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Coalition-Preclusion Contracts and Moderate Policies

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# Coalition-Preclusion Contracts and Moderate Policies\*

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## Abstract

We examine the effects of a novel political institution, which we call *Coalition-Preclusion Contracts*, on elections, policies, and welfare. Coalition-Preclusion Contracts enable political parties to credibly commit before the elections not to form a coalition after the elections with one or several other parties specified in the contract. We consider a political game in which three parties compete to form the government and study when contracts of the above type will be written. We find that in most circumstances Coalition-Preclusion Contracts with a single-party exclusion rule defend the interests of the majority by moderating the policies implemented. Moreover, they yield welfare gains for a large set of parameter values. We discuss the robustness of the results in more general settings and study how party-exclusion rules have to be adjusted when more than three parties compete in an election.

Keywords: coalition formation, political contracts, elections, government formation

JEL: D72, D82, H55

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“How many times do I have to repeat it, Mr. Markwort? You will not receive any other answer tonight than the one I have given over during the past weeks and months: There will be no cooperation whatsoever with the [party] die Linken.” (Andrea Ypsilanti, SPD, 2008)<sup>1</sup>

## 1 Introduction

In democracies with proportional representation, coalition formation is essential for government since one single party rarely obtains a majority of seats. On occasion, government coalitions are made up of conventional parties plus small parties perceived to be extreme or unfavorable by a large majority of voters, e.g. in Austria (ÖVP + FPÖ in 2000) or the Netherlands (VVD + CDA + PVV in 2010).<sup>2</sup> Although in political campaigns prior to an election conventional parties typically try to persuade voters that they will not form a coalition government with extreme parties, they do not always stick to their promise.<sup>3</sup> A case in point is illustrated by the above quote from Andrea Ypsilanti, who after the election in Hesse in 2008 was willing to renege on her promise. Such a breach of promise may be undesirable per se, but it may also affect welfare as a government coalition with extreme parties could lead to more extreme policies. What would be the consequences in the government formation and the policies implemented if, before the election, parties could bindingly commit *not* to form a government with a particular set of parties? Would welfare be improved? These are the central questions this paper poses. For this purpose, we examine a novel political institution that we call *Coalition-Preclusion Contracts* (henceforth simply *CPC*).<sup>4</sup>

In a CPC, a party specifies a list of other parties that it commits not to form a government coalition with after the next elections. Such promises are certified by a public authority.<sup>5</sup> If the party violates the CPC, i.e. if it forms a government with a party listed in its contract, the party is severely punished.<sup>6</sup> For instance, it may not be allowed to nominate candidates

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<sup>1</sup>Andrea Ypsilanti was the SPD candidate for the position of minister-president (Ministerpräsident) in the state elections in Hesse, Germany, in 2008. See [http://www.focus.de/magazin/tagebuch/tagebuch-schauspielerin-ypsilanti\\_aid\\_263564.html](http://www.focus.de/magazin/tagebuch/tagebuch-schauspielerin-ypsilanti_aid_263564.html), retrieved on 6 March 2008.

<sup>2</sup>In the last few decades, government coalitions with extreme parties have formed in several European countries. Examples include Denmark (SF in 1964 and 1966), Italy (RC in 1996), and Sweden (VP in 2002). (Source: Social Science Research Center Berlin (WZB).)

<sup>3</sup>In other cases, conventional parties kept their promise. For instance, the German SPD stated before the general election in 2005 that it would not form a coalition government involving the party “Die Linke” and stuck to its promise in that case.

<sup>4</sup>CPC are a new type of political contract as surveyed in Gersbach (2012).

<sup>5</sup>Either a new authority would be created or an existing authority would be entrusted with certification of CPC. In Germany, the Federal President could act as a CPC certifier.

<sup>6</sup>The punishment would extend to the party attempting to form a minority government by counting on the votes of the party excluded in the contract.

for cabinet positions, or its public funding may be considerably reduced. We assume that violations of CPC are punished so gravely that CPC will never be violated, making CPC a commitment enabling a party to credibly promise not to form a government with some particular party.

To examine the consequences of allowing parties to write CPC, we consider a political game in which three parties compete first for votes and second to form the government after the elections. There are two conventional parties with platforms on the moderate left and right with regard to political issues such as tax policy and public-good provision. Additionally, there is an *extreme* party whose defining characteristic is that it advocates a substantial policy change in a second policy dimension orthogonal to the conventional policy dimension mentioned earlier. Examples of such a policy change would be leaving NATO or the monetary union, closing the borders to immigrants, or breaking up large banks. Such a shift away from the status quo in the second, orthogonal policy dimension is preferred only by a minority of voters, at most one-third of the electorate. These voters feel strongly about the policy change advocated by the extreme party and always vote for it. In the first policy dimension the extreme party may have moderate preferences.

As a consequence, the median voter in the conventional policy dimension, who does not support the extreme policy change, would prefer a grand coalition of the conventional parties over a coalition government formed by one conventional party and the extreme party, if the latter would then be able to implement the extreme policy change. Conventional parties, however, are often tempted to engage in coalitions with the extreme party as the latter is willing to offer substantial power and perks in return for the implementation of the extreme policy change. Even though to attract voters conventional parties may have an incentive to rule out a coalition with the extreme party before the election, such parties may be tempted to break their promises after the election if they cannot form a single-party government and thus require a coalition partner. Accordingly, a natural idea to prevent coalitions between conventional and extreme parties would be to enable parties to credibly commit before the election not to form a coalition government with the specified parties after the election.

We analyze the conditions under which CPC will be written in a parliamentary democracy and what their welfare implications are. For this purpose, we first study a simple model with the three parties introduced previously and three voters, each of them with political leanings towards one of the parties. While the extreme voter will always vote sincerely for the extreme party, conventional voters may think about proceeding strategically. We show that voters vote sincerely in the situation without CPC leading to a coalition government between a conventional party and the extreme party.

When CPC are introduced, we find that whether the conventional parties will exclude the extreme party from a government coalition depends crucially on the probability of conventional voters coordinating their votes on the party that ruled out the extreme party if only one conventional party has done so. We characterize the equilibria depending on this probability and show that both conventional parties will exclude the extreme party if this probability is sufficiently large. In this case, a grand coalition will result. However, we find that another equilibrium may occur where conventional parties exclude any coalition government whatsoever, thus making a single-party government inevitable. Such equilibria can be prevented by allowing each party to exclude a maximum of one other party in its coalition contract.

In the second part of the paper, we introduce a micro-founded, more general election model with a large number of voters with different policy preferences and a specific bargaining protocol regarding government formation. We establish a close link between equilibria in the general model and equilibria in the simple game. Moreover, we perform a welfare analysis in the general model. In particular, we show that with the single-party exclusion rule, the introduction of CPC involves welfare gains for a large set of parameter values. Finally, in the last part of the paper, we discuss several interesting extensions. For instance, we allow for multiple extreme parties, asymmetric ideal points of parties or uncertainty about the election outcome for the extreme party. These and other extensions enhance the robustness of the results. In particular, we identify how exclusion rules need to be adapted when more than three parties participate in an election.

The paper is organized as follows: In Section 2 we relate our work to the existing literature. In Section 3 we present a stylized model for elections in a parliamentary democracy that captures the main tensions arising between parties and voters when there is the possibility of writing CPC. For this stylized model we consider two different rules specifying how contracts have to be written, and we characterize the set of equilibria in either case. In Section 4 we introduce a micro-founded model for elections in a parliamentary democracy when elections are held within a large electorate, and we present the solution concept we propose for solving it. In Section 5 we prove that if we consider the equilibrium concept previously introduced this more general model can be embedded in the previous stylized model. We exploit this finding to solve the more sophisticated model. In Section 6 we study the welfare implications of CPC in the light of the results from the previous section. In Section 7 we discuss the robustness of our results by studying several extensions of the micro-founded model. Section 8 concludes and discusses the predicament posed by the actual implementation of CPC in a parliamentary democracy.

## 2 Relation to the Literature

An attempt to make campaign promises regarding coalition partners credible occurred in Catalonia, Spain, in the regional elections that took place in 2006.<sup>7</sup> At the time of the elections there were two large parties, CiU and PSC, which were the only parties that had any chance of winning the elections, either outright or by heading a coalition. There were four other smaller parties of different sizes. In an unprecedented move, CiU stated in a document signed before a notary previous to the elections that it would not establish any kind of agreement with PP, possibly in order to become more attractive to centrist voters.<sup>8</sup> While such moves do increase the credibility of promises, they are not legally binding and hence lack the credibility potential of CPC.

As mentioned in the introduction, we consider in this paper a three-party political game with an underlying conflict between voters and parties on the one hand and among parties on the other. Our paper is related to several strands in the literature.

### *Coalition politics*

In a multi-party system with proportional seat allocation, it is very common for parties to be unable to form a government alone, so after the elections a coalition of parties emerges and takes over. Two aspects of this phenomenon have come in for particular attention in the literature, namely *(i)* how parties bargain in order to form a coalition government, and *(ii)* how this information – and other information regarding coalitions – is anticipated by the electorate.

The problem in *(i)* boils down to a bargaining situation among members with some opposing preferences. We build on the theory of government coalition formation as surveyed by Bandyopadhyay and Chatterjee (2006) and Austen-Smith and Banks (2005). Our approach involves the conventional parties as possible *formateurs* and the Nash bargaining solution with bargaining power proportional to the share of seats in parliament (see also Roemer, 2001).

With respect to *(ii)*, voters reckon the probability that each coalition is formed and anticipate the policy compromises that will result (see e.g. Austen-Smith and Banks 1988). There is some empirical support for the view that voters are influenced by possible government coalitions that might emerge after the elections (e.g. Blais et al., 2006, and Meffert and Gschwend, 2010, show that voters are responsive to coalition signals).

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<sup>7</sup>We are grateful to Albert Falcó-Gimeno for information and conversations on this issue.

<sup>8</sup>The document can be found (in Catalan, see 6.2) at <http://www.ciu.cat/media/9890.pdf>. The information was retrieved on 26 February 2014.

Without CPC parties try other ways of making campaign promises credible. For instance, Aragonés et al. (2007) consider a model for repeated elections with completely informed and ideological voters and conclude that, in equilibrium, the degree to which promises are credible increases with the reputation of the candidate. Debus (2009) shows that pre-electoral (non-bidding) announcements of possible coalitions can influence the outcome of the government formation game.

CPC offer a more powerful, supplementary way for parties to make credible promises regarding possible government coalitions. They work in situations in which reputation concerns are weak and campaign promises are thus cheap talk. Moreover, they do not rely on punishment threats by voters who, in turn, require that voters do not believe the announcements of parties once they have reneged. With CPC, parties can invariably make credible promises regarding possible partners in a coalition government.

### 3 Simple Model

In this section we analyze a stylized model of our political game. The analysis of the more general framework can be found in Section 4. We consider two conventional parties,  $L$  and  $R$ , an extreme party,  $E$ , and three voters,  $l$ ,  $r$ , and  $e$ . After the election, coalitions can be formed if no party has a majority. As a tie-breaking rule, we assume that, whenever the two conventional parties are in a completely symmetric situation regarding votes and the range of potential coalitions, the tie is fairly broken, i.e., with probability  $1/2$  each of the conventional parties takes the lead as a formateur.

The government formed after the elections implements a policy denoted by  $p$ . A single-party government  $k \in \{L, R\}$  implements its preferred policy  $p_k$ . A “grand coalition” government implements policy  $p_{LR}$ , and a coalition involving the extreme party and one conventional party  $k \in \{L, R\}$  implements either policy  $p_E$  or policy  $p_{kE}$ , depending on the bargaining power of  $E$  with respect to the conventional parties.<sup>9</sup> If both conventional parties compete for a coalition with  $E$ , the extreme party has *strong bargaining power*. The tie-breaking rule implies that in this case each conventional party has a probability  $1/2$  of taking the lead as formateur and thus being part of the government together with

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<sup>9</sup>In this simple model a “grand coalition” between  $L$  and  $R$  has the same number of votes/parliamentary seats as a coalition between either  $L$  or  $R$  and  $E$ . Typically, the vote share of extreme parties is less than that of the large conventional parties, as implied in our reference to a coalition between  $L$  and  $R$  as the “grand coalition”. This will be the case in the more general model in Section 4.



$E$ . In both cases, however, the resulting coalition implements the same policy, i.e.,  $p_E$ .<sup>10</sup> If only one conventional party  $k \in \{L, R\}$  is able to form a coalition with  $E$ , the extreme party has *weak bargaining power*, and the resulting coalition will implement policy  $p_{kE}$ . If no group of parties can agree to form a coalition government, a caretaker government takes over power and implements policy  $p_{ct}$ .

Voters and parties derive utility from the policy implemented by the government. Moreover, parties also derive utility from the perks associated with being in government. We refer to the utility that every voter and every party obtains from the implemented policy as  $p \in \{p_L, p_R, p_{LR}, p_{LE}, p_{RE}, p_E, p_{ct}\}$ .<sup>11</sup> The utility of party  $k \in \{L, R, E\}$  when policy  $p \in \{p_L, p_R, p_{LR}, p_{LE}, p_{RE}, p_E, p_{ct}\}$  is implemented is denoted by  $V_k(p)$ , and the utility of voter  $i \in \{l, r, e\}$  when policy  $p$  is implemented is denoted by  $v_i(p)$ .

We assume that it is more attractive for a conventional party to form a coalition with  $E$  than with the other conventional party, as it involves a larger amount of perks.<sup>12</sup> This means that, for  $k \in \{L, R\}$ , it holds that  $V_k(p_{kE}) > V_k(p_{LR})$ . As a conventional party has a probability 1/2 of being in the government when policy  $p_E$  is implemented, this latter policy is perceived by conventional parties as being worse than any policy that grants them a place in the government with certainty but better than any policy in which they remain out of government with certainty. In particular, for  $k, k' \in \{L, R\}$  with  $k \neq k'$ , it holds that  $V_k(p_{LR}) > V_k(p_E) > V_k(p_{k'})$ . Furthermore, we consider that a conventional party is better off when the other conventional party is in a single-party government than when the latter forms a coalition with the extreme party. A rationale for this assumption is the following: party  $E$  is in favor of some “extreme” policy, which it is willing to implement in exchange for helping a conventional party obtain a majority. Thus, for  $k, k' \in \{L, R\}$  with  $k \neq k'$ , it holds that  $V_k(p_{k'}) > V_k(p_{k'E})$ . Lastly, a policy implemented by a caretaker government is the worst possible outcome for both conventional parties. Summing up, we have

$$V_L(p_L) > V_L(p_{LE}) > V_L(p_{LR}) > V_L(p_E) > V_L(p_R) > V_L(p_{RE}) > V_L(p_{ct}), \quad (1)$$

$$V_R(p_R) > V_R(p_{RE}) > V_R(p_{LR}) > V_R(p_E) > V_R(p_L) > V_R(p_{LE}) > V_R(p_{ct}). \quad (2)$$

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<sup>10</sup>The policy implemented by such a coalition will depend on the particular bargaining protocol. With the bargaining protocol given in Section 4 it will be  $E$ 's ideal policy.

<sup>11</sup>As parties value being in government, e.g. because of the perks associated with it, there should be two utility values for each party for the situation when both conventional parties are able to form a coalition with  $E$  instead of just  $V_k(p_E)$ : one where  $k$  forms the coalition with  $E$ , and one where  $k' \neq k$  does so. This distinction is, however, not important for the incentives to sign CPC, which is the focus of the model. What we need is that the expected utility after the election outcome allows both conventional parties to compete for a coalition with  $E$  (before the government is formed). We capture the latter by  $V_k(p_E)$ . Note that for the voters it is not important which conventional party forms a coalition with  $E$ , as they are only concerned with public policy (which is the same in both cases) and not with perks.

<sup>12</sup>The role of perks is discussed in more detail in Section 4.

