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## Optimum Growth and Carbon Policies with Lags in the Climate System

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# Optimum Growth and Carbon Policies with Lags in the Climate System

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## Abstract

We study the optimal carbon tax in an economy in which climate change, stemming from polluting non-renewable resource, affects the economy's growth potential. Our main contribution is to introduce and explore the natural time lag of the climate system between emissions and damages to capital accumulation in an endogenous growth setting. This allows us to investigate how optimal climate policy, and its interplay with climate dynamics, affect long-run growth and the transition of the economy towards it. Without pollution decay, a higher speed of emissions diffusion steepens the growth profile of the economy. With pollution decay, this leads to lower short-run but higher long-run economic growth during transition. Poor understanding of the emissions diffusion process leads to suboptimal carbon taxes, resource extraction and growth.

**Keywords:** Climate Policy; Non-Renewable Resource Dynamics; Pollution Diffusion Lag; Optimum Growth

**JEL classification:** Q54; O11; Q52; Q32

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## 1. Introduction

Climate change has certain characteristics that impede the implementation of optimal environmental policies: it has a global dimension, necessitating difficult international negotiations and agreements; it requires mitigation policies that create economic costs and benefits which are substantial and unevenly distributed across different countries, and finally; it asks for a policy design that necessitates consideration of a very long time horizon. This poses a major challenge for a usually myopic political decision making process: past environmental policies were mostly implemented after major environmental damages had been publicly observed, creating political necessity to act.<sup>1</sup>

The effects of climate change will only be fully visible after several decades because greenhouse gas emissions cause economic damages only with a major time lag. The Stern Review states “climate models project that the world is committed to a further warming...over several decades due to past emissions.”, (Stern, 2007, p. 15). Looking into the future and the potentially large damages from climate change, one would expect a time lag of about 50 to 150 years, depending on the scenario followed, (Stern, 2007, p. 178). A certain degree of uncertainty remains in any case, an example for which is prominently given in the new IPCC fifth assessment report: “...due to natural variability, trends based on short records are very sensitive to the beginning and end dates and do not in general reflect long-term climate trends. As one example, the

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<sup>1</sup>From example the Montreal Protocol on the ozone layer or the ban of asbestos.

rate of warming over the past 15 years [...] is smaller than the rate calculated since 1951” (IPCC, 2013). The existence and form of this delay in the natural system has major implications for optimum growth and carbon policies, which we study in this paper.

The model is motivated by the evidence that natural disasters have a substantial impact on the economy, destroying part of its physical capital stock (Stern, 2013; Bretschger and Valente, 2011). At the same time, economic growth exacerbates the impact of natural disasters as the economy accumulates capital, so that each new event has a higher damaging potential. Since 1900, reported economic damages related to weather phenomena and climate change such as floods, droughts, storms, extreme temperatures, and wildfires account for about 75% of all the natural disasters recorded (EM-DAT The International Disasters Database, 2015). Moreover reported damages have increased greatly since the late 1980s.

This paper develops a theoretical model of a growing economy that is harmed by climate change. The model framework used in the paper is based on the endogenous growth approach of Rebelo (1991), enhanced by a polluting non-renewable resource as an essential input to production. We incorporate relevant features such as carbon emissions from non-renewable resources, the slow adjustment of the stock of pollution to emissions, and climate change that affects capital depreciation. Using this endogenous growth setup we characterize the optimal carbon tax when climate change affects the economy’s growth potential. We also study how climate dynamics interact with resource extraction and growth in the case of optimal and suboptimal policies. Our main contribution in the theoretical literature is twofold.

First, with our specification of damages in capital accumulation - linear to the level of pollution - and logarithmic utility, the optimal tax is proportional to current consumption, in line with the literature; for instance Gerlagh and Liski (2012), Golosov et al. (2014), Grimaud and Rouge (2014), van den Bijgaart et al. (2016). In the case of a more general CRRA utility, it asymptotically approaches this behavior. Climate change policy postpones resource extraction and consumption, and induces economic growth to start from a higher level, converging asymptotically to a lower positive constant, the latter being unaffected by policy. If all carbon in the atmosphere is removed through carbon decay, there is no climate problem in the long run; if carbon decay is absent, the long-run growth rate is affected by cumulative extraction.

Second, we introduce in continuous time a well-specified time lag between emissions from polluting non-renewable resources and the damages they cause. With our specification, a unit of emissions follows a diffusion process in which it only gradually increases the stock of harmful pollution; taken together with carbon decay this allows for a hump-shaped impulse response function. This process proves to be crucial for the transition of the economy towards its steady state: without pollution decay, a higher speed of emissions diffusion steepens the growth profile of the economy; with pollution decay this leads to lower short-run but higher long-run economic growth during transition. It follows that poor understanding of the emissions diffusion process can lead to suboptimal carbon taxes, resource extraction and growth. We use this result to argue that if emission taxes are not set by the social planner but by a regular political process, there is a risk of setting tax rates at too low a level.

To the best of our knowledge, this is the first contribution which combines endogenous growth with polluting non-renewable resources to derive the impact of a time lag in pollution dissemination in terms of closed-form solutions. Several contributions have studied the dynamic response of the economy to pollution. Withagen (1994) shows that the introduction of pollution from non-renewable resources in the utility function delays optimum resource extraction. Hoel and Kverndokk (1996) abstract from the finiteness of non-renewable resources by focusing on the economic recoverability of the resource stock. They also note that in the presence of greenhouse effects it will be optimal to slow down extraction and spread it over a longer period. Tahvonen (1997) additionally allows for a non-polluting backstop technology and defines different switching regimes between non-renewable resources and the backstop, which depend on initial

pollution and the price of non-renewable resources and the backstop. These models, in partial equilibrium, abstract from capital accumulation, which is crucial for growth, and capital destruction due to climate change, which represents climate damages in a more realistic way.

Sinclair (1994) argues, however, that "If global warming is taken to be a serious phenomenon, [...] interest rates need to be co-endogenized with other relevant variables", and studies the impact of environmental pollution in general equilibrium. The impact of pollution on growth has also been studied by Bovenberg and Smulders (1995) and Michel and Rotillon (1995).<sup>2</sup> In a Ramsey growth model, van der Ploeg and Withagen (2010) analyze optimal climate policy based on the social cost of carbon and the existence of renewable resources. Ikefuji and Horii (2012) develop a model with capital destruction due to climate change and conclude that growth is sustainable only if the tax rate on the polluting input increases over time. Contrary to our model they abstract from resource finiteness and the inherent time lag in climate change. Using an endogenous growth model, Bretschger and Valente (2011) show that less developed countries are likely to be hurt more than developing ones, with greenhouse gas emissions inducing negative growth deficits and possible unsustainability traps. Grimaud and Rouge (2014) analyze how the availability of an abatement technology affects optimal climate policies using an endogenous growth model based on the expansion-in-varieties framework and show that when such a technology is available, the optimal carbon tax that postpone resource extraction is uniquely determined. Another related paper is Golosov et al. (2014), which introduces non-renewable resources as in our model but abstracts from any capital stock.<sup>3</sup> Including the stock of capital is crucial for our approach to capture both endogenous growth and climate damage.

Time lags in the climate system are usually implemented in integrated climate assessment models. Prominent examples are Nordhaus (1992, 2011) that calibrate a Ramsey growth model to show a significant Pareto-improvement due to climate mitigation investment. Most theoretical models on climate change have sidestepped time lags in the climate system. Important exceptions are the contributions of Gerlagh and Liski (2012) and van den Bijgaart et al. (2016). In the former the authors rely on the assumption of full capital depreciation in each period and using quasi-hyperbolic preferences find that the equilibrium carbon price exceeds the imputed externality cost by multiple degrees of magnitude. The latter derives the social cost of carbon in closed-form for a general neoclassical economy whose development is approximated by a balanced growth path.

The remainder of the paper is organized as follows. Section 2 presents the climate dynamics and the technologies of our economy. In Section 3 we characterize the social cost of carbon, i.e. the first best (Pigouvian) per-unit tax that restores the socially optimal allocation. In section 4 we solve for the decentralized equilibrium. Section 5 analyzes the effect of climate dynamics and different taxation policies on economic growth. In section 6 we provide simulations in the case of a general CRRA utility and explain our results. Section 7 concludes.

## 2. The Basic Model

### 2.1. Climate System

Producers of consumption goods use polluting non-renewable resources,  $R_t$ , which generate a flow of emissions  $\phi R_t$ ;  $\phi \geq 0$  denotes the carbon content of the resource and  $t$  the time index. Emissions add to

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<sup>2</sup>For a survey of the literature on the relationship between environmental pollution and growth, see Brock and Taylor (2005)

<sup>3</sup> In this paper the closed-form solution of the Pigouvian tax depends on the assumption of constant savings rate all along the optimal path. This can be ensured if capital depreciates fully each period which makes it a flow rather than a stock variable.

the stock of harmful pollution  $P_t$ , which depreciates at rate  $\theta \geq 0$ . In our model the pollution accumulation process differs from the usual assumption of instantaneous emissions diffusion. We realistically assume that emissions slowly diffuse into the stock of harmful pollution, reflecting the inherent time lag of the climate system.

Take first the usual assumption of instantaneous emissions diffusion and let  $Z_t = \phi R_t$  be the flow of emissions that effectively adds to the stock of pollution according to  $\dot{P}_t = Z_t + \theta(\bar{P} - P_t)$ ;  $P_0$  given. A dot denotes the time derivative. Thus, at each date  $t$ , the stock of carbon increases by the flow of emissions,  $Z_t$ , and decreases by the natural removal  $\theta(\bar{P} - P_t)$ ; with  $\bar{P} \in (0, P_0]$  we proxy the long-run level of carbon concentration when  $\theta \neq 0$ ; we set it to  $P_0$  without loss of generality.<sup>4</sup>

Let us now include a distributed time lag formulation for the flow of emissions, i.e.  $Z_t \equiv \int_{-\infty}^t \kappa e^{-\kappa(t-s)} \phi R_s ds$ . Variable  $Z_t$  represents now the history of man-made emissions that effectively adds to the stock of pollution with a lag. Parameter  $\kappa \geq 0$  is the speed of this diffusion process; limiting cases are instantaneous diffusion ( $\kappa \rightarrow \infty$ ), i.e.  $Z_t = \phi R_t$ , and no diffusion ( $\kappa \rightarrow 0$ ), i.e.  $Z_t = 0$  at all times. We show in Appendix A that, given  $P_0 \geq 0$  and  $Z_0 = 0$ , the dynamic evolution of the climate system follows

$$\begin{cases} \dot{P}_t = Z_t + \theta(\bar{P} - P_t), \\ \dot{Z}_t = \kappa(\phi R_t - Z_t). \end{cases} \quad (1)$$

From the solution of (1), the marginal increase in the stock of carbon in period  $\nu$  from a marginal unit of emissions in period  $t$ , i.e. its impulse response, reads:

$$\frac{dP_\nu}{d(\phi R_t)} \equiv f_{\nu t} = \kappa \frac{e^{-\theta(\nu-t)} - e^{-\kappa(\nu-t)}}{\kappa - \theta} > 0, \quad \text{for all } \nu \geq t. \quad (2)$$

The impulse response function (2) is hump-shaped with a peak at  $\nu - t = \ln(\kappa/\theta)/(\kappa - \theta)$ ; see Figure 1. The maximum emissions-damage response reads  $(\kappa/\theta)^{\frac{1}{1-\kappa/\theta}}$  and is therefore a monotonically increasing concave function in  $\kappa/\theta$ , which converges to unity as  $\kappa/\theta$  grows to infinity. For a constant speed of emissions diffusion  $\kappa$ , a decrease in the decay rate  $\theta$  increases the maximum emissions-damage response and shifts it towards the future; see Figure 1a. Conversely, for constant decay rate  $\theta$ , an increase in  $\kappa$  increases the emissions-damage peak, shifts it towards the present but puts a relatively larger damaging impact on the short run in comparison to the long run; see Figure 1b.

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<sup>4</sup>We thereby assume that even if carbon emissions seize, the stock of harmful pollution will not decrease further than its initial level; see for example Grimaud and Rouge (2014) for an equivalent treatment.

