies that costs are strictly convex in the difference between the policies adopted in the two periods. On the other, $\eta \leq 2$ implies that the relative utility losses of voters and candidates from policies that differ from their ideal policy positions is higher than the relative increase of costs to engineer the policy change.

We can show the following theorem:[54]

Theorem 7

Let $\eta \in (1, 2]$ and $c > 0$. Then, the following holds:

(i) There exists $\eta^* \in (1, 2)$ such that, if $\eta \in (1, \eta^*)$ and $c < 2\Pi$, Theorem 1 holds.

(ii) There exists $\delta^{**} \in (0, \Pi)$ such that any extra-hurdle $\delta \in \left(\delta^{**}, \frac{1}{2}\right]$ is P-reducing.

(iii) If $\eta = 2$:

(iii.a) Any extra-hurdle $\delta \in \left(0, \frac{1}{2}\right]$ is P-reducing, and $W(\delta)$ is constant for all $\delta \in \left[0, \frac{1}{2}\right]$.

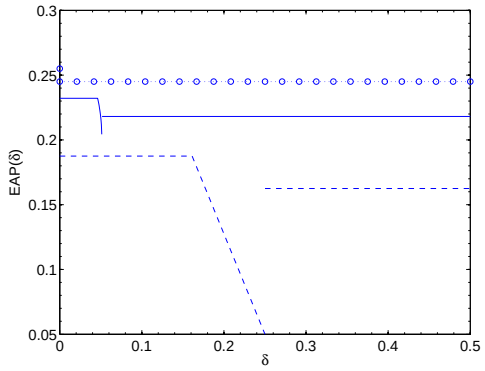
(iii.b) The set of extra-hurdles that are both W- and P-optimal is given by

$$\underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) \cap \underset{\delta \in \left[0, \frac{1}{2}\right]}{\operatorname{argmax}} W(\delta) = \left(0, \frac{1}{2}\right].$$

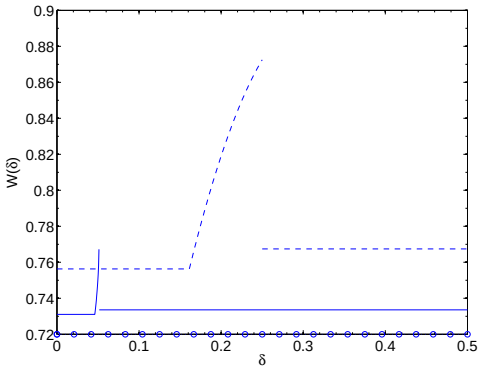
The results in Theorem 7 add to the robustness of Theorem 1. First, as stated in *(i)*, if costs of change are moderately convex, the results of Theorem 1 are unchanged. This is due to the continuity of $EAP(\delta)$ and $W(\delta)$ with respect to η. Second, in the special case where $\eta = 2$, the results of Theorem 1 also hold, except that the set of extra-hurdles that are both W- and P-optimal is not a singleton. Third, *(ii)* shows that for arbitrary values of $\eta \in (1, 2)$, $\delta = 0$ is never optimal in terms of ex-ante policy polarization, because any sufficiently large extra-hurdle

[53] Case $\eta > 2$ remains for further investigation.

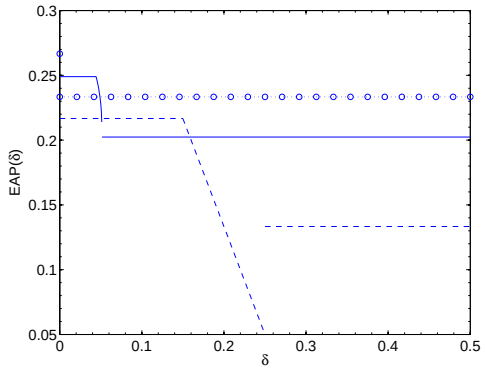
[54] The detailed proof is available upon request. Note that for convex costs of change, there is no explicit expression for the best response of the second-period office-holder. Thus, the main challenge consists in finding an expression for $i^*_{R1}(\delta)$, based on a set of implicit functions.



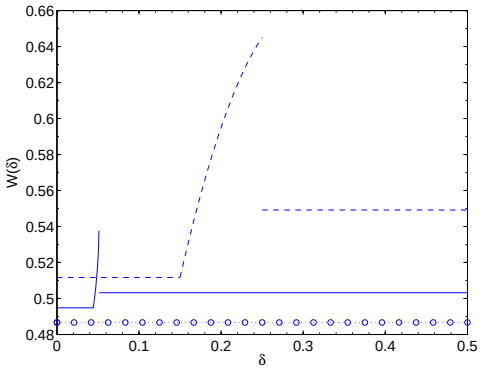
(a) Ex-ante policy polarization with $\rho = 0.1$.



(b) Welfare with $\rho = 0.1$.



(c) Ex-ante policy polarization with $\rho = \frac{1}{3}$.



(d) Welfare with $\rho = \frac{1}{3}$.

Figure 9: Equilibrium values of ex-ante policy polarization and welfare as a function of $\delta \in \left[0, \frac{1}{2}\right]$, using the parameter values $\mu_R = 0.8$, $c = 0.5$, $A = 2$, and $\rho = 0.1$ (resp. $\rho = \frac{1}{3}$). Each figure shows three different values of η: $\eta = 1$ (dashed), $\eta = 1.5$ (solid), and $\eta = 2$ (dotted).

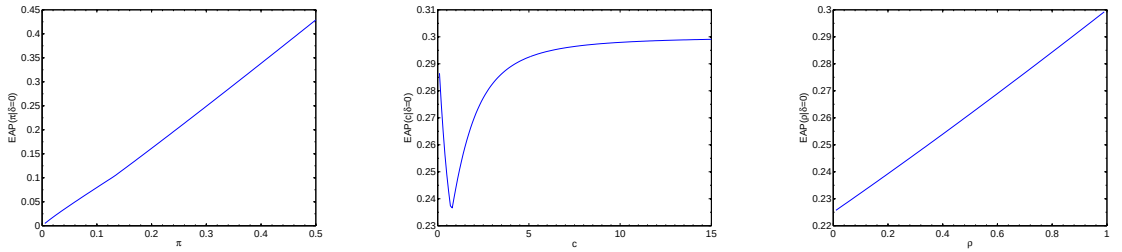
yields a lower ex-ante policy polarization than $\delta = 0$. Figures 9(a)–9(d) illustrate the results of Theorem 7. The graphs show welfare and ex-ante policy polarization for both linear ($\eta = 1$) and convex costs of change ($\eta = 1.5$ and $\eta = 2$).[55] The plots for $\eta = 1.5$ (solid lines) suggest that, for arbitrary $\eta \in (1, 2)$, even stronger results than statement *(ii)* of Theorem 7 might hold. More precisely, the solid plots of Figure 9(b) and Figure 9(d) suggest that, even if there are convex costs of change, $\delta = 0$ cannot be optimal in terms of welfare, because any sufficiently large extra-hurdle is W-increasing. Moreover, Figures 9(a)–9(d) suggest that for $\eta \in (1, 2)$, any non-zero extra-hurdle is both weakly W-increasing and weakly P-reducing. Thus, although for convex costs of change there does not always exist an extra-hurdle that is both W- and P-optimal, there is numerical evidence for the fact that any W-optimal extra-hurdle is P-reducing

[55]The equilibrium policy choices (and the resulting values of ex-ante policy polarization and welfare) in the case of $\eta = 1.5$ have been computed numerically. Details of the calculations are available upon request.

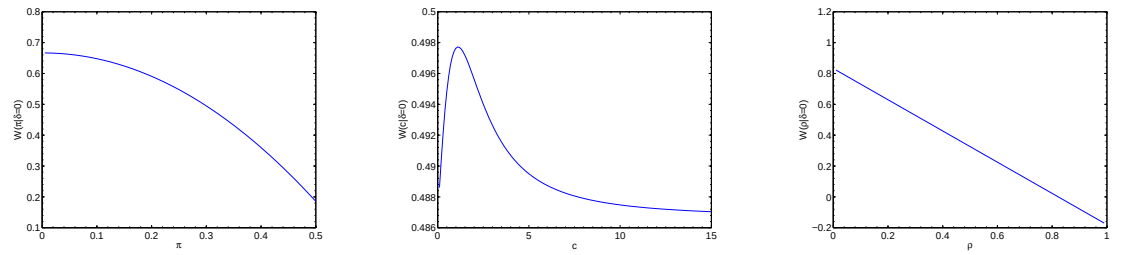
and any P-optimal extra-hurdle is W-increasing. An example for a situation where there is no extra-hurdle that is both W- and P-optimal is given in Figure 9(c) and Figure 9(d) for the case of $\eta = 1.5$. Indeed, in this case,

$$\underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) \cap \underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmax}} W(\delta) = \emptyset.$$

It is also helpful to consider some comparative statics results in the case of convex costs of change. In Figures 10(a)–10(f), we plot ex-ante policy polarization and welfare as a function of Π, c, and ρ, for $\delta = 0$ (i.e. in the absence of extra-hurdles) and $\eta = 1.5$. The graphs reveal the same qualitative comparative statics results as in Section 5.3 for linear costs of change. In particular, Theorem 2 is robust in a setting with convex costs of change.



(a) $EAP(\Pi|\delta = 0)$. (b) $EAP(c|\delta = 0)$. (c) $EAP(\rho|\delta = 0)$.



(d) $W(\Pi|\delta = 0)$. (e) $W(c|\delta = 0)$. (f) $W(\rho|\delta = 0)$.

Figure 10: Comparative statics for $\delta = 0$. Behavior of ex-ante policy polarization and welfare as a function of $\Pi \in \left[0, \frac{1}{2}\right]$, $c > 0$, and $\rho \in (0, 1)$, respectively. In all three plots we use $\eta = 1.5$ and $A = 2$. In the plots where the parameters μ_R, c, and ρ are fixed, we have set them to $\mu_R = 0.8$, $c = 0.5$, and $\rho = \frac{1}{3}$, respectively.

As noted above, we have not yet considered a situation where the relative utility losses of voters and candidates from policies that differ from their ideal policy positions is lower than the relative increase of costs to engineer the policy changes.[56] In order to analyze whether our results are robust to frameworks where this is the case, it is of interest to analyze a situation

[56] We have in fact assumed that $\eta \in [1, 2]$, and that the utility of voters and candidates from the policy is quadratic in the distance between the policy choice and their ideal position.

where voters and candidates derive a utility of

$$U_i^I(i_{kt}) = -|i_{kt} - i|$$

from the policy choice i_{kt} in period t and costs of change are given by

$$U^c(i_{k1}, i_{k'2}) = -c \cdot (i_{k1} - i_{k'2})^2,$$

if i_{k1} and $i_{k'2}$ are the policies chosen in the first and the second period, respectively. It is easy to show that in this framework, at least for small values of c, $EAP(\delta) \geq EAP(0)$ and $W(\delta) \leq W(0)$ for all $\delta > 0$.[57] Thus, $\delta = 0$ is both W- and P-optimal in this case.

10 Conclusion

We have presented a novel model of electoral competition in which changes of policies are costly for voters and candidates and these costs increase with the magnitude of the policy change. The model has allowed us to endogenize the degree of policy polarization in two-candidate elections. We have found that moderate costs of change yield lower ex-ante policy polarization and higher welfare than large costs of change or no costs at all. We have further suggested a way to curb ex-ante policy polarization without reducing welfare, namely by setting the re-election hurdle higher than 50%. Of course, numerous extensions beyond those already analyzed in the paper can be pursued. For instance, exploring the impact of costs of change in democracies with more than two parties, where governments typically consist of a coalition of parties, could reveal how such costs influence the say of parties in government policy-making. Another interesting avenue for further inquiry would be to allow fluctuations of the citizens' and the median voter's ideal policy position. This would imply that some policy changes would occur simply because electorate preferences change. These and other extensions can be expected to further enrich our understanding of policy-making in democracy.

[57] The detailed proof is available on request.

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Appendix A

This appendix contains all proofs from the main body of the paper.