The central tension in the model arises from three major differences in the preferences of conventional voters with respect to the preferences of the parties on the same political side (i.e. left or right). First, we assume that conventional voters will rank a grand coalition higher than any other coalition. Such a situation will arise, for instance, when there is a significant mass of voters with ideal policies centered around the median policy and conventional parties are polarized, meaning that they have substantially different ideal points. Second, we assume that when the extreme party has strong bargaining power a conventional voter will prefer the policy that would be carried out by a coalition made up of the conventional party on the political side she favor plus the extreme party to the ideal policy of the conventional party on the other political side. Third, we assume, inversely, that when the extreme party has weak bargaining power a conventional voter will prefer the ideal policy of the conventional party on the other political side to the policy that would be carried out by a coalition made up of the conventional party she favors plus the extreme party when the latter has weak bargaining power. That is, we impose

$$v_k(p_E) > v_k(p_{k'}) \text{ for } k, k' \in \{l, r\}, k \neq k', \quad (3)$$

and

$$v_k(p_{k'}) > v_k(p_{kE}) \text{ for } k, k' \in \{l, r\}, k \neq k'. \quad (4)$$

Equations (3) and (4) capture those situations in which the extreme party can substantially moderate the policy implemented by conventional parties in the first dimension, but only when it has strong bargaining power.<sup>13</sup> Moreover, in Section 3.1 we show (3) and (4) must hold if coalitions between a conventional party and the extreme party are to be formed in equilibrium (in the absence of CPC) and CPC is not to be innocuous. As we are interested in the role of CPC in preventing the formation of coalition governments in which a member of the coalition is a extreme party, we henceforth assume (3) and (4). For the sake of completeness, we assume that the different policy outcomes can be completely ordered by the conventional voters. The complete orders are

$$v_l(p_{LR}) > v_l(p_L) > v_l(p_E) > v_l(p_R) > v_l(p_{LE}) > v_l(p_{RE}) > v_l(p_{ct}), \quad (5)$$

$$v_r(p_{LR}) > v_r(p_R) > v_r(p_E) > v_r(p_L) > v_r(p_{RE}) > v_r(p_{LE}) > v_r(p_{ct}). \quad (6)$$

We consider a sequential game with an election in an initial stage and government formation in a subsequent stage. Then we investigate the impact on the outcome of the game if, before the election stage, parties can credibly rule out some coalition government options in a so-called Coalition-Preclusion Contract.

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<sup>13</sup>A more elaborate argument is presented in the model in Section 4. Note, however, that a framework where (3) and (4) hold cannot arise when the policy space is one-dimensional, as the extreme party cannot make any compromises regarding the implementation of its preferred policy.

The situation without Coalition-Preclusion Contracts is simple. If citizens vote sincerely, none of the parties will be able to form a single-party government, and since both conventional parties will prefer a coalition with the extreme party to a grand coalition government, they will compete for  $E$  as a coalition partner. Consequently,  $E$  possesses strong bargaining power leading to policy  $p_E$  within a coalition government with one of the conventional parties. Does a conventional voter have an incentive to deviate and vote strategically? The answer is negative. If one of the conventional voters votes strategically, the result will be a single-party government run by the conventional party they do not support. According to voters' preferences as given in (5) and (6), both conventional voters will prefer a coalition between  $L$  or  $R$  and  $E$ , who will implement policy  $p_E$ , to a single-party government by the conventional party they do not support. Hence we conclude that there is a unique equilibrium in pure strategies in which all citizens vote sincerely.<sup>14</sup>

Now we introduce *Coalition-Preclusion Contracts (CPC)*. A Coalition-Preclusion Contract enables a party to credibly commit before the election to not forming a government coalition after the election with the parties specified in the contract.<sup>15</sup> We initially assume a *one-party exclusion rule*, i.e., parties can exclude one other party at the most. Will this possibility of excluding another party from a coalition government be used by the parties? To answer this question, we now examine the equilibria of the game.

### 3.1 Characterization of the equilibria

We use backward induction to solve the sequential game, denoted by  $\mathcal{G}$ , where the conventional parties decide on whether or not to sign CPC before the election takes place. At the government formation stage, a single-party government will only come about if one of the conventional parties obtains both votes from the two conventional voters. By contrast, if voters vote sincerely, a coalition between a conventional party and the extreme party will result given that at least one of the conventional parties has not excluded the extreme party in its coalition contract. Only if both  $L$  and  $R$  have excluded  $E$  in their contract will a grand coalition come about.

At the election stage, conventional voters have to decide between voting sincerely or strategically. As argued previously, without coalition contracts voters will vote sincerely. By the same line of argument, voters will vote sincerely when both conventional parties have excluded  $E$  in their contract. Voting sincerely in this case will result in a grand coalition,

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<sup>14</sup>It is straightforward to verify that there is no equilibrium in which some voters will act strategically.

<sup>15</sup>We stress that a coalition contract not only precludes a coalition government with all parties specified in the contract, it also precludes any coalition government with at least one of the parties specified in the contract.

which is preferred by both voters to a single-party government with their less preferred conventional party. Strategic voting will only occur when one conventional party, say  $k$ , has excluded  $E$  from a coalition government, while the other,  $h \neq k$ , has not. In this case, there are two equilibria in pure strategies, where one conventional citizen will vote sincerely and the other strategically. As a matter of fact, if both conventional voters voted either sincerely or strategically, this would bring about a coalition between the conventional party that did not exclude  $E$  and the extreme party. Party  $E$  would possess weak bargaining power leading to policy  $p_{hE}$ . Conventional voters value this policy less than the policy advocated by a single-party government of their less preferred conventional party. Note that in the situation where only one party has excluded  $E$ , both conventional voters will prefer the equilibrium in which the other conventional voter acts strategically.

The restriction on the possible preferences of conventional parties given by (3) and (4) encompasses the only interesting cases regarding the role of CPC as a tool for preventing the formation of coalition governments between a conventional party and the extreme party. On the one hand, if (3) does not hold, conventional parties will not form in equilibrium a coalition with the extreme party (in the absence of CPC), as voters will find it more profitable to coordinate their votes on either of the conventional parties. On the other hand, if (4) does not hold, it will be a dominated strategy for parties to exclude  $E$  as voters will find it profitable to coordinate their votes on the conventional party which that has not excluded  $E$ , so in equilibrium no CPC will be written by conventional parties.

More generally, as the possibility is always available for conventional voters to coordinate their votes on either conventional party (with or without CPC), neither a caretaker government nor a coalition government with a conventional party and the extreme party will form in equilibrium when the latter has weak bargaining power. Note that these possibilities can only arise under sincere voting (or, symmetrically, under strategic voting by both conventional voters), but this strategy profile will never be an equilibrium, as each conventional voter will always find it profitable to deviate. Hence, policies  $p_{ct}$ ,  $p_{LE}$  and  $p_{RE}$  cannot arise in equilibrium, only  $p_{LR}$ ,  $p_L$ ,  $p_R$ , and  $p_E$ . The question, however, remains as to which of these latter policies may arise. We note that, from the point of view of the voters,  $p_{LR}$  Pareto-dominates  $p_L$ ,  $p_R$ , and  $p_E$ , while the latter policies are not pairwise Pareto-comparable. In the previous section we saw that, in the absence of CPC, conventional voters unanimously preferring a grand coalition will not be able to bring this outcome about via voting because the parties' preferences regarding the grand coalition are different from the voters' preferences.

The central part of the discussion in the rest of the section is devoted to the analysis of the incentives for conventional parties to sign a coalition contract when such a possibility is

available.<sup>16</sup> Note that if the conventional parties only exclude each other from a coalition after the election, CPC will not change the outcome relative to the situation without CPC. More generally, from the preferences of voters and parties given in (1), (2), (5), and (6) it follows that, for party  $L$ , excluding no party or excluding  $R$  will lead to the same outcome no matter what the choice of party  $R$  is. In other words, for party  $L$  the strategies  $\emptyset$  and  $\{R\}$  are equivalent. The same holds *mutatis mutandis* for party  $R$ . As a tie-breaking rule, we assume that  $\emptyset$  will be chosen over  $\{L\}$  and  $\{R\}$  for party  $R$  and  $L$  respectively, and that  $E$  will be excluded rather than not excluded by a conventional party when the latter is indifferent between the two options. We stress that if the parameters in (1), (2), (5) and (6) are drawn from a non-degenerate distribution, the latter event has probability zero. Hence, under the one-party exclusion rule we can summarize the contract choice game played by the conventional parties in the following table, where  $k \in \{L, R\}$ :

		Party $R$	
		$\emptyset$	$\{E\}$
Party $L$	$\emptyset$	$V_k(p_E)$	$q_k \cdot V_k(p_R) + (1 - q_k) \cdot V_k(p_L)$
	$\{E\}$	$q_k \cdot V_k(p_L) + (1 - q_k) \cdot V_k(p_R)$	$V_k(p_{LR})$

Table 1: Conventional parties' contract choice game

In Table 1,  $q_L$  and  $q_R$  respectively denote the beliefs of parties  $L$  and  $R$  that the conventional voters will coordinate their votes on the party that excludes a coalition with the extreme party, conditional on only one conventional party excluding a coalition with  $E$ .<sup>17</sup> In particular, we observe that an equilibrium of contract choices depends crucially on these parties' beliefs, i.e., on the exact value of  $(q_L, q_R) \in [0, 1] \times [0, 1]$ . In the main part of the paper, our examination will be based on equilibria where both conventional parties share the same beliefs on the probability that excluding  $E$  will prompt coordination on the excluding party. Formally, this implies  $q_L = q_R := q$ . We refer to these equilibria as *rational belief equilibria*. In Section 7.2 we characterize all possible equilibria without imposing  $q_L = q_R$ .

First, suppose without loss of generality that  $L$  excludes  $E$ . From the previous discussion

<sup>16</sup>We only consider conventional parties as writers of CPC, since the extreme party has no incentive to exclude a conventional party. This would only reduce its bargaining power at the government formation stage.

<sup>17</sup>Note that we assume that both parties' beliefs on the coordination probability do not depend on which party excludes  $E$ . A conventional party, say  $L$ , believes that the probability that coordination will occur in its favor when it is the only party that excludes  $E$  is the same as the probability that coordination occurs in favor of  $R$  if the latter is the only party excluding  $E$ . This is a cautious assumption. One might argue that voters are more likely to coordinate on the party that excludes  $E$  as such exclusion could itself be interpreted as a coordination device.

we know that when both conventional parties exclude the extreme party, voters will vote sincerely and a grand coalition will result. In the case where  $R$  does not exclude the possibility of forming a coalition with  $E$ , party  $R$  believes that with probability  $q_R$  a single-party government of  $L$  will be established and with the complementary probability  $R$  will be able to form a single-party government on its own. Consequently, signing a coalition contract excluding  $E$  given that  $L$  has excluded  $E$  is only profitable ex-ante for  $R$  if  $q \cdot V_R(p_L) + (1 - q) \cdot V_R(p_R) \leq V_R(p_{LR})$ , that is, if<sup>18</sup>

$$q \geq \frac{V_R(p_R) - V_R(p_{LR})}{V_R(p_R) - V_R(p_L)} \equiv q_R^c. \quad (7)$$

With regard to the incentive of  $L$  to exclude  $E$  given that  $R$  does so, the symmetric condition needs to be satisfied, i.e.,

$$q \geq \frac{V_L(p_L) - V_L(p_{LR})}{V_L(p_L) - V_L(p_R)} \equiv q_L^c. \quad (8)$$

Note that the right-hand sides of inequalities (22) and (23) are strictly smaller than one, implying that both parties excluding  $E$  is an equilibrium if, from the parties' perspectives, the probability that the voters will coordinate on the sole conventional party excluding  $E$  is sufficiently high.

Second, given that the other conventional party does not sign a coalition contract, we obtain the following conditions for a party to exclude the extreme party from a coalition:

$$q \geq \frac{V_R(p_E) - V_R(p_L)}{V_R(p_R) - V_R(p_L)} \equiv q_R^n, \quad (9)$$

$$q \geq \frac{V_L(p_E) - V_L(p_R)}{V_L(p_L) - V_L(p_R)} \equiv q_L^n. \quad (10)$$

As indicated, we use  $q_R^c$ ,  $q_L^c$ ,  $q_R^n$ , and  $q_L^n$  to denote the critical values that make the parties indifferent between excluding  $E$  and not signing a contract.

Using conditions (7) – (10), we can directly infer that both parties excluding the extreme party will be an equilibrium whenever  $q \geq q_L^c$  and  $q \geq q_R^c$ . Moreover, neither conventional party signing a coalition contract will be an equilibrium if  $q < q_L^n$  and  $q < q_R^n$ .

In our analysis, we further assume that conventional parties are symmetric with respect to all preference parameters.

**Symmetry Condition 1 (SC1):**  $q_L^n = q_R^n := q^n$  and  $q_L^c = q_R^c := q^c$

In terms of the primitives of the model, this condition is equivalent to

$$\frac{V_R(p_R) - V_R(p_{LR})}{V_L(p_L) - V_L(p_{LR})} = \frac{V_R(p_E) - V_R(p_L)}{V_L(p_E) - V_L(p_R)} = \frac{V_R(p_R) - V_R(p_L)}{V_L(p_L) - V_L(p_R)}.$$

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<sup>18</sup>Note that the weak inequality sign follows from the tie-breaking rule for events of probability zero.

For a complete characterization of the equilibria, the relations between  $q_k^n$  and  $q_k^c$ ,  $k \in \{L, R\}$ , play a crucial role. We distinguish two cases depending on the latter relation.<sup>19</sup>

**Symmetry Condition 2A (SC2A):**  $q_k^n < q_k^c$  for  $k \in \{L, R\}$

Under SC1 and SC2A, the equilibria outcomes depending on parties' beliefs on voter coordination are

$$(C_L, C_R) = \begin{cases} (\emptyset, \emptyset) & \text{if } 0 \leq q < q^n, \\ (\emptyset, E), (E, \emptyset) & \text{if } q^n \leq q < q^c, \\ (E, E) & \text{if } q^c \leq q \leq 1. \end{cases} \quad (11)$$

From conditions (22)–(25), it follows that SC2A is equivalent to

$$V_k(p_{LR}) + V_k(p_E) < V_k(p_k) + V_k(p_h), \quad k \neq h \in \{L, R\}.$$

Note that the above inequality can be further rewritten as

$$V_k(p_k) - V_k(p_{LR}) - (V_k(p_{LR}) - V_k(p_h)) > V_k(p_E) - V_k(p_{LR}), \quad k \neq h \in \{L, R\}. \quad (12)$$

Condition (12) stipulates that, for each party, the utilities derived from the single-party governments should be distributed asymmetrically enough around the utility obtained from a grand coalition and thus sets a lower bound on the difference in the relative utilities derived from the single-party governments with respect to the grand coalition. This bound is given by the utility difference between a coalition government with the extreme party and a grand coalition. We emphasize that SC2A requires certain symmetry properties *within* each party's utility profile but does not imply a statement about the relation between the parties' utility profiles, e.g. about the relation of  $q_L^c$  and  $q_R^c$ .

**Symmetry Condition 2B (SC2B):**  $q_k^n \geq q_k^c$  for all  $k \in \{L, R\}$

Under SC1 and SC2B, the equilibria outcomes depending on parties' beliefs about voter coordination are

$$(C_L, C_R) = \begin{cases} (\emptyset, \emptyset) & \text{if } 0 \leq q < q^c, \\ (\emptyset, \emptyset), (E, E) & \text{if } q^c \leq q < q^n, \\ (E, E) & \text{if } q^n \leq q \leq 1. \end{cases} \quad (13)$$

The interpretation of SC2B is symmetric to SC2A.

### 3.2 The equilibrium concept

Next, we pause to specify the precise meaning of equilibrium in our context. We start by noting that there are only two subgame-perfect equilibria in pure strategies. Then we specify the notion of correlated equilibrium that we use for our analysis.

<sup>19</sup>Note that we do not consider the case  $q_k^n > q_k^c$  and  $q_h^c > q_h^n$  for  $k \neq h \in \{L, R\}$  since this contradicts SC1.

### Proposition 1

In the sequential game  $\mathcal{G}$  there are only two subgame-perfect equilibria in pure strategies. They can be characterized as follows:

- (1) No party signs CPC, voters vote sincerely and the government formation results in a coalition between a conventional party, and the extreme party implementing policy  $p_E$ .<sup>20</sup>
- (2) Both parties exclude  $E$ , voters vote sincerely, and the government formation results in a grand coalition implementing policy  $p_{LR}$ .

In our previous description of the model, the two subgame-perfect equilibria in pure strategies correspond to  $q = 0$  and  $q = 1$ , respectively. Allowing for coordination probabilities in the interval  $(0, 1)$ , our equilibrium notion takes the form of a *correlated equilibrium*. We assume that, directly before the election and after the parties have signed CPC, a correlation device suggests coordinating votes on one of the conventional parties. This implies that one conventional voter is prompted to vote strategically. The probability with which the device suggests coordination on  $L$  or  $R$  may depend on the parties' contract choices at the previous game stage. Now we have to check whether the suggestions by the correlation device are incentive-compatible, i.e., whether the voter requested to vote strategically likes to follow the device's suggestion.

First, consider the situation where only one conventional party excludes the extreme party in its coalition contract at the first stage. Then at the second stage the correlation device will suggest coordinating on the party excluding  $E$  with probability  $q$  and coordinating on the party not excluding  $E$  with probability  $1 - q$ . Suppose one conventional voter does not follow the suggestion by the correlation device. Then, each conventional party will obtain one vote, and the party not excluding  $E$ , say  $k$ , will form a coalition with the extreme party implementing policy  $p_{kE}$ . By following the suggestion from the correlation device, a single-party government of either  $L$  or  $R$  would form. According to the voters' preferences given in (5) and (6), both conventional voters prefer a single-party government of their less preferred conventional party to a coalition government between a conventional party and  $E$  implementing  $p_{kE}$ . Consequently, the correlation device's suggestions are incentive-compatible in the case where only one conventional party excludes  $E$  from forming a coalition government with it.