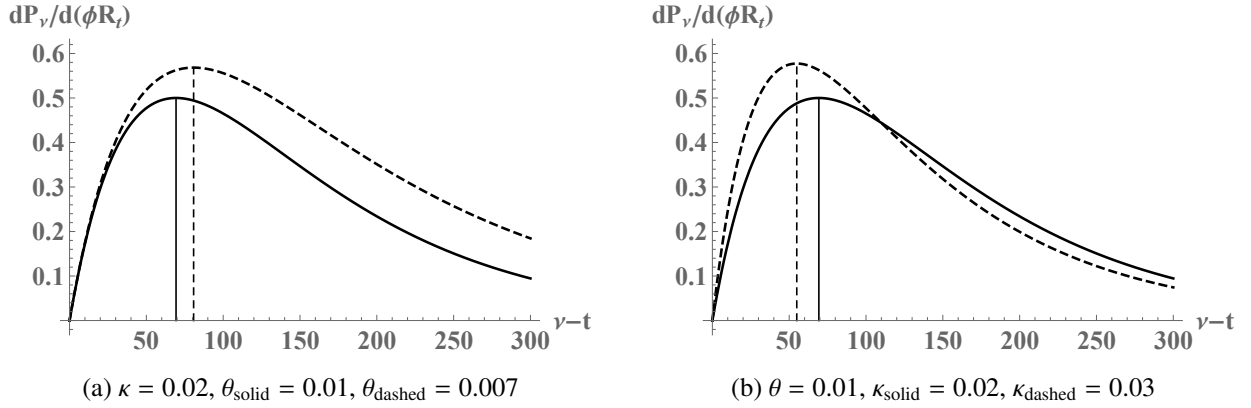


Figure 1: Marginal increase in the stock of carbon following a marginal increase in emissions for different values of  $\kappa$  and  $\theta$ . Left for constant  $\kappa$ , right for constant  $\theta$ .

Gerlagh and Liski (2012) arrive at the discrete-time equivalent expression of (2). Using their values for the parameters of the climate system,  $\kappa = 0.02$  and  $\theta = 0.01$ , we confirm their result of a peak emissions-damage response of about 70 years. It is important to note that the speed of emissions diffusion has a dual effect on the marginal damages from the extraction and use of the polluting resource: a level effect on the magnitude of marginal damages and a delay / discounting effect. This can be seen as follows.

For very small time intervals this equation can be approximated by  $f_{vt} \approx \kappa(\nu - t)$ ; a marginal unit of resources extracted and burned increases harmful pollution within this small time interval by  $\kappa$ . It follows that in the very short run our specification has a relatively larger damaging impact of a marginal increase in emissions the larger  $\kappa$  is. For a given decay rate  $\theta$ , this will lead to a higher peak of the pollution response, closer to the current date. For longer time periods, the marginal increase of harmful emissions will result in a marginal increase in the stock of pollution determined by the adjustment term  $\frac{e^{-\theta(\nu-t)} - e^{-\kappa(\nu-t)}}{\kappa - \theta}$ , which accounts for pollution decay and the slow diffusion of emissions into the stock of harmful pollution. If pollution decay is disregarded, i.e.  $\theta = 0$ , the damage response accounts only for the probability that a marginal unit of emissions emitted in period  $t$  has reached the stock of pollution in period  $\nu$ ,  $f_{vt} = 1 - e^{-\kappa(\nu-t)}$ . These effects of  $\kappa$  have a big impact on the transition of the economy towards its steady state, which we study here.<sup>5</sup>

## 2.2. Aggregate economy

Markets are fully competitive. Production in each period  $t$  is based on constant returns to scale technologies and on two inputs: capital  $K_t$ , and polluting non-renewable resources  $R_t$ . The stock of capital is a generic reproducible factor in this economy that includes both physical and human capital; we will call it “capital” for convenience. As proposed by Stern (2013) and Bretschger and Valente (2011) physical capital is exposed to climate disasters. Natural events like floods, droughts, wildfires, and extreme temperatures caused by anthropogenic climate change can destroy buildings, equipment, crops, roads, and public infrastructure; this puts a natural drag on economic growth, since part of the economic resources have to be allocated to fixing these damages. Conversely, there are durable forms of capital like human skills that

<sup>5</sup>Our emissions-damage response does not allow for a thick-tailed concentration of the carbon stock, where some part of the stock stays in the atmosphere for thousands of years, as proposed by natural scientists. We could have captured such a behavior with a richer “multi-box” climate module as in Gerlagh and Liski (2012). This added complexity would however not alter the results of the present paper in any fundamental way while it would make the model less tractable.

cannot be depleted; we will therefore allow for only a part  $\eta$  of the capital stock to be affected by harmful pollution.<sup>6</sup>

Following Rebelo (1991) there are two production sectors in this economy: the consumer goods sector and the investment sector. The consumer good is the numeraire, and is produced with both inputs; the investment good sector is assumed to be capital intensive and uses only capital. The economy features the following aggregate production functions for the consumption good  $Y_t$ , and the investment good  $I_t$ ,

$$Y_t = A(\epsilon_t K_t)^\alpha R_t^{1-\alpha}, \quad (3)$$

$$I_t = B(1 - \epsilon_t)K_t, \quad (4)$$

where  $\epsilon_t \equiv K_{Y_t}/K_t \in [0, 1]$  is the aggregate fraction of capital devoted to the consumption good, and  $R_t$  the total demand for the non-renewable resource. Investment leads to capital accumulation according to

$$\dot{K}_t = I_t - D(P_t)\eta K_t, \quad (5)$$

with  $K_0 > 0$ , and  $\eta$  the share of non-durable capital, which we assume to be constant. The part of capital that is exposed to wear decays according to the damage function  $D(P_t)$  due to natural depreciation and higher pollution levels. Costless resource extraction  $R_t$  depletes the existing stock of the non-renewable energy resource  $S_t$  (with  $S_0 > 0$ ), according to the standard law of motion and the stock constraint

$$\dot{S}_t = -R_t, \quad \int_0^\infty R_t dt \leq S_0. \quad (6)$$

Finally, the economy admits a representative household with preferences  $U(C_t)$  that owns all the financial wealth, i.e. capital and energy resources. In the general CRRA form we have  $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$  while with  $\sigma = 1$  we get the logarithmic form, i.e.  $U(C_t) = \log(C_t)$ ; parameter  $\sigma$  is the intertemporal elasticity of substitution.

**Assumption 1.** *The utility function is logarithmic, i.e.  $U(C_t) = \log(C_t)$ .*

The assumption of log-utility ( $\sigma = 1$ ) is the most widely used case in the literature of endogenous growth with polluting non-renewable resources, as it allows for closed-form solutions and a full characterization of the macroeconomic model features. We will also use it in the basic approach so that we can directly compare our results with the relevant literature. As an extension, we treat and discuss the case of  $\sigma \neq 1$  in section 6. In particular, we will derive how the interplay between the time lag in emissions diffusion and the substitution and income effect that arise in the non-log-utility case affect the dynamics and the steady state of the economy. We will show that when  $\sigma = 1$  and capital damages are linear to the stock of pollution, the optimal emissions tax rate grows with consumption while this condition is asymptotically reached with  $\sigma \neq 1$ .

**Assumption 2.** *Capital damages are linear to the level of pollution, i.e.  $D(P_t) = \delta + \chi P_t$ .*

Parameter  $\delta$  is the natural depreciation of the capital stock and  $\chi$  the damage sensitivity to pollution; see Ikefuji and Horii (2012) for a similar specification.

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<sup>6</sup>In a previous version, in order to capture this idea, we differentiated between a physical and a knowledge capital stock, the latter being unaffected by pollution. Physical capital was accumulated as in the present version, while we assumed that creation of new knowledge was knowledge intensive and used only itself as an input. However the widely-used Cobb-Douglas specification for (3), implying constant expenditure shares among inputs, makes the use of two differentiated stocks inessential; the results are qualitatively identical as in the current approach, while the model is now more tractable. For an endogenous growth framework with knowledge capital (but no physical capital) and flow pollution directly affecting utility see Grimaud and Rouge (2005).



### 2.3. Discussion about the model

In equilibrium, aggregate demand for the consumption good must equal its total supply, i.e.  $C_t = Y_t$ . The resource stock is finite and extraction and use of the non-renewable resource has to stop in finite or infinite time. This puts an upper bound on pollution and capital damages under all assumptions regarding the decay of the pollution stock. Furthermore, due to the specification of the production function (3), and the fact that damages due to pollution accumulation are bounded, the resource stock is essential in the sense that an additional unit of resources used in the production of the consumption good is always welfare enhancing. Accordingly, resource extraction will be positive in each time period and the resource stock will only be asymptotically depleted so that (6) holds with equality; see for example Daubanes and Grimaud (2010) for a similar argumentation. Following the same logic, the share of capital allocated to the consumption good sector has to obey  $\epsilon_t \in (0, 1)$  for all  $t \geq 0$ ; formal proofs are given in Appendix C. In the face of pollution, the economy at hand is always in transition. The only possible steady state is the one where resources are asymptotically depleted and pollution asymptotically reaches its steady state value,  $P_\infty = P_0 + \phi S_0$ , if  $\theta = 0$ , or  $P_\infty = \bar{P} = P_0$ , if  $\theta > 0$ . When that happens the growth rate of resource extraction,  $g_{Rt} \equiv \dot{R}_t/R_t$ , and the share  $\epsilon_t$  must have also reached their steady state values.<sup>7</sup> It follows from our specifications for the consumption good and capital accumulation that in the steady state the economy asymptotically reaches a balanced growth path which can be defined as follows:

**Definition 1.** *An equilibrium path is an asymptotic balanced growth path, if capital allocation and the growth rate of resource extraction are asymptotically constant, i.e.  $\lim_{t \rightarrow \infty} \epsilon_t = \epsilon_\infty$ , and  $\lim_{t \rightarrow \infty} \dot{R}_t/R_t = g_{R\infty}$ ; then  $\lim_{t \rightarrow \infty} \dot{K}_t/K_t = g_{K\infty}$  and  $\lim_{t \rightarrow \infty} \dot{C}_t/C_t = g_{C\infty}$ , asymptotically constant.*

Below we solve the planning problem and characterize the social cost of carbon. In section 4 we show that this is the first-best carbon tax that optimally corrects for the externality.

### 3. Social Optimum

The social planner chooses the fraction of capital allocated to the consumption good,  $\epsilon_t$ , and the resource extraction,  $R_t$ , in order to maximize  $\int_0^\infty U(C_t)e^{-\rho t} dt$  with  $C_t = Y_t$ , subject to equations (1), (3)-(6). Let  $\lambda_{Ct}$ ,  $\lambda_{St}$ ,  $\lambda_{Zt}$  be respectively the shadow prices for the consumption good  $C_t$ , the stock of the non-renewable resource  $S_t$ , and the history of lagged emissions  $Z_t$ . The first-order condition for resource extraction follows:

$$(1 - \alpha) \frac{C_t}{R_t} = \frac{\lambda_{St}}{\lambda_{Ct}} - \phi \kappa \frac{\lambda_{Zt}}{\lambda_{Ct}}. \quad (7)$$

According to equation (7), in each point of time, the marginal benefit from extracting and using the resource (left-hand-side) equals the marginal cost of resource use (right-hand-side), in terms of the consumption good. The cost consists of the scarcity cost of the exhaustible resource,  $\lambda_{St}/\lambda_{Ct}$ , i.e. its producer price in a competitive market, and of the social cost of carbon (SCC), i.e. the marginal externality damage of an additional unit of emissions,  $X_t \equiv -\phi \kappa \lambda_{Zt}/\lambda_{Ct}$ .  $X_t$  captures the externality from carbon emissions and as we show in section 4 is equal to the optimal Pigouvian tax. We prove in Appendix B that it can be written as

$$X_t = C_t \frac{\alpha \eta \phi}{\rho} \kappa \int_t^\infty \left[ \int_s^\infty D'(P_v) \left( \frac{\bar{\epsilon}}{\epsilon_v} \right) \left( \frac{C_t}{C_v} \right)^{\sigma-1} e^{-(\rho+\theta)(v-s)} dv \right] e^{-(\rho+\kappa)(s-t)} ds, \quad (8)$$

<sup>7</sup> In general we define  $g_V \equiv \dot{V}/V$  the growth rate of variable  $V$ .

with  $\bar{\epsilon} = \rho/B$ . The intuitive explanation of (8) is the following. The remaining portion in year  $v \geq s$ , after decay, of a marginal unit of emissions from year  $t$ , that has reached the stock of pollution in year  $s \geq t$ , has a negative impact in all years  $v \geq t$ . The first term inside the square brackets is the marginal damage of pollution on capital accumulation,  $D'(P_v)$ . The second term comes from the utility denominated shadow price of capital and is responsible for allocating capital between the consumption and the investment sector, while the third term reflects preferences of agents regarding intertemporal consumption smoothing. The exponential terms reflect the delay/decay structure of the climate system:  $e^{-\theta(v-s)}$  is the share of emissions remaining in year  $v$  from emissions that reached the stock of pollution in year  $s$ , while  $\kappa e^{-\kappa(s-t)}$  accounts for the slow adjustment of the stock of pollution from the marginal unit emitted in year  $t$ .