Proof of Proposition 1

Let $k \in R$ and $k' \in R \cup L$ be the office-holders in $t = 1$ and $t = 2$, respectively. For a given first-period policy choice i_{k1}, the second-period office-holder k' chooses his policy $i_{k'2} \in [0, 1]$ so that his second period utility,

$$\begin{aligned} v_{i_{k1}}(i_{k'2}) &:= U^P(g_{k'2}) + U^I_{k'}(i_{k'2}) + U^c(i_{k1}, i_{k'2}) + b \\ &= a_{k'} - (i_{k'2} - \mu_{k'})^2 - c|i_{k'2} - i_{k1}| + b, \end{aligned}$$

is maximized. We note that $v_{i_{k1}}(i_{k'2})$ is differentiable with respect to $i_{k'2}$ on $(0, 1)\backslash\{i_{k1}\}$. We distinguish two cases, depending on the relative size of i_{k1} and $\mu_{k'}$.

<u>*Case 1:*</u> $i_{k1} < \mu_{k'}$

In this case,

$$\frac{dv_{i_{k1}}(i_{k'2})}{di_{k'2}} = \begin{cases} -2(i_{k'2} - \mu_{k'}) + c & \text{if } 0 < i_{k'2} < i_{k1}, \\ -2(i_{k'2} - \mu_{k'}) - c & \text{if } i_{k1} < i_{k'2} < 1. \end{cases}$$

Therefore $v_{i_{k1}}(i_{k'2})$ is strictly increasing if and only if

$$0 < i_{k'2} < max\left\{\mu_{k'} - \frac{c}{2}, i_{k1}\right\},$$

which implies that

$$\underset{i_{k'2} \in [0,1]}{\text{argmax}}\, v_{i_{k1}}(i_{k'2}) = \left\{max\left\{\mu_{k'} - \frac{c}{2}, i_{k1}\right\}\right\}.$$

<u>*Case 2:*</u> $i_{k1} \geq \mu_{k'}$

Analogous reasoning leads to

$$\underset{i_{k'2} \in [0,1]}{\text{argmax}}\, v_{i_{k1}}(i_{k'2}) = \left\{min\left\{\mu_{k'} + \frac{c}{2}, i_{k1}\right\}\right\}.$$

Finally, combining *Case 1* and *Case 2* yields

$$\underset{i_{k'2} \in [0,1]}{\text{argmax}}\, v_{i_{k1}}(i_{k'2}) = \left\{min\left\{max\left\{\mu_{k'} - \frac{c}{2}, i_{k1}\right\}, \mu_{k'} + \frac{c}{2}\right\}\right\}.$$

This completes the proof.

□

Proof of Proposition 2

Let $i \in [0,1]$ be an arbitrary voter. The ability of the incumbent, $k \in R$, is common knowledge when citizens vote the second time. However, the ability of the challenging left-wing candidate, say $k' \in L$, is not known, because he is a new candidate. According to (1), $E[a_{k'}] = 0$. If k wins the second election, i will expect the policy to be $i^*_{R2}(i_{k1})$, while i will expect $i^*_{L2}(i_{k1})$ if k' wins. Thus, since i will vote for k if he strictly prefers office-holder k to win the second election, he will vote for k if

$$a_k - (i^*_{R2}(i_{k1}) - i)^2 - c|i^*_{R2}(i_{k1}) - i_{k1}| > E[a_{k'}] - (i^*_{L2}(i_{k1}) - i)^2 - c|i^*_{L2}(i_{k1}) - i_{k1}|.$$

The above inequality is equivalent to

$$a_k > \underbrace{(i^*_{R2}(i_{k1}) - i)^2 + c|i^*_{R2}(i_{k1}) - i_{k1}| - (i^*_{L2}(i_{k1}) - i)^2 - c|i^*_{L2}(i_{k1}) - i_{k1}|}_{=:(\dagger)}. \tag{24}$$

Under (2) and (6), we have

$$i^*_{R2}(i_{k1}) > i^*_{L2}(i_{k1}) \text{ for all } i_{k1} \in [0,1]. \tag{25}$$

Indeed, by (2) and (6), $\mu_R - \frac{c}{2} > \mu_L + \frac{c}{2}$ holds and by (5) we know that $i^*_{R2}(i_{k1}) \geq \mu_R - \frac{c}{2}$ and $i^*_{L2}(i_{k1}) \leq \mu_L + \frac{c}{2}$. From (25) it follows that (†) defined in (24) is strictly decreasing in i, since

$$\frac{d(\dagger)}{di} = 2\left(i^*_{L2}(i_{k1}) - i^*_{R2}(i_{k1})\right) < 0.$$

Thus, because an incumbent is re-elected in the second election if he receives a vote-share of $\frac{1}{2} + \delta$ or larger, the critical voter in the second election is[58]

$$i = \frac{1}{2} - \delta. \tag{26}$$

Inserting (26) into (24) proves that office-holder k will be re-elected if and only if (7) holds.[59]

□

Proof of Proposition 3

For given δ, office-holder k aims at maximizing his expected utility at the beginning of $t = 1$, which depends on i_{k1}, his policy choice in the first period. It will be useful to introduce the following notation:

[58] This builds on the assumption that voters are uniformly distributed on $[0,1]$. Note that, qualitatively, the main results of our paper would also hold for any other symmetric distribution of the voters, as detailed in Section 7.

[59] Whether indifferent voters vote for the incumbent or the challenger does not influence the outcome of the election. Indeed, the critical voter has measure zero, so the election is tied whenever he is indifferent between both candidates, independently of his voting behavior.

- $EU_\delta(i_{k1})$ denotes k's expected utility at the beginning of $t = 1$ as a function of i_{k1} and parametrized by $\delta \in \left[0, \frac{1}{2}\right]$, and
- $p_\delta(i_{k1})$ denotes k's re-election probability in the second election as a function of i_{k1} and parametrized by $\delta \in \left[0, \frac{1}{2}\right]$.[60]

By Proposition 2, we have

$$p_\delta(i_{k1}) = P[a_k \geq a_\delta(i_{k1})].$$

Moreover, we have assumed in Section 3.4 that A is sufficiently large, so

$$-A < a_\delta(i_{k1}) < A$$

for all $\delta \in \left[0, \frac{1}{2}\right]$ and all $i_{k1} \in [0,1]$. Therefore, for every fixed value of δ, $p_\delta(\cdot)$ is a piecewise constant function with $p_\delta(i_{k1}) \in \left\{\frac{1-\rho}{2}, \frac{1+\rho}{2}\right\}$ for all $i_{k1} \in [0,1]$. More precisely,

$$p_\delta(i_{k1}) = \begin{cases} \frac{1+\rho}{2} & \text{if } a_\delta(i_{k1}) \leq 0, \\ \frac{1-\rho}{2} & \text{if } a_\delta(i_{k1}) > 0. \end{cases} \tag{27}$$

We can now formulate the maximization problem that k faces in $t = 1$. For a given $\delta \in \left[0, \frac{1}{2}\right]$, k chooses $i_{k1} \in [0,1]$ so that

$$\begin{aligned} EU_\delta(i_{k1}) =& p_\delta(i_{k1}) \cdot \Big\{2b + 2E[a_k | a_k \geq a_\delta(i_{k1})] \\ & - (i_{k1} - \mu_R)^2 - (i^*_{R2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{R2}(i_{k1})|\Big\} \\ & + (1 - p_\delta(i_{k1})) \cdot \Big\{b + E[a_k | a_k < a_\delta(i_{k1})] \\ & - (i_{k1} - \mu_R)^2 - (i^*_{L2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{L2}(i_{k1})|\Big\} \end{aligned} \tag{28}$$

is maximized. Since b is large and the ability distribution is discrete, maximization of the re-election probability $p_\delta(i_{k1})$ is a necessary condition to maximize $EU_\delta(i_{k1})$. Hence office-holder k's re-election probability is given by

$$p^*(\delta) = \max_{i_{k1} \in [0,1]} p_\delta(i_{k1}). \tag{29}$$

We now define $I^*(\delta)$ as

$$I^*(\delta) = \operatorname*{argmax}_{i_{k1} \in [0,1]} p_\delta(i_{k1}). \tag{30}$$

We next observe that solving

$$\operatorname*{argmax}_{i_{k1} \in [0,1]} EU_\delta(i_{k1})$$

[60] Note that this is the re-election probability as perceived before a_k has been realized. Furthermore, $p_\delta(i_{k1})$ is to be understood conditionally on k being in office in $t = 1$.

is equivalent to solving

$$\underset{i_{k1}\in I^*(\delta)}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)),$$

where $EU_\delta(i_{k1}|p^*(\delta))$ denotes k's expected utility at the beginning of $t = 1$, conditional on $p_\delta(i_{k1})$ being equal to $p^*(\delta)$. Subsequently, we therefore proceed in three steps to maximize $EU_\delta(i_{k1})$. In Step 1, we compute $p^*(\delta)$ and $I^*(\delta)$ as a function of $\delta \in \left[0, \frac{1}{2}\right]$. Then, in Step 2, we solve

$$\underset{i_{k1}\in[0,1]}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)).$$

Finally, in Step 3, we restrict the solutions of Step 2 to $I^*(\delta)$, that is we compute

$$\underset{i_{k1}\in I^*(\delta)}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)).$$

Step 1: Computation of $p^*(\delta)$ and $I^*(\delta)$ as functions of $\delta \in \left[0, \frac{1}{2}\right]$

First, it is useful to define

$$\mathcal{A}^\delta := \{i_{k1} \in [0,1] \mid a_\delta(i_{k1}) \leq 0\}, \tag{31}$$

for each $\delta \in \left[0, \frac{1}{2}\right]$. $\mathcal{A}^\delta$ is the set of policies that ensure re-election when the office-holder turns out to have zero ability.[61] By (27), (29), and (30), we know that

$$p^*(\delta) = \begin{cases} \frac{1+\rho}{2} & \text{if } \mathcal{A}^\delta \neq \emptyset, \\ \frac{1-\rho}{2} & \text{if } \mathcal{A}^\delta = \emptyset \end{cases} \tag{32}$$

and

$$I^*(\delta) = \begin{cases} \mathcal{A}^\delta & \text{if } \mathcal{A}^\delta \neq \emptyset, \\ [0,1] & \text{if } \mathcal{A}^\delta = \emptyset. \end{cases} \tag{33}$$

Second, in order to determine $\mathcal{A}^\delta$, it is useful to define the sets

$$I_1 := \left[0, max\left\{\mu_L - \frac{c}{2}, 0\right\}\right), \tag{34}$$

$$I_2 := \left[max\left\{\mu_L - \frac{c}{2}, 0\right\}, \mu_L + \frac{c}{2}\right], \tag{35}$$

$$I_3 := \left(\mu_L + \frac{c}{2}, \mu_R - \frac{c}{2}\right), \tag{36}$$

$$I_4 := \left[\mu_R - \frac{c}{2}, min\left\{\mu_R + \frac{c}{2}, 1\right\}\right], \tag{37}$$

$$I_5 := \left(min\left\{\mu_R + \frac{c}{2}, 1\right\}, 1\right], \tag{38}$$

since the expressions in (5) for $i^*_{R2}(i_{k1})$ and $i^*_{L2}(i_{k1})$ depend on whether $i_{k1} \in I_1$, $i_{k1} \in I_2$, $i_{k1} \in I_3$, $i_{k1} \in I_4$ or $i_{k1} \in I_5$.[62] Since

$$I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5 = [0,1],$$

[61] We stress that the office-holder does not know his ability when he chooses i_{k1}. Moreover, recall that an office-holder with ability A will always be re-elected and an office-holder with ability $-A$ will never be.

[62] Note that $[0,0)$ and $(1,1]$ are empty sets.

we can use the decomposition

$$\mathcal{A}^\delta = \underbrace{\left\{\mathcal{A}^\delta \cap I_1\right\}}_{\text{Step 1a}} \cup \underbrace{\left\{\mathcal{A}^\delta \cap I_2\right\}}_{\text{Step 1b}} \cup \underbrace{\left\{\mathcal{A}^\delta \cap I_3\right\}}_{\text{Step 1c}} \cup \underbrace{\left\{\mathcal{A}^\delta \cap I_4\right\}}_{\text{Step 1d}} \cup \underbrace{\left\{\mathcal{A}^\delta \cap I_5\right\}}_{\text{Step 1e}} \tag{39}$$

to determine $\mathcal{A}^\delta$. Subsequently, in Steps 1a to 1e, we compute expressions for the subsets of $\mathcal{A}^\delta$ listed in (39). In Step 1f, we use Steps 1a to 1e to obtain expressions for $p^*(\delta)$ and $I^*(\delta)$, for all $\delta \in \left[0, \frac{1}{2}\right]$.