On the other hand, in the cases where no party excludes  $E$  or both conventional parties commit to not forming a coalition with the extreme party, the correlation device's suggestion to coordinate the votes on one conventional party is not incentive compatible for the

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<sup>20</sup>The party chosen by fair randomization.



voter requested to vote strategically: both conventional voters prefer a coalition between a conventional party and  $E$  implementing policy  $p_E$  or a grand coalition to a single-party government by their less preferred conventional party.

In summary, voters will follow the correlation device's suggestions to coordinate their voting behavior if and only if one party has excluded  $E$  in its CPC. Otherwise the correlation device's suggestions are not incentive-compatible and voters will vote sincerely. This reflects the voting behavior described in the previous subsection.

A comprehensive description of the CPC written in each correlated equilibria is given when SC2A holds (or when SC2B holds) by Equation (11) (or Equation (13)). That is, we have proved the following result:

**Theorem 1**

*In the sequential game  $\mathcal{G}$ , any equilibrium of the game can be described for each ex interim coordination signal  $q \in [0, 1]$  as follows:*

- (i) conventional parties choose CPC according to Equations (11) and (13),*
- (ii) voters follow the recommendation of the correlation device if and only if only one conventional party excluded  $E$  and vote sincerely otherwise,*
- (iii) government formation results in a grand coalition implementing policy  $p_{LR}$  if  $(C_L, C_R) \in \{(\emptyset, \emptyset), (E, E)\}$  and otherwise in a coalition-government between  $E$  and the only conventional party, say  $k$ , that did not exclude  $E$  implementing  $p_{kE}$ .*

### 3.3 General coalition-preclusion contracts

In the previous section, we discussed a particular specification of CPC where each party was allowed to preclude only one other party from forming a coalition. If there is no restriction on the number of parties that can be precluded in a coalition contract, we may encounter additional equilibria. The contract choice game for the conventional parties without the one-party exclusion rule is shown in Table 2, where  $V_k^x(p_L, p_R) := x \cdot V_k(p_L) + (1-x) \cdot V_k(p_R)$  for  $x \in [0, 1]$ .

Before studying the possible equilibria in the game defined in Table 2, it will be convenient to specify another condition.

**Risk Condition (RC):**  $V_k(p_{LR}) \geq V_k^{\frac{1}{2}}(p_L, p_R)$  for all  $k \in \{L, R\}$

		Party $R$			
		$\emptyset$	$\{E\}$	$\{L\}$	$\{L, E\}$
Party $L$	$\emptyset$	$V_k(p_E)$	$V_k^{1-q}(p_L, p_R)$	$V_k(p_E)$	$V_k^{1-q}(p_L, p_R)$
	$\{E\}$	$V_k^q(p_L, p_R)$	$V_k(p_{LR})$	$V_k^q(p_L, p_R)$	$V_k^{\frac{1}{2}}(p_L, p_R)$
	$\{R\}$	$V_k(p_E)$	$V_k^{1-q}(p_L, p_R)$	$V_k(p_E)$	$V_k^{1-q}(p_L, p_R)$
	$\{R, E\}$	$V_k^q(p_L, p_R)$	$V_k^{\frac{1}{2}}(p_L, p_R)$	$V_k^q(p_L, p_R)$	$V_k^{\frac{1}{2}}(p_L, p_R)$

Table 2: Conventional parties' contract choice game with  $k \in \{L, R\}$

This condition on the parties' utilities has the following appeal. If the policy outcomes of a single-party government are distributed symmetrically in the policy space around the grand-coalition policy outcome and parties have a concave utility function, then RC will hold. A concave utility function for the parties implies that they are risk-averse in connection with the policy outcome.

Next we make two important remarks regarding the game displayed in Table 2. First, for party  $L$  the strategies  $\emptyset$  and  $\{R\}$  are payoff-equivalent, so its strategy set consists effectively of only three strategies. Second, under RC  $\{R, E\}$  is weakly dominated by  $\{E\}$ . We assume, as a tie-breaking rule for events of probability zero, that in the case of indifference between excluding only  $E$  or any other possible contract choice, strategy  $\{E\}$  will be chosen. Analogous comments hold for party  $R$ . Hence we have proved the following result:

**Proposition 2**

*Under RC, if conventional parties play no weakly dominated strategy, the equilibria of game  $\mathcal{G}$  with the one-party exclusion rule are the same as the equilibria of game  $\mathcal{G}$  without the one-party exclusion rule given by Theorem 1.*

Proposition 2 implies that the one-party exclusion rule is not needed if conventional parties play no weakly dominated strategies. However, further equilibria arise without the one-party exclusion rule if conventional parties were to consider playing weakly dominated strategies. As shown in Section 6, allowing conventional parties to play weakly dominated (or, equivalently, imposing the one-party exclusion rule or not when parties may be considering weakly dominated strategies) also has consequences on welfare, as now the parties can force single-party governments to come about in equilibrium, which might bring about less utility for the voters than policy  $p_E$ . We derive all additional equilibria with weakly dominated strategies in Appendix A.

## 4 A Micro-founded Model for Elections

In the remaining part of the main body of this paper we examine the role of CPC in elections with a large electorate within a parliamentary democracy with three political parties and proportional seat allocation. To analyze the impact of this new political institution in a more realistic scenario, we consider a micro-founded sequential game with a set of players made up by the parties and the voters. Essentially, the game consists of the same steps as in Section 3, i.e., (1) CPC are written by the parties, (2) the electorate votes for the parties, and (3) a government is formed. We consider a solution concept and identify some circumstances regarding the preference profiles of voters and parties that permit the sequential game to be embedded in the more simple model analyzed in Section 3. Then we use the latter to derive results on the more general scenario. In the following we give a full description of the game.

### 4.1 Voters and policies

The electorate, denoted by  $\Omega$ , is made up of a finite odd number of voters,  $n$ , which we assume to be very large. Throughout the text, voters are indexed alternatively by  $i$  or  $j$ . Voters elect a parliament, which in turn elects the government with the ministers of the executive branch.

The government itself may be formed by a single party or a coalition of parties and it selects an element from a two-dimensional policy space  $\mathcal{T} \times \mathcal{D}$ , where  $\mathcal{T} \subseteq \mathbb{R}$  and  $\mathcal{D} = \{0, \bar{d}\}$ . A typical policy is denoted by  $p = (t, d)$ . The first component,  $t$ , can be varied continuously in  $\mathcal{T}$  and may represent an economic variable such as a tax rate, the size of the public budget, or expenditures on social welfare. The second policy dimension,  $d$ , is binary and represents an indivisible choice, e.g. membership of a monetary union, closing borders against immigrants or asylum seekers, legalizing or abolishing abortion, or adopting an international treaty. For convenience, we refer to  $d = 0$  as the status quo, while  $d = \bar{d}$  describes the discontinuous shift of policy  $d$ .

The parties in the government derive utility from the policy they implement and from the perks they obtain. Perks include all sources of utilities for parties in power beyond  $p$ , such as exerting power, ego rents, administrative or leadership position of party members, or public expenditures targeted at the interest groups supporting the parties. Some of these perks use public funds and thus lower utility of the voters in the electorate at large. Accordingly, we assume that the preferences of voter  $i \in \Omega$  can be described by the utility

function

$$U_i(t, d, b, v_i) = u_i(|t - t_i|) - \delta_i \cdot d - \theta \cdot b + \varepsilon_i \cdot \mathbb{1}_{v_i^*}(v_i), \quad (14)$$

where

- $(t, d) \in \mathcal{T} \times \mathcal{D}$  is the policy executed by the government,
- $b > 0$  are the total perks of the parties in government,
- $t_i$  is the ideal point of voter  $i$  with respect to policy  $t$ ,
- $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$  is strictly decreasing,
- $\delta_i \in \{-1, 1\}$  describes whether voter  $i$  is in favor of the status-quo policy in  $\mathcal{D}$  ( $\delta_i = 1$ ) or the discontinuous shift ( $\delta_i = -1$ ),
- $\theta$  is the utility loss of a voter per unit of perks from the parties in power,
- $v_i \in \{L, R, E\}$  is the vote cast by  $i$ ,  $v_i^* \in \{L, R, E\}$  is the party whose ideal point in policy  $\mathcal{T}$  is closest to  $i$ 's ideal point, and  $\varepsilon_i > 0$  captures  $i$ 's ideological gain from voting sincerely, or equivalently,  $-\varepsilon_i$  is the ideological burden of voting strategically.<sup>21</sup>

The indicator function  $\mathbb{1}_x(\cdot)$  is defined by  $\mathbb{1}_x(y) = 1$  if  $x = y$  and  $\mathbb{1}_x(y) = 0$  otherwise. In the remainder of this subsection, we provide a detailed description and justification of the utility function. We note that the formulation of voters' utility involves the assumption that the government formation will yield a fixed amount of perks,  $b$ .<sup>22</sup> The median voter with respect to policy  $t$  and her ideal point are denoted by  $m$  and  $t_m$ , respectively. For simplicity we let  $\delta_m = -1$ . We also assume that  $\{u_i(|t - t_i|)\}_{i \in \Omega}$  satisfies the *strong single-crossing property*, which means that for all  $i, j \in \Omega$  such that  $t_i < t_j$  and  $t, t' \in \mathcal{T}$  with  $t < t'$ , we have, for each  $x \geq 0$ ,

$$(a) \quad u_i(|t' - t_i|) - u_i(|t - t_i|) > x \Rightarrow u_j(|t' - t_j|) - u_j(|t - t_j|) > x,$$

$$(b) \quad u_j(|t - t_j|) - u_j(|t' - t_j|) > x \Rightarrow u_i(|t - t_i|) - u_i(|t' - t_i|) > x.$$

We stress that if  $u_i(\cdot) = u(\cdot)$ , where  $u(\cdot)$  is non-increasing and concave, then  $\{u_i(|t - t_i|)\}_{i \in \Omega}$  satisfies the strong single-crossing property. To facilitate the presentation of the results we also assume that  $t_i \neq t_j$  for all  $i, j \in \Omega$  such that  $i \neq j$ .<sup>23</sup> We call voters with  $\delta_i = 1$

<sup>21</sup>We assume that for each voter  $i$  with  $\delta_i = 1$  the party whose ideal point is closest is either  $L$  or  $R$ , whereas for any other voter the party whose ideal point is closest is  $E$ .

<sup>22</sup>In Section 7 we discuss the implications for our conclusions when the total perks vary depending on parliamentary support for the government.

<sup>23</sup>This assumption does not affect the results, but it is useful as it allows us to refer later to some specific voters that would otherwise not be uniquely defined.

*conventional voters* and voters with  $\delta_i = -1$  *extreme voters*. The set of conventional voters is denoted by  $\Omega^C$ . We assume that there is a share  $\rho < 1/3$  of extreme voters<sup>24</sup> in the electorate with  $0 < \rho < \frac{1}{3}$ .<sup>25</sup> We further assume that being in favor of the extreme policy is uncorrelated with the preferences on policy  $\mathcal{T}$ , so the extreme voters' ideal points  $\{t_i\}_{i \in \Omega \setminus \Omega^C}$  are distributed symmetrically with respect to  $t_m$ . The notion of extreme voters is used to describe the preferences of a minority that desires a discontinuous and large change of the status quo against more than two-thirds of the electorate.

Beyond their preferences on outcomes  $p = (t, d)$ , we also assume that voters may obtain some (dis)utility from the actual vote that they cast on the ballots. The factor behind this assumption is the tension between *sincere voting* and *strategic voting* (see e.g. Austen-Smith 1988). Whereas under sincere voting voters are assumed to take only ideological information – i.e., parties' ideal points – into account when casting their vote, under strategic voting voters care about eventual policy outcomes, so their beliefs about the probability that they are pivotal influences their vote. In our model we assume the existence of the so-called *ideological burden* of voting strategically, which lowers (albeit perhaps very slightly) the utility of a voter when she votes *strategically*, i.e., when she votes for a party whose ideal point is not the closest to her own ideal point. The larger the burden of voting strategically is for a voter, the more likely it is that she votes *sincerely*, i.e., for the party whose ideal point is closest to her ideal point. Specifically,  $\varepsilon_i$  denotes the utility gain of voter  $i$  when she will vote sincerely instead of strategically. This loss enters additively into the voter utility. We assume that the ideological burden  $\varepsilon_i$  for extreme voters  $i$  is very large, as they are committed to the single issue of the extreme party. Hence, they always vote for the extreme party. By contrast, we assume that  $\varepsilon_i > 0$  is very low for conventional voters  $i$ . Moreover, we assume that for any two conventional voters  $i$  and  $j$ , it holds that  $\varepsilon_i > \varepsilon_j$  if  $|t_j - t_m| < |t_i - t_m|$ , i.e., the cost of voting strategically, is smaller for those voters whose ideal point is closer to that of median voter  $m$ .

Lastly, for notational convenience we introduce, for each voter  $i \in \Omega$ , the set

$$\mathcal{F}^i := \{j \in \Omega \mid t_i < t_j\}, \quad (15)$$

which consists of all voters with an ideal point larger than  $t_i$ . Throughout the text we use the operator  $|\cdot|$  to denote the cardinality of a set.

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<sup>24</sup>Here a technical precision needs to be addressed: we assume that  $\rho \cdot n$  is an integer number. By considering  $n$  to be large enough, any  $\rho$  can be approximated as closely as desired.

<sup>25</sup>In Section 7 we discuss how uncertainties regarding  $\rho$  impact our results.

## 4.2 Parties

There are three parties, denoted by  $L$ ,  $R$  and  $E$ , competing for a fixed number of seats in parliament. Each party  $k \in \{L, R, E\}$  is characterized by its political orientation  $(t_k, d_k)$ , which typically reflects the ideal policy of the party's median member. We refer to  $(t_k, d_k)$  as the ideal point of party  $k$ . We assume that  $(t_L, 0)$  and  $(t_R, 0)$  are respectively the ideal points of parties  $L$  and  $R$ , which are henceforth called *conventional parties*. The median party member differs from the median voter. Specifically, we assume that  $t_L < t_m < t_R$  and

$$t_m - t_L = t_R - t_m. \quad (16)$$

While the former condition ensures that the ideal points of conventional parties are on opposite sides of the distribution  $\{t_i\}_{i \in \Omega}$ , the latter condition requires those ideal points to be symmetric with respect to  $t_m$ . Accordingly, party  $L$  (or  $R$ ) is the *left-wing* (or *right-wing*) party. Furthermore,  $(t_m, \bar{d})$  is the ideal point of party  $E$ , which is henceforth referred to as the *extreme party*. Note that, as the extreme voters are symmetrically distributed with respect to  $t_m$  on the dimension  $\mathcal{T}$ , their ideal point in the latter policy is precisely  $t_m$ .

Let  $s_k$  denote the share of seats in the parliament for party  $k$  and  $s_G$  the share of seats in the parliament of the coalition supporting the government. We assume that for each party  $k$  its ideal point corresponds to the median member  $i_k$  of the party regarding the policy  $t$ , which we denote by  $i_k$ . Hence,  $t_k = t_{i_k}$ . The utility of a party  $k$  with ideal point  $(t_k, d_k)$  is then given by

$$V_k(t, d, b, s_k, s_G) = u_{i_k}(|t - t_{i_k}|) - \alpha_k \cdot d + I_k \cdot \beta_k \cdot \frac{s_k}{s_G} \cdot b, \quad (17)$$

where

$$\alpha_k = \begin{cases} 1 & \text{if party } k \text{ is conventional,} \\ -1 & \text{otherwise,} \end{cases}$$

$s_G$  is the share of seats in parliament for politicians benefiting from the perks associated with holding power,

$$\beta_k = \begin{cases} 1 & \text{if party } k \text{ can extract perks when holding power,} \\ 0 & \text{otherwise,} \end{cases}$$

and  $I_k$  indicates whether party  $k$  is part of the government or not:

$$I_k = \begin{cases} 1 & \text{if party } k \text{ takes part in the government,} \\ 0 & \text{otherwise.} \end{cases}$$

The share  $s_G$  of the coalition in the parliament that can extract perks can be written as

$$s_G = \sum_{k \in \{L, R, E\}} I_k \cdot \beta_k \cdot s_k.$$

We assume that only conventional parties are able to extract perks, e.g. because they are better connected to powerful groups, so the sole remaining *leit motif* for party  $E$  is to implement the extreme policy. In particular, we assume for simplicity that  $\beta_L = \beta_R = 1$  and  $\beta_E = 0$ .<sup>26</sup> Moreover, we assume that  $E$  is better off not entering a government when policy  $d = 0$  is chosen.

To summarize, the misalignment between the interests of the voters (represented by the median voter) and the interests of the two conventional parties is fueled by three different components. For  $k \in \{L, R\}$ ,

- (i) party  $k$ 's ideal point is  $t_{i_k}$ , which is different from  $t_m$ ;
- (ii) party  $k$  is able to extract perks (if it is part of the government coalition), but perks generate disutility for voters;
- (iii) if  $b$  is large enough, then party  $k$  is office-oriented in the sense that under any circumstances it will prefer to form a government without the other conventional party, even if a different option is the most preferred government coalition for a majority of voters.