The cost of the externality is greater, when the following are larger: the emissions intensity parameter  $\phi$ , the part of non-durable capital  $\eta$ , and the share of capital in the production of the consumption good  $\alpha$ . It is also greater when the following are smaller: the discount rate  $\rho$ , the pollution decay  $\theta$ , and, *ceteris paribus*, the path of capital allocated to consumption  $\{\epsilon_v\}_t^\infty$ , since higher investment translates to a larger stock of capital in subsequent periods, creating larger damaging potential in the future.<sup>8</sup> Finally, we point out the effect of the slow emissions diffusion as a suppressing factor on the magnitude of marginal damages by the multiplicative term in the beginning of (8). As discussed in section 2.1, the speed of emissions diffusion has a dual effect on the marginal damages from the extraction and use of the polluting resource: i) a level effect on the magnitude of marginal damages and ii) a delay effect.

At this point, a direct comparison of our results to the literature seems appropriate. A similar expression to (8) has been found in van den Bijgaart et al. (2016). There the authors consider a general neoclassical economy, with climate dynamics similar to ours, where climate change destroys part of the final output. We show instead that similar results can be obtained in an endogenous growth framework, and in the case where pollution harms capital accumulation. Moreover, in several models of growth with polluting non-renewable resources the marginal externality damage is a linear function of the consumption good all along the optimal path, irrespective of whether lags in emissions dissemination are considered or not; see for example Gerlagh and Liski (2012), Golosov et al. (2014).

The linearity of the marginal externality damage in the consumption good stems from three factors: first, from the log-utility assumption; second, from the damage specification; third, from a constant savings rate at all times. While in the case of the general neoclassical economy one needs to impose the last condition (by assuming full capital depreciation in each period), as in the aforementioned contributions, in the case of endogenous growth, as in Grimaud and Rouge (2014) or in the present paper, this condition is immediately satisfied with logarithmic utility: take (8) with Assumption 1, i.e.  $\sigma = 1$ . We show in Appendix B that in this case  $\epsilon_t = \bar{\epsilon}$  at all times, i.e. there will be a constant fraction of capital allocated to investment; the equivalent of the constant savings rate in the neoclassical economy. Equation (8) now reads

$$X_t = C_t \frac{\alpha \eta \phi}{\rho} \kappa \int_t^\infty \left( \int_s^\infty D'(P_v) e^{-(\rho+\theta)(v-s)} dv \right) e^{-(\rho+\kappa)(s-t)} ds.$$

The linearity of  $X_t$  in  $C_t$  is granted if  $D(P_t)$  is also linear in pollution. Applying Assumption 2 readily leads to the following proposition:

**Proposition 1.** *Given Assumptions 1 and 2, the marginal externality damage of emissions is proportional to the consumption good and given by*

$$X_t = \tilde{X} C_t \quad \text{with} \quad \tilde{X} = \kappa \frac{\alpha \eta \phi \chi}{\rho(\rho + \theta)(\rho + \kappa)}; \quad (9)$$

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<sup>8</sup>We will discuss the effect of  $\sigma \neq 1$  on the SCC, and thus on the Pigouvian tax, in section 6.

$\tilde{X}$  is an increasing and concave function of  $\kappa$ , independent of time.

**Proof** See last paragraph above. ■

In the log-utility case, with linear and separable damages due to climate change in the utility function, as in Grimaud and Rouge (2014), or multiplicative exponential damages in the production function, as in Gerlagh and Liski (2012) and Golosov et al. (2014), or even linear damages in capital accumulation, as in the present approach, the social cost of the externality,  $X_t$ , is a linear function of the consumer good. In addition, we find that the fraction  $\tilde{X}$  is increasing in the speed of adjustment between emissions and pollution,  $\kappa$ , in a concave way reaching its upper limit,  $\frac{\alpha\eta\phi\chi}{\rho(\rho+\theta)}$ , as  $\kappa \rightarrow \infty$ .

Below we proceed by characterizing the decentralized equilibrium. We show that  $X_t$  is the Pigouvian tax needed to optimally correct for the externality, and study how the economy responds to more general taxation policies.

## 4. Decentralized Equilibrium

### 4.1. Firms

Each sector in the economy is populated by a unit mass of competitive firms  $j \in [0, 1]$ . Specifically, the production of consumption good  $Y_{jt}$  uses capital  $K_{Yjt}$ , and resources  $R_{jt}$ , according to  $Y_{jt} = AK_{Yjt}^\alpha R_{jt}^{1-\alpha}$ . The production of the investment good  $I_{jt}$  reads  $I_{jt} = BK_{Ijt}$ .  $A, B$  are productivity parameters. A producer of consumer good  $Y_{jt}$  solves

$$\max_{K_{Yjt}, R_{jt}} \{AK_{Yjt}^\alpha R_{jt}^{1-\alpha} - p_{Kt}K_{Yjt} - (p_{Rt} + \tau_t)R_{jt}\},$$

while one in the investment good sector solves

$$\max_{K_{Ijt}} \{p_{It}BK_{Ijt} - p_{Kt}K_{Ijt}\},$$

with  $p_{Kt}$  the rental price of capital,  $p_{It}$  the price of investment,  $p_{Rt}$  the producer price of the non-renewable resource and  $\tau_t$  a per-unit tax on resource extraction. Because production has constant returns to scale, firms face identical factor input ratios. Hence, the economy admits a representative firm active in both sectors with  $Y_t \equiv \int_0^1 Y_{jt}dj$  for total production,  $K_t \equiv \int_0^1 (K_{Yjt} + K_{Ijt})dj$  for the total stock of capital demanded, and  $\epsilon_t = K_{Yt}/K_t$ , the aggregate fraction of capital allocated to the consumption good. The first order conditions of these maximizations give the demand functions for non-renewable resources and capital in the consumption good sector, and a no-arbitrage condition which equates returns from the two usages of capital in this economy, i.e. in the consumption good sector and in the investment sector, namely,

$$p_{Rt} + \tau_t = (1 - \alpha)\frac{Y_t}{R_t}, \quad p_{Kt} = \alpha\frac{Y_t}{\epsilon_t K_t}, \quad p_{Kt} = p_{It}B. \quad (10)$$

### 4.2. Households

There is a continuum of infinitely lived households  $i \in [0, 1]$  that have the option to allocate their income to consumption, through the consumption good sector, or to additional capital formation, through the investment sector. The representative household  $i$  owns a share of the stock of energy resources  $S_{it}$ , and capital,  $K_{it}$ . In each time period a share of resources  $R_{it}$  is extracted and sold to firms at a price  $p_{Rt}$ . Furthermore,  $K_{it}$  is rented to firms at prices  $p_{Kt}$ . With  $T_t$  denoting lump-sum transfers, individual

income amounts to  $p_{Kt}K_{it} + p_{Rt}R_{it} + T_t$  while expenditures equal  $C_{it} + p_{It}H_{it}$ , with  $C_{it}$  denoting the flow of consumption and  $H_{it}$  reflecting the purchase of additional capital through the investment sector at price  $p_{It}$ . Capital and resource stocks evolve according to

$$\dot{K}_{it} = H_{it}^K - D(P_t)\eta K_{it}, \quad \dot{S}_{it} = -R_{it}, \quad (11)$$

while income equals expenditure, so that the income balance reads

$$p_{Kt}K_{it} + p_{Rt}R_{it} + T_t = C_{it} + p_{It}H_{it}. \quad (12)$$

Differentiating the household's assets,  $a_{it} = p_{It}K_{it} + p_{Rt}S_{it}$  with respect to time, using (11), (12), and the fact that  $p_{Kt} = p_{It}B$ , yields the household's dynamic budget constraint

$$\frac{\dot{a}_{it}}{a_{it}} = \beta_{it}^S \frac{\dot{p}_{Rt}}{p_{Rt}} + (1 - \beta_{it}^S) \left[ \frac{\dot{p}_{Kt}}{p_{Kt}} + B - \eta D(P_t) \right] - \frac{C_{it}}{a_{it}} + \frac{T_t}{a_{it}}, \quad (13)$$

with  $\beta_{it}^S \equiv p_{Rt}S_{it}/a_{it}$ , the share of the individual's resource wealth in her total assets. The household's objective is to choose the time path of consumption and share  $\beta_{it}^S$  which maximize its lifetime utility

$$\int_0^\infty U(C_{it})e^{-\rho t} dt,$$

subject to the budget constraint (13). In the general CRRA form we have  $U(C_{it}) = \frac{C_{it}^{1-\sigma}}{1-\sigma}$  while with  $\sigma = 1$  we get the logarithmic form, i.e.  $U(C_{it}) = \log(C_{it})$ . From combining the first order conditions of household optimization we find

$$\sigma \frac{\dot{C}_{it}}{C_{it}} = r_t - \rho, \quad (14)$$

$$\frac{\dot{p}_{Rt}}{p_{Rt}} = r_t, \quad (15)$$

$$\frac{\dot{p}_{Kt}}{p_{Kt}} + B - D(P_t)\eta = r_t. \quad (16)$$

These are the Keynes-Ramsey rule for consumption growth, the Hotelling rule for resource price development, and the return on investing in capital formation, with  $r_t$  being the economy-wide interest rate. By equating (15) with (16) we see that both assets, i.e. non-renewable resources and capital, should yield equal returns. The optimization is complemented by the appropriate transversality condition, reading

$$\lim_{t \rightarrow \infty} a_{it} C_{it}^{-\sigma} e^{-\rho t} = \lim_{t \rightarrow \infty} \left( \frac{p_{Kt}}{B} K_{it} + p_{Rt} S_{it} \right) C_{it}^{-\sigma} e^{-\rho t} = 0. \quad (17)$$

Finally we need to impose the restriction that  $\chi$  satisfies  $\alpha(B - \eta(\delta + \chi P_\infty)) > \rho$  so that households have enough incentives to invest in capital formation.

#### 4.3. Equilibrium

In equilibrium total demand for the consumption good must equal its total supply, i.e.  $C_t = \int_0^1 C_{it} di = Y_t$ . Given the initial values  $K_0, S_0, P_0$  and the dynamic evolution of the tax rate, the dynamics of the climate system (1), capital accumulation (5), resource depletion (6), the first order conditions for the representative firm (10), the aggregate version of the Keynes-Ramsey rule (14), the Hotelling rule for the price evolution of the non-renewable resource (15), the return on investment in capital formation (16), and the transversality condition (17), completely characterize the dynamic behavior of the decentralized economy.

#### 4.4. The Pigouvian tax

In section 3 we characterized the socially optimal solution and derived the expression for the social cost of carbon,  $X_t$ . Here we show that this is in fact the Pigouvian tax in the decentralized equilibrium that produces the first-best allocation.

As shown in Appendix E, with  $\sigma = 1$ , the capital share  $\epsilon_t$  immediately jumps to its optimal steady state value  $\bar{\epsilon} = \rho/B$  also in the decentralized case. By comparing the social planner's optimality condition (7) with its equivalent from (10), using  $C_t = Y_t$ , it is straightforward to see that the resource extraction will follow its optimal path if the producer's price for the non-renewable resource equals its scarcity rent ( $p_{Rt} = \lambda_{St}/\lambda_{Ct}$ ), and if the per-unit carbon tax equals the marginal externality damage of emissions found in (9) ( $\tau_t = X_t$ ). This is the optimal tax which we denote by  $\tau_t^o$ .