Step 1a: Computation of $\mathcal{A}^\delta \cap I_1$

If $i_{k1} \in I_1$, then, by (5),

$$i^*_{R2}(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i^*_{L2}(i_{k1}) = \mu_L - \frac{c}{2} > i_{k1}.$$

Thus, by (8),

$$\begin{aligned} a_\delta(i_{k1}) =& \left(\mu_R - \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_R - \frac{c}{2} - i_{k1}\right) \\ & - \left(\mu_L - \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 - c\left(\mu_L - \frac{c}{2} - i_{k1}\right) \\ =& \left(\mu_R - \frac{c}{2}\right)^2 - \left(\mu_L - \frac{c}{2}\right)^2 - 2\left(\frac{1}{2} - \delta\right)(\mu_R - \mu_L) + c(\mu_R - \mu_L) \\ =& (2\mu_R - 1)(1 - c) - (2\mu_R - 1)(1 - c) + 2\delta(2\mu_R - 1) \\ =& 2\delta(2\mu_R - 1), \end{aligned}$$

where we have used $\mu_L = 1 - \mu_R$. By (31), this yields

$$\mathcal{A}^\delta \cap I_1 = \begin{cases} I_1 & \text{if } \delta = 0, \\ \emptyset & \text{if } \delta \in \left(0, \frac{1}{2}\right]. \end{cases} \tag{40}$$

Step 1b: Computation of $\mathcal{A}^\delta \cap I_2$

If $i_{k1} \in I_2$, then, by (5),

$$i^*_{R2}(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i^*_{L2}(i_{k1}) = i_{k1}.$$

Hence, by (8),

$$\begin{aligned} a_\delta(i_{k1}) =& \left(\mu_R - \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_R - \frac{c}{2} - i_{k1}\right) - \left(i_{k1} - \left(\frac{1}{2} - \delta\right)\right)^2 \\ =& - i_{k1}^2 + (1 - 2\delta - c)i_{k1} + \left(\mu_R - \frac{c}{2}\right)^2 - \left(\mu_R - \frac{c}{2}\right)(1 - 2\delta - c). \end{aligned}$$

The discriminant of

$$-i_{k1}^2 + (1-2\delta-c)i_{k1} + \left(\mu_R - \frac{c}{2}\right)^2 - \left(\mu_R - \frac{c}{2}\right)(1-2\delta-c)$$

is

$$\Delta := (2\delta + 2\mu_R - 1)^2.$$

Hence,

$$\begin{aligned}\mathcal{A}^\delta \cap I_2 &= \left\{\left(-\infty, \mu_L - \frac{c}{2} - 2\delta\right] \cup \left[\mu_R - \frac{c}{2}, +\infty\right)\right\} \cap I_2 \\ &= \begin{cases} \left\{\mu_L - \frac{c}{2}\right\} & \text{if } \delta = 0 \text{ and } \mu_L - \frac{c}{2} \geq 0, \\ \emptyset & \text{otherwise.} \end{cases}\end{aligned} \tag{41}$$

Step 1c: Computation of $\mathcal{A}^\delta \cap I_3$

If $i_{k1} \in I_3$, then, by (5),

$$i_{R2}^*(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i_{L2}^*(i_{k1}) = \mu_L + \frac{c}{2} < i_{k1},$$

and, by (8),

$$\begin{aligned}a_\delta(i_{k1}) =& \left(\mu_R - \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_R - \frac{c}{2} - i_{k1}\right) \\ & - \left(\mu_L + \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_L + \frac{c}{2} - i_{k1}\right) \\ =& \underbrace{\left(\mu_R - \frac{c}{2}\right)^2 - \left(\mu_L + \frac{c}{2}\right)^2}_{=2\mu_R - 1 - c} - 2\left(\frac{1}{2} - \delta\right)(2\mu_R - 1 - c) + c(1 - 2i_{k1}) \\ =& 2\delta(2\mu_R - 1 - c) - 2i_{k1}c + c.\end{aligned}$$

Hence,

$$a_\delta(i_{k1}) \leq 0 \Leftrightarrow i_{k1} \geq \frac{2\delta(2\mu_R - 1 - c) + c}{2c} \tag{42}$$

and

$$\mathcal{A}^\delta \cap I_3 = \left[\frac{2\delta(2\mu_R - 1 - c) + c}{2c}, +\infty\right) \cap I_3.$$

We observe that

$$\frac{2\delta(2\mu_R - 1 - c) + c}{2c} \geq \frac{1}{2} > \mu_L + \frac{c}{2},$$

for all $\delta \geq 0$, and

$$\frac{2\delta(2\mu_R - 1 - c) + c}{2c} < \mu_R - \frac{c}{2} \Leftrightarrow \delta < \frac{c}{2}.$$

Therefore,

$$\mathcal{A}^\delta \cap I_3 = \begin{cases} \left[\frac{2\delta(2\mu_R - 1 - c) + c}{2c}, \mu_R - \frac{c}{2}\right) & \text{if } \delta \in \left[0, \frac{c}{2}\right), \\ \emptyset & \text{if } \delta \in \left[\frac{c}{2}, \frac{1}{2}\right]. \end{cases} \tag{43}$$

Step 1d: Computation of $\mathcal{A}^\delta \cap I_4$

If $i_{k1} \in I_4$, then, by (5),

$$i^*_{R2}(i_{k1}) = i_{k1} \quad \text{and} \quad i^*_{L2}(i_{k1}) = \mu_L + \frac{c}{2} < i_{k1},$$

and thus, by (8),

$$\begin{aligned} a_\delta(i_{k1}) &= \left(i_{k1} - \left(\frac{1}{2} - \delta\right)\right)^2 - \left(\mu_L + \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_L + \frac{c}{2} - i_{k1}\right) \\ &= i_{k1}^2 + (2\delta - 1 - c)i_{k1} - \left[\left(\mu_L + \frac{c}{2}\right)^2 + \left(\mu_L + \frac{c}{2}\right)(2\delta - 1 - c)\right]. \end{aligned}$$

The discriminant of

$$i_{k1}^2 + (2\delta - 1 - c)i_{k1} - \left[\left(\mu_L + \frac{c}{2}\right)^2 + \left(\mu_L + \frac{c}{2}\right)(2\delta - 1 - c)\right]$$

is

$$\Delta := (2\delta - 2\mu_R + 1)^2.$$

Hence,

$$\begin{aligned} \mathcal{A}^\delta \cap I_4 &= \left[\frac{1 + c - 2\delta - |2\delta - 2\mu_R + 1|}{2}, \frac{1 + c - 2\delta + |2\delta - 2\mu_R + 1|}{2}\right] \cap I_4 \\ &= \begin{cases} \left[\mu_L + \frac{c}{2}, \mu_R + \frac{c}{2} - 2\delta\right] \cap I_4 & \text{if } \delta < \mu_R - \frac{1}{2}, \\ \left[\mu_R + \frac{c}{2} - 2\delta, \mu_L + \frac{c}{2}\right] \cap I_4 & \text{if } \delta \geq \mu_R - \frac{1}{2}. \end{cases} \\ &= \begin{cases} \left[\mu_L + \frac{c}{2}, \mu_R + \frac{c}{2} - 2\delta\right] \cap I_4 & \text{if } \delta < \mu_R - \frac{1}{2}, \\ \emptyset & \text{if } \delta \geq \mu_R - \frac{1}{2}, \end{cases} \end{aligned} \tag{44}$$

where the last equality follows since $\mu_L + \frac{c}{2} < \mu_R - \frac{c}{2}$. Moreover, since

$$\mu_R + \frac{c}{2} - 2\delta \geq \mu_R - \frac{c}{2} \Leftrightarrow \delta \leq \frac{c}{2}$$

and $\mu_L + \frac{c}{2} < \mu_R - \frac{c}{2}$, the case where $\delta < \mu_R - \frac{1}{2}$ can be reformulated as

$$\mathcal{A}^\delta \cap I_4 = \begin{cases} \left[\mu_R - \frac{c}{2}, min\left\{\mu_R + \frac{c}{2} - 2\delta, 1\right\}\right] & \text{if } \delta \in \left[0, \frac{c}{2}\right], \\ \emptyset & \text{if } \delta \in \left(\frac{c}{2}, \mu_R - \frac{1}{2}\right). \end{cases} \tag{45}$$

Finally, combining (44) and (45) yields

$$\mathcal{A}^\delta \cap I_4 = \begin{cases} \left[\mu_R - \frac{c}{2}, min\left\{\mu_R + \frac{c}{2} - 2\delta, 1\right\}\right] & \text{if } \delta \in \left[0, \frac{c}{2}\right], \\ \emptyset & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \end{cases} \tag{46}$$

Step 1e: Computation of $\mathcal{A}^\delta \cap I_5$

If $i_{k1} \in I_5$, then, by (5),

$$i^*_{R2}(i_{k1}) = \mu_R + \frac{c}{2} < i_{k1} \quad \text{and} \quad i^*_{L2}(i_{k1}) = \mu_L + \frac{c}{2} < i_{k1}.$$

Thus, by (8),

$$\begin{aligned}
a_\delta(i_{k1}) =& \left(\mu_R + \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 - c\left(\mu_R + \frac{c}{2} - i_{k1}\right) \\
&- \left(\mu_L + \frac{c}{2} - \left(\frac{1}{2} - \delta\right)\right)^2 + c\left(\mu_L + \frac{c}{2} - i_{k1}\right) \\
=& \left(\mu_R + \frac{c}{2}\right)^2 - \left(\mu_L + \frac{c}{2}\right)^2 - 2\left(\frac{1}{2} - \delta\right)(\mu_R - \mu_L) - c(\mu_R - \mu_L) \\
=& (2\mu_R - 1)(1 + c) - (2\mu_R - 1)(1 + c) + 2\delta(2\mu_R - 1) \\
=& 2\delta(2\mu_R - 1).
\end{aligned}$$

By (31), this yields

$$\mathcal{A}^\delta \cap I_5 = \begin{cases} I_5 & \text{if } \delta = 0, \\ \emptyset & \text{if } \delta \in \left(0, \frac{1}{2}\right]. \end{cases} \tag{47}$$

Step 1f: Computation of $p^*(\delta)$ and $I^*(\delta)$

Let us now combine (40), (41), (43), (46), and (47) to obtain compact expressions for $\mathcal{A}^\delta$. These expressions will enable us to compute $p^*(\delta)$ and $I^*(\delta)$ with the help of (32) and (33). We distinguish three different cases.

Case 1: $\delta = 0$

We consider two subcases, depending on the value of the parameter c.

Case 1a: $c \leq 2(1 - \mu_R)$

In this case, $\mu_L - \frac{c}{2} \geq 0$, so

$$\mathcal{A}^0 = \left[0, \mu_L - \frac{c}{2}\right] \cup \left[\frac{1}{2}, 1\right],$$

which, by (32) and (33), yields

$$p^*(0) = \frac{1 + \rho}{2} \quad \text{and} \quad I^*(0) = \left[0, \mu_L - \frac{c}{2}\right] \cup \left[\frac{1}{2}, 1\right]. \tag{48}$$

Case 1b: $c > 2(1 - \mu_R)$

In this case, $\mu_L - \frac{c}{2} < 0$, so

$$\mathcal{A}^0 = \left[\frac{1}{2}, 1\right],$$

which, by (32) and (33), yields

$$p^*(0) = \frac{1 + \rho}{2} \quad \text{and} \quad I^*(0) = \left[\frac{1}{2}, 1\right]. \tag{49}$$

Case 2: $\delta \in \left(0, \frac{c}{2}\right]$

In this case, (40), (41), (43), (46), and (47) yield

$$\mathcal{A}^{\delta} = \left[\frac{2\delta(2\mu_R - 1 - c) + c}{2c}, min\left\{\mu_R + \frac{c}{2} - 2\delta, 1\right\}\right],$$

which in turn implies

$$p^*(\delta) = \frac{1+\rho}{2} \quad \text{and} \quad I^*(\delta) = \left[\frac{2\delta(2\mu_R - 1 - c) + c}{2c}, min\left\{\mu_R + \frac{c}{2} - 2\delta, 1\right\}\right], \tag{50}$$

by (32) and (33).