### 4.3 Elections and information

We consider proportional elections, i.e. the number of seats a party obtains in parliament is proportional to its vote share in the election. The utility of each voter is private information. The distribution of ideal points  $\{t_i\}_{i \in \Omega}$  and the share  $\rho$  of extreme voters are common knowledge.

### 4.4 The political process

We consider a political game that involves four main stages:

*Stage 1: Coalition-Preclusion Contracts.*

*Stage 2: Coordination Signal.*

*Stage 3: Elections.*

*Stage 4: Government Formation.*

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<sup>26</sup>This assumption only simplifies the analysis, it does not affect the results qualitatively. The analysis is more cumbersome with  $\beta_L = \beta_R = \beta_E = 1$ . As the extreme party can implement its preferred policy  $\bar{d}$  when in power,  $\beta_E = 0$ ,  $\beta_R = \beta_L = 1$  compensates the conventional parties for the acceptance of such policies.

First, all parties simultaneously write a coalition-preclusion contract. CPC are available to every party, in particular to party  $E$ . However, by excluding one conventional party,  $E$  weakens its position in bargaining for the implementation of extreme policy  $\bar{d}$ , which is its only concern. Hence we assume that  $E$  will behave as a dummy player at this stage and consequently will not write any contract. We denote the coalition contract written by party  $k$  by  $C_k \in \{\emptyset, \{L\}, \{R\}, \{E\}\} \setminus \{\{k\}\}$ . We further stress that any coalition contract is only effective at Stage 4. If there is no possibility of writing CPC, as is currently the general rule in all democracies, we can simply assume that Stage 1 is skipped.

Second, we assume that prior to the elections there is a public signal observed by voters and parties. This signal can be of any kind, from common memories of previous governments, polls in the media, personal scandals involving some politicians, etc. Moreover, we assume that the probability distribution of this public signal may depend on the parties' contract choices at the previous game stage, for example because signing such a contract involves more TV coverage for the party involved. Because the signal is publicly observed by all voters and interpreted as a recommendation to follow and these two facts are common knowledge, it can be used by the voters to coordinate their votes on one of the conventional parties. Formally, let  $(\{L, R\}, \pi)$  be the probability distribution of the signal, where  $L$  (or  $R$ ) is the recommendation to coordinate on party  $L$  (or  $R$ ).

Third, voters cast a vote. Since for the extreme voters the cost of strategic voting is too large, we assume that they are dummy players voting for party  $E$  no matter what.

Fourth and last, to formulate the government formation process, we make two assumptions. First, only conventional parties will take the lead as formateur since we assume that a conventional party always has the largest share of votes. Second, the extreme party cannot make a compromise on policy  $d$  if it gets into government. As  $d = \bar{d}$  is its defining characteristic, accepting  $d = 0$  would lead to a "collapse". In other words,  $d = \bar{d}$  acts as an ideological constraint on the mobility of the extreme party (see Muller 2003 and Benabou 2008).

Government formation is thus structured as follows: If one conventional party obtains a majority of seats ( $s_k \geq \frac{1}{2}$  for  $k \in \{L, R\}$ ), it will form a single-party government, which in turn will select a policy  $p = (t, d)$  that will be approved in parliament by a vote of confidence. Otherwise, i.e., if no conventional party has a majority of seats, both conventional parties will try to establish a coalition government in line with the following stages:

*Stage 4.1: Proposal Round.* Conventional parties that have not excluded  $E$  in their contract simultaneously offer a coalition government to  $E$  by suggesting a policy



$p = (t, d)$  for implementation.

*Stage 4.2: Acceptance Round.* Party  $E$  decides which offer to accept (if any) from the policies proposed.

*Stage 4.3: Grand Coalition Bargaining.* Only if a grand coalition is feasible according to the CPC written in Stage 1 and if  $E$  either rejects all offers or no proposal is made to  $E$ , then both conventional parties will bargain over the desired policy  $p = (t, d)$  for implementation by maximizing<sup>27</sup>

$$s_L \cdot V_L(t, d, b, s_L, s_G) + s_R \cdot V_R(t, d, b, s_R, s_G). \quad (18)$$

*Stage 4.4: Vote of Confidence.* The proposed coalition gains power to execute the agreed policy  $p = (t, d)$  if it receives a majority of votes in parliament.

*Stage 4.5: Caretaker Government.* If the vote of confidence fails, then a “caretaker government” will take over the duties of the executive branch.

Typically, the caretaker government will consist of bureaucrats ensuring that operations in the executive branch keep running. In our context, this means that a caretaker government would stick to  $d = 0$  and implement some policy  $t$ . However, we assume that parties and voters suffer from a sufficiently high utility loss when a caretaker government runs the executive branch. Accordingly, both conventional parties are always better off forming a grand coalition, which is in turn preferred to a caretaker government by the voters. Moreover, since any two parties in our model will have a majority, if they reach an agreement on some policy  $p$  at stages 4.2 or 4.3, the agreed policy will receive the majority in the vote of confidence. These two observations enable us to simplify the government formation subgame, which we consider to be effectively made up only of Stages 4.1, 4.2 and 4.3.

## 4.5 The equilibrium concept

In the political game introduced in the previous section a strategy profile is a combination of CPC and government formation offers (for the parties) and voting strategies (for the voters). As with the simple game, we use the concept of a correlated sequential equilibrium. More precisely, we refer to a profile of pure strategies as an *equilibrium* if

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<sup>27</sup>This bargaining procedure yields the Nash Bargaining solution with bargaining power proportional to the share in the parliament.

- (a) voters vote according to their signal from the correlation device if and only if exactly one conventional party has excluded E at the first stage,
- (b) for each subset of voters,  $S \subseteq \Omega$ , there exists no profile of strategies for voters in  $S$  such that all of them obtain a larger utility by changing their strategies to it, provided that voters in  $\Omega \setminus S$  do not change their strategies,
- (c) given (a) and (b), the strategies are subgame perfect.

Additionally, we say that an equilibrium is *in weakly undominated strategies* if parties eliminate weakly dominated strategies pure strategies. One detail is worth referring to explicitly with respect to (b). In voting games like the one considered in this paper, voters are usually confronted with a coordination problem, typically deciding to vote for the same second-best in order to prevent a Condorcet loser from arising as outcome (see e.g. Andonie and Kuzmics 2007, or Ekmekci 2009). In this paper, however, we avoid any coordination friction by considering strong Nash Equilibria, so we assume that voters can effectively communicate, agree on their strategies, and commit to them. As a consequence, there is no possibility of coordination failure. However, it remains the problem of selecting one equilibrium when there are two or more equilibria that are not Pareto comparable. The public signal carries out such selection in our framework.

## 5 Relation between the General Model and the Simple Model

In this section we prove that when we consider the solution concept introduced in Section 4.5, the sequential game described in Section 4.4 boils down to the simple model analyzed in Section 3. We do so by proving a series of results, the combination of which enables us to draw conclusions regarding the role of CPC in the political model of Section 4 from the results regarding the model of Section 3.

First we focus our attention on the analysis of the government formation process. When a (conventional) party obtains a majority in the elections, it is clear that it is optimal for it to implement its ideal point.

### Lemma 1

*Assume that party  $k \in \{L, R\}$  obtains a majority in the elections. Then policy  $p_k$  is implemented.*

When no party obtains a majority in the elections different cases need to be considered

depending on which CPC have been written. We start with the case where no contract has been written.

**Proposition 3**

Assume that  $C_L = C_R = \emptyset$  and no party obtains a majority in the elections. Then

- (a) both conventional parties offer  $p_E = (t_m, \bar{d})$  to party  $E$  at Stage 4.1,
- (b) party  $E$  accepts one of the offers at Stage 4.2 with equal probability,<sup>28</sup>
- (c) policy  $p_E$  is implemented.

**Proof:**

See Appendix B.

□

Next we analyze the case where only one conventional party writes a coalition contract, in which it excludes party  $E$ .

**Proposition 4**

Assume that  $C_h = \{E\}$  and  $C_k = \emptyset$  for  $k \neq h \in \{L, R\}$  and that no party obtains a majority in the election. Then

- (a) party  $k$  offers  $p_{kE} = (t_k, \bar{d})$  to party  $E$  at Stage 4.1,
- (b) party  $E$  accepts  $p_{kE}$  at Stage 4.2,
- (c) policy  $p_{kE}$  is implemented.

**Proof:**

See Appendix B.

□

The case where both conventional parties exclude party  $E$  is straightforward.

**Proposition 5**

Assume that  $C_L = C_R = \{E\}$  and no party obtains a majority in the elections. Then both conventional parties agree on  $p_{LR} = (t^*, 0)$ , where  $t^*$  is the solution of the bargaining problem in (18).

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<sup>28</sup>We note that, in fact, any mixed strategy that consists in accepting either proposal by the conventional parties with a certain probability is optimal for party  $E$ . Thus, there is a continuum of optimal responses by  $E$ . As a tie-breaking rule, we assume that  $E$  will accept either offer with probability  $\frac{1}{2}$ .

Second, we show that according to our equilibrium concept and the assumptions of the model, the voting game by the whole electorate can be reduced to a two-player game. For that purpose we denote by  $i_R^{crit}$  the conventional voter such that  $\mathcal{F}^{i_R^{crit}}$  is minimal with respect to inclusion among the sets  $\mathcal{F}^i$  as defined in Equation (15), where  $i$  is a conventional voter, with the property that  $|(\mathcal{F}^i \cup \{i\}) \cap \Omega^C| \geq \frac{1}{2}|\Omega|$ . That is,  $i_R^{crit}$  is the conventional voter  $i$  with the largest  $t_i$  such that all conventional voters  $j$  with  $t_j \geq t_i$  account for at least half of the population. Analogously, we define  $i_L^{crit}$  such that  $\Omega \setminus \mathcal{F}^{i_L^{crit}}$  is minimal with respect to inclusion among the sets  $\Omega \setminus \mathcal{F}^i$ , where  $i$  is a conventional voter, with the property that  $|\Omega^C \setminus \mathcal{F}^i| \geq \frac{1}{2}|\Omega|$ . That is,  $i_L^{crit}$  is the conventional voter  $i$  with the smallest  $t_i$  such that all conventional voters  $j$  with  $t_j \leq t_i$  account for at least half of the population.

**Proposition 6**

- (a) All extreme voters and all conventional voters in  $\left[\Omega \setminus \left(\mathcal{F}^{i_R^{crit}} \cup \{i_R^{crit}\}\right)\right] \cup \mathcal{F}^{i_L^{crit}}$  vote sincerely.
- (b) All conventional voters in  $\mathcal{F}^{i_R^{crit}} \setminus [\mathcal{F}^m \cup m]$  cast the same vote as  $i_R^{crit}$ .
- (c) All conventional voters in  $\mathcal{F}^m \setminus \mathcal{F}^{i_L^{crit}}$  cast the same vote as  $i_L^{crit}$ .
- (d) If the median voter  $m$  is a conventional voter, she votes for  $L$  with probability  $\frac{1}{2}$  and for  $R$  with probability  $\frac{1}{2}$ .

**Proof:**

See Appendix B. □

Third and last, we show that the whole political game in Section 4 boils down to the more simple game in Section 3. To that end, we make a number of assumptions on the relationship between utility gains/losses in the first and second policy dimensions. In particular, we consider  $\bar{d}$  to lie within a certain range, so that there are circumstances in which it is appealing for conventional voters that policy  $E$  be implemented and there are other circumstances in which other possibilities are preferable for them. As in the case of the simple model, these assumptions capture the only political situations in our three-party framework in which (i) coalitions between party  $E$  and one conventional party arise in equilibrium without CPC, and (ii) CPC alter the outcome of the elections. Specifically, we assume that

$$(H1) \quad \bar{d} < u_{i_R^{crit}}(|t_m - t_{i_R^{crit}}|) - u_{i_R^{crit}}(|t_{i_R} - t_{i_R^{crit}}|),$$

$$(H2) \quad \bar{d} > u_{i_R^{crit}}(|t_{i_L} - t_{i_R^{crit}}|) - u_{i_R^{crit}}(|t_{i_R} - t_{i_R^{crit}}|),$$

$$(H1') \quad \bar{d} < u_{i_L^{crit}}(|t_m - t_{i_L^{crit}}|) - u_{i_L^{crit}}(|t_{i_L} - t_{i_L^{crit}}|),$$

$$(H2') \quad \bar{d} > u_{i_L^{crit}}(|t_{i_R} - t_{i_L^{crit}}|) - u_{i_L^{crit}}(|t_{i_L} - t_{i_L^{crit}}|).$$

We have further remarks to make regarding the above conditions. First, Assumptions  $(H1)$  and  $(H2)$  imply

$$u_{i_R^{crit}}(|t_m - t_{i_R^{crit}}|) > u_{i_R^{crit}}(|t_{i_L} - t_{i_R^{crit}}|), \quad (19)$$

i.e., in policy dimension  $\mathcal{T}$  the critical voter  $i_R^{crit}$  is closer to the median voter,  $m$ , than to the ideal point of the left-wing party,  $t_L$ . Symmetrically, in policy dimension  $\mathcal{T}$  the critical voter  $i_L^{crit}$  is closer to the median voter,  $m$ , than to the ideal point of the right-wing party,  $t_R$ . Second, since  $\{u_i(\cdot)\}_{i \in \Omega}$  satisfies the strong single-crossing property, Assumptions  $(H1)$  and  $(H2)$  hold if we replace voter  $i_R^{crit}$  by any conventional voter  $i \in \mathcal{F}^{i_R^{crit}} \setminus \mathcal{F}^m$ . Symmetrically, Assumptions  $(H1')$  and  $(H2')$  hold if we replace voter  $i_L^{crit}$  by any conventional voter  $i \in \mathcal{F}^m \setminus \mathcal{F}^{i_L^{crit}}$ . Third, Assumptions  $(H1)$  and  $(H1')$  require a change in policy  $d$  to be not so extreme that voters that are located close to the median voter,  $m$ , will prefer  $(t_m, \bar{d})$  over the implementation of the ideal point of their less preferred conventional party. Fourth, Assumptions  $(H2)$  and  $(H2')$  are independent of the those above and require that voters located close to the median voter prefer the implementation of their less preferred conventional party's ideal in policy in  $\mathcal{T}$  to the implementation of  $\bar{d}$  together with the implementation of their most preferred conventional party's ideal in policy in  $\mathcal{T}$ . Lastly, from the above comments it follows that  $t_L < t_{i_R^{crit}} < t_m < t_{i_L^{crit}} < t_R$ , which is graphically represented in Figure 1.

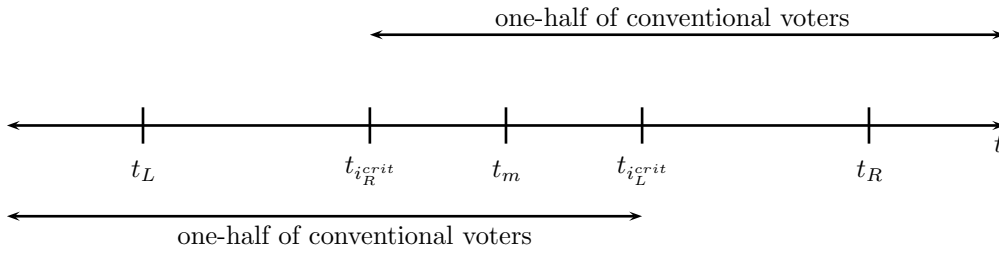


Figure 1: The critical voters.

We can now state and prove the main result connecting the two models.

**Proposition 7**

*Under Assumptions  $(H1)$ ,  $(H1')$ ,  $(H2)$ , and  $(H2')$ , the expected utilities of the conventional parties and the critical voters satisfy the relations in (1), (2), (5), and (6).*

**Proof:**

See Appendix B. □

From Lemma 1 and Propositions 3–7 we obtain our main result in this section.