Since  $\tau_t^o \equiv X_t$ , when Assumptions 1 and 2 are satisfied, the optimal tax is a constant fraction of the consumption good. The important point about this result is that it provides appropriate incentives to the economy to stretch the path of resource extraction. To be more precise, the per-unit tax that postpones extraction has to grow at a slower rate than the price of the non-renewable resource. Then, the unit price paid for the resource by consumers increases less rapidly than the price received by producers, which grows at the market's interest rate, giving them the incentive to postpone extraction: with  $\sigma = 1$  the price received by producers  $p_{Rt}$ , grows at the rate  $r_t$  (from (15)) while  $\tau_t^o$  grows with consumption, i.e. at  $r_t - \rho$  (from the aggregate version of (14)).<sup>9</sup>

Furthermore, it is a known result from the theory of non-renewable resource taxation that any term in the optimal per-unit tax that grows with the interest rate has no effect on the extracting behavior of the economy, suggesting that there is an infinite number of optimal taxes that give the same resource extraction incentives; see Dasgupta and Heal (1979), and Gaudet and Lasserre (2013).<sup>10</sup> We show in Appendix F that this is also the case here.

#### 4.5. Response to taxation

In light of the previous discussion, we will only study taxation policies proportional to consumption according to the following assumption:

**Assumption 3.** *All taxes considered are proportional to consumption:  $\tau_t = \tilde{\tau}C_t$ , with  $\tilde{\tau}$  constant.*

**Proposition 2.** *Suppose that Assumptions 1, 2, and 3 apply. Then in a decentralized equilibrium,*

- (i) the fraction of consumption  $\tilde{\tau}$  determines the dynamics of resource extraction; a higher value stretches resource extraction to the future;  $\tilde{\tau} = 0$  (no tax) results in the fastest equilibrium extraction,*
- (ii) economic growth starts from a higher level, the higher  $\tilde{\tau}$  is, converging asymptotically to a positive constant  $g_{C\infty}$ , which is lower than initial growth and unaffected by policy.*

<sup>9</sup>In fact this implies an equivalence between the per-unit tax that grows with consumption, as in our case, and a decreasing ad-valorem tax, as usually proposed by growth models with polluting resources, e.g. Groth and Schou (2007). To see this, note that the consumer price for the resource is  $p_{R,t} + \tau_t^o = \pi_t p_{R,t}$ , with  $\pi_t \equiv 1 + \tau_t^o/p_{R,t}$ , i.e. a decreasing ad-valorem tax rate.

<sup>10</sup>Grimaud and Rouge (2014), however, using a model of endogenous growth with polluting non-renewable resources, show that in the presence of Carbon-Capture-and-Storage (CCS) activity the optimal tax rate is linear in consumption, yet unique. In the presence of a CCS activity agents should be indifferent between instruments as long as they have the same results in protecting from climate change, which uniquely pins down the optimal tax rate.

**Proof** (i) Following the same procedure as in Appendix F, the time path of resource extraction and its growth rate can be calculated to be only dependent on  $\tilde{\tau}$  as,

$$R_t(\tilde{\tau}) = \frac{1 - \alpha}{\tilde{\tau} \left[ 1 + e^{\rho t} \left( e^{\frac{S_0 \rho}{1 - \alpha} \tilde{\tau}} - 1 \right)^{-1} \right]} > 0, \quad (18)$$

$$g_{Rt}(\tilde{\tau}) = \frac{-\rho}{1 + e^{-\rho t} \left( e^{\frac{S_0 \rho}{1 - \alpha} \tilde{\tau}} - 1 \right)} < 0. \quad (19)$$

With our assumptions  $\tilde{\tau}$  is decisive for the dynamics of resource extraction: with tax,  $g_{Rt} > -\rho$  and  $g_{R\infty} = \lim_{t \rightarrow \infty} g_{Rt} = -\rho$ ; zero tax entails the fastest resource depletion,  $g_{Rt} = -\rho$  in all time periods. When environmental policy is implemented, resource extraction is stretched to the future:  $dg_{Rt}/d\tilde{\tau} > 0$  (i.e. a flatter resource extraction profile)

(ii) By log-differentiating (3), with  $\epsilon_t = \bar{\epsilon} = \rho/B$ , using (5), we get the growth rate of consumption in the decentralized equilibrium,  $g_{Ct} \equiv \dot{C}_t/C_t$ , as a function of  $\tilde{\tau}$

$$g_{Ct}(\tilde{\tau}) = \alpha [B - \rho - \eta D(P_t)] + (1 - \alpha)g_{Rt}(\tilde{\tau}). \quad (20)$$

With  $\epsilon$  jumping immediately to its optimal steady state and  $P_0$  given, differentiating (20) at  $t = 0$  w.r.t.  $\tilde{\tau}$  implies  $dg_{C0}/d\tilde{\tau} = (1 - \alpha)dg_{R0}/d\tilde{\tau} > 0$ , i.e. a higher value for  $\tilde{\tau}$  induces the economy to start from a higher level of economic growth, converging to the positive constant  $g_{C\infty} = \alpha [B - \rho - \eta D(P_\infty)] - (1 - \alpha)\rho$ . Furthermore, because  $P_\infty = P_0 + \phi S_0$ , if  $\theta = 0$ , or  $P_0$  if  $\theta > 0$ , and  $g_{R0} > -\rho$ , the steady state level of economic growth is always lower than initial growth. ■

Two things are worth noting here. First, resource extraction is independent of climate damages. In general since pollution affects capital accumulation and the interest rate, one would anticipate damages to affect the path of resource extraction. This is not the case in the present setup due to logarithmic preferences: consider for convenience the FOC for  $R_t$  in (10) with a given ad-valorem tax,  $\pi_t$ , i.e.  $(1 - \alpha)C_t/R_t = \pi_t p_{Rt}$ . Log-differentiating this expression using the log-differentiated version of the second FOC in (10) along with (5), and (14)-(16) leads to  $\sigma g_{Rt} = -(\rho + (1 + \alpha(\sigma - 1))g_{\pi t} + \alpha(\sigma - 1)(B - \eta D(P_t)))$ ; with  $\sigma \neq 1$  resource extraction responds to pollution. With  $\sigma = 1$ , however, we get  $g_{Rt} = -\rho - g_{\pi t}$ .<sup>11</sup> In general this result, as well as the fact that  $\epsilon$  jumps immediately to its steady state, is the outcome of the substitution and income effect that arise due to pollution exactly offsetting each other when  $\sigma = 1$ ; we study this in more detail in section 6.

Second, it sounds counter-intuitive that higher taxation induces the economy to start from a higher point of economic growth. However, according to result (i) of the proposition, it is the constant  $\tilde{\tau}$  that determines the extraction path. Thus, higher taxes that stretch resource extraction to the future impose a lower drag on growth in earlier periods.

<sup>11</sup>Note also that, consistent with the literature, there are infinite ad-valorem taxes with the same dynamics (decreasing at the same rate) but different levels that give the same incentives to postpone extraction; see Dasgupta and Heal (1979), Grimaud and Rouge (2005), Gaudet and Lasserre (2013).

## 5. Effect of climate dynamics on growth

The level of harmful pollution at each time period, with  $\bar{P} = P_0$ , in the general case with pollution decay reads<sup>12</sup>

$$P_t = P_0 + \int_0^t f_{ts} \phi R_s ds, \quad (21)$$

with  $f_{ts}$  from (2) and  $R_t$  from (18); see Appendix A. In the no-tax case the stock of pollution in each time period reads

$$P_t = P_0 + \kappa \rho \phi S_0 \left[ \frac{e^{-\rho t}}{(\theta - \rho)(\kappa - \rho)} - \frac{e^{-\theta t}}{(\kappa - \theta)(\theta - \rho)} + \frac{e^{-\kappa t}}{(\kappa - \theta)(\kappa - \rho)} \right].$$

Next we discuss the transition process towards the steady state in the decentralized equilibrium. This will depend on the speed of emissions diffusion  $\kappa$ , the decay rate  $\theta$ , and the policy  $\tau_t$ , since these parameters govern the dynamics of resource extraction, of the climate system, and in turn affect the growth rate of the economy. We will thoroughly study the case of  $\theta = 0$  as only this case allows for a rigorous mathematical analysis. We will then present the results graphically and their intuition based on the presentation of the climate system in section 2.1 and equation (21).

### 5.1. Effects of climate dynamics on the decentralized equilibrium

The effects of pollution decay in the market solution are given in the following proposition and can be studied graphically in Figures 2 and 3.

**Proposition 3.** *Suppose that Assumptions 1, 2, and 3 apply. Then in a decentralized equilibrium,*

- (i) *without pollution decay,  $\theta = 0$ , the growth rate of consumption converges monotonically from above towards the steady state,  $g_{C\infty}$ ; higher  $\kappa$  speeds up the transition process and results in lower economic growth at all times,*
- (ii) *with positive decay,  $\theta > 0$ , the growth rate of consumption converges towards the steady state,  $g_{C\infty}$ , in a non-monotonic way (i.e. in a U-shaped manner); higher  $\kappa$  leads to a lower minimum growth, which is shifted forward to the present, and in lower short-run but higher long-run economic growth.*

**Proof** (i) When pollution decay is disregarded,  $\theta = 0$ , pollution starts from  $P_0$  and monotonically reaches its higher steady state  $P_\infty = P_0 + \phi S_0$  when resources are asymptotically depleted, i.e.  $\dot{P}_t > 0$  and  $\lim_{t \rightarrow \infty} \dot{P}_t = 0$ . Moreover from (19),  $\dot{g}_{Rt} < 0$  and  $\lim_{t \rightarrow \infty} \dot{g}_{Rt} = 0$ . From (20) this leads to  $\dot{g}_{Ct} < 0$ , with  $\lim_{t \rightarrow \infty} \dot{g}_{Ct} = 0$ , i.e. growth follows a monotonic path towards its steady state. A higher speed of emissions diffusion under the same tax policy will not affect resource extraction, i.e.  $dR_t/d\kappa = dg_{Rt}/d\kappa = 0$ ; from (18), (19). Moreover with  $\theta = 0$ ,  $df_{ts}/d\kappa > 0$  and  $\lim_{t \rightarrow \infty} df_{ts}/d\kappa = 0$ ; from (2). The previous lead to  $dg_{Ct}/d\kappa < 0$  and  $\lim_{t \rightarrow \infty} dg_{Ct}/d\kappa = 0$ ; from (20).

(ii) When pollution decay is taken into account,  $\theta > 0$ , the pollution stock is hump-shaped starting and finishing at  $P_0$ . From (20), the growth rate of consumption will have an inverse hump shape, i.e. a U-shape.

<sup>12</sup>The complexity of the climate cycle does not allow for an explicit analytical solution. We can, however, approximate the solution, using any mathematical software, as an infinite sum of terms according to  $P_t = P_0 + \kappa \frac{1-\alpha}{\kappa-\theta} \tilde{\tau}^{-1} \sum_{n=0}^{\infty} \left( \frac{1}{1-e^{-\frac{\rho}{1-\alpha}t}} \right)^n \left( \frac{e^{-\kappa t} - e^{-\rho n t}}{\kappa + \rho n} - \frac{e^{-\theta t} - e^{-\rho n t}}{\theta + \rho n} \right)$ . The interested reader can validate this expression to get the qualitative features of our climate system.

We show in Appendix A that a higher  $\kappa$ , will lead, *ceteris paribus*, to a higher pollution peak which will be also brought closer to the present; moreover, it still holds that  $dR_t/d\kappa = dg_{Rt}/d\kappa = 0$ ; from (18), (19). From the last two points and equation (20) it follows that higher  $\kappa$  leads to a lower minimum growth, shifted forward to the present. ■

Proposition 3 can be understood intuitively by considering the cases for  $\theta$ : if there is no pollution decay,  $\theta = 0$ , higher speed of emissions diffusion,  $\kappa$ , will increase the marginal effect of emissions from all preceding periods on the current pollution level. Taking together the finiteness of the resource, this will speed up the transition process towards the lower steady state resulting in lower economic growth at all times. A lower  $\tilde{\tau}$  would have the same effect on growth: the lower the tax is, the lower the initial level of economic growth and the faster the non-renewable resource extraction in earlier periods; see Proposition 2. Resource depletion is brought forward to date and so does pollution accumulation and its harmful effect on growth. If  $\theta > 0$ , with higher  $\kappa$ , the marginal emissions-damage response will be relatively higher in the short-run but relatively lower in the long-run, and the stock of pollution will follow a lower trajectory towards its steady state in later time periods; see Appendix A. Since resource extraction will be unaffected when the  $\tilde{\tau}$  fraction stays constant, a higher speed of emissions diffusion,  $\kappa$ , will result in economic growth of the decentralized equilibrium being lower in the short run, reaching a minimum level when pollution peaks and converging at a higher rate towards  $g_{C\infty}$ . A higher  $\tilde{\tau}$  smooths out such behavior: resource extraction and use is stretched to the future, which, for the same decay structure, will lead to a lower peak of pollution occurring at a later time period. Accordingly, when  $\theta > 0$ , there are two counter-acting forces on growth from  $\kappa$  and  $\tilde{\tau}$ .