Case 3: $\delta \in \left(\frac{c}{2}, \frac{1}{2}\right]$

For such extra-hurdles

$$\mathcal{A}^{\delta} = \emptyset,$$

so, by (32) and (33),

$$p^*(\delta) = \frac{1-\rho}{2} \quad \text{and} \quad I^*(\delta) = [0,1]. \tag{51}$$

Step 2: Solution to $\text{argmax}_{i_{k1} \in [0,1]} EU_{\delta}(i_{k1}|p^*(\delta))$

We obtain from (28) that

$$\begin{aligned} EU_{\delta}(i_{k1}|p^*(\delta)) =& p^*(\delta) \cdot \Big\{2b + 2E[a_k|k \text{ re-elected and } p_{\delta}(i_{k1}) = p^*(\delta)] \\ & - (i_{k1} - \mu_R)^2 - (i^*_{R2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{R2}(i_{k1})|\Big\} \\ & + (1 - p^*(\delta)) \cdot \Big\{b + E[a_k|k \text{ not re-elected and } p_{\delta}(i_{k1}) = p^*(\delta)] \\ & - (i_{k1} - \mu_R)^2 - (i^*_{L2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{L2}(i_{k1})|\Big\}. \end{aligned} \tag{52}$$

This expression of $EU_{\delta}(i_{k1}|p^*(\delta))$ can be simplified using

$$E[a_k|k \text{ re-elected and } p_{\delta}(i_{k1}) = p^*(\delta)] = \frac{A(1-\rho)}{2p^*(\delta)} \tag{53}$$

and

$$E[a_k|k \text{ not re-elected and } p_{\delta}(i_{k1}) = p^*(\delta)] = -\frac{A(1-\rho)}{2(1-p^*(\delta))}. \tag{54}$$

Recall that, by (27),

$$p^*(\delta) \in \left\{\frac{1-\rho}{2}, \frac{1+\rho}{2}\right\}.$$

Then, (53) follows from

$$\begin{aligned} E[a_k|k \text{ re-elected and } p_\delta(i_{k1}) = p^*(\delta)] &= \begin{cases} E[a_k|a_k = A] & \text{if } p^*(\delta) = \frac{1-\rho}{2}, \\ E[a_k|a_k \in \{0, A\}] & \text{if } p^*(\delta) = \frac{1+\rho}{2}, \end{cases} \\ &= \begin{cases} A & \text{if } p^*(\delta) = \frac{1-\rho}{2}, \\ \frac{A(1-\rho)}{1+\rho} & \text{if } p^*(\delta) = \frac{1+\rho}{2}, \end{cases} \\ &= \frac{A(1-\rho)}{2p^*(\delta)}. \end{aligned}$$

The proof of (54) is similar. Inserting (53) and (54) into (52) yields

$$\begin{aligned} EU_\delta(i_{k1}|p^*(\delta)) =& \frac{A(1-\rho)}{2} + b \cdot (1 + p^*(\delta)) \\ &+ p^*(\delta) \cdot \left[-(i_{k1} - \mu_R)^2 - (i^*_{R2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{R2}(i_{k1})|\right] \\ &+ (1 - p^*(\delta)) \cdot \left[-(i_{k1} - \mu_R)^2 - (i^*_{L2}(i_{k1}) - \mu_R)^2 - c|i_{k1} - i^*_{L2}(i_{k1})|\right]. \end{aligned} \tag{55}$$

We now analyze the behavior of $EU_\delta(i_{k1}|p^*(\delta))$, given by (55), as a function of $i_{k1} \in [0, 1]$. We observe from (5) and (55) that, for each $\delta \in \left[0, \frac{1}{2}\right]$, $EU_\delta(i_{k1}|p^*(\delta))$ is continuous in i_{k1} on $[0, 1]$ and differentiable on

$$(0, 1) \backslash \left\{\mu_L - \frac{c}{2}, \mu_L + \frac{c}{2}, \mu_R - \frac{c}{2}, \mu_R + \frac{c}{2}\right\}.$$

Accordingly, in Steps 2a to 2e we analyze the sign of

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$$

for $i_{k1} \in I^\circ_j$ and $j = 1, 2, \ldots, 5$. Recall that $I_1, I_2, \ldots, I_5$ are defined in (34)-(38) and, for each $j \in \{1, 2, \ldots, 5\}$, let I°_j denote the interior of I_j. In Step 2f, we combine Steps 2a to 2e in order to solve $\text{argmax}_{i_{k1} \in [0,1]} EU_\delta(i_{k1}|p^*(\delta))$.

Step 2a: $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ for $i_{k1} \in I^\circ_1$

If $i_{k1} \in I^\circ_1$, then, by (5),

$$i^*_{R2}(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i^*_{L2}(i_{k1}) = \mu_L - \frac{c}{2} > i_{k1}.$$

Thus, by (55),

$$\begin{aligned} \frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} =& p^*(\delta) \cdot [-2(i_{k1} - \mu_R) + c] + (1 - p^*(\delta)) \cdot [-2(i_{k1} - \mu_R) + c] \\ =& -2(i_{k1} - \mu_R) + c \\ >& 0, \end{aligned} \tag{56}$$

for all $i_{k1} < \mu_R + \frac{c}{2}$. From this property, it immediately follows that

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} > 0 \quad \text{for all} \quad i_{k1} \in I^\circ_1. \tag{57}$$

Step 2b: $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ for $i_{k1} \in I_2^\circ$

If $i_{k1} \in I_2^\circ$, then, by (5),

$$i_{R2}^*(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i_{L2}^*(i_{k1}) = i_{k1}.$$

Hence, by (55),

$$\begin{aligned}\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} =& p^*(\delta) \cdot [-2(i_{k1} - \mu_R) + c] \\ &+ (1 - p^*(\delta)) \cdot [-2(i_{k1} - \mu_R) - 2(i_{k1} - \mu_R)] \\ =& -2(i_{k1} - \mu_R)(2 - p^*(\delta)) + c \cdot p^*(\delta) \\ >& 0,\end{aligned} \tag{58}$$

for all $i_{k1} < \mu_R + \frac{c}{2} \cdot \frac{p^*(\delta)}{2-p^*(\delta)}$. Therefore,

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} > 0 \quad \text{for all} \quad i_{k1} \in I_2^\circ. \tag{59}$$

Step 2c: $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ for $i_{k1} \in I_3^\circ$

If $i_{k1} \in I_3^\circ$, then, by (5),

$$i_{R2}^*(i_{k1}) = \mu_R - \frac{c}{2} > i_{k1} \quad \text{and} \quad i_{L2}^*(i_{k1}) = \mu_L + \frac{c}{2} < i_{k1},$$

and, by (55),

$$\begin{aligned}\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} =& p^*(\delta) \cdot [-2(i_{k1} - \mu_R) + c] + (1 - p^*(\delta)) \cdot [-2(i_{k1} - \mu_R) - c] \\ =& -2(i_{k1} - \mu_R) + c(2p^*(\delta) - 1) \\ >& 0,\end{aligned} \tag{60}$$

for all $i_{k1} < \mu_R + \frac{c(2p^*(\delta)-1)}{2}$. Since $\mu_R + \frac{c(2p^*(\delta)-1)}{2} \geq \mu_R - \frac{c}{2}$, it follows that

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} > 0 \quad \text{for all} \quad i_{k1} \in I_3^\circ. \tag{61}$$

Step 2d: $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ for $i_{k1} \in I_4^\circ$

If $i_{k1} \in I_4^\circ$, then, by (5),

$$i_{R2}^*(i_{k1}) = i_{k1} \quad \text{and} \quad i_{L2}^*(i_{k1}) = \mu_L + \frac{c}{2} < i_{k1},$$

and thus, by (55),

$$\begin{aligned}\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} =& p^*(\delta)\cdot[-2(i_{k1}-\mu_R)-2(i_{k1}-\mu_R)]\\ &+(1-p^*(\delta))\cdot[-2(i_{k1}-\mu_R)-c]\\ =&-2(i_{k1}-\mu_R)(1+p^*(\delta))-c\cdot(1-p^*(\delta)),\end{aligned}\tag{62}$$

which is strictly positive if and only if $i_{k1} < \mu_R - \frac{c}{2}\cdot\frac{1-p^*(\delta)}{1+p^*(\delta)}$. Since, for $p^*(\delta)\in(0,1)$,

$$\mu_R - \frac{c}{2}\cdot\frac{1-p^*(\delta)}{1+p^*(\delta)} \in \left(\mu_R-\frac{c}{2},\mu_R\right)\subset I_4^\circ,$$

it follows that, for $i_{k1}\in I_4^\circ$,

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}\begin{cases}>0 & \text{if } i_{k1}<\mu_R-\frac{c}{2}\cdot\frac{1-p^*(\delta)}{1+p^*(\delta)},\\ =0 & \text{if } i_{k1}=\mu_R-\frac{c}{2}\cdot\frac{1-p^*(\delta)}{1+p^*(\delta)},\\ <0 & \text{if } i_{k1}>\mu_R-\frac{c}{2}\cdot\frac{1-p^*(\delta)}{1+p^*(\delta)}.\end{cases}\tag{63}$$

Step 2e: $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ for $i_{k1}\in I_5^\circ$

If $i_{k1}\in I_5^\circ$, then

$$i^*_{R2}(i_{k1}) = \mu_R+\frac{c}{2} < i_{k1} \quad\text{and}\quad i^*_{L2}(i_{k1}) = \mu_L+\frac{c}{2} < i_{k1},$$

by (5). Thus, by (55),

$$\begin{aligned}\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} =& p^*(\delta)\cdot[-2(i_{k1}-\mu_R)-c]+(1-p^*(\delta))\cdot[-2(i_{k1}-\mu_R)-c]\\ =&-2(i_{k1}-\mu_R)-c\\ <&0,\end{aligned}\tag{64}$$

for all $i_{k1} > \mu_R - \frac{c}{2}$. Therefore,

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} < 0 \quad\text{for all}\quad i_{k1}\in I_5^\circ.\tag{65}$$

Step 2f: Solution to $\text{argmax}_{i_{k1}\in[0,1]}\, EU_\delta(i_{k1}|p^*(\delta))$

Combining (57), (59), (61), (63), and (65) yields

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} > 0 \quad\text{for all}\quad i_{k1} < \mu_R - \frac{c}{2}\cdot\frac{(1-p^*(\delta))}{(1+p^*(\delta))}$$

and

$$\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}} < 0 \quad\text{for all}\quad i_{k1} > \mu_R - \frac{c}{2}\cdot\frac{(1-p^*(\delta))}{(1+p^*(\delta))},$$

whenever $\frac{dEU_\delta(i_{k1}|p^*(\delta))}{di_{k1}}$ exists. Due to the continuity of $EU_\delta(i_{k1}|p^*(\delta))$ in $i_{k1} \in [0,1]$, this yields

$$\underset{i_{k1}\in[0,1]}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R - \frac{c}{2}\cdot\frac{(1-p^*(\delta))}{(1+p^*(\delta))}\right\} \tag{66}$$

and

$$EU_\delta(i|p^*(\delta)) > EU_\delta(j|p^*(\delta)), \quad \text{for all } i \in \left[0, \mu_R - \frac{c}{2}\cdot\frac{(1-p^*(\delta))}{(1+p^*(\delta))}\right] \text{ and } j < i. \tag{67}$$

Step 3: Solution to $\operatorname{argmax}_{i_{k1}\in I^*(\delta)} EU_\delta(i_{k1}|p^*(\delta))$

From Step 1f we know that the value of $p^*(\delta)$ depends on whether $\delta \leq \frac{c}{2}$ or $\delta > \frac{c}{2}$. Let us therefore distinguish two cases.