**Theorem 2**

*Under Assumptions (H1), (H1'), (H2), and (H2'), a strategy profile in the political game of Section 4 is an equilibrium if and only if the strategy profile of parties and critical voters is an equilibrium in the game of Section 3 and the strategy profile fulfills Proposition 6.*

**5.1 Symmetry and risk conditions**

In the rest of the section, we analyze the requirements that the different conditions used in the model of Section 3 impose in this more general model. First, regarding SC1 we have

$$\begin{aligned} \frac{V_R(p_R) - V_R(p_{LR})}{V_L(p_L) - V_L(p_{LR})} &= \frac{V_R(p_E) - V_R(p_L)}{V_L(p_E) - V_L(p_R)} = \frac{V_R(p_R) - V_R(p_L)}{V_L(p_L) - V_L(p_R)} \\ \iff \frac{u_{i_R}(0) + \frac{b}{2} - u_{i_R}(|t_m - t_{i_R}|)}{u_{i_L}(0) + \frac{b}{2} - u_{i_L}(|t_m - t_{i_L}|)} &= \frac{u_{i_R}(|t_m - t_{i_R}|) + \frac{b}{2} - u_{i_R}(|t_{i_L} - t_{i_R}|) - \bar{d}}{u_{i_L}(|t_m - t_{i_L}|) + \frac{b}{2} - u_{i_L}(|t_{i_R} - t_{i_L}|) - \bar{d}} \\ &= \frac{u_{i_R}(0) + b - u_{i_R}(|t_{i_L} - t_{i_R}|)}{u_{i_L}(0) + b - u_{i_L}(|t_{i_R} - t_{i_L}|)}. \end{aligned}$$

Note that SC1 always holds in the limit when  $b$  becomes arbitrarily large. Second, regarding SC2A in the case of party  $L$  we have

$$\begin{aligned} V_L(p_L) - V_L(p_{LR}) &> V_L(p_E) - V_L(p_R) \\ \iff \frac{1}{2} \cdot u_{i_L}(0) + \frac{1}{2} \cdot u_{i_L}(|t_{i_R} - t_{i_L}|) &> u_{i_L}(|t_m - t_{i_L}|) - \frac{\bar{d}}{2}. \end{aligned}$$

Using Equation (16), it follows that SC2A will hold if  $u_{i_k}(\cdot)$  is convex or not strongly concave at  $|t_m - t_{i_k}|$ , for  $k \in \{L, R\}$ . Symmetrically, SC2B will hold if  $u_{i_k}(\cdot)$  is strongly concave at  $|t_m - t_{i_k}|$ , for  $k \in \{L, R\}$ .<sup>29</sup> Third, regarding RC in the case of party  $L$  note that

$$V_L(p_{LR}) \geq V_L^{\frac{1}{2}}(p_L, p_R) \iff u_{i_L}(|t_m - t_{i_L}|) \geq \frac{1}{2}u_{i_L}(0) + \frac{1}{2}u_{i_L}(|t_{i_R} - t_{i_L}|). \quad (20)$$

From Equation (16) it follows that a sufficient condition for RC to hold is that  $u_{i_L}(\cdot)$  and  $u_{i_R}(\cdot)$  be concave.

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<sup>29</sup>Formally, we require the second derivate of  $u(\cdot)$  to have a sufficiently small upper bound.

## 6 Welfare Analysis

Any analysis of the welfare implications of CPC will hinge on four key elements: whether the one-party exclusion rule applies or not, the consideration of rational beliefs in equilibrium, the criterion used to measure welfare, and the available information when we apply this criterion. The last-named element is important because the level of knowledge about the likelihood of coordination occurring in favor of a conventional party that excludes  $E$  depends on the stage at which we measure welfare. We assume rational beliefs throughout this section, i.e.,  $q = q_L = q_R$ . To analyze the different timing possibilities, it is useful to add a *Stage 0* to the political game described in Section 4.4, in which nature selects  $q$  from a given distribution. We can therefore analyze the impact of CPC on welfare *ex-ante*, i.e. before the game starts at Stage 0 or *ex-interim*, i.e. before coalition parties write the contracts at Stage 1, but as soon as the realization of  $q$  is common knowledge to all players.

In the following we assume that there is a welfare function  $f(p)$ , where  $p = (t, d)$ , that satisfies  $f(t, 0) > f(t, \bar{d})$  for all  $t \in \mathcal{T}$ . In particular, this means that  $f(p_{LR}) - f(p_E) > 0$ . Note that, since we have assumed that the level of total perks is constant, the measure of aggregate welfare depends on policy  $p$  only. The functions  $f(\cdot, 0)$  and  $f(\cdot, \bar{d})$  can be either convex, concave or neither. If  $f$  is convex (or concave) in  $t$ , then society is risk-loving (or risk-averse) with respect to policy  $t \in \mathcal{T}$ .

In Section 3 we showed that the only policies that can arise in equilibrium within the framework of the simple model are  $p_{LR}$ ,  $p_L$ ,  $p_R$ , and  $p_E$ . According to Proposition 7 this result translates into the micro-founded set-up. From the point of view of the two critical voters in the general model,  $p_L$ ,  $p_R$ , and  $p_E$  are not pairwise comparable. How society as a whole ranks these different policies is obviously relevant in assessing the impact that CPC have on welfare. More specifically, the following condition on  $f$  turns out to play a crucial role in determining whether CPC are welfare-improving or not:

$$\frac{1}{2}f(p_L) + \frac{1}{2}f(p_R) > f(p_E). \quad (21)$$

The left-hand side of the above inequality contains the expected societal utility when there is an equal probability that either conventional party will form a single-party government, whereas the right-hand side contains the societal valuation of  $p_E$ . Condition (21) ensures that society is not too risk-averse with respect to policy  $t \in \mathcal{T}$ . Indeed, note that the expected policy  $t$  when  $p_L$  and  $p_R$  are equally likely is  $t_m$ , so it coincides with policy  $t$  of  $p_E$ . Nevertheless, policy  $p_E$  implies  $\bar{d}$  in the other dimension. Condition (21) thus requires that the loss of societal utility due to uncertainty in the continuous policy be offset by the societal loss of carrying out the extreme policy.

Lastly, we assume that  $q$  is distributed with full support on  $[0, 1]$ . The main result regarding ex-ante welfare, i.e. welfare before the realization of  $q$  is known, is now stated and proved.

**Theorem 3**

*Under SC1 we obtain the following results.*

- (a) *CPC with the one-party exclusion rule are ex-ante welfare-improving when SC2B holds.*
- (b) *CPC with the one-party exclusion rule are ex-ante welfare-improving when SC2A holds if (21) applies.*

*In the micro-founded model, we obtain*

- (c) *CPC with the one-party exclusion rule are ex-ante welfare-improving if  $b$  is large enough.*
- (d) *CPC without the one-party exclusion rule may not be ex-ante welfare-improving even if  $b$  is large enough.*

**Proof:**

See Appendix B.

□

A formal proof can be found in Appendix B, but the intuition of the theorem can be summarized using Equations (11) and (13). First, consider case (a) of the theorem. If condition SC2B is satisfied, rational expectation equilibria with the one-party exclusion rule imply that either both conventional parties will not exclude any other party, in which case the introduction of CPC is without bite, or both conventional parties will exclude the extreme party. In the latter case, a grand coalition occurs which is preferable to policy outcome  $p_E$  without CPC. Consequently, the expected welfare gain from introducing CPC will be positive if the probability that  $q > q^c$  is positive.

Regarding item (b) of Theorem 3, if SC2A holds instead of SC2B, a single party government by a conventional party will be formed with equal probability for  $L$  and  $R$  when the coordination probability  $q$  is in the interval  $[q^n, q^c]$ . Condition (21) says that the expected welfare from single-party governments by conventional parties is higher than the welfare from the policy of a small coalition with strong bargaining power on the part of the extreme party,  $p_E$ . Then we can apply the same line of argument as before: If CPC have bite, they lead to more favorable policy outcomes in expectation than the policy that will be



implemented without them. Hence the expected welfare gain from introducing CPC will be positive.

However, if the expected welfare from single-party governments is smaller than that obtained from  $p_E$ , it will depend on the probability that  $q \in [q^n, q^c]$  whether the expected welfare gain from the introduction of CPC will be positive. In the micro-founded model, both probabilities  $q^n$  and  $q^c$  converge to  $\frac{1}{2}$  if  $b \rightarrow \infty$ . Consequently, we can find a sufficiently large amount of perks per seat  $b$  such that the interval  $[q^n, q^c]$  becomes small enough for the expected welfare gain associated with the introduction of CPC to be positive.<sup>30</sup>

In the case where parties play weakly dominated strategies, the conventional parties may force single-party governments to come about whenever  $q > q^n$ , while if  $q \leq q^n$  CPC are not used to exclude another party. If (21) holds, single-party governments are socially preferred to small coalitions with strong bargaining power on the part of the extreme party. Hence the introduction of CPC will again lead to expected welfare gains. However, if condition (21) does not hold, CPC will lead to lower social welfare. This will be true even for large amounts of perks, as  $q^n$  will converge to  $\frac{1}{2}$ , implying that single party governments will come about if  $q > \frac{1}{2}$ .

Regarding ex-interim welfare, i.e. welfare evaluated just after the realization of  $q$  is known, it is straightforward to check that the following result holds:

**Theorem 4**

*Under SC1 we obtain the following results:*

- (a) *CPC with the one-party exclusion rule are weakly ex-interim welfare-improving when SC2B holds.*
- (b) *CPC with the one-party exclusion rule are weakly ex-interim welfare-improving when SC2A holds if (21) holds.*

*In the micro-founded model, we obtain the following:*

- (c) *CPC with the one-party exclusion rule are ex-interim welfare-improving if  $b$  is large enough and  $q > \frac{1}{2}$ .*

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<sup>30</sup>The intuitive reason why the difference between  $q^n$  and  $q^c$  converges to zero if  $b$  approaches infinity is that the difference in expected perks between  $p_E$  and a single-party government as reflected in the numerator of  $q^n$  is the same as the difference between a single-party government and a grand coalition as captured in  $q^c$ . Consequently, the difference between  $q^n$  and  $q^c$  originates from the utility differences regarding policy. This latter part becomes less important if perks increase, and the utility difference becomes negligible if perks go to infinity. That the probabilities must converge to  $\frac{1}{2}$  follows from the fact that expected perks associated with  $p_E$  and with  $p_{LR}$  are half the size of those in a single party government. If we changed these assumptions, the critical probabilities would converge to different values in the limit.

(d) CPC without the one-party exclusion rule may not be ex-interim weakly welfare-improving even if  $b$  is arbitrarily large and  $q > \frac{1}{2}$ .

A more detailed look at the findings of the previous sections reveals further results regarding ex-interim welfare. First, not only CPC are quite often weakly ex-interim welfare-improving but in many cases they yield the *first-best outcome*  $p_{LR}$  whenever each conventional party excludes the extreme party.<sup>31</sup> First-best outcomes are not attainable without CPC. Second, if  $q^c, q^n < \frac{1}{2}$ , then CPC (with the one-party exclusion rule) yield the first-best outcome,  $p_{LR}$ , even in cases where the probability of voters coordinating on the sole party that has excluded  $E$ , i.e.  $q$ , is lower than a half.

## 7 Extensions

In this section we reconsider some of the assumptions made in Sections 3, 4, 5 and 6, and check whether our conclusions remain valid, at least qualitatively. We study several variations, the sole condition being that we only change one feature of the model at a time.

1. We consider two extreme parties,  $E_1$  and  $E_2$ , instead of just one.
2. We relax the assumption of rational expectations, i.e.  $q_L$  and  $q_R$  might be different.
3. A difference between conventional parties is introduced by allowing their corresponding ideal points in policy  $\mathcal{T}$  not to be symmetrically located with respect to the median voter's ideal point  $t_m$ .
4. We introduce uncertainty with respect to the share  $\rho$  of extreme voters.
5. We limit the power of party  $E$  in its bilateral negotiations with conventional parties by constraining the offers acceptable for these latter parties.
6. We consider the case where the total amount of perks increases with the parliamentary support for the government.
7. We analyze the long-term costs and benefits of CPC for parties and society when more than one election is considered.

We stress that for each of the above modifications affecting only elements introduced in Section 3, it suffices for our analysis to remain within the simpler framework.

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<sup>31</sup>We note that, in the framework of the micro-founded model, we have not imposed that  $p_{LR}$  should be the first-best outcome. Nevertheless, making such an assumption seems reasonable in most situations.

## 7.1 Multiple extreme parties

As in most parliamentary democracies there are several extreme parties that might enter parliament, we now examine the situation with more than one such party. This case is also interesting because even if there is only one extreme party and CPC were in place, the extreme party might have an incentive to split into two or more parties to bypass the effect of CPC – especially under the one-party exclusion rule.

We stay within the setup of Section 3, the only modification being that we now consider two extreme parties  $E_1$  and  $E_2$  instead of just one. We also assume that each extreme party obtains the same share of the votes, so that a coalition with either of them would be sufficient to form a government.<sup>32</sup> Note that if one extreme party had a higher share than the other and both conventional parties were symmetric, the extreme party with the lowest share would be irrelevant in the model, as this extreme party cannot form a majority with a conventional party.

As in the case of one extreme party only, an extreme party will possess *strong bargaining power* if neither conventional party has excluded it in its coalition contract. Accordingly, an extreme party will possess *weak bargaining power* if exactly one conventional party has excluded it in its coalition contract. We let both extreme parties be identical in the following senses. First, a coalition government between a conventional party and one of the extreme parties with strong bargaining power will implement policy  $p_E$ . Second, a coalition government between a conventional party and one of the extreme parties with weak bargaining power will implement policy  $p_{kE}$ , where  $k$  refers to the respective conventional party.

For the analysis of this modified setup, we need to specify the outcome at the government formation stage when one conventional party, say  $k$ , has excluded one of the extreme parties, e.g.  $E_1$ , while the other conventional party, say  $h \neq k$ , has excluded neither  $E_1$  nor  $E_2$ . In this case, there is competition regarding a coalition with  $E_2$ , and that competition grants  $E_2$  strong bargaining power. This implies that a coalition between a conventional party and  $E_2$  will implement policy  $p_E$ .<sup>33</sup> By contrast, as  $E_1$  is only able to form a coalition government with party  $h$ , it possesses weak bargaining power, which implies a government policy  $p_{hE}$ . Whether a coalition government will form with  $E_2$  or with  $E_1$  depends on the particular bargaining protocol that exists among the parties. In Table 3, we include both

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<sup>32</sup>To remain strictly within the setup of Section 3, we assume that the extreme voter is able to split her vote. In the case where each conventional party attempts to form a coalition with one extreme party, each coalition has a probability of 1/2 to form the government.

<sup>33</sup>We assume for simplicity that both extreme parties pursue the same extreme policy  $p_E$ . The analysis can also be applied to circumstances in which the two extreme parties pursue different policies  $P_{E_1}$  and  $p_{E_2}$  and both extreme policies impose the same utility loss on conventional voters.

possible bargaining protocols. In Table 3, we use blue to indicate the one where a coalition with the extreme party possessing strong bargaining power will be formed, and green to indicate the other one, where the extreme party with weak bargaining power becomes part of the government.

Throughout this section, we assume SC1, RC, rational expectations, and that parties do not use weakly dominated strategies. We are interested in characterizing the equilibrium outcomes under the above assumptions when CPC are available. It is worth making the following two remarks. On the one hand, at the election stage of the game, conventional voters will coordinate on one of the conventional parties if the outcome under sincere voting is either a policy  $p_{kE}$ , with  $k \in \{L, R\}$ , or a caretaker government implementing  $p_{ct}$ . On the other hand, under  $q^c < \frac{1}{2}$ , if parties do not use weakly dominated strategies, we can use the table below to describe the game at the first stage when the conventional parties sign their contracts. Here  $k \in \{L, R\}$ .

		Party $R$			
		$\emptyset$	$\{E_1\}$	$\{E_2\}$	$\{E_1, E_2\}$
Party $L$	$\emptyset$	$V_k(p_E)$	$V_k(p_E)$ $V_k^{1-q}(p_L, p_R)$	$V_k(p_E)$ $V_k^{1-q}(p_L, p_R)$	$V_k^{1-q}(p_L, p_R)$
	$\{E_1\}$	$V_k(p_E)$ $V_k^q(p_L, p_R)$	$V_k(p_E)$	$V_k^{\frac{1}{2}}(p_L, p_R)$	$V_k^{1-q}(p_L, p_R)$
	$\{E_2\}$	$V_k(p_E)$ $V_k^q(p_L, p_R)$	$V_k^{\frac{1}{2}}(p_L, p_R)$	$V_k(p_E)$	$V_k^{1-q}(p_L, p_R)$
	$\{E_1, E_2\}$	$V_k^q(p_L, p_R)$	$V_k^q(p_L, p_R)$	$V_k^q(p_L, p_R)$	$V_k(p_{LR})$

Table 3: Conventional parties' contract choice game with two extreme parties

We have written only those strategies that are payoff-different and have omitted all the strategies that are weakly dominated for both bargaining protocols. Indeed, let  $k, h \in \{L, R\}$  with  $k \neq h$  and  $l \in \{1, 2\}$ . Then, for party  $k$ , the following pairs of strategies are payoff-equivalent:  $\{h\}$  and  $\emptyset$ ,  $\{h, E_l\}$  and  $\{h, E_l\}$ ,  $\{h, E_1, E_2\}$ , and  $\{h, E_1, E_2\}$ . Moreover,  $\{h, E_1, E_2\}$  is weakly dominated by  $\{E_1, E_2\}$ . We stress that, as in the case of only one extreme party, the game in Table 3 is also obtained if we allow parties to use weakly dominated strategies but impose instead a two-party exclusion rule: in a contract each party can only exclude a coalition containing two other parties at the most. As a tie-breaking rule, we assume that conventional parties prefer to exclude extreme parties and not to exclude the other conventional party. Like in the previous section,  $V_k^x(p_L, p_R)$  stands for  $x \cdot V_k(p_L) + (1 - x) \cdot V_k(p_R)$ , where  $x \in [0, 1]$ , with the modification that  $q$  now denotes

the probability of conventional voters coordinating their votes on the conventional party that has excluded a larger number of extreme parties in its coalition contract. If both conventional parties have excluded the same number of extreme parties, we assume that the coordination probability for both parties is a half.