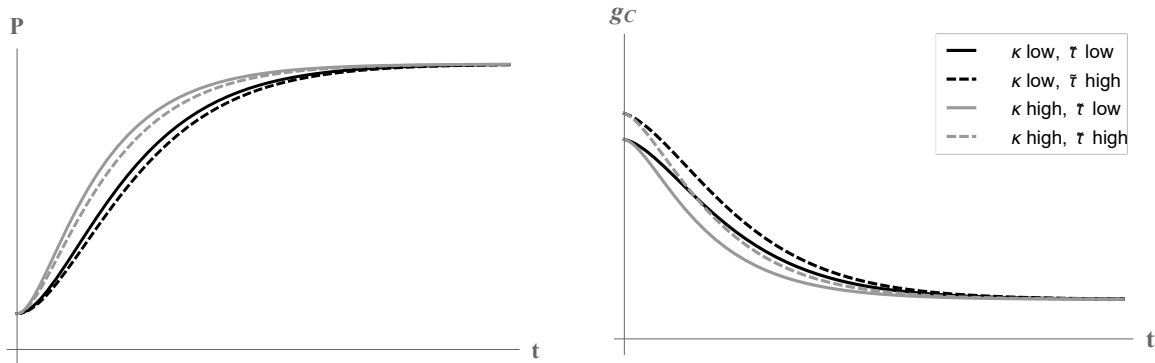


Figure 2: Pollution and consumption growth for different  $\tilde{\tau}$  and  $\kappa$ , ( $\theta = 0$  in both cases).



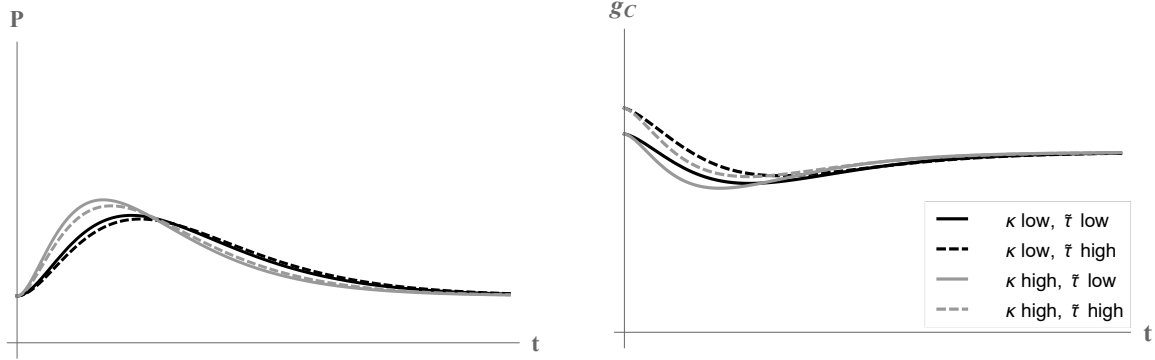


Figure 3: Pollution and consumption growth for different  $\tilde{\tau}$  and  $\kappa$ , ( $\theta > 0$  in both cases).

### 5.2. Effects of climate dynamics on the social optimum

Above we established that there are two counter-acting forces on growth arising from the speed of diffusion,  $\kappa$ , and the policy,  $\tilde{\tau}$ : when carbon decay is absent,  $\theta = 0$ , higher  $\kappa$  speeds up the transition process towards the lower steady state; higher  $\tilde{\tau}$  has a mitigating effect: it induces the economy to start from a higher level of economic growth and stretches resource extraction and pollution accumulation to the future which acts positively on growth. When  $\theta > 0$ , other things being equal, a higher speed of emissions diffusion induces a relatively higher marginal damaging impact in the short run relative to the long run and leads to a higher pollution peak, closer to the present. Economic growth that has a U-shape, reaches a lower minimum which is also brought forward. A higher  $\tilde{\tau}$  smooths out such a behavior. Since  $\tilde{\tau}^o = \tilde{X}$ , from Proposition 1,  $\tilde{\tau}^o$  is increasing in  $\kappa$ . Accordingly, the negative “direct” effect of a larger  $\kappa$  through its influence on climate dynamics, is mitigated by a positive “indirect” effect of  $\kappa$  through a higher optimal  $\tilde{\tau}^o$ . The previous can be summarized in the following proposition:

**Proposition 4.** *Given Assumptions 1 and 2, in a social optimum solution without pollution decay,  $\theta = 0$ , a larger  $\kappa$  steepens the growth profile of the economy; with  $\theta > 0$ , a larger  $\kappa$  creates ambiguous results on the timing and level of minimum economic growth.*

**Proof** Remember that  $\tilde{\tau}^o = \tilde{X}$ , given by (9). Differentiate the social optimum version of (20),  $g_C^o \equiv g_C(\tilde{\tau}^o)$ , w.r.t.  $\kappa$  to get

$$\frac{dg_C^o}{d\kappa} = -\alpha\eta\chi\phi \int_0^t \left[ \underbrace{\frac{df_{ts}}{d\kappa}}_{\text{direct}} R_s^o + f_{ts} \underbrace{\frac{dR_s^o}{d\tilde{\tau}^o} \frac{d\tilde{\tau}^o}{d\kappa}}_{\text{indirect}} \right] ds + (1 - \alpha) \underbrace{\frac{dg_{Rt}^o}{d\tilde{\tau}^o} \frac{d\tilde{\tau}^o}{d\kappa}}_{\text{indirect}}, \quad (22)$$

with  $f_{ts}$  from (2),  $R_s^o \equiv R_s(\tilde{\tau}^o)$ , from (18), and  $g_{Rt}^o \equiv g_{Rt}(\tilde{\tau}^o)$ , from (19). The two effects, direct and indirect, in the social optimum, tend to offset each other and in general create ambiguous results about the timing and the magnitude of minimum economic growth when  $\theta > 0$ . In the case of no pollution decay,  $\theta = 0$ , since pollution peaks only in the steady state, the direct effect of a larger  $\kappa$  is only about current emissions translating faster into pollution destroying capital. Hence, according to Proposition 2, the economy starts from a higher level of economic growth due to a higher optimal tax, and transitions faster to the lower steady

state. A larger  $\kappa$  then only steepens the growth profile of the economy. ■

For our discussion above on the impact of emissions diffusion,  $\kappa$ , on the transition of economic growth towards its steady state in the social optimum we provide Figure 4 as an illustration. Note also that for the same value of  $\kappa$ , the economy starts from a higher level of economic growth for  $\theta = 0$ . This is due to the discounting character of the pollution decay: from Proposition 1, other things being equal,  $\theta = 0$  results in a higher optimal tax than in the  $\theta > 0$  case because the discounted value of marginal damages is higher. This results in a higher optimal tax which according to Proposition 2 induces the economy to start from a higher level of economic growth.

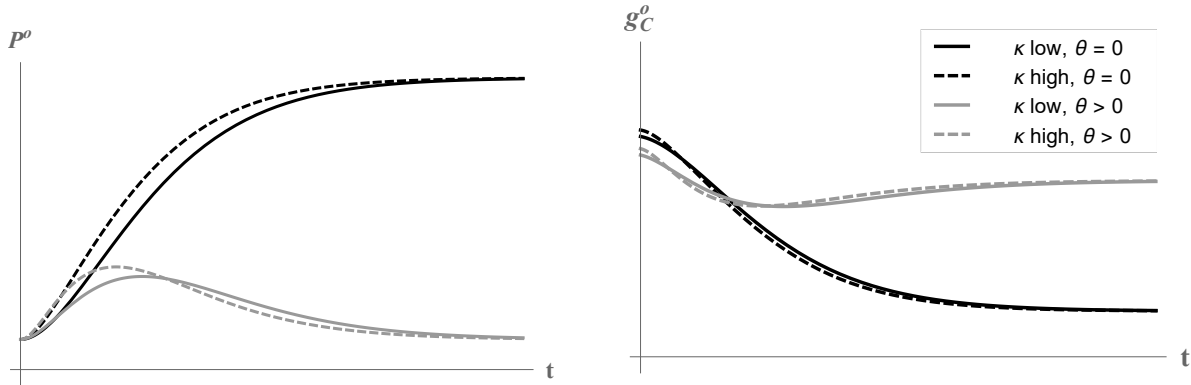


Figure 4: Optimal level of pollution and consumption growth for different values of  $\kappa$  in the  $\theta = 0$  and  $\theta > 0$  case.

Below, we discuss as an extension the case of CRRA utility and the interplay between the climate dynamics and the risk aversion of the representative household. Since pollution affects the return on investment in capital formation, with CRRA utility the substitution and income effect that arise do not necessarily cancel out. The Pigouvian tax rule does not anymore start off growing with consumption, even though it asymptotes to such behavior. Whether it starts off below or above the long term value depends on the intertemporal elasticity of substitution.

## 6. Non-Logarithmic CRRA Utility

As we established in section 3, the common feature of our model and those in the literature, of the optimal tax rate being a constant fraction of the consumption good all along the optimal path is a consequence of assuming log-utility function. In this section we will study the case of non-logarithmic utility. Since pollution affects the return on investment in capital formation, with a general CRRA utility the substitution and income effect that arise do not necessarily cancel out. We will see that the Pigouvian tax rule does not anymore start off growing with consumption, even though it asymptotes to such behavior. Whether it starts off above or below its steady state value will depend on the intertemporal elasticity of substitution.

In the steady state where resources have been asymptotically depleted, and the share  $\epsilon$  has already reached its steady state value and consumption grows at a constant rate, the optimal tax rate asymptotically becomes a constant fraction of consumption according to  $\lim_{t \rightarrow \infty} \tau_t^o / C_t = \tilde{\tau}^o$ , with

$$\tilde{\tau}^o = \kappa \frac{\alpha \eta \chi \phi}{\left[ \frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma} \right] \left[ \frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma} + \theta \right] \left[ \frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma} + \kappa \right]}, \quad (23)$$

with  $\Theta_\infty = B - \eta(\delta + \chi P_\infty)$ , and  $P_\infty = P_0 + \phi S_0$ , if  $\theta = 0$ , or  $P_\infty = P_0$ , if  $\theta > 0$ ; see Appendix G.

The assumption of non-logarithmic utility does not allow for further analytical solutions; hence, we will confine ourselves to numerical simulations. To this end, we can rewrite the dynamic system of the social planner in variables which are asymptotically constant on a balanced growth path and then linearize the model in the proximity to the steady state. Our calculation procedure is explained in detail in the Appendix G, while here we present only the main results of our simulation.

We consider a simplified version of the basic model without pollution decay for simplicity, i.e.  $\theta = 0$ . Pollution starts from  $P_0$ , asymptotically reaching  $P_0 + \phi S_0$  when resources have been depleted. From the no-arbitrage condition (16) we see that climate change affects the interest rate of the economy. This in principle creates a counteracting substitution and income effect. By combining the budget constraint (13), the Hotelling rule (15), and (16), we get the usual household budget constraint as,  $\dot{a}_t = r_t a_t - c_t + T_t$ . Let's think of an average interest rate between times 0 and  $t$  as  $\bar{r}_t = (1/t) \int_0^t r_s ds$ . The propensity to consume out of wealth is determined from<sup>13</sup>

$$\int_0^\infty e^{-(\bar{r}_t(\sigma-1)/\sigma + \rho/\sigma)t} dt. \quad (24)$$

A decreasing average interest rate due to pollution accumulation makes future consumption increasingly expensive compared to consumption today, motivating households to shift consumption from future to the present, i.e. an intertemporal substitution effect. This results in a falling capital share  $\epsilon_t$ . On the other hand, agents experience a decreasing interest rate income which tends to reduce consumption levels in all periods. In the latter case, capital allocation in the investment sector is decreasing, indicating an increasing share  $\epsilon_t$ . Which effect dominates will depend on the intertemporal elasticity of substitution,  $1/\sigma$ . From (24), if  $\sigma > 1$ , the propensity to consume out of wealth is increasing with falling  $\bar{r}_t$ , i.e. the substitution effect dominates. If  $\sigma < 1$  the propensity to consume out of wealth decreases with falling  $\bar{r}_t$ , i.e. the income effect dominates. If  $\sigma = 1$  they both cancel out and the shares jump to their steady state values as in sections 3 and 4. A slow diffusion of emissions into the stock of harmful pollution tends to mitigate these effects: if the full effects of pollution on capital accumulation appear with a time lag, the reduction in the interest rate is purely delayed.