Case 1: $\delta \in \left[0, \frac{c}{2}\right]$

In this case, (48), (49), and (50) yield

$$p^*(\delta) = \frac{1+\rho}{2}.$$

Hence, by (66),

$$\underset{i_{k1}\in[0,1]}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho}\right\}. \tag{68}$$

From (48) and (49), we obtain that, for $\rho \in (0,1)$,

$$\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho} \in \left(\mu_R - \frac{c}{6}, \mu_R\right) \subset \left[\frac{1}{2}, 1\right] \subset I^*(0).$$

Moreover, since

$$\frac{2\delta(2\mu_R - 1 - c) + c}{2c} \leq \mu_R - \frac{c}{2} < \mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho},$$

for all $\delta \leq \frac{c}{2}$, and

$$\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho} \leq \mu_R + \frac{c}{2} - 2\delta \Leftrightarrow \delta \leq \frac{c}{3+\rho},$$

it follows from (50) that

$$\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho} \in I^*(\delta) \quad \text{for } \delta \in \left[0, \frac{c}{3+\rho}\right]. \tag{69}$$

Combining (68) and (69) yields

$$\underset{i_{k1}\in I^*(\delta)}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho}\right\} \quad \text{for } \delta \in \left[0, \frac{c}{3+\rho}\right]. \tag{70}$$

For $\delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]$,

$$\underset{i_{k1}\in[0,1]}{\operatorname{argmax}} EU_\delta(i_{k1}|p^*(\delta)) \cap I^*(\delta) = \emptyset,$$

or, more precisely, the unique element in $\text{argmax}_{i_{k1}\in[0,1]} EU_\delta(i_{k1}|p^*(\delta))$ is larger than every element in $I^*(\delta)$. Hence, by (67), the office-holder will choose his policy by moving to the right as long as he stays in $I^*(\delta)$, the area that guarantees high re-election probability. That is,

$$\underset{i_{k1}\in I^*(\delta)}{\text{argmax}}\, EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R + \frac{c}{2} - 2\delta\right\} \quad \text{for } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]. \tag{71}$$

Case 2: $\delta \in \left(\frac{c}{2}, \frac{1}{2}\right]$

In this case,

$$p^*(\delta) = \frac{1-\rho}{2},$$

by (51), and (66) implies that

$$\underset{i_{k1}\in[0,1]}{\text{argmax}}\, EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R - \frac{c}{2}\cdot\frac{1+\rho}{3-\rho}\right\}.$$

Since $I^*(\delta) = [0,1]$, by (51), this immediately implies that

$$\underset{i_{k1}\in I^*(\delta)}{\text{argmax}}\, EU_\delta(i_{k1}|p^*(\delta)) = \left\{\mu_R - \frac{c}{2}\cdot\frac{1+\rho}{3-\rho}\right\} \quad \text{for } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \tag{72}$$

Combining (70), (71), and (72) yields (9). This completes the proof of Proposition 3.

□

Proof of Theorem 1

We will show that

(a) $EAP(\delta) \leq EAP(0)$, for all $\delta \in \left(0, \frac{1}{2}\right]$,

(b) $W(\delta) \geq W(0)$, for all $\delta \in \left(0, \frac{1}{2}\right]$,

(c) $\text{argmin}_{\delta\in\left[0,\frac{1}{2}\right]} EAP(\delta) = \left\{\frac{c}{2}\right\}$,

(d) $\text{argmax}_{\delta\in\left[0,\frac{1}{2}\right]} W(\delta) = \left\{\frac{c}{2}\right\}$,

from which Theorem 1 follows, with $\delta^* = \frac{c}{2}$. In Part 1, we analyze ex-ante policy polarization and prove *(a)* and *(c)*. Part 2 is devoted to welfare and the proofs of *(b)* and *(d)*.

Part 1: Analysis of $EAP(\delta)$

From (11), we know that

$$i^*_{R1}(\delta) = i^*_{R2}(\delta) > \frac{1}{2}, \quad \text{and}$$
$$i^*_{L2}(\delta) = \mu_L + \frac{c}{2} = 1 - \mu_R + \frac{c}{2} < \frac{1}{2}.$$

Inserting these equalities into (14) yields

$$EAP(\delta) = \frac{1+p^*(\delta)}{2}\left(i^*_{R1}(\delta) - \frac{1}{2}\right) + \frac{1-p^*(\delta)}{2}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right). \tag{73}$$

Thus, by (9) and (10), $EAP(\delta)$ is constant for $\delta \leq \frac{c}{3+\rho}$ and $\delta > \frac{c}{2}$ and decreasing in δ for $\delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]$. Moreover, $EAP(\delta)$ has a single discontinuity at $\delta = \frac{c}{2}$. It therefore suffices to compare $EAP\left(\frac{1}{2}\right)$ to both $EAP(0)$ and $EAP\left(\frac{c}{2}\right)$ to show that *(a)* and *(c)* hold. First, inserting (9) and (10) into (73) implies that

$$\begin{aligned} EAP\left(\frac{1}{2}\right) - EAP(0) =& \frac{3-\rho}{4}\cdot\left(\mu_R - \frac{c}{2}\cdot\frac{1+\rho}{3-\rho} - \frac{1}{2}\right) + \frac{1+\rho}{4}\cdot\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ & - \frac{3+\rho}{4}\cdot\left(\mu_R - \frac{c}{2}\cdot\frac{1-\rho}{3+\rho} - \frac{1}{2}\right) - \frac{1-\rho}{4}\cdot\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ =& -\frac{\rho}{2}\left(\mu_R - \frac{1}{2}\right) - \frac{\rho}{2}\cdot\frac{c}{2} + \frac{\rho}{2}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ =& -\frac{\rho c}{2} < 0, \end{aligned} \tag{74}$$

which establishes statement *(a)*. Second, again by inserting (9) and (10) into (73), we obtain

$$\begin{aligned} EAP\left(\frac{1}{2}\right) - EAP\left(\frac{c}{2}\right) =& \frac{3-\rho}{4}\cdot\left(\mu_R - \frac{c}{2}\cdot\frac{1+\rho}{3-\rho} - \frac{1}{2}\right) + \frac{1+\rho}{4}\cdot\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ & - \frac{3+\rho}{4}\cdot\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) - \frac{1-\rho}{4}\cdot\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ =& -\frac{\rho}{2}\left(\mu_R - \frac{1}{2}\right) + \frac{c}{4} + \frac{\rho}{2}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \\ =& \frac{(1-\rho)c}{4} > 0. \end{aligned} \tag{75}$$

This proves statement *(c)*. In this proof, we did not need an explicit expression for $EAP(\delta)$. However, for the sake of completeness, we can obtain an explicit expression for ex-ante policy polarization by inserting (9) and (10) into (73) and using simple algebraic manipulations:

$$EAP(\delta) = \begin{cases} \mu_R - \frac{1}{2} - \frac{c(1-\rho)}{4} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \mu_R - \frac{1}{2} + \frac{c(1+\rho)}{4} - \frac{(3+\rho)\delta}{2} & \text{if } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right], \\ \mu_R - \frac{1}{2} - \frac{c(1+\rho)}{4} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \end{cases}$$

Part 2: Analysis of $W(\delta)$

Now we investigate how welfare behaves as a function of δ. First, we compute $EU^P(\delta)$. For this purpose, suppose that $k \in R$ is the office-holder in $t = 1$ and that $k' \in L$ is his challenger in the second election. Because $E[a_{k'}] = 0$, and from (53) and (54),

$$E\left[a_k | k \text{ re-elected and } p_\delta(i_{k1}) = p^*(\delta)\right] = \frac{A(1-\rho)}{2p^*(\delta)} \quad \text{and}$$

$$E\left[a_k | k \text{ not re-elected and } p_\delta(i_{k1}) = p^*(\delta)\right] = -\frac{A(1-\rho)}{2(1-p^*(\delta))},$$

it follows by (16) that the expected lifetime utility from the public projects is constant:

$$EU^P(\delta) = \frac{A(1-\rho)}{2}.$$

Second, inserting (11) into (17) and (18) yields

$$EU^I_{\frac{1}{2}}(\delta) = -(1+p^*(\delta))\left(i^*_{R1}(\delta) - \frac{1}{2}\right)^2 - (1-p^*(\delta))\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2$$

and

$$EU^c(\delta) = -(1-p^*(\delta)) \cdot c\left(i^*_{R1}(\delta) - \left(\mu_L + \frac{c}{2}\right)\right),$$

respectively. Therefore, according to (15),

$$\begin{aligned} W(\delta) =& \frac{A(1-\rho)}{2} - (1+p^*(\delta))\left(i^*_{R1}(\delta) - \frac{1}{2}\right)^2 \\ & - (1-p^*(\delta))\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[i^*_{R1}(\delta) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}. \end{aligned} \tag{76}$$

By (9) and (10), it follows that $W(\delta)$ is constant for both ranges $\delta \leq \frac{c}{3+\rho}$ and $\delta > \frac{c}{2}$ and that $W(\delta)$ has a single discontinuity at $\delta = \frac{c}{2}$. Moreover, $W(\delta)$ is increasing in δ for $\delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]$, since

$$\frac{dW(\delta)}{d\delta} = \underbrace{\left[-2(1+p^*(\delta)) \cdot \left(i^*_{R1}(\delta) - \frac{1}{2}\right) - c(1-p^*(\delta))\right]}_{<0} \cdot \underbrace{\frac{di^*_{R1}(\delta)}{d\delta}}_{<0} > 0,$$

which follows from (76), and (9) and (10) for $\delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]$. It is therefore sufficient to compare $W\left(\frac{1}{2}\right)$ to both $W(0)$ and $W\left(\frac{c}{2}\right)$ in order to prove statements *(b)* and *(d)*. First, inserting (9) and (10) into (76), yields

$$\begin{aligned} W\left(\frac{1}{2}\right) - W(0) =& -\frac{3-\rho}{2}\left(\mu_R - \frac{c(1+\rho)}{2(3-\rho)} - \frac{1}{2}\right)^2 \\ & - \frac{1+\rho}{2}\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[\mu_R - \frac{c(1+\rho)}{2(3-\rho)} - \left(\mu_L + \frac{c}{2}\right)\right]\right\} \\ & + \frac{3+\rho}{2}\left(\mu_R - \frac{c(1-\rho)}{2(3+\rho)} - \frac{1}{2}\right)^2 \\ & + \frac{1-\rho}{2}\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[\mu_R - \frac{c(1-\rho)}{2(3+\rho)} - \left(\mu_L + \frac{c}{2}\right)\right]\right\}. \end{aligned}$$

Thus,

$$
\begin{aligned}
& W\left(\frac{1}{2}\right) - W(0) \\
=& -\frac{3-\rho}{2}\left(\mu_R - \frac{1}{2} - \frac{c(1+\rho)}{2(3-\rho)}\right)^2 \\
& -\frac{1+\rho}{2}\left\{\left(\mu_R - \frac{1}{2} - \frac{c}{2}\right)^2 + c\left[2\left(\mu_R - \frac{1}{2}\right) - \frac{2c}{3-\rho}\right]\right\} \\
& +\frac{3+\rho}{2}\left(\mu_R - \frac{1}{2} - \frac{c(1-\rho)}{2(3+\rho)}\right)^2 \\
& +\frac{1-\rho}{2}\left\{\left(\mu_R - \frac{1}{2} - \frac{c}{2}\right)^2 + c\left[2\left(\mu_R - \frac{1}{2}\right) - \frac{2c}{3+\rho}\right]\right\} \\
=& \left(\mu_R - \frac{1}{2}\right)^2 \cdot \left(-\frac{3-\rho}{2} - \frac{1+\rho}{2} + \frac{3+\rho}{2} + \frac{1-\rho}{2}\right) \\
& +\left(\frac{c}{2}\right)^2 \cdot \left[-\frac{(1+\rho)^2}{2(3-\rho)} - \frac{1+\rho}{2} + \frac{4(1+\rho)}{3-\rho} + \frac{(1-\rho)^2}{2(3+\rho)} + \frac{1-\rho}{2} - \frac{4(1-\rho)}{3+\rho}\right] \\
& +c\left(\mu_R - \frac{1}{2}\right) \cdot \left[\frac{1+\rho}{2} + \frac{1+\rho}{2} - (1+\rho) - \frac{1-\rho}{2} - \frac{1-\rho}{2} + (1-\rho)\right] \\
=& \left(\frac{c}{2}\right)^2 \cdot \frac{(1+\rho)(3+\rho)(-1-\rho-3+\rho+8) + (1-\rho)(3-\rho)(1-\rho+3+\rho-8)}{2(3-\rho)(3+\rho)} \\
=& \left(\frac{c}{2}\right)^2 \cdot \frac{2(1+\rho)(3+\rho) - 2(1-\rho)(3-\rho)}{(3-\rho)(3+\rho)} \\
=& \frac{4\rho c^2}{(3-\rho)(3+\rho)},
\end{aligned}
\tag{77}
$$

which is strictly positive for $\rho \in (0,1)$. This proves statement *(b)*. Second, again by making use of (9) and (10) in (76), we obtain

$$
\begin{aligned}
W\left(\frac{1}{2}\right) - W\left(\frac{c}{2}\right) =& -\frac{3-\rho}{2}\left(\mu_R - \frac{c(1+\rho)}{2(3-\rho)} - \frac{1}{2}\right)^2 \\
& -\frac{1+\rho}{2}\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[\mu_R - \frac{c(1+\rho)}{2(3-\rho)} - \left(\mu_L + \frac{c}{2}\right)\right]\right\} \\
& +\frac{3+\rho}{2}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right)^2 \\
& +\frac{1-\rho}{2}\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[\mu_R - \frac{c}{2} - \left(\mu_L + \frac{c}{2}\right)\right]\right\}.
\end{aligned}
$$