The characterization of equilibria depending on the parties' perceived coordination probability  $q$  has been relegated to Appendix C. Here we state the main results regarding the welfare implications of CPC in this modified framework in the light of the results from the setup with only one extreme party. The proposition below easily follows from the analysis contained in Appendix C. By default, we assume that no rule limits the maximum number of parties to be precluded in a contract.

**Proposition 8**

*Consider the framework with two extreme parties of equal size, and assume SC1, RC, rational expectations, and that parties do not use weakly dominated strategies. Then we obtain the following results:*

- (a) *CPC are ex-ante welfare-improving under both protocols if inequality (21) holds.*
- (b) *If inequality (21) holds, the ex-ante welfare associated with the blue protocol is larger than the ex-ante welfare associated with the green protocol.<sup>34</sup>*
- (c) *CPC are ex-interim welfare-improving under both protocols if  $b$  is large enough and  $q > \frac{1}{2}$ . Moreover, in this latter case the policy implemented is the first-best outcome  $PLR$ .*
- (d) *CPC with the one-party exclusion rule always yield worse outcomes than without the one-party exclusion rule. Moreover, if the reverse of inequality (21) holds, CPC are (ex-ante and ex-interim) welfare-decreasing.*

In the basic setup with only one extreme party, we saw that the simple one-party exclusion rule was sufficient to prevent any equilibrium in which all coalitions are ruled out. In that setting, the same outcomes are reached without the one-party exclusion rule if conventional parties play no weakly dominated strategies. In the case with two extreme parties, we can specify a two-party exclusion rule, or, equivalently, assume that no conventional party will play a weakly dominated strategy. Limiting the number of parties that may be excluded could give parties an incentive to split into two or more parties to relax the constraints of the coalition contract and still be able to be part of a government coalition albeit under

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<sup>34</sup>For this result to hold, it is sufficient to assume that all possible equilibrium outcomes are equally likely.

a different party name. But parties typically have to register a certain period before the election. After registration, the maximum number of parties that may be excluded in a contract is fixed, and an exclusion rule based on that number cannot be bypassed by party splitting.<sup>35</sup>

## 7.2 No rational expectations

Next we assume that parties' beliefs on voter coordination may not be rational, i.e.,  $q_R = q_L$  need not hold. Throughout the section, we assume SC1. On the one hand, in Table 1 we characterize all equilibria of the game when rational expectations are assumed away. Analogously to Section 4, it can be shown that signing a coalition contract excluding  $E$  given that  $L$  has excluded  $E$  is only profitable ex-ante for  $R$  if

$$q_R \geq \frac{V_R(p_R) - V_R(p_{LR})}{V_R(p_R) - V_R(p_L)} \equiv q_R^c, \quad (22)$$

while signing a coalition contract excluding  $E$  given that  $R$  has excluded  $E$  is only profitable ex-ante for  $L$  if

$$q_L \geq \frac{V_L(p_L) - V_L(p_{LR})}{V_L(p_L) - V_L(p_R)} \equiv q_L^c. \quad (23)$$

Similarly, given that the other conventional party does not sign a coalition contract, we obtain the following conditions for a party to exclude the extreme party from a coalition:

$$q_R \geq \frac{V_R(p_E) - V_R(p_L)}{V_R(p_R) - V_R(p_L)} \equiv q_R^n, \quad (24)$$

$$q_L \geq \frac{V_L(p_E) - V_L(p_R)}{V_L(p_L) - V_L(p_R)} \equiv q_L^n. \quad (25)$$

Recall that we use  $q_R^c, q_L^c, q_R^n,$  and  $q_L^n$  to denote the critical values that make the parties indifferent between excluding  $E$  and signing no contract and that for a complete characterization of the equilibria, the relations between  $q_k^n$  and  $q_k^c, k \in \{L, R\}$ , play a crucial role. We distinguish two cases depending on the latter relation. First, we assume SC2A. Figure 2 contains the equilibria outcomes when SC2A holds.

For coordination beliefs  $(q_L, q_R)$  such that  $(q_L, q_R) \in Q_{E, \emptyset} \cup Q_{\emptyset, E}$ , where  $Q_{E, \emptyset} = [q_L^n, 1] \times [0, q_R^n] \cup [q_L^c, 1] \times [q_R^n, q_R^c]$  and  $Q_{\emptyset, E} = [0, q_L^n] \times [q_R^n, 1] \cup [q_L^n, q_L^c] \times [q_R^c, 1]$ , there is a unique equilibrium which is asymmetric and where one conventional party excludes  $E$  and conventional voters coordinate their votes on one of the parties as described by the following

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<sup>35</sup>We note that a law should not be based on the identity of parties. Hence, from a legal point of view, an “extreme” party is no different from a conventional party, except for its share. However, in a political system with two well-established conventional parties, the number of extreme parties is  $\eta - 2$ , where  $\eta$  is the total number of parties. A law could therefore be based on  $\eta$ .

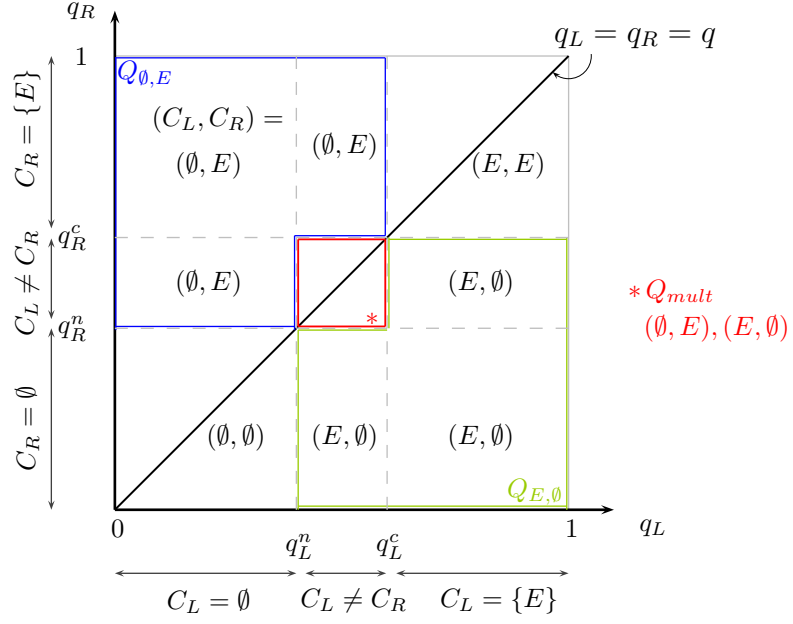


Figure 2: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2A.

strategy profiles:

$$(C_L, C_R) = \begin{cases} (\{E\}, \emptyset) & \text{if } (q_L, q_R) \in Q_{E, \emptyset}, \\ (\emptyset, \{E\}) & \text{if } (q_L, q_R) \in Q_{\emptyset, E}. \end{cases}$$

Moreover, when  $(q_L, q_R) \in Q_{mult} \equiv [q_L^n, q_L^c] \times [q_R^n, q_R^c]$  both asymmetric contract choices are the only equilibria. In Figure 3 we depict the case if the following condition holds.

Second, we assume SC2B. Figure 2 contains the equilibria outcomes when SC2B holds.

Now the area where asymmetric CPC are chosen reduces to  $Q_{E, \emptyset} := [q_L^n, 1] \times [0, q_R^c)$  and  $Q_{\emptyset, E} := [0, q_L^c) \times [q_R^n, 1]$ , while in the area  $Q_{mult} := [q_L^c, q_L^n] \times [q_R^c, q_R^n)$  both symmetric contract choices are feasible in equilibrium.<sup>36</sup> Note that rational belief equilibria are again indicated by the bisectrix in Figure 3.

On the other hand, we analyze welfare implications of CPC when we do not impose the rational expectations assumption. To that end, we assume that  $(q_L, q_R)$  is distributed on  $[0, 1] \times [0, 1]$  and that the value of the true probability of how voters coordinate,  $q$ , is not

<sup>36</sup>We note that in the case where  $q_k^n \geq q_k^c$  while  $q_h^n < q_h^c$ , for  $k \neq h \in \{L, R\}$ , the respective equilibrium characterization can be easily constructed with the help of Figures 2 and 3. In particular, for  $q_L^n \geq q_L^c$  and  $q_R^n < q_R^c$  it can be checked that there is no pure strategy equilibrium in the area  $Q_{mix} := [q_L^c, q_L^n] \times [q_R^n, q_R^c)$ , where only a mixed strategy equilibrium exists.

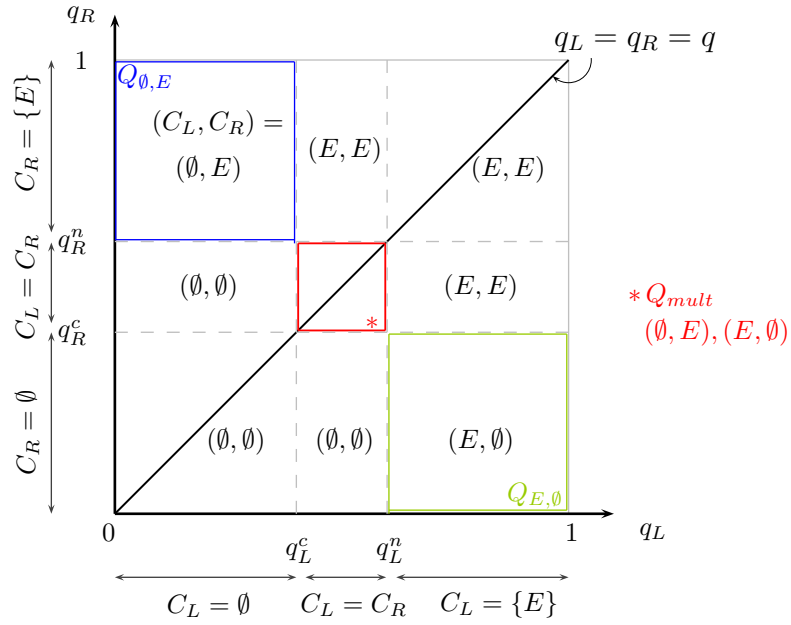


Figure 3: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2B.

known by the conventional parties before they write the contracts.<sup>37</sup> Parties might partially update their beliefs after the realization of  $q$ . Most importantly, relaxing the assumption of rational expectations may have major consequences on welfare, as we should now consider the whole area in Figures 2 and 3 instead of the bisectrix. Particularly relevant is the following fact: even if  $b$  is arbitrarily large (which implies  $q_R^c, q_L^c, q_R^n, q_L^n \simeq \frac{1}{2}$ ) there are regions in which asymmetric equilibria occur. These regions cover those circumstances in which both conventional parties believe that being the sole party excluding  $E$  only benefits coordination on that party for one of the conventional parties.

### 7.3 Asymmetric ideal points for conventional parties

Let us now assume that conventional parties' ideal points are not located symmetrically with respect to the median voter's ideal point, i.e. (16) does not hold. This fact implies that, under sincere voting, one conventional party, say  $L$ , will always be the party with the higher share of votes since more than half of the conventional voters' ideal points are closer to party  $L$ 's ideal point than to party  $R$ 's ideal point. When the asymmetry is so large

<sup>37</sup>We stress that, in Stage 2 of the game defined in Section 4, the value of  $q$  is used as a selection device for choosing the party on which conventional voters will be prompted to coordinate given that it is the sole party that has excluded  $E$ .



that under sincere voting party  $L$  obtains a share larger than  $\frac{1}{2}$ , it will obtain a majority in the parliament and thus, either with or without CPC, a single-party government by  $L$  will be formed to implement its ideal point.

In all other cases, the effect of CPC on the outcome of the elections is not so clear-cut, since both conventional parties are interested in forming a coalition with the extreme party at Stage 4.1. Observe that the larger the share of party  $L$  with respect to party  $R$  under sincere voting is, the “easier” it would be for voters to coordinate on  $L$  rather than  $R$ , for in the former case a smaller number of voters would be required to vote strategically.<sup>38</sup> As a consequence, we should not expect that probability  $q$  of coordinating on the only party that excludes  $E$  to be independent of which conventional party does so, even under rational expectations. This requires a complete analysis.

Let  $x_L$  denote the probability that coordination occurs in favor of  $L$  when  $C_L = \{E\}$  and  $C_R = \emptyset$ . Let also  $x_R$  denote the probability that coordination occurs in favor of  $R$  when  $C_R = \{E\}$  and  $C_L = \emptyset$ . We assume that  $x_R$  and  $x_L$  are common knowledge among the two conventional parties. We summarize the contract choice game played by the conventional parties with the one-party exclusion rule in the following table, where  $k \in \{L, R\}$ .

		Party $R$	
		$\emptyset$	$\{E\}$
Party $L$	$\emptyset$	$V_k(p_E)$	$(1 - x_R) \cdot V_k(p_L) + x_R \cdot V_k(p_R)$
	$\{E\}$	$x_L \cdot V_k(p_L) + (1 - x_L) \cdot V_k(p_R)$	$V_k(p_{LR})$

Table 4: Conventional parties’ contract choice game under asymmetric ideal points for the conventional parties.

Analogously to the analysis in Section 7.2, we can identify four relevant critical values. As a tie-breaking rule for events of probability zero in which parties are indifferent among two choices of CPC, we assume that they will always exclude  $E$ . For party  $L$ ,

$$x_R^n := \frac{V_L(p_E) - V_L(p_R)}{V_L(p_L) - V_L(p_R)} \text{ and } x_R^c := \frac{V_L(p_L) - V_L(p_{LR})}{V_L(p_L) - V_L(p_R)}.$$

That is, given that party  $R$  has not excluded  $E$ , party  $L$  will prefer to exclude  $E$  if and only if  $x_L \geq x_R^n$ , whereas given that party  $R$  has excluded  $E$ , party  $L$  will prefer to exclude  $E$  if and only if  $x_R \geq x_R^c$ . Note that  $x_R^n = q_L^n$  and  $x_R^c = q_L^c$ . Analogously, for party  $R$  we have

$$x_L^n := \frac{V_R(p_E) - V_R(p_L)}{V_R(p_R) - V_R(p_L)} \text{ and } x_L^c := \frac{V_R(p_R) - V_R(p_{LR})}{V_R(p_R) - V_R(p_L)}.$$

<sup>38</sup>A symmetric argument could be carried out for the opposite case.

That is, given that party  $L$  has not excluded  $E$ , party  $R$  prefers excluding  $E$  if and only if  $x_R \geq x_L^n$ , whereas given that party  $L$  has excluded  $E$ , party  $R$  prefers excluding  $E$  if and only if  $x_L \geq x_R^c$ .

Under SC1 we can define  $x^n := x_R^n = x_R^n$  and  $x^c := x_L^c = x_R^c$ . There are two possible cases that lead to different equilibria in the choice game defined in Table 4.

**Case I:**<sup>39</sup>  $x^n < x^c$

Figure 4 contains the equilibrium outcomes in this case.

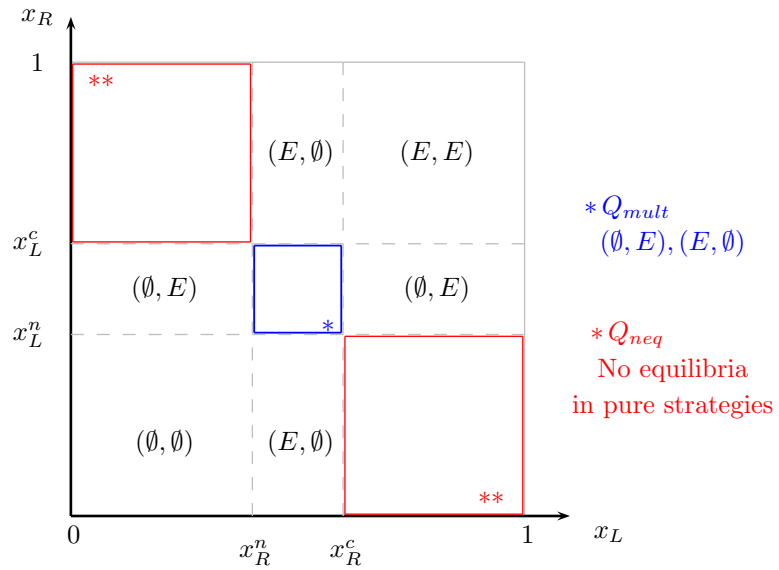


Figure 4: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2A when parties' ideal points are not symmetric.

**Case II:**<sup>40</sup>  $x^c \geq x^n$

Figure 5 contains the equilibrium outcomes in this case.