The same reasoning can be applied to the demand for the non-renewable resource and by extension to the carbon tax rate. Because of the Cobb-Douglas specification of the consumption good, indicating constant expenditure shares, a forward shift of consumption, for  $\sigma > 1$ , will result in a relatively higher demand for the non-renewable resource in earlier time periods, disregarding its scarcity. The social planner will then have to set a low  $\tilde{\tau}_t^\rho$  which is increasing as  $\epsilon_t$  falls. Following (8), a higher  $\kappa$  will have a magnifying effect on the net present value of marginal damages so the tax rate will be shifted upwards. In the appendix we solve for the socially optimal allocation when  $\sigma \neq 1$ . The model is then linearized and solved computationally. The choice of the values for the parameters and the initial conditions is explained in Appendix G, while Figure 5 provides the results to illustrate our previous discussion for the standard case of  $\sigma > 1$ , as commonly used in the literature of endogenous growth; see Ikefuji and Horii (2012). Finally, as explained in Appendix A, the speed of emissions diffusion is the reciprocal of the mean time lag. We choose a low value to reflect a time lag of 50 years, i.e.  $\kappa = 0.02$ , and a high value to reflect a time lag of 25 years, i.e.

<sup>13</sup>See, Barro and Sala-i-Martin (2003), Ch. 2.1.

$\kappa = 0.04$ ; see van den Bijgaart et al. (2016).<sup>14</sup>

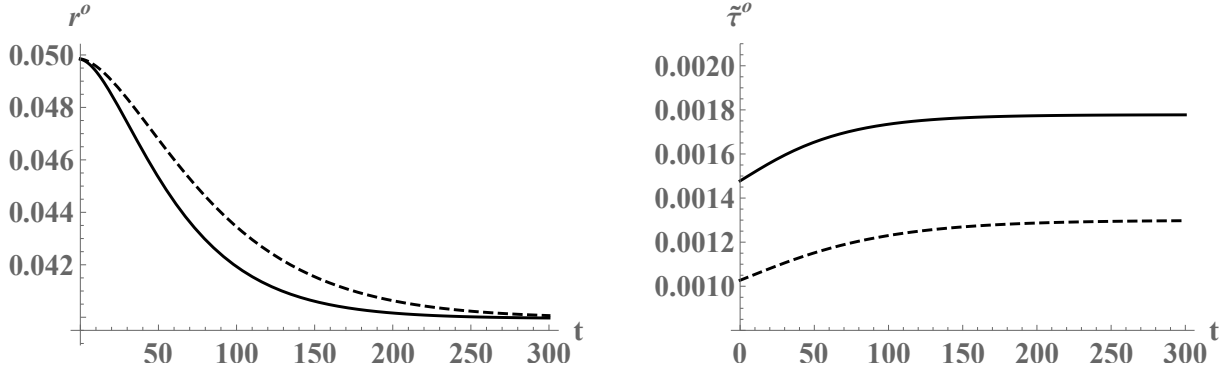


Figure 5:  $t_0 = 2010$ ,  $\rho = 0.015$ ,  $\sigma = 1.5$ ,  $\alpha = 0.9$ ,  $\theta = 0$ ,  $\delta = 0.05$ ,  $B = 0.106$ ,  $\chi = 1.7 \times 10^6$  \$/GtC,  $\phi = 1$ ,  $\eta = 1$ ,  $\kappa_{\text{low}} = 0.02$  (dashed),  $\kappa_{\text{high}} = 0.04$  (solid)

## 7. Conclusion

We use an endogenous growth model to study the effects of climate change caused by the extraction and use of nonrenewable resources. The central feature of the paper is the inclusion of a lag between greenhouse gas emissions and their effect on the stock of harmful pollution, which follows a well-defined time pattern. The time lag between emissions and their impact on the economy, here on capital accumulation, although important, has in general drawn little attention. The standard assumption in the literature of an instantaneous diffusion is the limiting case in our model.

Confirming results in the literature, with logarithmic utility, and our specification of damages to capital from the stock of pollution, the Pigouvian tax is a constant proportion of the consumption good in each time period. We therefore focus on general policies proportional to consumption and find that with log-utility, resource extraction is only determined by the tax rate. We also derive the crucial impact of climate dynamics on growth and resource extraction in private and social optimum. As regards optimal policy, the optimal per-unit emission tax rate increases in the dissemination speed; higher dissemination speed induces the economy to start at a higher level of economic growth. When pollution decay is not considered economic growth converges monotonically from above to its lower steady state, which is unaffected by policy; when pollution decay is considered, it may exceed the optimal level in the long-run. Finally, we study the effect of a more general CRRA utility function on the optimal carbon tax. We find that for a relevant value of the elasticity of intertemporal substitution above unity, and no pollution decay, the optimal tax grows initially faster than consumption while they asymptotically reach the same growth rate.

Political action is usually triggered only after environmental damages become visible. Therefore, a wrong perception of the speed of diffusion, e.g. a lower value for  $\kappa$ , can lead to a suboptimal taxation policy, i.e. a lower tax rate. We draw from our results in the decentralized equilibrium and note that, in the general case of  $\theta > 0$ , an environmental policy that mistakenly sets a lower than optimal tax will force the economy to start from a point of lower economic growth, reach faster a relatively lower level of minimum growth but

<sup>14</sup>In 2010 global consumption was around 49.8 billion US\$ (about 76% of global GDP); World Bank Indicators, 2015. With this value, our calibration implies a carbon tax in 2010 between 50 \$/tC ( $\kappa = 0.02$ ) and 75 \$/tC ( $\kappa = 0.04$ ).

then recover at a faster rate towards the steady state; in the case of pollution decay economic growth might exceed the social optimum during transition in the long run. If no pollution decay is considered,  $\theta = 0$ , an erroneously set environmental policy will result in a lower than optimal economic growth at all times. In this case the economy will start from a low point of economic growth while resource extraction will be brought forward and the harmful results of extracting and using the polluting non-renewable resource will arrive sooner.

We argue that if emission taxes are not set by the social planner but by a regular political process, there is a risk of setting tax rates at too low a level when actors underestimate the true pollution dissemination speed. Underestimation of climate change and pollution dissemination has different reasons. The usually observed myopia of decision makers and short-run targets like elections are one component. Moreover, climate sciences provide results and predictions which naturally include a certain degree of uncertainty because they concern the very long run. Finally, reactions and decisions might rely on cognitive experience. When environmental damages become visible they have the best conditions to trigger political action. Because this is not yet the case for climate change, the concerns of too little political action appear to be warranted.

## Appendix A. Time lags in the climate system

In this part of the Appendix we present the mathematic modeling of distributed time lags in the climate system and its properties. For the analysis we rely largely on MacDonald (1978). Take first the case of instantaneous diffusion of emissions. In this case the usual assumption is that the use of a pollutant,  $R_t$ , increases the harmful stock of emissions  $P_t$  at a rate

$$\dot{P}_t = \phi R_t + \theta(\bar{P} - P_t), \quad P_0 \geq 0 \text{ given}, \quad (\text{A.1})$$

with  $\phi > 0$  representing the carbon intensity of the polluting energy resource,  $\theta \geq 0$  the carbon decay parameter, and  $\bar{P} \in (0, P_0)$  the pre-industrial level of carbon concentration in the atmosphere. Now let's introduce a distributed lag in the model in order to relax the usual assumption of instantaneous pollution accumulation and let this process depend on the history of resource use

$$\dot{P}_t = \int_{-\infty}^t G_{t-s} \phi R_s ds + \theta(\bar{P} - P_t), \quad P_0 \geq 0 \text{ given}. \quad (\text{A.2})$$

With this formulation (A.1) becomes an integro-differential equation. The function  $G_x$  represents the memory of the system (or the delaying function) with  $\int_0^\infty G_x dx = 1$ . Function  $G_x$  could be also interpreted as the probability density function of the inherent time lag of the particular system so that the mean time lag  $\bar{T}$  for a given memory function would read  $\bar{T} = \int_0^\infty x G_x dx$ . With a special choice of the memory function one can replace (A.2) with a set of linear differential equations. For this purpose it is a standard approach to exploit the properties of the exponential functions by using the exponential distribution

$$G_x = \kappa e^{-\kappa x}, \quad \kappa > 0. \quad (\text{A.3})$$

The parameter  $\kappa$  measures the speed of emissions diffusion, or speed of adjustment, and is the reciprocal of the mean time lag  $\bar{T}$  from the same memory function, i.e.  $\kappa = \bar{T}^{-1}$ . We can then define the lagged history of carbon emissions as  $Z_t \equiv \int_{-\infty}^t G_{t-s} \phi R_s ds$ , and by using the Leibniz rule of integration to get the familiar equivalent system of differential equations, with  $P_0$  and  $Z_0$  given:

$$\begin{cases} \dot{P}_t = Z_t + \theta(\bar{P} - P_t), \\ \dot{Z}_t = \kappa(\phi R_t - Z_t). \end{cases} \quad (\text{A.4})$$

It can be proven that the corresponding system is globally stable for the relevant range of the parameters  $\kappa, \phi, \theta > 0$  and  $\bar{P} > 0$ . Since in general the initial value of  $Z_t$  cannot be defined,  $Z_0$  is chosen such that  $\lim_{\kappa \rightarrow \infty} P_{t=0} = P_0$ , as expected, i.e.  $Z_0 = 0$ . The solution for the climate system given the rate of resource extraction for each time period,  $R_t$ , now reads

$$P_t = P_0 + (\bar{P} - P_0)(1 - e^{-\theta t}) + \int_0^t \kappa \frac{e^{-\theta(t-s)} - e^{-\kappa(t-s)}}{\kappa - \theta} \phi R_s ds. \quad (\text{A.5})$$

The limiting cases for  $\kappa \rightarrow 0$  and  $\kappa \rightarrow \infty$  follow readily from the last equation. From (A.5), the marginal increase in the stock of carbon in period  $\nu$  from a marginal unit of emissions in period  $t$  reads:

$$\frac{dP_\nu}{d(\phi R_t)} \equiv f_{\nu t} = \kappa \frac{e^{-\theta(\nu-t)} - e^{-\kappa(\nu-t)}}{\kappa - \theta} > 0, \quad \text{for all } \nu \geq t. \quad (\text{A.6})$$

## Appendix B. Social optimum

The social planner chooses the share  $\epsilon_t$ , and resource extraction  $R_t$  in order to maximize lifetime utility  $\int_0^\infty U_t e^{-\rho t} dt$ , with  $U_t = \log(C_t)$  if  $\sigma = 1$  and  $U_t = \frac{C_t^{1-\sigma}}{1-\sigma}$  otherwise, subject to equations (3) - (6). The Hamiltonian of the social planner reads

$$H_t = \frac{C_t^{1-\sigma}}{1-\sigma} + \lambda_{Ct} [A(\epsilon_t K_t)^\alpha R_t^{1-\alpha} - C_t] \\ + \lambda_{Kt} K_t [B(1 - \epsilon_t) - D(P_t)] - \lambda_{St} R_t + \lambda_{Pt} [Z_t + \theta(\bar{P} - P_t)] + \lambda_{Zt} \kappa [\phi R_t - Z_t],$$

with  $\lambda_{Ct}, \lambda_{Kt}, \lambda_{St}, \lambda_{Pt}, \lambda_{Zt}$ , the shadow prices of the consumption good,  $C_t$ , capital stock,  $K_t$ , stock of non-renewable resources,  $S_t$ , stock of pollution,  $P_t$ , and the lagged history of emissions,  $Z_t$ . Assuming an internal solution, the first order conditions w.r.t. the  $C_t, \epsilon_t, R_t$ , i.e.  $\partial H_t / \partial(\cdot) = 0$  imply

$$C_t^{-\sigma} = \lambda_{Ct}, \quad (\text{B.1})$$

$$\alpha \frac{C_t}{K_t} = \frac{\lambda_{Kt}}{\lambda_{Ct}} B \epsilon_t, \quad (\text{B.2})$$

$$(1 - \alpha) \frac{C_t}{R_t} = \frac{\lambda_{St}}{\lambda_{Ct}} - \phi \kappa \frac{\lambda_{Zt}}{\lambda_{Ct}}. \quad (\text{B.3})$$