So,

$$
\begin{aligned}
& W\left(\frac{1}{2}\right) - W\left(\frac{c}{2}\right) \\
=& -\frac{3-\rho}{2}\left(\mu_R - \frac{1}{2} - \frac{c(1+\rho)}{2(3-\rho)}\right)^2 \\
& -\frac{1+\rho}{2}\left\{\left(\mu_R - \frac{1}{2} - \frac{c}{2}\right)^2 + c\left[2\left(\mu_R - \frac{1}{2}\right) - \frac{2c}{(3-\rho)}\right]\right\} \\
& +\frac{3+\rho}{2}\left(\mu_R - \frac{1}{2} - \frac{c}{2}\right)^2 \\
& +\frac{1-\rho}{2}\left\{\left(\mu_R - \frac{1}{2} - \frac{c}{2}\right)^2 + c\left[2\left(\mu_R - \frac{1}{2}\right) - c\right]\right\} \\
=& \left(\mu_R - \frac{1}{2}\right)^2 \cdot \left(-\frac{3-\rho}{2} - \frac{1+\rho}{2} + \frac{3+\rho}{2} + \frac{1-\rho}{2}\right) \\
& +\left(\frac{c}{2}\right)^2 \cdot \left[-\frac{(1+\rho)^2}{2(3-\rho)} - \frac{1+\rho}{2} + \frac{4(1+\rho)}{3-\rho} + \frac{3+\rho}{2} + \frac{1-\rho}{2} - 2(1-\rho)\right] \\
& +c\left(\mu_R - \frac{1}{2}\right) \cdot \left[\frac{1+\rho}{2} + \frac{1+\rho}{2} - (1+\rho) - \frac{3+\rho}{2} - \frac{1-\rho}{2} + (1-\rho)\right] \\
=& \frac{c^2\left(-\rho^2+4\rho+1\right)}{2(3-\rho)} - c\left(\mu_R - \frac{1}{2}\right)(1+\rho) \\
=& \frac{c^2\left(-\rho^2+4\rho+1\right) - c(2\mu_R-1)\left(-\rho^2+2\rho+3\right)}{2(3-\rho)} < -\frac{c^2(1-\rho)}{3-\rho} < 0, \qquad (78)
\end{aligned}
$$

where the first inequality in the last line holds since $-\rho^2 + 2\rho + 3 > 0$ for all $\rho \in (0,1)$ and $c < (2\mu_R - 1)$. This establishes statement *(d)*. For the sake of completeness, we derive the following explicit expression for welfare by inserting (9) and (10) into (76):

$$
W(\delta) = \begin{cases} \frac{A(1-\rho)}{2} - \frac{(2\mu_R-1)^2}{2} + \frac{c^2(1-\rho)}{2(3+\rho)} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \frac{A(1-\rho)-(1-2\mu_R-c)^2}{2} - 2(3+\rho)\delta^2 + \left[4c + (3+\rho)(2\mu_R-1)\right]\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right], \\ \frac{A(1-\rho)}{2} - \frac{(2\mu_R-1)^2}{2} + \frac{c^2(1+\rho)}{2(3-\rho)} & \text{if } \delta \in \left(\frac{c}{2}, \frac{1}{2}\right]. \end{cases}
$$

□

Proof of Theorem 2

From the expressions of $EAP(\delta)$ and $W(\delta)$ given in the proofs of Theorem 1 and Theorem 8, we immediately obtain expressions for $EAP(c|\delta = 0)$ and $W(c|\delta = 0)$, for all $c > 0$. The analysis of these expressions allows to show statements *(i)* and *(ii)*. Note that

$$\begin{aligned}\{c^*\} &= \operatorname*{argmin}_{c\geq 0} EAP(c|\delta=0) \cap \operatorname*{argmax}_{c\geq 0} W(c|\delta=0)\\ &= [2\Pi, (3+\rho)\Pi] \cap \{(3+\rho)\Pi\}\\ &= \{(3+\rho)\Pi\}.\end{aligned}$$

□

Proof of Theorem 3

For $i \in [0,1]$ let $EU_i^k(\delta_k)$ (resp. $EU_i^{k'}(\delta_{k'})$) denote the expected utility of voter i, immediately after candidate k (resp. k') has been elected for the first term in period $t=1$, and k (resp. k') has offered the extra-hurdle δ_k (resp. $\delta_{k'}$). Voter i will strictly prefer k to win the first election if and only if $EU_i^k(\delta_k) - EU_i^{k'}(\delta_{k'}) > 0$. The proof of the theorem is now divided into two steps. In Step 1, we show that $i = \frac{1}{2}$ is the critical voter in the first election. That is, the candidate whom voter $i = \frac{1}{2}$ supports in the first election will be in office in $t=1$.[63] In Step 2, we use this result to show which extra-hurdles are chosen in equilibrium and what the outcome of the first election will be.

Step 1: Critical voter in the first election

Analogously to (76), we can represent $EU_i^k(\delta_k)$ by

$$\begin{aligned}EU_i^k(\delta_k) =& \frac{A(1-\rho)}{2} - (1+p^*(\delta_k))\left(i_{R1}^*(\delta_k) - i\right)^2\\ &- (1-p^*(\delta_k))\left\{\left(\mu_L + \frac{c}{2} - i\right)^2 + c\left[i_{R1}^*(\delta_k) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}.\end{aligned} \tag{79}$$

By (2), and because a_k and $a_{k'}$ are drawn from the same distribution, the latter expression also holds for candidate k':

$$\begin{aligned}EU_i^{k'}(\delta_{k'}) =& \frac{A(1-\rho)}{2} - (1+p^*(\delta_{k'}))\left(1 - i_{R1}^*(\delta_{k'}) - i\right)^2\\ &- (1-p^*(\delta_{k'}))\left\{\left(1 - \left(\mu_L + \frac{c}{2}\right) - i\right)^2 + c\left[i_{R1}^*(\delta_{k'}) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}.\end{aligned} \tag{80}$$

Voter i will strictly prefer k to win the first election if and only if $EU_i^k(\delta_k) - EU_i^{k'}(\delta_{k'}) > 0$. Deducting (80) from (79) yields

$$\begin{aligned}&EU_i^k(\delta_k) - EU_i^{k'}(\delta_{k'}) =\\ &- (1+p^*(\delta_k))\left(i_{R1}^*(\delta_k) - i\right)^2 + (1+p^*(\delta_{k'}))\left(1 - i_{R1}^*(\delta_{k'}) - i\right)^2\\ &- (1-p^*(\delta_k))\left\{\left(\mu_L + \frac{c}{2} - i\right)^2 + c\left[i_{R1}^*(\delta_k) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}\\ &+ (1-p^*(\delta_{k'}))\left\{\left(1 - \left(\mu_L + \frac{c}{2}\right) - i\right)^2 + c\left[i_{R1}^*(\delta_{k'}) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}.\end{aligned} \tag{81}$$

[63] If the critical voter is indifferent between both candidates, the first election is tied.

From (9) and (10) we know that (81) yields different expressions, depending on whether $\delta_k \in \left[0, \frac{c}{3+\rho}\right]$, $\delta_k \in \left(\frac{c}{3+\rho}, \frac{c}{2}\right]$, or $\delta_k \in \left(\frac{c}{2}, \frac{1}{2}\right]$, and similarly for $\delta_{k'}$. Analyzing the nine resulting expressions of (81) separately shows that $EU_i^k(\delta_k) - EU_i^{k'}(\delta_{k'})$ is strictly increasing in i for all $(\delta_k, \delta_{k'}) \in \left[0, \frac{1}{2}\right] \times \left[0, \frac{1}{2}\right]$. For instance, if $\delta_k \in \left[0, \frac{c}{3+\rho}\right]$ and $\delta_{k'} \in \left[0, \frac{c}{3+\rho}\right]$, then, by (9),

$$i^*_{R1}(\delta_k) = i^*_{R1}(\delta_{k'}) = \mu_R - \frac{c}{2} \cdot \frac{1-\rho}{3+\rho}, \tag{82}$$

and, by (10),

$$p^*(\delta_k) = p^*(\delta_{k'}) = \frac{1+\rho}{2}. \tag{83}$$

Inserting (82) and (83) into (81), and after some simple algebraic manipulations, we find that

$$\frac{d\left[EU_i^k(\delta_k) - EU_i^{k'}(\delta_{k'})\right]}{di} = 2(2\mu_R - 1)(1+\rho),$$

which is strictly positive, since $2\mu_R - 1 > 0$ and $\rho > 0$. The proofs of the other eight cases are analogous. Hence, we conclude that the critical voter in the first election is $i = \frac{1}{2}$.

Step 2: Choice of extra-hurdle in equilibrium and outcome of first election

By the definition of welfare, as given in (15), we know that

$$EU^k_{\frac{1}{2}}(\delta) = EU^{k'}_{\frac{1}{2}}(\delta) = W(\delta).$$

Therefore, by Step 1:

- k wins the first election with probability 1, if $W(\delta_k) - W(\delta_{k'}) > 0$.
- k' wins the first election with probability 1, if $W(\delta_k) - W(\delta_{k'}) < 0$.
- k and k' each win the first election with probability equal to $\frac{1}{2}$, if $W(\delta_k) - W(\delta_{k'}) = 0$.

Because b is assumed to be large (see Section 3.4), candidate k (resp. k') offers δ_k (resp. $\delta_{k'}$) such that, given the extra-hurdle of the other candidate, his probability of winning the first election is maximized. Therefore, in equilibrium, candidate k suggests some $\delta_k \in \operatorname{argmax}_{\delta \in \left[0, \frac{1}{2}\right]} W(\delta)$, and candidate k' suggests some $\delta_{k'} \in \operatorname{argmax}_{\delta \in \left[0, \frac{1}{2}\right]} W(\delta)$, because otherwise one candidate could increase his re-election chances by offering a different extra-hurdle. Hence, by Section 5.2, both candidates commit to $\delta^* = \frac{c}{2}$ and win the election with a probability equal to $\frac{1}{2}$. This completes the proof of Theorem 3.