We note that in both figures there are two regions in which there are no equilibria in pure strategies. This is in sharp contrast with the equilibria in Section 7.2. From the two figures above we deduce, however, that as long as the difference between  $x_L$  and  $x_R$  is not very significant – geometrically that means that we stay close to the bisectrix – then the results

<sup>39</sup>In terms of the conditions on the parameters of the model, this case is equivalent to SC2A.

<sup>40</sup>In terms of the conditions on the parameters of the model, this case is equivalent to SC2B.

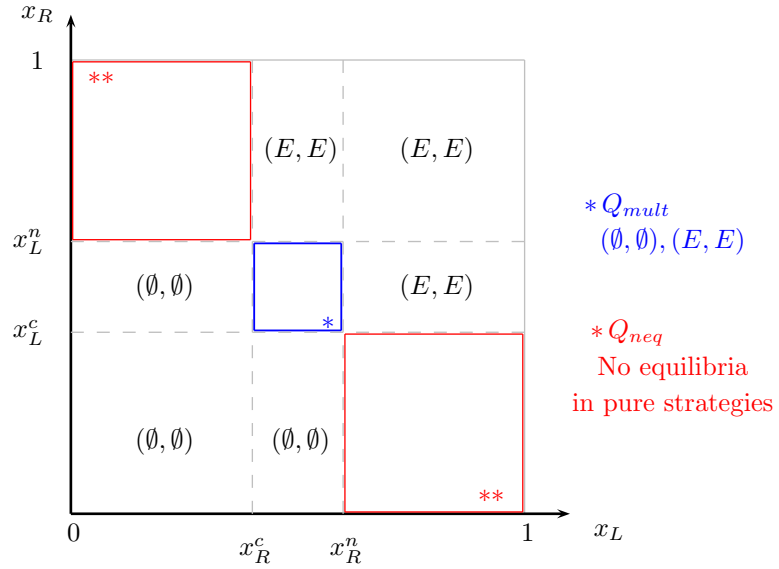


Figure 5: Equilibrium outcomes depending on parties’ beliefs about voter coordination under SC2B when parties’ ideal points are not symmetric.

concerning equilibria and welfare remain essentially the same as those in the main body of the paper.

#### 7.4 Uncertainty about the extreme party’s vote share

One of the key assumptions of our model is that the share of the extreme party  $E$  is perfectly foreseeable. There are many reasons why this could be the case, e.g. policy  $d$  might be an issue that is not subject to political fluctuations with a public opinion that is stable along time. However, it is interesting to speculate about the robustness of our results if the share of the extreme party is stochastic. A simple way of doing that is to assume that, prior to the game, it is common knowledge that with probability  $\pi_E \in (0, 1)$  party  $E$  will get into the parliament with a share  $\rho$  and with probability  $1 - \pi_E$  it will not reach the threshold of votes needed to enter the parliament, resulting in zero share. If the extreme party stayed out of the parliament, coordination on one of the conventional parties would result in a (super)majority for this party, which would implement its ideal point. Therefore, if it is certain that  $E$  will not get any seats in the parliament, no voter would find it profitable to vote strategically and, moreover, CPC would have no effect. In that event, the unique equilibria of the political game – with Stages 4.1 and 4.2 skipped – would then consist of sincere voting and a single-party government of either  $L$  or  $R$  (each

with probability  $\frac{1}{2}$ ) to respectively implement  $(t_L, 0)$  or  $(t_R, 0)$ .

Under the uncertainty modeled by  $\pi_E$ , however, voters would need to balance the expected benefits/costs of coordinating on one conventional party when only one conventional party writes a coalition contract against the expected benefits/costs of voting sincerely. In particular, there will exist a threshold  $\pi_E^*$  such that, if and only if  $\pi_E > \pi_E^*$ , then the critical voters  $i_L^{crit}$  and  $i_R^{crit}$  would still find it profitable to coordinate on one conventional party if only one conventional party excludes  $E$ . Parties would anticipate voters' behavior, so all results would remain true when  $\pi_E > \pi_E^*$ . That is, if the variance on party  $E$ 's share is small, the conclusions regarding the impact of CPC on welfare prevail.

## 7.5 Less power for the extreme party

Another important assumption of the paper is that, whereas conventional parties are free to accept any possible bargaining outcome with other parties, the extreme party can only accept bargains that offer  $\bar{d}$ . We might also assume, however, the existence of ideological constraints on the mobility of conventional parties. A polar case is to imagine that a conventional party can only accept  $\bar{d}$  in exchange for its ideal point in the policy dimension  $\mathcal{T}$ . In that case we would obtain that  $p_E$  coincides with  $\frac{1}{2}p_{LE} + \frac{1}{2}p_{RE}$ , which, given (5) and (6), implies that  $v_l(p_E) < v_l(p_R)$  and  $v_r(p_E) < v_r(p_L)$ . Thus without CPC no government coalition in which the extreme party is a member can form in equilibrium. In such circumstances CPC become redundant.

## 7.6 Increasing perks

Throughout the paper, the amount of perks has been assumed to be exogenously fixed and independent of the exact composition of the government. However, there seems to be widespread evidence that very large parliamentary support for the government reduces the opposition, not only at the political level but also at the media level.<sup>41</sup> With reduced opposition, parties in the government may be able to increase the amount of perks they get. In anticipation of such behavior, voters might not grant a super-majority to parties in the government. However, our results would not change as long as the ordinal preferences in (1), (2), (5), and (6) remained invariant.

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<sup>41</sup>Because of the existing links between power and the media.

## 7.7 Reputation effects

Elections are often considered a one-shot game, as they cannot be compared with each other. Candidates might change, socioeconomic circumstances might be very different, etc. However, parties are institutions that last for many years. In particular, they have long-term strategies. When more than one election is considered, writing a coalition contract, even if it is not effective, might have an effect on the reputation of a party's commitment credibility, i.e., voters might believe the party's announcements more (or less) than before. To account for this extension, a complete formal argument would be required. Nevertheless, in a repeated game setting we could possibly justify a coordination probability  $q = 1$  via a reputational argument as follows: In the repeated game, the described static game will be played in every period and voters in the case of asymmetric coalition contract choices could play the strategy to always coordinate on the party excluding  $E$ . In a repeated game setting such deviations would involve reputational losses and hence worse payoffs in the future.

## 8 Conclusions

Coalition-Preclusion Contracts (CPC) are a simple device that can affect how democracies with multiple parties operate. As coalition formation is observable and verifiable, CPC are easy to implement. We have suggested that on balance these contracts coupled with the single-party exclusion rule actually moderate policies and improve welfare.<sup>42</sup> Additionally, we have pursued a variety of extensions to explore the robustness of our findings. Yet numerous further issues wait to be explored. Combining CPC with endogenous platform choices in campaigns or exploring the consequences of such contracts in systems with more than two conventional parties are obvious candidates. Our results suggest that CPC could be introduced on an experimental base in democracy.

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<sup>42</sup>Of course, CPC have to honor constitutional rights of minorities.

# A Appendix

Under RC and rational expectations, the equilibria of the game  $\mathcal{G}$  without the one-party exclusion rule when conventional parties might play weakly dominated strategies depend on the relation between  $q, q^n, q^c$ , and  $\frac{1}{2}$ . We present the different cases in the tables below.

Note that RC can be rewritten as

$$q^c < \frac{1}{2}. \quad (26)$$

We distinguish three cases.

## Case I: SC2A

All possible policy outcomes that may appear in equilibrium in this case are summarized in Table 5.

$0 \leq q < q^n$	$q^n \leq q < q^c$	$q^c \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
$(\emptyset, \emptyset)$	$(\emptyset, \{E\})$ $(\{E\}, \emptyset)$ $(\{R, E\}, \emptyset)$ $(\emptyset, \{L, E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \emptyset)$ $(\emptyset, \{L, E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$

Table 5: Equilibria under SC2A without the one-party exclusion rule

## Case II: SC2B and $q^n < \frac{1}{2}$

All possible policy outcomes that may appear in equilibrium in this case are summarized in Table 6.

$0 \leq q < q^c$	$q^c \leq q < q^n$	$q^n \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
$(\emptyset, \emptyset)$	$(\emptyset, \emptyset)$ $(\{E\}, \{E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \emptyset)$ $(\emptyset, \{L, E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$

Table 6: Equilibria under SC2B and  $q^n < \frac{1}{2}$  without the one-party exclusion rule

## Case III: SC2B and $q^n \geq \frac{1}{2}$

All possible policy outcomes that may appear in equilibrium in this case are summarized in Table 7.

$0 \leq q < q^n$	$q^n \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q < q^n$	$q^n \leq q \leq 1$
$(\emptyset, \emptyset)$	$(\emptyset, \emptyset)$ $(\{E\}, \{E\})$	$(\emptyset, \emptyset)$ $(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$

Table 7: Equilibria under SC2B and  $q^n \geq \frac{1}{2}$  without the one-party exclusion rule

We note that, in all cases, if  $q$  is large enough, two different equilibria may arise: one in which any coalition with the extreme party is precluded and another in which all coalitions are precluded. By definition, this latter equilibria cannot arise if the one-party exclusion rule is in place.

## B Appendix

### Proof of Proposition 3:

We solve the subgame that starts at Stage 4 by backward induction.

#### Stage 4.2

Let  $s_L, s_R < \frac{1}{2}$  be the share of votes obtained respectively by  $L$  and  $R$  in the elections, and let  $p_L = (t'_L, d'_L)$  and  $p_R = (t'_R, d'_R)$  denote the corresponding policies offered to  $E$  by the two conventional parties at Stage 4.1. At Stage 4.2,  $E$  can either accept the policy proposed by  $L$ , accept the policy proposed by  $R$ , or reject both. With some abuse of notation, we denote those three actions by  $p_L$ ,  $p_R$ , and  $\emptyset$  respectively. The fact that  $d = \bar{d}$  acts as an ideological constraint on the mobility of the extreme party in the formation of a coalition implies that  $E$  will prefer any offer  $p' = (t', d')$  with  $d' = \bar{d}$  to any other offer  $p'' = (t'', d'')$  with  $d'' = 0$ . For notational simplicity, we write the lexicographic preference of party  $E$  for policies as follows:

$$p' \succ_E p'' \Leftrightarrow (d'_R = \bar{d} \wedge d''_R = 0) \vee ([d'_R = d''_R] \wedge [|t_m - t'| < |t_m - t''|]).$$

The optimal response, denoted by  $a_E$ , for the extreme party can then be written as

$$a_E = \begin{cases} p_L & \text{if } p_L \succ_E p_R \text{ and } d'_L = \bar{d}, \\ p_R & \text{if } p_R \succ_E p_L \text{ and } d'_R = \bar{d}, \\ \frac{1}{2}p_L + \frac{1}{2}p_R & \text{if } p_L \not\succeq_E p_R, p_R \not\succeq_E p_L \text{ and } d'_L = d'_R = \bar{d}, \\ \emptyset & \text{otherwise,} \end{cases} \quad (27)$$

where the term  $\frac{1}{2}p_L + \frac{1}{2}p_R$  means that  $E$  is indifferent between the two offers. Therefore it will randomize between them. As a tie-breaking rule, we assume in such cases that  $E$  will accept either offer with probability  $1/2$ .

#### Stage 4.1

Given the optimal response of  $E$  in (27), we now prove that both conventional parties compete “à la Bertrand” so that they offer  $p_E = (t_m, \bar{d})$  to  $E$ . We denote the actions for both parties in this simultaneous-move game by  $p_L = (t'_L, d'_L)$  and  $p_R = (t'_R, d'_R)$ . With some abuse of notation, we also denote the expected utility of party  $k \in \{L, R\}$  by  $V_k(p_k, p_h)$ , where  $h$  denotes the conventional party different from  $k$ . Then, since  $s_R, s_L < \frac{1}{2}$ ,



each conventional party  $k$  faces the following utility function:

$$V_k(p_k, p_h) = \begin{cases} u_{i_k}(|t'_k - t_{i_k}|) - \bar{d} + b & \text{if } p_k \succ_E p_h \text{ and } d'_k = \bar{d}, \\ u_{i_k}(|t'_h - t_{i_k}|) - \bar{d} & \text{if } p_k \prec_E p_h \text{ and } d'_h = \bar{d}, \\ \frac{1}{2} [u_{i_k}(|t'_k - t_{i_k}|) + u_{i_k}(|t'_h - t_{i_k}|)] - \bar{d} + \frac{1}{2}b & \text{if } p_k \not\prec_E p_h, p_k \not\prec_E p_h \text{ and } d'_k = d'_h = \bar{d}, \\ u_{i_k}(|t^* - t_{i_k}|) + \frac{s_k}{s_k + s_h}b & \text{otherwise,} \end{cases} \quad (28)$$

where  $t^*$  is the solution of the bargaining process in (18). It is clear that, for a sufficiently large  $b > 0$ ,  $p_L = p_R = p_E$  is an equilibrium. Moreover, it is the unique equilibrium. Indeed, let  $(p_L, p_R) \neq (p_E, p_E)$  be an equilibrium. We assume that  $b$  is large enough and distinguish three cases.

*Case 1:*  $p_k \not\prec_E p_h$  and  $d'_h = 0$

Note that  $V_k(p_k, p_h) = u_{i_k}(|t^* - t_{i_k}|) + \frac{s_k}{s_k + s_h}b < u_{i_k}(|t^* - t_{i_k}|) - \bar{d} + b = V_k(p, p_h)$  for  $p = (t^*, \bar{d})$ , so  $k$  gains by deviating from  $p_k$  to  $p$ .

*Case 2:*  $p_k \prec_E p_h$  and  $d'_h = \bar{d}$

Note that  $V_k(p_k, p_h) = u_{i_k}(|t'_h - t_{i_k}|) - \bar{d} < u_{i_k}(|t'_h - t_{i_k}|) - \bar{d} + \frac{1}{2}b = V_k(p_h, p_h)$ , so  $k$  gains by deviating from  $p_k$  to  $p_h$ .

*Case 3:*  $p_k \not\prec_E p_h$ ,  $p_k \not\prec_E p_h$  and  $d'_h = \bar{d}$

Note that  $V_k(p_k, p_h) = u_{i_k}(|t'_k - t_{i_k}|) - \bar{d} + \frac{1}{2}b < u_{i_k}(|t - t_{i_k}|) - \bar{d} + b = V_k(p, p_h)$  for  $p = (t, \bar{d})$  such that  $t \in \mathcal{T}$  is closer to  $t_m$  than  $t'_k$  but arbitrarily close to the latter. Therefore  $k$  gains by deviating from  $p_k$  to  $p$ .

□

#### Proof of Proposition 4:

Assume without loss of generality that  $k = R$ . As before, we analyze the subgame that starts at Stage 4 by backward induction.

##### Stage 4.2

Let  $p_R = (t'_R, d'_R)$  be the policy offered by  $R$  to  $E$ . Recall that  $L$  cannot offer anything to  $E$  since it is not allowed by the coalition contract. At Stage 4.2,  $E$  can either accept the policy proposed by  $R$  or reject it. With some abuse of notation, we denote those two

actions by  $p_R$  and  $\emptyset$  respectively. In this case, the fact that  $d = \bar{d}$  acts as an ideological constraint on the mobility of the extreme party implies that the optimal response  $a_E$  for the extreme party is

$$a_E = \begin{cases} p_R & \text{if } d'_R = \bar{d}, \\ \emptyset & \text{otherwise.} \end{cases} \quad (29)$$

#### Stage 4.1

Given the optimal response of  $E$  in (29), party  $R$  will offer  $p^* = (t_R, \bar{d})$  to  $E$  since, for  $b$  sufficiently large, it will prefer to secure the support of  $E$  by ceding  $\bar{d}$  in exchange for higher perks.

□

#### Proof of Proposition 6:

First, we stress that the outcome of the election depends only on the shares of the conventional parties, namely  $s_L$  and  $s_R$ . Thus, since we consider Strong Nash equilibria and there is a positive cost of voting strategically, the set of voters that vote strategically, denoted by  $S^*$ , will be such that either  $S^* \subset \mathcal{F}^m$  or  $S^* \subset \Omega \setminus \mathcal{F}^m$ , and either  $|S^*| = 0$  or  $|S^*| = \frac{1}{2}\rho|\Omega|$ .

Second, as the above observation suggests, there may be different sets  $S^*$  that form in equilibrium and lead to the same voting outcome. However, according to our notion of equilibrium, it is necessarily the case that  $S^*$  is either the empty set or a subset of size  $\frac{1}{2}\rho|\Omega|$  of conventional voters with ideal points closest to  $t_m$ . This statement follows from two facts. On the one hand, since in the policy dimension  $\mathcal{T}$  agents have preferences that satisfy the strong single-crossing property, if a conventional voter has no incentive to deviate from sincere voting, then no other conventional voter on her political side who is farther away from the median will have such an incentive either. On the other hand, the closer a conventional voter's ideal point  $t_i$  is to  $t_m$ , the less she will suffer from the policy shift toward the ideal point of her least preferred conventional party.