Moreover  $\partial H_t / \partial(\cdot) = \rho q_t - \dot{q}_t$  for every state variable,  $K_t, S_t, P_t, Z_t$ , with  $q_t$  its shadow price. This leads to

$$(\widehat{\lambda_{Kt} K_t}) = -\alpha \frac{\lambda_{Ct} C_t}{\lambda_{Kt} K_t} + \rho, \quad (\text{B.4})$$

$$\hat{\lambda}_{St} = \rho, \quad (\text{B.5})$$

$$\hat{\lambda}_{Pt} = D'(P_t) K \frac{\lambda_{Kt}}{\lambda_{Pt}} + \theta + \rho, \quad (\text{B.6})$$

$$\hat{\lambda}_{Zt} = -\frac{\lambda_{Pt}}{\lambda_{Zt}} + \kappa + \rho, \quad (\text{B.7})$$

Finally, the relevant transversality conditions read

$$\lim_{t \rightarrow \infty} \lambda_{S_t} S_t e^{-\rho t} = 0, \quad (\text{B.8})$$

$$\lim_{t \rightarrow \infty} \lambda_{K_t} K_t e^{-\rho t} = 0, \quad (\text{B.9})$$

$$\lim_{t \rightarrow \infty} \lambda_{P_t} P_t e^{-\rho t} = 0, \quad (\text{B.10})$$

$$\lim_{t \rightarrow \infty} \lambda_{Z_t} Z_t e^{-\rho t} = 0. \quad (\text{B.11})$$

From (B.5) we get that  $\hat{\lambda}_{S_t} = \rho$ , i.e. the Hotelling rule for the extraction of the non-renewable resource. Equation (B.2) shows indifference about allocating capital between the two activities: producing the consumption and the investment good. When  $\sigma = 1$  we can combine (B.2), with  $\lambda_{C_t} C_t = 1$ , from (B.1), and (B.4), to get  $\dot{\epsilon}_t = B\epsilon_t^2 - \rho\epsilon_t$ . With the use of the transversality condition (B.9), we get that the capital share jumps immediately to its steady state value,  $\bar{\epsilon} = \rho/B$ .

### Appendix C. Asymptotic constancy of capital share and resource depletion rate

In this part of the Appendix we derive the asymptotic constancy of the capital share  $\epsilon$ , and the resource depletion rate  $u = R/S$ , in the general case of  $\sigma \neq 1$ . In the main text we explained that the economy at hand is always in transition while it reaches a balanced growth path at the limit when resources get asymptotically depleted and pollution reaches its steady state value. The transversality condition (B.9) implies that  $\widehat{\lambda_{K_t} K_t} - \rho < 0$  while (B.4) with (B.2) can be rewritten as  $\widehat{\lambda_{K_t} K_t} = -B\epsilon_t + \rho$ . We combine these two conditions to get that  $\lim_{t \rightarrow \infty} \epsilon_t > 0$ . From (5), asymptotic constancy of  $\hat{K}_t$  implies  $\lim_{t \rightarrow \infty} \hat{\epsilon}_t \leq 0$ . Since  $\epsilon_t$  is strictly positive, the last inequality implies that  $\lim_{t \rightarrow \infty} \hat{\epsilon}_t = 0$ . From the transversality condition for the stock of resources, (B.8), we get that  $\hat{\lambda}_{S_t} + \hat{S}_t - \rho < 0$ . We substitute  $\hat{\lambda}_{S_t} = \rho$  and  $\hat{S}_t = -u_t$  to get  $\lim_{t \rightarrow \infty} u_t > 0$ . We then log-differentiate the production function for the consumption good, (3), with constant  $\epsilon$  at the limit. Asymptotic constancy of  $\hat{C}_t$  then demands that  $\lim_{t \rightarrow \infty} \hat{u}_t \leq 0$ . The last two conditions indicate that  $\lim_{t \rightarrow \infty} \hat{u}_t = 0$ , i.e.  $u$  asymptotically constant and positive, i.e.  $g_R$  asymptotically constant and negative. The upper bound  $\epsilon_t, u_t < 1$  follows from the essentiality of the resource and capital in the production function.

### Appendix D. The Social Cost of Carbon

We defined as  $X_t = -\phi\kappa\lambda_{Z_t}/\lambda_{C_t}$  the marginal externality damage from burning an additional unit of polluting non-renewable resource. We will now prove that this is equivalent to expression (8). From (A.5), it follows that  $P_\nu \geq P_s e^{-\theta(\nu-s)}$ , for each  $\nu \geq s$ . We combine the previous inequality with the transversality condition (B.10) to get that  $0 = \lim_{\nu \rightarrow \infty} \lambda_{P_\nu} P_\nu e^{-\rho\nu} \geq \lim_{\nu \rightarrow \infty} \lambda_{P_\nu} P_s e^{-\theta(\nu-s)} e^{-\rho\nu}$  or that

$$\lim_{\nu \rightarrow \infty} \lambda_{P_\nu} e^{-(\rho+\theta)(\nu-s)} = 0, \quad \text{for all } \nu \geq s. \quad (\text{D.1})$$

Following the same procedure for the transversality condition (B.11) we get

$$\lim_{s \rightarrow \infty} \lambda_{Z_s} e^{-(\rho+\kappa)(s-t)} = 0, \quad \text{for all } s \geq t. \quad (\text{D.2})$$

We multiply equation (B.6) with  $e^{-(\rho+\theta)(v-s)}$  to get that  $\dot{\lambda}_{Pv}e^{-(\rho+\theta)(v-s)} - (\rho+\theta)\lambda_{Pv}e^{-(\rho+\theta)(v-s)} = \eta D'(P_v)\lambda_{Kv}K_v e^{-(\rho+\theta)(v-s)}$ , i.e.  $d[\lambda_v e^{-(\rho+\theta)(v-s)}]/dv = \eta D'(P_v)\lambda_{Kv}K_v e^{-(\rho+\theta)(v-s)}$ . Using (D.1) we can then calculate the definite integral from  $v = s$  to  $v \rightarrow \infty$  as

$$-\lambda_{Ps} = \int_s^\infty \eta D'(P_v)\lambda_{Kv}K_v e^{-(\rho+\theta)(v-s)} dv, \quad (D.3)$$

while the same procedure for (B.7) gives

$$\lambda_{Zt} = \int_t^\infty \lambda_{Ps} e^{-(\rho+\kappa)(s-t)} ds. \quad (D.4)$$

Substituting  $\lambda_{Pt}$  from (D.3) into (D.4) and using (B.1) we get that

$$X_t = -\kappa \phi \frac{\lambda_{Zt}}{\lambda_{Ct}} = \frac{\kappa \phi}{\lambda_{Ct}} \int_t^\infty \left( \int_s^\infty \eta D'(P_v)\lambda_{Kv}K_v e^{-(\rho+\theta)(v-s)} dv \right) e^{-(\rho+\kappa)(s-t)} ds. \quad (D.5)$$

Using (B.1) and (B.2) we get expression (8). If  $\sigma = 1$ ,  $X_t$  rewrites as in (9), all along the optimal path. If  $\sigma \neq 1$ , on the asymptotic BGP, due to constancy of the  $\epsilon$  share, it holds, by combining (B.4) with (B.2), that  $\lim_{t \rightarrow \infty} \widehat{\lambda_{Ct} C_t} = \lim_{t \rightarrow \infty} \widehat{\lambda_{Kt} K_t} = -B\epsilon_\infty + \rho$ , constant. Accordingly, the double integral in (D.5) gives at the limit  $\frac{\chi \eta}{(B\epsilon_\infty + \theta)(B\epsilon_\infty + \kappa)} \lambda_{Kt} K_t$ , and with (B.2), (D.5) can be written as

$$\lim_{t \rightarrow \infty} X_t / C_t = \kappa \frac{\alpha \eta \phi \chi}{B\epsilon_\infty (B\epsilon_\infty + \theta)(B\epsilon_\infty + \kappa)}. \quad (D.6)$$

In Appendix G we will calculate the steady state value  $\epsilon_\infty = (B\sigma)^{-1}(\rho + \alpha(\sigma - 1)(B - \eta(\delta + \chi P_\infty)))$  in the general case of  $\sigma \neq 1$ . Substituting the result in (D.6) leads to equation (23) in the main text.

## Appendix E. Decentralized equilibrium with log-utility

For ease of exposition we define  $\psi \equiv p_{Rt}/(p_{Rt} + \tau_t)$ , the fraction of the producers' price in the total price for the non-renewable resource. Log-differentiating this expression gives  $\hat{\psi}_t = (1 - \psi)(\hat{p}_{Rt} - \hat{\tau}_t)$ , where we define  $\hat{V}_t = g_{Vt} \equiv \dot{V}_t/V_t$  the growth rate of variable  $V_t$ . The equations that characterize the decentralized economy can be found by log-differentiating the production function (3) with  $Y_t = C_t$ , the FOC (10) for the capital share and resource demand, together with the aggregate version of the Keynes-Ramsey rule, (14), the Hotelling rule, (15), the no-arbitrage condition, (16), and the aggregate capital accumulation, (5). Solving the occurring system in  $g_C, g_K, g_R, g_\epsilon, g_\psi, g_{pK}, g_{pR}, r$  in the case of  $\sigma = 1$  leads to the following dynamic equation for  $\epsilon$ :

$$\dot{\epsilon}_t = B\epsilon_t^2 - \rho\epsilon_t \quad (E.1)$$

with a solution given by  $\epsilon_t = \left( \frac{B}{\rho}(1 - e^{\rho t}) + \frac{\epsilon_0}{\rho} \right)^{-1}$ . Combining the demand for capital in (10) and the aggregate version of the transversality condition (17) for capital pins down the initial level of the capital shares  $\epsilon_0 = \rho/B$ . Substituting this back to the solution we get that  $\epsilon_t = \bar{\epsilon} = \rho/B$  for every  $t$ .



## Appendix F. Dynamics of the optimal per-unit tax

To see why any term in the optimal per-unit tax that grows with the interest rate has no effect on the extracting behavior of the economy proceed as follows. Given Assumptions 1 and 2, apply the optimal tax on the FOC for the non-renewable resource, (10), with  $C_t = Y_t$  in equilibrium:  $R_t^o = (1 - \alpha) \left( \frac{p_{Rt}}{C_t} + \frac{\tau_t^o}{C_t} \right)^{-1}$ ;  $R_t^o$  is the optimal path of resource extraction. Substituting  $p_{Rt} = p_{R0} e^{\int_0^t r_s ds}$ , from (15), and  $C_t = C_0 e^{\int_0^t (r_s - \rho) ds}$ , from (14), gives  $R_t^o = (1 - \alpha) \left( \frac{p_{R0}}{C_0} e^{\rho t} + \frac{\tau_t^o}{C_t} \right)^{-1}$ . Now consider a different tax  $\tau_t^o + \Delta e^{\int_0^t r_s ds}$  with  $\Delta > -p_{R0}$ . It is straightforward to verify that in this case  $R_t^o = (1 - \alpha) \left( \frac{p_{R0} + \Delta}{C_0} e^{\rho t} + \frac{\tau_t^o}{C_t} \right)^{-1}$ . Since  $\tau_t^o/C_t$  is constant, and the resource is essential, we can use the feasibility constraint  $\int_0^\infty R_t dt = S_0$  to calculate in both cases the same optimal resource extraction path as  $R_t^o = (1 - \alpha) \left[ \frac{\tau_t^o}{C_t} \left( 1 + e^{\rho t} \left( e^{\frac{S_0 \rho}{1 - \alpha} \frac{\tau_t^o}{C_t}} - 1 \right) \right) \right]^{-1}$ .