□

Proof of Theorem 4

Let

$$F_\alpha(x) := \int_0^x f_\alpha(t)dt$$

denote the cumulative distribution function of $f_\alpha(\cdot)$, and let $k \in R$ be in office in $t = 1$. Then, by the same reasoning as in the proof of Proposition 2, it can be shown that the critical voter in the second election is $i^* = F_\alpha^{-1}\left(\frac{1}{2} - \delta\right)$. By simple algebraic manipulations, it follows that, for any $\delta \in \left[0, \frac{1}{2}\right]$,

$$i^* = \frac{1}{2} - h_\alpha(\delta),$$

where we define

$$h_\alpha(\delta) := \begin{cases} \delta & \text{if } \alpha = 0, \\ \frac{\sqrt{(1-\alpha)^2+8\alpha\delta}+\alpha-1}{4\alpha} & \text{if } \alpha \in (0,1]. \end{cases}$$

Again, by the same arguments as in the proof of Proposition 2, it can be verified that $k \in R$ will be re-elected in the second election if and only if $a_k \geq a_{\delta,\alpha}(i_{k1})$, where

$$\begin{aligned} a_{\delta,\alpha}(i_{k1}) = &\left(i^*_{R2}(i_{k1}) - \left(\frac{1}{2} - h_\alpha(\delta)\right)\right)^2 + c\left|i^*_{R2}(i_{k1}) - i_{k1}\right| \\ &- \left(i^*_{L2}(i_{k1}) - \left(\frac{1}{2} - h_\alpha(\delta)\right)\right)^2 - c\left|i^*_{L2}(i_{k1}) - i_{k1}\right|. \end{aligned} \tag{84}$$

Moreover, for any $\alpha \in [0,1]$, $h_\alpha(\cdot)$ is a bijection from $\left[0, \frac{1}{2}\right]$ to $\left[0, \frac{1}{2}\right]$ and hence invertible. Indeed, $h_\alpha(0) = 0$, $h_\alpha\left(\frac{1}{2}\right) = \frac{1}{2}$, and $h_\alpha(\delta)$ is strictly increasing in δ. Using (84) and the invertibility of $h_\alpha(\cdot)$ in the proof of Proposition 3, yields

$$i^*_{R1,\alpha}(\delta) := \begin{cases} \mu_R - \frac{c}{2} \cdot \frac{1-\rho}{3+\rho} & \text{if } \delta \in \left[0, h_\alpha^{-1}\left(\frac{c}{3+\rho}\right)\right], \\ \mu_R + \frac{c}{2} - 2h_\alpha(\delta) & \text{if } \delta \in \left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right], \\ \mu_R - \frac{c}{2} \cdot \frac{1+\rho}{3-\rho} & \text{if } \delta \in \left(h_\alpha^{-1}\left(\frac{c}{2}\right), \frac{1}{2}\right] \end{cases} \tag{85}$$

and

$$p^*_\alpha(\delta) := \begin{cases} \frac{1+\rho}{2} & \text{if } \delta \in \left[0, h_\alpha^{-1}\left(\frac{c}{2}\right)\right], \\ \frac{1-\rho}{2} & \text{if } \delta \in \left(h_\alpha^{-1}\left(\frac{c}{2}\right), \frac{1}{2}\right], \end{cases} \tag{86}$$

where $i^*_{R1,\alpha}(\delta)$ and $p^*_\alpha(\delta)$ denote the equilibrium values of k's first-period policy choice and his re-election probability in the second election, respectively. Analogously to (73) and (76), we obtain

$$EAP_\alpha(\delta) = \frac{1 + p^*_\alpha(\delta)}{2}\left(i^*_{R1,\alpha}(\delta) - \frac{1}{2}\right) + \frac{1 - p^*_\alpha(\delta)}{2}\left(\mu_R - \frac{c}{2} - \frac{1}{2}\right) \tag{87}$$

and

$$\begin{aligned} W_\alpha(\delta) = &\frac{A(1-\rho)}{2} - (1 + p^*_\alpha(\delta))\left(i^*_{R1,\alpha}(\delta) - \frac{1}{2}\right)^2 \\ &- (1 - p^*_\alpha(\delta))\left\{\left(\mu_L + \frac{c}{2} - \frac{1}{2}\right)^2 + c\left[i^*_{R1,\alpha}(\delta) - \left(\mu_L + \frac{c}{2}\right)\right]\right\}. \end{aligned} \tag{88}$$

We observe that $EAP_0(\delta) = EAP(\delta)$ and $W_0(\delta) = W(\delta)$, where $EAP(\delta)$ and $W(\delta)$ correspond to the baseline model and are given in (73) and (76). From (85) and (86), it is obvious that $EAP_\alpha(\delta)$ and $W_\alpha(\delta)$ are constant for

$$\delta \in \left[0, h_\alpha^{-1}\left(\frac{c}{3+\rho}\right)\right] \quad \text{and} \quad \delta \in \left(h_\alpha^{-1}\left(\frac{c}{2}\right), \frac{1}{2}\right]$$

and have a single discontinuity at $\delta = h_\alpha^{-1}\left(\frac{c}{2}\right)$. Moreover, since $h_\alpha(\delta)$ is strictly increasing in δ, $EAP_\alpha(\delta)$ is strictly decreasing in δ and $W_\alpha(\delta)$ is strictly increasing in δ, for

$$\delta \in \left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right].$$

We can now prove statements *(i)-(iv)*. First, statement *(i)* follows from

$$EAP_\alpha\left(\frac{1}{2}\right) - EAP_\alpha(0) = EAP_0\left(\frac{1}{2}\right) - EAP_0(0) < 0 \tag{89}$$

and

$$W_\alpha\left(\frac{1}{2}\right) - W_\alpha(0) = W_0\left(\frac{1}{2}\right) - W_0(0) > 0. \tag{90}$$

The equalities in (89) and (90) hold because, by (85) and (86), $i^*_{R1,\alpha}(0)$, $i^*_{R1,\alpha}\left(\frac{1}{2}\right)$, $p^*_\alpha(0)$ and $p^*_\alpha\left(\frac{1}{2}\right)$ are independent of the value of α. The inequalities hold by (74) and (77). Second, statement *(ii)*, with $\delta^*_\alpha = h_\alpha^{-1}\left(\frac{c}{2}\right)$, follows from

$$EAP_\alpha\left(\frac{1}{2}\right) - EAP_\alpha\left(h_\alpha^{-1}\left(\frac{c}{2}\right)\right) = EAP_0\left(\frac{1}{2}\right) - EAP_0\left(\frac{c}{2}\right) > 0$$

and

$$W_\alpha\left(\frac{1}{2}\right) - W_\alpha\left(h_\alpha^{-1}\left(\frac{c}{2}\right)\right) = W_0\left(\frac{1}{2}\right) - W_0\left(\frac{c}{2}\right) < 0,$$

where the inequalities hold by (75) and (78). Third, since $\delta^*_\alpha = h_\alpha^{-1}\left(\frac{c}{2}\right)$, where

$$h_\alpha^{-1}(x) = \begin{cases} x & \text{if } \alpha = 0, \\ \frac{(4\alpha x - \alpha + 1)^2 - (1-\alpha)^2}{8\alpha} & \text{if } \alpha \in (0,1], \end{cases} \tag{91}$$

δ^*_α is continuous and decreasing in $\alpha \in [0,1]$. Indeed,

$$\lim_{\alpha \to 0} \frac{(4\alpha x - \alpha + 1)^2 - (1-\alpha)^2}{8\alpha} = x,$$

by l'Hôpital's rule, and

$$\frac{dh_\alpha^{-1}(x)}{d\alpha} = x(2x-1) < 0, \tag{92}$$

for all $\alpha \in (0,1)$ and for all $x \in \left(0, \frac{1}{2}\right)$. This completes the proof of *(iii)*. Fourth, recall that $EAP_\alpha(\delta)$ and $W_\alpha(\delta)$ are constant in δ for $\delta \in \left[0, h_\alpha^{-1}\left(\frac{c}{3+\rho}\right)\right]$. Moreover, for $\delta \in$

$\left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right]$, $EAP_\alpha(\delta)$ is strictly decreasing and $W_\alpha(\delta)$ strictly increasing in δ. Since, by (92), $h_\alpha^{-1}\left(\frac{c}{3+\rho}\right)$ and $h_\alpha^{-1}\left(\frac{c}{2}\right)$ are decreasing in α, statement *(iv)* therefore holds if

$$\frac{dEAP_\alpha(\delta)}{d\alpha} < 0 \tag{93}$$

and

$$\frac{dW_\alpha(\delta)}{d\alpha} > 0 \tag{94}$$

hold for all $\alpha \in (0,1)$ and all $\delta \in \left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right]$. Since for $\delta \in \left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right]$, (85), (86), (87), and (88) yield

$$\frac{dEAP_\alpha(\delta)}{d\alpha} = \frac{1+p_\alpha^*(\delta)}{2} \cdot \frac{di_{R1,\alpha}^*(\delta)}{d\alpha} = -(1+p_\alpha^*(\delta)) \cdot \frac{dh_\alpha(\delta)}{d\alpha}$$

and

$$\begin{aligned}\frac{dW_\alpha(\delta)}{d\alpha} &= -2 \cdot (1+p_\alpha^*(\delta)) \cdot \left(i_{R1,\alpha}^*(\delta) - \frac{1}{2}\right) \cdot \frac{di_{R1,\alpha}^*(\delta)}{d\alpha} - (1-p_\alpha^*(\delta)) \cdot c \cdot \frac{di_{R1,\alpha}^*(\delta)}{d\alpha} \\ &= 2 \cdot \underbrace{\left[2 \cdot (1+p_\alpha^*(\delta)) \cdot \left(i_{R1,\alpha}^*(\delta) - \frac{1}{2}\right) + (1-p_\alpha^*(\delta)) \cdot c\right]}_{>0} \cdot \frac{dh_\alpha(\delta)}{d\alpha},\end{aligned}$$

and since $\frac{dh_\alpha(\delta)}{d\alpha} > 0$ for all $\alpha \in (0,1)$ and all $\delta \in \left(0, \frac{1}{2}\right)$, (93) and (94) hold for all $\alpha \in (0,1)$ and all $\delta \in \left(h_\alpha^{-1}\left(\frac{c}{3+\rho}\right), h_\alpha^{-1}\left(\frac{c}{2}\right)\right]$. It remains to show that $\frac{dh_\alpha(\delta)}{d\alpha} > 0$ does indeed hold. For $\alpha \in (0,1)$,

$$\begin{aligned}\frac{dh_\alpha(\delta)}{d\alpha} &= \frac{4\alpha \cdot \left[\frac{\alpha-1+4\delta}{\sqrt{(1-\alpha)^2+8\alpha\delta}} + 1\right] - 4 \cdot \left[\sqrt{(1-\alpha)^2+8\alpha\delta} + \alpha - 1\right]}{16\alpha^2} \\ &= \frac{\alpha \cdot [\alpha - 1 + 4\delta] - \left[(1-\alpha)^2 + 8\alpha\delta - \sqrt{(1-\alpha)^2+8\alpha\delta}\right]}{4\alpha^2\sqrt{(1-\alpha)^2+8\alpha\delta}} \\ &= \frac{\alpha(1-4\delta) - 1 + \sqrt{(1-\alpha)^2+8\alpha\delta}}{4\alpha^2\sqrt{(1-\alpha)^2+8\alpha\delta}},\end{aligned}$$

which is strictly positive if and only if

$$\begin{aligned}&(1-\alpha)^2 + 8\alpha\delta - [(1-\alpha) + 4\alpha\delta]^2 > 0 \\ \Leftrightarrow &8\alpha\delta - 16\alpha^2\delta^2 - 2(1-\alpha)4\alpha\delta > 0 \\ \Leftrightarrow &8\alpha^2\delta(1-2\delta) > 0.\end{aligned}$$

The last inequality is satisfied for all $\alpha \in (0,1)$ and all $\delta \in \left(0, \frac{1}{2}\right)$.

□

Appendix B

In this appendix, we analyze the case where

$$c \geq 2\Pi,$$

in contrast to assumption (6) in the main body of the paper. For Proposition 2 to hold for $c \geq 2\Pi$, we assume here that, in the second election, every voter who is indifferent between the incumbent and the challenger will vote for the incumbent. This assumption is necessary for the following reason: If $i_{k1} \in [\mu_R - \frac{c}{2}, \mu_L + \frac{c}{2}]$, then (†) from the proof of Proposition 2 equals zero for all $i \in [0,1]$. This means that, for $a_k = 0$ and $i_{k1} \in [\mu_R - \frac{c}{2}, \mu_L + \frac{c}{2}]$, all voters are indifferent between k and his competitor. Therefore, if, for instance, indifferent voters randomize between both candidates with probability $\frac{1}{2}$, Proposition 2 only holds for $\delta = 0$, since, by Section 3.3, the incumbent will win the election if he receives a vote-share of $\frac{1}{2} + \delta$ or larger. With the above assumption regarding indifferent voters in place, we obtain

Proposition 4
Let $\delta \in \left[0, \frac{1}{2}\right]$. Then, the following holds:

(i) If $2\Pi \leq c < (3+\rho)\Pi$, then in $t = 1$, the incumbent $k \in R$ will choose

$$i^*_{R1}(\delta) = \begin{cases} \mu_R - \frac{c}{2} \cdot \frac{1-\rho}{3+\rho} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \mu_R + \frac{c}{2} - 2\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \Pi\right], \\ \mu_L + \frac{c}{2} & \text{if } \delta \in \left(\Pi, \frac{1}{2}\right] \end{cases}$$

in equilibrium of the game $\mathcal{G}^R$.