□

#### Proof of Proposition 7:

On the one hand, Propositions 3–5 describe the way in which parties fully anticipate the outcome of the government formation process – see Stage 4 – if they behave according to our equilibrium concept and this fact is public knowledge. Note that the different outcomes of this process coincide precisely with those considered in the model of Section 3. On the other hand, Proposition 6 identifies two critical voters,  $i_R^{crit}$  and  $i_L^{crit}$ , whose votes fully indicate the voting outcome of the whole electorate in equilibrium. Therefore, to solve the whole political game, we can simply focus on the choices of the parties regarding their contracts and on the voting choices of the critical voters. Note that this general game can therefore be reduced to the game considered in Section 3.

The rest of the proof consists in proving that the critical voters' expected utilities satisfy the ordinal relations in (5) and (6) and that the conventional parties' expected utilities satisfy the ordinal relations in (1) and (2).

First, let us consider party  $L$ . If  $b$  is large enough, we have the following expected utilities

$$\begin{aligned}
V_L(p_L) &= V_L\left(t_{i_L}, 0, b, \frac{1}{2}, \frac{1}{2}\right) = u_{i_L}(0) + b > \\
V_L(p_{LE}) &= V_L\left(t_{i_L}, \bar{d}, b, \frac{1-\rho}{2}, \frac{1-\rho}{2}\right) = u_{i_L}(0) - \bar{d} + b > \\
V_L(p_{LR}) &= V_L\left(t_m, 0, b, \frac{1-\rho}{2}, 1-\rho\right) = u_{i_L}(|t_m - t_{i_L}|) + \frac{b}{2} > \\
V_L(p_E) &= V_L\left(t_m, \bar{d}, b, \frac{1}{4}(1-\rho), \frac{1}{2}(1-\rho)\right) = u_{i_L}(|t_m - t_{i_L}|) - \bar{d} + \frac{b}{2} > \\
V_L(p_R) &= V_L\left(t_{i_R}, 0, b, 0, \frac{1}{2}\right) = u_{i_L}(|t_{i_R} - t_{i_L}|) > \\
V_L(p_{RE}) &= V_L\left(t_{i_R}, \bar{d}, b, 0, \frac{1-\rho}{2}\right) = u_{i_L}(|t_{i_R} - t_{i_L}|) - \bar{d} > \\
&V_L(p_{ct}).
\end{aligned}$$

The case of party  $R$  is symmetric and can be proved analogously. Second, let us consider the critical voter  $i_R^{crit}$ . We stress that  $i_R^{crit}$  is, by construction, closer to party  $L$ 's ideal point than to party  $R$ 's ideal point in policy  $\mathcal{T}$ , i.e.,

$$u_{i_R^{crit}}(|t_{i_L} - t_{i_R^{crit}}|) > u_{i_R^{crit}}(|t_{i_R} - t_{i_R^{crit}}|). \quad (30)$$

Then

$$\begin{aligned}
v_L(p_{LR}) &= U_{i_R^{crit}}(t_m, 0, b, L) = u_{i_R^{crit}}(|t_m - t_{i_R^{crit}}|) - b + \varepsilon_{i_R^{crit}} > \\
v_L(p_L) &= U_{i_R^{crit}}(t_{i_L}, 0, b, L) = u_{i_R^{crit}}(|t_{i_L} - t_{i_R^{crit}}|) - b + \varepsilon_{i_R^{crit}} > \\
v_L(p_E) &= U_{i_R^{crit}}(t_m, \bar{d}, b, L) = u_{i_R^{crit}}(|t_m - t_{i_R^{crit}}|) - \bar{d} - b + \varepsilon_{i_R^{crit}} > \\
v_L(p_R) &= U_{i_R^{crit}}(t_{i_R}, 0, b, R) = u_{i_R^{crit}}(|t_{i_R} - t_{i_R^{crit}}|) - b > \\
v_L(p_{LE}) &= U_{i_R^{crit}}(t_{i_L}, \bar{d}, b, L) = u_{i_R^{crit}}(|t_{i_L} - t_{i_R^{crit}}|) - \bar{d} - b + \varepsilon_{i_R^{crit}} > \\
v_L(p_{RE}) &= U_{i_R^{crit}}(t_{i_R}, \bar{d}, b, R) = u_{i_R^{crit}}(|t_{i_R} - t_{i_R^{crit}}|) - \bar{d} - b > \\
&v_L(p_{ct}),
\end{aligned}$$

where the first inequality holds by (19), the second inequality holds directly by Assumption (H2), the third inequality holds directly by Assumption (H1), the fourth inequality holds by Assumption (H2) and (30), and the fifth inequality follows immediately from (30). The case of the critical voter  $i_L^{crit}$  can be proved analogously.

□

### Proof of Theorem 3:

Parts (a) and (b) follow immediately from Figures 2 and 3 respectively, whereas Part (d) follows from Tables 5, 6, and 7. To prove Part (d) it suffices to realize that, from the micro-founded values for  $q^c$  and  $q^n$ , it follows that

$$\lim_{b \rightarrow \infty} q^c = \lim_{b \rightarrow \infty} \frac{\frac{b}{2} + u_{i_L}(0) - u_{i_L}(|t_m - t_{i_L}|)}{b + u_{i_L}(0) - u_{i_L}(|t_m - t_{i_L}|)} = \frac{1}{2}, \quad (31)$$

and

$$\lim_{b \rightarrow \infty} q^n = \lim_{b \rightarrow \infty} \frac{\frac{b}{2} + u_{i_L}(|t_m - t_{i_L} - u_{i_L}(|t_{i_R} - t_{i_L}|)|)}{b + u_{i_L}(0) - u_{i_L}(|t_m - t_{i_L}|)} = \frac{1}{2}. \quad (32)$$

□

## C Appendix

In the first part of this appendix, we assume that there is no rule that limits the number of parties that can be excluded and characterize the equilibria depending on the parties' perceived coordination probability  $q = q_L = q_R$  in the game with two equally-sized extreme parties and two symmetric conventional parties. The critical probabilities  $q^c$  and  $q^n$  defined in Section 3 are still useful in the extended framework. Indeed, for  $q \geq q^c$ , we have  $V_k(p_{LR}) \geq V_k^q(p_h, p_k)$ , while  $q \geq q^n$  implies  $V_k^q(p_k, p_h) \geq V_k(p_E)$ , where  $k \neq h \in \{L, R\}$ . With a larger number of parties than in the previous setup, we obtain more equilibria. However, not all the different equilibria lead to different policy outcomes. As such we are mainly interested in the latter, we focus our attention on the resulting equilibrium policy outcomes given the coordination probability  $q$  rather than on the equilibrium strategies. We use  $(q \cdot p_k, (1 - q) \cdot p_h)$ , with  $k \neq h \in \{L, R\}$ , to denote the outcome where with probability  $q$  conventional party  $k$  will form a single-party government and with the complementary probability party  $h$  will lead a single-party government of its own. We distinguish three cases.<sup>43</sup>

**Case I:**  $q^c \leq q^n \leq \frac{1}{2}$

All possible policy outcomes that can appear in equilibrium in this case are summarized in the following table:

	$0 < q < q^c$	$q^c < q < q^n$	$q^n < q < \frac{1}{2}$	$\frac{1}{2} < q < 1$
Blue protocol	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$ $p_{LR}$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$ $p_{LR}$	$p_{LR}$
Green protocol	$p_E$	$p_{LR}$ $p_E$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$ $p_{LR}$	$p_{LR}$

Table 8: Green versus blue protocol in the case of two extreme parties (Case I)

To prove the results contained in Table 8, we distinguish several subcases.

**Case I.1:**  $0 < q < q^c$

From  $0 < q < q^c \leq q^n \leq \frac{1}{2} < 1 - q$  and the definition of  $q^c$  and  $q^n$  it follows that, given  $k \neq h \in \{L, R\}$ ,

$$V_k^q(p_k, p_h) < V_k(p_E) \leq V_k^{\frac{1}{2}}(p_k, p_h) < V_k^{1-q}(p_k, p_h) \quad (33)$$

<sup>43</sup>As our main goal is to calculate welfare, which depends on variable  $q$ , we neglect indifference cases when  $q$  coincides with either  $q^c, q^n, 1 - q^n$ , or  $\frac{1}{2}$ .

and

$$V_k(p_{LR}) < V_k^{1-q}(p_k, p_h). \quad (34)$$

First, in the case of the blue bargaining protocol, it follows from (33) and (34) that the strategies  $\emptyset$  and  $\{E_1, E_2\}$  are weakly dominated for both conventional parties. Hence the coalition contract choice game for the conventional parties in undominated strategies reduces to

		Party $R$	
		$\{E_1\}$	$\{E_2\}$
Party $L$	$\{E_1\}$	$V_L(p_E), V_R(p_E)$	$V_L^{\frac{1}{2}}(p_L, p_R), V_R^{\frac{1}{2}}(p_L, p_R)$
	$\{E_2\}$	$V_L^{\frac{1}{2}}(p_L, p_R), V_R^{\frac{1}{2}}(p_L, p_R)$	$V_L(p_E), V_R(p_E)$

There are two equilibria:  $(\{E_1\}, \{E_2\})$  and  $(\{E_2\}, \{E_1\})$ . Nevertheless, according to the preferences of the voters in (5) and (6) and the fact that both extreme parties have weak bargaining power due to the contracts chosen by the parties, the policy outcomes associated with both equilibria are the same, namely  $(\frac{1}{2}p_L, \frac{1}{2}p_R)$ .

Second, in the case of the green bargaining protocol, it follows from (33) and (34) that the strategy  $\{E_1, E_2\}$  is weakly dominated for both conventional parties. Hence the coalition contract choice game for the conventional parties in undominated strategies reduces to

	$\emptyset$	$\{E_1\}$	$\{E_2\}$
$\emptyset$	$V_L(p_E), V_R(p_E)$	$V_L^q(p_R, p_L), V_R^q(p_R, p_L)$	$V_L^q(p_R, p_L), V_R^q(p_R, p_L)$
$\{E_1\}$	$V_L^q(p_L, p_R), V_R^q(p_L, p_R)$	$V_L(p_E), V_R(p_E)$	$V_L^{\frac{1}{2}}(p_L, p_R), V_R^{\frac{1}{2}}(p_L, p_R)$
$\{E_2\}$	$V_L^q(p_L, p_R), V_R^q(p_L, p_R)$	$V_L^{\frac{1}{2}}(p_L, p_R), V_R^{\frac{1}{2}}(p_L, p_R)$	$V_L(p_E), V_R(p_E)$

There is only one equilibrium:  $(\emptyset, \emptyset)$ . In this equilibrium, both extreme parties have strong bargaining power due to the coalition contracts chosen by the parties. As a consequence, according to the preferences of the voters in (5) and (6), the policy outcome associated with this equilibrium is  $p_E$ .

**Case I.2:**  $q^c < q < q^n$

From  $q^c < q < q^n < \frac{1}{2}$  and the definition of  $q^c$  and  $q^n$  it follows now that, given  $k \neq h \in \{L, R\}$ ,

$$V_k^q(p_k, p_h) < V_k(p_E) \leq V_k^{\frac{1}{2}}(p_k, p_h) < V_k^{1-q}(p_k, p_h)$$

and

$$V_k(p_{LR}) > V_k^{1-q}(p_k, p_h).$$

In the case of the blue bargaining protocol there are three equilibria:  $(\{E_1\}, \{E_2\})$ ,  $(\{E_2\}, \{E_1\})$  and  $(\{E_1, E_2\}, \{E_1, E_2\})$ . As in the previous case, we have a situation where the policy outcomes associated to the first two equilibria are the same, namely  $(\frac{1}{2}p_L, \frac{1}{2}p_R)$ . Regarding the latter equilibria, however, we find that the policy outcome is  $p_{LR}$ , as no extreme party can form a coalition with any of the conventional parties.

In the case of the green bargaining protocol, there are two equilibria:  $(\emptyset, \emptyset)$  and  $(\{E_1, E_2\}, \{E_1, E_2\})$ . In the first equilibrium, both extreme parties have strong bargaining power due to the contracts chosen by the parties, so the policy outcome associated with it is  $p_E$ . In the second equilibrium, the policy outcome is  $p_{LR}$ , as no extreme party can form a coalition with any of the conventional parties.

**Case I.3:**  $q^n < q < \frac{1}{2}$

From  $q^c \leq q^n < q < \frac{1}{2}$  and the definition of  $q^c$  and  $q^n$  it follows now that, given  $k \neq h \in \{L, R\}$ ,

$$V_k(p_E) < V_k^q(p_k, p_h) < V_k^{\frac{1}{2}}(p_k, p_h) < V_k^{1-q}(p_k, p_h)$$

and

$$V_k(p_{LR}) > V_k^{1-q}(p_k, p_h).$$

In the case of the blue bargaining protocol the following are the equilibria:  $(\{E_1\}, \{E_2\})$ ,  $(\{E_2\}, \{E_1\})$ , and  $(\{E_1, E_2\}, \{E_1, E_2\})$ , which lead respectively to policies  $p_E$ ,  $p_E$  and  $p_{LR}$ .

In the case of the green bargaining protocol, there are five equilibria:  $(\{E_1\}, \emptyset)$ ,  $(\{E_2\}, \emptyset)$ ,  $(\emptyset, \{E_1\})$ ,  $(\emptyset, \{E_2\})$  and  $(\emptyset, \emptyset)$ . The policy outcome associated with the first four equilibria is  $p_E$ , since we are considering the green bargaining protocol and in all cases there is an extreme party that possesses weak bargaining power, while the other extreme party possesses strong bargaining power. In the last equilibrium, the policy outcome is  $p_{LR}$ , as no extreme party can form a coalition with any of the conventional parties.

**Case I.4:**  $\frac{1}{2} < q$

From  $q^c \leq q^n < q < \frac{1}{2}$  and the definition of  $q^c$  and  $q^n$  it follows now that, given  $k \neq h \in \{L, R\}$ ,

$$V_k(p_E) < V_k^{\frac{1}{2}}(p_k, p_h) < V_k^q(p_k, p_h), \quad V_k^{1-q}(p_k, p_h) < V_k^{\frac{1}{2}}(p_k, p_h)$$

and

$$V_k(p_{LR}) > V_k^{1-q}(p_k, p_h).$$

With both bargaining protocols, the unique equilibrium is  $(\{E_1, E_2\}, \{E_1, E_2\})$ , which leads to policy  $p_{LR}$ .

**Case II:**  $q^c \leq \frac{1}{2} \leq q^n$

All possible policy outcomes that can appear in equilibrium in this case are summarized in the following table:<sup>44</sup>

	$0 < q < q^c$	$q^c < q < \frac{1}{2}$	$\frac{1}{2} < q < q^n$	$q^n < q < 1$
Blue protocol	$p_E$	$p_{LR}$ $p_E$	$p_{LR}$ $p_E$	$p_{LR}$
Green protocol	$p_E$	$p_{LR}$ $p_E$	$p_{LR}$ $p_E$	$p_{LR}$

Table 9: Green versus blue protocol in the case of two extreme parties (Case II)

**Case III:**  $q^n \leq q^c \leq \frac{1}{2}$

All possible policy outcomes that can appear in equilibrium in this case are summarized in the following table:<sup>45</sup>

	$0 \leq q < q^n$	$q^n \leq q < q^c$	$q^c \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
Blue protocol	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$ $p_{LR}$	$p_{LR}$
Green protocol	$p_E$	$(qp_L, (1-q)p_R)$ $((1-q)p_L, qp_R)$	$(qp_L, (1-q)p_R)$ $((1-q)p_L, qp_R)$ $p_{LR}$	$p_{LR}$

Table 10: Green versus blue protocol in the case of two extreme parties (Case III)

<sup>44</sup>A comprehensive proof of all cases can be provided by the authors upon request.

<sup>45</sup>As in Case II, a comprehensive proof of all cases can be provided by the authors upon request.



Lastly, we focus on the case where the one-party exclusion rule applies. From Table 3, it immediately follows that  $p_{LR}$  cannot arise in equilibrium as it is not an outcome of the game. The three tables below show the policies that arise in equilibrium when there are two extreme parties but the one-party exclusion applies.

	$0 < q < \frac{1}{2}$	$\frac{1}{2} < q < 1$
Blue protocol	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$
Green protocol	$p_E$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$

Table 11: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case I:  $q^c \leq q^n \leq \frac{1}{2}$ )

	$0 < q < q^n$	$q^n < q < 1$
Blue protocol	$p_E$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$
Green protocol	$p_E$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$

Table 12: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case II:  $q^c \leq \frac{1}{2} \leq q^n$ )

	$0 \leq q < q^n$	$q^n \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
Blue protocol	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$
Green protocol	$p_E$	$(qp_L, (1-q)p_R)$ $((1-q)p_L, qp_R)$	$(\frac{1}{2}p_L, \frac{1}{2}p_R)$

Table 13: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case III:  $q^n \leq q^c \leq \frac{1}{2}$ )

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