## Appendix G. Social optimum with non-logarithmic utility

This part of the Appendix provides the dynamic system used in the simulation for section 6, i.e. we treat the asymptotic balanced growth path and stability in the general case of  $\sigma \neq 1$ . It will be convenient to modify the dynamic system of the social planner in variables that converge to a constant at the limit. In Appendix C we proved asymptotic constancy of  $\epsilon$ , and  $u = R/S$ . Moreover, we showed that  $\tilde{\tau}_t^o = \frac{\tau_t^o}{C_t} = -\phi \kappa \frac{\lambda_{Zt}}{\lambda_{Ct} C_t}$  reaches also a constant value at the limit. From (D.3) and (B.2) with  $\lim_{t \rightarrow \infty} \widehat{\lambda_{Kt} K_t} = -B\epsilon_\infty + \rho$  same holds for  $\tilde{\gamma}_t = \frac{\lambda_{Pt}}{\lambda_{Ct} C_t}$ . Furthermore, we define, as in Appendix E,  $\psi_t \equiv p_{Rt}/(p_{Rt} + \tau_t^o) = (\lambda_{St}/\lambda_{Ct})/(\lambda_{St}/\lambda_{Ct} - \kappa \phi \lambda_{Zt}/\lambda_{Ct})$ , the fraction of the producers' price in the total price paid by consumers. It follows from the asymptotic constancy of  $\tau_t^o/C_t$ , the Hotelling rule,  $\hat{p}_{Rt} = r_t$ , and the Keynes-Ramsey rule,  $\sigma \hat{C}_t = r_t - \rho$ , that the term  $\tau_t^o/p_{Rt}$  grows at  $-((\sigma - 1)r_t + \rho)/\sigma < 0$ , for  $\sigma \geq 1$ , which we conventionally assume, so that  $\lim_{t \rightarrow \infty} \psi_t = 1$ . To get the dynamic system in  $\{\epsilon_t, u_t, \tilde{\tau}_t, \tilde{\gamma}_t, \psi_t, S_t, P_t, Z_t\}$  we proceed as follows.<sup>15</sup> By log-differentiating (3) with  $u_t = R_t/S_t$ , using (5) we get

$$\hat{C}_t - \alpha(\hat{\epsilon}_t + B(1 - \epsilon_t) - \eta(\delta + \chi P_t)) - (1 - \alpha)(\hat{u}_t - u_t) = 0. \quad (G.1)$$

With our definitions, equation (B.3) can be written as  $(1 - \alpha)C_t/(u_t S_t) = p_{Rt} + \tau_t^o$ . We log-differentiate this expression with  $\hat{p}_{Rt} = \hat{\lambda}_{St} - \hat{\lambda}_{Ct} = \rho + \sigma \hat{C}_t$  from (B.1), the Hotelling rule  $\hat{\lambda}_S = \rho$ , and  $\tilde{\tau}_t = \tau_t^o/C_t$ , to get

$$\hat{u}_t - u_t + \psi_t(\rho + (\sigma - 1)\hat{C}_t) + (1 - \psi_t)\hat{\tau}_t = 0. \quad (G.2)$$

Furthermore, we log-differentiate the definition for  $\psi_t$  which gives

$$\hat{\psi}_t - (1 - \psi_t)(\rho + (\sigma - 1)\hat{C}_t - \hat{\tau}_t) = 0. \quad (G.3)$$

From (B.1), (B.2) and (B.4) we get

$$\hat{\epsilon}_t + (\sigma - 1)\hat{C}_t + \rho - \epsilon_t B = 0. \quad (G.4)$$

<sup>15</sup>For convenience we will drop in this section the “ $o$ ” upper script, having however in mind that all results refer to the social optimum solution.

Finally, we substitute the growth rate of the shadow prices in the log-differentiated version of the definitions for  $\tilde{\tau}_t = \frac{\tau_t^p}{C_t} = -\kappa\phi \frac{\lambda_{Z_t}}{\lambda_{C_t}C_t}$  and  $\tilde{\gamma}_t = \frac{\lambda_{P_t}}{\lambda_{C_t}C_t}$  from (B.1), (B.6), and (B.7) to get

$$\hat{\tau}_t - (\sigma - 1)\hat{C}_t - \kappa \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right) - \rho = 0, \quad (\text{G.5})$$

$$\hat{\gamma}_t - (\sigma - 1)\hat{C}_t - \frac{\alpha\eta\chi}{B\epsilon_t \tilde{\gamma}_t} - \theta - \rho = 0. \quad (\text{G.6})$$

We then combine equations (G.1)-(G.6) to get the relevant dynamic system in  $\{\epsilon_t, u_t, \tilde{\tau}_t, \tilde{\gamma}_t, \psi_t, S_t, P_t, Z_t\}$ , as

$$\dot{\epsilon}_t = \epsilon_t \left( \kappa \frac{(\sigma - 1)(1 - \alpha)(1 - \psi_t)}{\sigma} \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right) - \frac{(\sigma - 1)\alpha\Theta_t + \rho}{\sigma} + B\epsilon_t \right), \quad (\text{G.7})$$

$$\dot{u}_t = u_t \left( -\frac{(\sigma - 1)\alpha\Theta_t + \rho}{\sigma} - \kappa \frac{(1 + \alpha(\sigma - 1))(1 - \psi_t)}{\sigma} \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right) + u_t \right), \quad (\text{G.8})$$

$$\dot{\tilde{\tau}}_t = \tilde{\tau}_t \left( \kappa \frac{(1 + \alpha(\sigma - 1))(1 - \psi_t) + \sigma\psi_t}{\sigma} \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right) + \frac{(\sigma - 1)\alpha\Theta_t + \rho}{\sigma} \right), \quad (\text{G.9})$$

$$\dot{\tilde{\gamma}}_t = \tilde{\gamma}_t \left( -\kappa \frac{(\sigma - 1)(1 - \alpha)(1 - \psi_t)}{\sigma} \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right) + \frac{\alpha\eta\chi}{B\epsilon_t \tilde{\gamma}_t} + \frac{(\sigma - 1)\alpha\Theta_t + \rho}{\sigma} + \theta \right), \quad (\text{G.10})$$

$$\dot{\psi}_t = \psi_t(\psi_t - 1)\kappa \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right), \quad (\text{G.11})$$

along with (1) and (6), with  $\Theta_t = B - \eta(\delta + \chi P_t)$ , and  $R_t = u_t S_t$ . The steady state values read

$$\epsilon_\infty = \frac{\rho + \alpha(\sigma - 1)(B - \eta(\delta + \chi P_\infty))}{B\sigma}, \quad (\text{G.12})$$

$$u_\infty = B\epsilon_\infty, \quad (\text{G.13})$$

$$\tilde{\tau}_\infty = \kappa \frac{\alpha\eta\phi\chi}{B\epsilon_\infty(B\epsilon_\infty + \theta)(B\epsilon_\infty + \kappa)}, \quad (\text{G.14})$$

$$\tilde{\gamma}_\infty = -\frac{\alpha\eta\chi}{B\epsilon_\infty(B\epsilon_\infty + \theta)}, \quad (\text{G.15})$$

$$\psi_\infty = 1, \quad (\text{G.16})$$

$$S_\infty = 0, \quad (\text{G.17})$$

$$P_\infty = P_0, \text{ if } \theta > 0 \text{ and } P_0 + \phi S_0, \text{ if } \theta = 0, \quad (\text{G.18})$$

$$Z_\infty = 0. \quad (\text{G.19})$$

The eigenvalues of the jacobian matrix of the corresponding system, calculated at the steady state values, are

$\{-\theta, -\kappa, -\frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma}, -\frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma}, \frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma}, \frac{\rho + \alpha(\sigma - 1)\Theta_\infty}{\sigma}, \frac{\rho + \theta\sigma + \alpha(\sigma - 1)\Theta_\infty}{\sigma}, \frac{\rho + \kappa\sigma + \alpha(\sigma - 1)\Theta_\infty}{\sigma}\}$ , with  $\Theta_\infty = B - \eta(\delta + \chi P_\infty)$ , and  $P_\infty = P_0 + \phi S_0$ , if  $\theta = 0$ , or  $\bar{P} = P_0$ , if  $\theta > 0$ , i.e. four negative and four positive eigenvalues, implying a saddle-path stability around the steady state. The growth rate of consumption can be calculated by (G.1) to be

$$g_{Ct} = \frac{\alpha\Theta_t - \rho}{\sigma} - \frac{\kappa(1 - \alpha)(1 - \psi_t)}{\sigma} \left( 1 + \phi \frac{\tilde{\gamma}_t}{\tilde{\tau}_t} \right), \quad (\text{G.20})$$

with a steady state value of

$$g_{C\infty} = \frac{\alpha(B - \eta(\delta + \chi P_{\infty})) - \rho}{\sigma}. \quad (\text{G.21})$$

#### Appendix G.1. Linearized Version of the Model for $\sigma \neq 1$

In order to simulate the model we use the standard linearization technique: the linearized version of our autonomous dynamic system in  $\mathbf{x}_t = \{\epsilon_t, u_t, \tilde{\tau}_t, \tilde{\gamma}_t, \psi_t, S_t, P_t, Z_t\}^\top$  can be obtained by using the jacobian matrix  $\mathbf{J}$  evaluated at the steady states presented above as  $d(\mathbf{x}_t - \mathbf{x}_{\infty})/dt \approx \mathbf{J}(\mathbf{x}_t - \mathbf{x}_{\infty})$ , with  $\mathbf{x}_{\infty} = \{\epsilon_{\infty}, u_{\infty}, \tilde{\tau}_{\infty}, \tilde{\gamma}_{\infty}, \psi_{\infty}, S_{\infty}, P_{\infty}, Z_{\infty}\}^\top$ , which allows for an easy solution of the system of linear homogeneous equations. For the simulation we use as initial conditions  $\{\psi_0 = 0.65, S_0 = 6000\text{GtC}, P_0 = 830\text{GtC}, Z_0 = 0\}$ , while we calculate the initial level of the control variables  $\{\epsilon_0, u_0, \tau_0, \gamma_0\}$  such that the constants of integration associated with the unstable roots (positive eigenvalues) are zero. The parameters chosen are  $\{\rho = 0.015, \sigma = 1.5, \alpha = 0.9, \theta = 0, \delta = 0.05, B = 0.106, \chi = 1.7 \times 10^6 \$/\text{GtC}, \phi = 1, \eta = 1, \kappa_{\text{low}} = 0.02, \kappa_{\text{high}} = 0.04\}$ . Below we justify our choices for the numerical exercise.

#### Appendix G.2. Choice of initial conditions and parameters

We choose 2010 to be our  $t = 0$ . An approximation for  $\psi_0$ , the share of the before tax price to the total price paid by consumers, can be taken from the IEA Monthly Oil Statistics, IEA (2015), to be around 0.65 as an average for the period 2006-2015.  $S_0 = 6000\text{GtC}$  follows estimates from the 2010 version of the DICE model.  $P_0 = 830\text{GtC}$  was retrieved by data from the European Environmental Agency.<sup>16</sup>  $Z_0 = 0$  is chosen according to our discussion in Appendix A. The values of  $\rho$ ,  $\sigma$  and  $\delta$  are standard in the literature of endogenous growth. We normalize  $\phi = 1$ , so that a unit of resource use equals a unit of emissions; we also set  $\eta = 1$  in the numerical exercise. In this model real GDP equals to  $C_t + p_{It}I_t$ . Worldwide gross capital formation,  $p_{It}I_t/\text{GDP}_t$ , is on average 0.24 for the period 1960-2014, World Bank Indicators, 2015. Accordingly  $C_t/\text{GDP}_t = 0.76$ . Energy expenditure as share of GDP in the US for the period 1949-2011 is in the range of 0.04–0.1, EIA (2015). We choose then a value of 0.08. Using now the FOC (10) for energy we can calculate  $\alpha = 1 - \frac{(p_{Rt} + \phi\tau_t)R_t/\text{GDP}_t}{C_t/\text{GDP}_t} \approx 0.9$ . The value of natural disasters reported in the period 2005-2015 amounts on average to  $139 \times 10^9$  \$/year (EM-DAT The International Disasters Database, 2015). Using the value for the stock of capital from the 2010 version of the DICE model (DICE2010) and  $P_0 = 830\text{GtC}$ , we get that  $\chi = 1.7 \times 10^6 \$/\text{GtC}$ .  $B$  was chosen so that the initial level of the interest rate is about 5% (as in DICE2010) and that it satisfies the condition  $\alpha(B - \eta(\delta + \chi(P_0 + \phi S_0))) > \rho$ , from (G.21), equivalent to the one assumed in section 4.2. Parameter  $\kappa$  is the reciprocal of the mean time lag. We choose a low value to reflect a time lag of 50 years, and a high value to reflect a time lag of 25 years.

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