(ii) If $(3+\rho)\Pi \leq c < 4\Pi$, then in $t = 1$, the incumbent $k \in R$ will choose

$$i^*_{R1}(\delta) = \mu_L + \frac{c}{2}$$

in equilibrium of the game $\mathcal{G}^R$.

(iii) If $c \geq 4\Pi$, then in $t = 1$, the incumbent $k \in R$ will choose

$$i^*_{R1}(\delta) = \mu_R$$

in equilibrium of the game $\mathcal{G}^R$.

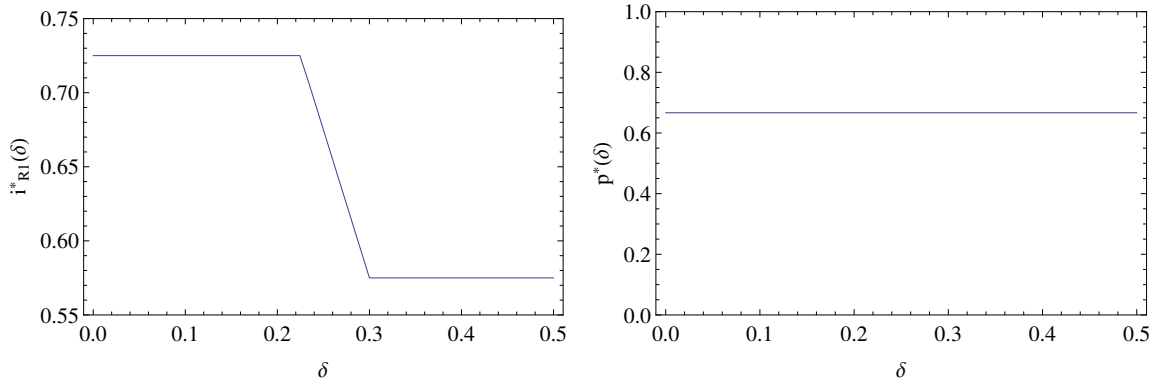
The proof of Proposition 4 is very similar to the proof of Proposition 3.[64] From Proposition 4 we see that, for $c \geq 2\Pi$, $i^*_{R1}(\delta)$ is weakly decreasing in δ. However, there is no unique value of

[64] The detailed calculations are available upon request.

δ that minimizes $i^*_{R1}(\delta)$. For instance, if $c \geq (3+\rho)\Pi$, then $i^*_{R1}(\delta)$ is constant in δ. Due to the assumption that every voter who is indifferent between incumbent and challenger will vote for the incumbent, k's re-election probability is equal to $\frac{1+\rho}{2}$ if he chooses $i_{k1} \in [\mu_R - \frac{c}{2}, \mu_L + \frac{c}{2}]$. Thus, if $c \geq 2\Pi$ (in contrast to the case where $c < 2\Pi$), the re-election probability in equilibrium is

$$p^*(\delta) = \frac{1+\rho}{2} \tag{95}$$

for all $\delta \in \left[0, \frac{1}{2}\right]$.[65] This is why $i^*_{R1}(\delta)$ is continuous in δ, if $c \geq 2\Pi$. The behavior of $i^*_{R1}(\delta)$ and $p^*(\delta)$, for parameter values satisfying $2\Pi \leq c < (3+\rho)\Pi$, is illustrated in Figures 11(a) and 11(b).



(a) Equilibrium policy choice of right-wing policy-maker in $t = 1$.

(b) Re-election probability in equilibrium.

Figure 11: Illustration of the equilibrium analysis in $t = 1$, using the parameter values $\mu_R = 0.8$, $c = 0.75$, $A = 2$, and $\rho = \frac{1}{3}$, satisfying $2\Pi \leq c < (3+\rho)\Pi$.

Finally, from Propositions 4 and 1 it follows that, if $2\Pi \leq c < (3+\rho)\Pi$, then

$$i^*_{R2}(\delta) = i^*_{R1}(\delta), \text{ and} \tag{96}$$

$$i^*_{L2}(\delta) = \mu_L + \frac{c}{2}. \tag{97}$$

Accordingly, if $c \geq (3+\rho)\Pi$, then

$$i^*_{R2}(\delta) = i^*_{L2}(\delta) = i^*_{R1}(\delta). \tag{98}$$

This completes the description of the equilibrium outcomes of the game $\mathcal{G}^R$, for $c \geq 2\Pi$. Analogously to Section 4.3, the equilibrium policy choices of the game $\mathcal{G}^L$ follow immediately, because ideal policy positions of candidates are distributed symmetrically around $\frac{1}{2}$ and candidates' ability distribution is independent of their policy orientation.

[65] Recall that, by (10), $p^*(\delta)$ is not constant in δ for $c < 2\Pi$.

In the following, we analyze the dependence of ex-ante policy polarization and welfare on $\delta \in \left[0, \frac{1}{2}\right]$. $EAP(\delta)$ and $W(\delta)$ are defined by (14) and (15) respectively, with $i^*_{R1}(\delta)$ given by Proposition 4, $i^*_{R2}(\delta)$ and $i^*_{L2}(\delta)$ given by (96), (97), and (98), and $p^*(\delta)$ given by (95). Let us now state the counterpart of Theorem 1.

Theorem 8

In equilibrium of $\mathcal{G}$, the following holds:

(i) Any extra-hurdle $\delta \in \left(0, \frac{1}{2}\right]$ is both weakly W-increasing and weakly P-reducing.

(ii) If $2\Pi \leq c < (3+\rho)\Pi$, then

$$\underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) = \underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmax}} W(\delta) = \left[\Pi, \frac{1}{2}\right].$$

(iii) If $c \geq (3+\rho)\Pi$, then

$$\underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmin}} EAP(\delta) = \underset{\delta \in \left[0,\frac{1}{2}\right]}{\operatorname{argmax}} W(\delta) = \left[0, \frac{1}{2}\right].$$

Proof

The proof is analogous to the proof of Theorem 1.[66] It is useful though to mention the following intermediate results:

First, if $2\Pi \leq c < (3+\rho)\Pi$, then

$$EAP(\delta) = \begin{cases} \frac{(2\mu_R - 1)(1+\rho)}{4} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \frac{(2\mu_R - 1)(1+\rho)}{4} + \frac{c - (3+\rho)\delta}{2} & \text{if } \delta \in \left(\frac{c}{3+\rho}, \mu_R - \frac{1}{2}\right], \\ \frac{1 - 2\mu_R + c}{2} & \text{if } \delta \in \left(\Pi, \frac{1}{2}\right] \end{cases}$$

and

$$W(\delta) = \begin{cases} \frac{A(1-\rho)}{2} - \frac{(2\mu_R - 1)^2}{2} + \frac{c^2(1-\rho)}{2(3+\rho)} & \text{if } \delta \in \left[0, \frac{c}{3+\rho}\right], \\ \frac{A(1-\rho) - (1 - 2\mu_R - c)^2}{2} - 2(3+\rho)\delta^2 + \left[4c + (2\mu_R - 1)(3+\rho)\right]\delta & \text{if } \delta \in \left(\frac{c}{3+\rho}, \Pi\right], \\ \frac{A(1-\rho)}{2} - \frac{(1 - 2\mu_R + c)^2}{2} & \text{if } \delta \in \left(\Pi, \frac{1}{2}\right]. \end{cases}$$

Second, if $(3+\rho)\Pi \leq c < 4\Pi$, then

$$EAP(\delta) = \frac{1 - 2\mu_R + c}{2}$$

and

$$W(\delta) = \frac{A(1-\rho) - (1 - 2\mu_R + c)^2}{2}.$$

[66] The detailed calculations are available upon request.

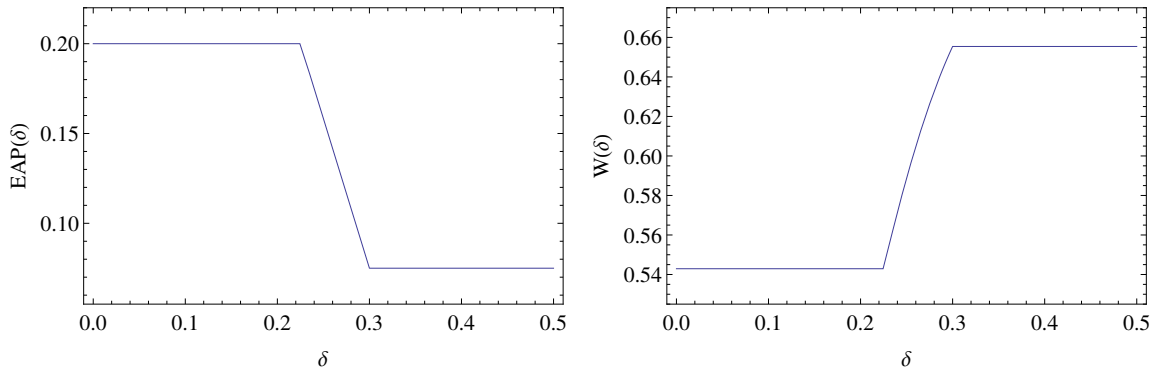
Third, if $c \geq 4\Pi$, then

$$EAP(\delta) = \mu_R - \frac{1}{2}$$

and

$$W(\delta) = \frac{A(1-\rho)}{2} - 2\left(\mu_R - \frac{1}{2}\right)^2.$$

□



(a) Ex-ante policy polarization.

(b) Welfare.

Figure 12: Illustration of the equilibrium values of ex-ante policy polarization and welfare, using the parameter values $\mu_R = 0.8$, $c = 0.75$, $A = 2$, and $\rho = \frac{1}{3}$, satisfying $2\Pi \leq c < (3+\rho)\Pi$.

Note that statement *(i)* of Theorem 8 is the same as *(i)* in Theorem 1. That is, compared to the case of $\delta = 0$, the introduction of any non-zero extra-hurdle will still weakly improve welfare and weakly reduce ex-ante policy polarization. According to *(ii)* and *(iii)* of Theorem 8, the set of W-optimal extra-hurdles is still identical to the set of P-optimal extra-hurdles. However, in contrast to the main part of the paper, this set is now a continuum, i.e. there is a range of values of δ that are both W- and P-optimal. Theorem 8 is illustrated in Figures 12(a) and 12(b), which show plots of $EAP(\delta)$ and $W(\delta)$, for $\mu_R = 0.8$, $c = 0.75$, $A = 2$, and $\rho = \frac{1}{3}$. These parameter values satisfy $2\Pi \leq c < (3+\rho)\Pi$. The figures show that $EAP(\delta)$ is weakly decreasing and $W(\delta)$ weakly increasing in δ. For $c \geq (3+\rho)\Pi$, ex-ante policy polarization and welfare are constant, because in this case $i^*_{R1}(\delta)$, $i^*_{R2}(\delta)$, $i^*_{L2}(\delta)$ and $p^*(\delta)$ are constant in δ.

Concerning the extension where extra-hurdles are selected by candidates themselves, note that, because for $c \geq 2\Pi$ there is a continuum of values of δ that are both W- and P-optimal, Theorem 3 has to be reformulated as follows:

Theorem 9

In equilibrium of $\mathcal{G}'$, $k \in R$ and $k' \in L$ will commit to extra-hurdles $\hat{\delta}$ and $\tilde{\delta}$, respectively, with $\hat{\delta}$ and $\tilde{\delta}$ each being W- and P-optimal, and both candidates will win the first election with a probability equal to $\frac{1}{2}$.

Proof

Much as in the proof of Theorem 3, in equilibrium candidate k will commit to some W-optimal $\hat{\delta}$, as the latter maximizes expected lifetime utility of the median voter. Analogously, candidate k' will commit to some W-optimal $\tilde{\delta}$. Both candidates will win the first election with probability equal to $\frac{1}{2}$. Since equilibrium policy choices and re-election probabilities are the same for all $\delta \in \text{argmax}_{\delta \in \left[0, \frac{1}{2}\right]} W(\delta)$, none of the two candidates has a preference for one specific W-optimal extra-hurdle. By Theorem 8, the set of W-optimal extra-hurdles is identical to the set of P-optimal extra-hurdles. This completes the proof.

□

Theorem 9 shows that, for $c \geq 2\Pi$, there is a continuum of values of δ that can be implemented in equilibrium. Those values are W- and P-optimal.

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